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A CRACKED SHEET STIFFENED BY SEVERAL PARTIALLY DEBONDED INTACT OR BROKEN STRINGERS

BY

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A CRACKED SHEET STIFFENED BY SEVERAL PARTIALLY DEBONDED INTACT OR BROKEN STRINGERS

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ABSTRACT

The effect of several stringers on the stress intensity factors at the tips of a crack is considered. The stringers which are continuously attached to the plate and placed perpendicular to the crack may be partially debonded due to high stress concentrations. Since the stringers may even break under excessive loading conditions, both intact and broken stringers are considered to investigate the effect of rupture. The continuity of displacements along the bond lines leads to an integral equation which is solved to give the shear stress distribution in the adhesive and the stress intensity factors at the crack tips.

Introduction

Stiffened panels, i.e. metal sheets with stringers continuously bonded through an adhesive have long been of major interest [1], [5]. Greif and Sanders have given the solution of a stringer perfectly bonded to a cracked sheet [1]. On the other hand, the case with riveted stringers - both intact and broken - has been treated by Poe [2] and [3]. Furthermore, the problem of a cracked isotropic plate stiffened by a stringer which may be partially debonded has been considered by Arin [4]. It has been concluded that the debonding process as well as the stringer placing are quite important as far as the stiffening effect of the stringer is concerned. In a separate work [5] the effect of lateral bending stiffness of the stringer has been investigated.

However, due to high load levels stringer breakage can occur in addition to debonding. Also, in actual structures several stringers are present instead of one. Hence the problem of several intact or broken stringers will be considered in this paper. The method employed here is the same as the one used in [4] and therefore most of the results will be used without derivation.

For intact stringers the case where the loads are applied on the crack surfaces will be considered (see Fig. 1). This will also give the singular part of the solution around the crack tips. The actual problem where the loads are applied at infinity can be obtained by a simple superposition. However, for broken stringers the actual problem will be treated as it is (see Fig. 2) due to difficulties involved in superposition. In all these cases loads will be considered uniform.

The technique used here makes it possible to consider any number of stringers located at arbitrary locations. For the sake of simplicity the numerical results will be given for uniformly spaced stringers all located along the positive x axis. However, the results for the stringers located along the negative x axis can be obtained by simply substituting for d_0 - the distance of the first stringer to the y axis - its value with a negative sign.

Formulation of the Problem

The problem will be formulated using the same notation* as in [4]. Also, due to symmetry only the upper half of the plate will be considered. The adhesive will be treated as a shear spring and the shear stresses will be considered as body forces in the plate solution (generalized plane stress). Let q represent the uniform pressure applied on the crack surfaces in the case of intact stringers (see Fig. 1) and the uniform tension applied at infinity in the case of broken stringers (see Fig. 2). Then the continuity of displacements can be written as [4]

$$v_p(z) - v_s(z) = \frac{h_a}{d_s \mu_a} P(z) \quad , \quad z \text{ on } L \tag{1}$$

Here, L denotes the union of straight lines L_j defined by $x = c_j$, $b_j \leq y < \infty$; $j = 1,\ldots n_s$ where n_s is the number of stringers, b_j is the half debond length of the j th stringer. For uniformly spaced stringers we have

$$c_j = d_o + (j-1)d_1 \tag{2}$$

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* (E,ν): Elastic constants of the plate. $\mu_p = E/2(1+\nu)$ $\kappa = (3-\nu)/(1+\nu)$ for generalized plane stress.
(E_s,A_s): Elastic modulus and cross-sectional area of the stringer.
μ_a: Shear modulus of the adhesive.
(h_p,h_a): Thicknesses of the plate and the adhesive.
a: Half crack length
d_s: Stringer width
$v_p(z)v_s(z)$: Displacements of the plate and the corresponding stringer at z location.
$P(z)$: Shear stress in the adhesive at z location.

where d_1 is the stringer spacing and d_0 is the distance of the first stringer to the mid-point of the crack.

The displacements can be expressed as [4]

$$v_p(z) = qk_0(z) + \int_L k_p(z,z_0)P(z_0)dy_0$$

$$v_s(z) = \int_L k_s(z,z_0)P(z_0)dy_0 + C, \; z \text{ on } L$$

$$y_0 = \mathrm{Im}(z_0) \qquad (3)$$

Note that C represents the rigid body displacement and assumes a different constant value on each stringer. Hence if z is on L_j then $C = C_j$, $j = 1,\ldots n_s$.

Kernels in (3) are given as follows:

<u>$k_0(z)$:</u> For either intact or broken stringers.

$$k_0(z)=\begin{cases} k^*(z), & \text{for crack surface loading} \\ k^*(z) + \frac{1+\kappa}{8\mu_p}\, y, & \text{for loading at infinity} \end{cases}$$

$$y = \mathrm{Im}(z) \qquad (4)$$

and

$$k^*(z) = \frac{1}{4\mu_p}\left[(\kappa+1)\,\mathrm{Im}\sqrt{z^2-a^2} + (1-\kappa)y - 2y\,\mathrm{Re}\,\frac{z}{\sqrt{z^2-a^2}}\right] \qquad (5)$$

$\underline{k_p(z,z_o)}$: From [4]

$$k_p(z,z_o) = \frac{1}{4\pi\mu_p h_p(1+\kappa)}[\kappa[\log(z-z_o) + \log(\bar{z}-\bar{z}_o)]$$

$$- \kappa[\log(z-\bar{z}_o) + \log(\bar{z}-z_o)]$$

$$- \mathrm{Re}\{(\frac{1+\kappa}{2})[\theta_1(\bar{z},\bar{z}_o) - \theta_1(\bar{z},z_o) + \kappa\theta_1(z,z_o)$$

$$- \kappa\theta_1(z,\bar{z}_o)]$$

$$+ (1+\kappa)[\theta_4(z,z_o) - \theta_4(z,\bar{z}_o)] + \kappa[\theta_3(z,\bar{z}_o)$$

$$- \theta_3(z,z_o)]$$

$$+ \theta_3(\bar{z},z_o) - \theta_3(\bar{z},\bar{z}_o) + (\frac{z_o-\bar{z}_o}{\bar{z}-z_o})\theta_5(z,z_o)$$

$$- (\frac{\bar{z}_o-z_o}{\bar{z}-\bar{z}_o})\theta_5(z,\bar{z}_o)\}] \tag{6}$$

where

$$\theta_1(z,z_o) = \log\left[zz_o - a^2 + \sqrt{z_o^2-a^2}\sqrt{z^2-a^2}\right]$$

$$\theta_2(z) = \log[z + \sqrt{z^2-a^2}]$$

$$\theta_3(z,z_o) = \frac{1}{2}\left(\frac{\bar{z}_o-z_o}{z-\bar{z}_o}\right)\left[1 + \frac{\sqrt{z^2-a^2}\sqrt{\bar{z}_o^2-a^2}}{a^2-\bar{z}_o^2}\right]$$

$$\theta_4(z,z_o) = \frac{1}{2}\left(\frac{z-\bar{z}}{\bar{z}-\bar{z}_o}\right)\left[\frac{I(\bar{z}) - I(\bar{z}_o)}{\sqrt{\bar{z}^2-a^2}}\right]$$

$$\theta_5(z,z_o) = \theta_4(z,\bar{z}_o) - \frac{1}{2}\frac{(z-\bar{z})}{\sqrt{\bar{z}^2-a^2}}J(z_o)$$

$$I(z_0) = \sqrt{z_0^2 - a^2} - z_0$$

$$J(z_0) = \frac{z_0}{\sqrt{z_0^2 - a^2}} - 1 \tag{7}$$

$\underline{k_s(z,z_0)}$: Defining $y = \mathrm{Im}(z)$, $y_0 = \mathrm{Im}(z_0)$ for intact stringers

$$k_s(z,z_0) = \begin{cases} \left.\begin{array}{ll} \dfrac{y}{A_s E_s}, & y < y_0 \\[2ex] \dfrac{y_0}{A_s E_s}, & y > y_0 \end{array}\right\} & \text{if } z \text{ and } z_0 \text{ are on the same stringer} \\[2ex] 0 & \text{otherwise} \end{cases} \tag{8}$$

and $C \equiv 0$. Also note that due to symmetry the equilibrium conditions for stringers are automatically satisfied.

For broken stringers, from the solution of a one dimensional elastic body we obtain

$$k_s(z,z_0) = \begin{cases} \left.\begin{array}{ll} 0, & y < y_0 \\[2ex] \dfrac{y_0 - y}{A_s E_s}, & y > y_0 \end{array}\right\} & \text{if } z \text{ and } z_0 \text{ are on the same stringer} \\[2ex] 0 & \text{otherwise} \end{cases} \tag{9}$$

and C_j, $j = 1,\ldots n_s$ are unknown constants. For these n_s additional unknowns we consider the following n_s equilibrium equations for the stringers to obtain a compatible system.

$$\int_{L_j} P(z_0)\,dy_0 = \begin{cases} -\dfrac{1+\kappa}{8\mu_p} E_s A_s q & \text{if the load at infinity is also transferred to the stringer} \\ 0 & \text{if the end of the stringer at infinity is stress free} \end{cases}$$

$$j = 1, \ldots n_s \tag{10}$$

Hence from (1) and (3) the integral equation of the problem can be obtained as

$$P(z) + \int_L k(z,z_0)P(z_0)\,dy_0 + \frac{d_s\mu_a}{h_a} C = \frac{d_s\mu_a}{h_a} q\, k_0(z), \quad z \text{ on } L \tag{11}$$

which will be considered together with equations (10) and solved for the shear stress distribution. Note that C in (11) actually represents n_s unknowns.

