A COMPUTER PROGRAM FOR FITTING SMOOTH SURFACES TO THREE-DIMENSIONAL AIRCRAFT CONFIGURATIONS

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SUMMARY

This paper describes a computer program developed at the Langley Research Center to fit smooth surfaces to the component parts of three-dimensional aircraft configurations. The resulting equation definition of an aircraft numerical model is useful in obtaining continuous two-dimensional cross section plots in arbitrarily defined planes, local tangents, enriched surface plots and other pertinent geometric information.

At Langley Research Center and other government and industrial facilities, the geometry organization used as input to the program has become known as the Harris Wave Drag Geometry. An early version of this point surface definition was introduced in a NASA TM X-947. (See ref. 1.)

INTRODUCTION

An important part of the design, analysis, and construction of aerospace vehicles, as well as ships, automobiles, etc., is the construction of surface models for the required study. Mathematical models are the most useful because of their versatility and exactness of definition. Many more different mathematical models may be analyzed by a computer program than would be possible by any other method within a given time frame.

A set of planes defined by points to approximate a curved surface is the simplest type of mathematical model. A large number of points on the surface must be given to achieve an accurate definition using a set of planes.

The surface fitting technique used in this paper is based on approximating complex surfaces by piecing together smaller surface "patches." The surface equations were conceived and developed by Steven Coons. The term "Coons' Patch" is associated with this piecewise definition. (See ref. 2.) The computer program under discussion uses the bicubic form of the patch equation. (See ref. 3.) This form of the patch equation requires 48 parameters consisting of the coordinate position of the corners of the patch (12 parameters), the derivatives of the variables at the corners with respect to the parametric

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independent variables (24 parameters), and the cross derivatives with respect to the parametric variables (12 parameters).

Although patch surface definition has been applied in many computer programs, the process of determining the derivatives has been varied. One process has been to apply phantom points near the corner points to approximate the derivatives by finite differencing. Many programs, including the one described herein, set the cross derivatives equal to zero. The LRC surface program uses parametric cubic spline functions to approximate the derivatives of the space variables with respect to the parametric independent variables.

The organization of the geometry input data to the program is identical to the geometry used for several standardized aerodynamic analysis computer programs. (See ref. 4.) By using the same input format throughout the design and analysis procedure, the designer can be assured of a consistent numerical model.

It should be pointed out that the program is not limited to aircraft configurations since a component part such as a fuselage can be some other object. Also, it is a simple matter to substitute an alternate data input format if this might prove more convenient.

Current program options that are available are to present on an XY plotter or on a cathode ray tube orthographic projections and cross section or contour plots. The patch equations are used to compute the required vectors.

A complete description of the program and its usage is documented in NASA TM X-3206. (See ref. 5.)

SYMBOLS

A,B,C,D	parametric cubic spline coefficients
a,b,c,d	plane equation coefficients
$\bar{B}$	boundary matrix
G	matrix equation of the solution of a patch and a plane
L	chord length
M	blending function matrix
P	a vector whose components are functions of t
S	component of surface patch equation
t	independent variable in cubic equation

u,w	independent variables in patch equation
V	a vector whose components are functions of u and w
x,y,z	coordinates of a point
θ	pitch angle
ϕ	roll angle
ψ	yaw angle

PROBLEM DESCRIPTION AND METHOD OF SOLUTION

A functional definition of an aircraft numerical model is useful for obtaining pertinent geometric information for use in aeronautical analysis programs or for building models for wind-tunnel testing. A computer program has been developed to provide this functional definition in surface patch equations.

The organization of the input data for the program is broken down into various component parts of an aircraft configuration such as wing, fuselage, nacelle, vertical and horizontal tails. A typical input configuration is given in figure 1. Sufficient data points with no abrupt changes in curvature must be supplied as the modeling technique is designed to approximate a smooth surface.

A surface patch is a portion of surface bounded by space curves and is a function of two variables, u and w, which can have only the values between 0.0 and 1.0. It is necessary to supply 48 parameters for each patch equation. A 4×4 boundary matrix is constructed for each component of the patch equation (X,Y,Z). It is called a boundary matrix because the parameters are functions of the patch boundaries: corner points, corner derivatives and cross derivatives. The cross derivatives are set equal to zero in this application.

The boundary matrix is then pre- and postmultiplied by constant-valued blending functions and stored for further use in this form. The blending functions provide a blending effect from one surface patch to an adjoining patch. Each surface patch is represented in exactly the same matrix form which proves to be very convenient for computer processing. The basic patch equations are given in figure 2.

The corner derivatives for the surface patch equations are obtained by using a three-dimensional parametric cubic spline curve fit technique. The coordinate points describing the curve are expressed as cubic functions of one variable. Consecutive points in the u-direction are fitted in this manner and also in the w-direction.

In the curve fit method, a series of adjacent polynomial segments between each given point is used to represent the curve. The adjacent cubic segments are related by setting the second derivatives equal at the common points.

A matrix equation is developed where the slopes are the unknowns. The second derivatives at the first and last points on a curve are made equal to zero since there are two less equations than unknowns. Since the matrix equation is tridiagonal, the Thomas Algorithm, which is equivalent to Gaussian Elimination without pivoting, is used for the solution. The Thomas Algorithm is a rapid, recursive method for solving a tridiagonal system of equations. Figure 3 shows the basic equations used.

Parametric cubic splines were chosen because of their several advantages. Parametric curves are not sensitive to infinite slopes, spline curves approximate smoothness satisfactorily, and cubics are the lowest order polynomial able to twist through space.

To create orthographic projections of the given body surface, angles of yaw, pitch, and roll must be supplied. The three-dimensional rotation equations are applied directly to the patch equations, where the rotated patch equation has only two components for the desired paper plane.

The surface plots may be enriched to any degree. Two equations in w may be obtained by giving u a value between 0.0 and 1.0. These equations may be solved by giving w values between 0.0 and 1.0 for additional body surface lines in the w-direction. Similarly, two equations in u may be obtained by giving w a value between 0.0 and 1.0, and solving by varying u from 0.0 to 1.0 for additional curves in the u-direction.

A partial "hidden line" test may be used to delete points that face away from the viewer. A surface normal vector is computed from the rotated and projected patch equation at each requested point. The computed point is visible and plotted if the surface vector is positive. If the surface vector is negative, the point is not plotted. Figure 4 gives the basic equations for the surface plots and figure 5 presents some examples.

