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AIRSHIP STRESSES DUE TO VERTICAL VELOCITY GRADIENTS AND ATMOSPHERIC TURBULENCE

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ABSTRACT: Munk's potential flow method is used to calculate the resultant moment experienced by an ellipsoidal airship. This method is first used to calculate the moment arising from basic maneuvers considered by early designers, and then expended to calculate the moment arising from vertical velocity gradients and atmospheric turbulence. This resultant moment must be neutralized by the transverse force of the fins. The results show that vertical velocity gradients at a height of 6000 feet in thunderstorms produce a resultant moment approximately three to four times greater than the moment produced in still air by realistic values of pitch angle or steady turning. Realistic values of atmospheric turbulence produce a moment which is significantly less than the moment produced by maneuvers in still air.

INTRODUCTION

At one time airship design was a highly organized and systematic activity, and hundreds of papers have been written on the subject. The period of greatest activity was from 1910 to 1938. However, in spite of careful efforts several notable disasters occurred. Some were at least partly the result of political considerations; examples are the American ship Shenandoah and the British ship R-101. The most spectacular of all, the Hindenburg disaster, was of course due to the use of hydrogen as lifting gas. With the exception of the inadvisable use of hydrogen and the deterioration of the hull of the R-101, most well-known dirigible disasters were connected either with atmospheric turbulence or vertical wind currents in storms or above mountains[1].

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The British ship R-38 buckled in the middle and broke in two because of a strong wind gust (1922). At the time the airship was already experiencing significant stresses arising from a sharp turning maneuver. The Shenandoah perished in a 70 mph squall (1924). As the result of a navigational error the Akron was drawn into the center of a storm. While maneuvering upward to offset a downdraft, its lower rudder hit the ocean and the airship fell into the sea (1933). The Macon lost its top rudder during a squall and was also lost at sea (1935).

As a result of the R-38 crash the Royal Aeronautical Society established the R-38 Memorial Prize. In response to this competition three exceptionally detailed airship design papers were published[2-4]. This was in 1923, and taken together they constitute probably the most detailed airship design analyses available in English. Most later work was a refinement of methods discussed in these articles. One can even view the design of the Graf Zeppelin and her sister ships (1928-1938) within the context of the methods presented by these British and American authors. Of course the principal ingredient missing from these relatively early papers is the practical experience and full-scale data obtained by the German designers. However, there were no basic changes in the relevant technology in the years from 1923 to 1938.

An important part of the early design work was the highly ingenious description of the aerodynamic forces on airship hulls devised by Munk[5-6]. His theory is based on an ideal (non-viscous) fluid and Kelvin impulses. Under most conditions Munk's theory is in surprisingly close agreement with full-scale experiments[2].

As pointed out in several recent articles[7-9], the technology relevant to airship design has undergone an extraordinary expansion along with all other aerospace activity. Modern computers and modern knowledge of structural dynamics permit analyzing the airship's structure as a whole. It is essential to apply our current knowledge of atmospheric turbulence and vertical wind currents to these structural calculations. Safety is the overwhelming design consideration applicable to future airships, and relating atmospheric hazards to structural integrity holds the greatest promise of assuring safe operation. It might be argued that damaging atmospheric effects can usually be avoided, particularly during non-scheduled flights. The record of the German pilots serves to establish this to some degree. But the importance of scheduled operations also requires that atmospheric hazards be given careful consideration.

The purpose of this paper is to show how our present knowledge of the atmosphere can be combined with Munk's equations to calculate the resultant moment on an airship arising from vertical currents and atmospheric turbulence. Approximate results are given for the resultant moment experienced by a 1,000 foot long ellipsoidal airship with a fineness (length-to-diameter) ratio of 5. This is the shape suggested for a "basic" airship considered in a recent design study by Mowforth[10]. These results are compared with the moments arising from pitch angles and steady turning rates in still air which were taken into consideration by the early designers.