Here

$$k(z,z_0) = \frac{d_s\mu_a}{h_a}[k_s(z,z_0) - k_p(z,z_0)] \tag{12}$$

The stress intensity factor will be defined as

$$K = \lim_{x\to a} [\sqrt{2(x-a)}]\sigma_y(x,0) \tag{13}$$

and given as [4]

$$\frac{K}{\sqrt{a}} = q + \frac{2}{a_0}\int_L \alpha(z_0)P(z_0)\,dy_0 \quad , \quad y_0 = \mathrm{Im}(z_0) \tag{14}$$

where

$$\alpha(z_o) = \frac{1}{2\pi h_p(1+\kappa)} \mathrm{Im}\{\left(\frac{z_o-\bar{z}_o}{a_o-z_o}\right) J(z_o)$$

$$- \left(\frac{a_o+I(z_o)}{a_o-z_o}\right) \left[1 + \kappa + \left(\frac{\bar{z}_o-z_o}{a_o-z_o}\right)\right]\} \qquad (15)$$

and

$$a_o = \begin{cases} a & \text{for right tip} \\ -a & \text{for left tip} \end{cases} \qquad (16)$$

Numerical Results and Conclusions

The numerical results are obtained for an aluminum plate of elastic constants $\nu = 0.30$, $E = 69.0$ GN/m^2 (10.0×10^6 psi) and thickness $h_p = 2.3$ mm (0.09 in.). The cross-sectional area of the stringer and the elastic modulus are assumed to be $A_s = 106$ mm^2 (0.165 in^2) and $E_s = 85.5$ GN/m^2 (12.4×10^6 psi) respectively. The adhesive is supposed to have a shear modulus of $\mu_a = 1.14$ GN/m^2 (0.165×10^6 psi) and thickness of $h_a = 0.1$ mm (0.004 in.). Results are given for both left and right crack tips. It is also assumed that q = constant.

Intact stringers: Fig. 4 shows the effect of debonding on the stress intensity factors. With the first stringer located at $d_o = 0.5a$, the stringers have little stiffening effect for $b_1/a > 2$ even if the other stringers are still perfectly bonded to the sheet. As expected [4], the stress intensity factor appears to be quite insensitive to the third stringer due to its location being rather far from the crack tip. Fig. 5 illustrates the way the stringer spacing influences the stress intensity factors. Namely more stringers located between the crack tips result in appreciably smaller stress intensity factors (compare Fig. 4 and Fig. 5). In this case, the third stringer being located away from the crack tip, again has no significant effect. One interesting result is that as long as there is at least one perfectly bonded stringer between the crack tips, the debonding process occurring in any other stringer will result in only a small drop in the stiffening effect but not a drastic one. Again note that by the time $b_1/a \approx 2$ most of the first stringer's stiffening effect is already diminished. The effect of b_2, the debond length of the second stringer is shown in Fig. 6, keeping b_1 and b_3 constant. For similar reasons the third stringer is unimportant as far as K factors are concerned. Note that $K/q\sqrt{a}$ values approximately approach to those of a single stringer with $b_1/a = 1.0$, $d_0/a = 0.5$ (compare Figures 4 and 6). $K/q\sqrt{a}$ vs. b_2/a variation appears to be similar to $K/q\sqrt{a}$ vs. b_1/a. In both cases a considerable loss of stiffening effect of the stringers is observed with the increasing b_1 or b_2. It is also possible to show the dependence on b_3 in a similar fashion. However if the third stringer is located away from the crack tip, K values will not be affected significantly. For example

$$\left.\begin{array}{l} K_{left} \cong 0.891,\ K_{right} \cong 0.801 \text{ for } d_0/a = 0.5 \\ \\ K_{left} \cong 0.945,\ K_{right} \cong 0.868 \text{ for } d_0/a = 1.0 \end{array}\right\} \begin{array}{l} n_s = 3,\ d_1/a = 1.0 \\ b_1/a = b_2/a = 1.0 \\ d_s/a = 0.2 \end{array}$$

for all b_3 values.

The effect of the location of the first stringer namely d_0 is shown in Figures 7 and 8. As expected, whatever the number of stringers, the spacings and the debond lengths are, $K/q\sqrt{a}$ values rapidly approach unity starting around $d_0/a \approx 2$. In all these curves it is apparent that the main factor for low $K/q\sqrt{a}$ values is to have as many perfectly bonded stringers as possible between the two crack tips. For one stringer lowest K occurs if the stringer is placed on the crack and approximately one fourth of the crack length away from the corresponding crack tip. But for two or more stringers this depends on the other parameters. However the fact that the lowest K values will be obtained by the maximum number of stringers critically placed on the crack still remains the same. That is why in most cases lowest K occurs for $d_0 \cong 0$. The dependence of K on the crack length is illustrated in Figures 9 and 10. For all the curves, the minimum K for the right tip occurs when $d_0/a \cong 0.30 - 0.50$, which is in agreement with similar conclusions drawn previously for a single stringer [4].

<u>Broken Stringers</u>: In the case of broken stringers the actual problem , i.e. with the loads applied at infinity will be considered directly due to the fact that superposition will

not simplify the solution. We will consider uniform loading at infinity, i.e. q = constant (see Fig. 2). However for the sake of comparison with the intact stringers the case where the loads are applied on the crack surfaces will also be considered to determine the extent to which the stiffening effect of the stringers is lost due to breakage. One immediate observation from Figures 11, 13 and 15 is that the stress intensity factors are considerably higher than those of the intact stringer results. This suggests that the loss of stiffening effect is quite significant. Other than this, similar trends will be observed for broken stringers. However the adverse effect of stringer breakage does not end here. As can be seen from Figures 12, 14 and 16 the stress intensity factors increase much further beyond unity thereby making a stiffened structure even more susceptible to fracture if the breakage occurs. This would be the same whether the loads applied at infinity are transmitted through the plate only or through the plate and the stringers simultaneously. High $K/q\sqrt{a}$ values can be attributed to the pulling effect of the broken stringers (for intact stringers pulling works to the advantage of the structure by reducing the stress intensity factors).

The results in Fig. 11 have similarity with Figures 4 and 5 and therefore can be interpreted in an identical manner. Fig. 12 also illustrates the $K/q\sqrt{a}$ dependence on b_1/a in the case of loads applied at infinity. The stress intensity factors for the right tip now become higher than those of the left tip. And as expected the curves corresponding to a single stringer tend to approach unity as b_1 increases indefinitely. The relation $K/q\sqrt{a}$ vs. d_0/a shown in Fig. 13 for the crack surface loading can also be explained as in Fig. 7. However $K/q\sqrt{a}$ vs. d_0/a illustrated in Fig. 14 indicates values significantly higher than unity for the loads applied at infinity. One important observation is that the K values will shoot up appreciably if one of the stringers is placed on or very close to one of the tips and debonding in that particular stringer is either very small or non-existent. This phenomenon is the opposite of the one observed in the case of intact stringers (see Fig. 7 and 8). It should also be noted that all $K/q\sqrt{a}$ values approach unity as d_0 increases. Fig. 15 illustrates similar findings for $K/q\sqrt{a}$ vs. half crack length as in Figs. 9 and 10 with the exception of higher values for $K/q\sqrt{a}$. Same relation is given in Fig. 16 for the loads applied at infinity. Finally, to give an idea as to how d_s affects the stress intensity factors Fig. 17 shows this relation for the parameters taken the same as in [4] for the sake of comparison. As seen from Fig. 17, $K/q\sqrt{a}$ does not change more than approximately 2%.

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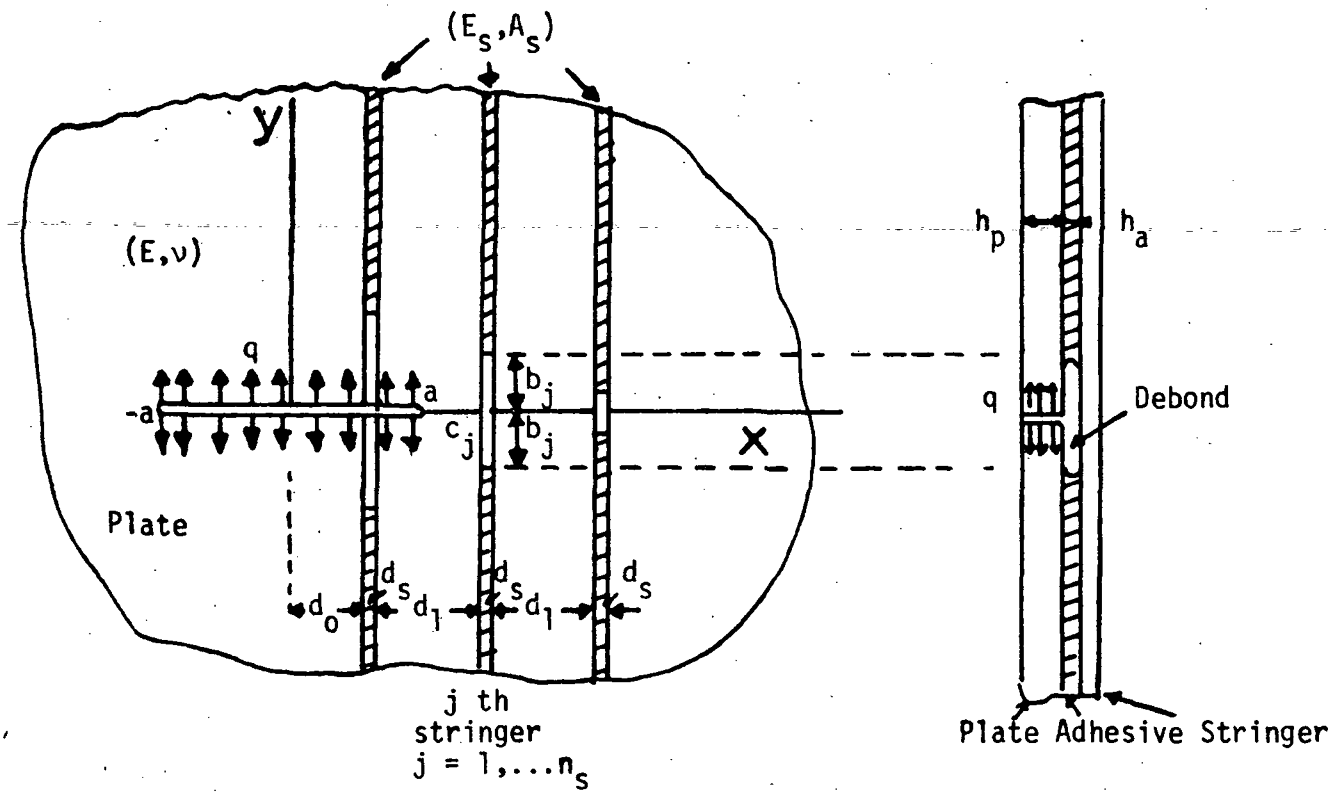


Fig. 1. Geometry of the problem (intact stringers)