The simultaneous solution of the patch equations and the equation of a plane is used to provide cross section or contour plots. The cutting plane is defined by three points and may be at any orientation. The result is a 4×4 matrix composed of constant elements and may be evaluated for an equation in the two variables u and w. The problem then becomes that of finding the zeros of a cubic polynomial in one of the parametric variables. Since there are three zeros, the program has logic in it to pick one of the points. If an error is made, it normally stands out immediately. The points defining the curve of intersection are then rotated and translated so that the intersection plane coincides with the YZ-plane of the paper. Figure 6 gives the basic equations for obtaining the cross section or contour plots and some typical output plots are given in figure 7.

CONCLUDING REMARKS

A computer program has been written that fits smooth surfaces to the component parts of an aircraft configuration. The resulting surface patch equations can be useful anywhere an equation definition may be required. Program options currently provide plotting of the enriched surface at any desired viewing angle and the solution of a number of patches and a plane for generating cross section or contour plots through the given surfaces.

Although the program was written to accept a point surface definition of an aircraft as input, it is a simple matter to substitute another input format to describe another object.

The program has been used for on-line display on a cathode ray tube and to drive Gerber, Varian, and Calcomp plotting devices.

REFERENCES

1. Harris, Roy V., Jr.: An Analysis and Correlation of Aircraft Wave Drag. NASA TM X-947, 1964.

2. Coons, Steven A.: Surfaces for Computer-Aided Design of Space Forms. MAC-TR-41 (Contract No. AF-33(600)-42859), Massachusetts Inst. Technol., June 1967. (Available from DDC as AD 663 504.)

3. Eshleman, A. L.; and Meriwether, H. D.: Graphic Applications to Aerospace Structural Design Problems. Douglas Paper 4650, McDonnell Douglas Corp., Sept. 1967.

4. Craidon, Charlotte B.: Description of a Digital Computer Program for Airplane Configuration Plots. NASA TM X-2074, 1970.

5. Craidon, Charlotte B.: A Computer Program for Fitting Smooth Surfaces to an Aircraft Configuration and Other Three-Dimensional Geometries. NASA TM X-3206, 1975.

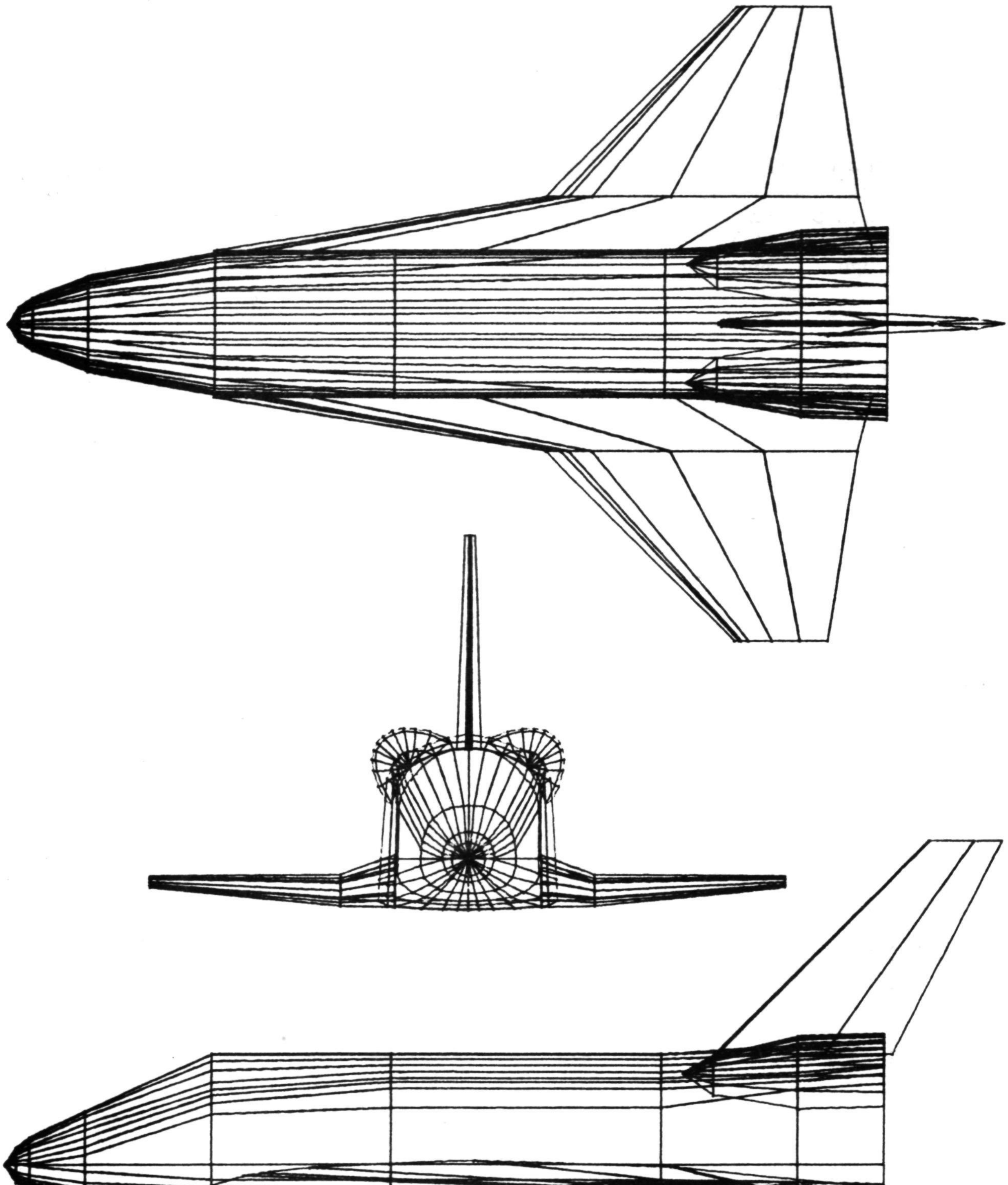
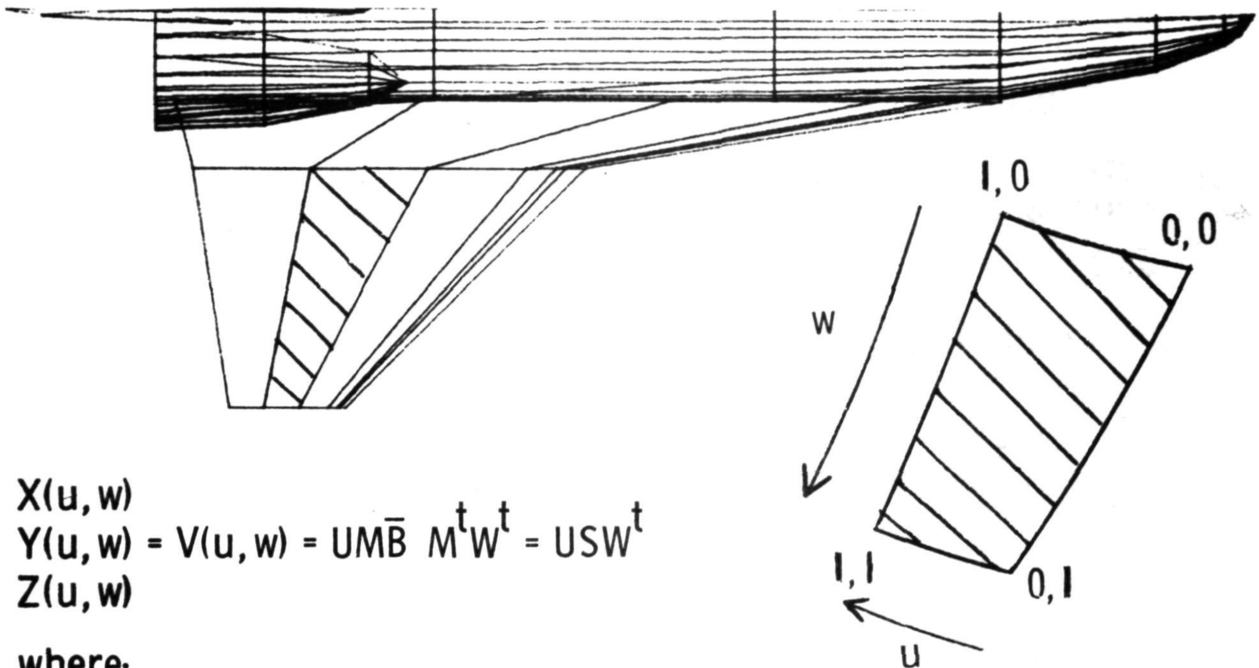


