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SATELLITE LIFETIME ROUTINE USER'S MANUAL

• by H. U. Everett and T. R. Myler

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SATELLITE LIFETIME ROUTINE

USER'S MANUAL

By H. U. Everett and T. R. Myler
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SUMMARY

This report describes a FORTRAN coded computer program which determines secular variations in mean orbital elements of Earth satellites and the lifetime of the orbit. The dynamical model treats a point mass satellite subject to solar and lunar disturbing gravitational fields, second, third and fourth harmonics of the Earth's oblate potential, Earth's atmospheric drag and solar radiation pressure. Each of these disturbing functions may be selectively simulated. Data preparation instructions, a sample problem and definitions of output quantities are included.

1.0 INTRODUCTION

This document presents a computing procedure for determining long period variations in the orbital elements of an Earth satellite. A time history of orbital elements yields the orbital lifetime when the perigee altitude decreases to near zero. Thus, one of the primary purposes of this computing procedure is to determine the lifetime of an Earth orbit.

Instantaneous derivatives of the orbital elements are calculated at equal time increments over one orbit period and averaged to obtain the secular derivatives which are integrated to yield mean orbital element time histories. The disturbing forces causing the orbital element changes are due to the sun, moon, Earth oblateness, Earth atmospheric drag and solar pressure. The secular derivatives of elements due to each of the disturbances, except solar pressure, are individually calculated and summed to obtain the total derivatives.

A computer program similar to the procedure described herein was obtained from the NASA Goddard Space Flight Center and is described in Reference 1. The program of Reference 2 is an extension and modification of the NASA version. Extensions were made to permit simulation of solar radiation pressure, third and fourth Earth oblateness harmonics and atmospheric drag. Modifications were to provide automatic stepsize and error control for numerical integration and to simplify program input and output. The current program is a modification of Reference 2 to remove computational singularities in time rates of change of eccentricity and argument of perigee for circular orbits (zero eccentricity).

2.0 NOTATION

Symbols used in this report, excluding the appendices, are listed below with their definitions and units for program computations.

A_D	atmospheric drag acceleration, $R_E\,day^{-2}$
A_X, A_Y, A_Z	Cartesian components of disturbing acceleration in the inertial equatorial frame, R_E day^{-2}
a	orbital semimajor axis, R_E
C	circumferential component of disturbing acceleration, R_E day^{-2}
C_D	coefficient of drag
E	eccentric anomaly, radians
e	orbital eccentricity
GM	universal gravity constant, $R_E{}^3$ day^{-2}
g	mean anomaly
H	coefficient of third harmonic of Earth oblate potential
h	e sin ω
I	orbital inclination, radians
J	coefficient of second harmonic of Earth oblate potential
K	coefficient of fourth harmonic of Earth oblate potential
k	e cos ω
L_S	Solar mean longitude, radians
M	mass of gravitating body, Earth mass M_E is the unit
p	orbital semilatus rectum, R_E, $a(1-e^2)$
R	radial component of disturbing acceleration, R_E day^{-2}
R_E	Earth equatorial radius, also a unit of length
r	radius, R_E
S	aerodynamic reference area, m^2
t	time since Jan. 0, 1961, days .
u	argument of latitude, radians, $\nu + \omega$
V	inertial velocity, R_E day^{-2}
W	component of disturbing acceleration directed normal to the orbital plane, R_E day^{-2}
w	sine of satellite equatorial latitude
X, Y, Z	Cartesian coordinates of position, R_E

α	angle between geocentric radii to the satellite and to the sun, radians
β	angle between geocentric radii to the satellite and to the tangent point on the Earth surface of a line from the satellite to the Earth-sun line, radians
γ	satellite illumination angle, radians
ϵ	obliquity of the ecliptic (angle between equatorial and ecliptic planes), radians
λ	satellite equatorial latitude, radians
λ_S	true longitude of the sun, radians
ρ	atmospheric density, kg m^{-3}
Φ	oblate Earth gravity potential, $R_E{}^2$ day $^{-2}$
ν	true anomaly, radians
Ω	right ascension of the ascending node, radians
Ω_E	inertial Earth rotation rate, 6.300383 radians day^{-1}
ω	argument of perigee, radians

Subscripts

D	quantity pertaining to atmospheric drag
E	quantity pertaining to Earth
M	quantity pertaining to moon
O	quantity pertaining to Earth oblateness
R	earth relative
S	quantity pertaining to sun
T	total
X, Y, Z	component in coordinate direction

Superscripts

$(\dot{\ })$	derivative with respect to time
$(\hat{\ })$	unit vector
$(\ )'$	components in ecliptic plane

3.0 PHYSICAL ENVIRONMENT

3.1 Earth Model

3.1.1 Gravity. — Earth gravitational potential giving a disturbance to two-body motion is defined by terms through the fourth harmonic:

$$\Phi = \frac{GM_E}{r}\frac{R_E}{r^2}^2 \left[\frac{J}{3}(1-3\sin^2\lambda) - \frac{HR_E}{5r}(3-5\sin^2\lambda)\sin\lambda + \frac{K}{30}\frac{R_E}{r^2}^2(3-30\sin^2\lambda + 35\sin^4\lambda)\right] \tag{3.1}$$

where

Earth equatorial radius, $R_E = 6378.166$ km

Earth mass, $M_E = 1$ Earth mass

Coefficient of 2nd harmonic, $J = 0.00162345$

Coefficient of 3rd harmonic, $H = -5.95$ E-6

Coefficient of 4th harmonic, $K = 7.95$ E-6

Universal gravitation constant, $GM_E = 11467.849\ R_E{}^3/\text{day}^2$

λ is the latitude of the satellite position

$$\sin\lambda = \frac{Z}{r}$$

3.1.2 Atmosphere. — A static atmosphere model is used to compute the atmospheric density for altitudes below 120 kilometers. The calculations are based on an altitude-temperature profile that approximates the 1962 U. S. Standard Atmosphere Model. A dynamic atmosphere model is available for computing densities at altitudes above 120 kilometers. The dynamic model varies with time, location, and solar activity and is based on the 1969 NASA model presented in Reference (3). The computational algorithms used for both models are discussed in Appendix A.

3.2 Lunar Ephemeris

The lunar ephemeris is defined by the following expressions for mean elements presented in Reference (4):

$$\Lambda_M = 270.434358 + 13.1763965268d - 0.001133T^2 + 0.0000019T^3 \text{ deg}$$
$$\Gamma_M = 334.329653 + 0.1114040803d - 0.010325T^2 - 0.000012T^3 \text{ deg}$$
$$\Omega_M = 259.183275 - 0.0529539222d + 0.002078T^2 + 0.000002T^3 \text{ deg}$$

where

Λ_M is mean longitude

Γ_M is mean longitude of perigee

Ω_M is longitude of mean ascending node
d is ephemeris days from epoch of 1900 Jan. 0.5 Ephemeris Time (E. T.)
T is Julian centuries of 36525 ephemeris days from epoch of 1900 Jan 0.5 E. T.
M_M = mean anomaly = $\Lambda_M - \Gamma_M$
ν_M = true anomaly = $M_M + 2e_M \sin M_M + \frac{5}{4} e_M{}^2 \sin 2M_M + \ldots$
ω_M = argument of perigee = $\Gamma_M - \Omega_M$

$$r_M = \text{radius} = \frac{p_M}{1 + e_M \cos\nu_M}$$

p_M = semilatus rectum
e_M = eccentricity = 0.054900489

Remaining constants for the moon or its ephemeris are
semimajor axis, $a_M = 60.2681\ R_E$
inclination to ecliptic, $\sin I_M = 0.089683448$
$\cos I_M = 0.99597032$
mass, $M_M = M_E/81.335$.