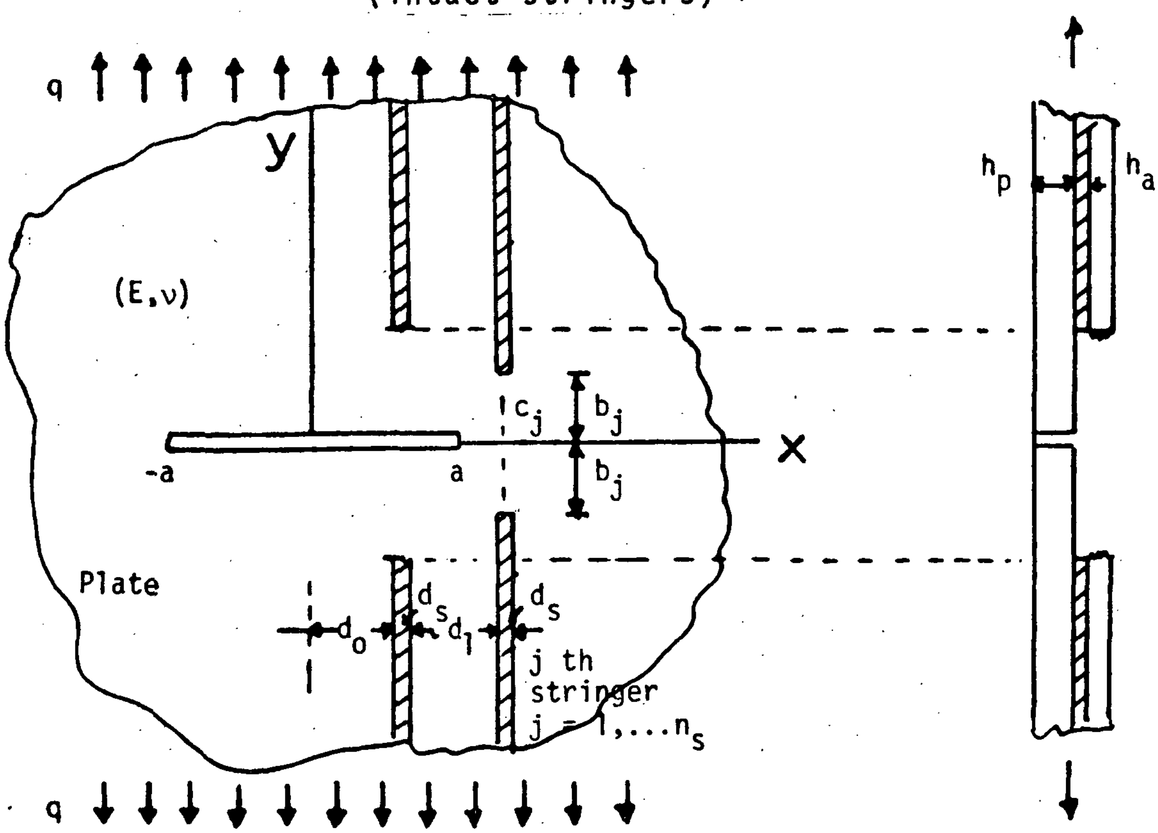
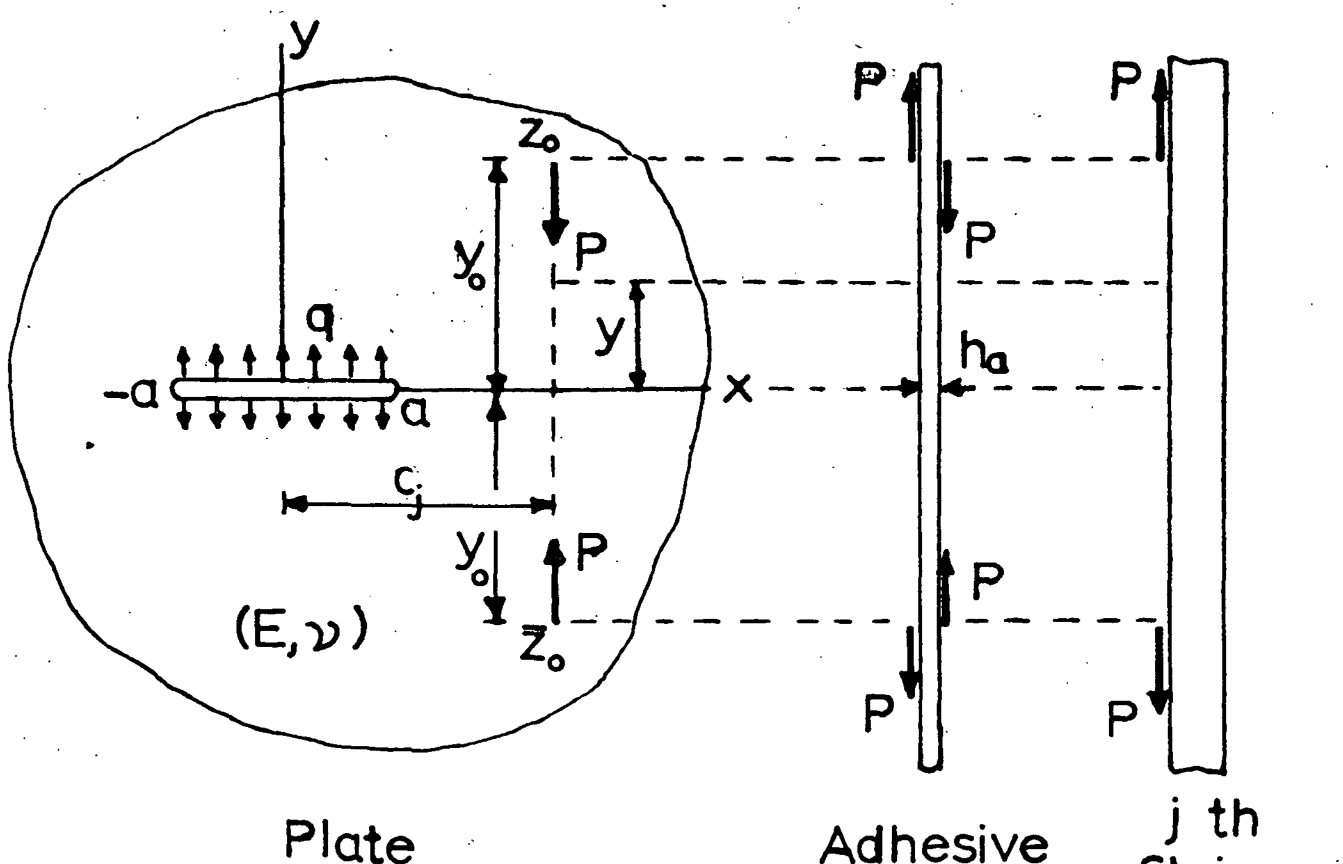


Fig. 2. Geometry of the problem (broken stringers)



$$z_o = c_j + iy_o$$

For uniformly spaced stringers: $c_j = d_o + (j-1)d_1 \quad , \quad j = 1, \ldots n_s$

Fig. 3. Free-body diagrams

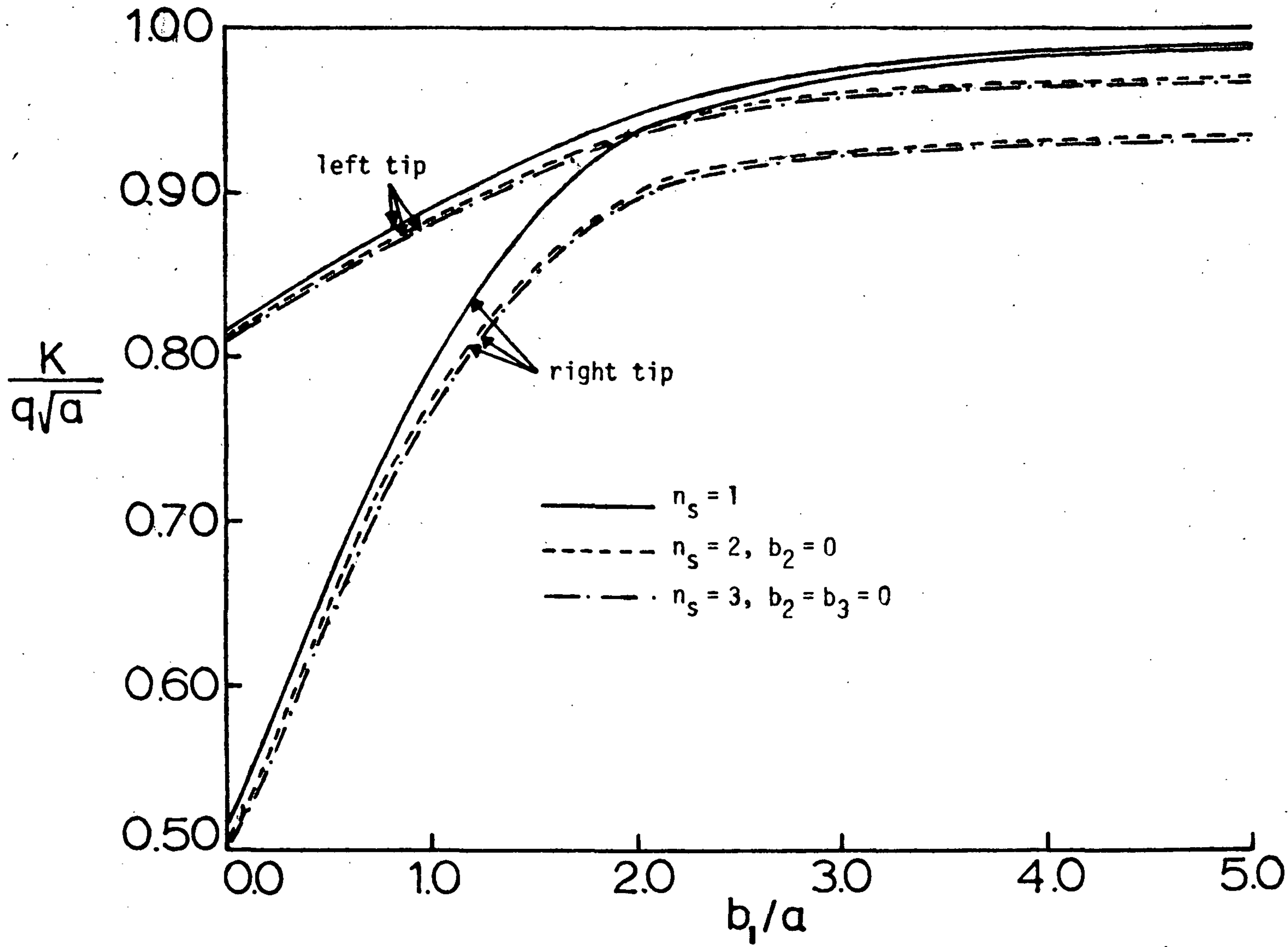


Fig. 4. $K/q\sqrt{a}$ vs. b_1/a
(intact stringer, $d_0/a = 0.5$, $d_1/a = 1.0$, $d_s/a = 0.2$)

