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COLLEGE OF ENGINEERING

MARQUETTE UNIVERSITY

MILWAUKEE, WISCONSIN 53233

ATTITUDE DETERMINATION

OF A

HIGH ALTITUDE BALLOON SYSTEM

PART II

DEVELOPMENT OF THE PARAMETER

DETERMINATION PROCESS

by

Marquette University Systems Group

N. J. Nigro Associate Professor, Mechanical Engineering

A. F. Elkouh Associate Professor, Mechanical Engineering

P. Nimityongskul Graduate Student

K. S. Shen Graduate Student

V. N. Jhaveri Graduate Student

A. Sethi Graduate Student

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CHAPTER I

INTRODUCTION

1.1 Motivation and Relevance of Report In April of 1974 the National Aeronautics and Space Administration conducted a high altitude balloon experiment called LACATE (Lower Atmosphere Composition and Temperature Experiment) which employed an infrared radiometer to sense remotely vertical profiles of the concentrations of selected atmospheric trace constituents and temperature. The constituents were measured by inverting infrared radiance profile of the earth's horizon. The radiometer line of sight was scanned vertically across the horizon at approximately 0.25° per second, requiring 30 seconds to acquire a complete radiance profile. The specifications required that the relative vertical position of the data points making up a profile be known to approximately 30 arc seconds. The general description of the balloon system for accomplishing the mission is given in reference (1), refer figures 1.1-1 and 1.1-2.

In order to fix the orientation of the line of sight of the radiometer, it is necessary to be able to determine the configuration of the platform in space, i.e. the attitude of the system. This can be accomplished by simulating the balloon system and using the gyro output in conjunction with a parameter estimation process. Simulation of the balloon system requires a mathematical model plus analysis of the model. The required mathematical model has already been developed for use with the system simulation process and the details are described in reference (1) pages 9-38. The attitude of the balloon system can be determined once the initial conditions; i.e. the initial state, is known.

1.2 Objective of the Report The main objective of this report will be to develop a process to determine the unknown initial state parameters by employing the output of the system mathematical model in conjunction with

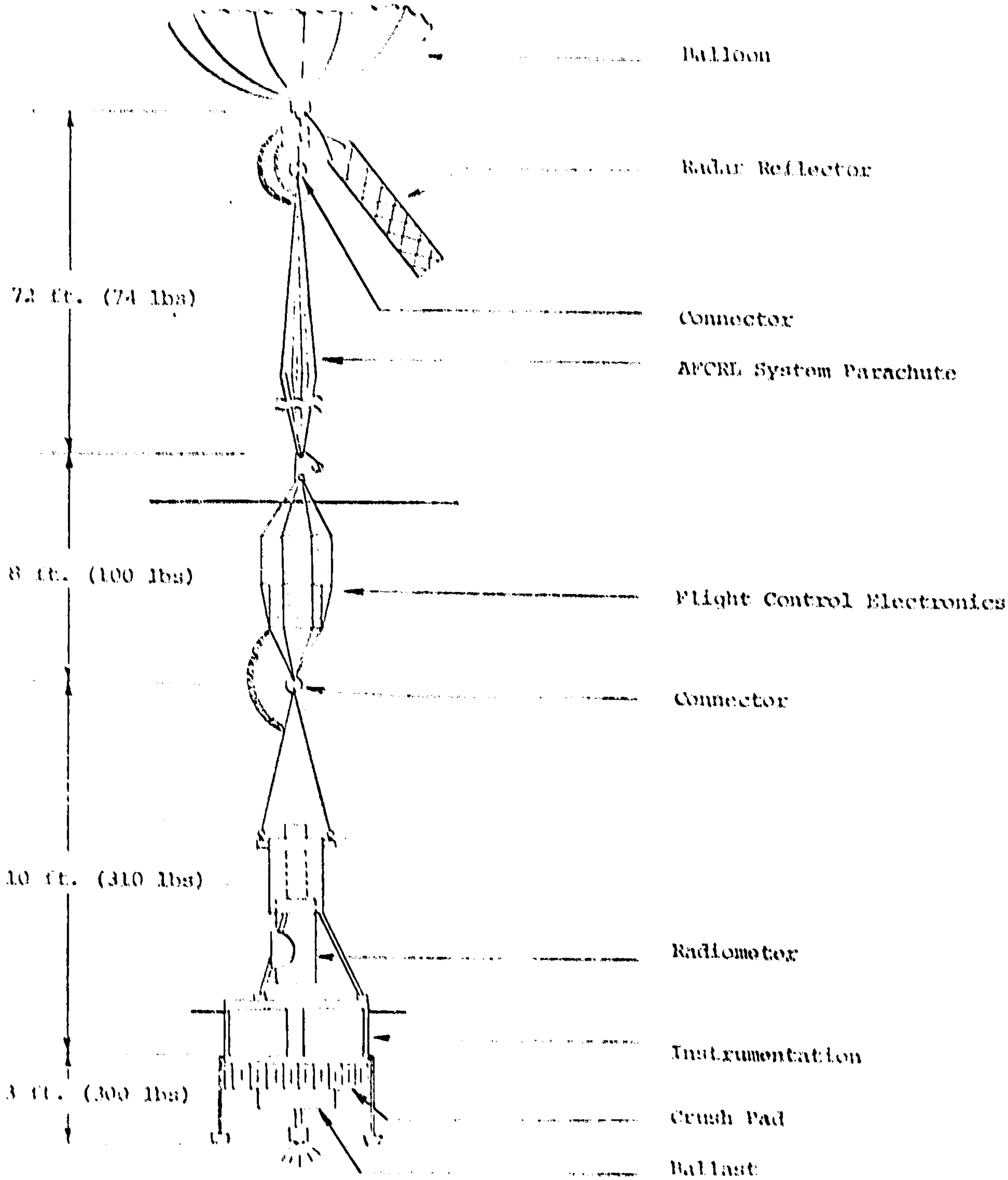


Figure 1.1-1 LACATE Balloon System

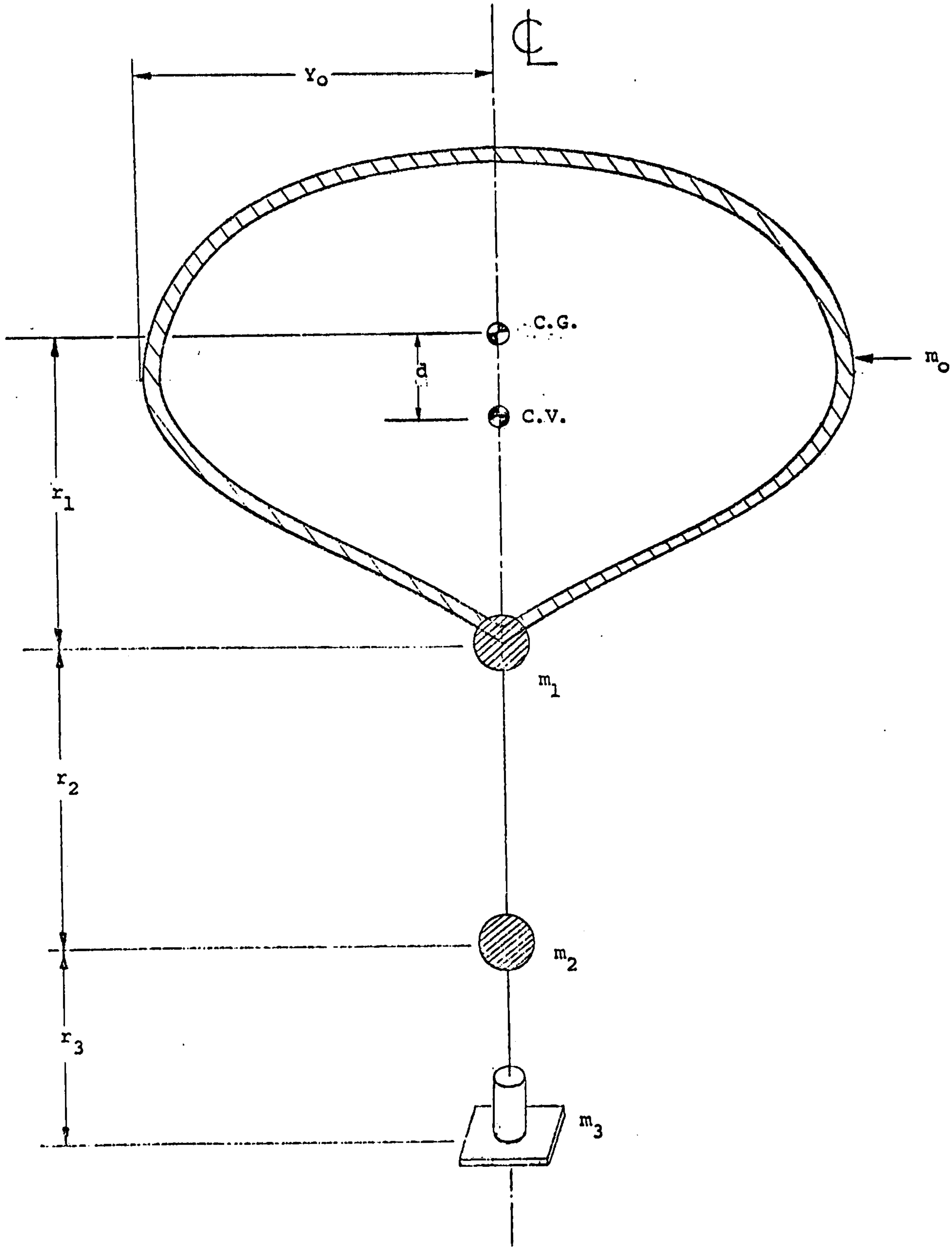


Figure 1.1-2 Idealized Balloon System

the output obtained from the instrumentation system. This system consisted of three orthogonally oriented rate gyros and magnetometer which were fixed to the balloon platform. The development of this process involves 2 major steps. First, a parameter determination process must be developed which solves for the unknown initial state parameters of the system which, in turn, give the best fit to the data obtained from the instrumentation system. Second, a simulation process must be developed for computing the state of the balloon system which can then be compared with the actual state of the system.

The body of this report consists of 2 major parts. The first part discusses the attitude determination process in general. The details of this part include

(i) System Simulation Discussion of the method of solution and computation of the output of interest (i.e. angular velocity components along the platform axis).

(ii) Computation of System Natural Frequencies. Discussion of a method for solving system eigenvalues.

(iii) Optimization Technique. Discussion of the Hooke and Jeeves direct search method.

(iv) Verification Process. Discussion of a method for verification of the attitude determination process.

In the second part of the report all the numerical data and results are presented and discussed.

CHAPTER II

ATTITUDE DETERMINATION PROCESS

2.1 Introduction

The attitude (state) of the balloon system can be determined as a function of time if (a) a method for simulating the motion of the system is available and (b) the initial state is known. The system motion can be simulated once the system model is determined. The initial state can then be obtained by fitting the system motion (as measured by sensors) to the corresponding output predicted by the mathematical model. In the case of the LACATE experiment the sensors consisted of three orthogonally oriented rate gyros and a magnetometer all mounted on the research platform. The initial state was obtained by fitting the angular velocity components measured with the gyros to the corresponding values obtained from the solution of the math model.

A block diagram illustrating the attitude determination process employed for the LACATE experiment is shown in figure 2.1-1. The process consists of three essential parts; i.e., a process for simulating the balloon system (block 1), an instrumentation system (block 2) for measuring the output, and a parameter estimation process (block 3) for systematically and efficiently solving the initial state. A more detailed discussion of each of these parts is presented below.

2.2 System Simulation Process

The main steps in the system simulation process are shown in the block diagram of figure 2.2-1. They consist of (a) development of a system model to predict state (block 1), (b) solution of the model (block 2) and (c) developing a math model for computing the angular velocity components of the research platform. Each of these steps will be discussed in greater detail below.

