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**COMPUTER PROGRAM  
FOR DESIGN ANALYSIS  
OF RADIAL-INFLOW TURBINES**

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16. Abstract Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor radius ratios, and rotor-exit tangential velocity distribution. Losses are determined by an internal loss model. The program output includes diameters, efficiencies, temperatures, pressures, velocities, and flow angles. Presented are the loss model, the analysis equations, a description of input and output, the FORTRAN program listing and variable list, and sample cases.					
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# COMPUTER PROGRAM FOR DESIGN ANALYSIS OF RADIAL-INFLOW TURBINES

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## SUMMARY

This report presents a computer program for the design analysis of radial-inflow turbines. Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor-exit-tip to rotor-inlet radius ratio, rotor-exit-hub to tip radius ratio, and the magnitude and radial distribution of rotor-exit tangential velocity. The turbine losses, which are determined by an internal loss model, include those due to stator and rotor boundary layers, tip clearance, disk friction, and exit velocity. The program output includes diameters, total and static efficiencies, and all absolute and relative temperatures, pressures, velocities, and flow angles at stator inlet, stator exit, rotor inlet, and rotor exit. At the rotor exit, these values are presented at any number of radial positions up to a maximum of 17.

Presented in this report are the loss model, the analysis equations, an explanation of input and output, and the FORTRAN program listing and variable list. Sample cases are included to illustrate use of the program.

## INTRODUCTION

The analysis of a power or propulsion system involves many repetitive calculations of component performance and geometry over a range of conditions. Such calculations are most easily and quickly done by a computer. One component of interest for small gas turbine systems is the radial-inflow turbine. Radial-inflow turbine geometries for achieving maximum static efficiency are presented as a function of specific speed in reference 1. However, there appeared to be no readily available computer program for performing the velocity-diagram analysis required for determining geometry and estimating performance.

A computer program for the design analysis of radial-inflow turbines was, therefore, developed. Input design requirements are power, mass flow rate, inlet temperature and pressure, and rotative speed. The design variables include stator-exit angle, rotor-exit-tip to rotor-inlet radius ratio, rotor-exit-hub to tip radius ratio, and the magnitude and radial distribution of rotor-exit tangential velocity. The turbine losses include those due to stator and rotor boundary layers, tip clearance, disk friction, and exit velocity. The program output includes diameters, total and static efficiencies, and all absolute and relative temperatures, pressures, velocities, and flow angles at stator inlet, stator exit, rotor inlet, and rotor exit. At the rotor exit, these values can be presented at any number of radial positions up to a maximum of 17.

This radial-inflow turbine design analysis computer program is described herein. Presented in this report are the loss model, the analysis method, an explanation of input and output, and the FORTRAN program listing and variable list. Sample cases are included to illustrate use of the program.

## LOSS MODEL

An important part of any turbine design problem is the estimation of losses. The loss model used for this analysis is a modification and extension of the model used in reference 1. Accounted for by this model are the three-dimensional (profile plus end wall) viscous losses in the stator and the rotor, the disk-friction loss on the back side of the rotor, the loss due to the clearance between the rotor tip and the outer casing, and the exit velocity loss.

### Viscous Loss

The stator and rotor viscous losses are each expressed in terms of a kinetic-energy loss coefficient, which is defined as the loss in kinetic energy as a fraction of the ideal kinetic energy of the blade row actual flow. In terms of boundary layer parameters (see ref. 2), the two-dimensional kinetic-energy loss coefficient is expressed as

$$\bar{e}_{2D} = \frac{\psi_{tot}}{s \cos \Phi - \delta_{tot} - t} \quad (1)$$

where the flow angle  $\Phi$  is equal to  $\alpha_1$  for the stator and  $\beta_2$  for the rotor. The symbols are defined in appendix A. The station designations are indicated on the

schematic cross section of a radial-inflow turbine shown in figure 1. An example velocity diagram indicating angle designations is shown in figure 2. Substituting

$$\psi = E \theta \quad (2)$$

and

$$\delta = H\theta \quad (3)$$

into equation (1), introducing the surface length  $l$ , and dividing numerator and denominator by blade-row exit spacing  $s$  yield

$$\bar{e}_{2D} = \frac{E \left( \frac{\theta_{tot}}{l} \right) \left( \frac{l}{s} \right)}{\cos \Phi - \frac{t}{s} - H \left( \frac{\theta_{tot}}{l} \right) \left( \frac{l}{s} \right)} \quad (4)$$

The momentum thickness per unit of surface length is assumed to be expressed as a function of Reynolds number as

$$\frac{\theta_{tot}}{l} = C \left( \frac{\theta_{tot}}{l} \right)_{ref} \left( \frac{Re}{Re_{ref}} \right)^{-0.2} \quad (5)$$

The factor  $C$  is introduced as a convenience for modifying the loss level if desired. It is further assumed that the ratio of three-dimensional loss to two-dimensional loss is equal to the ratio of three-dimensional (blade surface plus end wall) surface area to two-dimensional (blade wall) surface area (see ref. 2); that is,

$$\bar{e}_{3D} = \bar{e}_{2D} \left( \frac{A_{3D}}{A_{2D}} \right) \quad (6)$$

Combining equations (4), (5), and (6) yields

$$\bar{e}_{3D} = \frac{EC \left( \frac{\theta_{tot}}{l} \right)_{ref} \left( \frac{Re}{Re_{ref}} \right)^{-0.2} \left( \frac{l}{s} \right) \left( \frac{A_{3D}}{A_{2D}} \right)}{\cos \Phi - \frac{t}{s} - HC \left( \frac{\theta_{tot}}{l} \right)_{ref} \left( \frac{Re}{Re_{ref}} \right)^{-0.2} \left( \frac{l}{s} \right)} \quad (7)$$

Equation (7) is used to determine the three-dimensional viscous losses in the stator and the rotor. Evaluation of the various terms in equation (7) is as follows.

Flow angle. - As mentioned previously,

$$\Phi_s = \alpha_1 \quad (8)$$

and

$$\Phi_r = \beta_2 \quad (9)$$

Energy and form factors. - The energy and form factors are obtained from equations (7-14) and (7-13), respectively, of reference 2 using a velocity profile exponent of 0.2. Sufficient numbers of terms are used for each series such that additional terms affect the series sum by less than 0.1 percent. The resultant equations are

$$E = \frac{2 \left( \frac{1}{1.92} + \frac{Q}{3.2} + \frac{Q^2}{4.8} + \frac{Q^3}{6.72} \right)}{\frac{1}{1.68} + \frac{Q}{2.88} + \frac{Q^2}{4.4} + \frac{Q^3}{6.24}} \quad (10)$$

and

$$H = \frac{\frac{1}{1.2} + \frac{3Q}{1.6} + \frac{5Q^2}{2.0} + \frac{7Q^3}{2.4} + \frac{9Q^4}{2.8}}{\frac{1}{1.68} + \frac{Q}{2.88} + \frac{Q^2}{4.4} + \frac{Q^3}{6.24}} \quad (11)$$

For the stator,

$$Q_s = \frac{\gamma - 1}{\gamma + 1} \left( \frac{V}{V_{cr1}} \right)^2 \quad (12)$$

and for the rotor,

$$Q_r = \frac{\gamma - 1}{\gamma + 1} \left( \frac{W}{W_{cr2}} \right)^2 \quad (13)$$

Reference loss coefficients. - The reference values of momentum thickness per unit length and Reynolds number were combined into reference loss coefficients, which were evaluated by matching the program results with the experimental values of stator loss coefficient ( $\bar{e}_{3D,s} = 0.064$ , which was obtained from unpublished data) and total efficiency ( $\eta' = 0.88$ ) for the turbine of reference 3. This is considered to be representative state-of-the-art performance for a carefully designed turbine. With the coefficients  $C_s$  and  $C_r$  equal to unity, the reference loss coefficients are

$$\left( \frac{\theta_{tot}}{l \text{ Re}^{-0.2}} \right)_{\text{ref},s} = 0.03734 \quad (14)$$

and

$$\left( \frac{\theta_{tot}}{l \text{ Re}^{-0.2}} \right)_{\text{ref},r} = 0.11595 \quad (15)$$

Reynolds number. - Blade-row chord is used as the characteristic length. For the stator,

$$\text{Re}_s = \frac{\rho_1 V_1 c_s}{\mu} \quad (16)$$

and for the rotor

$$\text{Re}_r = \frac{\rho_2 W_2 c_r}{\mu} \quad (17)$$

Expressing velocity in terms of its throughflow component, multiplying both numerator and denominator by annulus area, and applying the continuity equation yield

$$Re_s = \frac{w}{\pi \left(\frac{h}{c}\right)_s \mu D_1 \cos \alpha_1} \quad (18)$$

and

$$Re_r = \frac{w}{\pi \left(\frac{h}{c}\right)_r \mu D_{2,m} \cos \beta_2} \quad (19)$$

Length to spacing ratio. - For the stator, the surface length to exit spacing ratio is expressed as

$$\left(\frac{l}{s}\right)_s = \left(\frac{l}{c}\right)_s \left(\frac{c}{s}\right)_s \quad (20)$$

A stator solidity  $(c/s)_s$  is input to the program and internally adjusted to correspond to an integral number of vanes. The stator surface length to chord ratio  $(l/c)_s$  is unity for an uncambered vane and obtained from equation (B10) of appendix B for a cambered vane.

For the rotor, the surface length is obtained from equation (C2) of appendix C. The mean spacing at the rotor exit is

$$s_{2,m} = \frac{2\pi r_{2,m}}{n_r} = \frac{\pi(r_{2,h} + r_{2,t})}{n_r} \quad (21)$$

Trailing-edge thickness to spacing ratio. - Trailing-edge thickness is specified as a fraction of blade height using existing turbines as a guide. For the stator, the trailing-edge thickness is 5 percent of the stator blade height; that is,

$$\left(\frac{t}{s}\right)_s = 0.05 \left(\frac{h}{s}\right)_s = 0.05 \left(\frac{h}{c}\right)_s \left(\frac{c}{s}\right)_s \quad (22)$$

The solidity  $c/s$  is an input value, while the aspect ratio  $h/c$  is either input or computed from the stator geometry model depending on the case.



For the rotor, the trailing-edge thickness is 4 percent of the rotor blade exit height; that is,

$$t = 0.04(r_{2,t} - r_{2,h}) \quad (23)$$

Dividing equation (23) by spacing as expressed by equation (21) yields

$$\left(\frac{t}{s}\right)_r = \frac{0.04 n_r \left(1 - \frac{r_{2,h}}{r_{2,t}}\right)}{\pi \left(1 + \frac{r_{2,h}}{r_{2,t}}\right)} \quad (24)$$

Three-dimensional- to two-dimensional-area ratio. - Stator vane surface area for one passage is

$$A_{b,s} = 2l_s h_s = 2\left(\frac{l}{c}\right)_s c_s h_s \quad (25)$$

Stator end-wall surface area for one passage is

$$A_{w,s} = \frac{2}{n_s} \pi (r_0^2 - r_1^2) \quad (26)$$

Expressing number of stator vanes as

$$n_s = \frac{2\pi r_1}{s_s} \quad (27)$$

and substituting equation (27) into equation (26) yield

$$A_{w,s} = \frac{s_s}{r_1} (r_0^2 - r_1^2) \quad (28)$$

Since

$$\frac{A_{3D}}{A_{2D}} = \frac{A_b + A_w}{A_b} \quad (29)$$

substitution of equations (25) and (28) into equation (29) yields

$$\left(\frac{A_{3D}}{A_{2D}}\right)_s = 1 + \frac{r_0^2 - r_1^2}{2r_1 \left(\frac{l}{c}\right)_s \left(\frac{c}{s}\right)_s h_s} \quad (30)$$

For the rotor,

$$\left(\frac{A_{3D}}{A_{2D}}\right)_r = 1 + \frac{A_{w,t,r} + A_{w,h,r}}{A_{b,r}} \quad (31)$$

where  $A_{b,r}$ ,  $A_{w,t,r}$ , and  $A_{w,h,r}$  are obtained from equations (C5), (C13), and (C17), respectively, of appendix C.

#### Disk-Friction Loss

The disk-friction loss is calculated using equations (8-7), (8-10), and (8-15) of reference 4 with the coefficient  $C_{IV} = 0.085$ . For one side of a disk, the friction loss is, therefrom, expressed as

$$L_{df} = \frac{0.02125 \rho_{1a} U_{1a}^3 r_{1a}^2}{gJ \left(\frac{\rho U r}{\mu}\right)_{1a}^{0.2} w} \quad (32)$$

#### Tip Clearance Loss

In accordance with the tip clearance discussion in reference 5, it is assumed that the fractional loss due to clearance is equal to the ratio of clearance to passage height at the rotor exit. Clearance is input to the program as a fraction of diameter. The loss due to clearance is, therefore, computed from

$$\frac{L_c}{\Delta h'_{VD, av}} = \frac{h_c}{r_{2,t} - r_{2,h}} = \left(\frac{h_c}{D_{t/2}}\right) \frac{2r_{2,t}}{r_{2,t} - r_{2,h}} = \left(\frac{h_c}{D_{t/2}}\right) \left(\frac{2}{1 - \frac{r_{2,h}}{r_{2,t}}}\right) \quad (33)$$

where  $\left(\frac{h_c}{D_{t/2}}\right)$  is the input parameter.

### Exit Velocity Loss

The kinetic-energy loss associated with the turbine exit velocity is

$$L_{ex} = \frac{V_2^2}{2gJ} \quad (34)$$

### ANALYSIS METHOD

The flow analysis is one dimensional at the stator inlet, stator exit, and rotor inlet, each of these calculation stations being at a constant radius. At the rotor exit, where there is a variation in flow-field radius, an axisymmetric two-dimensional analysis is made using constant height sectors. Simple radial equilibrium is used to establish the static pressure gradient at the rotor exit.

The fluid energy corresponding to the shaft power requirement is

$$\Delta h'_{shft} = \frac{C_P P}{Jw} \quad (35)$$

Disk friction and clearance losses, which are expressed by equations (32) and (33), respectively, are added to the shaft work to yield the average total fluid energy extraction; that is,

$$\Delta h'_{VD, av} = \Delta h'_{shft} + L_{df} + L_c \quad (36a)$$

which can also be expressed as

$$\Delta h'_{VD, av} = \frac{\Delta h'_{shft} + L_{df}}{1 - \frac{L_c}{\Delta h'_{VD, av}}} \quad (36b)$$

The numerical values needed for evaluating  $L_{df}$  are not immediately known, and an iteration is used starting with  $L_{df} = 0$ .

