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#### Abstract

A microscopic model has been developed to study the microwave emission from snow. In this model the individual snow particles are considered to be the scattering centers. Mie scattering theory for spherical particles is then used to compute the volume scattering and extinction coefficients of the closely packed scattering spheres, which are assumed not to interact coherently. The results of the computations show significant volume scattering effects in the microwave region which result in low observed emissivities from cold, dry snow. In the case of wet snow, the microwave emissivities are increased considerably, in agreement with earlier experimental observations in which the brightness temperatures have increased significantly at the onset of melting.


## I. INTRODUCTION

Recently, satellite detection of melting snow and melting ice by using a combination of visible and near infrared data has achieved some success (Strong, et al., 1971). The utility of this technique is limited by frequent cloud cover over areas of interest. The Electrically Scanned Microwave Radiometers (ESMR) on board the NIMBUS 5 and 6 satellites provides an additional ability to sense through the cloudy sky. Therefore, it is needed to develop an applicable analytical model for explanation of the microwave emission from snow.

A microwave model has been developed to study the microwave emission from snow and glacier ice (Chang, et al., 1975). It was assumed that the snow field or snow cover consists of closely packed scattering spheres which do not interact coherently. Comparing with the macroscopic multilayer or variable dielectric model, this method in addition provides the scattering processes in the radiative transfer equation. Since each individual snow grain is considered as a scattering center, the Mie scattering theory (Stratton, 1941) was utilized to compute the scattering cross-sections. In this paper, the particular signature of melting snow is studied.

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## II. THEORETICAL APPROACH

The rigorous solution for the diffraction of a plane monochromatic wave by a homogeneous dielectric sphere was first obtained by Mie (1908). The more detailed development of the theory can be found in the literature (Stratton, 1941; Van de Hulst, 1957).

Consider a uniform dielectric sphere of radius $r$ and the complex index of refraction $n$ in the presence of a linearly polarized plane wave with wavelength $\lambda$ and electric field vector $E_{1}$. The electric field $E$ observed at a distance $R \gg r$ from the sphere is the vector sum of incident field and scattered field. It can be expressed as:

$$
\begin{equation*}
E=E_{1}+\bar{S} E_{1} \frac{e^{1 k R}}{i k R} \tag{1}
\end{equation*}
$$

where $k$ is the propergation constant $2 \pi / \lambda$ and $\vec{S}$ is the scattering matrix. The solution takes the form of an expansion in spherical wave functions with complex scattering coefficients $a_{m}$ and $b_{m}$ determined by the boundary and the angular dependence of each multipole. The extinction cross-sections for spherical ice particles with refractive index $n=1.78$ +i 0.0024 , which corresponds to the values in the centimeter wavelength range of pure water ice at $0^{\circ} \mathrm{C}$ (Sweeny and Colbeck, 1974) is calculated for several wavelengths and particle radii. The results of the calculations are shown in Figure 1.

In order to develop the model for the case of melting ice sphere, it is assumed that the sphere consists of a central core of ice and a surrounding shell of water. The solution of scattering of electromagnetic waves from these concentric spheres has been solved by Aden and Kerker (1951). In this study the thickness of the water layer is set to be one tenth of the radius of the sphere. The index of refraction for water is calculated according to the results of Lane and Saxton (1952). The extinction crosssection for water coated ice particle with refraction index of $n=1.78$ +i 0.0024 for ice cover, several wavelengths and particle radii are shown in Figure 2.

The Rayleigh-Jeans approximation for the intensity of thermal radiation from a blackbody is applicable at microwave frequencies and at temperatures typical of the earth surface and its atmosphere; thus the radiative transfer equation (Chandrasekhar, 1950), with axial symmetry, may be written as:

$$
\begin{align*}
\cos \theta \frac{d T_{B}(\theta)}{d X}+\gamma_{A B S}\left(T_{B}(\theta)\right. & -T(X))  \tag{2}\\
& =\frac{\gamma_{S C A}}{2} \int_{0}^{\pi} T_{B}\left(\theta_{S}\right) F\left(\theta, \theta_{S}\right) \sin \theta_{S} d \theta_{S}-\gamma_{S C A} T_{B}(\theta)
\end{align*}
$$



Fig. 1-Extinction cross-section as a function of microwave wavelength for several snow particle radii.


Fig. 2-Extinction cross-section as a function of microwave wavelength for several wet snow particle radii.