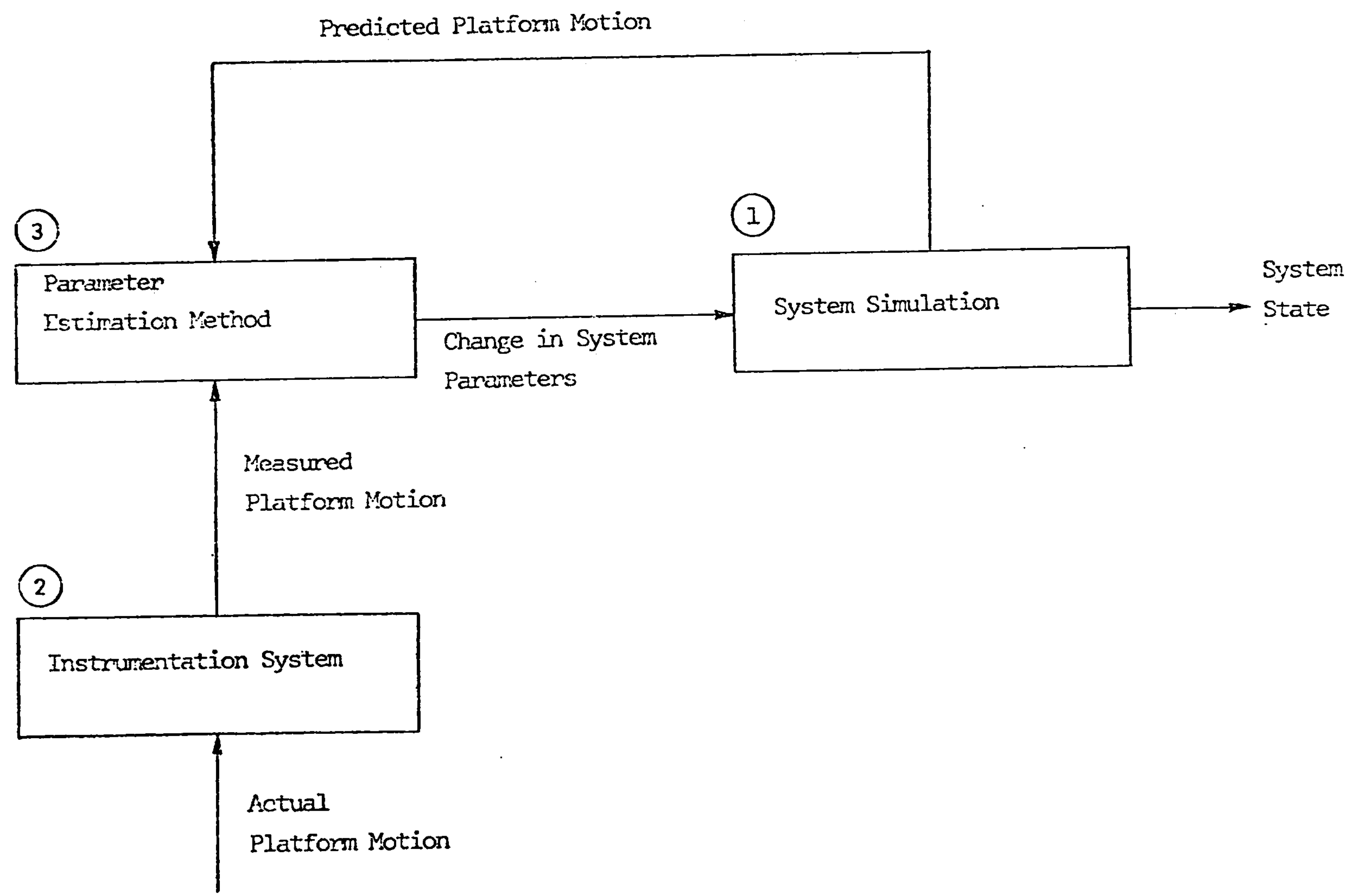


Figure 2.1-1 Attitude Determination Process

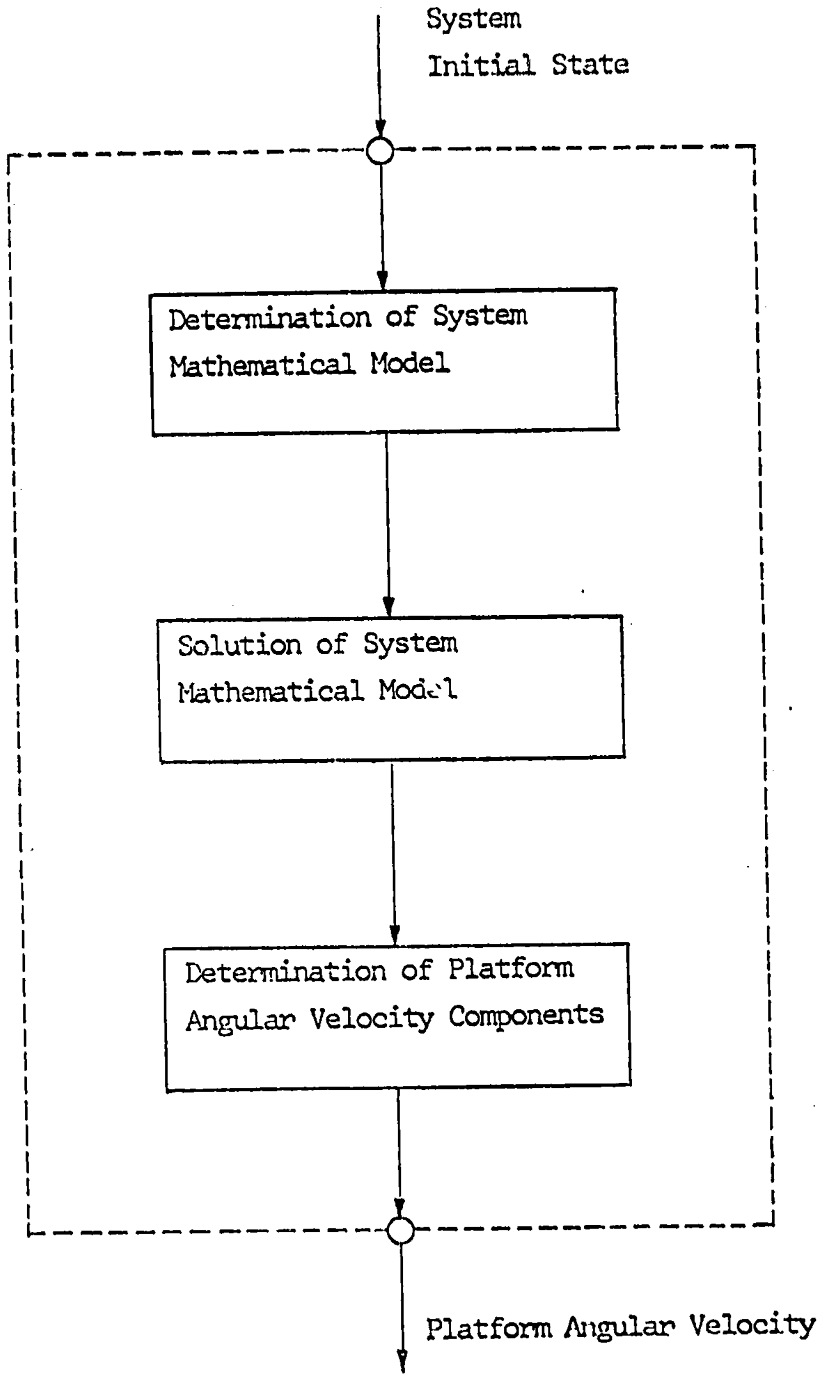


Figure 2.2-1 System Simulation Process

System math model. A system math model must be obtained which enables one to predict the motion of the system at float altitude since all forms of output (e.g. angular velocity, tension in the cables, etc.) can be determined once this is known. This motion is very complex and involves various types of oscillation including bounce, pendulation and spin. Moreover, the complexity of the motion is increased with increasing number of subsystems.

The math model of the balloon system is complicated primarily by the two important factors. They are as follows:

(a) the balloon itself is a distributed parameter system which has motion in an infinite fluid media. Hence, it is necessary to first idealize it as an equivalent rigid body in order to develop a lumped parameter model for the entire system.

(b) The balloon system is subjected to nondeterministic wind gusts which result in forces acting externally on the system. At the present time very little is known about the nature of these gusts.

The exact dynamic model for the balloon itself consists of the equations of motion for the solid (i.e. balloon fabric) and the fluid dynamic equations. These equations are coupled through the boundary conditions which must be satisfied at the interface of the solid and fluid media. The resulting model is extremely complex and consists of a system of coupled partial differential equations. The system model is simplified by treating the balloon as a lumped parameter (rigid body) element. This is accomplished by developing approximate expressions for the aerodynamic forces and torques which result due to the interaction between the balloon and fluid media. These forces and torques are then treated as external reactions on the solid system.

A linear systems model which includes the effect of the aerodynamic reactions has been developed (3) by neglecting the effect of second order terms. An equivalent form of this model which involves only the pendulation angles in two orthogonally oriented planes is given as follows

$$\ddot{\bar{\theta}} + A\bar{\theta} = \bar{0} \;, \qquad 2.2\text{-}1$$

$$\ddot{\bar{\psi}} + A\bar{\psi} = \bar{0} \;, \text{ and} \qquad 2.2\text{-}2$$

$$\ddot{\phi}_3 = 0 \;, \text{ where} \qquad 2.2\text{-}3$$

$$\bar{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \;, \qquad 2.2\text{-}4$$

$$\bar{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \;, \qquad 2.2\text{-}5$$

θ_i, ψ_i = pendulation angles in two orthogonally oriented planes, and

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \;. \qquad 2.2\text{-}6$$

The elements a_{ij} are defined in terms of the system parameters in reference (3). The pendulation angles θ_i and ψ_i and the spin angle ϕ are illustrated in figure 2.2-2 and figure 2.2-3.

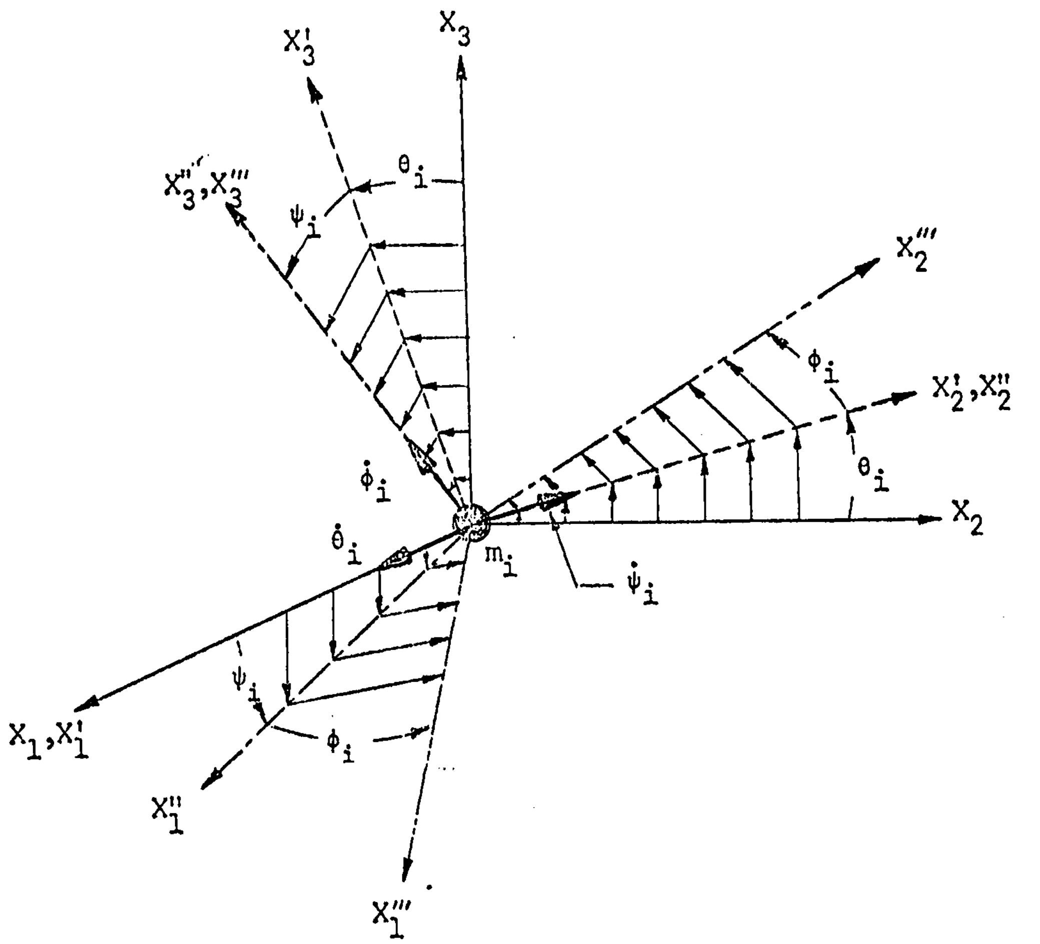


Figure 2.2-2 Eulerian Angles

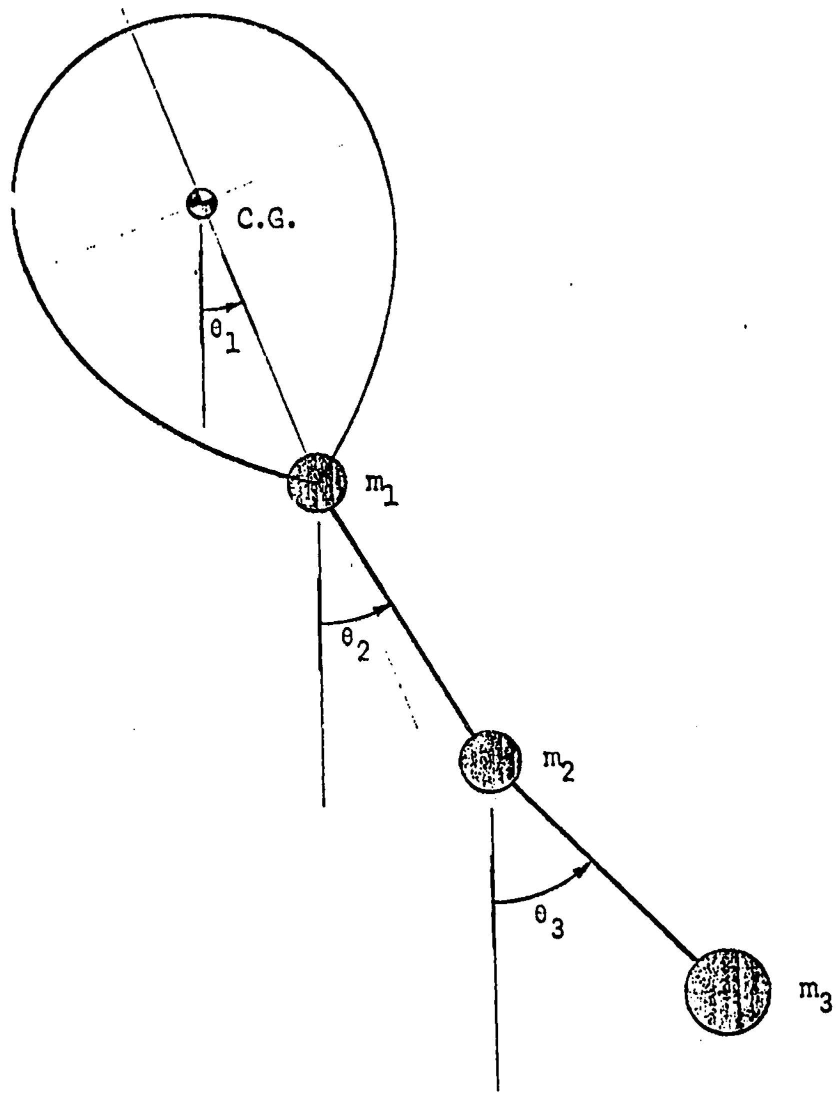


Figure 2.2-3 System Pendulation Angles in X_2X_3 Plane

<u>Solution of System Model</u>. The solution to equation 2.2-1 can be obtained by assuming the following form for $\bar{\theta}$; i.e.

$$\bar{\theta} = \bar{X} \sin \Omega t. \tag{2.2-7}$$

Substitution in to equation 2.2-1 yields the following eigenvalue problem, i.e.

$$A\bar{X} = \Omega^2\bar{X}, \quad \text{where} \tag{2.2-8}$$

$$\bar{X}_i(i = 1, 2, 3) = \begin{matrix} X_{1i} \\ X_{2i} \\ X_{3i} \end{matrix} \quad \text{are the eigenvectors and}$$

$\Omega_i(i = 1,2,3)$ are the eigenvalues.

Equation 2.2-8 can be solved numerically on the computer. A listing of the fortran program employed to solve this equation for the particular problem under study is given in appendix A.

The solution to equation 2.2-2 is obtained in the same way and yields the identical eigenvalue problem. The solution for equation 2.2-3 is easily obtained by integrating the equation twice with respect to time (t). The final closed form solution for $\bar{\theta}$, $\bar{\psi}$ and ϕ_3 is given as

$$\begin{aligned} \bar{\theta}(t) = {} & \bar{X}_1(c_1 \sin \Omega_1 t + c_2 \cos \Omega_1 t) \\ & + \bar{X}_2(c_3 \sin \Omega_2 t + c_4 \cos \Omega_2 t) \\ & + \bar{X}_3(c_5 \sin \Omega_3 t + c_6 \cos \Omega_3 t) \ , \end{aligned} \tag{2.2-9}$$

$$\bar{\psi}(t) = \bar{X}_1(c_7 \sin \Omega_1 t + c_8 \cos \Omega_1 t)$$
$$+ \bar{X}_2(c_9 \sin \Omega_2 t + c_{10} \cos \Omega_2 t)$$
$$+ \bar{X}_3(c_{11} \sin \Omega_3 t + c_{12} \cos \Omega_3 t) \text{ , and} \qquad 2.2\text{-}10$$

$$\phi_3(t) = at + \phi_3(to) \text{ , where} \qquad 2.2\text{-}11$$

$c_j (j = 1,2,\ldots,12)$ = unknown constants which are determined from the initial states; i.e. $\bar{\theta}(to)$, $\bar{\psi}(to)$, $\dot{\bar{\theta}}(to)$ and $\dot{\bar{\psi}}(to)$,

a = constant rate of spin, and

$\phi_3(to)$ = initial spin displacement.