Ecliptic Cartesian components of the moon position are

$$\begin{bmatrix} X_M' \\ Y_M' \\ Z_M' \end{bmatrix} = \begin{bmatrix} B_M' \end{bmatrix}^{-1} \begin{bmatrix} r_M \\ 0 \\ 0 \end{bmatrix}$$

where

$$B_{M_{11}}' = \cos(\omega_M + \nu_M)\cos\Omega_M - \sin(\omega_M + \nu_M)\sin\Omega_M \cos I_M$$

$$B_{M_{12}}' = -\sin(\omega_M + \nu_M)\cos\Omega_M - \cos(\omega_M + \nu_M)\sin\Omega_M \cos I_M$$

$$B_{M_{13}}' = \sin\Omega_M \sin I_M$$

The equatorial Cartesian components are

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} X_M' \\ Y_M' \\ Z_M' \end{bmatrix} \qquad (3.2)$$

where

$$A_{11} = 1$$
$$A_{12} = 0$$
$$A_{13} = 0$$
$$A_{21} = 0$$
$$A_{22} = \cos \epsilon$$
$$A_{23} = -\sin \epsilon$$
$$A_{31} = 0$$
$$A_{32} = \sin \epsilon$$
$$A_{33} = \cos \epsilon$$

3.3 Solar Ephemeris

Reference 5, page 98 gives the true longitude of the sun as a function of time (t) in days from 1961 Jan. 0.0 as

$$\lambda_S = L_S + 2e_S \sin g_S$$

where mean longitude is given by

$$L_S = -1.4062711 + 0.0172027914t \text{ radians}$$

Solar mean anomaly is

$$g_S = -0.0496208 + 0.0172019697t \text{ radians}$$

and solar orbital eccentricity (e_S) is 0.0167255.

Instantaneous geocentric solar distance is given by

$$\left(\frac{a_S}{r_S}\right)^3 = \left(1-e_S{}^2\right)^{-\frac{3}{2}} + 3e_S \cos g_S$$

according to reference 6. Remaining constants pertaining to the sun or its ephemeris are

semimajor axis, $a_S = 23454.708\ R_E$

argument of perigee, $\omega_S = 4.923277$ radians

obliquity of ecliptic, $\epsilon = 23.452294 - 0.0130125T - 0.00000164T^2 + 0.000000503T^3$ degrees

where T is Julian centuries of 36525 ephemeris days from the epoch of 1900 0.5 E. T.

mass, $M_S = 3.32951.3M_E$

The equatorial coordinates of the sun are

$$X_S = r_S \cos \lambda_S$$

$$Y_S = r_S \sin \lambda_S \cos \epsilon$$

$$Z_S = r_S \sin \lambda_S \sin \epsilon$$

4.0 COORDINATE SYSTEMS

4.1 Description

Equations of motion are applied in an inertial equatorial frame as shown in Figure 1. The positive X axis is along the mean equinox of date, Z is collinear with the mean North polar axis, and Y is in the equatorial plane completing a right-hand orthogonal system.

The lunar and solar mean elements are referenced to the ecliptic frame which is also illustrated in Figure 1. The X′ axis is collinear with the X axis and the mean equinox of date, the Z′ axis is normal to the ecliptic plane in the direction of the Earth's orbital angular momentum vector, and the Y′ axis completes the right-hand orthogonal system. The angle between the equatorial and ecliptic planes is ϵ as defined in Section 3.3. The coordinate transformation from ecliptic to equatorial components of a vector is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

where $\begin{bmatrix} A \end{bmatrix}$ is defined in equation (3.2).

4.2 Disturbing Acceleration Transformation

Transformation of the inertial equatorial components of disturbing acceleration (A_X, A_Y, A_Z) to a satellite trajectory relative set is given by

$$\begin{bmatrix} R \\ C \\ W \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} A_X \\ A_Y \\ A_Z \end{bmatrix} \tag{4.2}$$

where

$$B_{11} = \cos u \cos \Omega - \sin u \sin \Omega \cos I$$
$$B_{12} = -\sin u \cos \Omega - \cos u \sin \Omega \cos I$$
$$B_{13} = \sin \Omega \sin I$$
$$B_{21} = \cos u \sin \Omega + \sin u \cos \Omega \cos I$$
$$B_{22} = -\sin u \sin \Omega + \cos u \cos \Omega \cos I$$
$$B_{23} = -\cos \Omega \sin I$$
$$B_{31} = \sin u \sin I$$
$$B_{32} = \cos u \sin I$$
$$B_{33} = \cos I$$

Component R is along the radius, positive away from the origin; C is in the orbital plane and normal to R, positive in the direction of satellite motion; and W is normal to the orbital plane, positive in the direction of angular momentum. Accelerations R,C,W are based on the accelerations A_X, A_Y, A_Z of individual disturbances which are defined in Section 5.0.