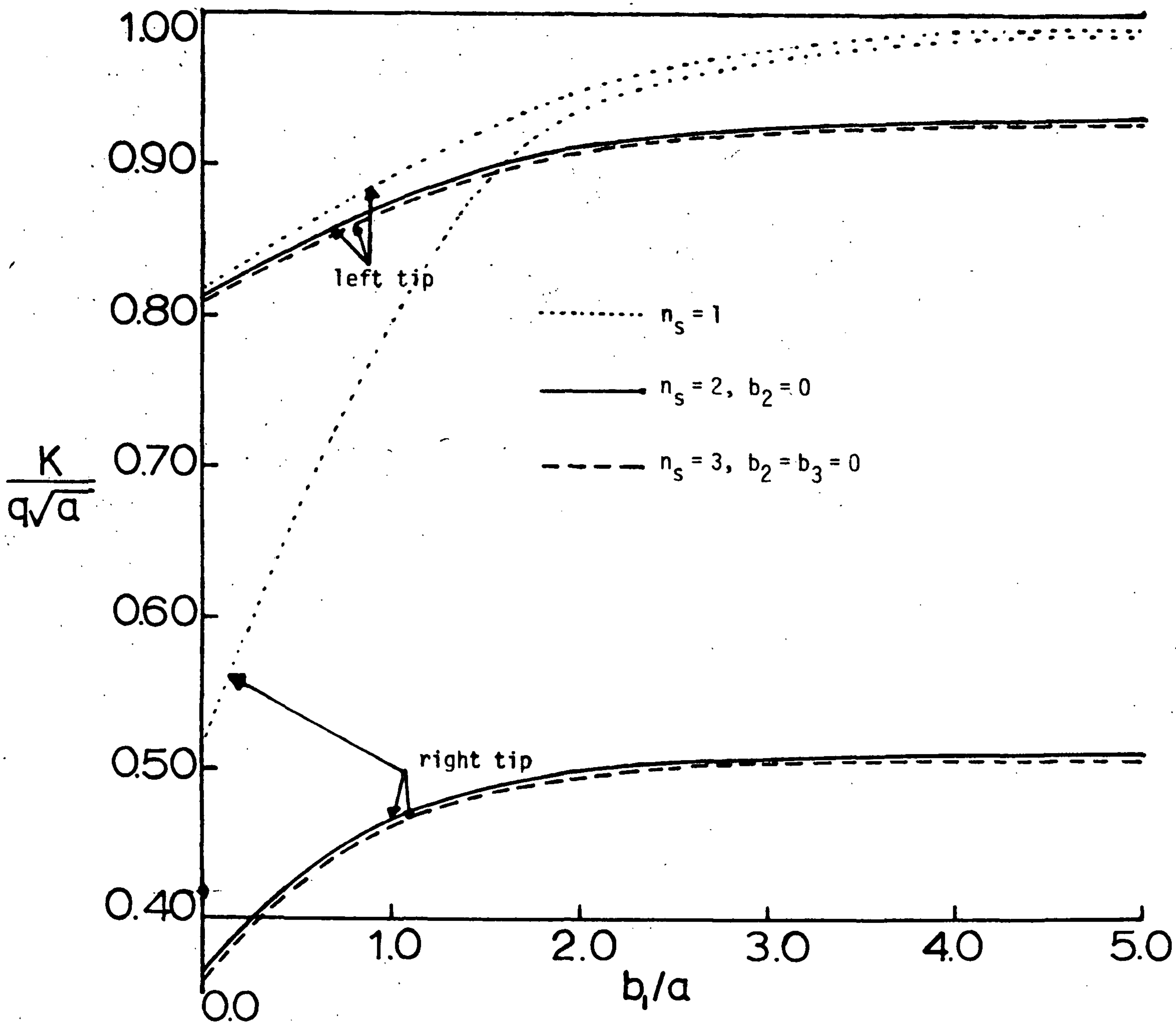


Fig. 5. $K/q\sqrt{a}$ vs. b_1/a
(intact stringer, $d_0/a = d_1/a = 0.5$, $d_s/a = 0.2$)

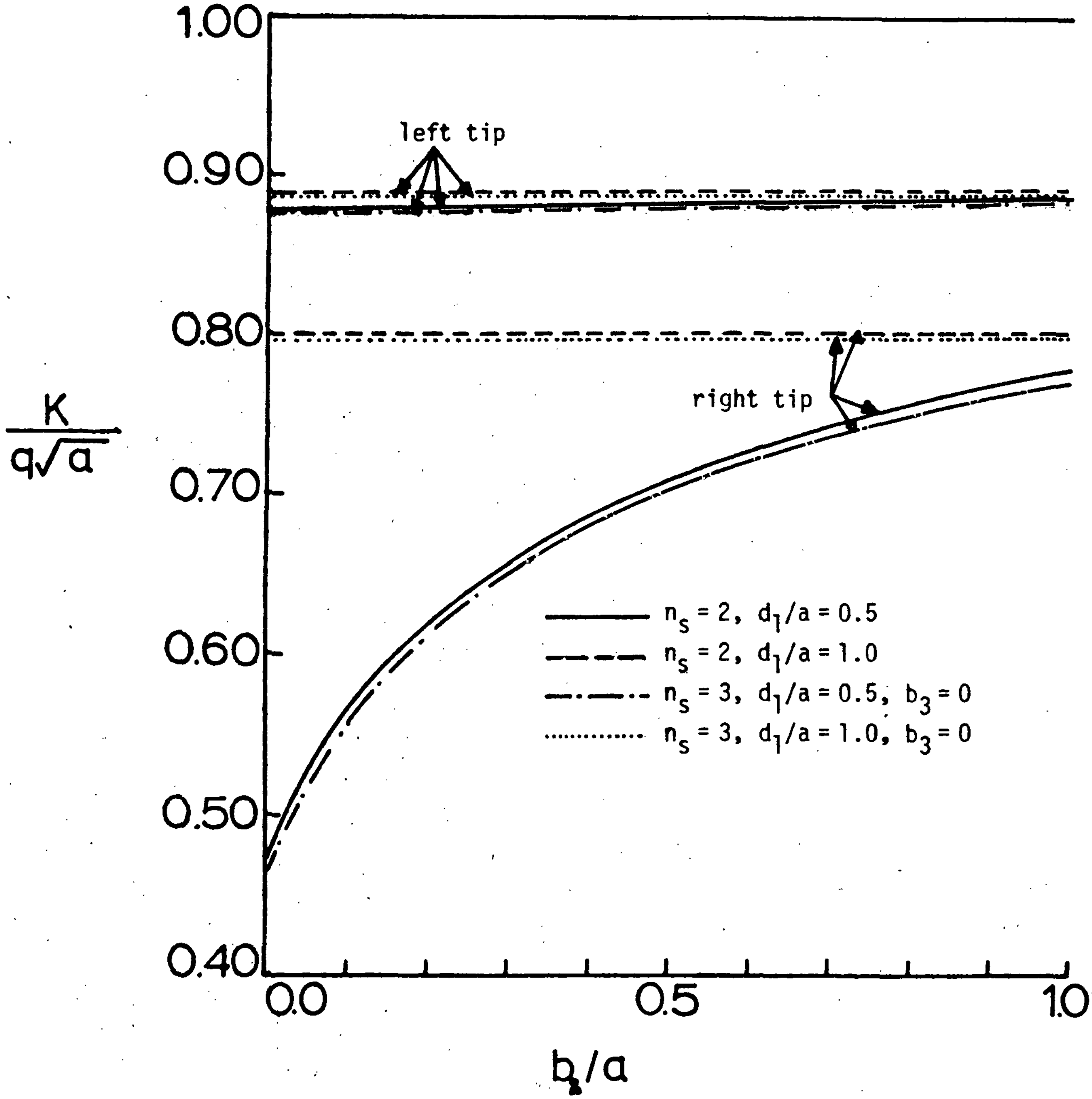


Fig. 6. $K/q\sqrt{a}$ vs. b_2/a
(intact stringer, $d_0/a = 0.5$, $b_1/a = 1.0$, $d_s/a = 0.2$)