Figure 1. - A typical input configuration.



$$\left.\begin{array}{l} X(u,w) \\ Y(u,w) \\ Z(u,w) \end{array}\right. = V(u,w) = UM\bar{B}\ M^tW^t = USW^t$$

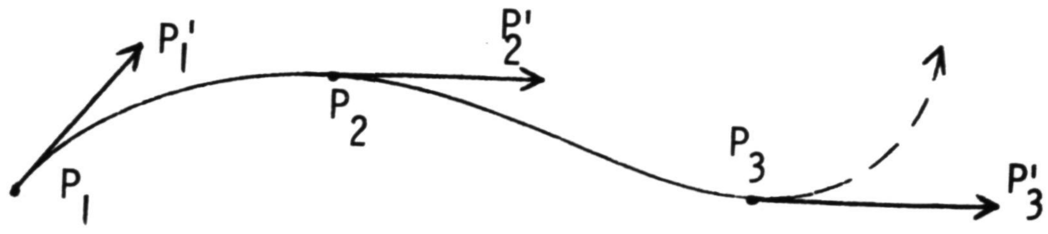
where:

$$U = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} w^3 & w^2 & w & 1 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} V(0,0) & V(0,1) & V_w(0,0) & V_w(0,1) \\ V(1,0) & V(1,1) & V_w(1,0) & V_w(1,1) \\ V_u(0,0) & V_u(0,1) & V_{uw}(0,0) & V_{uw}(0,1) \\ V_u(1,0) & V_u(1,1) & V_{uw}(1,0) & V_{uw}(1,1) \end{bmatrix}$$

Figure 2. - Surface patch equation.



Cubic Equation:

$$\left.\begin{array}{l} X(t) = \\ Y(t) = P(t) = At^3 + Bt^2 + Ct + D \\ Z(t) = \end{array}\right.$$

Cubic Coefficients in Terms of End Points and Slopes (for one segment):

$$A_1 = \frac{2(P_1 - P_2)}{L_1^3} + \frac{P_1'}{L_1^2} + \frac{P_2'}{L_1^2}$$

$$B_1 = \frac{3(P_2 - P_1)}{L_1^2} - \frac{2P_1'}{L_1} - \frac{P_2'}{L_1}$$

$$C_1 = P_1'$$

$$D_1 = P_1$$

System of Equations to Solve for Unknown Slopes:

$$\begin{bmatrix} L_2 & 2(L_1 + L_2) & L_1 & \bigcirc & \\ & L_3 & 2(L_2 + L_3) & L_2 \\ & \bigcirc & \bullet & \bullet \end{bmatrix} \mathrm{X} \begin{bmatrix} P_1' \\ P_2' \\ \bullet \\ \bullet \\ P_n' \end{bmatrix} = \begin{bmatrix} f(L_1, L_2, P_1, P_2, P_3) \\ f(L_2, L_3, P_2, P_3, P_4) \\ \bullet \end{bmatrix}$$

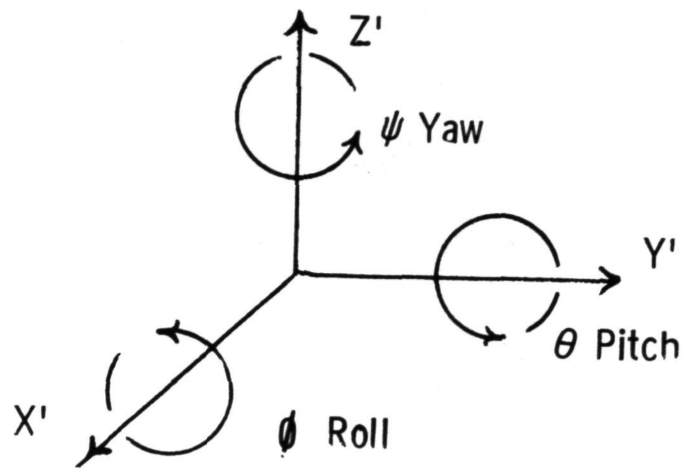
Figure 3. - Parametric cubic spline curve fitting.

