X-920-75-299
PREPRINT

NASA TM X- 71035

SPACEBORNE EARTH APPLICATIONS RANGING SYSTEM SPEAR

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DECEMBER 1975

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

Earth surface motions on the order of one to several cm per year have been observed. These motions could be due to land subsidance, i.e. the gulf coast of Texas subsides about 5 to 10 cm per year; likewise, similar values hold for portions of the Florida coast), crustal uplift (i.e. dilatency which is a phenomena observed to precede earthquakes primarily along thrust faults), and loading (i.e. due to large dams, etc.). Knowledge of these motions is of practical importance to government and local agencies.

A technique is discussed here for the accurate (i.e. to within fractions of cm/yr) detection of these motions utilizing the latest space technology. It is shown that over a six day period and assuming a 50% cloud cover (i.e. as experienced over the last few years of laser operation) utilizing spaceborne precision ranging systems, intersite distances on the order of 5 to 15 km (dependent mostly on the beam width of the laser) can be determined in the vertical and horizontal components with errors in the 0.5 to 1.5 cm range. These errors are almost independent of ground survey errors up to 0.25 meters and orbit errors up to 200 meters.

A spaceborne laser ranging system is assumed to range simultaneously to two or more ground emplaced retroreflectors. The fundamental advantage derived from simultaneous ranging is the elimination to first order of errors due to the system. This means elimination of (a) bias errors in the ranging system, (b) errors due to propagation effects, and (c) errors associated with the spacecraft's motion in its orbit.

In conclusion, it can be stated that horizontal and vertical errors in relative distance determination up to 25 km are expected to be in the 0.5 to 1.5 cm range using only a 6 day mission. This assumes very modest errors in the ranging noise σ of 5 cm (present GSFC laser system precision), orbital errors of 200 meters, and a priori knowledge in the intersite distances of 0.25 meters. These results make this system very attractive as a new tool for monitoring very small earth surface motions.

SPACEBORNE EARTH APPLICATIONS RANGING SYSTEM
SPEAR

INTRODUCTION

Accurate determination of intersite distances has been pursued at Goddard for many years. This work began in 1964 when the first satellite (BE-B) equipped with laser corner reflectors was launched. Continuous improvement of ground based laser ranging systems made it possible to determine intersite distances of a thousand km to within perhaps tens of centimeters. As early as 1968, J. Rosenberg suggested reversing the system; that is, using corner cubes on the ground and precision laser systems in orbit. Serious consideration was given to such a system during 1971/72 at Goddard. At this time work began to design a precision laser ranging system for the space shuttle laboratory. Note that the following analysis applies to both electronic transponders as well as laser corner cube retroflectors. The analysis considers only ranges which are independent of the systems used for their determination. An analysis using possible range rates will be published at a later date.

One impetus came from the possible use of the application of such a system for earthquake prediction. C. Scholz, L. Sykes and Y. Aggarwal presented a paper at the AGU meeting in Washington, D. C. (Ref. 1) in the spring of 1973 stating that dilatancy, among other phenomena, is a real precursor to earthquakes. As outlined in their paper, the ground rises around an active zone. A well documented case is, to quote the example used by Scholz, the 1964 Niigata earthquake ($M \doteq 7.5$)* (See Figure 1). Measurements indicated rather clearly a constant vertical motion of the ground of about 12 cm over 60 years (0.2 cm/yr). This is followed by a quite rapid change of 5 cm over 3 years (1.7 cm/yr). Subsequently, the motion stops for 3 to 4 years till the earthquake occurred. A system as described here could perform such measurements. Accurate (cm-level) relative intersite distances are needed in many other practical applications. Such a system could further be installed to monitor small motions near or on large dams, major construction sites, shore facilities and structures, etc.

I. SYSTEMS CONCEPT

As was shown by Scholz, et al. (1973) Ref. 1, the region of rock dilatancy extends one fault length (L) on each side of an active fault zone (total length $\doteq$ 3L) and one

*M ≡ Earthquake magnitude as measured on Richter Scale.