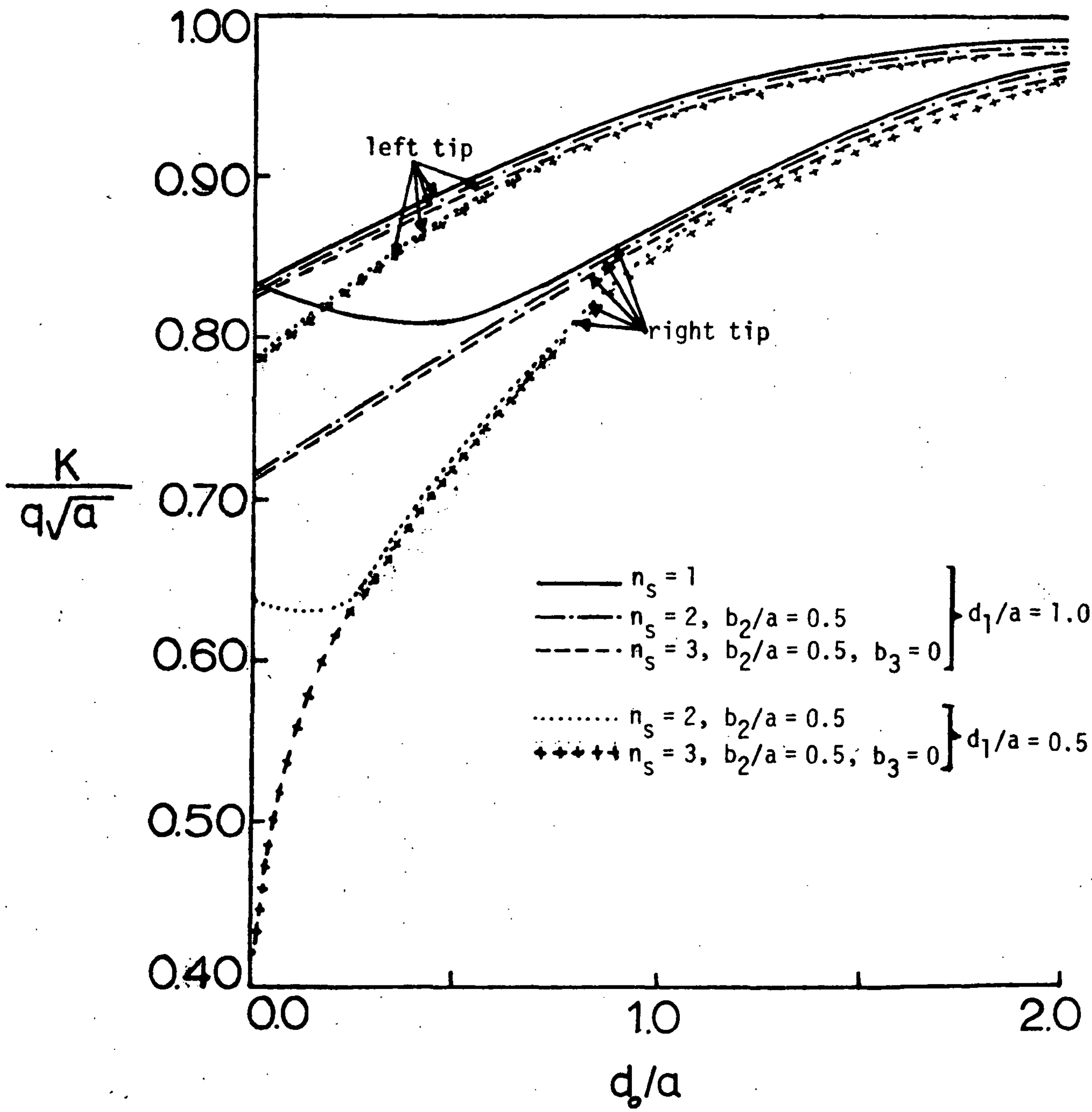


Fig. 7. $K/q\sqrt{a}$ vs. d_0/a
(intact stringer, $b_1/a = 1.0$, $d_s/a = 0.2$)

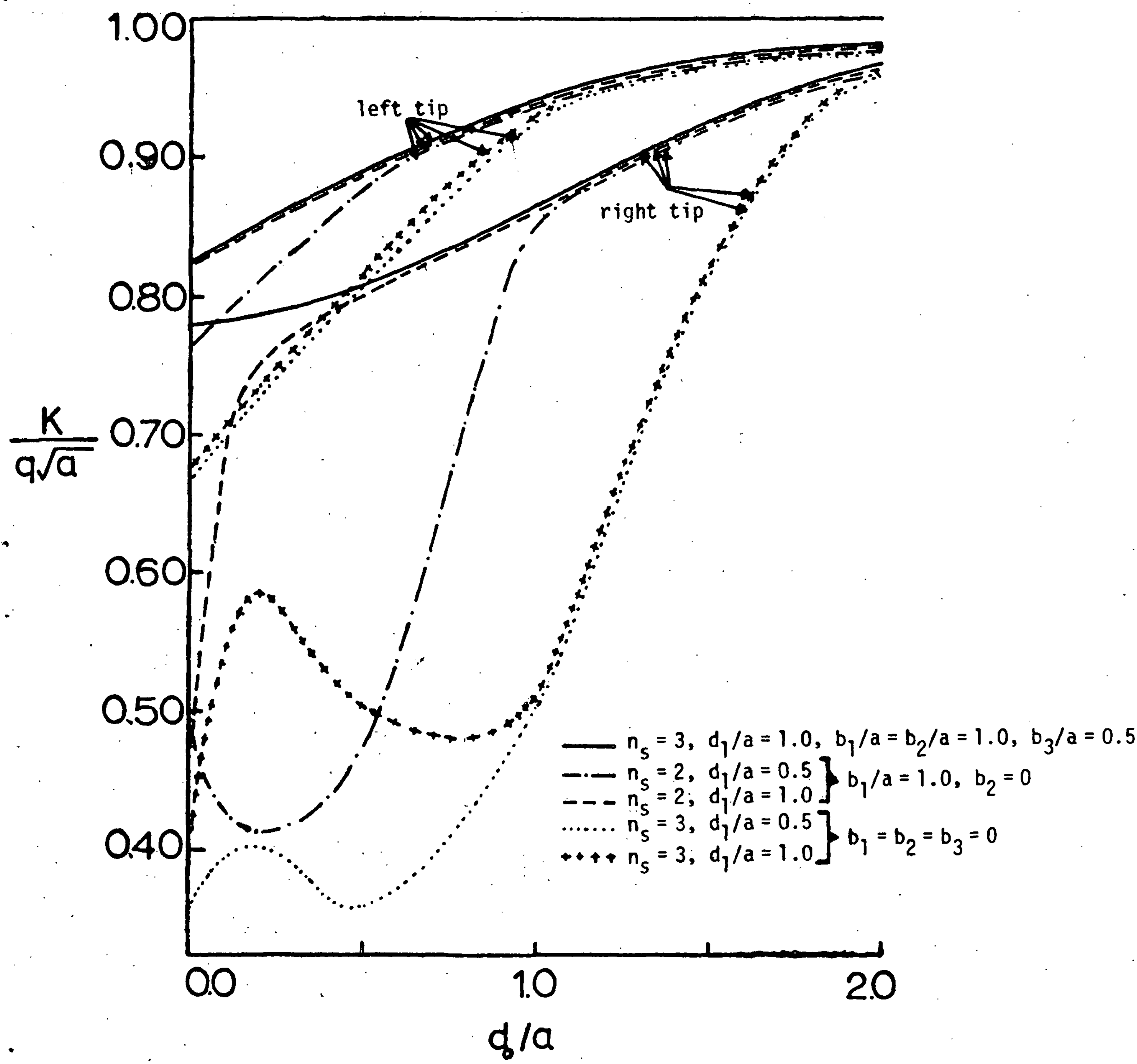


Fig. 8. $K/q\sqrt{a}$ vs. d_o/a
(intact stringer, $d_s/a = 0.2$)

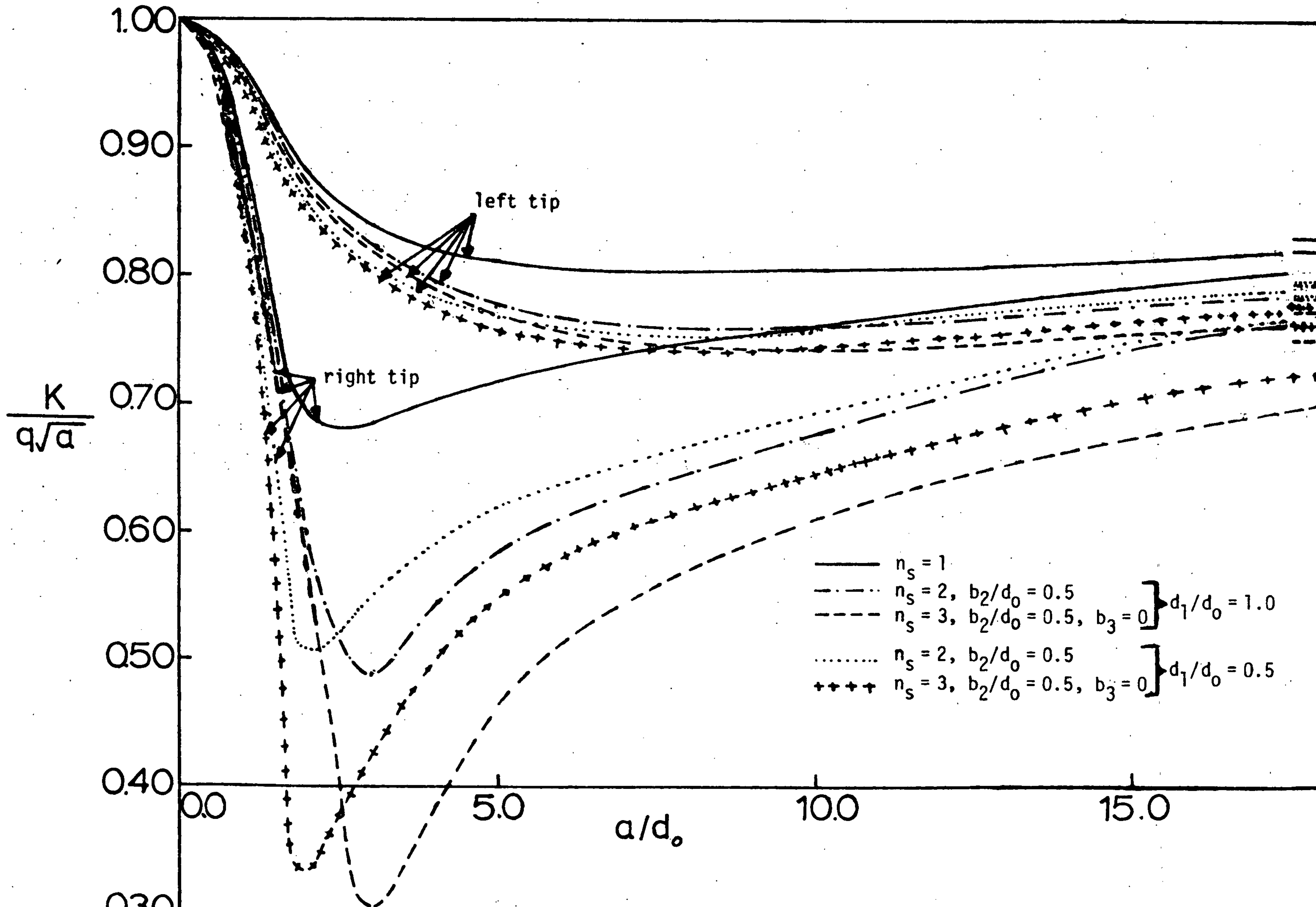


Fig. 9. $K/q\sqrt{a}$ vs. a/d_o
(intact stringer, $b_1/d_o = 1.0$, $d_s/d_o = 0.2$)