Two Component Rotated Patch Equation:

$$S'_x = S_x f_1(\phi,\theta,\psi) + S_y f_2(\phi,\theta,\psi) + S_z f_3(\phi,\theta,\psi)$$

$$S'_y = S_x f_4(\phi,\theta,\psi) + S_y f_5(\phi,\theta,\psi) + S_z f_6(\phi,\theta,\psi)$$

where:



Enriched, Rotated Patch:

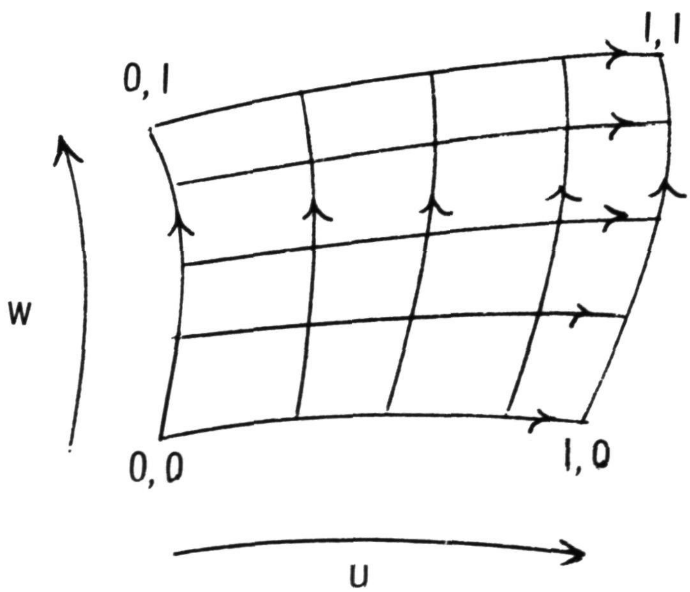


Figure 4. - Rotated and enriched patch.

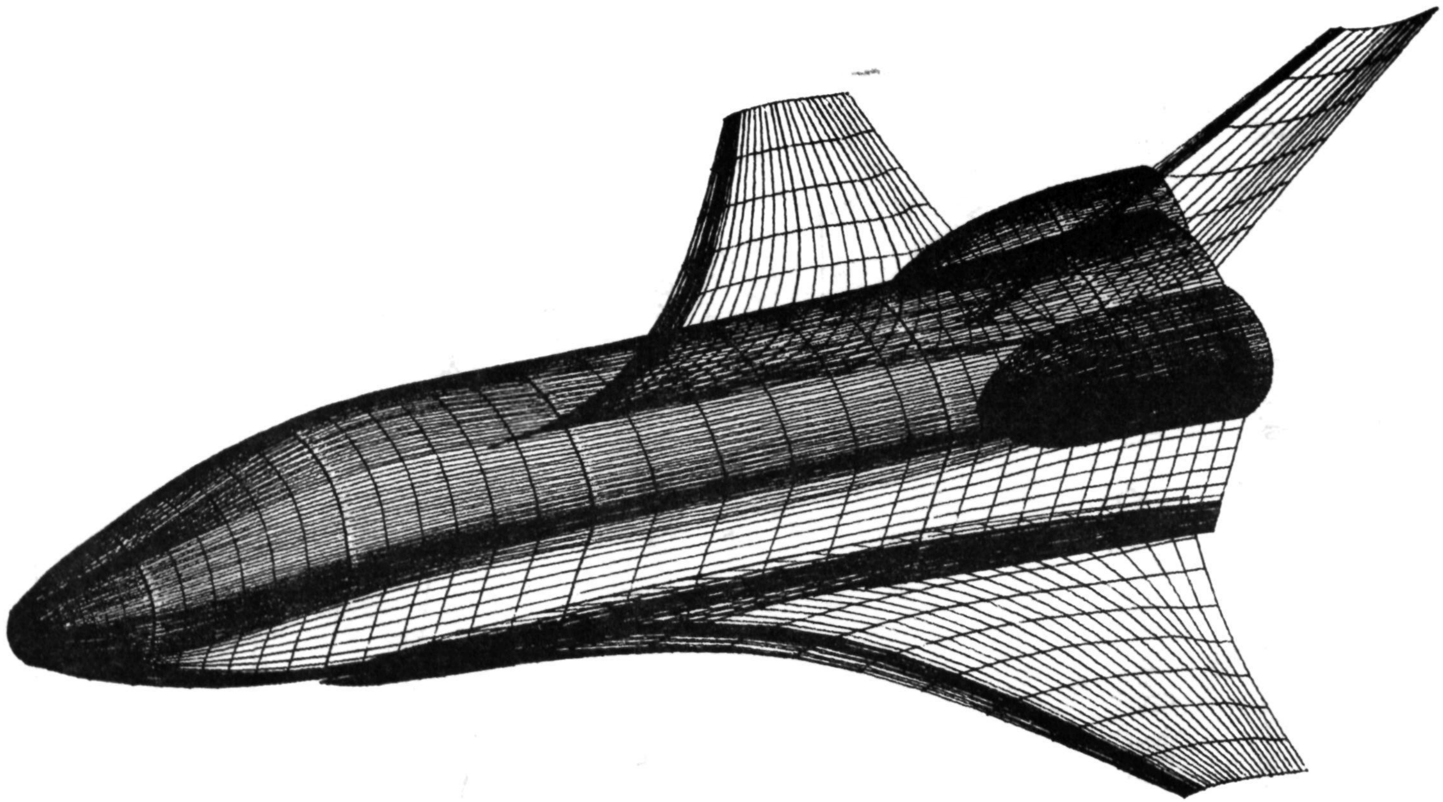


Figure 5. - Surface plot examples.