Computation of Balloon Angular Velocities. The relationship between the platform motion (θ_3, $\dot{\theta}_3$, ψ_3, $\dot{\psi}_3$, ϕ_3 and $\dot{\phi}_3$) and the platform angular velocity components is obtained through the application of the Euler angle transformations to the system platform shown in figure 2.2-4. The transformation equations are given as follows:

$$\omega_1 = \dot{\theta}_3 \cos \psi_3 \cos \phi_3 + \dot{\psi}_3 \sin \phi_3 \text{ ,}$$
$$\omega_2 = -\dot{\theta}_3 \cos \psi_3 \sin \phi_3 + \dot{\psi}_3 \cos \phi_3 \text{ , and}$$
$$\omega_3 = \dot{\theta}_3 \sin \psi_3 + \dot{\phi}_3 \text{ , where} \qquad 2.2\text{-}12$$

$\omega_i (i = 1,2,3)$ = angular velocity components of the platform along $\bar{e}_i$ direction,

θ_3 = pendulation angle in the $\bar{e}_2$ $\bar{e}_3$ plane,

ψ_3 = pendulation angle in the $\bar{e}_1$ $\bar{e}_3$ plane, and

ϕ_3 = spin angle about the $\bar{e}_3$ axis.

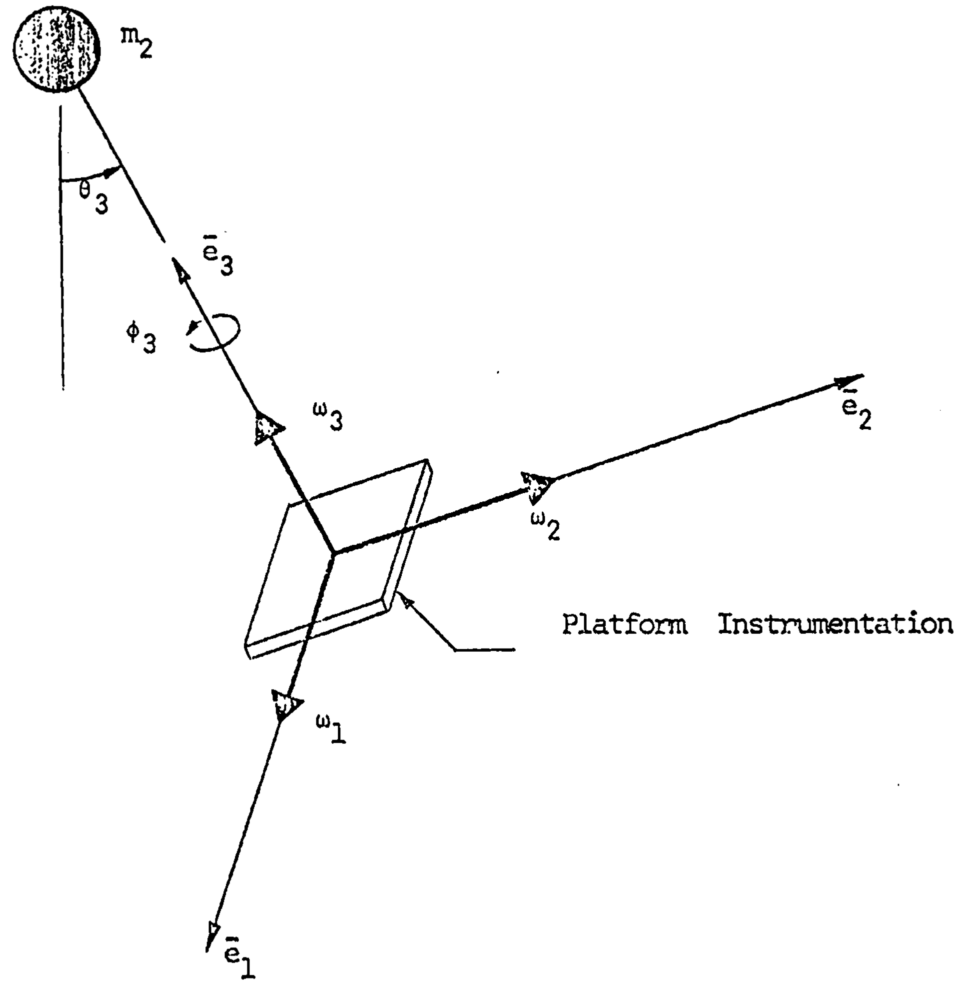


Figure 2.2-4 Platform Angular Velocities

2.3 Parameter Estimation Method

The main object of the parameter estimation method is to determine the initial system state ($\bar{\theta}_o$, $\bar{\psi}_o$ and ϕ_{3o}) such that the rates ($\tilde{\bar{\omega}}$) obtained from the rate gyros fit (over some time interval $0 \leq t \leq T$), in an optimal sense, those rates ($\bar{\omega}$) predicted from the system model. With this initial state determined, the instantaneous system state is obtained simply as the output ($\bar{\theta}(t)$, $\bar{\psi}(t)$ and $\phi_3(t)$) of the system model. Hence, the problem is basically one of the parameter determination in which the initial state parameters ($\bar{\theta}_o$, $\bar{\psi}_o$ and ϕ_{3o}) play the role of the unknown parameters. For the purpose of this work, the platform rates will be fit in a least square sense; i.e., a performance function (Φ) will be formed and the initial state determined such that this function is minimized. The process will be repeated (i.e., $\bar{\theta}_o$, $\bar{\psi}_o$ and ϕ_{3o} will be updated) every T seconds. Initially T will be set equal to 30 seconds (time required to acquire a complete radiance profile) although a study (to be conducted in the future) will be made to determine the minimum T required such that the necessary precision and accuracy are satisfied.

In this research, the function Φ is formed as follows:

$$\Phi = \sum_{j=1}^{N} \sum_{i=1}^{3} (\omega_i - \tilde{\omega}_i)_j^2 \quad , \text{ where} \qquad 2.3\text{-}1$$

N = number of data points taken in $0 \leq t \leq T$,

ω_i = angular velocity computed from system model, and

$\tilde{\omega}_i$ = angular velocity given by the rate gyros and to be compared with ω_i.

The function Φ is clearly dependent on the initial state. This initial state is obtained from the condition that Φ take on a minimum; i.e., by solving the following optimization problem

$$\min.\ \Phi = \Phi(\bar{\omega}). \qquad 2.3\text{-}2$$

The angular velocity components (ω_i) are obtained from the transformation equations 2.2-12 and since $\bar{\theta}$, $\bar{\psi}$, ϕ_3 are functions of the initial state ($\bar{\theta}_o$, $\bar{\psi}_o$ and ϕ_3o), equation 2.3-2 can be written as

$$\min \Phi = \Phi(\bar{\theta}_o, \bar{\psi}_o, \phi_3o) \qquad 2.3\text{-}3$$

Since the values of ϕ are obtained numerically ($\tilde{\bar{\omega}}$ is given as a discrete data point) it will be necessary to employ some direct search technique to solve the above optimization problem. In general, the algorithm for any direct search techniques is given as follows

$$\bar{X}_o^{k+1} = \bar{X}_o^{k} + \delta\bar{X}_o^{k}, \quad (k = 1,2,3,\ldots), \quad \text{where} \qquad 2.3\text{-}4$$

$\bar{X}_o^k$ is the vector of old values,

$\delta\bar{X}_o^k$ is a vector of increments, and

$\bar{X}_o^{k+1}$ is the vector of improved values.

The vector $\delta\bar{X}_o^k$ is found such that $\phi(\bar{X}_o^{k+1}) < \phi(\bar{X}_o^k)$. The value of k is incremented until $\bar{X}_o$ converges; i.e., until the norm of $\delta\bar{X}_o$ satisfies some error criteria. Figure 2.3-1 illustrates the application of equation 2.3-4 to minimize ϕ .

There are many direct search techniques for systematically determining $\delta\bar{X}_o$. After comparing several of these, the direct search method of Hooke and Jeeves was chosen for the following reasons, i.e., (a) the special feature in accelerating of distance, so called pattern search, and (b) this method is already available in subroutine form (5) and was employed with the process outlined in figure 2.3-1. The discussion of Hooke and Jeeves direct search algorithm is presented in section 2.4.

2.4 Hooke and Jeeves - Direct Search Method.

The Hooke and Jeeves [2] method is a simple and powerful univariant method for finding the minimum of a function. A modification of the basic univariant numerical search method, it involves trial explorations and then

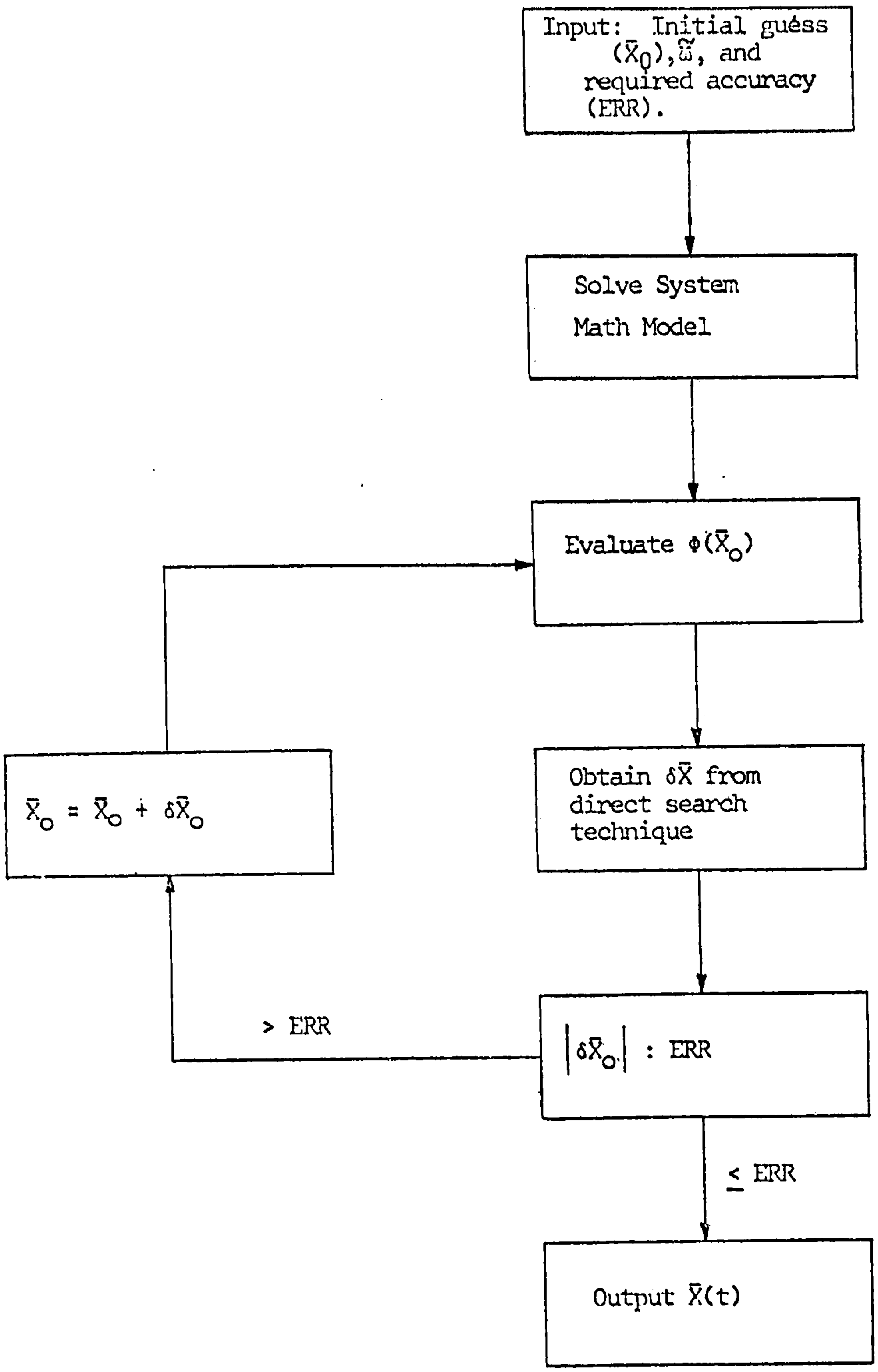


Figure 2.3-1 Block Diagram Illustrating Process for Attitude Determination of LACATE Mission

ever-expanding steps, called pattern moves, in the direction indicated by the explorations. The method is designed to follow a ridge, so that it does not suffer from the main disadvantages of the basic univariant approach which can not effectively cope with ridges or sharp valleys.

The Hooke and Jeeves algorithm consists of two major phases, an "exploratory search" around the current base point and a "pattern search" in the direction selected for minimization. Figure 2.4-1 is a simplified informational flow diagram for the algorithm as implemented by Wook [4]. The steps (blocks) in this figure are as follow:

Block 1 The initial estimates for all decision variables (X_i) as well as initial incremental changes or step sizes (ΔX_i) in the decision variables are provided.

Block 2 The objective function, $\Phi(\bar{X})$ is evaluated at the base point ($\bar{X}$) which is the vector of initial guesses (X_i) of the decision variables.

Block 3 An exploratory search (type 1) is performed next, i.e. each decision variable (X_i) is changed in rotation, one at a time, by the incremental amount (ΔX_i) and the objective function is evaluated at the new point. If this incremental fails to improve the objective function then X_i is changed by ($-\Delta X_i$) and the value of Φ ($\bar{X}$) again evaluated as before. If the objective function is still not improved then X_i is left unchanged, and the same procedure is employed again with X_{i+1}. This process is repeated until all the decision variables (X_1, $X_2, \ldots, X_n$) have been so changed. Figure 2.4-2 illustrates the steps in an exploratory search for a two dimensional problem. For each change in the decision variable, the value of the objective function $\Phi(\bar{X})$ is compared with its value at the previous point. If, upon completion of the exploratory search, none of the changes yields an improved $\Phi(\bar{X})$ (i.e. $X_1, X_2, \ldots, X_n$) remain unchanged , then the stages in block 7 are performed next; otherwise block 4 is implemented.