The turbine energy transfer equation is

$$\Delta h'_{VD, av} = \frac{1}{gJ} \left( U_{1a} V_{u, 1a} - \frac{1}{w} \sum_{i=1}^k w_i U_{2, i} V_{u, 2, i} \right) \quad (37)$$

Substituting

$$U_{2, i} = \frac{r_{2, i}}{r_{1a}} U_{1a} \quad (38)$$

into equation (37) and manipulating terms yield

$$\Delta h'_{VD, av} = \frac{U_{1a} V_{u, 1a}}{gJ} \left( 1 - \frac{1}{w} \sum_{i=1}^k w_i \frac{r_{2, i} V_{u, 2, i}}{r_{1a} V_{u, 1a}} \right) \quad (39)$$

The average change in tangential momentum is

$$\Delta(rV_u)_{av} = r_{1a} V_{u, 1a} - \frac{1}{w} \sum_{i=1}^k w_i r_{2, i} V_{u, 2, i} \quad (40)$$

As discussed in reference 5, the rotor inlet gas tangential velocity and blade speed can be related as

$$\frac{V_{u, 1a}}{U_{1a}} = 1 - \frac{2}{n_r} \quad (41)$$

Substituting equations (40) and (41) into equation (39) and solving for blade speed yield

$$U_{1a} = \left[ \frac{gJ\Delta h'_{VD,av} (rV_u)_{1a}}{1 - \frac{2}{n_r} \Delta(rV_u)_{av}} \right]^{1/2} \quad (42)$$

The  $rV_u$  ratio is specified as input, and  $n_r$  is determined from equation (C19) of appendix C. With  $U_{1a}$  known, equation (41) is used to determine  $V_{u,1a}$ . Rotor inlet diameter is

$$D_{1a} = C_N \frac{U_{1a}}{\pi N} \quad (43)$$

#### Stator Exit

The conditions at the stator exit, station 1, are computed from the following equations. The radius ratio  $r_1/r_{1a}$  and the stator exit angle  $\alpha_1$  are known input values. Stator loss coefficient  $\bar{e}_s$  can be either input or computed from the previously presented loss model. If the loss model is used, the stator-exit calculation is iterative between equations (50) to (54).

$$D_1 = \left( \frac{r_1}{r_{1a}} \right) D_{1a} \quad (44)$$

$$V_{u,1} = \left( \frac{r_{1a}}{r_1} \right) V_{u,1a} \quad (45)$$

$$V_1 = \frac{V_{u,1}}{\sin \alpha_1} \quad (46)$$

$$V_{rad,1} = \frac{V_{u,1}}{\tan \alpha_1} \quad (47)$$

$$T'_1 = T'_0 \quad (48)$$

$$T_1 = T'_1 - \frac{V_1^2}{2gJc_p} \quad (49)$$

$$V_{1, id}^2 = \frac{V_1^2}{1 - \bar{e}_s} \quad (50)$$

$$p_1 = \left(1 - \frac{V_{1, id}^2}{2gJc_p T'_0}\right)^{\gamma/(\gamma-1)} p'_0 \quad (51)$$

$$p'_1 = p_1 \left(\frac{T'_1}{T_1}\right)^{\gamma/(\gamma-1)} \quad (52)$$

$$\rho_1 = \frac{p_1}{RT_1} \quad (53)$$

$$h_s = \frac{w}{\rho_1 V_{rad, 1} \pi D_1} \quad (54)$$

$$V_{cr, 1} = \sqrt{\frac{2\gamma}{\gamma + 1} gRT'_1} \quad (55)$$

### Stator Inlet

Stator-inlet diameter  $D_0$  and flow angle  $\alpha_0$  are either input or calculated from the stator geometry model, appendix B. Stator height  $h_s$  is assumed constant, and the following four equations are solved simultaneously for the stator-inlet, station 0, conditions  $V_0$ ,  $T_0$ ,  $p_0$ , and  $\rho_0$ :

$$V_0 = \frac{w}{\pi D_0 h_s \rho_0 \cos \alpha_0} \quad (56)$$

$$T_0 = T'_0 - \frac{V_0^2}{2gJc_p} \quad (57)$$

$$p_0 = p'_0 \left( \frac{T_0}{T'_0} \right)^{\gamma/(\gamma-1)} \quad (58)$$

$$\rho_0 = \frac{p_0}{RT_0} \quad (59)$$

### Rotor Inlet

Assuming no change in total temperature or total pressure between stator exit and rotor inlet (i. e.,  $T'_{1a} = T'_1$  and  $p'_{1a} = p'_1$ ), we can solve the following set of six equations for the variables indicated on the left sides:

$$V_{1a} = \frac{V_{u, 1a}}{\sin \alpha_{1a}} \quad (60)$$

$$T_{1a} = T'_{1a} - \frac{V_{1a}^2}{2gJc_p} \quad (61)$$

$$p_{1a} = p'_{1a} \left( \frac{T_{1a}}{T'_{1a}} \right)^{\gamma/(\gamma-1)} \quad (62)$$

$$\rho_{1a} = \frac{p_{1a}}{RT_{1a}} \quad (63)$$

$$V_{\text{rad}, 1a} = V_{\text{rad}, 1} \frac{\rho_1 D_1}{\rho_{1a} D_{1a}} \quad (64)$$

$$\alpha_{1a} = \tan^{-1} \frac{V_{u,1a}}{V_{rad,1a}} \quad (65)$$

At this point there is sufficient information for evaluation of disk friction loss from equation (32), and the calculation cycles back to equation (36b) until a convergence is obtained.

The rotor-inlet conditions relative to the rotor are

$$W_{u,1a} = V_{u,1a} - U_{1a} \quad (66)$$

$$\beta_{1a} = \tan^{-1} \frac{W_{u,1a}}{V_{rad,1a}} \quad (67)$$

$$W_{1a} = \frac{V_{rad,1a}}{\cos \beta_{1a}} \quad (68)$$

$$T'_{1a} = T_{1a} + \frac{W_{1a}^2}{2gJc_p} \quad (69)$$

$$p'_{1a} = p_{1a} \left( \frac{T'_{1a}}{T_{1a}} \right)^{\gamma/(\gamma-1)} \quad (70)$$

$$W_{cr,1a} = V_{cr,1} \sqrt{\frac{T'_{1a}}{T_{1a}}} \quad (71)$$

#### Rotor Exit

At the rotor exit, station 2, the annulus is divided into equal-height sectors, with the sectors being related by simple radial equilibrium. The solution at the rotor exit has to satisfy two input requirements, average specific work and flow. This is done by means of two iteration loops. The outer-loop iteration is started by setting a value for the mean-sector tangential velocity  $V_{u,2,m}$ . The inner loop is then cycled by varying



the mean-sector axial velocity  $V_{x,2,m}$  until continuity is satisfied. The average specific work is then checked. If necessary,  $V_{u,2,m}$  is adjusted and the iteration process repeated until both loops are simultaneously satisfied.

The rotor-exit hub and tip diameters are known from the calculated rotor-inlet diameter and input diameter ratios. For the  $k$  sectors, therefore, the sector mean diameters  $D_{2,i}$  are known. Without knowledge of the rotor-exit solution, we can then calculate for all  $i$

$$U_{2,i} = \frac{\pi D_{2,i}^N}{C_N} \quad (72)$$

$$T''_{2,i} = T''_{1a} + \frac{U_{2,i}^2 - U_{1a}^2}{2gJc_p} \quad (73)$$

$$W_{cr,2,i} = W_{cr,1a} \sqrt{\frac{T''_{2,i}}{T''_{1a}}} \quad (74)$$

$$p''_{2,id,i} = p''_{1a} \left( \frac{T''_{2,i}}{T''_{1a}} \right)^{\gamma/(\gamma-1)} \quad (75)$$

The rotor loss coefficient  $\bar{e}_r$  can be either input or computed from the previously presented loss model using mean-sector conditions. With the rotor-exit mean-sector tangential and axial absolute-velocity component values set by the iteration loops previously discussed, the rotor-exit calculation proceeds as follows, starting with the mean sector:

$$W_{u,2,m} = V_{u,2,m} - U_{2,m} \quad (76)$$

$$\beta_{2,m} = \tan^{-1} \frac{W_{u,2,m}}{V_{x,2,m}} \quad (77)$$

$$W_{2,m} = \frac{V_{x,2,m}}{\cos \beta_{2,m}} \quad (78)$$

$$T_{2,m} = T_{2,m}'' - \frac{W_{2,m}^2}{2gJc_p} \quad (79)$$

$$W_{2,id,m}^2 = \frac{W_{2,m}^2}{1 - \bar{e}_r} \quad (80)$$

$$p_{2,m} = \left( 1 - \frac{W_{2,id,m}^2}{2gJc_p T_{2,m}''} \right)^{\gamma/(\gamma-1)} p_{2,id,m}'' \quad (81)$$

$$\rho_{2,m} = \frac{p_{2,m}}{RT_{2,m}} \quad (82)$$

For the other sectors, the rotor loss coefficient is assumed constant and equal to the mean-sector value. The angular momentum distribution  $(rV_u)_{2,i}/(rV_u)_{2,m}$  is specified as input. The calculation then proceeds from the mean sector into the hub and from the mean sector out to the tip; thus,

$$V_{u,2,j} = V_{u,2,m} \frac{(rV_u)_{2,j}}{(rV_u)_{2,m}} \frac{D_{2,m}}{D_{2,j}} \quad (83)$$

In the following, the subscript  $j$  refers to all values of  $i$  except  $i = m$ :

$$W_{u,2,j} = V_{u,2,j} - U_{2,j} \quad (84)$$

$$p_{2,j} = p_{2,j\pm 1} + \frac{\rho_{2,j\pm 1}}{2g} \left( \frac{V_{u,2,j}^2}{D_{2,j}} + \frac{V_{u,2,j\pm 1}^2}{D_{2,j\pm 1}} \right) (D_{2,j} - D_{2,j\pm 1}) \quad (85)$$

$$W_{2,id,j}^2 = 2gJc_p T_{2,j}'' \left[ 1 - \left( \frac{p_{2,j}}{p_{2,id,j}''} \right)^{(\gamma-1)/\gamma} \right] \quad (86)$$

$$W_{2,j} = \sqrt{W_{2,id,j}^2 (1 - \bar{e}_r)} \quad (87)$$

$$\beta_{2,j} = \sin^{-1} \frac{W_{u,2,j}}{W_{2,j}} \quad (88)$$

$$V_{x,2,j} = W_{2,j} \cos \beta_{2,j} \quad (89)$$

$$T_{2,j} = T'_{2,j} - \frac{W_{2,j}^2}{2gJc_p} \quad (90)$$

$$\rho_{2,j} = \frac{p_{2,j}}{RT_{2,j}} \quad (91)$$

After equations (83) to (91) are evaluated for all values of  $j$ , the mass flow rate at the rotor exit is calculated as

$$w_{\text{calc}} = \pi \frac{D_{2,t} - D_{2,h}}{2k} \sum_{i=1}^k \rho_{2,i} V_{x,2,i} D_{2,i} \quad (92)$$

If  $w_{\text{calc}} \neq w$ , the value of  $V_{x,2,m}$  is adjusted, and the calculation is cycled back to equation (77). When  $w_{\text{calc}} = w$ , the energy transfer determined from equation (37) is compared with the work requirement from equation (36b) and, if necessary, the value of  $V_{u,2,m}$  is adjusted and the calculation cycled back to equation (76). When the flow and work requirements are simultaneously satisfied, the rotor-exit solution is established. The following parameters are then calculated:

$$\Delta h'_{VD,i} = \frac{1}{gJ} (U_{1a} V_{u,1a} - U_{2,i} V_{u,2,i}) \quad (93)$$

$$T'_{2,i} = T'_0 - \frac{\Delta h'_{VD,i}}{c_p} \quad (94)$$

$$V_{cr,2,i} = V_{cr,1a} \sqrt{\frac{T'_{2,i}}{T'_{1a}}} \quad (95)$$

$$\alpha_{2,i} = \tan^{-1} \frac{V_{u,2,i}}{V_{x,2,i}} \quad (96)$$

$$p'_{2,i} = p_{2,i} \left( \frac{T'_{2,i}}{T_{2,i}} \right)^{\gamma/(\gamma-1)} \quad (97)$$

$$V_{2,i} = \frac{V_{x,2,i}}{\cos \alpha_{2,i}} \quad (99)$$

### Performance

There are five types of losses considered in this analysis. The disk-friction and clearance losses are obtained from equations (32) and (33), respectively. The stator, rotor, and leaving losses are

$$L_s = \frac{V_{1,id}^2 - V_1^2}{2gJ} \quad (100)$$

$$L_{r,av} = \sum_{i=1}^k \frac{w_i}{w} \frac{W_{2,id,i}^2 - W_{2,i}^2}{2gJ} \quad (101)$$

$$L_{ex,av} = \sum_{i=1}^k \frac{w_i}{w} \frac{V_{2,i}^2}{2gJ} \quad (102)$$

The sector ideal specific works are

$$\Delta h'_{id, i} = c_p T'_0 \left[ 1 - \left( \frac{p'_{2, i}}{p'_0} \right)^{(\gamma-1)/\gamma} \right] \quad (103)$$

$$\Delta h_{id, i} = c_p T'_0 \left[ 1 - \left( \frac{p_{2, i}}{p_0} \right)^{(\gamma-1)/\gamma} \right] \quad (104)$$

The sector efficiencies based on velocity diagram work are

$$\eta'_{VD, i} = \frac{\Delta h'_{VD, i}}{\Delta h'_{id, i}} \quad (105)$$

$$\eta_{VD, i} = \frac{\Delta h_{VD, i}}{\Delta h_{id, i}} \quad (106)$$

The overall velocity diagram efficiencies are

$$\eta'_{VD, av} = \frac{\Delta h'_{VD, av}}{\sum_{i=1}^k \frac{w_i}{w} \Delta h'_{id, i}} \quad (107)$$

$$\eta_{VD, av} = \frac{\Delta h_{VD, av}}{\sum_{i=1}^k \frac{w_i}{w} \Delta h_{id, i}} \quad (108)$$

while the net overall efficiencies are

$$\eta'_{shft} = \eta'_{VD, av} \frac{\Delta h'_{shft}}{\Delta h'_{VD, av}} \quad (109)$$

$$\eta_{\text{shft}} = \eta_{\text{VD, av}} \frac{\Delta h'_{\text{shft}}}{\Delta h'_{\text{VD, av}}} \quad (110)$$

A design parameter that is computed by the program is specific speed

$$N_{\text{sp}} = \frac{N(w/\rho_2)^{1/2}}{(J \Delta h'_{\text{id, av}})^{3/4}} \quad (111)$$

## DESCRIPTION OF INPUT AND OUTPUT

This section presents a detailed description of the program input, normal output, and error messages. The input and corresponding printed output for an example problem are included for illustrative purposes.

### Input

The program input consists of a title card and a data set in NAMELIST form for each case. The title, which is printed as a heading on the output listing, can be located anywhere in columns 1 to 78 on the title card. A title card, even if it is left blank, must be the first card for each case.

The data are input in data records having the NAMELIST name INPUT. The variables that comprise INPUT along with descriptions, units, and special remarks are presented in the list to follow. Either SI units or U.S. customary units may be used with this program. Values for some of the variables in the input list are internally preset by the program before reading the input. Thus, if a preset value (see input list for values) is appropriate, that particular variable does not have to be specified in the input.

- |       |   |
|-------|---|
| IU    | units indicator (preset value = 2):             |
|       | 1 - SI units                                    |
|       | 2 - U.S. customary units                        |
| NSTAR | stator geometry indicator:                      |
|       | 0 - cambered vane with specified diameter ratio |
|       | 1 - cambered vane with specified aspect ratio   |
|       | 2 - uncambered vane                             |

ALPHA0 stator-inlet flow angle from radial direction, deg (input only for NSTAR = 0 or 1)

ALPHA1 stator-exit flow angle from radial direction, deg

CDT2 ratio of clearance gap to blade tip diameter at rotor exit (preset value = 0.)

CR rotor loss-coefficient multiplier (preset value = 1.)

CS stator loss-coefficient multiplier (preset value = 1.)

EBARR rotor loss coefficient (input only if it is desired that internal loss model not be used for computing  $\bar{e}_r$ )

EBARS stator loss coefficient (input only if it is desired that internal loss model not be used for computing  $\bar{e}_s$ )

GAM specific heat ratio

K number of radial sectors at rotor exit

MU gas viscosity, (N)(sec)/m<sup>2</sup>; lb/(sec)(ft)

N rotative speed, rad/sec; rev/min

POW shaft power, kW; hp

PTIN inlet total pressure, N/cm<sup>2</sup>; lb/in.<sup>2</sup>

R gas constant, J/(kg)(K); (ft)(lb)/(lbm)(°R)

RH2RT2 rotor-exit-hub - to tip-radius ratio

RT2R1A rotor-exit-tip - to rotor-inlet-radius ratio

RV1AAV ratio of rotor-inlet angular momentum to average change in angular momentum (preset value = 1.)