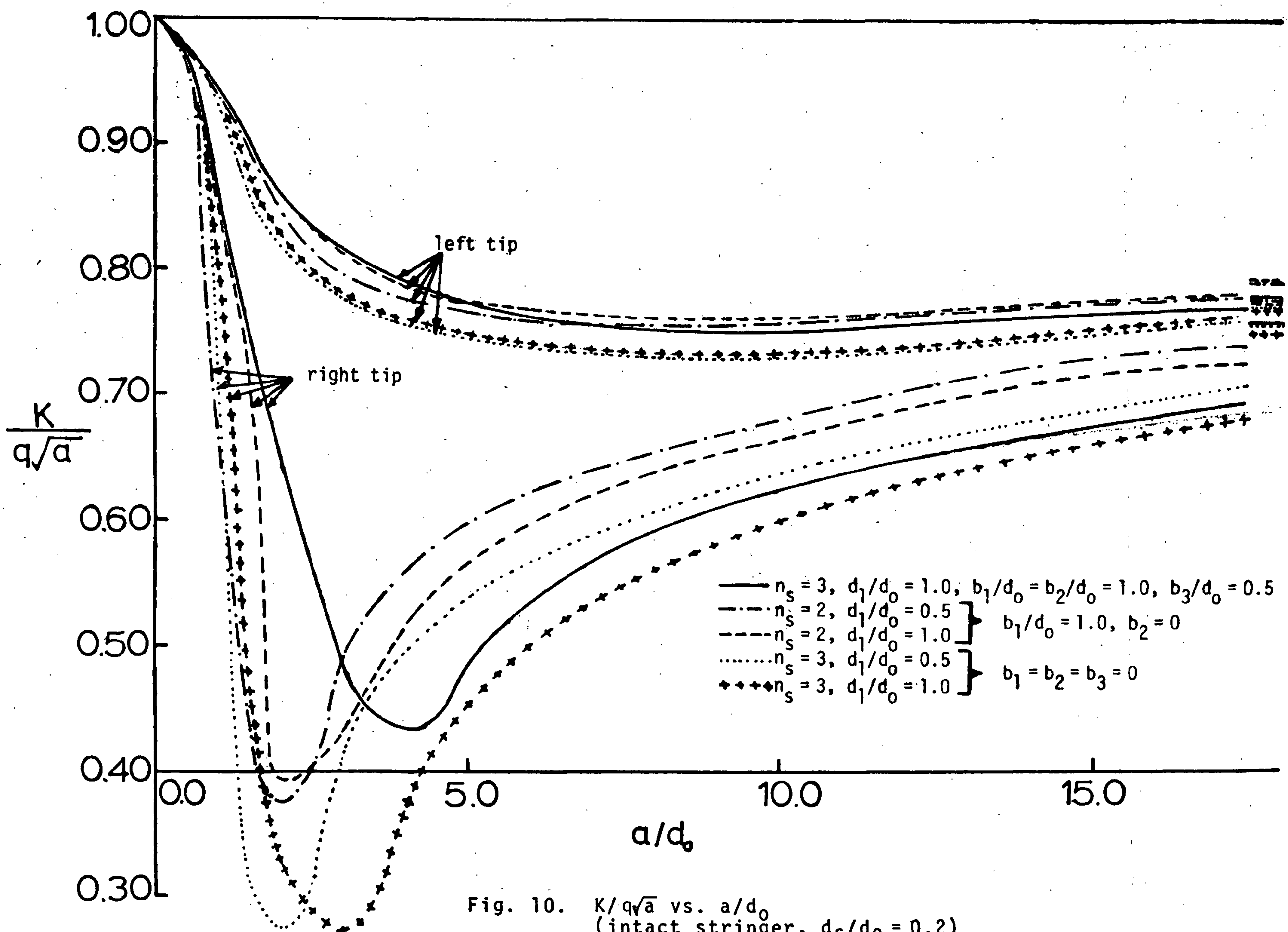


Fig. 10. $K/q\sqrt{a}$ vs. a/d_0
(intact stringer, $d_S/d_0 = 0.2$)

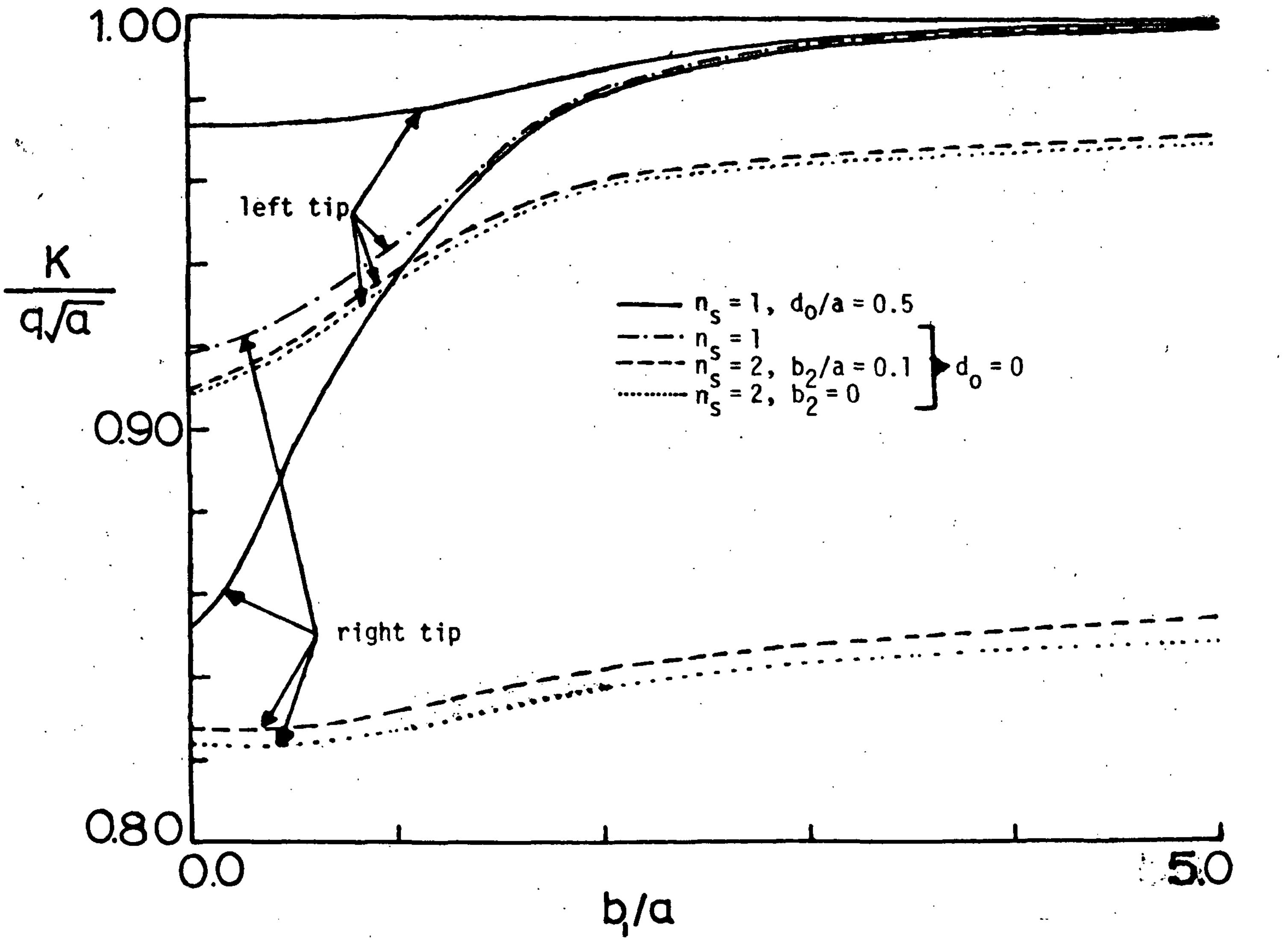


Fig. 11. $K/q\sqrt{a}$ vs. b_1/a
(broken stringer, loading on the crack surfaces, $d_1/a = 0.5$, $d_s/a = 0.2$)

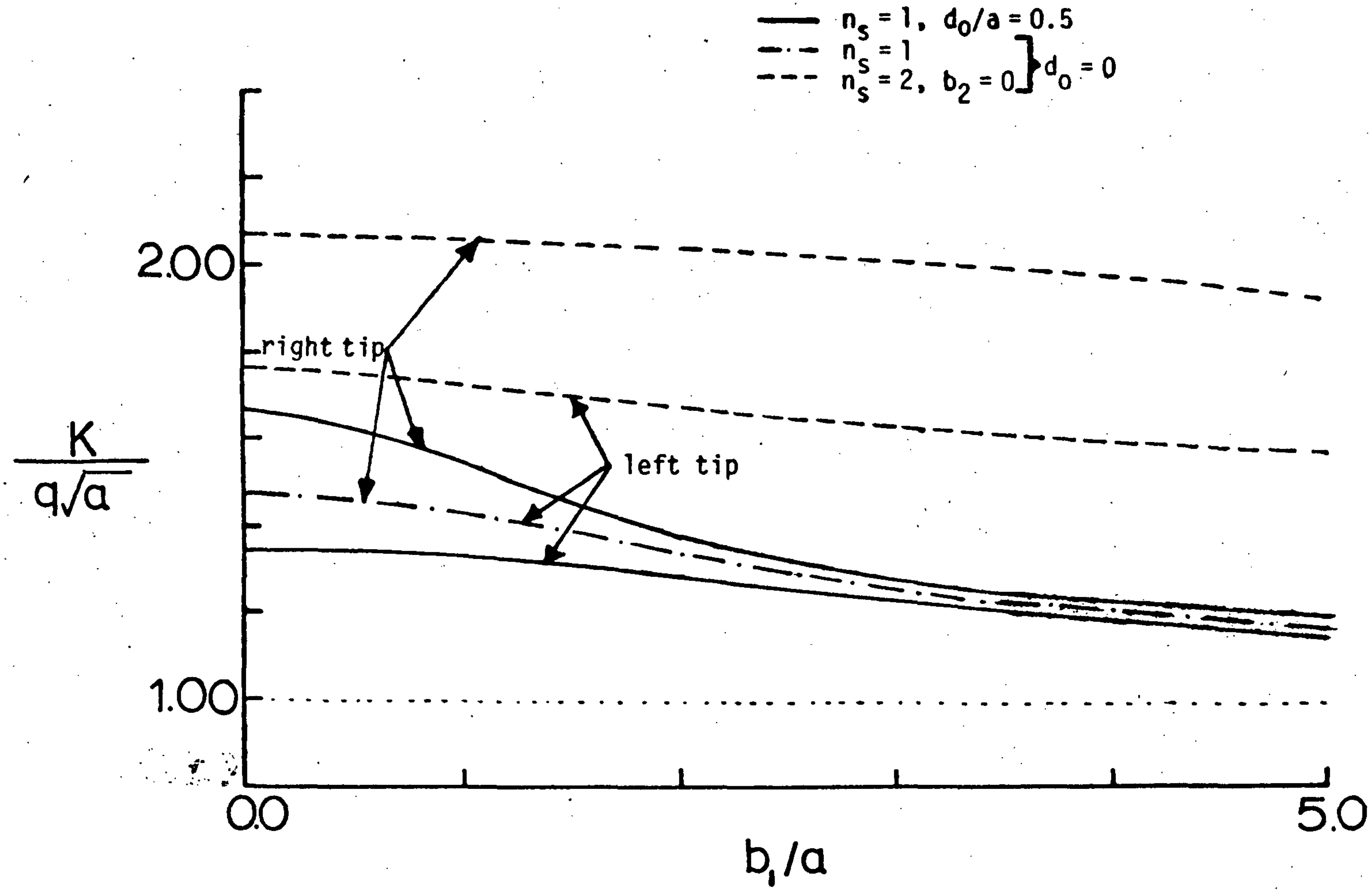


Fig. 12. $K/q\sqrt{a}$ vs. b_1/a
(broken stringer, loading at infinity, $d_1/a = 0.5$, $d_s/a = 0.2$)