For a non-scattering medium, the left-hand side of the equation would be equated to zero; the right-hand side represents the angular distribution of radiation resulting from the scattering. To solve this equation the snow layer is first divided into a number of smaller layers. An initial estimate is then made of the brightness temperature for the upwelling radiance from the lower boundary. This upwelling component is then scattered by the medium of next small layer and the radiance redistributed according to the scattering phase function. The upwelling brightness temperature can then be calculated from the bottom layer upward order. The downwelling components are then similarly calculated from the top most layer downward. The reflected downwelling radiance from the bottom underlying surface attribute to non-neglectable amount of radiance in the microwave observation, it is necessary to take into account the effect of the reflecting surface. Unfortunately, then is little data or theory available to help in determining this surface effect, so approximations are used. The simplest approximation is to assume a specular surface. On the other hand, the surface can be assumed infinitely rough and apply the Lambertian approximation (Peake, et al., 1966). The reflected radiances are then added to the upwelling radiance and the process is repeated until a satisfactory convergence is obtained. The method of solution is a variation of the Gauss-Seidal interaction (Hildebrand, 1946).

## III. COMPUTATIONAL RESULTS

To illustrate the scattering effects on the upwelling microwave brightness temperature, a simplified snow field model is constructed. A layer of uniformly sized spheres (the snow cover) is situated on top of the earth. The earth surface is assumed to be a Lambertian surface with index of reflection of $1.75+\mathrm{i} 0.03$ for dry soil surface and $3.0+\mathrm{i} 0.1$ for wet soil surface. The particle density $N(r)$ for different particle radius $r$ is represented analytically as

$$
\begin{equation*}
N(r)=\left(\frac{1}{2 r}\right)^{3} \tag{3}
\end{equation*}
$$

The particle density corresponds to a typical moderately packed snow with mass density ( $4 / 3 \pi \mathrm{r}^{3} \mathrm{~N}(\mathrm{r}) \rho$ ) of $470 \mathrm{~kg} / \mathrm{m}^{3}$. The physical temperature for the snow layer is assumed to be uniform at $273^{\circ} \mathrm{K}$.

To demonstrate the effect of snow melting in the field, the model is chosen to be a snow particle coated with a thin layer of water. The thickness of the water layer is one tenth of the radius of the snow particle. A set of brightness are then calculated by using the formula derived in Section II. The results for dry and wet snow layer at wavelength of 1.55 cm are displayed in Figures 3, 4, and 5.

When the particle size parameter $\alpha \equiv 2 \pi r / \lambda$ is small, the calculated extinction cross-section for the melting snow particle is approximately


Fig. 3-Surface brightness temperature as a function of snow particle radius. The thicknesses of the snow layer are: (a) 10 cm , (b) 50 cm , (c) 1 m , (d) 2 m and (e) 5 m .
two orders of magnitude larger than that of dry snow particle. The increase of $\sigma_{\mathrm{SCA}}$ the scattering cross-section for melting snow particle is not as fast as $\sigma_{\mathrm{EXT}}$ the extinction cross-section, therefore the ratio $\sigma_{\mathrm{SCA}}$ / $\sigma_{\mathrm{EXT}}$ decreases for the melting snow particle as compared to the dry snow particle. Since less radiance is scattered by the snow particle; the upwelling brightness temperature becomes higher for melting snow temperature becomes higher for melting snow (Edgerton, et al., 1971; Gloersen, et al., 1974).

## IV. CONCLUSIONS

A microscopic model, consisting of the individual snow particles as the scattering centers for Mie scattering, has been used to explain


Fig. 4-Surface brightness temperature as a function of wet snow particle radius on dry earth surface. The thicknesses of the snow layer are: (a) 10 cm , (b) 50 cm , (c) 1 m , (d) 2 m and (e) 5 m .


Fig. 5-Surface brightness temperature as a function of wet snow particle radius on wet earth surface. The thicknesses of the snow layer are: (a) 10 cm , (b) 50 cm , (c) 1 m , (d) 2 m and (e) 5 m .
qualitatively the brightness temperatures over dry and wet snowfields. The computational results show that scattering from the individual snow particles is a dominant factor in the measured upwelling brightness temperature in the case of dry snow. When the snow thickness is larger than the penetration depth, the earth surface, whether dry or wet, will not contribute to the upwelling brightness temperature. For wet snow layers of 50 cm or more, the brightness temperatures approach the physical temperature of the snow layer. The distinction between these two signatures may be used to detect the dry and wet snow cover.

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