Block 4 After completing the type I exploratory search, the new base

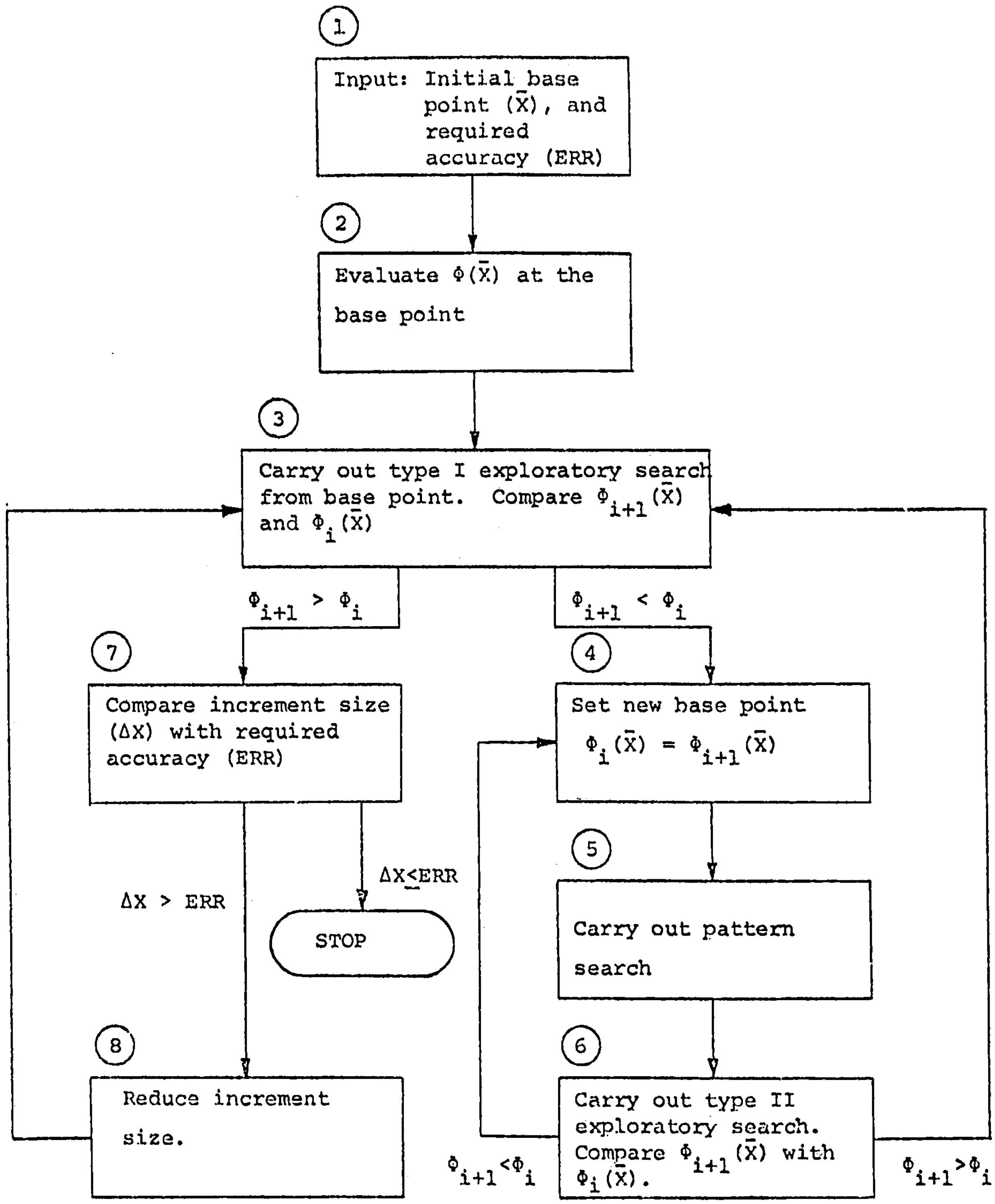


Figure 2.4-1 Block Diagram for Pattern Search Method.

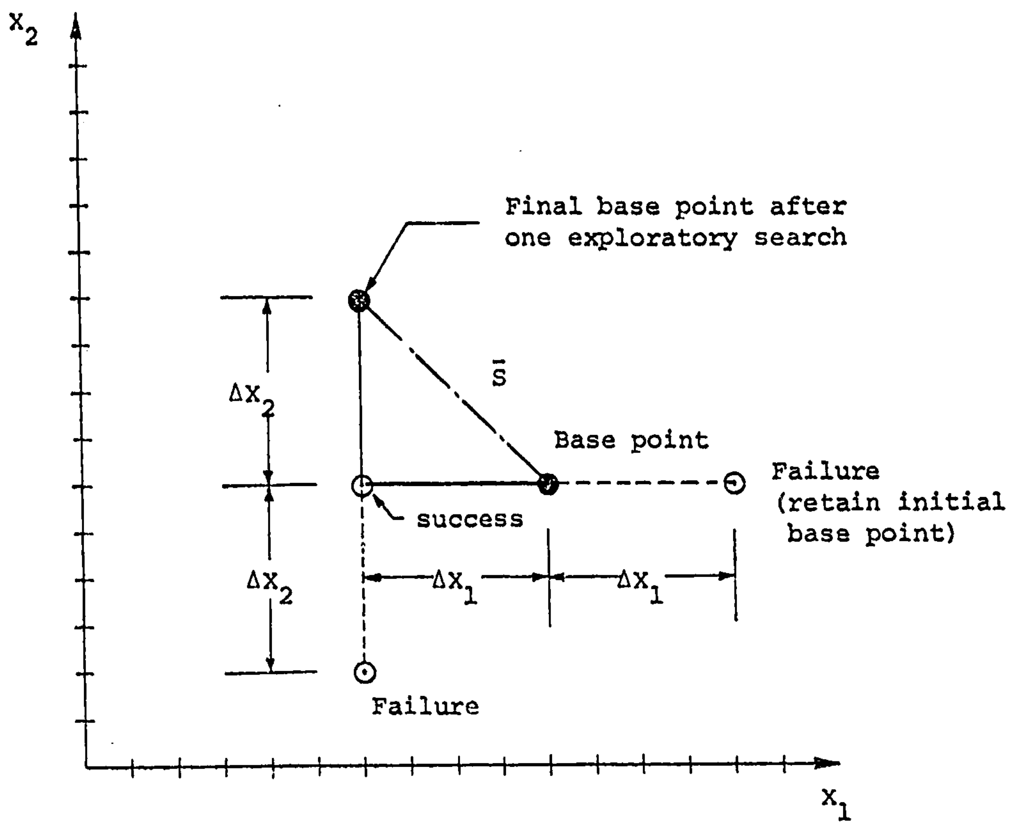


Figure 2.4-2 Exploratory Search Strategy

point is set equal to the final base point obtained from block 3.

<u>Block 5</u> After completing the type I exploratory search and obtaining a new base point, a "pattern search" is made. The new value of the decision variables define a vector, S(see figure 2.4-2), that represent a successful direction for minimization. A series of pattern searches is now made along this vector, usually in increments of $2|\bar{S}|$ until $\Phi(\bar{X})$ no longer decreases. The magnitude of the step sizes for the pattern search (i.e. $\bar{S}'_1$ in figure 2.4-3) is roughly proportional to the number of successes previously encountered in each coordinate direction during the exploratory searches for the previous cycle. The success or failure of a pattern move is not established until after a type II exploratory search (block 6) has been completed. If $\Phi(\bar{X})$ does not decrease after the type II exploratory search, then the pattern search has failed and a new type I exploratory search (block 3) is made in order to define a new successful direction. The base point for a pattern search in the case of a two dimensional problem is illustrated in figure 2.4-3.

<u>Block 6</u> The type II exploratory search (figure 2.4-3) is made after a temporary exploration point is obtained from a pattern search. The chief difference between the type I and type II exploratory searches is the magnitude of the step sizes (ΔX_i). In the case of the type II exploratory search, ΔX_i is taken as some multiple of the ΔX_i (i.e. $C_i(\Delta X_i)$, see figure 2.4-3) used in the type I exploratory search. This is done in order to accelerate the search.

<u>Block 7</u> If the type I exploratory search fails to give a new successful direction, then the current $|\Delta x_i|$ is compared to some preset allowable tolerance (error input in block 1). If $|\Delta x_i|$ is larger than the allowable error, then block 8 is implemented. Failure to improve $\Phi(\bar{X})$ for $|\Delta x_i|$ smaller than the allowable error indicates that a local optimum has been

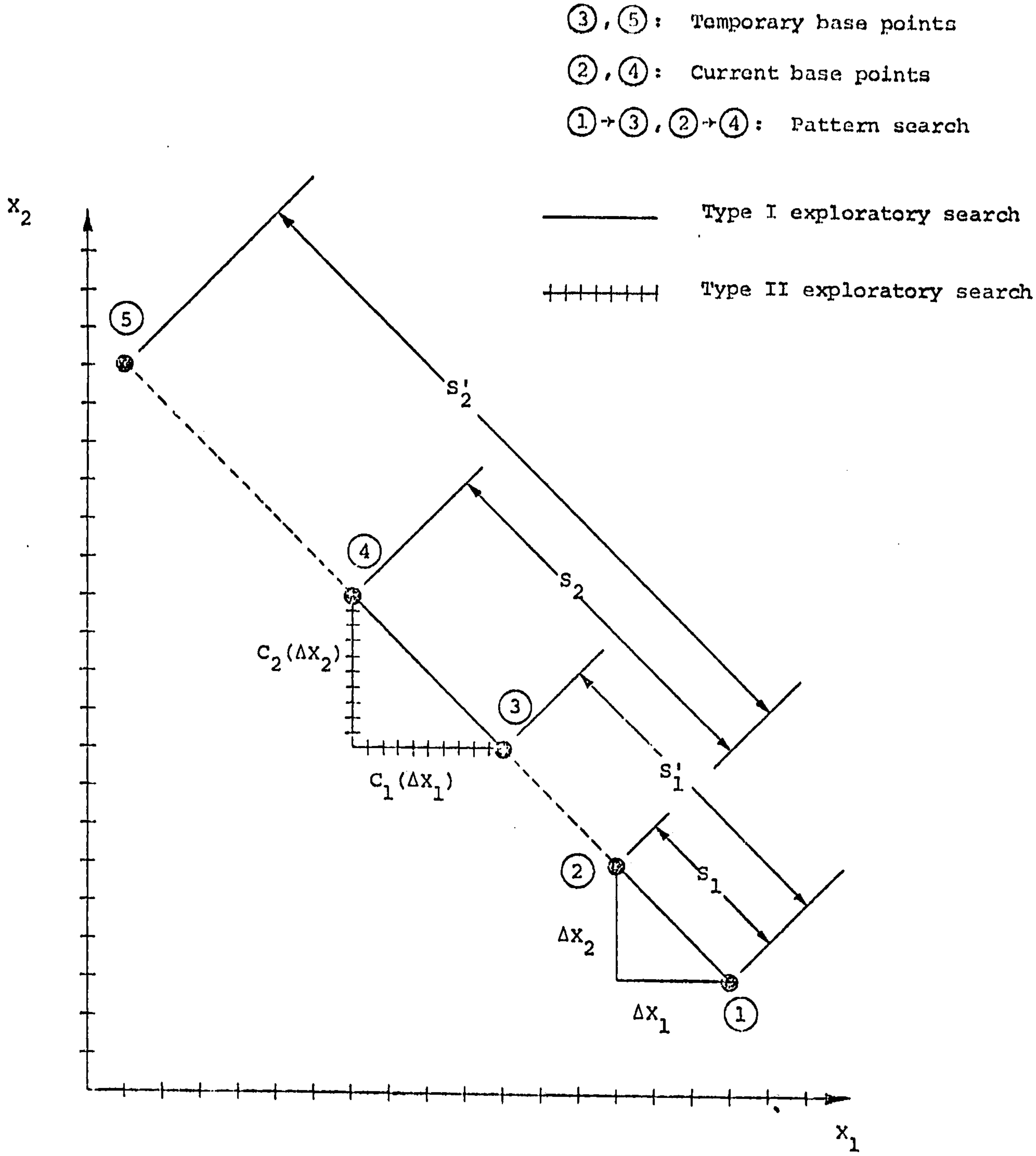


Figure 2.4-3 General Search Strategy

reached and the search is terminated.

Block 8 If $|\Delta x_i|$ is still larger than the prespecified error, then $|\Delta x_i|$ is reduced gradually until the type I exploratory search can be employed to define a new successful direction.

In order to terminate the search, two additional basic tests must be satisfied. These are described as follows:

(i) After each exploratory and pattern search, the increment in the objective function $|\Delta\Phi|$ is compared with a prescribed test value. If this increment is less than the test value, then the exploratory or pattern search is said to have failed. In this case block 3 or 7 is implemented.

(ii) If $|\Delta\Phi|$ is greater than the prescribed test value, then a test is made to determine if the objective has increased (a failure) or decreased (a successful search). This second test ensures that the values of the objective function is always being improved.

The fortran coding for the Hooke and Jeeves method has been provided (with some minor revision) by M.I.T. Joint Computer Facility. This is given in appendix B. The program is available in subroutine form with various parameters in the calling sequence. These parameters include

(i) the number of decision variables (n),

(ii) the initial guesses of the variables $(x_1,x_2,\ldots,x_n)$,

(iii) the required accuracy (ERR), and

(iv) the allowable number of allowable iterations.

2.5 Test Problem

In order to verify the accuracy and precision of proposed parameter estimation process (refer section 2.3) it is necessary to employ the process to a test problem in which the initial state is already known. Preferably, the test problem model should be identical to the one employed in this study; i.e. equations 2.2-1 to 2.2-3. With the initial state known, the resulting values of θ_3, ψ_3, ϕ_3, $\dot{\theta}_3$, $\dot{\psi}_3$ and $\dot{\phi}_3$ which determines the state of the platform (as a function of time) can be obtained from equations 2.2-9 to 2.2-11. With these, the values of ω_1, ω_2 and ω_3 can be computed from equation 2.2-12.

The testing process will consist of the following steps.

(a) Assign fixed values to the initial state parameters $\bar{\theta}(0)$, $\bar{\psi}(0)$, $\dot{\bar{\theta}}(0)$ and $\dot{\bar{\psi}}(0)$, or equivalently to the unknown constants $c_1, c_2, \ldots, c_{12}$ in equations 2.2-9 and 2.2-10.

(b) Compute the orientation (state) of the platform θ_3, $\dot{\theta}_3$, ψ_3, $\dot{\psi}_3$, ϕ_3 and $\dot{\phi}_3$ as a function of time by employing equations 2.2-9 to 2.2-11.

(c) Determine the values of $\tilde{\omega}_1$, $\tilde{\omega}_2$ and $\tilde{\omega}_3$ as a function of time by substituting the results from part (b) into equation 2.2-12.

(d) Employ the results (sampled at various times) from part (c) as input to the parameter estimation process and utilize this process to recover the initial state values (or unknown constants $c_1, c_2, \ldots, c_{12}$).

The accuracy and precision of the process can be determined by comparing the results of part (d) to the corresponding assumed values of part (a). The testing procedure can be repeated for various sets of input values. A flow chart illustrating the test process is given in figure 2.5-1.