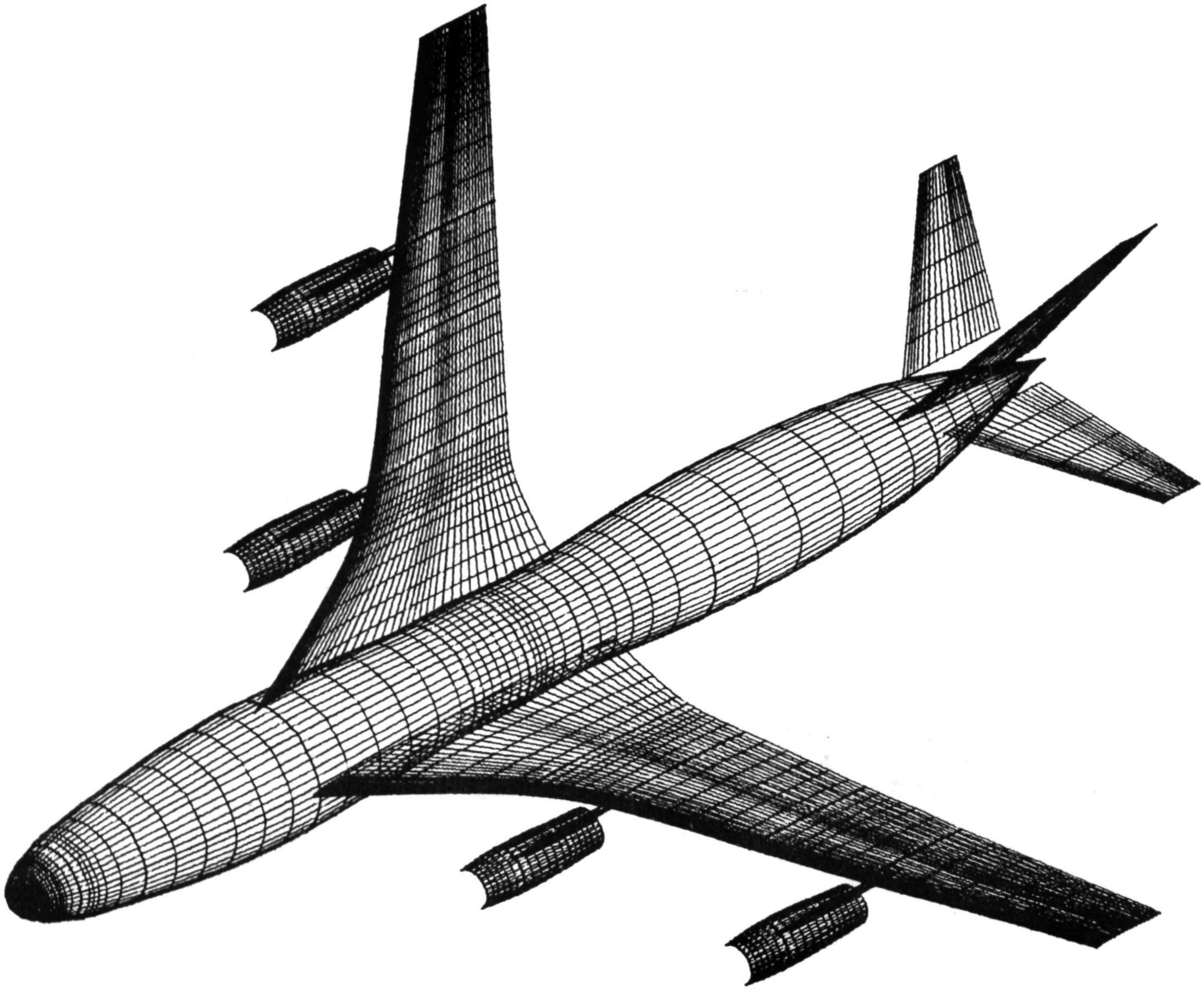
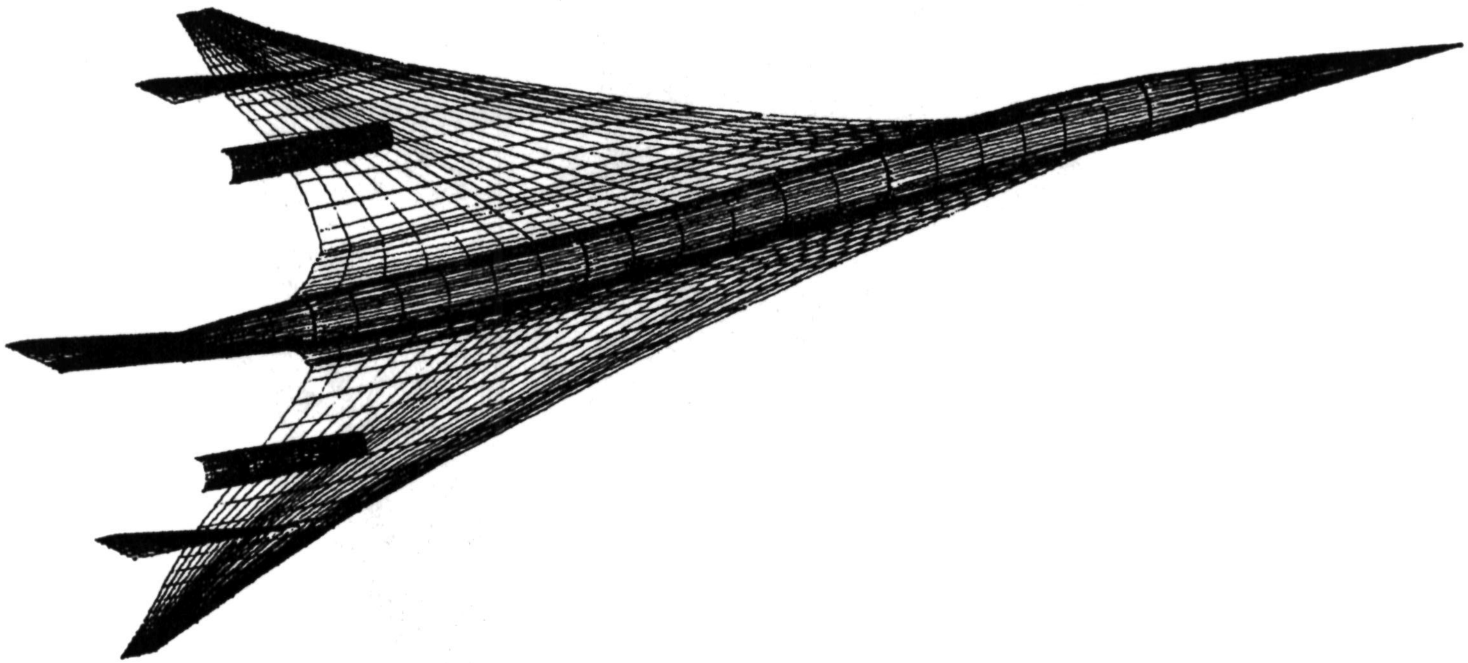


Figure 5. - Continued.