RV2I2M(I), ratio of i<sup>th</sup> sector to mean-sector rotor-exit angular momentum  
I = 1, K (preset value = 1. for all I)

R0R1A stator-inlet- to rotor-inlet radius ratio (input only for NSTAR = 0)

R1R1A stator-exit- to rotor-inlet radius ratio

SIGSIN stator solidity based on exit spacing (preset value = 1.35)

STAR stator aspect ratio (input only for NSTAR = 1 or 2)

TTIN inlet total temperature, K; °R

W mass flow rate, kg/sec; lb/sec

The program inputs for two sample cases are shown in table I. Both cases are for the same design requirements. The first case is with SI units and the second case is with U. S. customary units. The first card for each case is the previously described title card. After the title card is the NAMELIST data set containing all the variable values. For the second case, only those variables changing in value from the first case need be input. The output corresponding to this sample input is presented and described in the following section.

## Output

The program output consists of title headings, the input variable values, and the computed results. This section presents normal output. Error message output is described in the next section.

Tables II and III present the output that corresponds to the sample input shown in table I. Table II is for the case with SI units. The top line of output is a program identification title that is automatically printed with the first case of each data package. The second line is the title card message. The next three lines indicate the units, SI in this case, used for the different variables.

The heading \*INPUT\* is followed by eight lines showing the input values used for this case. Identification of all the items on the output listing is self-explanatory. The zero value shown for stator aspect ratio indicates that this was not an input for this case. Values of 1.0000 are shown for the stator and rotor loss coefficients to indicate that the actual values are computed using the internal loss model.

The heading \*OUTPUT\* is followed by the computed results. Absolute temperatures, pressures, flow angles, velocities, and velocity ratios, along with diameter, are shown for each calculation station. At the rotor exit, these values are shown for the mean diameter of each sector as well as for the hub and the tip. Additional output for the rotor-exit sectors include flow rate, specific work, and total and static efficiencies. Also shown are the computed loss coefficients for the stator and rotor, the stator height, number of stator vanes, and number of rotor blades.

Under the heading \*OVERALL PERFORMANCE\* are the turbine total-to-total and total-to-static pressure ratios, diagram specific work, and both diagram and net total and static efficiencies. Also shown are the individual loss components as fractions of the turbine ideal work and the specific speed.

Table III is similar to table II except that U. S. customary units are used, as indicated by the title message as well as by the specified units themselves. Note that the values for all the dimensionless variables are the same in table III as in table II.



## Error Messages

The program contains seven output messages indicating the nonexistence of a solution satisfying the specified input requirements. These messages are presented in this section, and their causes are discussed. In general, when one of these messages appear, the program input should be checked for errors.

(1) NO SOLUTION FOUND AFTER 100 ITERATIONS FOR CONTINUITY AT ROTOR EXIT - This message is caused by the program making 100 iterations through subroutine CONTIN without a solution being found. There is no obvious reason for this situation except possibly for input error in the specification of rotor-exit angular momentum.

(2) ROTOR EXIT CHOKES AT MAXIMUM MASS FLOW RATE = XXXX.XXXX - This message is caused by the choking mass flow rate for the rotor exit being less than the design flow rate specified as program input. If the input design requirements are correct, then possible corrective action includes reducing exit angular momentum, increasing rotor-exit-tip- to rotor-inlet-radius ratio, and decreasing rotor-exit-hub- to tip-radius ratio.

(3) REQUIRED SPECIFIC WORK GREATER THAN ENERGY AVAILABLE IN GAS - See item (4).

(4) SPECIFIC WORK REQUIRED IN SECTOR XX GREATER THAN ENERGY AVAILABLE IN GAS - These last two messages are caused by the turbine-exit total temperature, average value or any sector value, being less than zero. Possible corrective action includes increasing turbine inlet temperature, decreasing turbine power, or increasing mass flow rate.

(5) REQUIRED STATOR IDEAL KINETIC ENERGY GREATER THAN ENERGY AVAILABLE IN GAS - See item (6).

(6) ROTOR IDEAL RELATIVE KINETIC ENERGY REQUIRED IN SECTOR XX GREATER THAN ENERGY AVAILABLE IN GAS - These last two messages are caused by the computed ideal energy required by the stator or the rotor being greater than that available from an infinite expansion of the gas. The probable reason for this condition is an error in the computed or input loss coefficient.

(7) THE PROGRAM CAN NOT FIND A SOLUTION SIMULTANEOUSLY SATISFYING CONTINUITY, RADIAL EQ., AND THE LOSS MODEL AT THE ROTOR EXIT - This message is caused by rotor-exit relative velocity, as determined by radial equilibrium and radial distribution of loss, being less than its tangential component, as determined primarily by blade speed. Corrective action includes decreasing rotor-exit tip radius and increasing rotor-exit-hub to tip radius ratio.

## PROGRAM DESCRIPTION

This computer program consists of main program RIFTUD (Radial Inflow Turbine Design), blade loss coefficient subprograms EFFIC, SIMPS1, SHUB, and SHUB2, and rotor-exit continuity subprograms CONTIN and PABC. The entire program is written in FORTRAN IV language and has been run on both an IBM 7094 and a UNIVAC 1110. Running time on the UNIVAC 1110 is about 1 second per design case for a five-sector design. In this section, the functions of the main and subprograms are described, the program variables are defined, and the program listing is presented.

### Main Program RIFTUD

Main program RIFTUD performs all input and output operations as well as all of the flow analysis as presented in the section ANALYSIS METHOD.

Program variables. - The variables used in RIFTUD are defined in terms of the following symbols, which are defined in appendix A:

ALPHAA	$\alpha_{1a}$ (degrees)	BETA2(I)	$\beta_{2,i}$ (degrees)
ALPHA0	$\alpha_0$ (degrees)	BET1A	$\beta_{1a}$ (radians)
ALPHA1	$\alpha_1$ (degrees)	BET2(I)	$\beta_{2,i}$ (radians)
ALPHA2(I)	$\alpha_{2,i}$ (degrees)	BET2M	$\beta_{2,m}$ (radians)
ALPH0	$\alpha_0$ (radians)	CDT2	$(h_c/D_t)_2$
ALPH1	$\alpha_1$ (radians)	CHRD	c
ALPH1A	$\alpha_{1a}$ (radians)	CN	$C_N/\pi$
ALPH2(I)	$\alpha_{2,i}$ (radians)	CP	$c_p$
ALP0	$\alpha_0$ (degrees)	CPOW	$C_P$
ALP1A	previous value of $\alpha_{1a}$	CR	$C_r$
ALSTG	$\varphi$	CS	$C_s$
ALUNC	$\odot_{cam}$	DEL VX	increment in $(V_x/V_{cr})_{2,m}$
BETA1A	$\beta_{1a}$ (degrees)		

DHIDS(I)	$\Delta h_{id, i}$	ENT	truncated integer value of $n_r$ before roundoff
DHIDSA	$\Delta h_{id, av}$	ER	$\bar{e}_r$
DHIDT(I)	$\Delta h'_{id, i}$	ES	$\bar{e}_s$
DHIDTA	$\Delta h'_{id, av}$	ES1	previous value of $\bar{e}_s$
DHSHFT	$\Delta h'_{shft}$	ETAS	$\eta_{shft}$
DHVD(I)	$\Delta h'_{VD, i}$	ETASV	$\eta_{VD, av}$
DHVDAV	$\Delta h'_{VD, av}$	ETASVD(I)	$\eta_{VD, i}$
DOR	$180/\pi$	ETAT	$\eta'_{shft}$
DVUAVC	$(DV_u)_{2, av, calc}$	ETATV	$\eta'_{VD, av}$
DVUTOT	$w(DV_u)_{2, av, calc}$	ETATVD(I)	$\eta'_{VD, i}$
DVU2AV	$(DV_u)_{2, av, required}$	EX	$\gamma/(\gamma - 1)$
D0	$D_0$	F2(I)	$w_{2, i}$
D1	$D_1$	G	$g$
D1A	$D_{1a}$	GAM	$\gamma$
D2(I)	$D_{2, i}$	HR	$h_r$
D2M	$D_{2, m}$	HS	$h_s$
EBARR	$\bar{e}_r$ input value	I	dummy index, usually referring to sector number
EBARS	$\bar{e}_s$ input value	ID	sector number increment
EL	$h_{r, 2/k}$	II	sector number index for previous sector
EN	$n_r$	IND	solution indicator for rotor-exit continuity calculation
ENS	$n_s$	IST	iteration counter for stator calculation
ENST	truncated integer value of $n_s$ before roundoff		

ISTT	iteration counter for stator calculation	PS0	$P_0$
ITER	iteration counter for rotor-exit calculation	PS1	$P_1$
IU	units indicator - see section Input	PS1A	$P_{1a}$
J	J	PS2(I)	$P_{2,i}$
K	number of sectors	PTIN	$P'_0$
KK	$K + 2$	PTR1A	$P'_{1a}$
KP1	$K + 1$	PTR2(I)	$P'_{2,i}$
K1	indicator for print control	PTR2ID(I)	$P'_{2, id, i}$
LC	$L_c$	PT0	$P'_0$
LCDH	$L_c / \Delta h'_{VD, av}$	PT1	$P'_1$
LCDHIS	$L_c / \Delta h_{id, av}$	PT1A	$P'_{1a}$
LLDHIS	$L_{ex, av} / \Delta h_{id, av}$	PT2(I)	$P'_{2,i}$
LRDHIS	$L_{r, av} / \Delta h_{id, av}$	Q	$(w/\rho)_{2, av}$
LSDHIS	$L_s / \Delta h_{id, av}$	R	R
LW	$L_{df}$	RHOT0	$\rho'_0$
LWDHIS	$L_{df} / \Delta h_{id, av}$	RHOX	previous value of $\rho_0$
LWX	previous value of $L_{df}$	RHO0	$\rho_0$
M	mean sector index	RHO1	$\rho_1$
MU	$\mu$	RHO1A	$\rho_{1a}$
N	N	RHO2(I)	$\rho_{2,i}$
NSTAR	stator geometry indicator - see section Input	RH2RT2	$(r_h/r_t)_2$
PI	$\pi$	RLOSS	$L_{r, av}$
POW	P	RSTG	$r_{cm}$
		RT2R1A	$r_{2,t} / r_{1a}$

RV1AAV	$(rV_u)_{1a} / \Delta(rV_u)_{av}$	TTR2(I)	$T'_{2,i}$
RV2IM(I)	$(rV_u)_{2,i} / (rV_u)_{2,m}$	TT0	$T'_0$
RV2I2M(I)	$(rV_u)_{2,i} / (rV_u)_{2,m}$	TT1	$T'_1$
R0R1A	$r_0 / r_{1a}$	TT1A	$T'_{1a}$
R1R1A	$r_1 / r_{1a}$	TT2(I)	$T'_{2,i}$
SIGRV	$\sum_{i=1}^k [(rV_u)_{2,i} / (rV_u)_{2,m}] / k$	TT2AV	$T'_{2,av}$
SIGS	$(c/s)_s$	U1A	$U_{1a}$
SIGSIN	input value of $(c/s)_s$	U1ASQ	$U_{1a}^2$
SS	s	U2(I)	$U_{2,i}$
SSPD	$N_{sp}$	VCR1	$V_{cr,1}$
STAR	$(h/c)_s$	VCR1A	$V_{cr,1a}$
TGJCP	$2gJc_p$	VCR2(I)	$V_{cr,2,i}$
TITLE(I)	input/output array for title card message	VOVCRA	$(v/v_{cr})_{1a}$
TSPR	$p'_0 / p_{2,i}$	VOVCR0	$(v/v_{cr})_0$
TS0	$T_0$	VOVCR1	$(v/v_{cr})_1$
TS1	$T_1$	VOVCR2(I)	$(v/v_{cr})_{2,i}$
TS1A	$T_{1a}$	VR0	$V_{rad,0}$
TS2(I)	$T_{2,i}$	VR1	$V_{rad,1}$
TTIN	$T'_0$	VR1A	$V_{rad,1a}$
TTPR	$p'_0 / p'_{2,i}$	VUOU1A	$(v_u/U)_{1a}$
TTR1A	$T'_{1a}$	VU0	$V_{u,0}$
		VU1	$V_{u,1}$

VU1A	$V_{u,1a}$	WCR2(I)	$W_{cr,2,i}$
VU2O	previous value of $V_{u,2,m}$	WGIV	input value of $w$
VU2(I)	$V_{u,2,i}$	WOWCRA	$(W/W_{cr})_{1a}$
VXVCR	$(V_x/V_{cr})_{2,m}$	WOWCRM	$(W/W_{cr})_{2,m}$
VXVCRP	previous value of $(V_x/V_{cr})_{2,m}$	WOWCR2(I)	$(W/W_{cr})_{2,i}$
VX2(I)	$V_{x,2,i}$	WU1A	$W_{u,1a}$
V0	$V_0$	WU2(I)	$W_{u,2,i}$
V1	$V_1$	W1A	$W_{1a}$
V1A	$V_{1a}$	W2(I)	$W_{2,i}$
V1IDSQ	$V_{1,id}^2$	W2IDSQ(I)	$W_{2,id,i}^2$
V2(I)	$V_{2,i}$	W2TOT	$w_{calc}$
V2LOSS	$L_{ex,av}$	Z1	$(T/T')_{1,id}$
W	$w$	Z2	$(T/T'')_{2,id,m}$
WCALC	$w_{calc}$	Z3	$(T/T'')_{2,id,j}$
WCR1A	$W_{cr,1a}$		

Program listing. - The FORTRAN listing for main program RIFTUD is as follows:

```

REAL J,N,MU,LCDH,LW,LWX,LC,LCDHIS,LWDHIS,LLDHIS, LSDHIS,LRDHIS
DIMENSION DZ(17),U2(17),TTR2(17),WCR2(17),PTR2ID(17),VU2(17),
1WU2(17),DHVD(17),TT2(17),VCR2(17),BET2(17),W2(17),TS2(17),PS2(17),
2W2IDSQ(17),RH02(17),F2(17),RV2IM(17),VX2(17),ALPH2(17),V2(17),
3PT2(17),PTR2(17),VOVCR2(17),WOWCR2(17),DHIDT(17),DHIDS(17),
4ETATVD(17),ETASVD(17),ALPHA2(17),BETA2(17)
DIMENSION RV2I2M(17),TITLE(13)
COMMON/EFF/GAM,VOVCR1,W,PI,STAR,MU,D1,ALPH1,NSTAR,ALPH0,ALUNC,SIGS
1,DD,HS,CS,WOWCRM,D1A,D2M,HR,BET2M,EN,RH2RT2,CR
NAMELIST/INPUT/R,GAM,ALPHA0,ALPHA1,N,TTIN,PTIN,W,POW,RV1AAV,R1R1A,
1ROR1A,RT2R1A,RH2RT2,CDT2,MU,RV2I2M,K,IU,EBARS,EBARR,STAR,NSTAR
2,CS,CR,SIGSIN
EBARS=1.0
EBARR=1.0
CS=1.0
CR=1.0
SIGSIN= 1.35
ES=0.0
RV1AAV=1.0
CDT2=0.0
DO 1 I=1,17