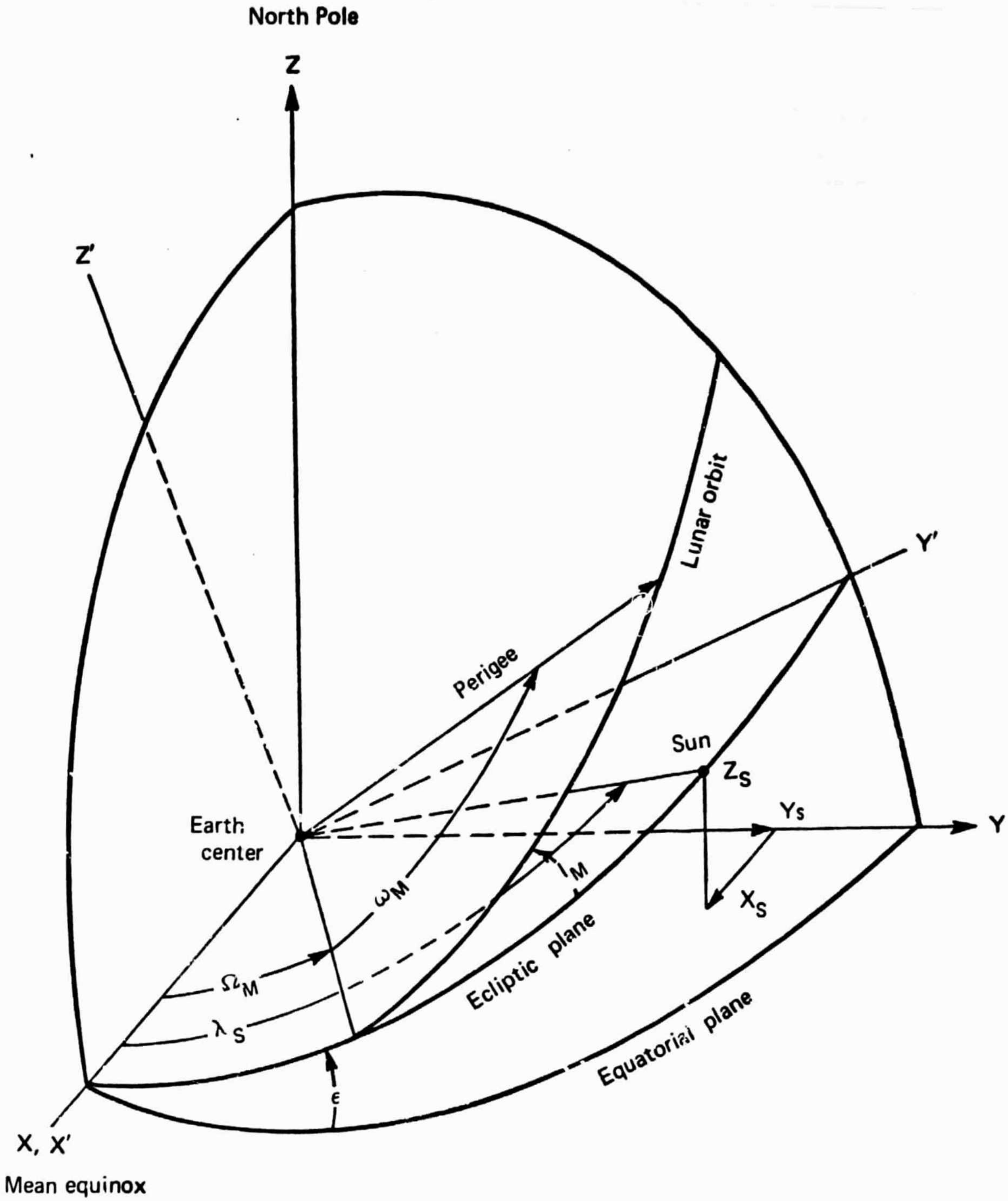

FIGURE 1. – LUNAR AND SOLAR ORBITAL GEOMETRY IN THE EQUATORIAL REFERENCE FRAME

5.0 PERTURBATION METHODS

The disturbing forces considered are those due to solar gravity, lunar gravity, Earth oblateness, Earth atmospheric drag and solar radiation pressure. The method used to simulate each of the disturbances is a technique of averaging the orbital element derivatives over an orbit period at equal intervals of mean anomaly. This method accounts for the inertial position of the moon and sun according to the date and time. Additionally, this method permits simulation of solar radiation pressure.

The classical set of orbital elements normally used for analyses include: a (semimajor axis), e (eccentricity), ω (argument of perigee), Ω (right ascension of ascending node) and I (inclination). However, because the derivatives of e and ω are undefined at zero eccentricity, parameters h and k as described in Reference (7) are numerically integrated in the computational procedure. These parameters are defined in terms of e and ω as follows:

$$h = e \sin \omega$$

$$k = e \cos \omega$$

Derivatives of h and k are taken from the above mentioned technical paper and are not a function of e or ω. Since h and k are integrated, e and ω are obtained from the following relationships

$$e = \sqrt{h^2 + k^2}$$

$$\omega = \tan^{-1}\left(\frac{h}{k}\right)$$

Derivatives of these quantities which are required for program output are

$$\dot{e} = \frac{h\dot{h} + k\dot{k}}{e}$$

$$\dot{\omega} = \frac{k\dot{h} - h\dot{k}}{e^2}$$

5.1 Lunar Disturbance

Inertial accelerations on the satellite due to the lunar gravity are as follows:

$$A_X = -GM_M \left(\frac{X - X_M}{\Delta_M{}^3} + \frac{X_M}{r_M{}^3}\right) \qquad X \longrightarrow Y, Z \qquad (5.1)$$

where

$$\Delta_M{}^2 = (X - X_M)^2 + (Y - Y_M)^2 + (Z - Z_M)^2$$

These inertial accelerations are transformed to the satellite trajectory relative set using equation (4.2). Then the instantaneous derivatives for the orbital elements are obtained from R, C, W as shown below. Derivatives for a, Ω and I are taken from Reference 8 and derivatives for h and k are taken from Reference 7.

$$
\left.
\begin{aligned}
\frac{da}{dt} &= \sqrt{\frac{p}{GM_E}}\;\frac{2a}{1-e^2}\left(eR \sin \nu + \frac{p}{r} C\right) \\
\frac{d\Omega}{dt} &= \sqrt{\frac{p}{GM_E}}\;\frac{1}{\sin I}\left(\frac{r}{p} \sin u\, W\right) \qquad I \neq 0 \\
\frac{dI}{dt} &= \sqrt{\frac{p}{GM_E}}\left(\frac{r}{p} \cos u\, W\right) \\
\frac{dh}{dt} &= \sqrt{\frac{p}{GM_E}}\left[-\cos u\, R + C\left(1 + \frac{r}{p}\right) \sin u + \frac{r}{p} hC - \frac{r}{p} kW \sin u \cot I\right] \\
\frac{dk}{dt} &= \sqrt{\frac{p}{GM_E}}\left[\sin u\, R + C\left(1 + \frac{r}{p}\right) \cos u + \frac{r}{p} kC + \frac{r}{p} hW \sin u \cot I\right]
\end{aligned}
\right\} \quad (5.2)
$$

Averaging the instantaneous derivatives over the orbit period is done in equal increments of mean anomaly; and therefore equal time intervals. The secular rate, averaged with respect to mean anomaly, is

$$\dot{\Omega}_{SEC} = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\Omega}{dt}\, dg \qquad \Omega \longrightarrow I, a, h, k$$

where g is mean anomaly. A transformation from mean to eccentric anomaly is made to avoid repeated solution of Kepler's equation for mean anomaly.

Since $g = E - e \sin E$

then $dg = (1 - e \cos E)\, dE = \frac{r}{a}\, dE$

therefore

$$\dot{\Omega}_{SEC} = \frac{1}{2\pi a} \int_0^{2\pi} \frac{r\, d\Omega}{dt}\, dE \qquad \Omega \longrightarrow I, a, h, k \qquad (5.3)$$

5.2 Solar Disturbance

5.2.1 Gravity. — Inertial accelerations of the satellite due to solar gravity are as follows:

$$A_X = - GM_S \left[(\text{[illegible]}\, T) \frac{X - X_S}{\Delta_S{}^3} + \frac{X_S}{r_S{}^3} \right] \qquad X \longrightarrow Y, Z$$

where

$$\Delta_S^2 = (X - X_S)^2 + (Y - Y_S)^2 + (Z - Z_S)^2$$

SFACT is solar radiation multiplier defined in Section 5.2.2.

These inertial accelerations are transformed to the satellite trajectory relative set using equation (4.2). Instantaneous derivatives of the orbital elements are obtained using equations (5.2) and the secular derivatives are obtained using equation (5.3).

5.2.2 Solar Radiation Pressure. – Solar radiation pressure is simulated by reducing the GM_S when calculating the acceleration on the satellite due to the sun. During satellite illumination

$$SFACT = 1 - \frac{\overline{GM}}{GM_S}$$

$$\overline{GM} = \frac{Sk}{W} a_S^2 g_o$$

S = satellite reference area towards sun in m^2

k = solar flux constant at 1 AU,
1.03034×10^{-6} lbf/m^2

W = satellite mass in lb.

a_S = semi-major axis of solar orbit in R_E

g_o = acceleration of gravity at Earth's surface in R_E/day^2

When the satellite is in the Earth's shadow, SFACT is defined to be unity. Figure 2 shows the geometry used to determine when the satellite is illuminated. Application of plane geometry laws gives

$$\gamma = \frac{\pi}{2} - \alpha + \beta$$

where

$$\cos\alpha = \hat{r} \cdot \hat{r}_S$$

$$\cos\beta = \frac{R_E}{r}$$

When the satellite illumination angle γ is negative, the sun is eclipsed by the Earth. Conversely, the satellite is illuminated when γ is positive.