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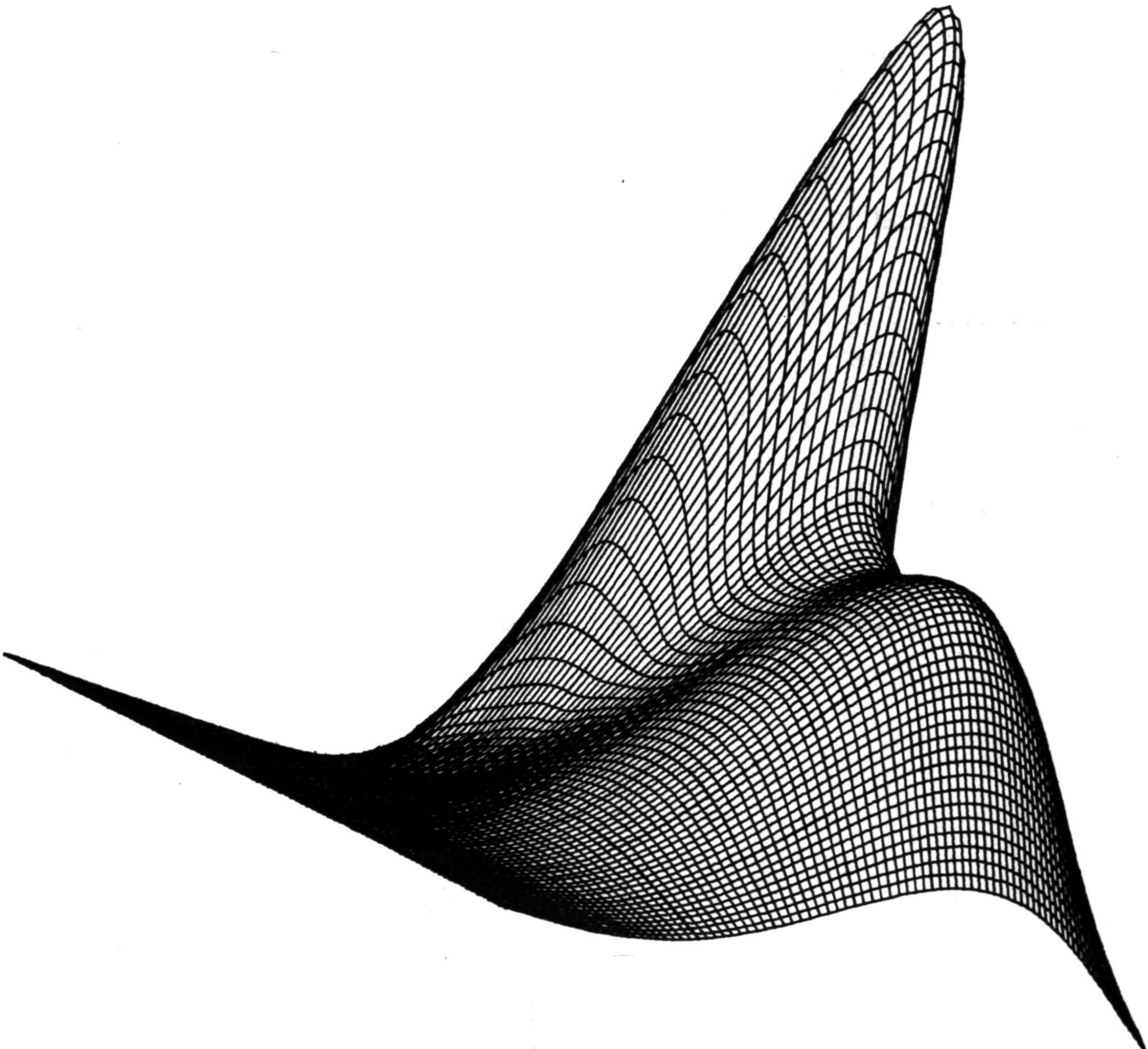

Figure 5. - Concluded.

Patch Equation:

$$V(u,w) = USW^t = \begin{matrix} X(u,w) \\ Y(u,w) \\ Z(u,w) \end{matrix} = \begin{matrix} U\,S_x\,W^t \\ U\,S_y\,W^t \\ U\,S_z\,W^t \end{matrix}$$

Equation of a Plane :

$$\alpha\, x + b\,y + c\,z - d = 0.$$

then:

$$U\left[\alpha\ S_x + b\,S_y + c\,S_z\right] W^t - d = 0.$$

with:

$$G = \alpha\ S_x + b\,S_y + c\,S_z$$

$$U\,G\,W^t = d = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} & & \\ & G & \\ & & \end{bmatrix} \begin{bmatrix} w^3 \\ w^2 \\ w \\ 1 \end{bmatrix}$$

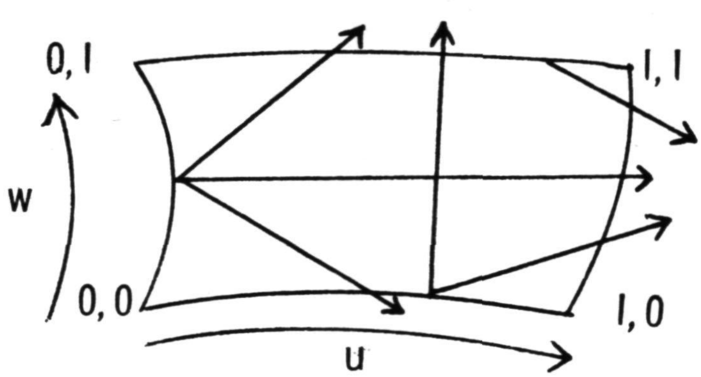

Figure 6. - Cross sections or contours through a patch.