```

```

1 RV2I2M(I)=1.0
  IU=2
  ITER=0
  PI=3.14159
  DOR=57.2958
  ROR1A=0.0
  ALPHA0=999.99
  WRITE(6,100)
100 FORMAT(1H1,39X,54HRADIAL INFLOW TURBINE VELOCITY DIAGPAM DESIGN AN
  1ALYSIS)
  2 READ (5,101) TITLE
101 FORMAT(13A6)
  IF(ITER.GT.0) WRITE(6,1011)
1011 FORMAT(1H1)
  WRITE(6,102) TITLE
102 FORMAT(1H ,13A6)
  READ (5,INPUT)
  IF (NSTAR.EQ.0) STAR=0.0
  IST=0
  VXVCRP=0.0
  DELVX=0.01
  ALPH0=ALPHA0/DOR
  ALPH1=ALPHA1/DOR
  IF(NSTAR.NE.2) ALSTG=(ALPH0+ALPH1)/2.
  EN=PI/30.*(110.-ALPHA1)*TAN(ALPH1)
  ENT=AINT(EN)
  IF(EN-ENT.LT..5) EN=ENT
  IF(EN-ENT.GE..5) EN=ENT+1.
  VUOU1A=1.-2./EN
  WGIV=W
  KP1=K+1
  WRITE (6,103)
103 FORMAT(39H0THIS OUTPUT IS IN THE FOLLOWING UNITS./
  1      129HOTEMPERATURE  PRESSURE      GAS CONST  ROT SPEED  MASS
  2FLOW      POWER      VISCOSITY  VELOCITY      SPFC WORK  DIAMETER
  3      ANGLE)
  GO TO (3,4),IU
  3 CPOW=1000.
  J=1.
  G=1.
  CN=2.
  PTO=PTIN*10000.
  WRITE (6,104)
104 FORMAT(3X6HKELVIN6X7HN/SQ CM4X8HJLS/KG-K5X7HRAD/SEC5X6HKG/SEC8X2HK
  1W6X10HN-SEC/SQ M5X5HM/SEC6X6HJLS/GM8X2HCM8X7HDEGREES)
  GO TO 5
  4 CPOW=550.
  J=777.649
  G=32.174
  CN=60./PI
  PTO=PTIN*144.
  WRITE (6,105)
105 FORMAT(12H DEG RANKINE2X8HLB/SQ IN4X8HBTU/LB-R5X7HREV/MIN5X6HLB/SE
  1C8X2HHP7X9HLB/FT-SEC4X6HFT/SEC6X6HBTU/LB8X2HIN8X7HDEGREES)
  5 DSHFT=CPOW/J*POW/W
  EX=GAM/(GAM-1.)
  CP=EX*R/J
  TGJCP=2.*G*J*CP
C
C      WRITE INPUT VALUES

```

```

C      WRITE (6,106) TTIN,ALPHA0,EBARS,R,PTIN,ALPHA1,EBARR,GAM,N,STAR,
      IRVIAAV,MU,W,K,POW,RDR1A
106  FORMAT(8H0*INPUT*/13H INLET TEMP =,F10.4,7X,18HSTATOR IN ANGLE =,
      1F6.2,6X,19HSTATOR KE LOS COEF=,F6.4,5X,14HGAS CONSTANT =F8.4/13H I
      2NLET PRESS=,F10.4,7X,18HSTATOR EX ANGLE =,F6.2,6X,19HROTOR KE LOS
      3S COEF=,F6.4,5X,14HSPEC HT RATIO=,F6.4/13H ROTAT SPEED=,F10.3,7X,1
      48HSTATOR ASPECT RAT=,F6.4,
      4          6X,19HROTOR IN/DEL RVU =,F6.4,5X,14HVISCOSIT
      5Y      =,E10.4/13H MASS FLOW =,F10.4,7X,11HDIAM RATIOS,19X,19HROTOR
      6 EX RAD SECTS=,I2/13H SHAFT POWER=,F10.3,8X,17HSTAT IN/ROT IN =,F
      76.4,6X,19HROT EX SECT/MN RVU=)
      K1=K
      IF(K.GT.10) K1=10
      WRITE (6,107) (RV2I2M(I),I=1,K1)
107  FORMAT(1H+,78X,F4.2,9F5.2)
      WRITE (6,108) R1R1A
108  FORMAT(31X,17HSTAT EX/ROT IN =,F6.4)
      IF(K.GT.10) WRITE (6,109) (RV2I2M(I),I=11,K)
109  FORMAT(1H+,77X,5F5.2)
      WRITE (6,110) RT2R1A,RH2RT2,CDT2
110  FORMAT(31X,17HROT EX TP/ROT IN=,F6.4/31X,17HROT EX HUB/TIP =F6.4,
      16X,19HCL HT/ROT EX TIP D=,F6.4)
      LCDH=CDT2*2./(1.-RH2RT2)
      LW=0.0
      6  DHVDAV=(DHSFT+LW)/(1.-LCDH)
      ISTT=0
      TT2AV=TTIN-DHVDAV/CP
      IF(TT2AV.LT.0.0) GO TO 203
      U1ASQ=G*J/VUOU1A*DHVDAV*RVIAAV
      U1A=SQRT(U1ASQ)
      VU1A=U1A*VUOU1A
      D1A=CN*U1A/N
C
C      STATION 1 - STATOR EXIT
C
      D1=R1R1A*D1A
      VU1=VU1A/R1R1A
      V1=VU1/SIN(ALPH1)
      VR1=VU1/TAN(ALPH1)
      TT1=TTIN
      TS1=TT1-V1*V1/TGJCP
25  ES1=ES
      ES=EBARS
      IF(EBARS.EQ.1.0.AND.IST.EQ.0) ES=.05
      IF(EBARS.EQ.1.0.AND.IST.GT.0) CALL EFFIC (ES,ER,1)
      IST=IST+1
      ISTT=ISTT+1
      7  V1IDSQ=V1*V1/(1.-ES)
      Z1=1.-V1IDSQ/TGJCP/TT1
      IF(Z1.LE.0.0) GO TO 204
      PS1=PT0*Z1**EX
      PT1=PS1*(TT1/TS1)**EX
      RH01=PS1/R/TS1
      HS=W/RH01/VR1/PI/D1
      VCR1=SQRT(2.*GAM/(GAM+1.)*G*R*TT1)
      VOVCR1=V1/VCR1
C
C      STATION 0 - STATOR INLET
C

```



```

IF(NSTAR.GT.0) GO TO 71
DO=D1A*RDR1A
CHRD=SQRT(DO**2+D1**2-SQRT((DO**2+D1**2)**2-(DO**2-D1**2)**2/
1COS(ALSTG)**2))/2.
STAR=HS/CHRD
ALUNC=ALPH1-ALPH0-A COS((DO**2+D1**2-4.*CHRD**2)/2./DO/D1)
70 CONTINUE
IF(IST.EQ.1) SIGS=SIGSIN
SS=CHRD/SIGS
ENS= PI*D1/SS
ENST=AIN(T(ENS)
IF(ENS-ENST.LT..5) ENS=ENST
IF(ENS-ENST.GE..5) ENS=ENST+1.
SS= PI*D1/ENS
SIGS=CHRD/SS
IF(EBARS.NE.1.0) GO TO 26
IF(ISTT.EQ.1) GO TO 25
IF(ABS(ES-ES1)/ES.GT..0001) GO TO 25
26 TTO=TTIN
RHOTD=PTD/R/TTO
RHOD=RHOTD
9 VO=W/PI/DO/HS/COS(ALPHD)/RHOD
TSO=TTO-VO**2/TGJCP
PSO=PTO*(TSO/TTO)**EX
RHOX=RHOD
RHOD=PSO/R/TSO
IF(ABS(RHOX-RHOD)/RHOD.GT..0001) GO TO 9
VUO=VO*SIN(ALPHD)
VRO=VO*COS(ALPHD)
VOVCRO=VO/VCR1

C
C
C
STATION 1A - ROTOR INLET

ALPH1A=ALPH1
8 VR1A=VU1A/TAN(ALPH1A)
V1A=VU1A/SIN(ALPH1A)
TT1A=TT1
TS1A=TT1A-V1A*V1A/TGJCP
PS1A=PT1*(TS1A/TT1)**EX
RH01A=PS1A/R/TS1A
VR1A=VR1*RH01/RH01A*R1R1A
ALP1A=ALPH1A
ALPH1A=ATAN (VU1A/VR1A)
IF(ABS(ALPH1A-ALP1A).GT..01/DOR) GO TO 8
LWX=LW
LW=.0061/6/J*RH01A*U1A**3*D1A**2/W/(RH01A*U1A*D1A/MU)**.2
IF(ABS(LW-LWX)/LW.GT..0001) GO TO 6
PT1A=PT1
WU1A=VU1A-U1A
BET1A=ATAN(WU1A/VR1A)
W1A=VR1A/COS(BET1A)
TTR1A=TS1A+W1A**2/TGJCP
PTR1A=PS1A*(TTR1A/TS1A)**EX
VCR1A=VCR1
WCR1A=VCR1A*SQRT(TTR1A/TT1A)
VOVCRA=V1A/VCR1A
WOWCRA=W1A/WCR1A

C
C
C
STATION 2 - ROTOR EXIT

```

```

M=(K+3)/2
KK=K+2
D2(KK)=D1A*RT2R1A
D2(1)=D2(KK)*RH2RT2
HR=(D2(KK)-D2(1))/2.
EL=HR/FLOAT(K)
SIGRV=0.0
DO 12 I=1,KK
IF(I.EQ.1.OR.I.EQ.KK) GO TO 11
D2(I)=D2(1)+(2.*FLOAT(I)-3.)*EL
RV2IM(I)=RV2IM(I-1)
SIGRV=SIGRV+RV2IM(I)/FLOAT(K)
11 U2(I)=D2(I)*N/CN
TTR2(I)=TTR1A+(U2(I)**2-U1ASQ)/TGJCP
WCR2(I)=WCR1A*SQRT(TTR2(I)/TTR1A)
12 PTR2ID(I)=PTR1A*(TTR2(I)/TTR1A)**EX
RV2IM(1)=RV2IM(2)-(RV2IM(3)-RV2IM(2))/2.
RV2IM(KK)=RV2IM(K+1)-(RV2IM(K)-RV2IM(K+1))/2.
DVU2AV=D1A*VU1A*(1.-1./RV1AAV)
VU2(M)=DVU2AV/D2(M)*RV2IM(M)/SIGRV
ITER=1
13 WU2(M)=VU2(M)-U2(M)
DHVD(M)=(U1A*VU1A-U2(M)*VU2(M))/G/J
TT2(M)=TTO-DHVD(M)/CP
IF(TT2(M).LT.0.0) GO TO 205
VCR2(M)=VCR1A*SQRT(TT2(M)/TT1A)
IF(ITER.EQ.1) VXVCR=0.5
IND=1
14 VX2(M)=VXVCR*VCR2(M)
BET2(M)=ATAN(WU2(M)/VX2(M))
W2(M)=VX2(M)/COS(BET2(M))
TS2(M)=TTR2(M)-W2(M)**2/TGJCP
D2M=D2(M)
BET2M=BET2(M)
WOWCRM=W2(M)/WCR2(M)
ER=EBARR
IF(EBARR.EQ.1.0) CALL EFFIC(ES,ER,2)
15 W2IDSQ(M)=W2(M)**2/(1.-ER)
Z2=1.-W2IDSQ(M)/TGJCP/TTR2(M)
IF(Z2.LE.0.0) GO TO 206
PS2(M)=PTR2ID(M)*Z2**EX
RH02(M)=PS2(M)/R/TS2(M)
F2(M)=PI*D2(M)*EL*RH02(M)*VX2(M)
IF(ITER.EQ.1) WCALC=F2(M)*FLOAT(K)
IF(ITER.EQ.1) GO TO 51
67 I=M
ID=-1
16 II=I
I=I+ID
VU2(I)=VU2(M)*RV2IM(I)*D2(M)/D2(I)
WU2(I)=VU2(I)-U2(I)
PS2(I)=PS2(II)+RH02(II)/G*(VU2(II)**2/D2(II)+VU2(I)**2/D2(I))/2.*(
ID2(I)-D2(II))
W2IDSQ(I)=TGJCP*TTR2(I)*(1.-(PS2(I)/PTR2ID(I))**(1./EX))
Z3=1.-W2IDSQ(I)/TGJCP/TTR2(I)
IF(Z3.LE.0.0) GO TO 207
W2(I)=SQRT(W2IDSQ(I))*(1.-ER)
IF(W2(I).LE.ABS(WU2(I))) GO TO 68
BET2(I)=A SIN(WU2(I)/W2(I))
VX2(I)=W2(I)*COS(BET2(I))

```

```

    TS2(I)=TTR2(I)-W2(I)**2/T6JCP
    RH02(I)=PS2(I)/R/TS2(I)
    IF(I.EQ.1) GO TO 17
    IF(I.EQ.KK) GO TO 18
    F2(I)=PI*D2(I)*EL*RH02(I)*VX2(I)
    GO TO 16
17 I=M
    ID=1
    GO TO 16
18 W2TOT=0.0
    DO 19 I=2,KP1
19 W2TOT=W2TOT+F2(I)
    WCALC=W2TOT
    IF(K.GT.1) GO TO 51
69 IF(DVU2AV .EQ. 0.0) GO TO 21
    DVUTOT=0.0
    DO 20 I=2,KP1
20 DVUTOT=DVUTOT+F2(I)*D2(I)*VU2(I)
    DVUAVC=DVUTOT/W
    IF(ABS(DVUAVC-DVU2AV)/DVU2AV.LE..0001) GO TO 21
    VU20=VU2(M)
    VU2(M)=VU20*DVU2AV/DVUAVC
    GO TO 13
21 V2LOSS=0.0
    DHIDTA=0.0
    DHIDSA=0.0
    Q=0.0
    RLOSS=0.0
    DO 23 I=1,KK
    DHVD(I)=(U1A*VU1A-U2(I)*VU2(I))/G/J
    TT2(I)=TTO-DHVD(I)/CP
    IF(TT2(I).LT.0.0) GO TO 208
    VCR2(I)=VCR1A*SQRT(TT2(I)/TT1A)
    ALPH2(I)=ATAN(VU2(I)/VX2(I))
    V2(I)=VX2(I)/COS(ALPH2(I))
    PT2(I)=PS2(I)*(TT2(I)/TS2(I))**EX
    PTR2(I)=PS2(I)*(TTR2(I)/TS2(I))**EX
    VOVC2(I)=V2(I)/VCR2(I)
    WOWCR2(I)=W2(I)/WCR2(I)

```

C  
C  
C

#### SECTOR PERFORMANCE

```

    DHIDT(I)=CP*TTO*(1.-(PT2(I)/PT0)**(1./EX))
    DHIDS(I)=CP*TTO*(1.-(PS2(I)/PT0)**(1./EX))
    IF(I.EQ.1.OR.I.EQ.KK) GO TO 22
    V2LOSS=V2LOSS+F2(I)/W*V2(I)**2/2./G/J
    RLOSS=RLOSS+F2(I)/W*(W2IDS0(I)-W2(I)**2)/2./G/J
    Q=Q+F2(I)/RH02(I)
    DHIDTA=DHIDTA+F2(I)/W*DHIDT(I)
    DHIDSA=DHIDSA+F2(I)/W*DHIDS(I)
22 ETATVD(I)=DHVD(I)/DHIDT(I)
23 ETASVD(I)=DHVD(I)/DHIDS(I)

```

C  
C  
C

#### OVERALL PERFORMANCE

```

    ETATV=DHVDV/DHIDTA
    ETASV=DHVDV/DHIDSA
    ETAT=DHSHFT/DHIDTA
    ETAS=DHSHFT/DHIDSA
    SSPD=N*SQRT(Q)/(J*DHIDTA)**.75