5.3 Earth Disturbance

5.3.1 Gravity. -- Disturbing acceleration components in the inertial equatorial frame are obtained by differentiating equation (3.1)

$$A_X = \frac{\partial \Phi}{\partial X} \qquad X \longrightarrow Y, Z$$

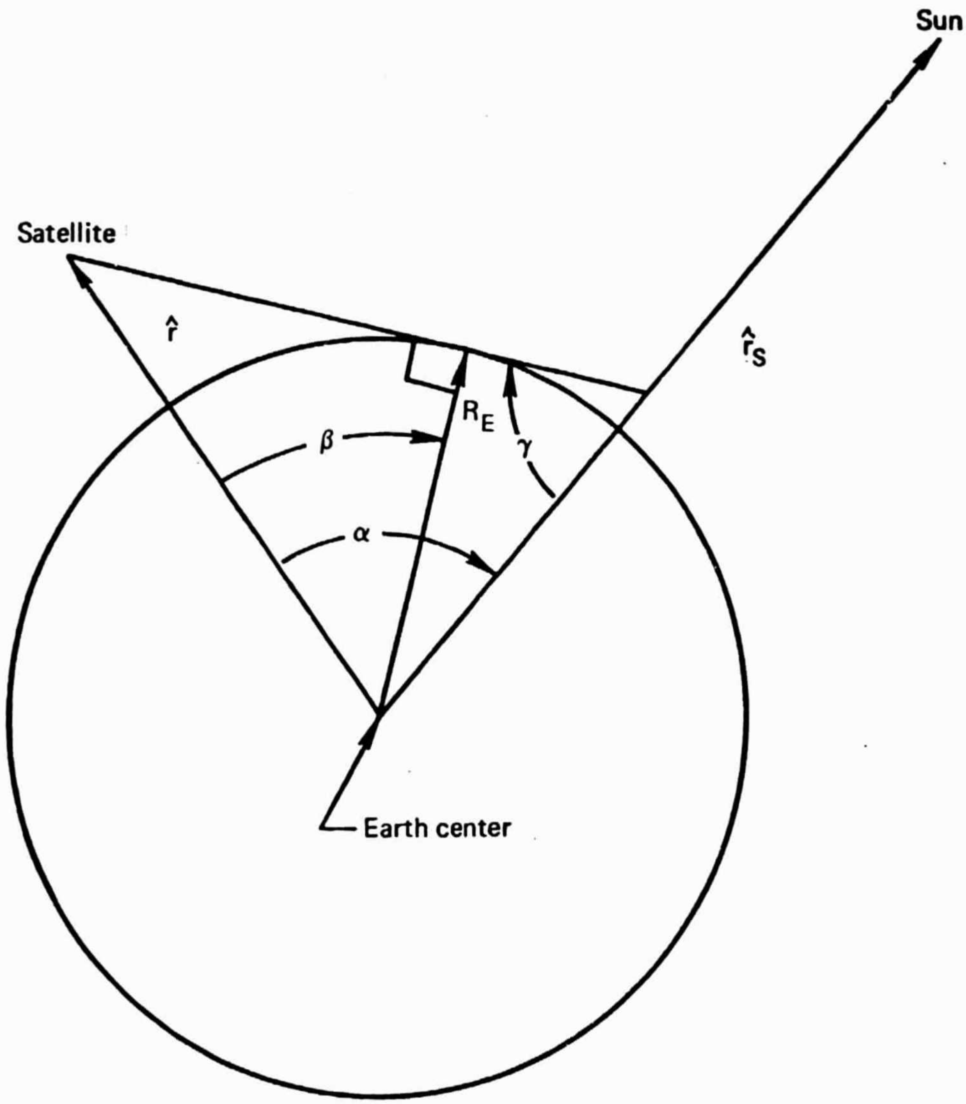

FIGURE 2. – GEOMETRY TO DETERMINE SATELLITE ILLUMINATION

or

$$A_X = -X\left[F(Z,r)\right]$$

$$A_Y = -Y\left[F(Z,r)\right]$$

$$A_Z = \frac{-GM_E}{r^2}\left(\frac{R_E}{r}\right)^2\left[J\,w(3-5w^2) + \frac{H}{5}\frac{R_E}{r}(30w^2-35w^4-3)\right.$$

$$\left.+\frac{k}{6}\left(\frac{R_E}{r}\right)^2 w\,(15-70w^2+63w^4)\right]$$

where

$$w = \frac{Z}{r}$$

$$\left[F(Z,r)\right] = \frac{GM_E}{r^3}\left(\frac{R_E}{r}\right)^2\left[J(1-5w^2) + H\frac{R_E}{r}(3-7w^2)w\right.$$

$$\left.+\frac{k}{6}\left(\frac{R_E}{r}\right)^2(3-42w^2+63w^4)\right]$$

The above inertial accelerations are transformed to the satellite trajectory relative set using equation (4.2). The instantaneous derivatives of the orbital elements are then obtained from equation (5.2). The secular derivatives are obtained using equation (5.3).

5.3.2 Atmosphere. – The satellite trajectory relative acceleration components due to atmospheric drag are obtained from

$$R = \frac{V}{V_R}\frac{-A_D\, e\sin\nu}{\sqrt{1+e^2+2e\cos\nu}}$$

$$C = \frac{-A_D}{V_R}\left[\frac{V(1+e\cos\nu)}{\sqrt{1+e^2+2e\cos\nu}} - r\Omega_E\cos I\right]$$

$$W = \frac{-A_D}{V_R}\,\Omega_E r\cos u\sin I$$

where

$$A_D = \frac{1}{2}\,\frac{\rho V_R^2 C_D S}{m}$$

$$V = \sqrt{GM_E\left(\frac{2}{r}-\frac{1}{a}\right)}$$

$$V_R = V - \Omega_E\, r\cos I$$

The instantaneous derivatives of the orbital elements are then obtained from equation (5.2). The secular derivatives are obtained using equation (5.3). Atmospheric effects are neglected if the satellite is above 1400 kilometers.

5.4 Total Derivatives of the Elements

Total derivatives for the orbit elements are obtained by simply summing contributions from the various sources of disturbance.

$$\dot{a}_T = \dot{a}_D + \dot{a}_S + \dot{a}_O + \dot{a}_M$$

$$\dot{i}_T = \dot{i}_D + \dot{i}_S + \dot{i}_O + \dot{i}_M$$

$$\dot{\Omega}_T = \dot{\Omega}_D + \dot{\Omega}_S + \dot{\Omega}_O + \dot{\Omega}_M$$

$$\dot{h}_T = \dot{h}_D + \dot{h}_S + \dot{h}_O + \dot{h}_M$$

$$\dot{k}_T = \dot{k}_D + \dot{k}_S + \dot{k}_O + \dot{k}_M$$

In most instances, the derivatives $\dot{a}_S$, $\dot{a}_O$ and $\dot{a}_M$ should vanish except for numerical inaccuracies and, therefore, are not usually used in the equation for $\dot{a}_T$. However, the $\dot{a}_S$ and $\dot{a}_M$ may be selectively used if desired by the user.

5.5 Numerical Integration Technique

A Runge-Kutta technique which includes error control and automatic interval sizing is used to numerically solve the differential equations for the orbital element secular rates. Time is the independent variable.

Development of the integration procedure occurred at the NASA Lewis Research Center and is described in Reference 9, Appendix D.

6 0 USER INSTRUCTIONS

6.1 Introduction

6.1.1 Limitations. – Neither a ground track nor a time history of satellite position in any inertial frame of reference may be obtained since satellite position in orbit is not integrated.

Mean orbital elements are used by the procedure; thus, short period variations in the elements having a frequency on the order of the satellite orbital period are not simulated.

Since the perturbation method uses the averaging technique, care should be taken that a sufficient number of samples are specified to yield an accurate average. Required number of sampling points will vary for different problems. One test for accuracy is to compare results from the same problem but with different numbers of samples per orbit. Probably a more satisfactory check is simply to verify the derivatives of semi-major axis that are small enough to be neglected—e.g., those due to solar and lunar disturbances for a near Earth orbit and that due to the oblate disturbance.

Inclination must not be zero during the calculations since inclination is in the denominator of the derivative of ascending node. The minimum value of inclination has not been determined but an input value of 0.01 degrees has been used with success. An increased number of integration steps results from near-zero inclination due to the large derivative of ascending node.