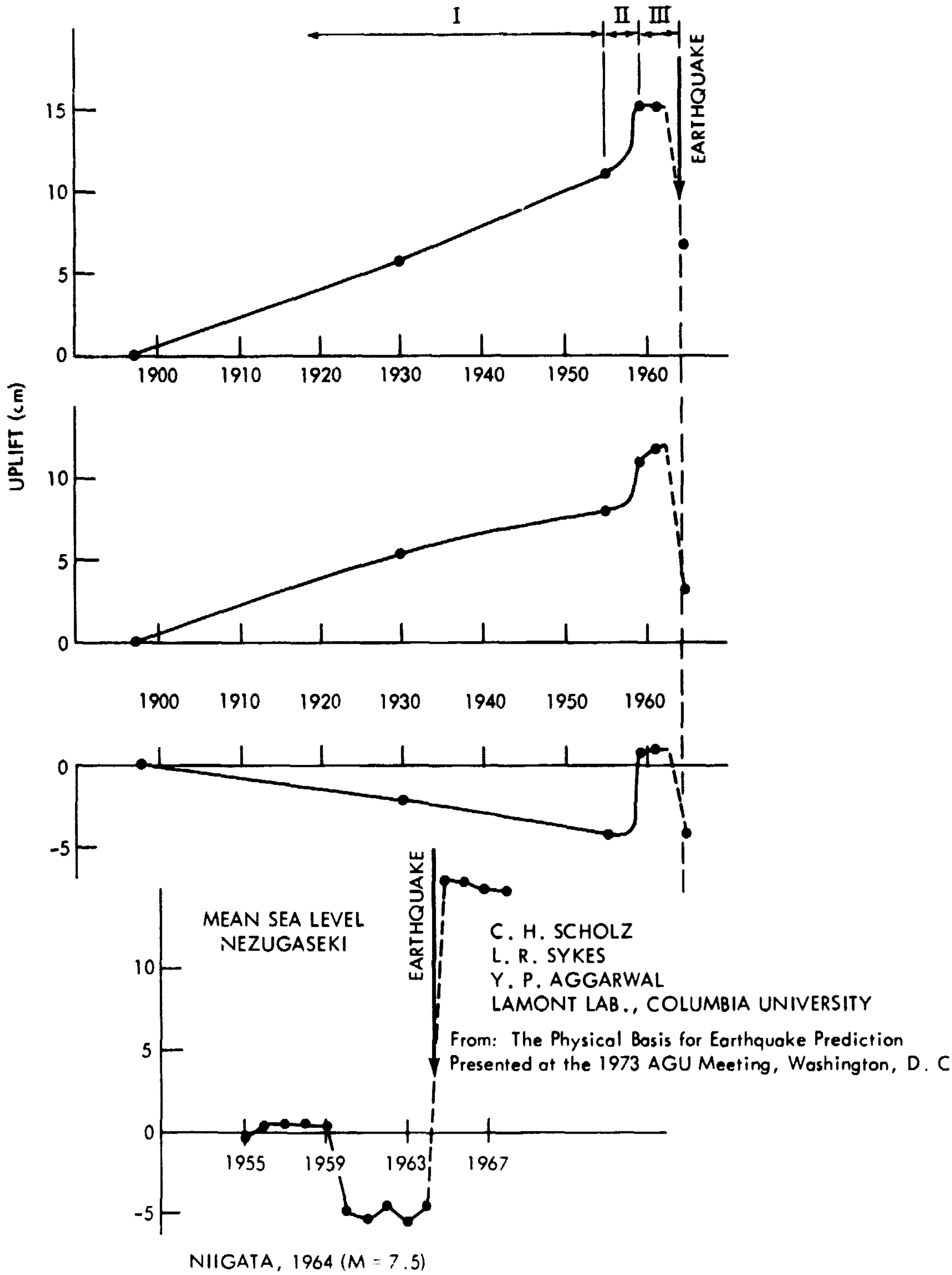

Figure 1. (Reference 1)

length on each side (width = 2L). Thus, if one wishes to observe the rise of the ground, an array of ground transponder cubes which covers this active area must be used. ($A \doteq 6L^2$) as shown schematically in Figure 2.

The smallness of the motions to be observed over several years requires the use of a rather specific ranging and/or tracking systems concept. In essence, one wants to determine very accurately (cm-range) a distance $|\vec{D}|$ or better, its changes between two ground points using satellite technology. In general, there are three major obstacles or error sources to overcome if one wishes to compute extremely accurate vector distances of 5 to 20 km, that is accurate to 0.5 to 1.5 cm from a spacecraft. These are:

1. Orbital errors of the spacecraft

2. Bias errors in the ranging system

3. Atmospheric propagation errors

These can be eliminated to first order by using range differencing. Practically, this means sending one pulse from the spacecraft to two or more ground stations, subtracting these measurements and using only their "differences" to compute $|\vec{D}|$, the distance between neighboring ground stations (i.e. transponders or laser corner cubes). It is thus evident by studying Figure 3 that the above mentioned error sources will not play any major role (only second order) since

$$\Delta\rho_{kj} \doteq (\rho_{kj} + \delta\rho_{kj}) - (\rho_{kj+1} + \delta\rho_{kj+1})$$