```

```

LC=LCDH*DHVDAV
LCDHIS=LC/DHIDSA
LWDHIS=LW/DHIDSA
LLDHIS=V2LOSS/DHIDSA
LSDHIS=(V1IDSQ-V1*V1)/2./G/J/DHIDSA
LPDHIS=RLOSS/DHIDSA
GO TO (30,32),IU
30 PTO=PTO/10000.
PSO=PSO/10000.
PT1=PT1/10000.
PS1=PS1/10000.
PT1A=PT1A/10000.
PS1A=PS1A/10000.
PTR1A=PTR1A/10000.
DO=DO*100.
D1=D1*100.
D1A=D1A*100.
HS=HS*100.
DO 31 I=1, KK
D2(I)=D2(I)*100.
PT2(I)=PT2(I)/10000.
PS2(I)=PS2(I)/10000.
PTR2(I)=PTR2(I)/10000.
31 DHVD(I)=DHVD(I)/1000.
DHVDAV=DHVDAV/1000.
GO TO 36
32 PTO=PTO/144.
PSO=PSO/144.
PT1=PT1/144.
PS1=PS1/144.
PT1A=PT1A/144.
PS1A=PS1A/144.
PTR1A=PTR1A/144.
DO=DO*12.
D1=D1*12.
D1A=D1A*12.
HS=HS*12.
DO 33 I=1, KK
D2(I)=D2(I)*12.
PT2(I)=PT2(I)/144.
PS2(I)=PS2(I)/144.
33 PTR2(I)=PTR2(I)/144.
C
C WRITE CALCULATED VALUES
C
36 WRITE (6,111)
111 FORMAT(9H *OUTPUT*/ 26X, 2(3HABS, 6X), 17X, 3(3HABS, 5X), 9X, 4HREL , 4(5X
1, 3HREL)/17X, 4HDIA-, 2(4X, 5HTOTAL), 4X, 2(6HSTATIC, 3X), 4HFLOW, 3X, 5HVEL
20-, 4X, 4HCRIT, 3X, 5HBLADE, 2(4X, 5HTOTAL), 4X, 4HFLOW, 3X, 5HVELO-, 4X, 4HCR
3IT/16X, 5HMETER, 2(5X, 4HTEMP, 4X, 5HPRESS), 3X, 5HANGLE, 4X, 4HCITY, 9H VE
4L RAT, 7H SPEED, 5X, 4HTEMP, 4X, 5HPRESS, 8H ANGLE, 4X, 13HCITY VEL RA
5T)
ALPO=ALPHO*DOR
ALPHAA=ALPHA*DOR
BETA1A=BETA1*DOR
DO 34 I=1, KK
ALPHA2(I)=ALPH2(I)*DOR
34 BETA2(I)=BET2(I)*DOR
WRITE (6,112)DO, TTD, PTD, TSD, PSD, ALP O, VD, VOVCRO

```

```

112 FORMAT(13H STATOR INLET,2(F9.3,F9.2),F9.3,F7.2,F9.2,F7.3)
WRITE(6,1121) ES,HS,ENS
1121 FORMAT(11H LOSS COEF=,F6.4,67X,11HSTATOR HGT=,F7.4,17H,NUMBER OF V
1ANES=,F5.1)
WRITE(6,113)D1,TT1,PT1,TS1,PS1,ALPHA1,V1,VOVCR1
113 FORMAT(13H STATOR EXIT ,2(F9.3,F9.2),F9.3,F7.2,F9.2,F7.3)
WRITE(6,114)D1A,TT1A,PT1A,TS1A,PS1A,ALPHA1A,V1A,VOVCRA,U1A,TTR1A,
1PTR1A,BETA1A,W1A,WOWCRA
114 FORMAT(13HROTOR INLET ,2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
1F9.3,F8.2,F9.2,F7.3)
WRITE(6,1141) ER
1141 FORMAT(11H LOSS COEF=,F6.4)
WRITE(6,115)(D2(I),TT2(I),PT2(I),TS2(I),PS2(I),ALPHA2(I),V2(I),
1VOVCR2(I),U2(I),TTR2(I),PTR2(I),BETA2(I),W2(I),WOWCR2(I),I=1,KK)
115 FORMAT(13H ROTOR EXIT ,2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
1F9.3,F8.2,F9.2,F7.3/(13X2(F9.3,F9.2)F9.3,F7.2,F9.2,F7.3,2F9.2,
2F9.3,F8.2,F9.2,F7.3))
F2(1)=0.0
F2(KK)=0.0
WRITE(6,1161)
1161 FORMAT(1H )
WRITE(6,116) EN
116 FORMAT(26X,4HMASS,5X,4HDIAG/17X,4HDIA-,5X,4HFLOW,5X,4HSPEC,4X,5HTO
1TAL,3X,6HSTATIC/16X,5HMETER,5X,4HRATE,5X,4HWORK,2(4X,5HEFFIC),3X,
223HNUMBER OF ROTOR BLADES=,F5.1)
WRITE(6,117)(D2(I),F2(I),DHVD(I),ETATVD(I),ETASVD(I),I=1,KK)
117 FORMAT(13X,F9.3,F9.4,F9.3,2F8.3)
TTPR=PTO/PT2(M)
TSPR=PTO/PS2(M)
WRITE(6,118)TTPR,LSOHHIS,TSPR,LRDHHIS,DHVDIV,LWDHHIS,ETATV,LCDHHIS,
1ETASV,LLDHHIS,ETAT,ETAS,SSPD
118 FORMAT(22HD*OVERALL PERFORMANCE*/48X,20HLOSS/IDEAL T-S DEL H/6X,24
1HTOT-TOT PRESSURE RATIO =,F8.4,13X,10HSTATOR =,F6.4/6X,24HTOT-ST
2AT PRESSURE RATIO=,F8.4,13X,10HROTOR =,F6.4/6X,24HDIAG AVG SPEC
3IFIC WORK =,F8.4,13X,10HWINDAGE =,F6.4/6X,24HDIAG TOTAL EFFICIENC
4Y =,F8.4,13X,10HCLEARANCE=,F6.4/6X,24HDIAG STATIC EFFICIENCY =,F8
5.4,13X,10HEXIT KE =,F6.4/6X,24HNET TOTAL EFFICIENCY =,F8.4/6X,
624HNET STATIC EFFICIENCY =,F8.4,8X,15HSPECIFIC SPEED=,F7.3)
GO TO 2
51 IF(IND.GE.6.AND.ABS(WGIV-WCALC)/WGIV.LE..0001) GO TO 65
CALL CONTIN(VXVCR,WCALC,IND,1,WGIV,.05)
IF(IND-10)14,61,61
61 IF(ITER-1)62,62,63
62 VXVCR=.9
IF(K.EQ.1) GO TO 63
IND=1
ITER=ITER+1
GO TO 14
63 IF(IND-10)201,201,202
65 IF(ITER-1)66,66,69
66 ITER=ITER+1
IND=1
GO TO 67
68 IF(VXVCR.GE.VXVCRP) GO TO 72
VXVCR=VXVCRP-DELVX
DELVX=DELVX/10.
IF(DELVX.LE..00001) GO TO 209
72 VXVCR=VXVCR+DELVX
VXVCRP=VXVCR

```

```

71 CHR D=HS/STAR
  IF(INSTAR.EQ.2) GO TO 76
  RSTG=(CHR D * COS(ALSTG) + SQRT(CHR D ** 2 * (COS(ALSTG) ** 2 - 1.) + D1 ** 2)) / 2.
  DO=(SQRT(CHR D ** 2 / 2. + 2. * RSTG ** 2 - D1 ** 2 / 4.)) * 2.
  ALUNC=ALPH1 - ALPH0 - A * COS((DO ** 2 + D1 ** 2 - 4. * CHR D ** 2) / 2. / DO / D1)
  GO TO 70
76 ALPHJ=ATAN(SIN(ALPH1) / (CHR D / D1 * 2. + COS(ALPH1)))
  ALSTG=(ALPH0 + ALPH1) / 2.
  DO=2. * SQRT(CHR D ** 2 + D1 ** 2 / 4. + CHR D * D1 * COS(ALPH1))
  GO TO 70
201 WRITE(6,120)WCALC
120 FORMAT(48H0 ROTOR EXIT CHOKES AT MAXIMUM MASS FLOW RATE = ,F9.4)
  GO TO 2
202 WRITE(6,121)
121 FORMAT(69H0 NO SOLUTION FOUND AFTER 100 ITERATIONS FOR CONTINUITY
  1AT ROTOR EXIT)
  GO TO 2
203 WRITE(6,122)
122 FORMAT(60H0REQUIRED SPECIFIC WORK GREATER THAN ENERGY AVAILABLE IN
  1 GAS)
  GO TO 2
204 WRITE(6,123)
123 FORMAT(74H0REQUIRED STATOR IDEAL KINETIC ENERGY GREATER THAN ENER
  1Y AVAILABLE IN GAS)
  GO TO 2
205 WRITE(6,124) M
124 FORMAT(33H0SPECIFIC WORK REQUIRED IN SECTOR, I3, 37H GREATER THAN EN
  1ERGY AVAILABLE IN GAS)
  GO TO 2
206 WRITE(6,125) M
125 FORMAT(55H0ROTOR IDEAL RELATIVE KINETIC ENERGY REQUIRED IN SECTOR,
  1 I3, 37H GREATER THAN ENERGY AVAILABLE IN GAS)
  GO TO 2
207 WRITE(6,125) I
  GO TO 2
208 WRITE(6,124) I
  GO TO 2
209 WRITE(6,128)
128 FORMAT(123H0THE PROGRAM CAN NOT FIND A SOLUTION SIMULTANEOUSLY SAT
  1ISFYING CONTINUITY, RADIAL EQ., AND THE LOSS MODEL AT THE ROTOR EX
  2IT)
  GO TO 2
999 STOP
  END

```

### Blade-Row Loss Coefficient Subprograms

The stator and rotor loss-coefficient calculations, as described in the section LOSS MODEL, are performed by subroutine EFFIC. Function subprograms SIMPS1, SHUB, and SHUB2 are used for the numerical integration required to determine the rotor hub wall area (eq. (C17)). SIMPS1 uses a modification of Simpson's rule wherein more intervals are placed in those regions requiring greater accuracy. SHUB and SHUB2 are the functions being integrated.

Program variables. - Variables transfer between main program RIFTUD and subroutine EFFIC by means of both the EFFIC(ES, ER, J) arguments and labeled common

block /EFF/. The arguments ES and ER are the stator and rotor loss coefficients, respectively, and J is the blade-row indicator (J = 1 for stator and J = 2 for rotor). The common block variables, which were defined in the RIFTUD variable list, are ALPH0, ALPH1, ALUNC, BET2M, CS, CR, D0, D1, D1A, D2M, EN, GAM, HR, HS, MU, NSTAR, PI, RH2RT2, SIGS, STAR, VOVCR1, W, and WOWCRM. The remaining variables in subroutine EFFIC are defined as follows:

A(X)	arithmetic statement function for Q where argument $X = \left( V/V_{cr} \right)_1$ or $\left( W/W_{cr} \right)_{2,m}$
AA	a
AR	$Q_r$
AS	$Q_s$
A2	a/2
A3A2R	$\left( A_{3D}/A_{2D} \right)_r$
A3A2S	$\left( A_{3D}/A_{2D} \right)_s$
BB	b
BWSR	$A_{b,r}$
CHRDR	$c_r$
D(Y)	arithmetic statement function for denominator of equation (10) or (11) where argument $Y = Q_s$ or $Q_r$
E(Y)	arithmetic statement function for E where argument $Y = Q_s$ or $Q_r$
ELOCS	$(l/c)_s$
ELOSR	$(l/s)_r$
ELOSS	$(l/s)_s$
EWSR	$A_{w,t,r} + A_{w,h,r}$
H(Y)	arithmetic statement function for H where argument $Y = Q_s$ or $Q_r$
K	error parameter for integration of function SHUB
K2	error parameter for integration of function SHUB2

REFR	$\left(\theta_{\text{tot}}/l \text{ Re}^{-0.2}\right)_{\text{ref, r}}$
REFS	$\left(\theta_{\text{tot}}/l \text{ Re}^{-0.2}\right)_{\text{ref, s}}$
RER	$\text{Re}_r$
RES	$\text{Re}_s$
SRH	$n_r A_{w, h, r}$
SRH1	part of SRH from first integral in equation (C17)
SRH2	part of SRH from second integral in equation (C17)
SRS	$n_r A_{w, t, r}$
TOSR	$(t/s)_r$
TOSS	$(t/s)_s$
YC	$r_{1a}$
Y2	$r_{1a} - b\sqrt{3}/2$

Variables transfer among *EFFIC*, *SIMPS1*, *SHUB*, and *SHUB2* by means of the function arguments and the labelled common block /SH/. The function arguments are *SIMPS1*(X1, X2, FUNC, KSIG), *SHUB*(X), and *SHUB2*(Y) where

X1	lower limit of integration
X2	upper limit of integration
FUNC	function being integrated, <i>SHUB</i> or <i>SHUB2</i> in this case
KSIG	error parameter, increases if integration is inaccurate
X	variable of integration
Y	variable of integration

The common block variables, which were defined in the *EFFIC* variable list, are *AA*, *BB*, and *YC*. These also are the only variables, aside from the arguments, used in *SHUB* and *SHUB2*.

A further description of *SIMPS1* and definition of its internal variables can be obtained from reference 6. The function described is called *SIMPS2*, but it becomes exactly *SIMPS1* upon deletion of the parameter *J* from the function arguments.