6.1.2 Computer Time Estimation. – Average computation time on the CDC 6600 is about six integration intervals per CP second. This time is applicable when simulating all four disturbances averaging at 30 points per orbit and includes the re-entry phase. For cases which do not include the re-entry phase, the computation time is about twelve integration intervals per CP second.

6.1.3 Control Cards. – The control cards and deck setup required to execute two problems on the CDC 6600 computer are shown below. Extension to three or more problems is straight-forward.

```
JOB, CM115000, T200.
PROJECT
ATTACH (GO, S7051B, ID=EVERETT)
GO.
7/8/9 multi-punched in column I
     Table of Names
$D=1
     Data Cards of Problem Set 1
$D=1
     Data Cards of Problem Set 2
$D=1
6/7/8/9 multi-punched in column I
```

6.2 Input Data

Program input is achieved by using the NASA input subroutine, reference 10, which utilizes arithmetic input statements. This subroutine provides flexibility and ease in inputting data. Rules for preparaing the input cards are shown below.

6.2.1 Rules for Data Card Preparation. –

(1) Input data have preassigned names and storage locations. Data are input by a statement of the type:

ISTART = –1, START (2) = 0, , 127.1/15, – 30E – 2 $$ SCOUT

This card will store – 1 in the location identified as ISTART, zero in the second location of the START array, will not disturb the third location of the START array, will store the quotient of the indicated division in the fourth location of the START array and –0.30 in the fifth location of the START array. The $$ causes the card to print on the output listing. The word SCOUT appears as comment only.

(2) The data card format is flexible. Blanks are ignored, except in the alphanumeric field. All 80 columns may be used. Decimal points are optional.

(3) If the $$ is omitted, the data is stored but the card is not printed with the output. Comments may be placed to the right of the $$, however, avoid any characters adjacent to the $$ since these may result in undesirable printer line spacing.

(4) A comma after the last value on a card is optional if the next card begins with a variable name.

(5) The following arithmetic operations are permitted:

Addition	Use the + character
Subtraction	Use the – character
Multiplication	Use the * character
Division	Use the / character

Parentheses to indicate order of arithmetic operations are not permitted. The order of operations is from left to right.

(6) As indicated in the example data card, each variable may be regarded as an array.

(7) Where built-in (BN) values are indicated in the input definitions, the parameter need not be input unless different values are required. All parameters are initialized zero unless a BN value is indicated.

(8) Preceding and following each problem set must be a $D=1 card. A single $D=1 card must separate problem sets.

(9) Alphanumeric or title information is input as follows:

LINE1=(A14)F-1 TRAJECTORY$$

Fourteen is the number of columns containing the title, excluding the $$.

(10) Units are kilometers, days, degrees, kilograms unless otherwise noted.

(11) Preceding the $D=1 card of the first problem set must be the table of names.

6.2.2 Table of Names. — The table of names assigns an input variable name to a single or several storage locations, which are assigned to variables internal to the computer program. All variable names appearing on the input data cards must be assigned a storage location in the table of names. The table of names is shown in Section 6.4 in the sample problem.

6.2.3 Epoch Conditions. —

JDATE (1)	Julian Date of the epoch, ends in 0.5
JDATE (2)	fractional day of the epoch
ISTART	= −1, mean orbital elements at the epoch JDATE are determined from the START array.
	=1, is the same as ISTART =-1 except START (4) is satellite right ascension
	=0, Mean orbital elements at the epoch are input directly as E, I, ARGP, NODE and A or N
START (1)	radius to satellite, feet
START (2)	inertial velocity of satellite, ft/sec
START (3)	geocentric latitude of satellite position
START (4)	Greenwich East longitude of satellite position
START (5)	heading of satellite inertial velocity
START (6)	flight path angle of satellite inertial velocity
A	satellite orbital semimajor axis
E	satellite orbital eccentricity
I	satellite orbital inclination, must not be zero
ARGP	satellite argument of perigee
NODE	right ascension of ascending node of the satellite orbit on the equatorial plane
N	satellite mean motion, rev/day, used if A=0 and ISTART = 0.

6.2.4 Perturbation Options. –

MOON =0, lunar disturbance is ignored

=1, uses a procedure which averages the lunar disturbance over the satellite orbit at equal intervals in satellite mean anomaly. (1BN)

=2, same as MOON = 1 except lunar disturbance effect on semi-major axis is included.

SUN =0, solar disturbance is ignored.

=1, uses a procedure which averages the solar disturbance over the satellite orbit at equal intervals in satellite mean anomaly. (1BN)

=2, same as SUN=1 except solar disturbance affect on semi-major axis is included.

OBLATE =0, earth oblateness disturbance is ignored.

=1, second, third and fourth harmonics of the earth's oblate potential are simulated by averaging the disturbance over the satellite orbit at equal intervals in satellite mean anomaly. (1BN)

DRAG =0, earth atmospheric drag is ignored.

=1, uses a procedure which averages the drag disturbance over the satellite orbit at equal intervals in satellite mean anomaly. (1BN)

NSUN integer number of equal increments in mean anomaly for averaging disturbances over the satellite orbit to obtain secular rates. $10 \leqslant NSUN \leqslant 360$

SPRESS satellite reference area used in solar pressure calculation, square meters. SPRESS is the satellite area perpendicular to the satellite – sun line. (1BN)

6.2.5 Atmosphere Definition. –

LUIGI specifies atmosphere definition. Cannot be changed in subsequent cases.

=0, atmospheric quantities are calculated based upon 1962 Standard Atmosphere.

=1 or 2, atmospheric quantities are calculated based on an exospheric temperature profile based on local time, season and time within the eleven year solar cycle. Table of FBAR must be supplied.

= –1, or –2, atmospheric quantities calculated from an input constant value of exospheric temperature, TEXCON.

	=±2, a table of density as a function of exospheric temperature and altitude is printed.
TEXCON	constant exospheric temperature, used when LUIGI = −1 or −2, must be between 650 and 2100 degrees Kelvin.
FBAR	table of solar flux ($\overline{F}_{10.7}$), 100 values maximum, units are 10^{-22} watts/in^2/cycle/sec.
TCYCLE	table of time for FBAR, units are years A. D.

6.2.6 Earth Model Definition. —

REQUAT	earth equatorial radius (6378.166 BN)
EESQRD	eccentricity squared of spheroidal earth model (0.0066934217 BN)
OBLATJ	coefficient of the second gravitational harmonic (0.00162345 BN)
OBLATH	coefficient of the third gravitational harmonic (−5.75E −6 BN)
OBLATK	coefficient of the fourth gravitational harmonic (7.95E −6 BN)
GM	gravitational constant, ft^3/sec^2 (1.4076576E16 BN)

6.2.7 Satellite Definition. —

CD	aerodynamic drag coefficient (2.5 BN)
SREF	aerodynamic reference area, square meters (1 BN)
MASS	satellite mass (1 BN)

6.2.8 Termination. —

TSTOP	time since JDATE at which case will terminate.
ITERM	selective parameter for terminating the case when FINALVALUE is reached. Case will terminate on earliest of TSTOP, FINALVALUE and STEPMX.

	Dependent Variable
=1	semi-major axis
=2	eccentricity
=3	inclination
=4	argument of perigee
=5	right ascension of ascending node
=6	perigee altitude
=7	time derivative of semi-major axis due to atmospheric drag, km/day.