$$\doteq (\rho_{kj} - \rho_{kj+1}) + (\delta\rho_{kj} - \delta\rho_{kj+1})$$

$$= (\rho_{kj} - \rho_{kj+1})$$

For bias errors in the ranging system

$$\delta\rho_{kj+1} = \delta\rho_{kj}$$

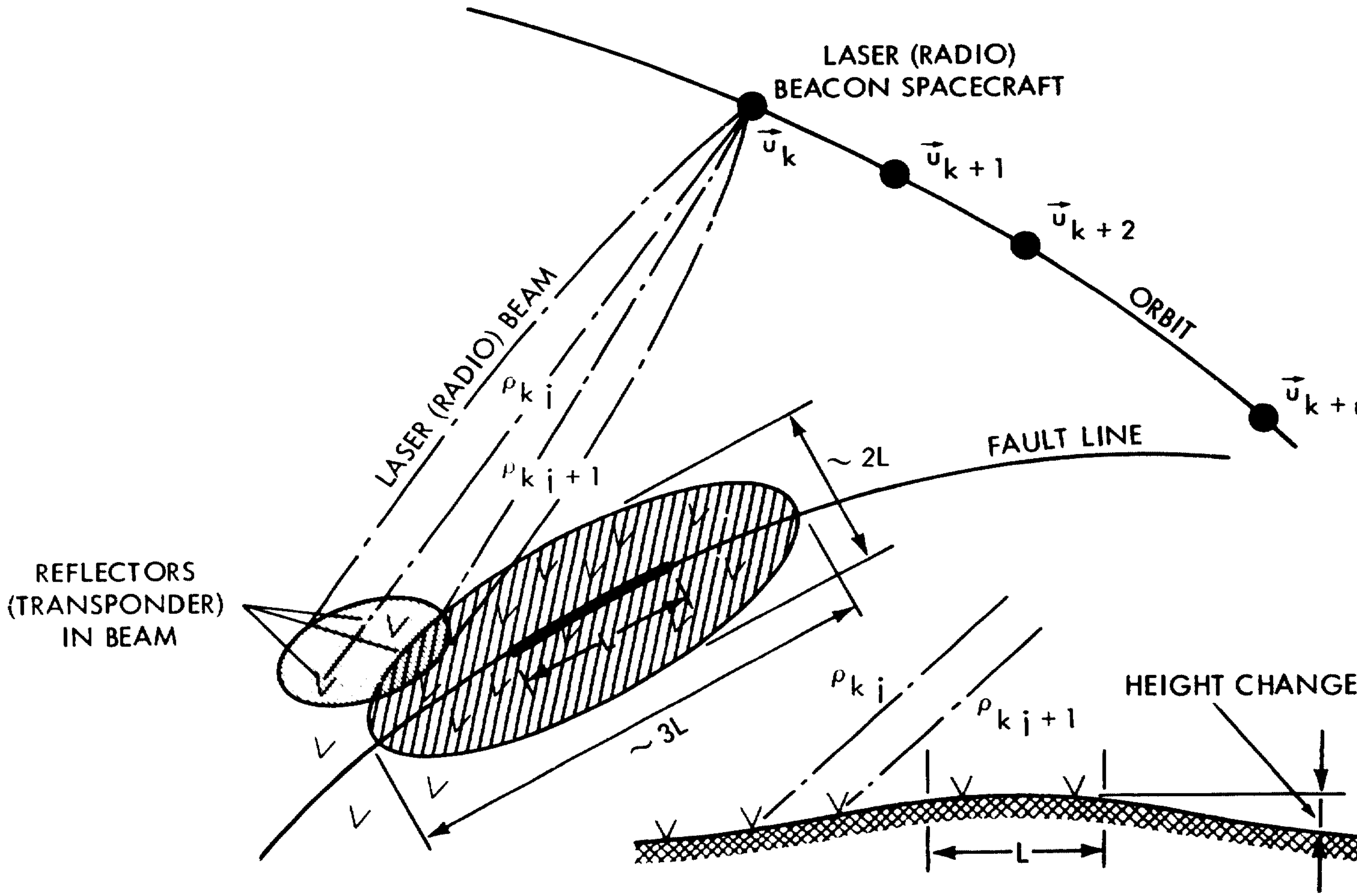

Figure 2. Rock Dilatancy Determination from Space

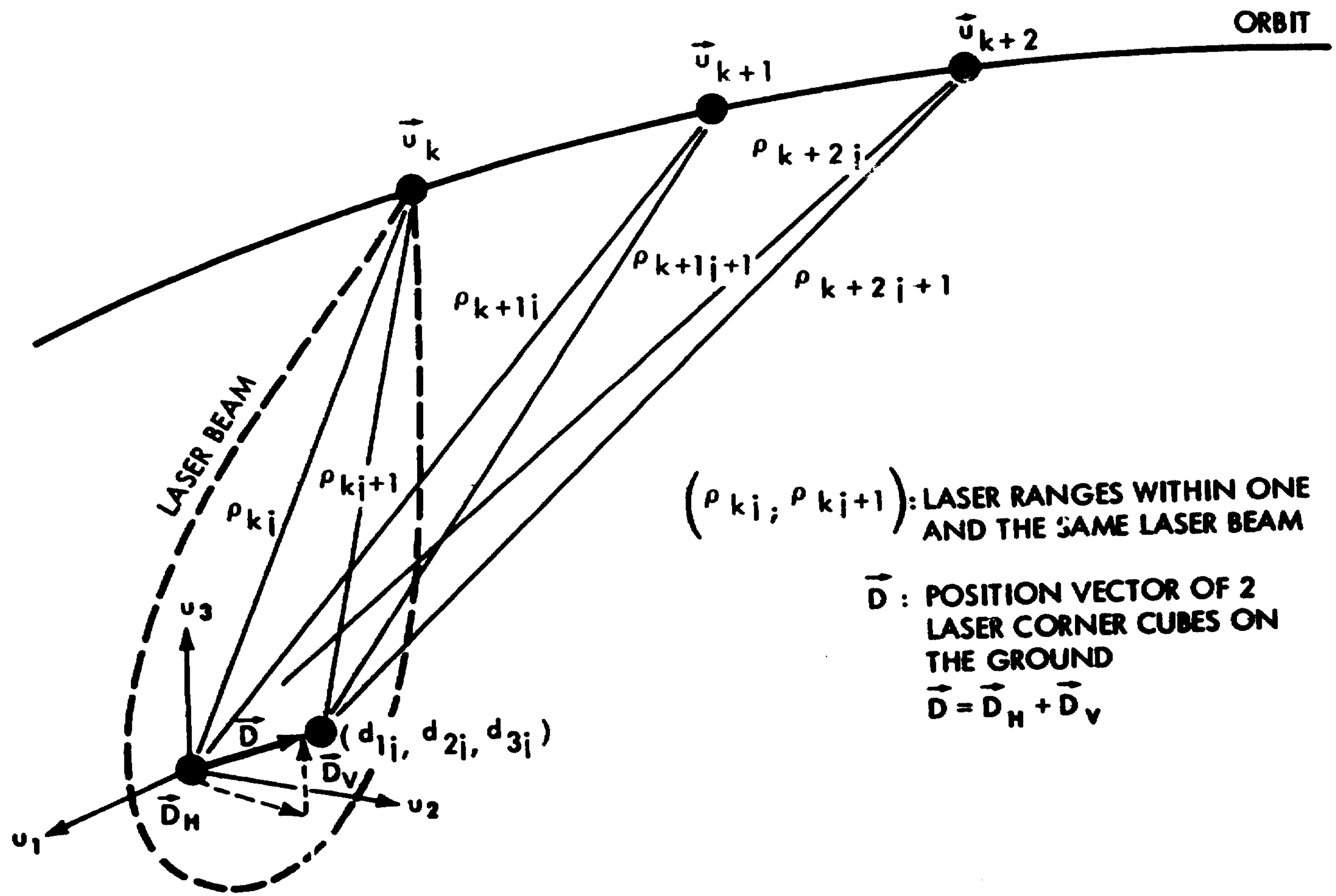

Figure 3. Schematic Spaceborne Ranging System Measurement Geometry

because both the j^{th} and $(j + 1)^{th}$ reflectors* are interrogated by the same signal (within the beam). For orbital errors and atmospheric refraction,

$$\delta\rho_{kj+1} = \delta\rho_{kj} + \text{(second order terms)}$$

In summary, using range differences as the basic "measured" quantity eliminates all first order errors in the determination of the distance $|\vec{D}|$.

Using one pulse to cover more than one corner cube (or transponder) unfortunately raises a power problem. From a signal to noise ratio vantage point it is preferable to send a single beam out at a time to one ground station. This does, however, create a problem by introducing the effects of errors 1 and 2 mentioned before. For instance, a two second time interval between pulses means their origin (spacecraft position) has separated in space by say 15 km. This means the orbit error introduced by those 15 km along the orbit pass will now increase the error in the determination of $|\vec{D}|$. The bias error in range, from firing to one cube and the subsequent firing to the other cube will further directly influence the determination of $|\vec{D}|$. Thus the control of bias errors in the ranging system and the orbit error becomes an important systems design factor. A detailed analysis of this second possibility will subsequently be published.