Program listing. - The FORTRAN listings for subroutine EFFIC and functions SIMPS1, SHUB, and SHUB2 are as follows:

```

SUBROUTINE EFFIC (ES,ER,J)
REAL MU
COMMON/EFF/GAM,VOVCR1,W,PI,STAR,MU,D1,ALPH1,NSTAR,ALPH0,ALUNC,SIGS
1,DO,HS,CS,WOWCRM,D1A,D2M,HR,BET2M,EN,RH2RT2,CR
COMMON/SH/AA,BB,YC
A(X)=(GAM-1.)/(GAM+1.)*X*X
D(Y)= 1./1.68+Y/2.88+Y*Y/4.4+Y*Y*Y/6.24
H(Y)=(1./1.2+3.*Y/1.6+5.*Y*Y/2.+7.*Y**3/2.4+9.*Y**4/2.8)/D(Y)
E(Y)=2.*(1./1.92+Y/3.2+Y*Y/4.8+Y**3/6.72)/D(Y)
GO TO (1,2),J
1 AS= A(VOVCR1)
REFS=.03734
RES= W/PI/STAR/MU/D1/COS(ALPH1)
ELOCS=1.
IF(NSTAR.NE.2.OR.ALUNC.NE.0.0) ELOCS=ALUNC/2./SIN(ALUNC/2.)
ELOSS= ELOCS*SIGS
A3A2S= 1.+(DO**2-D1**2)/4./SIGS/HS/D1/ELOCS
TOSS= .05*SIGS*STAR
ES=CS*E(AS)*REFS/RES**2+ELOSS*A3A2S/(COS(ALPH1)-CS*H(AS)*REFS/RES
1**2+ELOSS-TOSS)
GO TO 3
2 AR= A(WOWCRM)
REFR=.11595
CHRDR=0.5*SQRT((D1A-D2M-HR+HS)**2+(D1A-D2M)**2)
RER=W/PI*CHRDR/HR/MU/D2M/COS(BET2M)
ELOS=EN*CHRDR/2./SQRT(2.)/D2M
BWSR=PI/8.*(D1A-D2M-HR+2.*HS)*(D1A-D2M+HR)-(D1A-D2M-HR)**2)
SRS=PI/2.*(D1A-D2M-HR)*((PI/2.-1.)*D1A+D2M+HR)
EXTERNAL SHUB,SHUB2
K= 0
K2=0
AA= (D1A-D2M-HR+2.*HS)/2.
BB= (D1A-D2M+HR)/2.
YC=D1A/2.
A2=AA/2.
Y2=YC-BB/2.*SQRT(3.)
SRH1=2.*PI*SIMPS1(0.0,A2,SHUB,K)
IF(K.GT.0) WRITE(6,10) K
10 FORMAT(3H K=,I2)
SRH2=2.*PI*SIMPS1(Y2,YC,SHUB2,K2)
IF(K2.GT.0) WRITE(6,11) K2
11 FORMAT(4H K2=,I2)
SRH=SRH1+SRH2
EWSR=(SRS+SRH)/EN
A3A2R=1.+EWSR/BWSR
TOSR= 0.04/PI*EN*(1.-RH2RT2)/(1.+RH2RT2)
ER=CR*E(AR)*REFR/RER**2+ELOS*A3A2R/(COS(BET2M)-CR*H(AR)*REFR/RER
1**2+ELOS-TOSR)
3 RETURN
END

```

```

      FUNCTION SIMPS1 (X1,X2,FUNC,KSIG)
C.....THIS ROUTINE INTEGRATES FUNC(X) FROM X1 TO X2 USING A MULTIPLE
C.....INTERVAL SIMPSON#S RULE TECHNIQUE.
      LOGICAL SPILL
      DOUBLE PRECISION ANS,Q
      DIMENSION V(200),H(200),A(200),B(200),C(200),P(200),E(200)
      DATA TWO,THREE,FOUR,THIRTY/2.0,3.0,4.0,30.0/
      DATA T,NMAX,NSIG/3.0E-5,200,1/
C.....INITIALIZE FIRST ELEMENTS OF ARRAYS.
      V(1)= X1
      H(1)= (X2 - V(1))/TWO
      A(1)= FUNC ( V )
      B(1)= FUNC ( V(1)+H(1) )
      C(1)= FUNC ( X2 )
      P(1)= H(1)*(A(1) • FOUR*B(1) • C(1))
      E(1)= P(1)
      ANS= P(1)
      N=1
      FRAC=T
      SPILL=.FALSE.
1     TEST=ABS(FRAC*ANS)
      K=N
      DO 3 I=1,K
C.....TEST MAGNITUDE OF 4TH ORDER ERROR IN THIS INTERVAL.
      IF (ABS(E(I)).LE.TEST) GO TO 3
      IF (N.LT.NMAX) GO TO 2
C.....GO TO FINISH IF STORAGE IS FILLED UP.
      SPILL=.TRUE.
      KSIG=KSIG+NSIG
      GO TO 4
C.....SUBDIVIDE INTERVAL AGAIN TO REDUCE 4TH ORDER ERROR.
2     N=N+1
      V(N)=V(I)+H(I)
      H(N)=H(I)/TWO
      A(N)=B(I)
      B(N)=FUNC(V(N)+H(N))
      C(N)=C(I)
      P(N)=H(N)*(A(N)+FOUR*B(N)+C(N))
      H(I)=H(N)
      B(I)=FUNC(V(I)+H(I))
      C(I)=A(N)
      Q=P(I)
      P(I)=H(I)*(A(I)+FOUR*B(I)+C(I))
      Q=P(I)+P(N)-Q
      ANS=ANS+Q
      E(I)=Q
      E(N)=Q
3     CONTINUE
C.....TEST ALL INTERVALS AGAIN IF ANY WERE SUBDIVIDED THE LAST TIME.
      IF (N.GT.K) GO TO 1
4     Q=0.0
      DO 5 I=1,N
5     Q=Q+E(I)
C.....TIGHTEN ERROR LIMIT IF TOTAL ACCUMULATED ERROR TOO LARGE.
      IF (ABS(Q/T).LE.ABS(ANS).OR.SPILL) GO TO 6
      FRAC=FRAC/TWO
      GO TO 1
C.....FINISH OFF CALCULATION.
6     SIMPS1=(ANS+Q/THIRTY)/THREE
      RETURN
C.....THIS ENTRY USED TO GET AT INTERNAL VARIABLES.
      ENTRY SIMPX1 (TT,NN,QQ)
      T=TT
      NN=N
      QQ=Q
      RETURN
      END

```

```

FUNCTION SHUB(X)
COMMON/SH/AA, BB, YC
SHUB=(YC-BB/AA*SQRT(AA**2-X*X))*SQRT(1.+BB**2/AA**2*X*X/(AA**2-X*X
1))
RETURN
END

```

```

FUNCTION SHUB2(Y)
COMMON/SH/AA, BB, YC
SHUB2=Y*SQRT(1.+AA**2/BB**2*(Y-YC)**2/(BB**2-(Y-YC)**2)')
RETURN
END

```

### Rotor-Exit Continuity Subprograms

Subroutines CONTIN and PABC provide the means for obtaining the rotor-exit continuity solution; that is, they yield the value of  $\left(\frac{V_x}{V_{cr}}\right)_{2, m}$  that satisfies the input mass flow rate. CONTIN provides an estimate for the value of an independent variable X that satisfies a given value of the dependent variable Y by means of curve fitting. The curve is a parabola whose coefficients are calculated by PABC, which is called by CONTIN. The estimation continues until a solution is obtained within the desired tolerance. Subroutines CONTIN and PABC are described in detail in reference 7.

Program variables. - Variables transfer between main program RIFTUD and subroutine CONTIN by means of the CONTIN(XEST, YCALC, IND, JZ, YGIV, XDEL) arguments. Transfer of variables between CONTIN and PABC is by means of the PABC(X, Y, A, B, C) arguments. These arguments are defined as follows:

XEST     on call: value of X used to calculate YCALC  
          on return: value of X to be used to calculate next value of YCALC

YCALC    value of Y corresponding to XEST during call

IND       on call: controls sequence of calculation in CONTIN  
          on return: indicates when a choked solution is found or when no solution can be found

JZ        determines whether subsonic or supersonic solution will be obtained:  
          1 - subsonic solution  
          2 - supersonic solution

YGIV value of Y desired for solution  
 XDEL maximum permissible change in XEST between iterations  
 X independent variable  
 Y dependent variable  
 A coefficient A in  $y = Ax^2 + Bx + C$   
 B coefficient B in  $y = Ax^2 + Bx + C$   
 C coefficient C in  $y = Ax^2 + Bx + C$

The internal variables for CONTIN are defined in reference 7.

Program listing. - The FORTRAN listings for subroutines CONTIN and PABC are as follows:

```

      SUBROUTINE CONTIN(XEST,YCALC,IND,JZ,YGIV,XDEL)
      C
      C--CONTIN CALCULATES AN ESTIMATE OF THE RELATIVE FLOW VELOCITY
      C--FOR USE IN THE VELOCITY GRADIENT EQUATION
      C
      DIMENSION X(3),Y(3)
      NCALL = NCALL+1
      IF (IND.NE.1.AND.NCALL.GT.100) GO TO 160
      GO TO (10,30,40,50,60,110,150),IND
      C--FIRST CALL
      10 NCALL = 1
      XORIG = XEST
      IF (YCALC.GT.YGIV.AND.JZ.EQ.1) GO TO 20
      IND = 2
      Y(1) = YCALC
      X(1) = 0.
      XEST = XEST+XDEL
      RETURN
      20 IND = 3
      Y(3) = YCALC
      X(3) = 0.
      XEST = XEST-XDEL
      RETURN
      C--SECOND CALL
      30 IND = 4
      Y(2) = YCALC
      X(2) = XEST-XORIG
      XEST = XEST+XDEL
      RETURN
      40 IND = 5
      Y(2) = YCALC
      X(2) = XEST-XORIG
      XEST = XEST-XDEL
      RETURN
      C--THIRD OR LATER CALL - FIND SUBSONIC OR SUPERSONIC SOLUTION
      50 Y(3) = YCALC
      X(3) = XEST-XORIG
      GO TO 70
    
```

```

60 Y(1) = YCALC
   X(1) = XEST-XORIG
70 IF (YGIV.LT.AMIN1(Y(1),Y(2),Y(3))) GO TO (120,130),JZ
80 IND = 6
   CALL PABC(X,Y,APA,BPB,CPC)
   DISCR = BPB**2-4.*APA*(CPC-YGIV)
   IF (DISCR.LT.0.) GO TO 140
   IF (ABS(400.*APA*(CPC-YGIV)).LE.BPB**2) GO TO 90
   XEST = -BPB-SIGN(SQRT(DISCR),APA)
   IF (JZ.EQ.1.AND.APA.GT.0..AND.Y(3).GT.Y(1)) XEST = -BPB+
1  SQRT(DISCR)
   IF (JZ.EQ.2.AND.APA.LT.0.) XEST = -BPB-SQRT(DISCR)
   XEST = XEST/2./APA
   GO TO 100
90 IF (JZ.EQ.2.AND.BPB.GT.0.) GO TO 130
   ACB2 = APA/BPB*(CPC-YGIV)/BPB
   IF (ABS(ACB2).LE.1.E-8) ACB2=0.
   XEST = -(CPC-YGIV)/BPB*(1.+ACB2+2.*ACB2**2)
100 IF (XEST.GT.X(3)) GO TO 130
   IF (XEST.LT.X(1)) GO TO 120
   XEST = XEST+XORIG
   RETURN
C--FOURTH OR LATER CALL - NOT CHOKED
110 IF(XEST-XORIG.GT.X(3)) GO TO 130
   IF(XEST-XORIG.LT.X(1)) GO TO 120
   Y(2) = YCALC
   X(2) = XEST-XORIG
   GO TO 70
C--THIRD OR LATER CALL - SOLUTION EXISTS,
C--BUT RIGHT OR LEFT SHIFT REQUIRED
120 IND = 5
C--LEFT SHIFT
   XEST = X(1)-XDEL+XORIG
   XOSHFT = XEST-XORIG
   XORIG = XEST
   Y(3) = Y(2)
   X(3) = X(2)-XOSHFT
   Y(2) = Y(1)
   X(2) = X(1)-XOSHFT
   RETURN
130 IND = 4
C--RIGHT SHIFT
   XEST = X(3)+XDEL+XORIG
   XOSHFT = XEST-XORIG
   XORIG = XEST
   Y(1) = Y(2)
   X(1) = X(2)-XOSHFT
   Y(2) = Y(3)
   X(2) = X(3)-XOSHFT
   RETURN
C--THIRD OR LATER CALL - APPEARS TO BE CHOKED
140 XEST = -BPB/2./APA
   IND = 7
   IF (XEST.LT.X(1)) GO TO 120
   IF(XEST.GT.X(3)) GO TO 130
   XEST = XEST+XORIG
   RETURN
C--FOURTH OR LATER CALL - PPOBABLY CHOKED
150 IF (YCALC.GE.YGIV) GO TO 110
   IND = 10
   RETURN

```

```
C--NO SOLUTION FOUND IN 100 ITERATIONS
160 IND = 11
RETURN
END
```

```
      SUBROUTINE PABC(X,Y,A,B,C)
C
C--PABC CALCULATES COEFFICIENTS A,B,C OF THE PARABOLA
C--Y=A*X**2+B*X+C, PASSING THROUGH THE GIVEN X,Y POINTS
C
      DIMENSION X(3),Y(3)
      C1 = X(3)-X(1)
      C2 = (Y(2)-Y(1))/(X(2)-X(1))
      A = (C1*C2-Y(3)+Y(1))/C1/(X(2)-X(3))
      B = C2-(X(1)+X(2))*A
      C = Y(1)-X(1)*B-X(1)**2*A
      RETURN
      END
```

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, October 23, 1975,  
505-04.

## APPENDIX A

### SYMBOLS

A	area, $\text{m}^2$ ; $\text{ft}^2$
a	semiaxis of ellipse in x-direction, m; ft
b	semiaxis of ellipse in y-direction, m; ft
C	loss coefficient multiplier
$C_e$	circumference of ellipse, m; ft
$C_N$	dimensional constant, $2\pi$ rad/rev; 60 sec/min
$C_P$	dimensional constant, 1000 W/kW; 550 (ft)(lb)/(sec)(hp)
c	chord, m; ft
$c_p$	specific heat at constant pressure, J/(kg)(K); Btu/(lb)( $^{\circ}\text{R}$ )
D	diameter, m; ft
E	energy factor
$\bar{e}$	blade-row loss coefficient
g	dimensional constant, 1; 32.17 (lbm)(ft)/(lbf)( $\text{sec}^2$ )
H	form factor
h	blade or clearance height, m; ft
$\Delta h$	specific enthalpy difference, J/kg; Btu/lb
J	dimensional constant, 1; 778 (ft)(lb)/Btu
k	number of sectors
L	loss, J/kg; Btu/lb
l	surface length (leading edge to trailing edge), m; ft
N	rotative speed, rad/sec; rev/min
$N_{sp}$	specific speed, dimensionless; $(\text{ft}^{3/4})(\text{lbm}^{3/4})/(\text{min})(\text{sec}^{1/2})(\text{lbf}^{3/4})$
n	number of blades
P	shaft power, kW; hp
p	absolute pressure, $\text{N}/\text{m}^2$ ; $\text{lb}/\text{ft}^2$
Q	parameter defined by equation (12) or (13)

R	gas constant, J/(kg)(K); (ft)(lbf)/(lbm)( <sup>o</sup> R)
Re	Reynolds number
r	radius, m; ft
s	blade spacing at blade-row exit, m; ft
T	absolute temperature, K; <sup>o</sup> R
t	trailing-edge thickness, m; ft
U	blade speed, m/sec; ft/sec
V	absolute velocity, m/sec; ft/sec
W	relative velocity, m/sec; ft/sec
w	mass flow rate, kg/sec; lb/sec
x	x coordinate, m; ft
y	y coordinate, m; ft
$\alpha$	fluid absolute angle measured from radial direction at stations 0, 1, and 1a and from axial direction at station 2, deg
$\beta$	fluid relative angle measured from radial direction at stations 0, 1, and 1a and from axial direction at station 2, deg
$\gamma$	specific heat ratio
$\delta$	displacement thickness, m; ft
$\eta$	efficiency
$\Theta$	sector angle, deg
$\Theta_{cam}$	camber angle, deg
$\theta$	momentum thickness, m; ft
$\mu$	viscosity, (N)(sec)/m <sup>2</sup> ; lb/(ft)(sec)
$\rho$	density, kg/m <sup>3</sup> ; lb/ft <sup>3</sup>
$\Phi$	flow angle from throughflow direction, deg
$\phi$	stator vane stagger angle from radial direction, deg
$\Psi$	angle defined in figure 3, deg
$\psi$	energy thickness, m; ft



**Subscripts:**

av	average
b	blade surface
c	clearance
calc	calculated
cm	chord mean
cr	critical condition
df	disk friction
ex	exit
h	hub
i	sector number
id	ideal
j	sector number other than mean sector
m	mean sector
r	rotor
rad	radial direction
ref	reference
s	stator
shft	shaft
sr	surface of revolution
t	tip
tot	total
u	tangential direction
VD	velocity diagram
w	wall
x	axial direction
0	stator inlet
1	stator exit
1a	rotor inlet

2 rotor exit  
2D two dimensional  
3D three dimensional

Superscripts:

' absolute total value  
" relative total value

## APPENDIX B

### STATOR GEOMETRY MODEL

The purpose of the stator vane geometry model is to interrelate the stator inlet and exit radii, inlet and exit flow angles, chord, and surface length to chord ratio in a consistent manner. The basic geometry model is shown in figure 3. There are three specific cases considered herein: (1) an uncambered vane with known chord length, (2) a cambered vane with known chord length and inlet flow angle, and (3) a cambered vane with known inlet radius and flow angle. In all cases, the exit radius and flow angle are known.