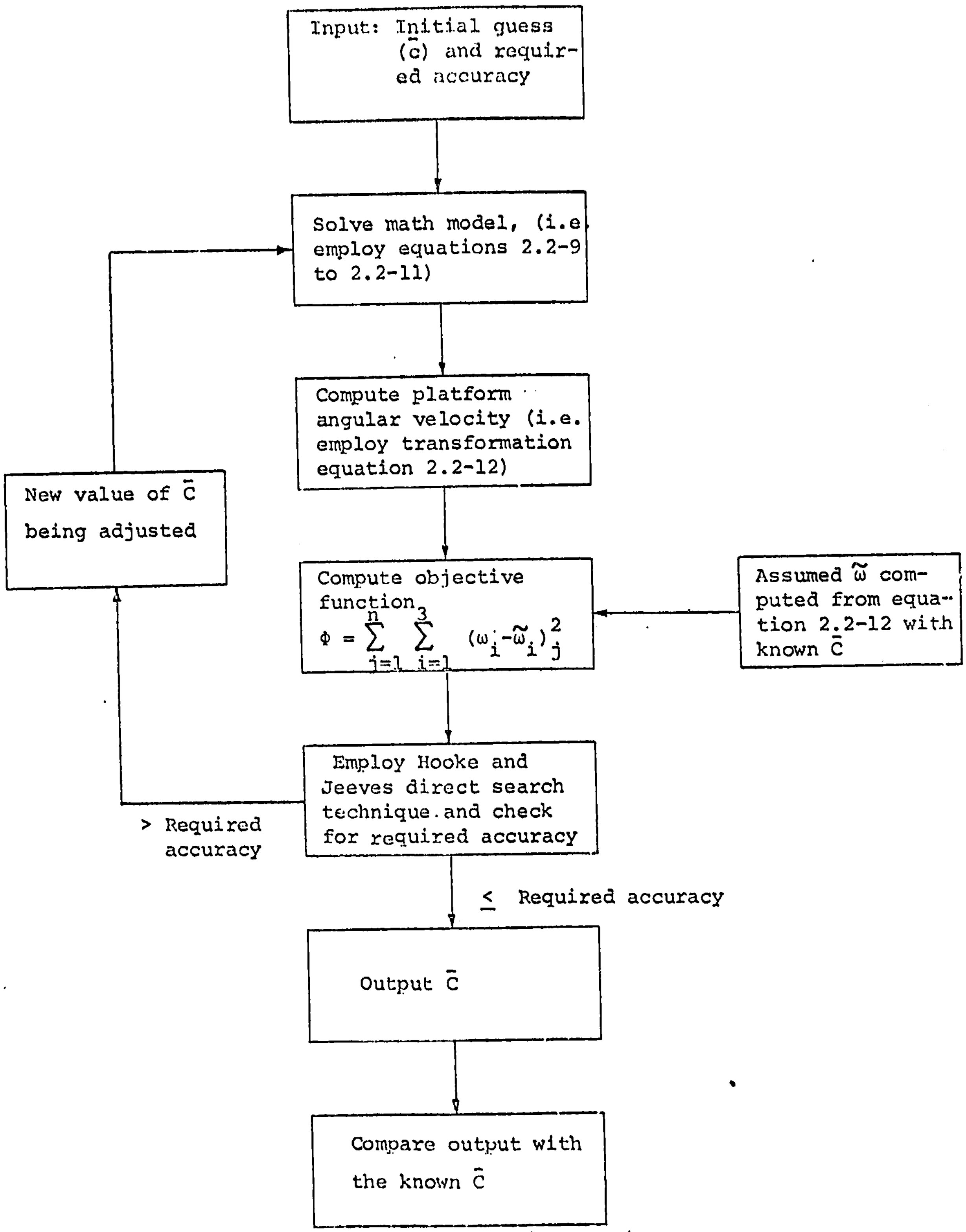


Figure 2.5-1 Flow Chart for Test Problem

CHAPTER III

RESULTS AND CONCLUSIONS

3.1 Data for LACATE Mission

Balloon and System Data. Figure 1.1-1 illustrates the actual Lacate balloon system and figure 1.1-2 illustrates the corresponding idealized system used in this study. The values for the various lengths and masses of the idealized system are given in table 3.1-1.

The actual design profile of the balloon at float altitude is illustrated in figure 3.1-1 and the corresponding dimensions are given in table 3.1-2. Values for (a) mass center, (b) center of volume, and (c) moment of inertia (at the mass center) of the balloon were computed and these values are presented in table 3.1-3 along with other balloon properties.

Standard Atmosphere Data. Figure 3.1-2 illustrates the U.S. Standard Atmosphere and the corresponding properties are presented in table 3.1-4. Table 3.1-5 presents the actual properties of the atmosphere at the float altitude (47 kilometers).

Math Model Data. The expressions for the element of A matrix of equations 2.2-1 and 2.2-2 for the system model are given in reference (3). The numerical values for these elements were computed based on the data given above and these values are presented in table 3.1-6.

Gyro Data. The platform coordinate axis which were employed for referencing the angular velocity components (as measured by the instrumentation package) do not coincide with the coordinate axis used in the Euler angle transformation equation (2.2-12). The relationship between these two coordinate systems is shown in figure 3.1-3. The resulting transformation equations which relate the angular velocity components are given as

$$\omega_1 = -\tilde{\omega}_1 \text{ ,} \qquad 3.1\text{-}1$$

$$\omega_2 = \tilde{\omega}_2 \text{ , and} \qquad 3.1\text{-}2$$

$$\omega_3 = -\tilde{\omega}_3 \text{ , where} \qquad 3.1\text{-}3$$

ω_i is the angular velocity component along the X_i axis, and

$\tilde{\omega}_i$ is the corresponding angular velocity component obtained from the gyro.

The numerical values of the angular velocity components obtained from the gyro system are presented in appendix C. Typical plots of the angular velocity components obtained from the gyros are illustrated in figures 3.1-4 to 3.1-6.

The azimuth angle ϕ_m obtained from the magnetometer was measured clockwise from magnetic north to the negative X_1 platform axis. However, the angle ϕ in equation 2.2-11 is measured counter-clockwise from north to the same negative X_1 platform axis. This is illustrated in figure 3.1-7. The transformation equation relating these two angles is given as

$$\phi = 360° - \phi_m \text{ , where} \qquad 3.1\text{-}4$$

ϕ is the spin displacement in equation 2.2-11, and

ϕ_m is the spin displacement measured by the magnetometer.

A typical plot of the azimuth angle ϕ_m is shown in figure 3.1-8.

Eigenvalue Problem. The solution to the eigenvalue problem (equation 2.2-8) was obtained by employing the computer program given in appendix A in conjunction with the coefficients presented in table 3.1-6. The solution for the eigenvalues Ω_i and corresponding eigenvectors is presented in table 3.1-7. The magnitude of Ω_i is equal to the natural frequency of the system.

TABLE 3.1-1

Idealized LACATE System Properties

r_o (maximum radius of the balloon) = 249.989 ft.

r_1 (distance from center of mass of balloon shell to mass m_1) = 222.091 ft.

r_2 (distance from mass m_1 to m_2) = 75 ft.

r_3 (distance from mass m_2 to m_3) = 15 ft.

d (distance from center of mass (C.G.) to center of volume (C.V.)) = 23.712 ft.

m_o (mass of balloon shell) = 2850 lb_m

m_1 (lumped mass) = 135 lb_m

m_2 (lumped mass) = 135 lb_m

m_3 (lumped mass) = 375 lb_m

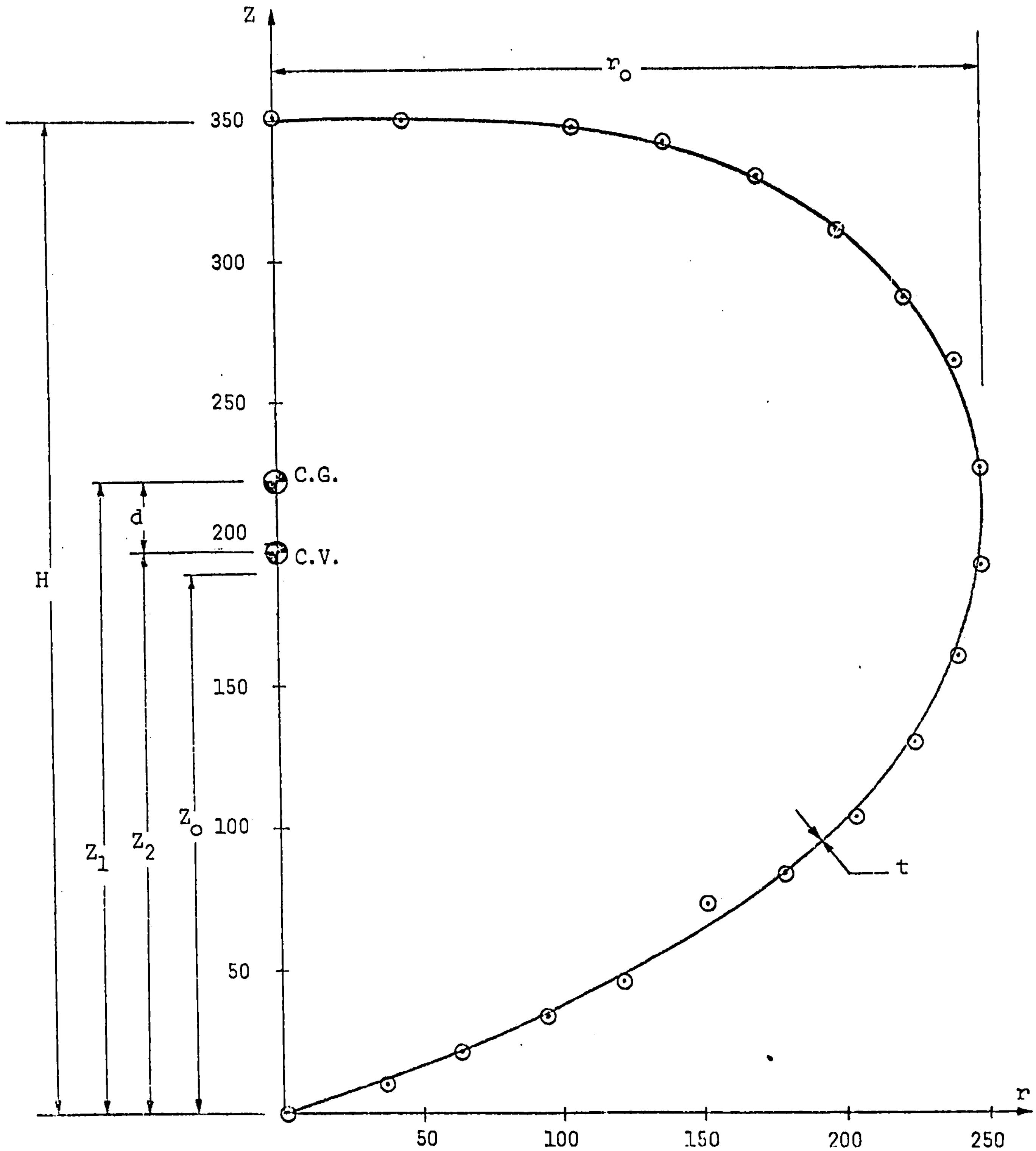


Figure 3.1-1 Actual Balloon Profile

Table 3.1-2

Balloon Profile Data

HEIGHT-Z (ft)	RADIUS-r (ft)	FABRIC WT.-W_Z (lbs)
.0	.2	.0
10.6	32.2	42.2
21.9	63.8	102.8
34.3	95.1	181.8
46.8	122.2	267.2
62.9	151.7	380.7
81.9	179.5	511.6
104.4	204.4	658.6
130.7	225.4	819.7
160.5	240.7	991.9
193.1	248.9	1171.3
226.6	248.8	1353.3
258.9	240.1	1532.5
287.9	223.1	1703.9
311.7	199.4	1862.8
329.4	170.9	2005.7
340.9	139.3	2130.2
347.4	106.4	2315.5
350.4	72.9	2491.0
351.0	.0	2850.0

TABLE 3.1-3

BALLOON PROPERTIES

r_o (maximum radius) = 249.989 ft.

Z_o (height, corresponded to r_o) = 209.526 ft.

V_H (inflated volume) = 45,378,282 cu. ft.

H (inflated height) = 350.1 ft.

m_o (total weight of balloon shell including top cap weight) = 2880 lbs.

Gore length = 674.83 ft.

Surface area = 635,711 sq. ft.

Z_1 (distance from bottom apex to the mass center of balloon shell) = 222.091 ft.

Z_2 (distance from nadir to center of volume) = 198.379 ft.

t (thickness of balloon shell (strato film®) = 0.0006 inch.

I_{o1} (moment of inertia at the mass center (C.G.) = 2.622374 X 10^6 slug-ft^2

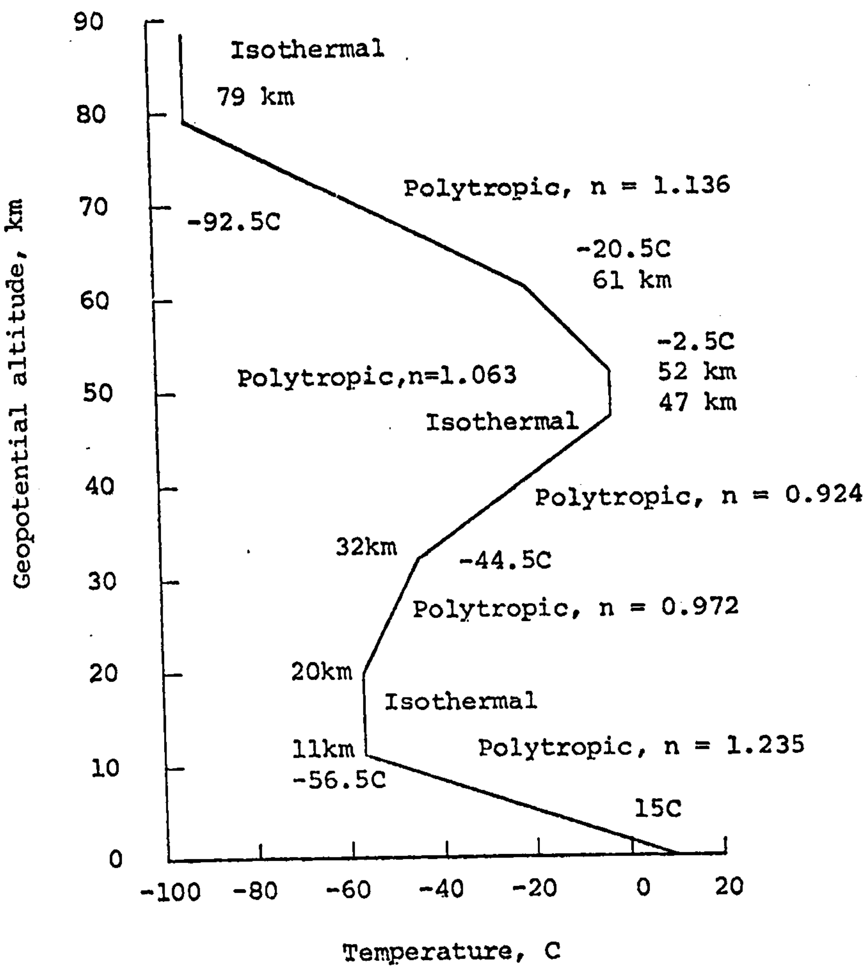


Figure 3.1-2 U.S. Standard Atmosphere (6)

Table 3.1-4

General Properties of the U.S. Standard Atmosphere (6)

ALTITUDE, (m)	TEMPERATURE (C)	TYPE OF ATMOSPHERE	LAPSE RATE (C/km)	$\bar{g}$, (m/sec^2)	n	PRESSURE p(N/m^2)	DENSITY ρ(kg/m^3)
0	15.0					1.013×10^5	1.225
		Polytropic	−6.5	9.790	1.235		
11,000	−56.5					2.263×10^4	3.639×10^{-1}
		Isothermal	0.0	9.759			
20,000	−56.5					5.475×10^3	8.804×10^{-2}
		Polytropic	+1.0	9.727	0.972		
32,000	−44.5					8.680×10^2	1.323×10^{-2}
		Polytropic	+2.8	9.685	0.924		
47,000	−2.5					1.109×10^2	1.427×10^{-3}
		Isothermal	0.0	9.654			
52,000	−2.5					5.900×10^1	7.594×10^{-4}
		Polytropic	−2.0	9.633	1.053		
61,000	−20.5					1.821×10^1	2.511×10^{-4}
		Polytropic	−4.0	9.592	1.136		
79,000	−92.5					1.038	2.001×10^{-5}
		Isothermal	0.0	9.549			
88,743	−92.5					1.644×10^{-1}	3.170×10^{-6}