FINALVALUE	value of dependent variable for case termination when ITERM is non-zero.
SLOPE	= −1, termination occurs when ITERM parameter is decreasing.
	=0, termination occurs first time ITERM parameter is attained.
	= 1, termination occurs when ITERM parameter is increasing.
XTOL	tolerance on the independent variable, time, that will cause case termination on a dependent variable when ITERM is non-zero. (0.0001 BN)

6.2.9 Integration Controls. –

BACKUP	=1, causes the integration to proceed backwards in time. All inputs remain positive.
EREF	reference value of normalized truncation error. (1E −4 BN)
ERRFAC	factor by which the normalized truncation error may exceed EREF before rejection of the interval occurs. (5 BN)
DSTART	Estimated initial integration interval. The integration interval is subsequently varied by the program to control truncation error. (1 BN)

6.2.10 Input and Output Control. –

NPROB	problem number printed with page headings. It is incremented by the program when successive problems are run (1 BN)
LINE1=(A67)	Line one title in columns 12 through 78. (blanks BN)
LINE2=(A67)	Line two title in columns 12 through 78. (blanks BN)
SAVE	=1, causes all input data to be saved for possible subsequent problems. If SAVE=0 all input locations are cleared and restored to their built-in values between problems.
DPRINT	print interval for time. If DPRINT=0, output will occur every nth successful interval where STEPS=n. Abnormal termination may occur with large values of DPRINT.

STEPS	number of integration steps between printouts when DPRINT=0. (1 BN)
CHKOUT	=1, prints out h, k, $\dot{h}$, $\dot{k}$ parameters (integration variables) as a function of time, on page C.
	=2, in addition to h, k, $\dot{h}$, $\dot{k}$ parameters, printout moon ephemeris (right ascension of ascending node and declination) at each time step.

6.2.11 Error Exit Control. –

DMIN	minimum continuous integration interval size. (0.5 BN)
STEPMX	maximum number of successful and unsuccessful integration intervals which may occur during one problem. (100 BN)
ESTART	=0, causes the program to dump COMMON data and exit the computer when an error is encountered
	=1, causes the program to dump COMMON data and proceed to the next problem when an error is detected.
	= –1, causes the program to proceed directly to the next problem when an error is detected.

6.3 Output Definitions

Input Options and Data	Self Explanatory
SFACT	1. $-\overline{GM}/GM_S$ where $\overline{GM}$ = solar gravitational constant acting on satellite in sunlight. See Section 5.2.2 GM_S = universal gravitational constant

Page A

TIME	time from the epoch, days.
ARG PER	satellite argument of perigee, deg. (zero is printed when eccentricity is zero)
ASC NODE	right ascension of the satellite ascending node in the equatorial plane, deg.
INCL	inclination of the satellite orbit to the equatorial plane, deg.
ECCEN	eccentricity of the satellite orbit
SEMIMJR	semimajor axis of the satellite orbit, km.
PER ALT	perigee altitude of the satellite orbit above a spherical Earth, km.

NERR	indicates which orbital element has the largest integration error

NERR	Element
2	semimajor axis
3	h parameter (e sin ω)
4	inclination
5	k parameter (e cos ω)
6	node

STEPS GOOD	count of successful integration intervals
STEPS BAD	count of unsuccessful integration intervals
SHADOW POINTS	count of NSUN points in the current orbit which are eclipsed by the Earth

DERIVATIVES OF SEMIMAJOR AXIS IN KM/DAY

LUNAR	lunar contribution to $\dot{a}$, km/day
SOLAR	solar contribution to $\dot{a}$, km/day
OBLATE	oblate Earth contribution to $\dot{a}$, km/day
DRAG	atmospheric drag contribution to $\dot{a}$, km/day

<u>Page B</u> Page B output is self explanatory

<u>Page C</u> Page C output is self explanatory. Page C applies to parameters h and k and is printed only if CHKOUT $\neq$ 0.

6.4 Sample Problem

Output of a typical problem is presented in this section. The problem is for an initial orbit of 250 km perigee, 1200 km apogee and an epoch of 20 November 1975. All orbit perturbations are considered and a solar flux time history is predicted for the atmospheric drag calculations.

The first page is the table of names, which precedes the data of the first problem set. The second page is a list of the data cards of the first problem set. These first two pages are printed since each card has the $S characters. Following these pages is a page showing input options and data, followed by Pages A, B, and C, as defined in Section 6.3.

```
$D=1,$T,1=C,1.=J,1=LINE1,13=LINE2,31=SAVE,32=FSTART,39=REQUAT,51=SUN,46=N,    $$1
 52.=MOON,53=OBLATE,55=A,56=E,57=I,58=ARGP,59=NODE,60=BACKUP,                 $$
61=ERFF,62=ERRFAC,77=DSTART,83=TSTOP,84=DPHINT,                               $$
88.=ITERM,89=SLOPE,90=FINALVALUE,91=XTOL, $$
95.=STEPMX,96.=STEPS,97=JDATE,99=DMIN,163=SPRESS,179.=NPROB,48.=NSUN,         $$
218=OBLATJ,219=OBLATK,220=OBLATH,161=GM,                                      $$
            224.=IST RT,225=START,371=DRAG,373=CD,374=SREF,375=MASS,          $$
378.=LUIGI,501=TCYCLE,601=FBAR,386=TEXCON,389=EESQRD,400=CHKOUT,              /$$
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
SD=1  $$1
LINE1=(A67)CHECKOUT                                                          $$
ISTART=0  $$
MOON=2,SUN=2,OBLATE= 1,DRAG=1,LUIGI=1,NSUN=30,STEPMX=500,DPRINT=30,ISTOP= 750 $$
CHKOUT=1 $$
EREF=1E-3  $$
ITERM=7,SLOPE=-1,-1000 $$
JDATE=2442736.5,.7916  $$  100 PST 20 NOV 1975
LINE2=(A67)250 X 1200 KM                                                     $$
A=250+1200+6378.2+6378.2/2.  $$
E=1200+6378.2/A-1 $$
I=90.0,ARGP=163.546,NODE=0  $$
MASS=77.1*.45359,CD=2.2,BREF=(.4536  $$
SAVE=1,ESTART=-1 $$
  $$  SOLAR FLUX DATA FROM NASA MSFC ES44/ORSE 13 AUG 1975
TCYCLE=  1975.25,  1975.50,  1975.75,  1976.00,  1976.25,  1976.50,  $$
         1976.75,  1977.00,  1977.25,  1977.50,  1977.75,  1978.00,  $$
         1978.25,  1978.50,  1978.75,  1979.00,  1979.25,  1979.50,  $$
         1979.75,  1980.00,  1980.25,  1980.50,  1980.75,  1981.00,  $$
         1981.25,  1981.50,  1981.75,  1982.00,  1982.25,  1982.50,  $$
         1982.75,  1983.00,  1983.25,  1983.50,  1983.75,  1984.00,  $$
         1984.25,  1984.50,  1984.75,  1985.00,  1985.25,  1985.50,  $$
         1985.75,  1986.00,  1986.25,  1986.50,  1986.75,  1987.00,  $$
         1987.25,  1987.50,  1987.75,  1988.00,  1988.25,  1988.50,  $$
         1988.75,  1989.00,  1989.25,  1989.50,  1989.75,            $$
FBAR=      79.81,    77.69,    77.09,    76.77,    76.23,    75.62,  $$
           75.20,    75.65,    76.90,    78.59,    80.78,    83.03,  $$
           85.30,    86.59,    89.02,    92.85,    97.64,   102.56,  $$
          107.71,   111.38,   114.76,   117.57,   118.77,   120.23,  $$
          120.22,   120.36,   121.94,   122.52,   120.77,   119.19,  $$
          115.44,   111.89,   110.69,   108.05,   105.84,   102.77,  $$
            8.56,    95.47,    92.24,    88.59,    85.37,    84.47,  $$
           82.94,    81.88,    80.67,    79.84,    79.54,    79.96,  $$
           81.13,    81.94,    82.69,    83.58,    84.63,    85.47,  $$
           88.34,    91.13,    93.97,    96.94,    99.93,            $$
```

```
PROBLEM NUMBER  1

INPUT OPTIONS AND DATA

SUN MOON OBLATE NMOON NSUN DRAG  SPRFSS ISTART EPOCH JULIAN DATE SAVE   CO   SREF    .  MASS
 2.   2    1.    30   30   1.     1.0       0 2442736.5, .79160  1. 2.20      .5     56.97

SFACT=   .999978
```