II. MATHEMATICAL MODELS

1. DIRECT BASELINE ESTIMATION USING SIMULTANEOUS RANGING

During the estimation of the accuracy of the intersite vector distance $|\vec{D}|$ from scalar range difference measurements, $(\rho_{kj} - \rho_{kj+1})$, two coordinate systems are employed. The first is a local topocentric system, $Z_{3\times1}$, centered at the observer. The second is a geocentric earth fixed system, $U_{3\times1}$, to which spacecraft motion is related. The intersite vector distance $|\vec{D}|$ is defined in the same $Z_{3\times1}$ system in which a surveyor would work. The variation in $|\vec{D}|$ is the quantity needed to determine small (cm-range) horizontal and vertical displacements in the earth's upper crust.

*If ground transponders are used, this is not so simple, since the delay time in each transponder is not the same. (1 nanosec $\doteq$ 30 cm).

As defined in Ref. 2 (Kahn, Vonbun, 1966), the Z-coordinate system is centered at the observer with the

z_1 -axis directed towards local east
z_2 -axis directed towards local north
z_3 -axis directed along normal to local horizon plane

In order to relate the local $Z_{3\times 1}$ coordinate system with the dynamics of a spacecraft, a transformation into a geocentric earth fixed system must be made. This system, named the U-coordinate system, is defined as follows:

u_1 -axis directed towards Greenwich Meridian
u_2 -axis normal to (u_1, u_3) axes
u_3 -axis directed along earth's axis of rotation.