#### Uncambered Vane

For this case, the vane surface length is assumed equal to the chord; that is,  $(L/c)_s = 1$ . The remaining unknowns are inlet flow angle and inlet radius. From the geometry of figure 3(a),

$$\tan \varphi_0 = \frac{r_1 \sin \varphi_1}{c + r_1 \cos \varphi_1} \quad (\text{B1})$$

Since the vane is uncambered,  $\alpha_0 = \varphi_0$  and  $\alpha_1 = \varphi_1$ . Therefore, equation (B1) becomes

$$\tan \alpha_0 = \frac{\sin \alpha_1}{\frac{c}{r_1} + \cos \alpha_1} \quad (\text{B2})$$

The inlet radius is found by using the law of cosines; that is,

$$r_0^2 = c^2 + r_1^2 - 2cr_1 \cos (180 - \alpha_1) \quad (\text{B3})$$

Since  $\cos \alpha_1 = -\cos (180 - \alpha_1)$ , we get

$$r_0 = \sqrt{c^2 + r_1^2 + 2cr_1 \cos \alpha_1} \quad (\text{B4})$$

## Cambered Vane

It is assumed that the surface length is a circular arc, as shown in figure 3(b). The arc length to chord ratio can be related to the sector angle  $\Theta$ . Arc length is

$$l = \frac{\pi r \Theta}{180} \quad (\text{B5})$$

Chord is

$$c = 2r \sin \frac{\Theta}{2} \quad (\text{B6})$$

Dividing equation (B5) by equation (B6) then yields

$$\frac{l}{c} = \frac{\pi \Theta}{360 \sin \frac{\Theta}{2}} \quad (\text{B7})$$

The sector angle  $\Theta$  can be related to the camber angle  $\Theta_{\text{cam}}$ . Remembering that a tangent is perpendicular to the radius at the point of tangency, referring to figure 3(b), we can write

$$(180 - \Theta_{\text{cam}}) + \Theta = 180 \quad (\text{B8})$$

Therefore,

$$\Theta = \Theta_{\text{cam}} \quad (\text{B9})$$

and equation (B7) becomes

$$\left(\frac{l}{c}\right)_s = \frac{\pi \Theta_{\text{cam}}}{360 \sin \frac{\Theta_{\text{cam}}}{2}} \quad (\text{B10})$$

Referring to figure 3(a), the camber angle can be expressed as

$$\Theta_{\text{cam}} = 180 - \alpha_0 - (180 - \alpha_1) - \Psi = \alpha_1 - \alpha_0 - \Psi \quad (\text{B11})$$

From the law of cosines,

$$\cos \Psi = \frac{r_0^2 + r_1^2 - c^2}{2r_0r_1} \quad (\text{B12})$$

Substituting equation (B12) into equation (B11) yields

$$\Theta_{\text{cam}} = \alpha_1 - \alpha_0 - \cos^{-1} \left( \frac{r_0^2 + r_1^2 - c^2}{2r_0r_1} \right) \quad (\text{B13})$$

Equations (B13) and (B10) are used to determine surface length to chord ratio once the inlet radius and chord are known.

To determine inlet radius or chord for the two cambered vane cases being considered, it is assumed that the stagger angle at the chord midpoint is equal to average flow angle; that is,

$$\varphi_{\text{cm}} = \frac{\alpha_0 + \alpha_1}{2} \quad (\text{B14})$$

Referring again to figure 3(a), the law of cosines yields

$$r_1^2 = r_{\text{cm}}^2 + \left(\frac{c}{2}\right)^2 - 2r_{\text{cm}}\left(\frac{c}{2}\right)\cos \varphi_{\text{cm}} \quad (\text{B15})$$

and, since  $\cos \varphi_{\text{cm}} = -\cos (180 - \varphi_{\text{cm}})$ , also

$$r_0^2 = r_{\text{cm}}^2 + \left(\frac{c}{2}\right)^2 + 2r_{\text{cm}}\left(\frac{c}{2}\right)\cos \varphi_{\text{cm}} \quad (\text{B16})$$

Adding equations (B15) and (B16) gives

$$r_0^2 + r_1^2 = 2r_{\text{cm}}^2 + 2\left(\frac{c}{2}\right)^2 \quad (\text{B17})$$

Known chord. - In this case, we are trying to find the inlet radius. With the chord known, equation (B15) is solved for  $r_{cm}$  using the quadratic formula

$$r_{cm} = \frac{1}{2} \left[ c \cos \varphi_{cm} + \sqrt{(c \cos \varphi_{cm})^2 + (4r_1^2 - c^2)} \right] \quad (B18)$$

With  $r_{cm}$  evaluated thusly, equation (B17) yields the inlet radius

$$r_0 = \sqrt{2r_{cm}^2 + \frac{c^2}{2} - r_1^2} \quad (B19)$$

Known inlet radius. - In this case, we are trying to find the chord. Subtracting equation (B15) from equation (B16) and solving for  $r_{cm}$  result in

$$r_{cm} = \frac{r_0^2 - r_1^2}{2c \cos \varphi_{cm}} \quad (B20)$$

Substituting equation (B20) into equation (B17) yields

$$c^4 - 2c^2 \left( r_0^2 + r_1^2 \right) + \left( \frac{r_0^2 - r_1^2}{\cos \varphi_{cm}} \right)^2 = 0 \quad (B21)$$

Using the quadratic formula and taking the positive root finally give

$$c = \sqrt{r_0^2 + r_1^2} - \sqrt{(r_0^2 + r_1^2)^2 - \left( \frac{r_0^2 - r_1^2}{\cos \varphi_m} \right)^2} \quad (B22)$$

## APPENDIX C

### ROTOR GEOMETRY MODEL

The purpose of the rotor geometry model is to provide a consistent basis for evaluating mean surface length, chord length, blade-surface and end-wall surface areas, and number of blades. Shown in figure 4 is the rotor geometry with dimensions expressed in terms of various radii and the stator blade height. The model is based on the meridional-plane projection of the rotor. The tip contour is assumed to be circular and the hub contour is assumed to be elliptical, each being a  $90^\circ$  arc. It is further assumed that the rotor consists of full blades only.

#### Surface and Chord Lengths

The mean surface length and mean chord are the elliptical arc and straight line, respectively, joining the midpoints (see fig. 4) of the rotor leading and trailing edges. The general formula for the circumference of an ellipse with semiaxes  $a$  and  $b$  is

$$C_e \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}} \quad (C1)$$

Therefore, the mean surface length of the rotor is

$$l_r = \frac{\pi}{2} \sqrt{\frac{1}{2} \left[ \left( r_{1a} - r_{2,t} + \frac{h_s}{2} \right)^2 + \left( r_{1a} - r_{2,m} \right)^2 \right]} \quad (C2)$$

From the Pythagorean theorem, the chord length is

$$c_r = \sqrt{\left( r_{1a} - r_{2,t} + \frac{h_s}{2} \right)^2 + \left( r_{1a} - r_{2,m} \right)^2} \quad (C3)$$

## Wall Areas

The blade surface area is the area between the elliptical hub and the circular tip. The general formula for the area within an ellipse is

$$A = \pi ab \quad (C4)$$

Therefore, the blade surface area for one passage (i. e., two sides) is

$$A_{b,r} = \frac{\pi}{2} \left[ (r_{1a} - r_{2,t} + h_s)(r_{1a} - r_{2,h}) - (r_{1a} - r_{2,t})^2 \right] \quad (C5)$$

The tip and hub wall areas are determined as the surfaces of revolution of the tip and hub curves around the turbine axis. The general formula for the surface of revolution of a curve  $y = f(x)$  between  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$A_{sr} = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \int_{y_1}^{y_2} y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \quad (C6)$$

With the turbine axis taken as  $y = 0$ , the centers of the tip and hub curves are at  $x = 0$  and  $y = r_{1a}$ . Therefore, the equation of the circular tip is

$$x^2 + (y - r_{1a})^2 = (r_{1a} - r_{2,t})^2 \quad (C7)$$

while that of the elliptical hub is

$$\frac{x^2}{a^2} + \frac{(y - r_{1a})^2}{b^2} = 1 \quad (C8)$$

where the semiaxes  $a$  and  $b$  are

$$a = r_{1a} - r_{2,t} + h_s \quad (C9)$$

and



$$b = r_{1a} - r_{2, h} \quad (C10)$$

For the circular tip, differentiation of equation (C7) yields

$$\frac{dy}{dx} = -\frac{x}{y - r_{1a}} \quad (C11)$$

Substitution of equations (C11) and (C7) into equation (C6) results in

$$A_{sr, t} = 2\pi(r_{1a} - r_{2, t}) \int_0^{r_{1a} - r_{2, t}} \left[ \frac{r_{1a}}{\sqrt{(r_{1a} - r_{2, t})^2 - x^2}} - 1 \right] dx \quad (C12)$$

Integrating equation (C12) and dividing by the number of rotor passages then gives the tip wall area for one passage

$$A_{w, t, r} = \frac{2\pi}{n_r} (r_{1a} - r_{2, t}) \left[ \left( \frac{\pi}{2} - 1 \right) r_{1a} + r_{2, t} \right] \quad (C13)$$

For the elliptical hub, the area of the surface of revolution cannot be analytically expressed because equation (C6) cannot be integrated analytically. Further, neither form of equation (C6) can be integrated numerically over its entire range because of an infinite slope at one limit. However, if we break the curve into two pieces, we can write

$$A_{sr, h} = 2\pi \int_0^{a/2} y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx + 2\pi \int_{r_{1a} - b \frac{\sqrt{3}}{2}}^{r_{1a}} y \sqrt{1 + \left( \frac{dx}{dy} \right)^2} dy \quad (C14)$$

From equation (C8), the two derivatives can be determined as

$$\frac{dy}{dx} = \frac{bx}{a\sqrt{a^2 - x^2}} \quad (C15)$$

and

$$\frac{dx}{dy} = -\frac{a(y - r_{1a})}{b\sqrt{b^2 - (y - r_{1a})^2}} \quad (C16)$$

Substituting equations (C8), (C15), and (C16) into equation (C14) and dividing by the number of rotor passages then give the following hub wall area for one passage:

$$A_{w, h, r} = \frac{2\pi}{n_r} \left\{ \int_0^{a/2} \left( r_{1a} - \frac{b}{a} \sqrt{a^2 - x^2} \right) \sqrt{1 + \frac{b^2 x^2}{a^2 (a^2 - x^2)}} dx + \int_{r_{1a} - \frac{\sqrt{3}}{2}}^{r_{1a}} y \sqrt{1 + \frac{a^2 (y - r_{1a})^2}{b^2 [b^2 - (y - r_{1a})^2]}} dy \right\} \quad (C17)$$

Equation (C17) can be integrated numerically. In the program, each of the integrals is evaluated by Simpson's rule.

#### Blade Number

From the approach of reference 8 along with the additional assumption that  $V_{u, 1a} = U_{1a}$ , a number of blades can be computed as

$$n_r = 2\pi \tan \alpha_{1a} \quad (C18)$$

This number of blades is based on not permitting the velocity to fall below zero anywhere within the rotor channel; the limiting condition, therefore, is zero velocity on the pressure surface at the rotor inlet. Equation (C18) yields a rather large number of blades, especially at the higher flow angles. Studies such as the one reported in reference 5 have shown that the number of blades can be reduced significantly from these high values without significant degradation in performance. Therefore, a reduction factor is used in equation (C18). Also, the small difference between  $\alpha_{1a}$  and  $\alpha_1$  is neglected. With these changes, the number of rotor blades is computed herein as

$$n_r = \frac{\pi}{30} (110 - \alpha_1) \tan \alpha_1 \quad (C19)$$

## REFERENCES

1. Rohlik, Harold E.: Analytical Determination of Radial Inflow Turbine Design Geometry for Maximum Efficiency. NASA TN D-4384, 1968.
2. Prust, Herman W., Jr.: Boundary-Layer Losses. Ch. 7 of Turbine Design and Application, Vol. 2, Arthur J. Glassman, ed., NASA SP-290, 1973, pp. 93-124.
3. Kofskey, Milton G.; and Holeski, Donald E.: Cold Performance Evaluation of a 6.02-Inch Radial Inflow Turbine Designed for a 10-Kilowatt Shaft Output Brayton Cycle Space Power Generation System. NASA TN D-2987, 1966.
4. Roelke, Richard J.: Miscellaneous Losses. Ch. 8 of Turbine Design and Application, Vol. 2, Arthur J. Glassman, ed., NASA SP-290, 1973, pp. 125-148.
5. Rohlik, Harold E.: Radial Inflow Turbines. Ch. 10 of Turbine Design and Application, Vol. 3, Arthur J. Glassman, ed., NASA SP-290, 1975, pp. 31-58.
6. Zuk, John; and Smith, Patricia J.: Quasi-One-Dimensional Compressible Flow Across Face Seals and Narrow Slots. NASA TN D-6787, 1972.
7. Katsanis, Theodore; and McNally, William D.: FORTRAN Program for Calculating Velocities and Streamlines on the Hub-Shroud Mid-Channel Flow Surface of an Axial- or Mixed-Flow Turbomachine. II - Programmer's Manual. NASA TN D-7344, 1974.
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TABLE I. - INPUT FORM WITH SAMPLE DATA

TITLE		PROJECT NUMBER	ANALYST	SHEET _____ OF _____
STATEMENT NUMBER	CONT	FORTRAN STATEMENT		IDENTIFICATION
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80
<p>THIS IS A SAMPLE CASE USING SI UNITS</p> <p>\$INPUT R=208.1,GAM=1.667,ALPHA0=55.6,ALPHA1=72.0,N=4031.7,TTIN=1083.33,                      PTIN=9.101,W=,2771,PØW=22.371,RIR1A=1.0216,ROR1A=1.2581,RT2R1A=0.7133,                      RH2RT2=0.3493,CDT2=.0023,MU=.5804E-04,K=5,IU=1,NSTAR=0 \$</p> <p>THIS IS THE SAME CASE USING U.S. CUSTØMARY UNITS</p> <p>\$INPUT R=38.68,N=38500.,TTIN=1950.,PTIN=13.2,W=.611,PØW=30.,MU=.39E-04,IU=2 \$</p>				
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80