CHECKOUT
250 X 1200 KM

LTV AEROSPACE CORP.
ROUTINE Y051, PROBLEM 1

TIME DAYS	ARG PER DEG	ASC NODE DEG	INCL DEG	ECCEN	SEMIMJR KM	PER ALT KM	NERR	STEPS GOOD	STEPS BAD	SHADOW POINTS	DERIVATIVES OF SEMIMAJOR AXIS, KM/DAY LUNAR	SOLAR	OBLATE	DRAG
0.000	163.546	0.000	90.000	.063871	7103.200	251.76	0	0	0	7	3.81E-15	5.77E-0[illegible]	-6.14E-09	-6.11E-01
30.000	61.352	359.989	89.997	.063645	7088.928	261.90	3	5	1	6	-3.88E-15	-1.82E-04	8.28E-09	-4.85E-01
60.000	317.320	359.970	89.996	.061404	7067.259	264.99	3	10	1	5	8.24E-15	1.58E-04	-9.72E-09	-6.35E-01
90.000	210.740	359.955	89.999	.059794	7053.743	259.41	3	14	2	11	-7.37E-15	7.79E-04	8.20E-09	-4.32E-01
120.000	105.374	359.947	89.998	.059188	7037.542	262.72	3	19	2	12	-5.21E-16	-9.36E-04	-4.76E-09	-5.56E-01
150.000	.009	359.931	89.993	.055005	7012.249	248.38	5	23	3	10	3.75E-17	5.08E-04	4.11E-11	-9.52E-01
180.000	250.524	359.892	89.988	.051850	6994.664	272.84	3	28	3	9	1.77E-15	1.13E-04	4.51E-09	-4.12E-01
210.000	141.149	359.834	89.984	.050493	6972.397	250.61	5	34	4	0	2.68E-15	5.09E-13	-6.77E-09	-8.88E-01
240.000	32.853	359.763	89.983	.047747	6951.826	248.05	5	39	5	9	-5.87E-15	6.41E-04	5.73E-09	-5.78E-01
270.000	280.935	359.692	89.983	.044317	6936.800	271.83	3	43	6	10	-9.03E-17	-7.34E-04	-2.06E-09	-3.99E-01
300.000	167.733	359.621	89.980	.041963	6909.516	242.38	5	48	6	11	2.19E-15	2.60E-04	-2.01E-09	-1.46E+00
330.000	55.732	359.531	89.976	.038120	6874.936	249.32	3	53	7	12	7.02E-16	4.36E-04	3.84E-09	-9.85E-01
360.000	299.710	359.413	89.968	.032076	6841.138	259.68	3	58	8	10	4.65E-15	-6.80E-04	-2.57E-09	-9.09E-01
390.000	179.520	359.264	89.963	.029455	6814.008	235.13	5	63	8	5	5.39E-15	2.18E-04	-7.45E-13	-1.08E+00
420.000	63.478	359.094	89.960	.026398	6782.717	242.63	3	68	9	9	-3.68E-15	-1.51E-04	1.61E-09	-1.38E+00
450.000	301.266	358.901	89.951	.016447	6718.036	245.02	3	74	9	11	-1.44E-15	4.61E-04	-6.96E-10	-2.10E+00
480.000	166.970	358.666	89.938	.007151	6617.945	193.55	5	80	11	13	-8.34E-16	1.41E-04	6.14E-12	-1.02E+01
483.436	120.988	358.631	89.919	.000693	6501.994	135.06	3	89	14	13	3.84E-15	2.05E-04	8.06E-11	-9.97E+02

DERIVATIVES OF ELEMENTS IN SEC OF ARC/DAY EXCEPT ECCENTRICITY IN 1/DAY

TIME	(ARGUMENT OF PERIGEE			)	(RT ASC OF ASC NODE			)	(INCLINATION			)	(ECCENTRICITY			)
DAYS	LUNAR	SOLAR	OBLATE	DRAG	LUNAR	SOLAR	OBLAT	DRAG	LUNAR	SOLAR	OBLAT	DRAG	LUNAR	SOLAR	OBLATE	DRAG
0.000	-2.5	1.8	-12333.	-7.	-.68	-.23	0.	.10	-.39	-.42	.00	-.41	1.74E-08	-1.24E-06	5.72E-05	-7.73E-05
30.000	-2.8	-2.8	-12287.	13.	-.59	-.32	-1.	-.14	.80	-.01	.00	-.13	-2.91E-07	2.23E-07	-3.11E-05	-6.10E-05
60.000	1.9	4.4	-12715.	-15.	-.14	-.29	-2.	.20	1.04	.39	.00	-.29	-1.35E-06	-2.20E-07	-3.94E-05	-8.05E-05
90.000	5.0	-3.2	-12769.	2.	-.05	-.10	-0.	-.12	-.36	.37	-.00	-.24	1.28E-06	-3.62E-07	4.76E-05	-5.50E-05
120.000	-3.7	-3.0	-12542.	3.	-.51	.00	-1.	.08	-1.13	.01	-.00	-.07	1.31E-07	1.02E-06	1.77E-05	-7.03E-05
150.000	-2.6	7.1	-12920.	-4.	-.73	-.07	-3.	-.01	-.12	-.35	.00	-.69	8.94E-08	-5.32E-07	-6.07E-05	-1.22E-04
180.000	-2.7	-3.4	-13234.	11.	-.43	-.21	-5.	-.09	.90	-.34	-.00	-.08	-2.46E-07	-4.18E-07	1.83E-05	-5.24E-05
210.000	2.2	1.0	-12995.	-23.	-.07	-.35	-7.	.27	.70	-.03	-.00	-.46	-9.28E-07	8.28E-07	5.17E-05	-1.15E-04
240.000	3.2	-2.3	-13139.	2.	-.03	-.25	-8.	-.17	-.24	.26	.00	-.32	7.94E-07	-7.73E-07	-5.58E-05	-7.49E-05
270.000	-2.9	-3.8	-13656.	11.	-.28	-.09	-8.	.03	-.82	.36	.00	-.06	1.02E-07	8.82E-07	-1.09E-05	-5.06E-05
300.000	-2.1	9.8	-13500.	-27.	-.55	-.01	-9.	.17	-.47	.09	-.00	-1.05	-3.40E-08	-4.01E-07	6.48E-05	-1.90E-04
330.000	-2.1	-4.6	-13483.	33.	-.54	-.04	-12.	-.28	.52	-.29	.00	-.35	-1.98E-07	-5.78E-07	-3.97E-05	-1.26E-04
360.000	-.8	-1.8	-14385.	-42.	-.21	-.22	-16.	.23	1.00	-.37	.00	-.30	-4.87E-07	1.09E-06	-3.15E-05	-1.16E-04
390.000	6.0	.3	-14217.	-28.	.00	-.31	-18.	-.01	.05	-.11	-.00	-.81	-6.73E-08	-8.29E-07	6.88E-05	-1.41E-04
420.000	-.7	-3.1	-13929.	221.	-.32	-.25	-20.	-.35	-1.09	.29	.00	-.51	4.63E-07	8.67E-07	-3.29E-05	-1.72E-04
450.000	-1.5	12.0	-15687.	-603.	-.70	-.11	-26.	.45	-.48	.37	.00	-.96	2.06E-07	-1.02E-06	-1.68E-05	-2.47E-04
480.000	-4.5	-34.3	-15212.	-1195.	-.52	-.00	-34.	.40	.80	.06	-.00	-6.45	-2.29E-08	2.15E-07	7.65E-05	-9.10E-04
483.436	7.5	-202.8	4912.	*********	-.04	-.00	-47.	9.37	.54	.01	-.00	******	-5.22E-08	1.09E-06	4.37E-05	-2.45E-02