The transformation needed later from the $Z_{(3\times 1)}$-coordinate system into the U-coordinate system is as follows (see reference 2):

$$Z_{(3\times 1)} = R_1(\pi/2 - \phi)\, R_3(\pi/2 + \lambda)\, [U_{(3\times 1)} - S_{(3\times 1)}] \qquad (1)$$

where

$S_{(3\times 1)}$: is the position vector of the observer in the U-system

and

$R_1(\pi/2 - \phi)$, $R_3(\pi/2 + \lambda)$: are rotation matrices,

The vector $\bar{D}$ is now to be computed from range difference measurements which are scalar quantities. It is to be shown how $\bar{D}$ is computed. The basic measurement, slant range ρ, from the ground stations to the spacecraft or vice versa can be expressed in terms of the coordinate systems defined in equation 1 in matrix notation as follows:

$$\rho = (Z^T Z)^{1/2} \qquad (2)$$

where

$$Z^T Z = U^T U + S^T S - 2 S^T U$$

and $(U^T U)^{1/2}$ = geocentric distance to spacecraft
$(S^T S)^{1/2}$ = geocentric distance to ground station

By assuming that at time t_k, a signal is sent out by the ranging system onboard the spacecraft and received at the j^{th} and $(j + 1)^{th}$ station. The range at each station is by (2) expressed as follows:

$$\rho_{kj} = [(U^T U)_k + (S^T S)_j - 2S_j^T U_k]^{1/2} \quad (3a)$$

$$\rho_{kj+1} = [(U^T U)_k + (S^T S)_{j+1} - 2S_{j+1}^T U_k]^{1/2} \quad (3b)$$

with similar expressions as (3a) and (3b) resulting for times $t_{k+1}, t_{k+2}, \ldots,$

The range difference measurement is obtained by subtracting equation 3b from 3a which then represents the fundamental observation equation, that is:

$$(\rho_{kj} - \rho_{kj+1}) = [(U^T U)_k + (S^T S)_j - 2S_j^T U_k]^{1/2} - [(U^T U)_k + (S^T S)_{j+1} - 2S_{j+1}^T U_k]^{1/2} \quad (4)$$

In arriving at equation 4, the assumption made is that both stations, the j^{th} and $(j + 1)^{th}$ simultaneously observe the laser pulse at time t_k.

Since the range difference measurement is subject to errors as all measurements are, and since $\vec{D}$ is derived from these measurements, $\vec{D}$ is also subject to errors. We estimate the errors of $\vec{D}$ due to errors in the range difference measurements $(\rho_{kj} - \rho_{kj+1})$. To do so, standard linear estimation theory (Ref. 3) is used for a set of k observations of the same cubes. That is, by varying (4) and using first order terms of the Taylor expansion one obtains:

$$\tilde{y}_{(k\times 1)} = A_{(k\times 6)} \underset{(6\times 1)}{\tilde{x}_0} + B_{(k\times 3)} \underset{(3\times 1)}{\tilde{s}_j} + C_{(k\times 3)} \underset{(3\times 1)}{\tilde{s}_{j+1}} + \varepsilon_{(k\times 1)} \quad (5)$$

where

$$\tilde{y}_{(k\times 1)} \equiv \delta(\rho_{kj} - \rho_{kj+1})_{(k\times 1)}$$

$$A_{(k\times 6)} \equiv \left\{\frac{1}{\rho_{kj}} [U_k - S_j]^T - \frac{1}{\rho_{kj+1}} [U_k - S_{j+1}]^T\right\} \left[\frac{\partial U_k}{\partial U_0}\right]$$

$$\underset{(6\times 1)}{\tilde{x}_0} \equiv \underset{(6\times 1)}{\delta U_0}$$

$$B_{(k\times 3)} \equiv -\frac{1}{\rho_{kj}} [U_k - S_j]^T$$

$$\underset{(3\times 1)}{\tilde{s}_j} \equiv \delta S_j$$

$$C_{(k\times 3)} \equiv \frac{1}{\rho_{kj+1}} [U_k - S_{j+1}]^T$$

$$\underset{(3\times 1)}{\tilde{s}_{j+1}} \equiv \delta S_{j+1}$$

ϵ $\equiv$ vector of observation errors.

2. INTERSITE DISTANCE DETERMINATION AND ITS ACCURACY

An estimation of the intersite distance between two ground emplaced corner cube and their errors is obtained from range difference measurements by using the standard least squares estimation technique (Ref. 3, 6, 7). In order to minimize the effects due to geopotential uncertainties, multiple short orbital arcs of 1 to 1-1/2 revolutions are used. In the least squares process, estimates are to be obtained simultaneously for each orbital arc's state vector as well as the coordinates of the corner cubes. The intersite distances and their errors are computed by a least squares solution. A mathematical description of the estimation process now follows.

The generalized matrix equation which represents all the observation equations obtained from simultaneous ranging from the spacecraft to two ground emplaced laser retroreflectors reads as follows:

$$\tilde{Y}_{\ell \times 1} = \Gamma_{\ell \times (6r+6)} T_{(6r+6) \times 1} + V_{\ell \times 1} \tag{6}$$

where

r = number of different orbital arcs for which the state vector is to be estimated $r = 1, 2, \ldots p$;

$$p \geq 1$$

$\ell = \sum_{i=1}^{r} k_i$; k_i = total number of observations associated with each orbital arc.

$$\ell >>> (6r + 6)$$

$$\tilde{Y}_{\ell \times 1} = \begin{bmatrix} \tilde{Y}_{1(k_1 \times 1)} \\ \hdashline \cdot \\ \cdot \\ \cdot \\ \hdashline \tilde{Y}_{r\,(k_r \times 1)} \end{bmatrix}_{\ell \times 1} ;$$