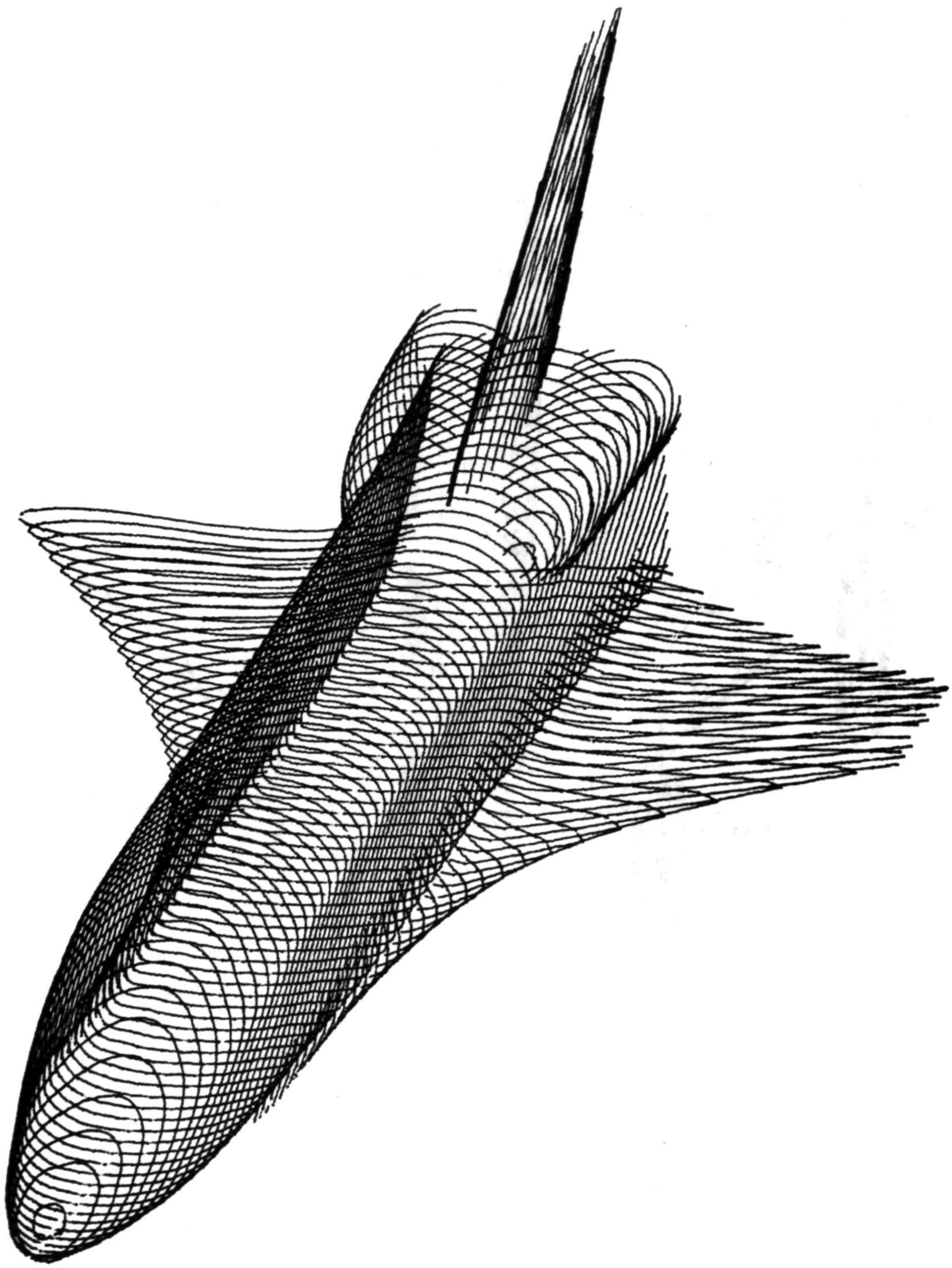

Figure 7. - Cross section plot examples.

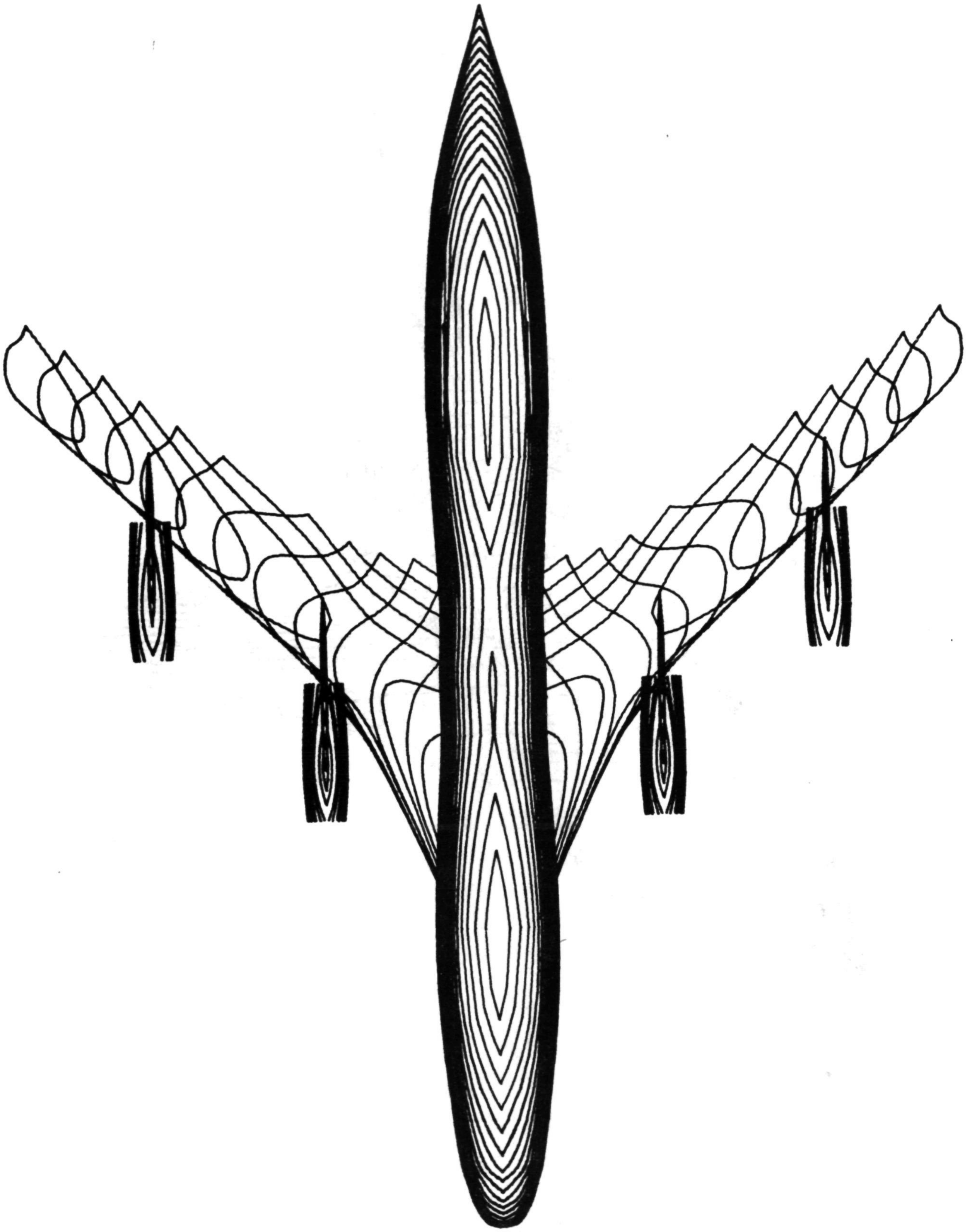

Figure 7. - Continued.