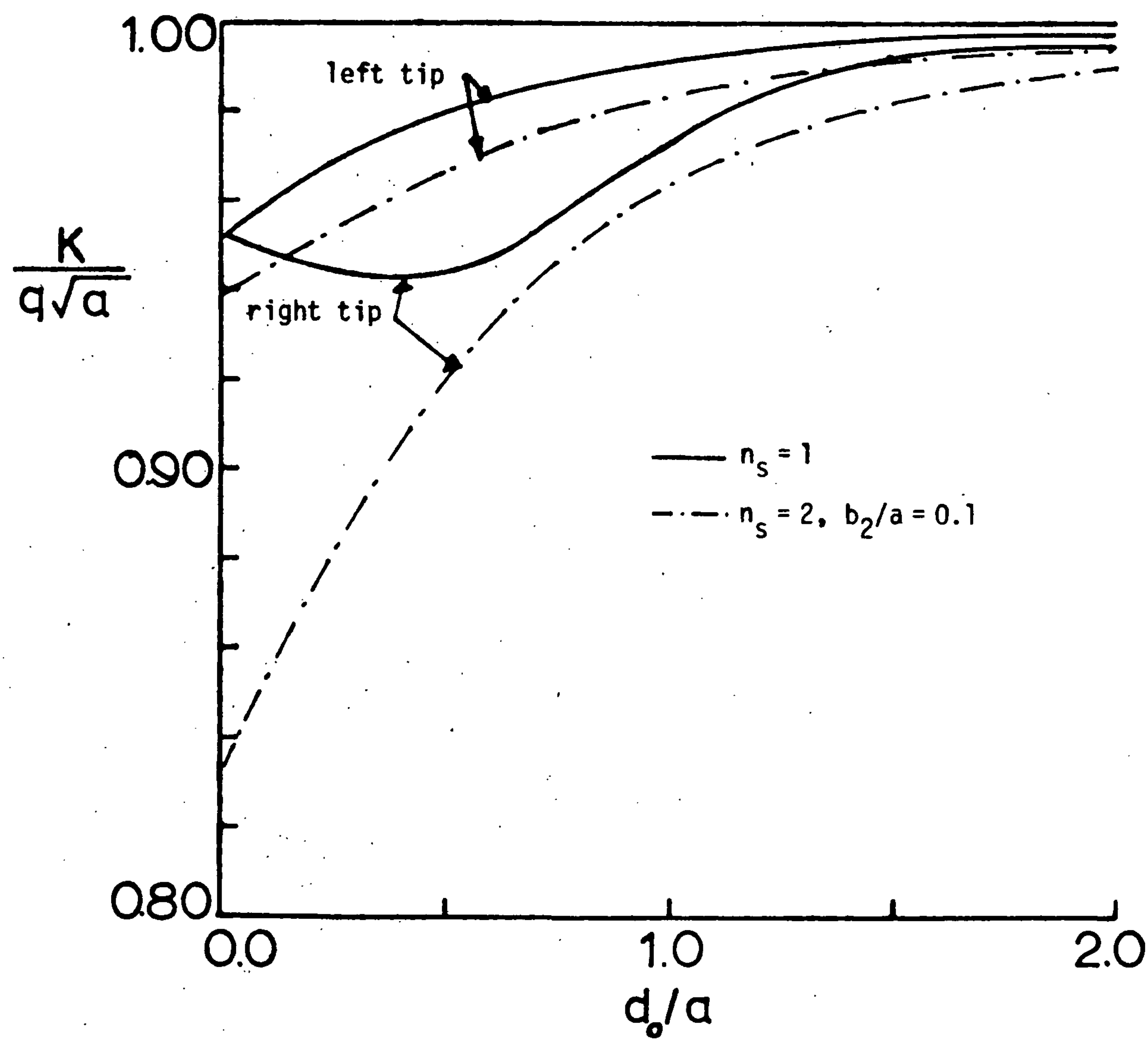


Fig. 13. $K/q\sqrt{a}$ vs. d_0/a
(broken stringer, loading on the crack surfaces, $b_1/a = 1.0$, $d_1/a = 0.5$, $d_s/a = 0.2$)

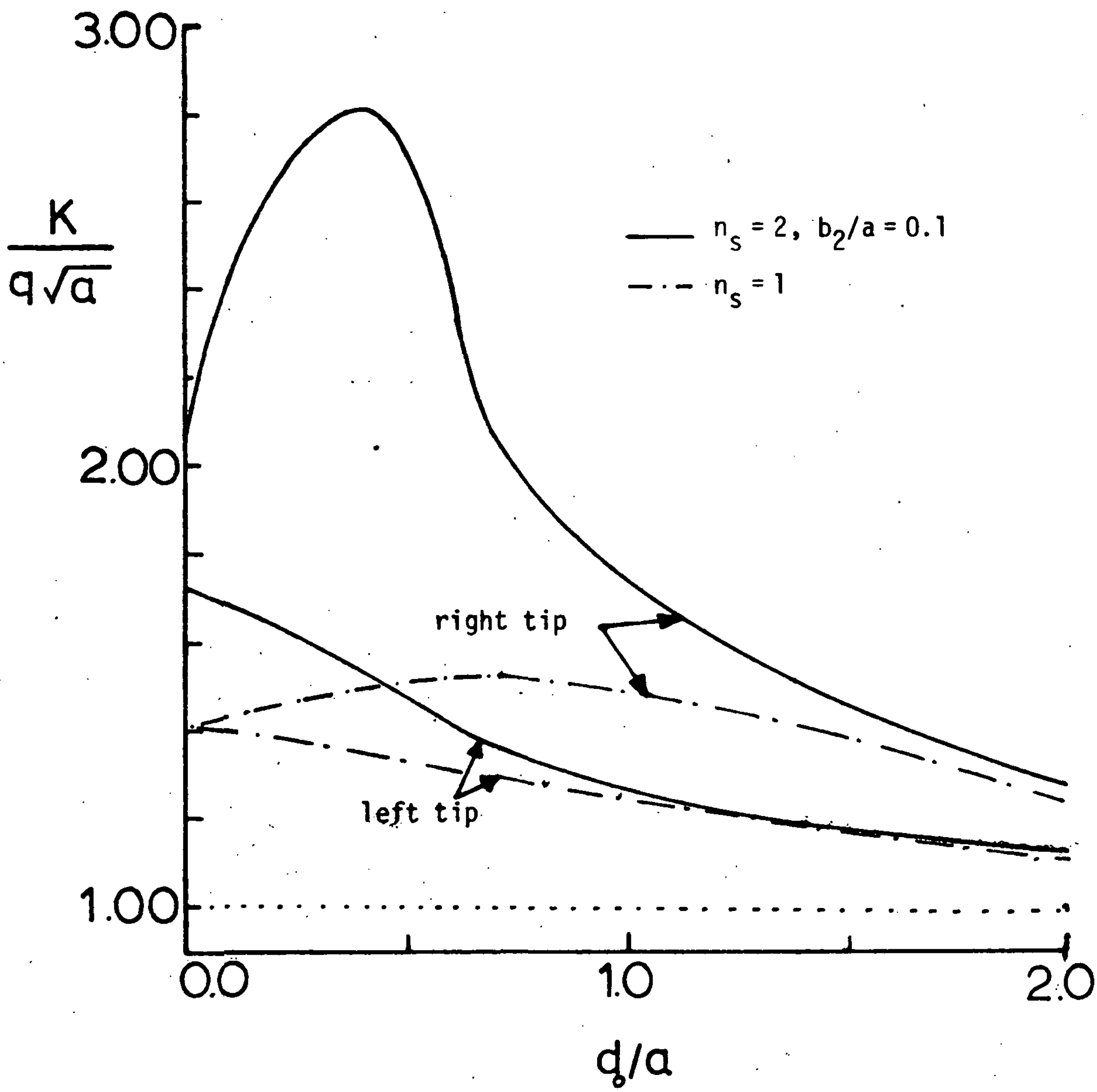


Fig. 14. $K/q\sqrt{a}$ vs. d_0/a
(broken stringer, loading at infinity, $d_1/a = 0.5$, $b_1/a = 1.0$, $d_s/a = 0.2$)