Vector of all measurement residuals observed during the entire 'SPEAR' data collection period.

$$\Gamma_{\ell \times (6r+6)} = \begin{bmatrix} \underset{(k_1 \times 6)}{A_1} & \underset{(k_1 \times 6)}{0} & \underset{(k_1 \times 6)}{0} & \cdots & \underset{(k_1 \times 6)}{0} & \underset{(k_1 \times 3)}{B_1} & \underset{(k_1 \times 3)}{C_1} \\ \underset{(k_2 \times 6)}{0} & \underset{(k_2 \times 6)}{A_2} & \underset{(k_2 \times 6)}{0} & \cdots & \underset{(k_2 \times 6)}{0} & \underset{(k_2 \times 3)}{B_2} & \underset{(k_2 \times 3)}{C_2} \\ \underset{(k_3 \times 6)}{0} & 0 & \underset{(k_3 \times 6)}{A_3} & \cdots & 0 & \underset{(k_3 \times 3)}{B_3} & \underset{(k_3 \times 3)}{C_3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \underset{(k_r \times 6)}{0} & 0 & 0 & \cdots & \underset{(k_r \times 6)}{A_r} & \underset{(k_r \times 3)}{B_r} & \underset{(k_r \times 3)}{C_r} \end{bmatrix}_{\ell \times (6r+6)}$$

Matrix of sensitivity coefficients associated with each measurement.

$$
T_{(6r+6)\times 1} \equiv \begin{bmatrix} \tilde{x}_0^1 \\ (6\times 1) \\ \hline \vdots \\ \hline \tilde{x}_0^r \\ (6\times 1) \\ \hline \tilde{s}_{j\,(3\times 1)} \\ \hline \tilde{s}_{j+1} \\ (3\times 1) \end{bmatrix}_{(6r+6)\times 1}
$$

Vector of error sources which are major contributors to the measurement residuals. These error sources are the states associated with each orbital arc used for calculation of measured minus calculated range differences (i.e. residuals) and corrections to the corner cube coordinates. This vector is estimated by the least square process.

$$
V_{\ell\times 1} \equiv \begin{bmatrix} \varepsilon_1 \\ (k_1\times 1) \\ \hline \vdots \\ \hline \varepsilon_r \\ (k_r\times 1) \end{bmatrix}_{(\ell\times 1)}
$$

Vector of residuals which reflect the neglect of second order terms of Taylor expansion made for representing the errors in the measurements.

The estimate of the vector T is obtained from ℓ range difference measurements with $\ell >> 1$ and a priori information about the state of each orbital arc and station survey. That is:

$$
T_{(6r+6)\times 1} = \underbrace{[\Gamma^T W^{-1}\Gamma + W_{T_0}^{-1}]^{-1}}_{(6r+6)\times(6r+6)} \; \underbrace{[\Gamma^T W^{-1}\tilde{Y} + W_{T_0}^{-1} T_0]}_{(6r+6)\times 1} \qquad (7)
$$

where

W^{-1} = A priori covariance matrix associated with system accuracy.

$W_{T_0}^{-1}$ = A priori covariance matrix associated with orbit accuracy and station survey for each retroreflector.

The errors associated with the estimate of $T_{\ell \times 1}$ are obtained from the covariance matrix given by the following equations

$$E(TT^T) = [\Gamma^T W^{-1} \Gamma + W_{T_0}^{-1}]^{-1}_{(6r+6)\times(6r+6)} \tag{8}$$

Equations (7) and (8) are now used to (a) compute the intersite distance and (b) its errors. The intersite distance, that is, the difference between the corner cube position vectors $(\tilde{s}_j - \tilde{s}_{j+1})$ is obtained by a coordinate transformation of the vector $\mathbf{T}$. This transformation reads as follows;

$$\vec{D} \equiv (\tilde{s}_j - \tilde{s}_{j+1})_{3\times 1} = Q_{3\times(6r+6)} \mathbf{T}_{(6r+6)\times 1} \tag{9}$$

where

$$\mathbf{Q} \equiv \left[\mathbf{0}_{(3\times 6r)} \,\middle|\, \mathbf{I}_{(3\times 3)} \,\middle|\, -\mathbf{I}_{(3\times 3)} \right]_{3\times(6r+6)}$$

$\mathbf{0}$ ≡ a null matrix

$\mathbf{I}$ ≡ the identity matrix

The error in the intersite distance is now given by the covariance matrix of the intersite distances, that is:

$$E[(\tilde{s}_j - \tilde{s}_{j+1})(\tilde{s}_j - \tilde{s}_{j+1})^T]_{(3\times 3)} = \{QE(TT^T)\,Q^T\}_{(3\times 3)} \tag{10}$$

which upon introducing Equation 8 finally reads:

$$E[(\tilde{s}_j - \tilde{s}_{j+1})(\tilde{s}_j - \tilde{s}_{j+1})^T]_{(3\times 3)} = \{Q[\Gamma^T W^{-1}\Gamma + W_{T_0}^{-1}]^{-1}\,Q^T\}_{(3\times 3)} \tag{11}$$

Equation 11 representing the intersite distance error, is the basic equation used for the SPEAR System error analysis. Numerical results using (11) are depicted in Figures 4 & 5.