C = temperature in degrees Centigrade

km = Killometers

n = polytropic exponent

$\bar{g}$ = local acceleration of gravity

TABLE 3.1-5

Properties of Atmosphere at float Altitude (47 km)

ρ_{oa} (density of air) = $2.78 \times 10^{-6} \frac{slug}{ft^3}$

ρ_{oH} (density of helium) = $3.834 \times 10^{-7} \frac{slug}{ft^3}$

P_o (pressure) = $2.254 \frac{lb}{ft^2}$

C_D (viscous drag coefficient) = .5

g (gravity) = $31.7 \frac{ft}{sec^2}$

μ_a (viscosity of air) = $3.57 \times 10^{-7} \frac{lb_m}{ft\ sec}$

μ_H (viscosity of helium) = $3.74 \times 10^{-7} \frac{lb_m}{ft.sec.}$

TABLE 3.1-6

Coefficients of A Matrix [Equation 2.2-1 & 2.2-2]

$a_{11} = 2.10463 \times 10^{-2}$

$a_{21} = -6.08078 \times 10^{-2}$

$a_{31} = -5.07147 \times 10^{-9}$

$a_{12} = -3.85241 \times 10^{-2}$

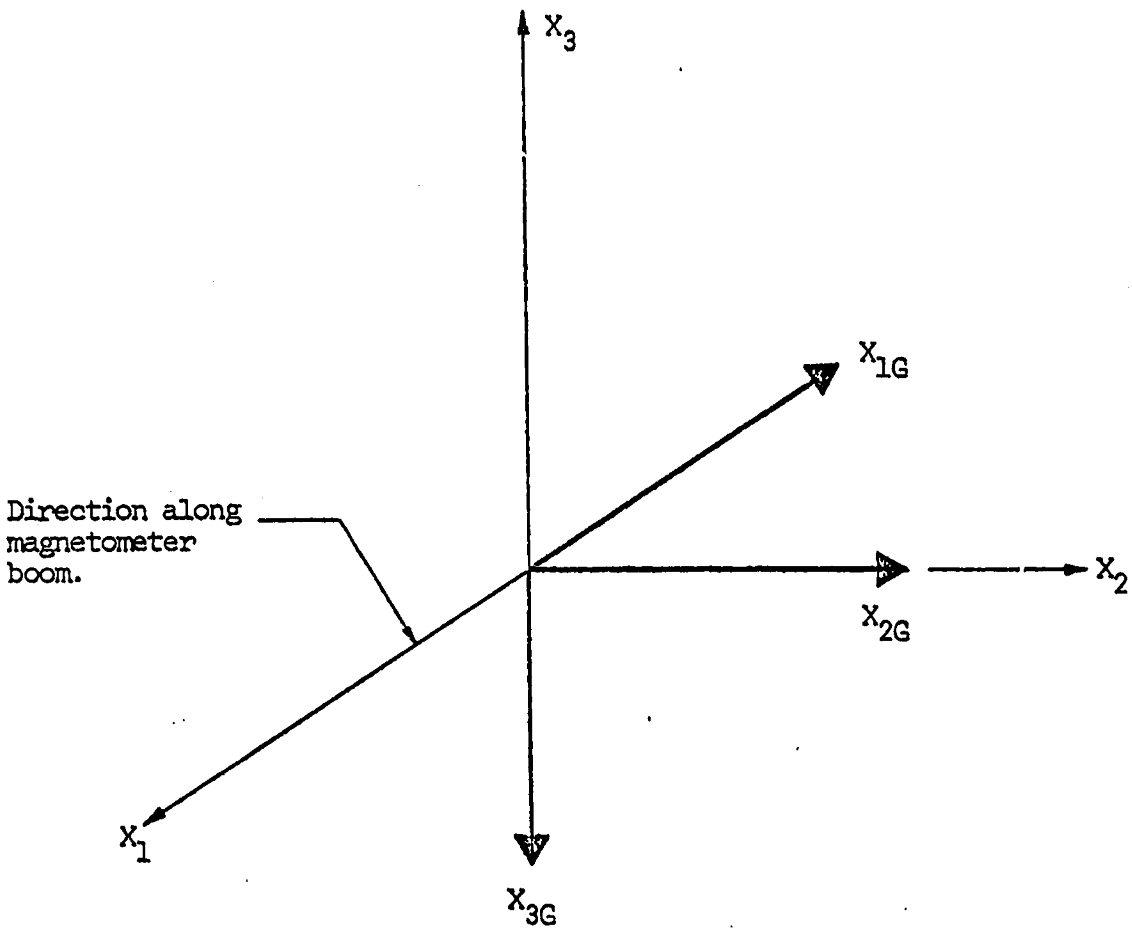
$a_{22} = 1.74685$

$a_{32} = -7.98370$

$a_{13} = 0.0$

$a_{23} = -1.17407$

$a_{33} = 7.98370$



X_i = coordinate system employed for purposes of math model.

X_{iG} = gyro coordinate system.