AERODYNAMIC CONSIDERATIONS

Munk's Equations

The motion of airship hulls gives rise to an air flow that is well approximated by potential flow. There may be a large resultant moment of the aerodynamic forces, but only a comparatively small lift and drag. With wings the conditions are different as there is considerable lift. Since the momentum of the flow is not necessarily in the direction of motion of the hull, a principal axis problem presents itself. Strictly speaking, we should distinguish between the momentum of the flow and the Kelvin impulse of the flow, but Munk himself disregarded this difference and we have no need to make the distinction here. The net resultant moment is expressed in terms of the volumes of the apparent additional masses of the hull. The apparent additional mass of a solid moving through a fluid along one of its principal axes is simply a proportionality constant expressing the resistance to accelerations along the axis offered by the fluid itself. Note that it is not a measure of the inertia of the solid, because the solid need not have any mass at all. In this case all of the energy is stored in the flow, and the apparent additional masses along each principal axis are equal to the apparent masses. The effect of the fluid surrounding the solid is, however, fully described by assigning to the solid an apparent additional mass in addition to its original or actual mass. The apparent mass of a circular cylinder in a uniform two-dimensional stream is $\rho\pi r^2$; and for a sphere in a three-dimensional uniform stream its value is $\frac{1}{2}(\frac{4}{3}\pi r^3)\rho$. Here r and ρ are radius and density. Apparent volume is obtained from apparent additional mass by dividing by the density.

Munk shows that an airship hull, flying steadily under an angle of attack α and with the velocity of flight V experiences a resultant couple of the magnitude

$$M = \frac{\rho}{2}V^2(K_2-K_1)\sin 2\alpha \tag{1}$$

where K_1 and K_2 denote the apparent volumes with respect to the longitudinal and transverse principal axes of the hull. This moment is unstable, consequently fins are required for stabilization. Munk also calculates the transverse force on an airship (with circular cross section) turning under an angle of yaw:

$$dF = dx[(k_2-k_1)\frac{dS}{dx}V^2\frac{\rho}{2}\sin 2\phi + k'V^2\frac{\rho}{R}\cos^2\phi + k'V^2\frac{\rho}{R}x\frac{dS}{dx}\cos^2\phi] \tag{2}$$

where

dF = Transverse force acting over a differential length along the longitudinal axis

dx = Differential length along the longitudinal axis

k_1 = (Hull volume)/K_1

k_2 = (Hull volume)/K_2

k' = Ratio of the apparent hull moment of inertia about the aerodynamic center to the moment of inertia of the displaced air

x = Position on the longitudinal axis relative to aerodynamic center

S = Area of circular cross section at x

ϕ = Yaw angle

R = Turning radius

V = Airship velocity

ρ = Density of air

This expression of course does not contain the air forces on the fins. Munk's theory also yields a closed form expression for the pressure distribution over any ellipsoid inclined at an arbitrary angle to the flow. The first term on the right-hand side of Equation 2 can be used to calculate the longitudinal distribution of forces resulting from a vertical gust. In this case the yaw angle in Equation 2 is identified with the angle of attack

$$\phi = \tan^{-1} \frac{u}{V} \qquad (3)$$

where u is the transverse velocity and V is the forward velocity. Munk assumes the airship has a variable effective angle of attack along its axis. The magnitude of the superposed angle is $\tan^{-1}(u/V)$, where u generally is variable. The momentum produced at each portion of the airship is the same as the air force at that portion if the entire airship had that particular angle of attack. Consequently, Equation 2 can be used to determine the moment experienced by an airship as it moves through a vertical velocity gradient. In this case we assume the pilot is able to hold the airship on a straight course in inertial space without yaw or pitch. Equation 2 will also be used to calculate the moment resulting from a turning maneuver. Equation 1 provides a direct method of calculating the bending moment when the only disturbing force is due to pitch.

Moment Response Function

Munk's theory can be extended to calculate the transverse forces caused by atmospheric turbulence. It is assumed the pilot is able to hold the airship on a straight course in inertial space without yaw or pitch. We begin by attributing to a circular cross-section of area S the virtual mass $S\rho dx$ just as if the cross-section were part of a circular cylinder immersed in two-dimensional flow. The transverse force acting on this cross section as a result of the velocity perturbation $u = u_0 e^{i\omega t}$ is

$$f = \rho S dx (i\omega) u_0 e^{i\omega t} \qquad (4)$$

Now

$$\omega = 2\pi f = 2\pi\frac{V}{\lambda} = k_p V \qquad (5)$$

where ω is the angular frequency of the perturbation, f is the cyclical frequency, V is the forward velocity of the airsh , λ is the wavelength in the forward direction, and k_p is the propagation constant for a particular wavelength. It is convenient to take the geometric center of the ellipsoid as the origin of our coordinate system. Then the moment experienced by the airship, per unit velocity perturbation, is given approximately by

$$\frac{M}{u_0} = V\rho k_p \int_{-L/2}^{+L/2} S(x) e^{i(\omega t+kx)}\ x\ dx \qquad (6)$$

where L is the length of the airship.