III. ERROR ANALYSIS RESULTS

In order to evaluate the accuracy with which intersite distances can be determined using the SPEAR concept, a parametric error analysis study was performed utilizing the mathematical model described by Equation 11. For this study very moderate systems errors were assumed. The results obtained, are presented in Figures 4 and 5. All assumptions concerning the source of errors and their magnitude are indicated on each figure.

Although orbital errors up to 200 meters were used in the error analysis, their contribution to the horizontal and vertical components of $\vec{D}$ consistently remained negligible. Furthermore, survey errors associated with the location of the retroreflectors also do not significantly change the errors in the vertical and horizontal components of $\vec{D}$. This means that no extra care has to be taken in the emplacement of ground equipment.

Figure 5 shows the dependency of the vertical and horizontal distance errors on the noise of the ranging systems used. As can be seen, a 5 cm system is adequate, producing errors in the 0.7 and 1.3 cm ranges. As stated earlier, 5 cm ground laser ranging systems are now operational at Goddard Space Flight Center, thus no difficulties are foreseen in achieving this precision for a spaceborne laser system.

IV. PRACTICAL APPLICATIONS

The SPEAR system can be developed as a payload for the shuttle applications program. As a matter of fact, a breadboard model is under development at Goddard (Ref. 5).

Such a system can easily be utilized in detecting small relative variations of the Earth's upper crust in the 0.3 to 0.5 cm/yr range. Monitoring, is thus possible, of land subsidance as it occurs at coastal regions of Florida and Texas, as well as construction sites such as dams and even large buildings. Dilatency, the vertical uplift along seismic zones, can further be followed and thus studied as a precursory effect which takes place before many shallow foci earthquakes (Ref. 1, 3, 4). The San Andreas fault area and the Niigata area in Japan are examples.