Figure 3.1-3 Gyro vs Math Model Coordinate System

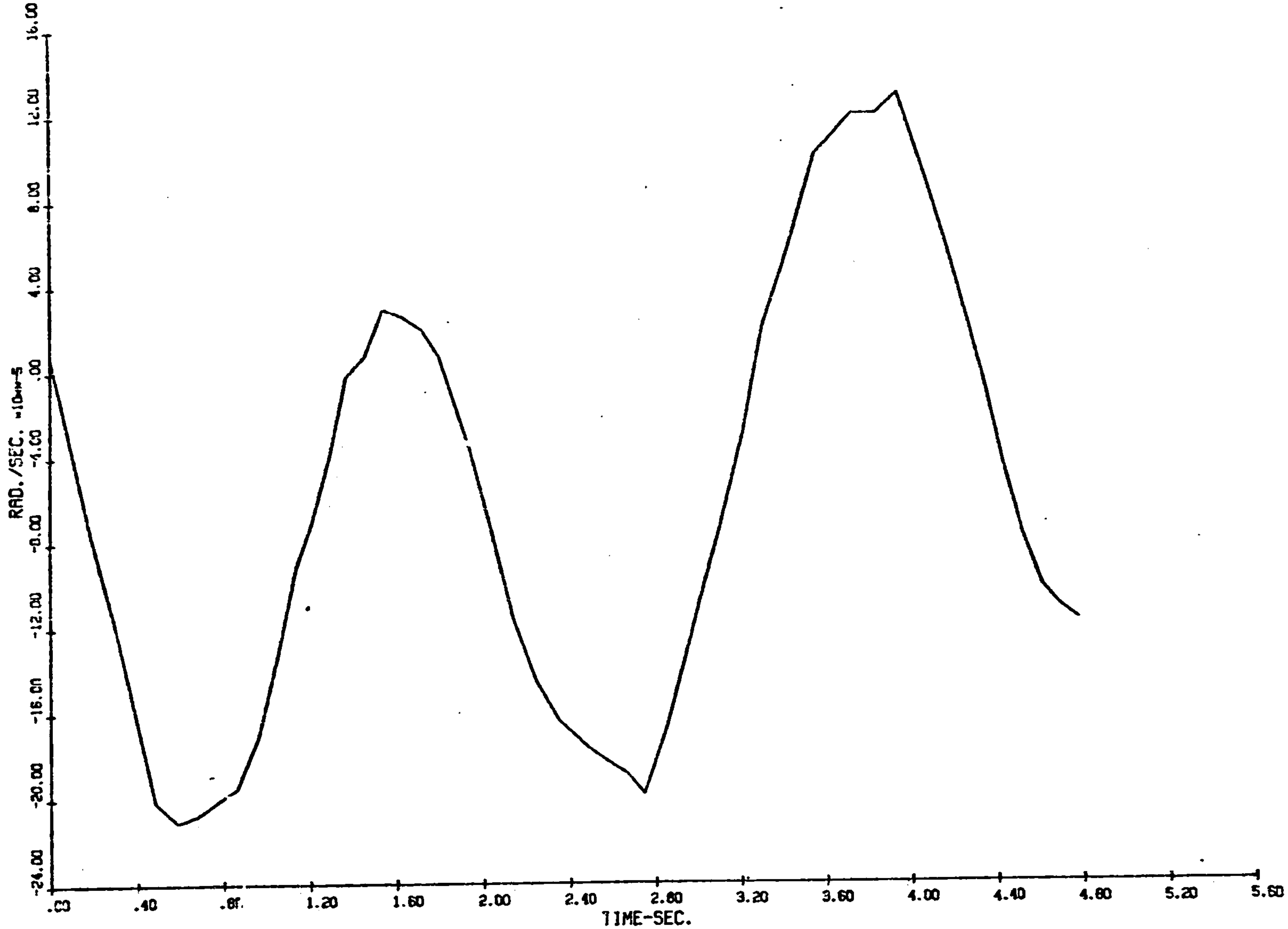


Figure 3.1-4 Angular Velocity ($\tilde{\omega}_1$) Data From Gyro

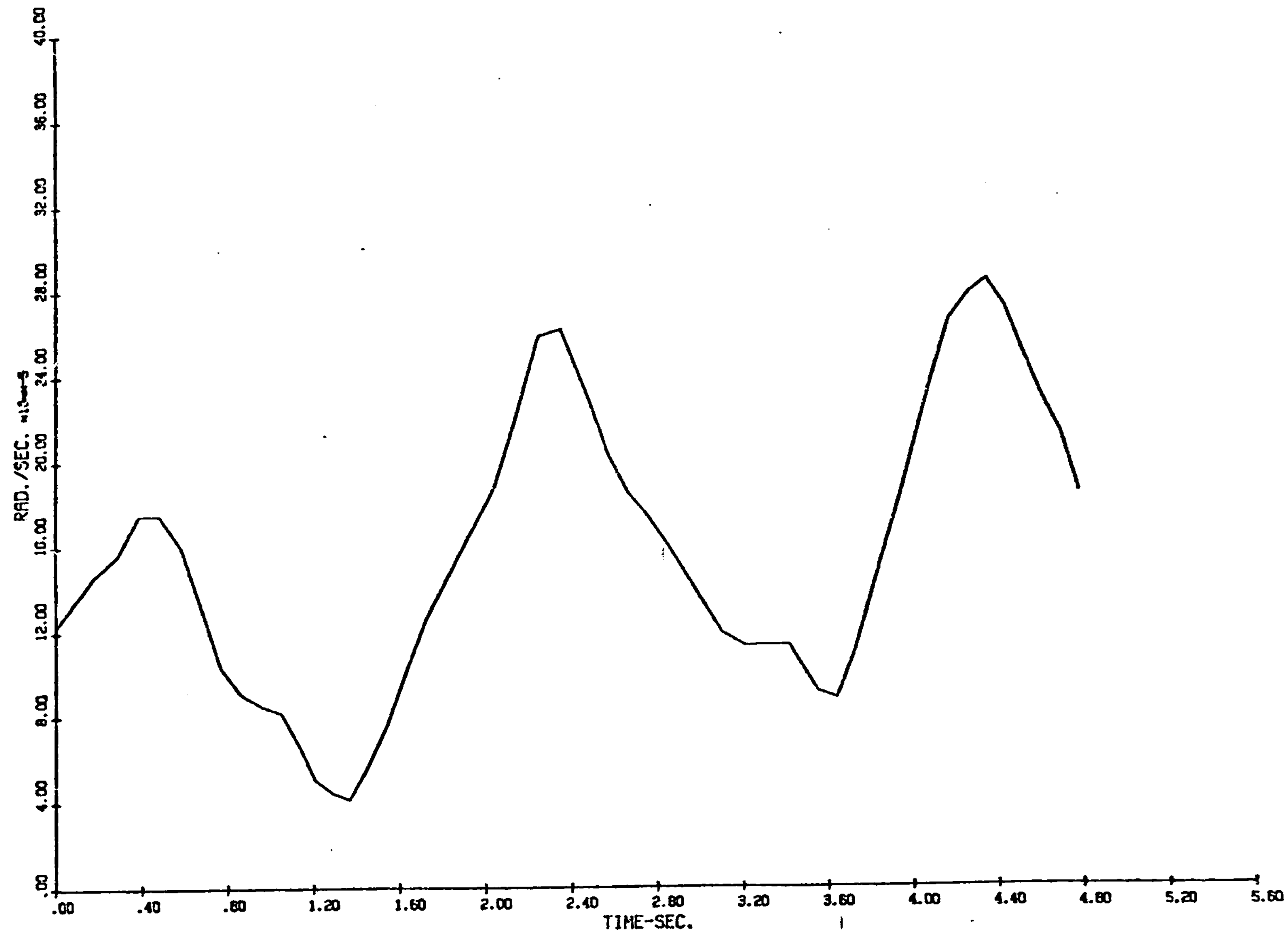


Figure 3.1-5 Angular Velocity ($\tilde{\omega}_2$) Data From Gyro

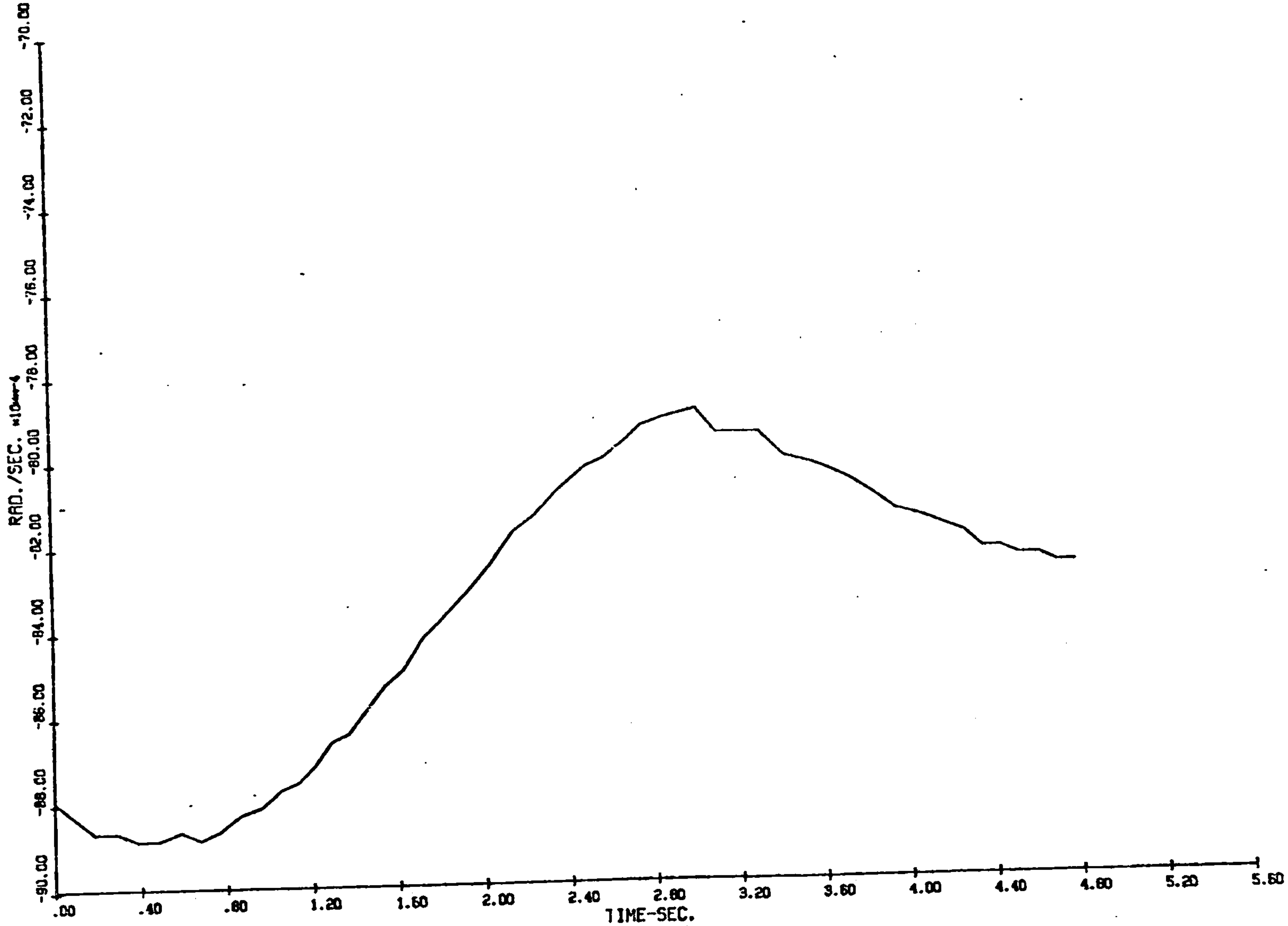


Figure 3.1-6 Angular Velocity ($\tilde{\omega}_3$) Data From Gyro

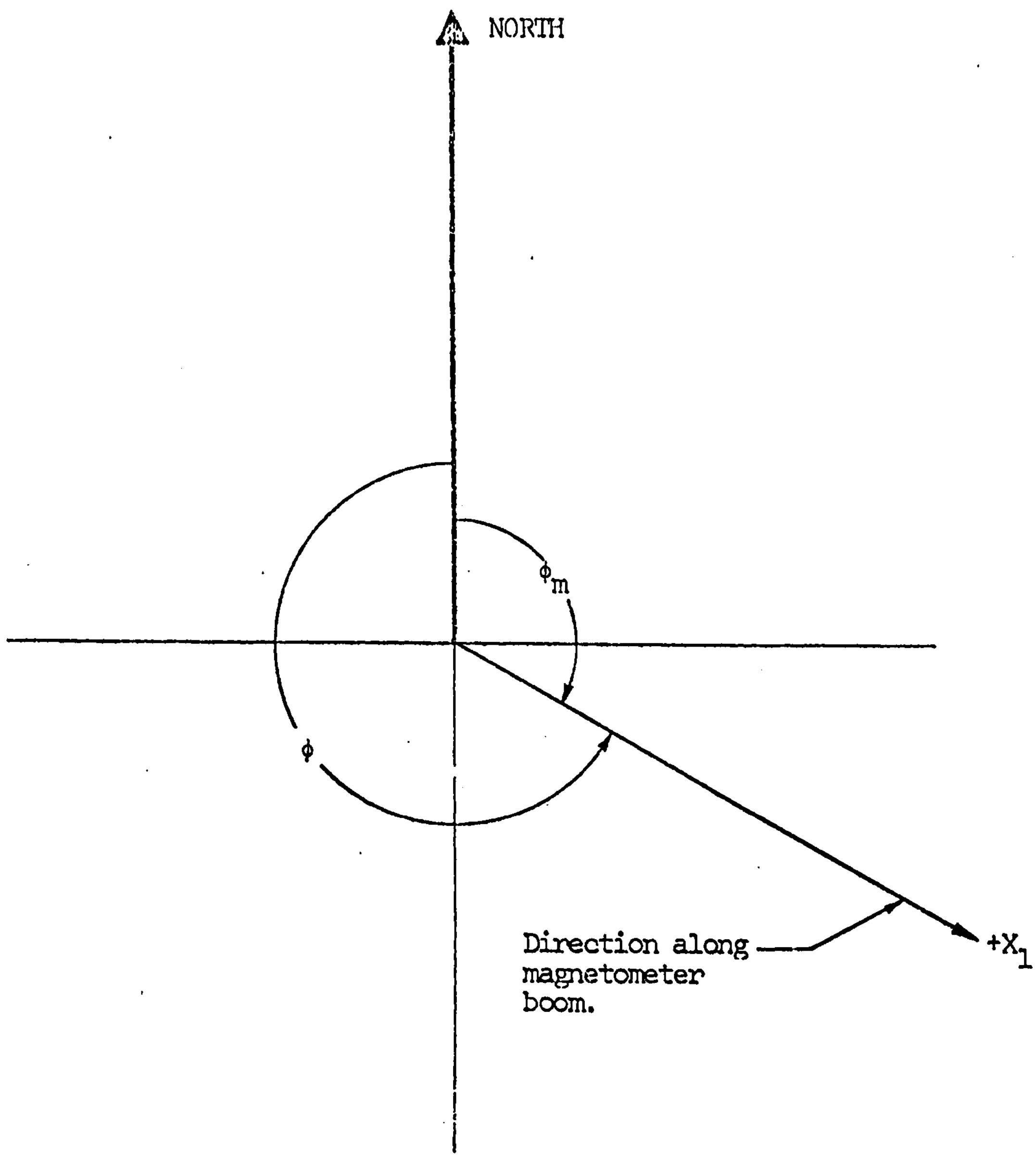


Figure 3.1-7 Measurement of Spin Displacement

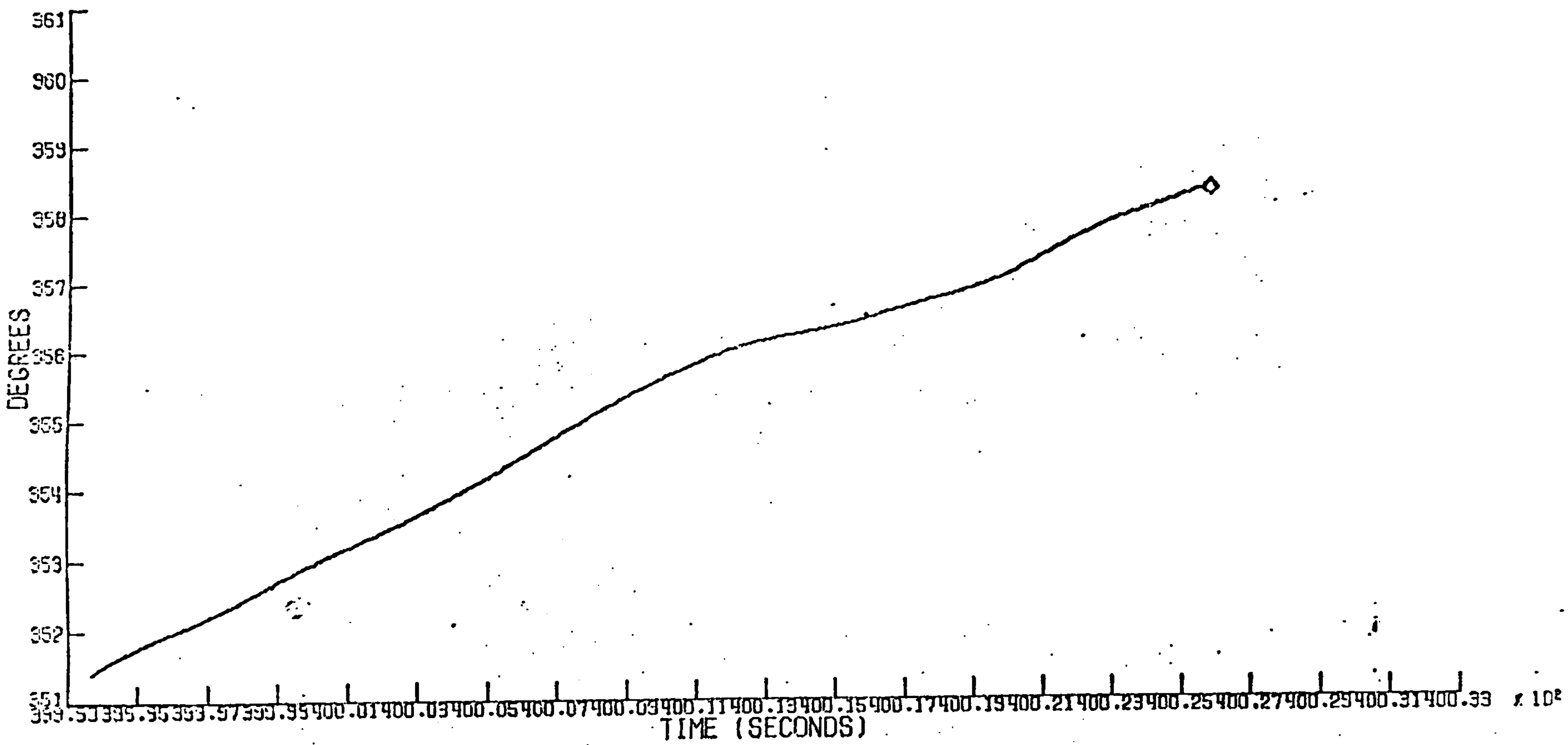


Figure 3.1-8 Azimuth Angle (ϕ_m) From Magnetometer

TABLE 3.1-7

Solution for the Eigenvalues Ω_i and Corresponding Eigenvectors.

i	Ω_i	$\bar{X}_i$
1	1.296692×10^{-1}	9.08376 0.997919 1.000
2	7.066434×10^{-1}	-7.550639×10^{-2} 0.937457 1.000
3	3.038977	6.555123×10^{-4} - .156788 1.000

3.2 Evaluation of Parameter Determination Process

The process for evaluating the attitude determination process by employing a test problem was discussed in section 2.5. The form of the math model for the test problem was identical to the system model given in equation 2.2-1 through 2.2-3. Several different sets of unknown constants ($\tilde{C}_i$) were assumed and the resulting angular velocity components ($\tilde{\omega}_i$) were computed by employing the solution equations 2.2-9 to 2.2-11 and the transformation equation 2.2-12. These values of $\tilde{\omega}_i$ were input to parameter determination process which was then employed to recover the unknown constants.

Table 3.2-1 gives the results for one test case. These results indicate that the process is capable of recovering the unknown constants c_i with good precision. The results presented in table 3.2-1 are typical of those obtained for other test cases. Hence one can conclude that the process is suitable for evaluating the attitude of the LACATE system.

During the next stage of the research, the process will be employed (in conjunction with the math model given in eqs. 2.2-1 - 2.2-3) to determine the angular velocity components of the platform of the LACATE balloon system. These values will then be compared to the actual data in order to evaluate the system model. Results will be presented in the next report.