Uniform S is not a candidate hull shape, but this case leads to the simplest form of the moment response function. If S is uniform, the result is

$$\left|\frac{M}{u_0}\right| = V\rho SL\left(\frac{\sin(\pi\zeta)}{\pi\zeta} - \cos(\pi\zeta)\right) \qquad (7)$$

where $\zeta = L/\lambda$. This is the long wavelength approximation, and approaches zero as ζ approaches zero. For short wavelengths, $\zeta \gg 1$, the bending moment at the longitudinal positions of maximum transverse velocity is the important consideration. In this case

$$\left|\frac{M}{u_0}\right| = \frac{S\rho V}{k_p}\left(\frac{\pi}{2} - 1\right) \qquad (8)$$

For an ellipsoidal airship with a fineness ratio of 5 we set

$$S(x) = \frac{\pi}{25}\left\{(L/2)^2 - x^2\right\} \qquad (9)$$

and use Equation 6 to obtain

$$\left|\frac{M}{u_0}\right| = \frac{\pi}{50}\ L^3 \rho V\left\{\frac{\sin\hat{k}}{\hat{k}} + \frac{3\cos\hat{k}}{\hat{k}^2} - \frac{3\sin\hat{k}}{\hat{k}^3}\right\} \qquad (10)$$

where $\hat{k} = Lk_p/2$, again this is the long wavelength approximation, and the right-hand side of Equation 10 tends to zero as $\hat{k}$ tends to zero. The short wavelength approximation, Equation 8, still applies provided

$$\frac{dS}{dx}\ \frac{\lambda}{S} \ll 1 \qquad (11)$$

Vertical Wind Gradients in Thunderstorms

Using Munk's equations we can calculate the force at each section of the hull of a representative airship for vertical wind currents known to exist in the atmosphere. Typical values for a thunderstorm are considered. Taken together Figures 1-3 enable us to obtain a good approximation of the vertical currents and horizontal scale of thunderstorms. Figure 1 shows information which was obtained to describe the atmospheric effects an airplane experiences as it flies through a thunderstorm. Figure 2 suggests that the vertical velocity profile given in Figure 1 is applicable above about twenty thousand feet. With the help of Figure 3, we can construct a similar thunderstorm profile for an altitude of approximately 6,000 feet, which is a typical operational altitude. These diagrams give no information about the severity of the turbulence; they can only be used to study the airship stresses arising from vertical currents. However, we can observe that the region of "violent turbulence" extends much further than the region of severe vertical currents. Figure 1 represents the vertical velocity profile in the plane of travel of the thunderstorm; the updraft usually has a fairly uniform cross section of about 10 miles traverse to its line of travel. Figure 3 shows that below about ten thousand feet the vertical flow is not quite as constricted as at higher altitudes. Let us therefore make the approximation that at 6,000 feet the horizontal scale of the currents is twice as large as shown in Figure 1. Accordingly, the magnitude of the vertical current distribution is cut in half (due to the transverse extent of the storm the flow is treated as two-dimensional). This means that the more severe vertical velocity gradients in the horizontal direction are of the order of 0.2 ft./sec./ft.

Atmospheric Turbulence

It is possible to describe the energy distribution of atmospheric turbulence as a function of wavelength by the same relatively simple formula for the following three forms of turbulence[11]:

(i) clear air turbulence near the ground,
(ii) turbulence near and in cumulus clouds,
(iii) thunderstorms.

The distribution of energy density $S_w(k)$ at different wavelengths $(1/k)$ of the vertical component of the turbulence may be given by

$$S_w(k) = 2L_s \sigma_w^2 \frac{1 + (8/3)\ (L_1 k)^2}{\left[1 + (L_1 k)^2\right]^{11/6}} \qquad (12)$$

where σ_w = Root mean square vertical velocity

L_s = Scale of turbulence

$L_1 = 1.339\ (2\pi L_s)$

for each patch of turbulence. Twice the total energy per unit mass of air equals the mean square of the turbulence velocity so that