TABLE II. - OUTPUT FOR SAMPLE CASE USING SI UNITS

## RADIAL INFLOW TURBINE VELOCITY DIAGRAM DESIGN ANALYSIS

THIS IS A SAMPLE CASE USING SI UNITS

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE KELVIN	PRESSURE N/SQ CM	GAS CONST JLS/KG-K	ROT SPEED RAD/SEC	MASS FLOW KG/SEC	POWER KW	VISCOSITY N-SEC/SQ M	VELOCITY M/SEC	SPEC WORK JLS/GM	DIAMETER CM	ANGLE DEGREES				
<b>*INPUT*</b>														
INLET TEMP =	1083.3300		STATOR IN ANGLE =	55.60	STATOR KE LOS COEF=	1.0000	GAS CONSTANT =				208.1110			
INLET PRESS=	9.1011		STATOR EX ANGLE =	72.00	ROTOR KE LOSS COEF=	1.0000	SPEC HT RATIO=				1.6670			
ROTAT SPEED=	4031.710		STATOR ASPECT RAT=	.0000	ROTOR IN/DEL RVU =	1.0000	VISCOSITY =				.5804-04			
MASS FLOW =	.2771		DIAM RATIOS		ROTOR EX RAD SECTS=	5								
SHAFT POWER=	22.371		STAT IN/ROT IN =	1.2581	ROT EX SECT/MN RVU=	1.00 1.00 1.00 1.00								
			STAT EX/ROT IN =	1.0216										
			ROT EX TP/ROT IN=	.7133										
			ROT EX HUB/TIP =	.3493	CL HT/ROT EX TIP D=	.0023								
<b>*OUTPUT*</b>														
	DIA- MEYER	ABS TOTAL TEMP	ABS TOTAL PRESS	STATIC TEMP	STATIC PRESS	ABS FLOW ANGLE	ABS VELO- CITY	ABS CRIT VEL RAT	BLADE SPEED	REL TOTAL TEMP	REL TOTAL PRESS	REL FLOW ANGLE	REL VELO- CITY	REL CRIT VEL RAT
STATOR INLET	19.550	1083.33	9.101	1072.02	8.865	55.60	108.49	.204		STATOR HGT= 1.8527, NUMBER OF VANES= 16.0				
LOSS COEF=	.0640													
STATOR EXIT	15.875	1083.33	8.995	1013.94	7.623	72.00	268.67	.506						
ROTOR INLET	15.539	1083.33	8.995	1010.85	7.565	71.92	274.59	.517	313.24	1020.45	7.746	-31.50	99.93	.194
LOSS COEF=	.2820													
ROTOR EXIT	3.872	926.12	5.925	908.32	5.644	.00	136.09	.277	78.05	931.98	6.019	-29.83	156.88	.319
	4.593	926.12	5.914	908.99	5.644	.00	133.49	.272	92.59	934.36	6.046	-34.74	162.46	.330
	6.035	926.12	5.886	910.68	5.644	.00	126.74	.258	121.66	940.35	6.115	-43.83	175.69	.355
	7.478	926.12	5.852	912.83	5.644	.00	117.60	.240	150.74	947.96	6.203	-52.04	191.19	.385
	8.920	926.12	5.810	915.43	5.644	.00	105.45	.215	179.82	957.21	6.310	-59.61	208.46	.418
	10.363	926.12	5.762	918.50	5.644	.00	89.06	.181	208.90	968.07	6.436	-66.91	227.09	.453
	11.084	926.12	5.735	920.20	5.644	.00	78.48	.160	223.44	974.11	6.507	-70.65	236.82	.470
		MASS FLOW RATE	DIAG SPEC WORK	TOTAL EFFIC	STATIC EFFIC	NUMBER OF ROTOR BLADES= 12.0								
	3.872	.0000	81.769	.919	.834									
	4.593	.0414	81.769	.916	.834									
	6.035	.0516	81.769	.907	.834									
	7.478	.0592	81.769	.896	.834									
	8.920	.0631	81.769	.883	.834									
	10.363	.0617	81.769	.868	.834									
	11.084	.0000	81.769	.860	.834									
<b>*OVERALL PERFORMANCE*</b>														
TOT-TOT PRESSURE RATIO =	1.5553	LOSS/IDEAL T-S DEL H												
TOT-STAT PRESSURE RATIO=	1.6125	STATOR = .0252												
DIAG AVG SPECIFIC WORK =	81.7686	ROTOR = .0779												
DIAG TOTAL EFFICIENCY =	.8915	WINDAGE = .0048												
DIAG STATIC EFFICIENCY =	.8339	CLEARANCE= .0059												
NET TOTAL EFFICIENCY =	.8800	EXIT KE = .0658												
NET STATIC EFFICIENCY =	.8232	SPECIFIC SPEED= .739												

TABLE III. - OUTPUT FOR SAMPLE CASE USING U. S. CUSTOMARY UNITS

THIS IS THE SAME CASE USING U. S. CUSTOMARY UNITS

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE DEG RANKINE	PRESSURE LB/SQ IN	GAS CONST BTU/LB-R	ROT SPEED REV/MIN	MASS FLOW LB/SEC	POWER HP	VISCOSITY LB/FT-SEC	VELOCITY FT/SEC	SPEC WORK BTU/LB	DIAMETER IN	ANGLE DEGREE S				
*INPUT*														
INLET TEMP =	1950.0000	STATOR IN ANGLE =	55.60	STATOR KE LOS COEF=	1.0000	GAS CONSTANT =	38.6800							
INLET PRESS=	13.2000	STATOR EX ANGLE =	72.00	ROTOR KE LOSS COEF=	1.0000	SPEC HT RATIO=	1.6670							
ROTAT SPEED=	38500.000	STATOR ASPECT RAT=	.0000	ROTOR IN/DEL RVU =	1.0000	VISCOSITY =	.3900-G4							
MASS FLOW =	.6110	DIAM RATIOS		ROTOR EX RAD SECTS=	5									
SHAFT POWER=	30.000	STAT IN/ROT IN =	1.2581	ROT EX SECT/MN RVU=	1.00 1.00 1.00 1.00 1.00									
		STAT EX/ROT IN =	1.0216											
		ROT EX TP/ROT IN=	.7133											
		ROT EX HUB/TIP =	.3493	CL HT/ROT EX TIP D=	.0023									
*OUTPUT*														
	DIA- METER	ABS TOTAL TEMP	ABS TOTAL PRESS	STATIC TEMP	STATIC PRESS	ABS FLOW ANGLE	ABS VELO- CITY	ABS CRIT VEL RAT	BLADE SPEED	REL TOTAL TEMP	REL TOTAL PRESS	REL FLOW ANGLE	REL VELO- CITY	REL CRIT VEL RAT
STATOR INLET	7.697	1950.00	13.200	1929.63	12.858	55.60	355.93	.204						
LOSS COEF=	.0640								STATOR HGT=	.7294	NUMBER OF VANES=	16.0		
STATOR EXIT	6.250	1950.00	13.046	1825.10	11.057	72.00	881.46	.506						
ROTOR INLET	6.118	1950.00	13.046	1819.53	10.973	71.92	900.88	.517	1027.71	1836.81	11.235	-31.50	327.85	.194
LOSS COEF=	.2820													
ROTOR EXIT	1.524	1667.02	8.593	1634.98	8.186	.00	446.49	.277	256.36	1677.56	8.729	-29.83	514.70	.319
	1.808	1667.02	8.577	1636.19	8.186	.00	437.98	.272	303.76	1681.86	8.769	-34.74	533.00	.330
	2.376	1667.02	8.537	1639.23	8.186	.00	415.83	.258	399.16	1692.64	8.869	-43.83	576.40	.355
	2.944	1667.02	8.487	1643.09	8.186	.00	385.83	.240	494.56	1706.34	8.996	-52.04	627.26	.385
	3.512	1667.02	8.427	1647.78	8.186	.00	345.96	.215	589.96	1722.97	9.152	-59.61	683.91	.418
	4.080	1667.02	8.357	1653.30	8.186	.00	292.19	.181	685.36	1742.53	9.335	-66.91	745.05	.453
	4.364	1667.02	8.318	1656.37	8.186	.00	257.47	.160	733.06	1753.41	9.438	-70.65	776.96	.470
	DIA- METER	MASS FLOW RATE	DIAG SPEC WORK	TOTAL EFFIC	STATIC EFFIC	NUMBER OF ROTOR BLADES= 12.0								
	1.524	.0000	35.178	.919	.834									
	1.808	.0914	35.178	.916	.834									
	2.376	.1138	35.178	.907	.834									
	2.944	.1305	35.178	.896	.834									
	3.512	.1392	35.178	.883	.834									
	4.080	.1361	35.178	.868	.834									
	4.364	.0000	35.178	.860	.834									
*OVERALL PERFORMANCE*														
TOT-TOT PRESSURE RATIO =	1.5553	LOSS/IDEAL T-S DEL H	STATOR	= .0252										
TOT-STAT PRESSURE RATIO=	1.6125	ROTOR	= .0779											
DIAG AVG SPECIFIC WORK =	35.1777	WINDAGE =	.0048											
DIAG TOTAL EFFICIENCY =	.8915	CLEARANCE=	.0059											
DIAG STATIC EFFICIENCY =	.8339	EXIT KE =	.0658											
NET TOTAL EFFICIENCY =	.8800													
NET STATIC EFFICIENCY =	.8232	SPECIFIC SPEED=	95.355											

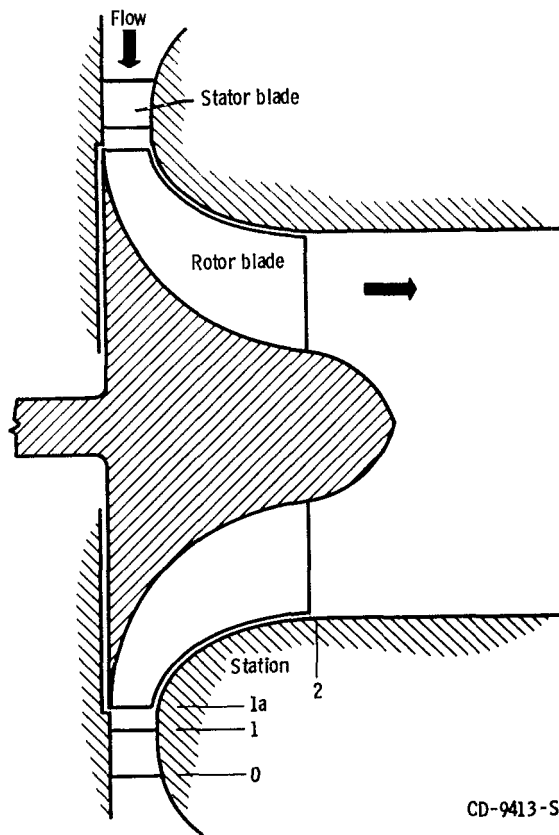


Figure 1. - Schematic cross section of radial-inflow turbine.

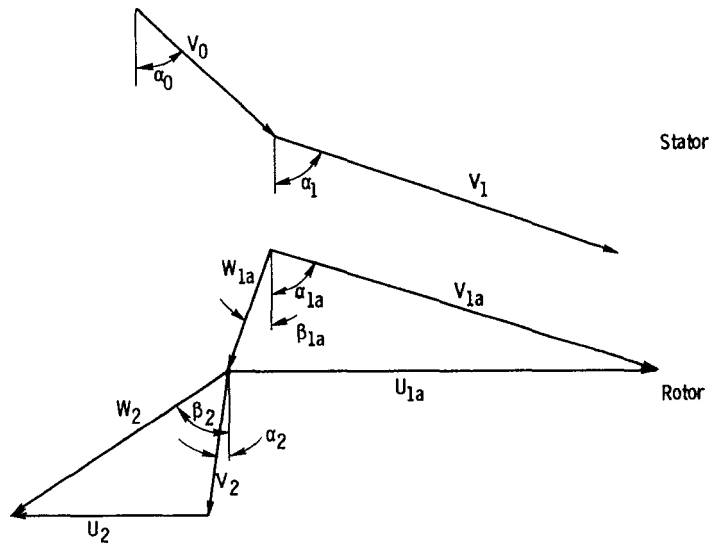
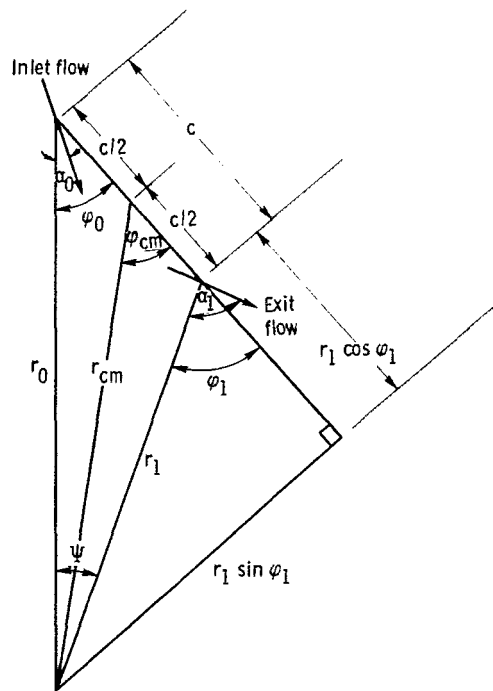
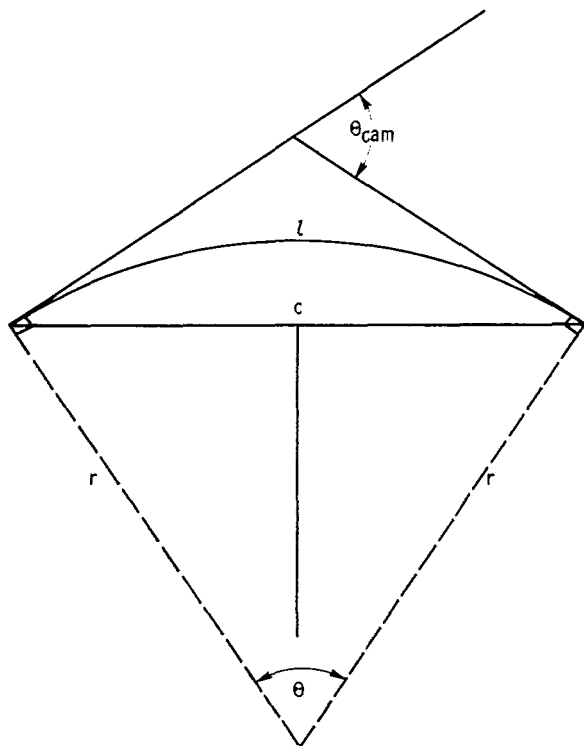


Figure 2. - Velocity diagram.





(a) Chord.



(b) Surface.

Figure 3. - Stator geometry model.

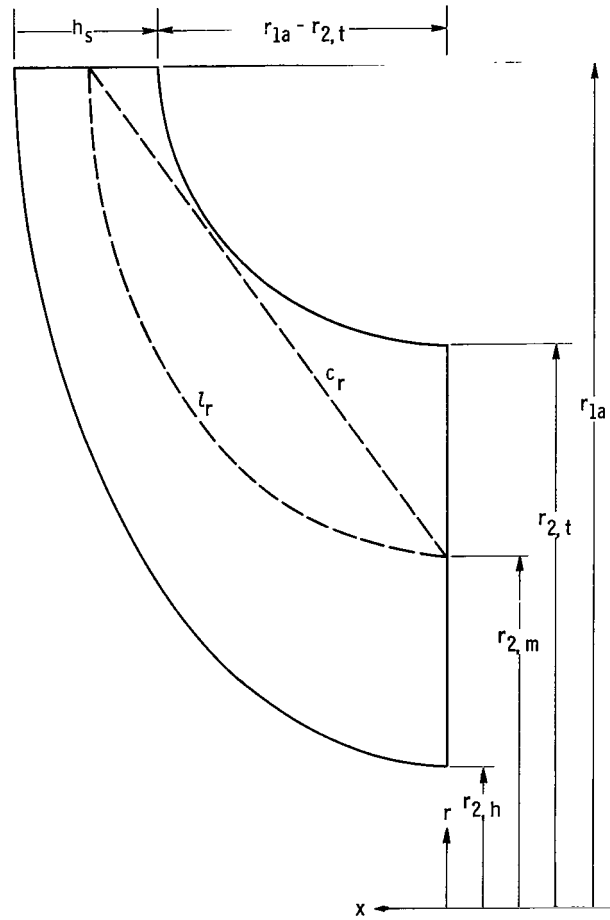


Figure 4. - Rotor geometry model.