APPENDIX A

Fortran Coding for Computation of Eigenvalues and Eigenvectors (Eq. 2.2-8).

```
      DIMENSION BMAT(7,7),AC(7,7),C(50,10),X(50,1)
      DIMENSION A(50),RR(50),RI(50),IANA(50),W(50),P(50),F(50)
      COMMON /BLKIN/ AI01,D,R(3)
      OUTPUT 'ENTER HERE--->AI01,D,R(1)'
      INPUT AI01,D,R(1)
      DO 1000 I=1,7
      DO 1000 J=1,7
      BMAT(I,J)=0.0
 1000 AC(I,J)=0.
      CALL COEFF(BMAT,AC)
      DO 4000 J=1,3
      A(J)=BMAT(J,1)
      A(J+3)=BMAT(J,2)
      A(J+6)=BMAT(J,3)
 4000 WRITE(108,5000) (BMAT(J,I),I=1,3)
 5000 FORMAT(/4(1PE15.5))
      OUTPUT ' ENTER M  ORDER OF THE MATRIX'
      INPUT M
      CALL HSBG(M,A,M)
      WRITE(108,70)
   70 FORMAT(//5X'TRIANGULAR (ALMOST!!) MATRIX A FROM HSBG'//)
      DO 40 L=1,M
   40 WRITE(108,10),(A(I),I=L,M*(M-1)+L,M)
   10 FORMAT(/4(1PE15.5))
      CALL ATEIG(M,A,RR,RI,IANA,M)
      WRITE(108,80)
   80 FORMAT(///'EIGENVALUE'/)
      OUTPUT ,(IANA(I),I=1,M)
      OUTPUT,(RR(I),I=1,M),(RI(I),I=1,M)
      WRITE(108,3000)
 3000 FORMAT(/'RESULTS ON  RR(I),OMEGA(I),REDIOD(I),FREQUENCY(I)'//)
      DO 2000 I=1,M
      W(I)=ABS(SQRT(RR(I)))
      P(I)=2.*3.1415926/W(I)
      F(I)=W(I)/(2.*3.1415926)
 2000 WRITE(108,6000) I,RR(I),W(I),P(I),F(I)
 6000 FORMAT(5X,I2,4(1PE16.6))
```

```
C
C
90    OUTPUT 'ENTER EIGENVALUE'
      INPUT E
      X(3,1)=1.00
      X(1,1)=(-BMAT(1,3)*(BMAT(2,2)-E)+BMAT(1,2)*BMAT(2,3))/((BMAT(1,1)
     1-E)*(BMAT(2,2)-E)-BMAT(1,2)*BMAT(2,1))
      X(2,1)=(-BMAT(2,3)*(BMAT(1,1)-E)+BMAT(1,3)*BMAT(2,1))/((BMAT(1,1)
     1-E)*(BMAT(2,2)-E)-BMAT(1,2)*BMAT(2,1))
      WRITE(108,7000) E
7000  FORMAT(///'HERE IS THE EIGENVALUE -->  'F10.5/,
     1'AND THE CORRESPONDING EIGENVECTOR :-  '/)
      OUTPUT X(1,1),X(2,1),X(3,1)
      OUTPUT   'ENTER 0 IF YOU HAVE MORE EIGENVECTOR TO COMPUTE'
      INPUT IKJ
      IF(IKJ.EQ.0) GO TO 90
      STOP 'EIGVEC'
      END
```

```
C
C     ..................................................................
C
C        SUBROUTINE HSBG
C
C        PURPOSE
C           TO REDUCE A REAL MATRIX INTO UPPER ALMOST TRIANGULAR FORM
C
C        USAGE
C           CALL HSBG(N,A,IA)
C
C        DESCRIPTION OF THE PARAMETERS
C           N      ORDER OF THE MATRIX
C           A      THE INPUT MATRIX, N BY N
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY
C                  A IN THE CALLING PROGRAM WHEN THE MATRIX IS IN
C                  DOUBLE SUBSCRIPTED DATA STORAGE MODE.  IA=N WHEN
C                  THE MATRIX IS IN SSP VECTOR STORAGE MODE.
C
C        REMARKS
C           THE HESSENBERG FORM REPLACES THE ORIGINAL MATRIX IN THE
C           ARRAY A.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SIMILARITY TRANSFORMATIONS USING ELEMENTARY ELIMINATION
C           MATRICES, WITH PARTIAL PIVOTING.
C
C        REFERENCES
C           J.H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -
C           CLARENDON PRESS, OXFORD, 1965.
C
C     ..................................................................
C
      SUBROUTINE HSBG(N,A,IA)
      DIMENSION A(1)
```

```
      DOUBLE PRECISION S
      L=N
      NIA=L*IA
      LIA=NIA-IA
C
C        L IS THE ROW INDEX OF THE ELIMINATION
C
   20 IF(L-3) 360,40,40
   40 LIA=LIA-IA
      L1=L-1
      L2=L1-1
C
C        SEARCH FOR THE PIVOTAL ELEMENT IN THE LTH ROW
C
      ISUB=LIA+L
      IPIV=ISUB-IA
      PIV=ABS(A(IPIV))
      IF(L-3) 90,90,50
   50 M=IPIV-IA
      DO 80 I=L,M,IA
      T=ABS(A(I))
      IF(T-PIV) 80,80,60
   60 IPIV=I
      PIV=T
   80 CONTINUE
   90 IF(PIV) 100,320,100
  100 IF(PIV-ABS(A(ISUB))) 180,180,120
C
C        INTERCHANGE THE COLUMNS
C
  120 M=IPIV-L
      DO 140 I=1,L
      J=M+I
      T=A(J)
      K=LIA+I
      A(J)=A(K)
  140 A(K)=T
C
C        INTERCHANGE THE ROWS
```

```
C
      M=L2-M/IA
      DO 160 I=L1,NIA,IA
      T=A(I)
      J=I-M
      A(I)=A(J)
  160 A(J)=T
C
C        TERMS OF THE ELEMENTARY TRANSFORMATION
C
  120 DO 200 I=L,LIA,IA
  200 A(I)=A(I)/A(ISUB)
C
C        RIGHT TRANSFORMATION
C
      J=-IA
      DO 240 I=1,L2
      J=J+IA
      LJ=L+J
      DO 220 K=1,L1
      KJ=K+J
      KL=K+LIA
  220 A(KJ)=A(KJ)-A(LJ)*A(KL)
  240 CONTINUE
C
C        LEFT TRANSFORMATION
C
      K=-IA
      DO 300 I=1,N
      K=K+IA
      LK=K+L1
      S=A(LK)
      LJ=L-IA
      DO 280 J=1,L2
      JK=K+J
      LJ=LJ+IA
  280 S=S+A(LJ)*A(JK)*1.0D0
  300 A(LK)=S
C
```

```
C        SET THE LOWER PART OF THE MATRIX TO ZERO
C
      DO 310 I=L,LIA,IA
  310 A(I)=0.0
  320 L=L1
      GO TO 20
  360 RETURN
      END
C
C     ...................................................................
C
C        SUBROUTINE ATEIG
C
C        PURPOSE
C           COMPUTE THE EIGENVALUES OF A REAL ALMOST TRIANGULAR MATRIX
C
C        USAGE
C           CALL ATEIG(M,A,RR,RI,IANA,IA)
C
C        DESCRIPTION OF THE PARAMETERS
C           M      ORDER OF THE MATRIX
C           A      THE INPUT MATRIX, M BY M
C           RR     VECTOR CONTAINING THE REAL PARTS OF THE EIGENVALUES
C                  ON RETURN
C           RI     VECTOR CONTAINING THE IMAGINARY PARTS OF THE EIGEN-
C                  VALUES ON RETURN
C           IANA   VECTOR WHOSE DIMENSION MUST BE GREATER THAN OR EQUAL
C                  TO M, CONTAINING ON RETURN INDICATIONS ABOUT THE WAY
C                  THE EIGENVALUES APPEARED (SEE MATH. DESCRIPTION)
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY A
C                  IN THE CALLING PROGRAM WHEN THE MATRIX IS IN DOUBLE
C                  SUBSCRIPTED DATA STORAGE MODE.
C                  IA=M WHEN THE MATRIX IS IN SSP VECTOR STORAGE MODE.
C
C        REMARKS
C           THE ORIGINAL MATRIX IS DESTROYED
C           THE DIMENSION OF RR AND RI MUST BE GREATER OR EQUAL TO M
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
```

```
C             NONE
C
C         METHOD
C             QR DOUBLE ITERATION
C
C         REFERENCES
C             J.G.F. FRANCIS - THE QR TRANSFORMATION---THE COMPUTER
C             JOURNAL, VOL. 4, NO. 3, OCTOBER 1961, VOL. 4, NO. 4, JANUARY
C             1962.  J. H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -
C             CLARENDON PRESS, OXFORD, 1965.
C
C     ..................................................................
C
      SUBROUTINE ATEIG(M,A,RR,RI,IANA,IA)
      DIMENSION A(1),RR(1),RI(1),PRR(2),PRI(2),IANA(1)
      INTEGER P,P1,Q
C
      E7=1.0E-8
      E6=1.0E-6
      E10=1.0E-10
      DELTA=0.5
      MAXIT=30
C
C        INITIALIZATION
C
      N=M
   20 N1=N-1
      IN=N1*IA
      NN=IN+N
      IF(N1) 30,1300,30
   30 NP=N+1
C
C        ITERATION COUNTER
C
      IT=0
C
C        ROOTS OF THE 2ND ORDER MAIN SUBMATRIX AT THE PREVIOUS
C        ITERATION
C
```

```
      DO 40 I=1,2
      PRR(I)=0.0
   40 PRI(I)=0.0
C
C        LAST TWO SUBDIAGONAL ELEMENTS AT THE PREVIOUS ITERATION
C
      PAN=0.0
      PAN1=0.0
C
C        ORIGIN SHIFT
C
      R=0.0
      S=0.0
C
C        ROOTS OF THE LOWER MAIN 2 BY 2 SUBMATRIX
C
      N2=N1-1
      IN1=IN-IA
      NN1=IN1+N
      N1N=IN+N1
      N1N1=IN1+N1
   60 T=A(N1N1)-A(NN)
      U=T*T
      V=4.0*A(N1N)*A(NN1)
      IF(ABS(V)-U*E7) 100,100,65
   65 T=U+V
      IF(ABS(T)-AMAX1(U,ABS(V))*E6) 67,67,68
   67 T=0.0
   68 U=(A(N1N1)+A(NN))/2.0
      V=SQRT(ABS(T))/2.0
      IF(T)140,70,70
   70 IF(U) 80,75,75
   75 RR(N1)=U+V
      RR(N)=U-V
      GO TO 130
   80 RR(N1)=U-V
      RR(N)=U+V
      GO TO 130
  100 IF(T)120,110,110
```

```
  110 RR(N1)=A(N1N1)
      RR(N)=A(NN)
      GO TO 130
  120 RR(N1)=A(NN)
      RR(N)=A(N1N1)
  130 RI(N)=0.0
      RI(N1)=0.0
      GO TO 160
  140 RR(N1)=U
      RR(N)=U
      RI(N1)=V
      RI(N)=-V
  160 IF(N2)1280,1280,180
C
C        TESTS OF CONVERGENCE
C
  180 N1N2=N1N1-IA
      RMOD=RR(N1)*RR(N1)+RI(N1)*RI(N1)
      EPS=E10*SQRT(RMOD)
      IF(ABS(A(N1N2))-EPS)1280,1280,240
  240 IF(ABS(A(NN1))-E10*ABS(A(NN))) 1300,1300,250
  250 IF(ABS(PAN1-A(N1N2))-ABS(A(N1N2))*E6) 1240,1240,260
  260 IF(ABS(PAN-A(NN1))-ABS(A(NN1))*E6)1240,1240,300
  300 IF(IT-MAXIT) 320,1240,1240
C
C        COMPUTE THE SHIFT
C
  320 J=1
      DO 360 I=1,2
      K=NP-I
      IF(ABS(RR(K)-PRR(I))+ABS(RI(K)-PRI(I))-DELTA*(ABS(RR(K))
     1    +ABS(RI(K)))) 340,360,360
  340 J=J+I
  360 CONTINUE
      GO TO (440,460,460,480),J
  440 R=0.0
      S=0.0
      GO TO 500
  460 J=N+2-J
```

```
      R=RR(J)*RR(J)
      S=RR(J)+RR(J)
      GO TO 500
  480 R=RR(N)*RR(N1)-RI(N)*RI(N1)
      S=RR(N)+RR(N1)
C
C        SAVE THE LAST TWO SUBDIAGONAL TERMS AND THE ROOTS OF THE
C        SUBMATRIX BEFORE ITERATION
C
  500 PAN=A(NN1)
      PAN1=A(N1N2)
      DO 520 I=1,2
      K=NP-I
      PRR(I)=RR(K)
  520 PRI(I)=RI(K)
C
C        SEARCH FOR A PARTITION OF THE MATRIX, DEFINED BY P AND Q
C
      P=N2
      IF (N-3)600,600,525
  525 IPI=N1N2
      DO 580 J=2,N2
      IPI=IPI-IA-1
      IF(ABS(A(IPI))-EPS) 600,600,530
  530 IPIP=IPI+IA
      IPIP2=IPIP+IA
      D=A(IPIP)*(A(IPIP)-S)+A(IPIP2)*A(IPIP+1)+R
      IF(D)540,560,540
  540 IF(ABS(A(IPI)*A(IPIP+1))*(ABS(A(IPIP)+A(IPIP2+1)-S)+ABS(A(IPIP2+2)
     1 )) -ABS(D)*EPS) 620,620,560
  560 P=N1-J
  580 CONTINUE
  600 Q=P
      GO TO 680
  620 P1=P-1
      Q=P1
      IF (P1-1) 680,680,650
  650 DO 660 I=2, P1
      IPI=IPI-IA-1
```

```
      IF(ABS(A(IPI))-EPS)680,680,660
  660 Q=Q-1
C
C        QR DOUBLE ITERATION
C
  680 II=(P-1)*IA+P
      DO 1220 I=P,N1
      II1=II-IA
      IIP=II+IA
      IF(I-P)720,700,720
  700 IPI=II+1
      IPIP=IIP+1
C
C        INITIALIZATION OF THE TRANSFORMATION
C
      G1=A(II)*(A(II)-S)+A(IIP)*A(IPI)+R
      G2=A(IPI)*(A(IPIP)+A(II)-S)
      G3=A(IPI)*A(IPIP+1)
      A(IPI+1)=0.0
      GO TO 780
  720 G1=A(II1)
      G2=A(II1+1)
      IF(I-N2)740,740,760
  740 G3=A(II1+2)
      GO TO 780
  760 G3=0.0
  780 CAP=SQRT(G1*G1+G2*G2+G3*G3)
      IF(CAP)800,860,800
  800 IF(G1)820,840,840
  820 CAP=-CAP
  840 T=G1+CAP
      PSI1=G2/T
      PSI2=G3/T
      ALPHA=2.0/(1.0+PSI1*PSI1+PSI2*PSI2)
      GO TO 880
  860 ALPHA=2.0
      PSI1=0.0
      PSI2=0.0
  880 IF(I-Q)900,960,900
```

```
  900 IF(I-P)920,940,920
  920 A(II1)=-CAP
      GO TO 960
  940 A(II1)=-A(II1)
C
C        ROW OPERATION
C
  960 IJ=II
      DO 1040 J=I,N
      T=PSI1*A(IJ+1)
      IF(I-N1)980,1000,1000
  980 IP2J=IJ+2
      T=T+PSI2*A(IP2J)
 1000 ETA=ALPHA*(T+A(IJ))
      A(IJ)=A(IJ)-ETA
      A(IJ+1)=A(IJ+1)-PSI1*ETA
      IF(I-N1)1020,1040,1040
 1020 A(IP2J)=A(IP2J)-PSI2*ETA
 1040 IJ=IJ+IA
C
C        COLUMN OPERATION
C
      IF(I-N1)1080,1060,1060
 1060 K=N
      GO TO 1100
 1080 K=I+2
 1100 IP=IIP-I
      DO 1180 J=Q,K
      JIP=IP+J
      JI=JIP-IA
      T=PSI1*A(JIP)
      IF(I-N1)1120,1140,1140
 1120 JIP2=JIP+IA
      T=T+PSI2*A(JIP2)
 1140 ETA=ALPHA*(T+A(JI))
      A(JI)=A(JI)-ETA
      A(JIP)=A(JIP)-ETA*PSI1
      IF(I-N1)1160,1180,1180
 1160 A(JIP2)=A(JIP2)-ETA*PSI2
```

```
 1180 CONTINUE
      IF(I-N2)1200,1220,1220
 1200 JI=II+3
      JIP=JI+IA
      JIP2=JIP+IA
      ETA=ALPHA*PSI2*A(JIP2)
      A(JI)=-ETA
      A(JIP)=-ETA*PSI1
      A(JIP2)=A(JIP2)-ETA*PSI2
 1220 II=IIP+1
      IT=IT+1
      GO TO 60
C
C         END OF ITERATION
C
 1240 IF(ABS(A(NN1))-ABS(A(N1N2))) 1300,1280,1280
C
C         TWO EIGENVALUES HAVE BEEN FOUND
C
 1280 IANA(N)=0
      IANA(N1)=2
      N=N2
      IF(N2)1400,1400,20
C
C         ONE EIGENVALUE HAS BEEN FOUND
C
 1300 RR(N)=A(NN)
      RI(N)=0.0
      IANA(N)=1
      IF(N1)1400,1400,1320
 1320 N=N1
      GO TO 20
 1400 RETURN
      END
```

APPENDIX B

Fortran Coding for Hooke and Jeeves Direct Search Method.

```
      SUBROUTINE OPTMIN(PSI,SSI,N,DEL,DLMIN,LIMIT,FUNCT)
      DIMENSION PSI(1),PHI(25),THT(25),EPS(25)
      ITNUM=0
      ALFA=1.02
C     EVALUATE AT INITIAL BASEPOINT.
      CALL FUNCT(PSI,SSI)
C     START AT BASEPOINT.
 100  S=SSI
      DO 101 I=1,N
 101  PHI(I)=PSI(I)
      ICALL=1
      IF(ITNUM.GT.LIMIT) WRITE(108,2000);RETURN
C     MAKE EXPLORATORY MOVES.
      GO TO 150
C     IS PRESENT VALUE LESS THAN BASEPOINT VALUE?
 160  IF(S.LT..9999*SSI) GO TO 200
      GO TO 300
C     SET NEW BASEPOINT.
200   SSI=S
      DO 201 I=1,N
      THT(I)=PSI(I)
      PSI(I)=PHI(I)
C     MAKE PATTERN MOVES.
201   PHI(I)=PHI(I)+ALFA*(PHI(I)-THT(I))
      CALL FUNCT(PHI,SPI)
      S=SPI
      ICALL=2
C     MAKE EXPLORATORY MOVES.
      IF(ITNUM.GT.LIMIT) WRITE(108,2000) LIMIT;RETURN
      GO TO 150
C     IS PRESENT VALUE LESS THAN BASEPOINT VALUE?
260   IF(S.LT..9999*SSI) GO TO 200
      GO TO 100
300   IF(DEL.LT.DLMIN) RETURN
```

```
      DEL=DEL/2.
      GO TO 100
C     MAKE EXPLORATORY MOVES.
 150  DO 180 K=1,N
      EPS(K)=.05*PHI(K)
      IF(EPS(K).EQ.0.) EPS(K)=.05
      PHI(K)=PHI(K)+EPS(K)*DEL
      CALL FUNCT(PHI,SPI)
      IF(SPI.LT.S) GO TO 179
      PHI(K)=PHI(K)-2.*EPS(K)*DEL
      CALL FUNCT(PHI,SPI)
      IF(SPI.LT.S) GO TO 179
      PHI(K)=PHI(K)+EPS(K)*DEL
      GO TO 180
 179  S=SI(
 180  CONT.NUE
      ITNUM=ITNUM+1
      GO TO (160,260),ICALL
2000  FORMAT(//// ****VALUES OF X(I) DO NOT CONVERGE IN ',I6,'  ITERATIO
     1NS****'///)
      END
```

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