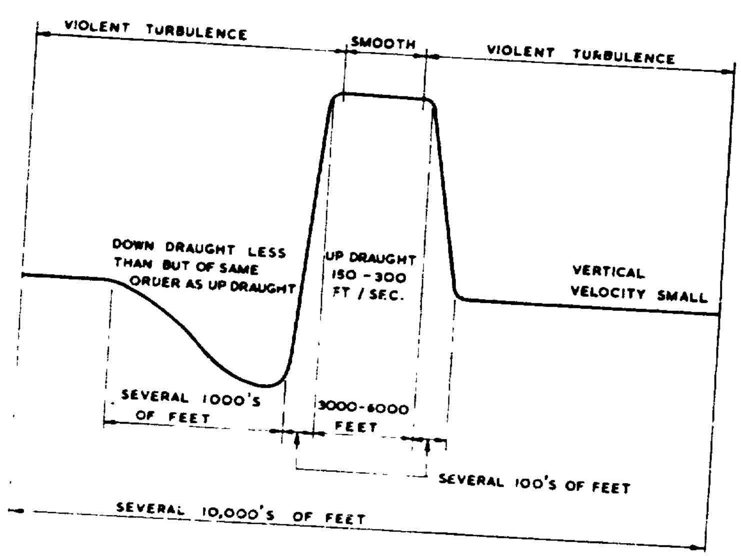

Figure 1. Variation of mean vertical velocity in line of travel of a thunderstorm a few thousand feet below tops of clouds (Not to scale, Ref. 11)

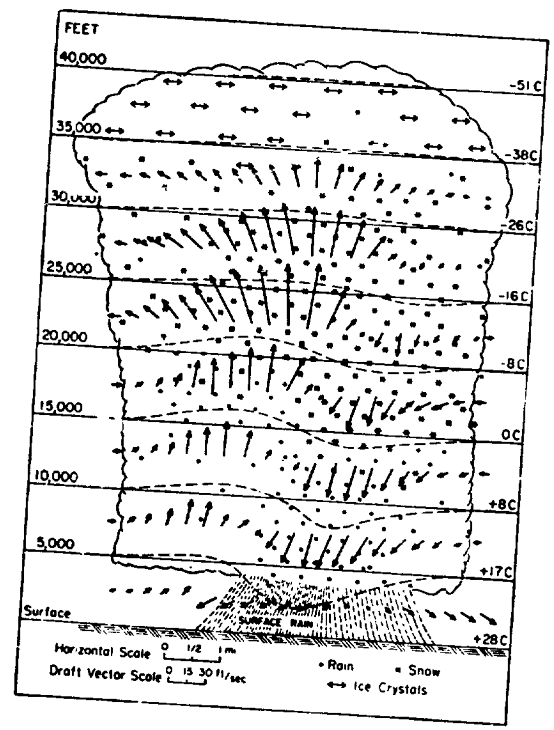

Figure 2. Mature Thunderstorm (Ref. 12)