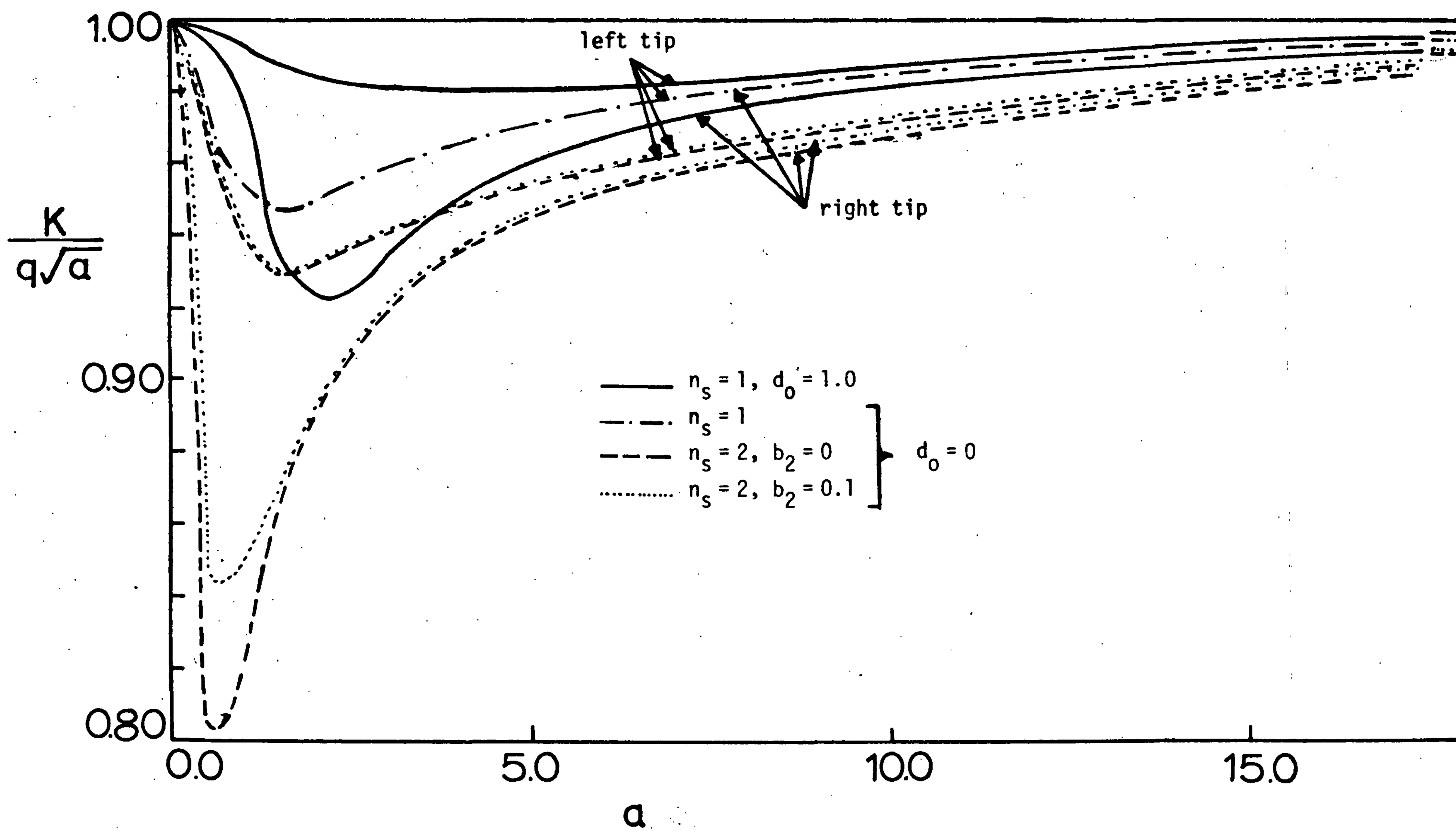


Fig. 15. $K/q\sqrt{a}$ vs. a
(broken stringer, loading on the crack surfaces,
$d_1 = 0.5$, $b_1 = 1.0$, $d_s = 0.2$)

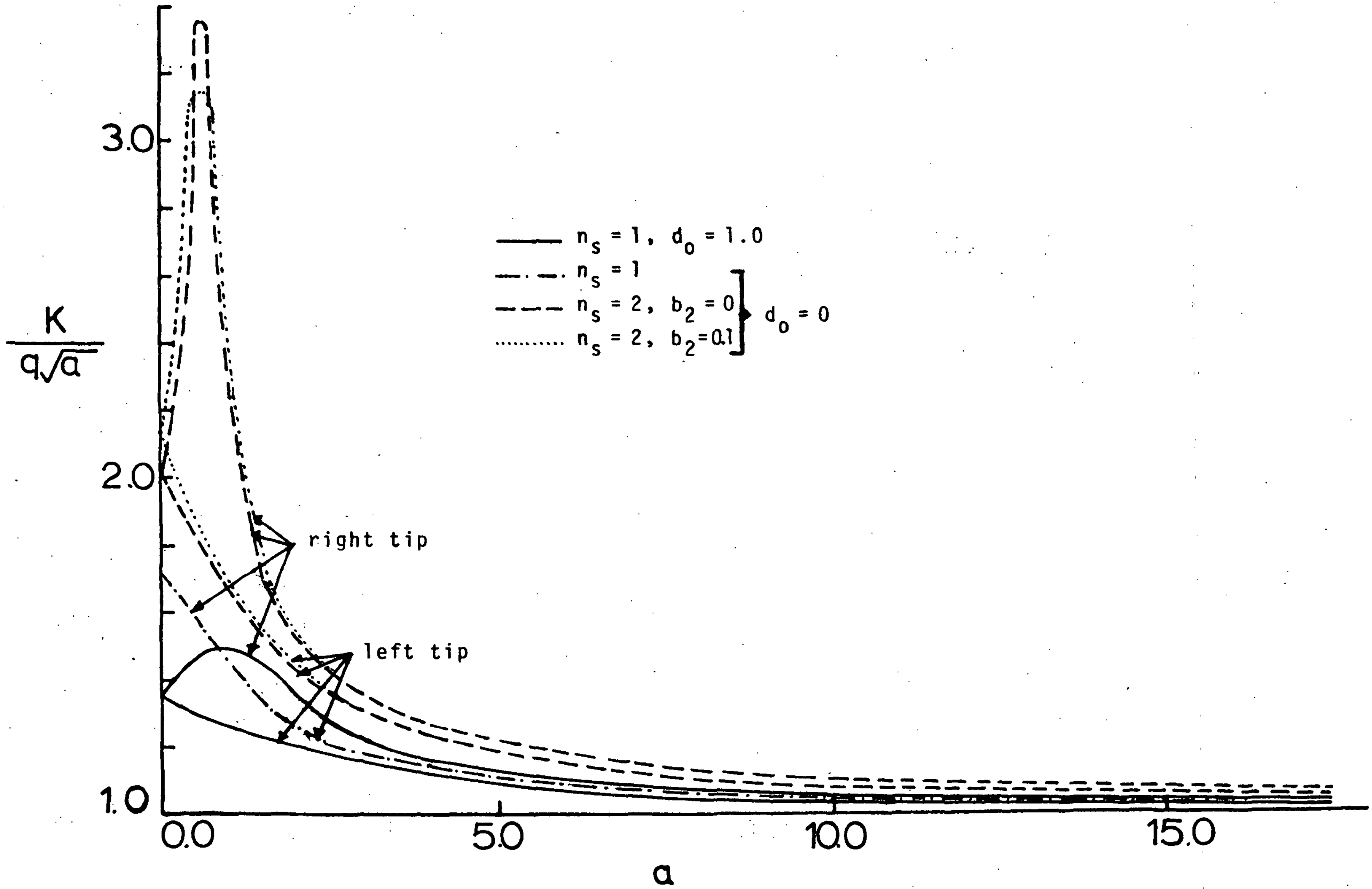


Fig. 16. $K/q\sqrt{a}$ vs. a
(broken stringer, loading at infinity,
$d_1 = 0.5$, $b_1 = 1.0$, $d_s = 0.2$)

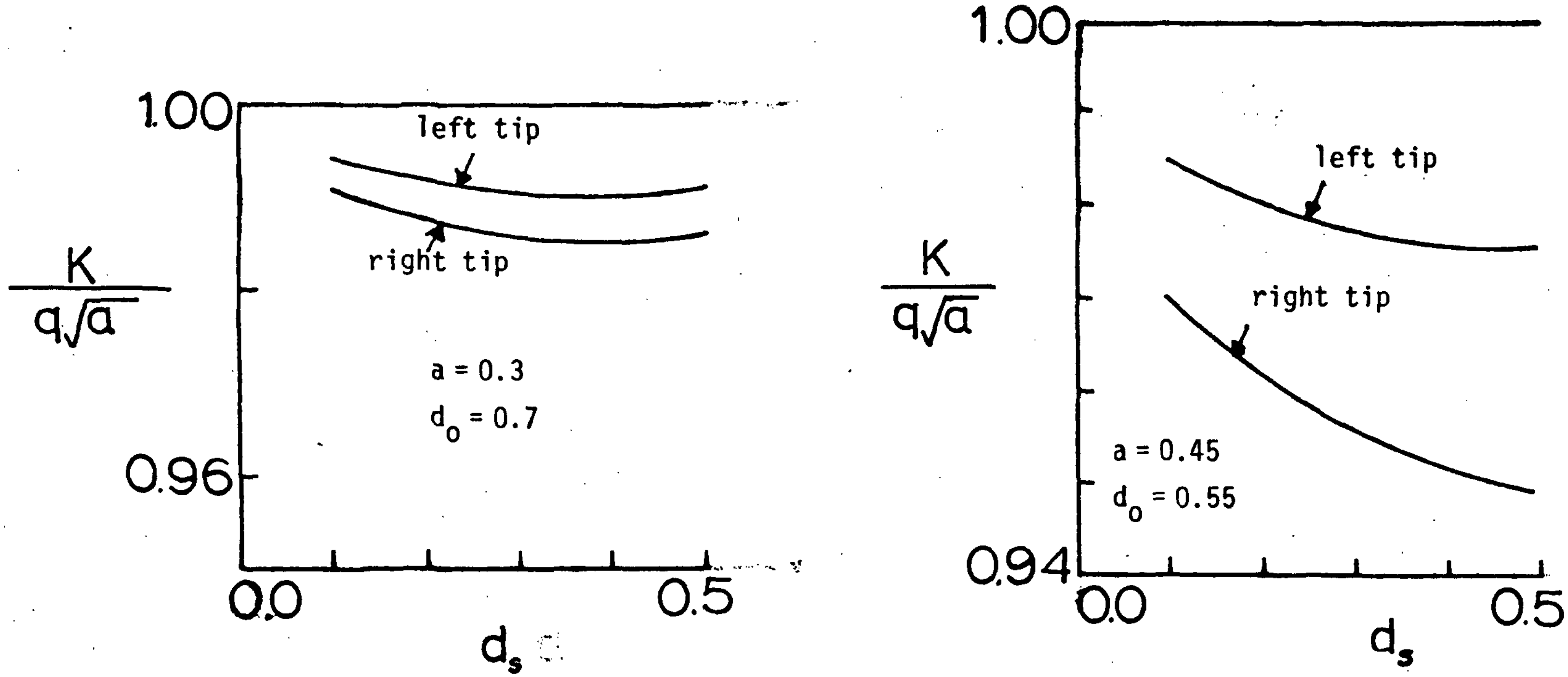


Fig. 17. $K/q\sqrt{a}$ vs. d_s
(broken stringer, $n_s = 1$, $d_1 = 0.5$, $b_1 = 0$)