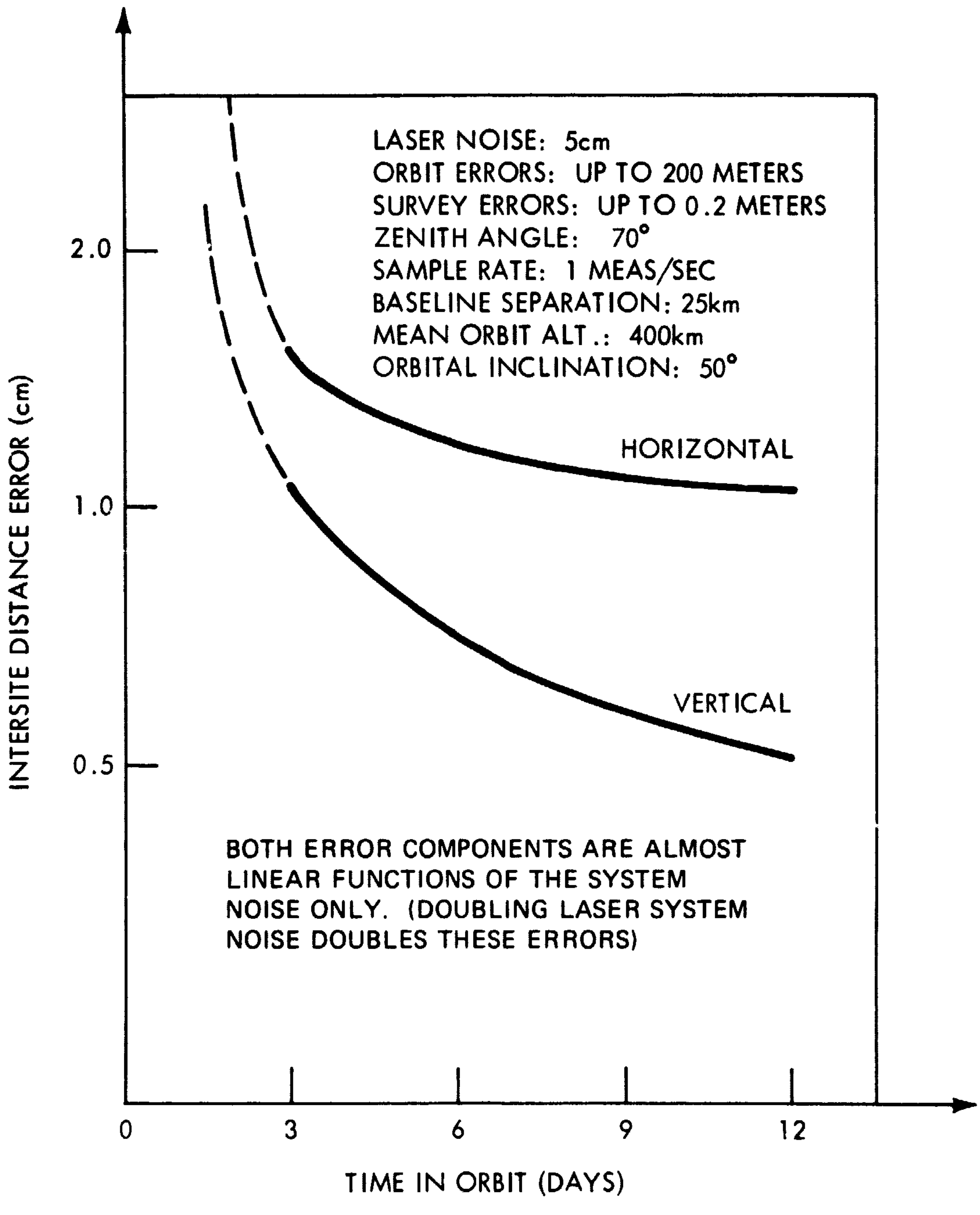

Figure 4. Intersite Distance Errors vs. Time in Orbit

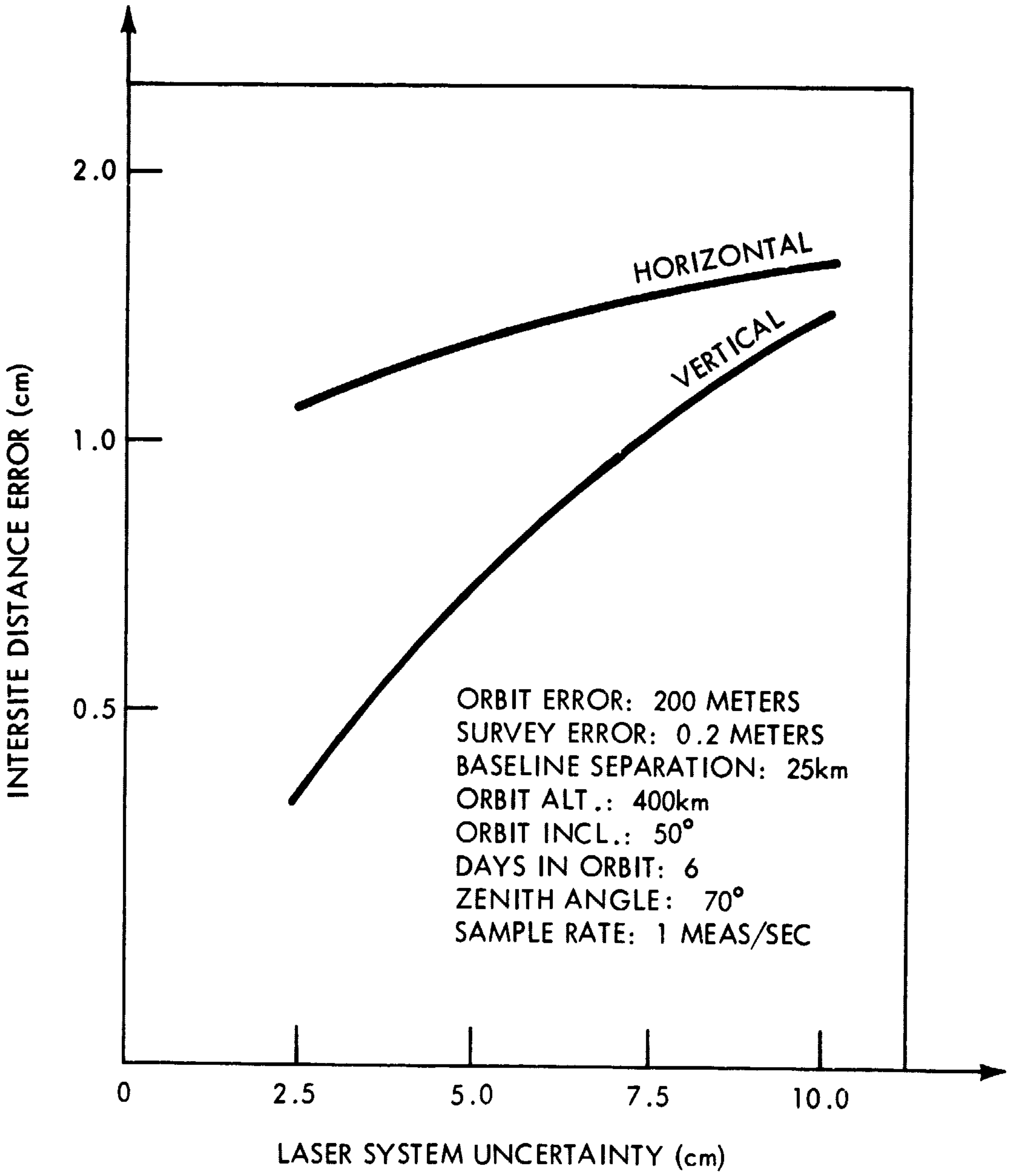

Figure 5. Intersite Distance Errors vs. Laser System Uncertainty

V. CONCLUSIONS

This paper has shown that the SPEAR system can be used to determine intersite distances up to 15 km depending only on the beam width or beam splitting of a spaceborne laser system within a 6 day shuttle mission to a precision 0.5 cm to 1.5 cm (assuming 50% cloud coverage). As anticipated, (a) survey errors (up to 0.25 m), (b) orbital errors (up to 200 m), and (c) range bias errors (many meters) play only a minor role, making the system potentially useful for many practical applications where very small relative motions are to be monitored.

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