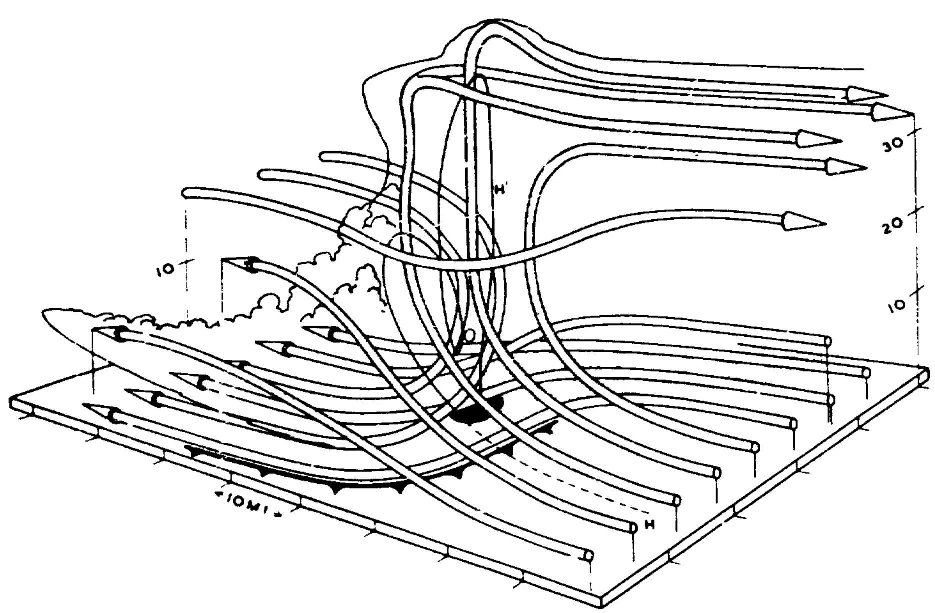

Figure 3. Model of air flow in a severe storm cumulonimbus. Stream lines are drawn of the motion relative to the storm, which moves from left to right, and are shaded where condensation has occured. A schematic outline is drawn of the anvil cloud and of a cumulus belt over the trailing edge of the squall front (marked on the ground as a cold front) Heights are marked in thousands of feet. (Reproduced from "Air Flow in Cumulonimdus" by F.H. Ludlam, Atmospheric Turbulence and its relation to Aircraft, H.M. S.O., 1963)

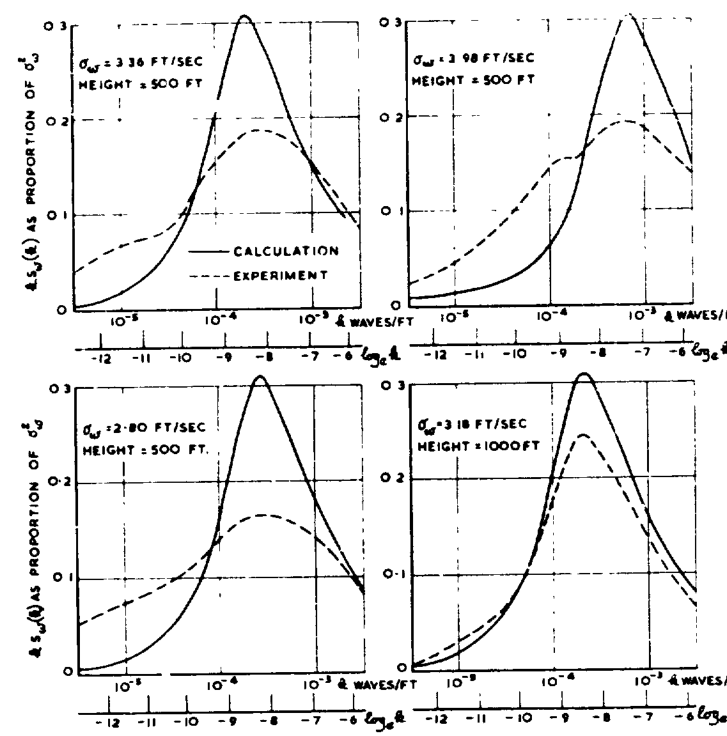

Figure 4. Distribution of energy per unit mass of air $\left[\frac{1}{2}S_w(k)\right]$ at different wavelengths (1/k). Measurements on ballon cables deduced from F.B. Smith (1961) and compared with

$$S_w(k)=2L_s\sigma_w^2\left[1+(8/3)(L_1k)^2\right]/\left[1+(L_1k)^2\right]^{11/6}$$

where $L_1=1.339(2\pi L_s)$; σ_w=R.M.S. vertical velocity. L_s is chosen so that calculated and experimental distributions have k·S(k) a maximum at the same k.

$$\sigma_w^2 = \int_0^{+\infty} S_w(k)\,dk = \int_{-\infty}^{+\infty} kS_w(k)\,d(\log_e k) \qquad (13)$$

Thus, twice the total energy per unit mass of air is given by the area under the curve of $kS_w(k)$ against $\log_e k$ and the area under the curve $kS_w(k)/\sigma_w^2$ is unity. Usually Equation 12 is adjusted to fit experimental data by selecting L_s so that the calculated and experimental distributions $kS_w(k)$ have a maximum at the same value of k. A comparison of theoretical and experimental distributions is shown in Figure 4[13]. We shall follow the common practice of referring to $S_w(k)$ as a power spectral density even though in reality it is a mean-square-value-density spectrum.

NUMERICAL RESULTS

Wind Velocity Gradients

The resultant moment experienced by the airship is evaluated from

$$M = \int_{-L/2}^{+L/2} \left(\frac{dF}{dx}\right) x\,dx \qquad (14)$$

by using Equations 2 and 3 and setting u = (du/dx) (L/2 - x). The results are shown in Table I for (du/dx) = 0.01, 0.1, 0.2, and 0.3.