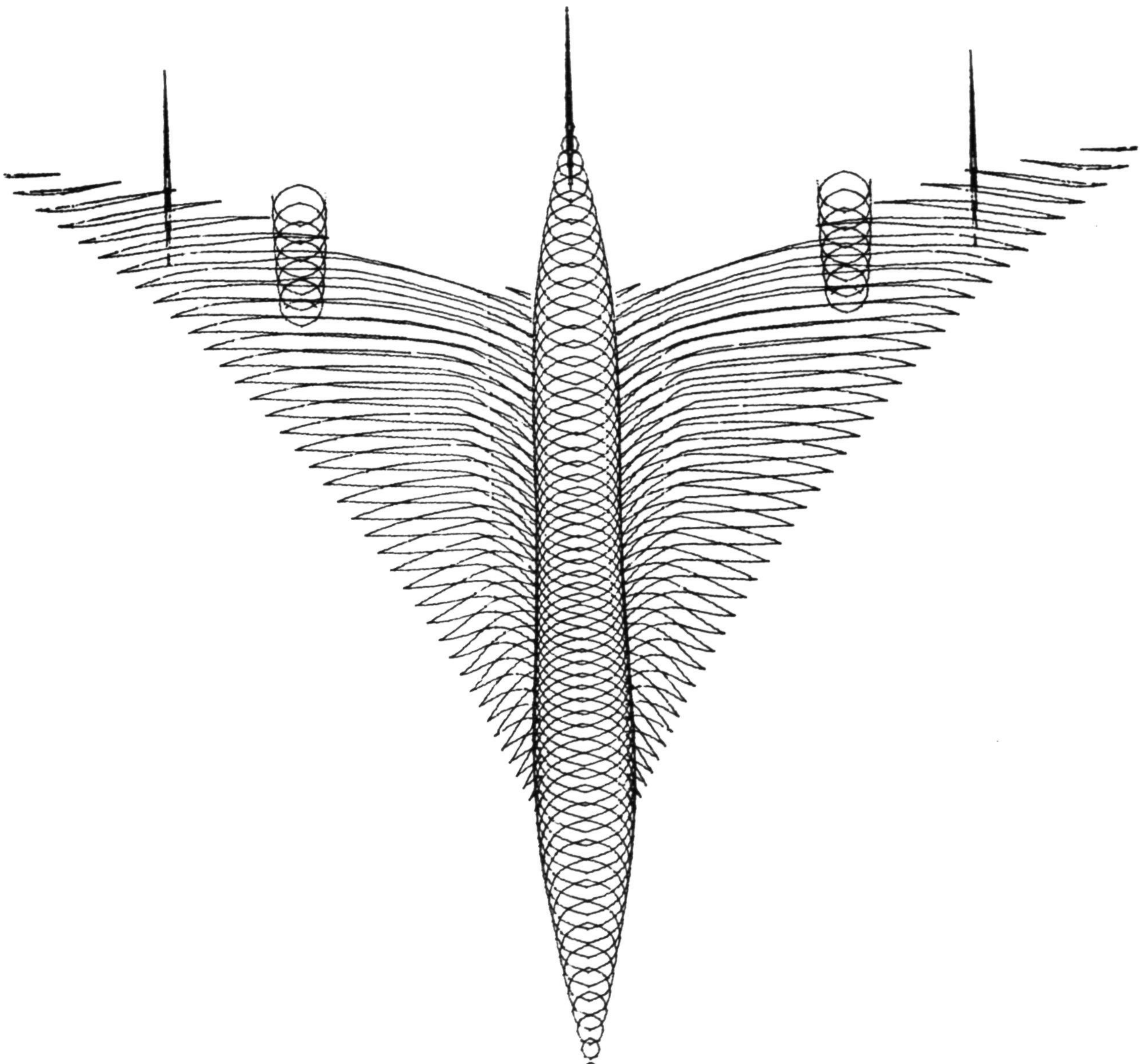

Figure 7. - Continued.

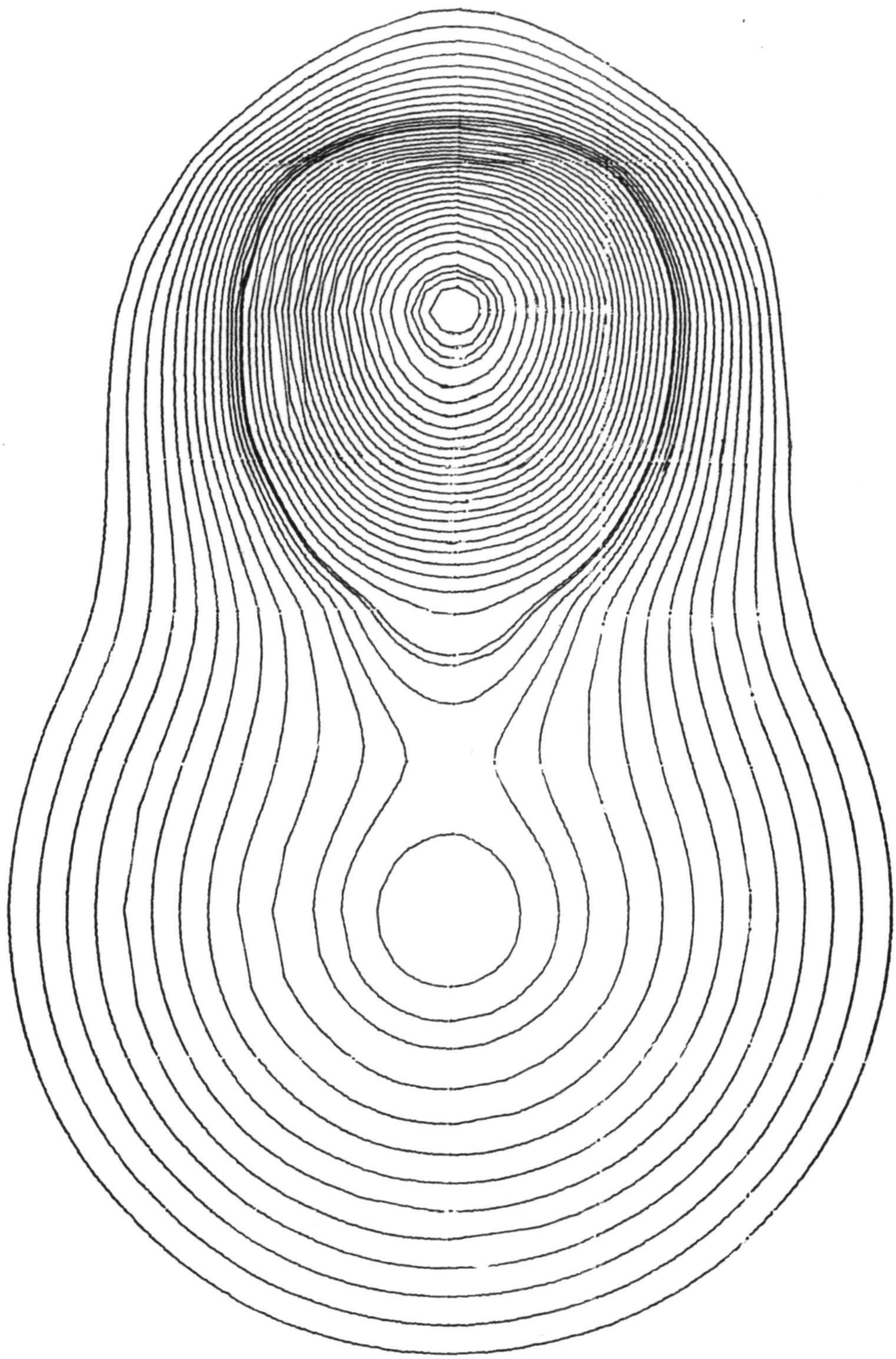

Figure 7. - Concluded.