Atmospheric Turbulence

If H(k) is the response function describing the resultant couple when the airship is subjected to a transverse velocity wave of unit amplitude, then the mean square value of this moment is given by

$$M^2_{r.m.s.} = \int_0^{+\infty} |H(k)|^2 \; S_w(k)\,dk \qquad (15)$$

if $S_w(k)$ is a stationary function. Using $dk = d\hat{k}/(\pi L)$ and $d\hat{k} = \hat{k}\,d(\log_e \hat{k})$ Equation 15 becomes

$$M^2_{r.m.s.} = \frac{1}{\pi L} \int_{-\infty}^{+\infty} |H(\hat{k})|^2 \; S_w(\hat{k})\,\hat{k}\,d(\log_e \hat{k}) \qquad (16)$$

After setting

$$|H(k)|^2 = \left|\frac{M}{u_c}\right|^2 \qquad (17)$$

Equation 16 was used to evaluate $M_{r.m.s.}$ in response to the atmospheric power spectral density function given by Equation 12. Two cases were considered[11]: (1) Captive balloon data taken at a height of 1,000 feet with σ_w = 3.54 fps, and (2) Data obtained from an airplane flight at 40,000 feet in a thunderstorm. The results are shown in Table I.

Summary

The resultant moments obtained by the various methods discussed in this paper are compared in Table I. Dashes are used where an entry is not applicable. These moments are an important measure of the airship's stress because they must be neutralized by the transverse force of the fins. The first five cases, which include rectilinear motion at a constant pitch angle and steady turning without pitch, are conditions in still air which were considered by the early designers. The angle of yaw corresponding to the turning radius R was obtained by Munk[5]

$$\phi = \frac{L}{2R} \frac{1}{k_2 - k_1} \tag{18}$$

Equation 1 was used in Case 1, and Equation 2 was used in Case 2. Agreement of these two cases serves as a check on the numerical methods and also confirms, with remarkable accuracy, the approximations Munk used in deriving Equation 2.

Cases Six through Eleven correspond to situations where our current knowledge of the atmosphere was used. When a uniform vertical velocity gradient was considered, the vertical velocity was assumed to be zero at the tail and increase in the direction of flight. The resultant moment for Case Nine is less than Case Eight because the sine of the angle 2ϕ contained in Equation 2 decreases as ϕ increases beyond 45°. The data for the thunderstorm[11] were obtained at 40,000 feet and are not fully satisfactory for our purpose. However, the density was adjusted to this height, and the result corresponding to a direct application of the power spectral density equations is included. These results show that the values of atmospheric turbulence found in the literature produce a moment which is significantly less than the moment produced by realistic maneuvers in still air. However, the vertical velocity gradients at an altitude of 6,000 feet in a thunderstorm produce a moment which is three to four times larger than the moment produced by maneuvers in still air.

Table I. Numerical Results

Case Number	1	2	3	4	5	6	7	8	9	10	11
Pitch Angle, degrees	6	6	0	0	0	0	0	0	0	0	0
Yaw Angle, degrees	0	0	3.4	6.9	13.8	0	0	0	0	0	0
Uniform Vertical Velocity Gradient, fps/ft	0	0	0	0	0	0.01	0.1	0.2	0.3	-	-
Turning Radius, 10^3 ft	∞	∞	10.	5.	2.5	∞	∞	∞	∞	∞	∞
Scale of Turbulence, ft	-	-	-	-	-	-	-	-	-	890.	5600.
R.M.S. Vertical Velocity, fps	-	-	-	-	-	-	-	-	-	3.54	9.40
Resultant Moment, 10^6 ft-lbs	36.1	36.2	19.0	37.8	80.1	17.2	106.	112.	106.	*2.45	*3.08
Equations Used	1	2,14	2, 14,18	2, 14,18	2, 14,18	2, 14,3	2, 14,3	2, 14,3	2, 14,3	10, 12,16	10, 12,16

* R.M.S.

Conditions Applicable to All Cases Unless Otherwise Noted

Forward Velocity of Airship = 100 fps

Density = 0.001988 slugs/ft^3 (Standard Atmosphere at 6000 ft)

In Case 11, Density = 0.000582 slugs/ft^3 (Standard Atmosphere at 40,000 ft)

Fineness Ratio = 5

Coefficients of Additional Mass of Ellipsoid: $k_1 = 0.059$, $k_2 = 0.895$, $k' = 0.701$

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