CHECKOUT
250 X 1200 KM

LTV AEROSPACE CORP.
ROUTINE 7051, PROBLEM 1

TIME DAYS	HDOT LUNAR	HDOT SOLAR	HDOT OBLATE	HDOT DRAG	KDOT LUNAR	KDOT SOLAR	KDOT OBLATE	KDOT DRAG	H	K
0.000	7.76E-07	-9.12E-07	3.85E-03	-1.96E-05	2.11E-07	1.02E-06	1.08E-03	7.48E-05	1.8941E-02	-6.4133E-02
30.000	-6.76E-07	-2.34E-07	-1.90E-03	-5.16E-05	6.32E-07	8.94E-07	3.42E-03	-3.29E-05	5.7609E-02	3.1472E-02
60.000	1.34E-06	1.12E-06	-2.76E-03	5.12E-05	-6.05E-07	7.35E-07	-2.59E-03	-6.22E-05	-4.1626E-02	4.5141E-02
90.000	-1.90E-06	9.98E-07	3.16E-03	2.76E-05	-3.63E-07	-1.49E-07	-1.93E-03	4.76E-05	-3.0564E-02	-5.1393E-02
120.000	4.08E-07	1.21E-06	9.71E-04	-6.80E-05	9.89E-07	5.55E-07	3.47E-03	1.79E-05	5.7070E-02	-1.5692E-02
150.000	-7.01E-07	1.89E-06	-3.45E-03	-1.02E-06	8.95E-08	-5.32E-07	-6.01E-05	-1.22E-04	8.1772E-06	5.5005E-02
180.000	4.60E-07	6.81E-07	1.09E-03	4.85E-05	-5.63E-07	-6.74E-07	-3.14E-03	2.00E-05	-4.8883E-02	-1.7287E-02
210.000	-9.93E-07	3.38E-07	2.51E-03	-6.75E-05	3.92E-07	-7.91E-07	1.96E-03	9.28E-05	3.1674E-02	-3.9323E-02
240.000	1.05E-06	-8.73E-07	-2.59E-03	-4.02E-05	2.68E-07	-3.56E-07	1.60E-03	-6.32E-05	2.5902E-02	4.0110E-02
270.000	-2.17E-07	-1.02E-06	-5.46E-04	5.02E-05	-5.86E-07	-6.26E-07	-2.88E-03	-7.26E-06	-4.3512E-02	8.4069E-03
300.000	4.12E-07	-2.03E-06	2.70E-03	-3.50E-05	1.24E-07	-3.04E-08	5.21E-04	1.87E-04	8.9157E-03	-4.1005E-02
330.000	-3.82E-07	-9.75E-07	-1.44E-03	-1.01E-04	2.09E-07	4.05E-07	2.00E-03	-7.62E-05	3.1503E-02	2.1464E-02
360.000	3.65E-07	-1.08E-06	-1.08E-03	9.72E-05	-3.43E-07	3.00E-07	-1.96E-03	-6.30E-05	-2.7860E-02	1.5898E-02
390.000	-8.59E-07	-5.60E-08	2.03E-03	2.80E-06	6.01E-08	8.29E-07	-5.17E-05	1.41E-04	2.4670E-04	-2.9454E-02
420.000	3.76E-07	5.98E-07	-8.25E-04	-1.42E-04	[illegible]	7.44E-07	1.58E-03	-1.02E-04	2.3620E-02	1.1788E-02
450.000	-2.39E-07	1.37E-06	-6.18E-04	1.86E-04	2.19E-09	2.90E-07	-1.09E-03	-1.69E-04	-1.4058E-02	8.5362E-03
480.000	1.48E-07	1.21E-06	5.31E-04	-1.65E-04	5.78E-08	5.88E-08	4.44E-05	8.96E-04	1.6123E-03	-6.9669E-03
483.036	-5.77E-08	1.28E-06	2.89E-05	-1.32E-02	5.25E-09	2.38E-08	-3.66E-05	2.56E-02	5.9429E-04	-3.5692E-04

$D=1 $$

APPENDIX A

ATMOSPHERE MODEL

Static Atmosphere Model

A static atmosphere model is used to calculate the density for altitudes below 120 kilometers. The atmosphere is defined by an assumed relationship between the temperature and the geopotential altitude and by a sea level pressure. Geopotential altitude is the altitude above a constant gravity earth which gives the same potential energy as altitude above an earth with inverse square gravity. Geopotential altitude is

$$H = h \left(\frac{R_o}{R_o + h} \right)$$

where R_o is the mean earth radius and h is the geometric altitude. A continuous function with linear segments is used as the relationship between the temperature and geopotential altitude. For each segment

$$T = T_n + L_n (H - H_n) \qquad H_n \leqslant H \leqslant H_{n+1}$$

where L_n is the slope of the linear segment and T_n is the temperature at altitude H_n.

If the perfect gas equation of state,

$$P = \frac{RT\rho}{M_o}$$

and the hydrostatic equation,

$$dP = -\rho g_o \, dH$$

and the linear temperature-altitude relation are combined and integrated, the pressure is expressed as

$$P = P_n \left[\frac{T_n}{T} \right]^{\frac{g_o M_o}{R L_n}} \qquad L_n \neq 0$$

$$P = P_n \exp \left[\left(\frac{-g_o M_o}{RT_n} \right) (H - H_n) \right] \qquad L_n = 0$$

The density is

$$\rho = \frac{PM_o}{RT}$$

and the speed of sound is

$$a = \left(\frac{\gamma P}{\rho} \right)^{1/2}$$

APPENDIX A

The following constants are used to approximate the 1962 U. S. Standard Atmosphere.

Sea Level Pressure	101325 newtons/m^2
Units constant, g_o	9.80665 m/sec^2
Sea Level Molecular Weight, M_o	28.9644
Universal gas constant, R	8314.32 joules/kg^{o}K
Mean earth Radius, R_o	6,356,766 m
Specific heat ratio, γ	1.4

Geopotential Altitude, km	Temperature ^{o}K
0	288.15
11	216.65
20	216.65
32	228.65
47	270.65
52	270.65
61	252.65
79	180.65
88.743	180.65
98.451	210.65
108.129	260.65
117.776	360.65
146.541	960.65
156.071	1110.65
165.571	1210.65
184.485	1350.65
221.967	1550.65
286.476	1830.65
376.312	2160.65
463.526	2420.65
548.230	2590.65
630.530	2700.65

APPENDIX A

Dynamic Atmosphere Model

Because of the dynamic nature of the upper atmosphere and the importance of the effect of aerodynamic drag on orbital motion, a dynamic atmosphere model is used for altitudes above 120 kilometers. The model is a slightly modified version of the 1969 NASA model, Reference (3), which is based on the model of Jacchia, Reference (11). The modifications are as recommended in Reference (12). The model requires an input table of 10.7 cm mean solar flux and a table of geomagnetic index. The computational algorithm, which is almost identical to that of Appendix A of Reference (3), is presented below without discussion.

A. Exospheric Temperature Computation

1. Angle between atmospheric bulge and computation point.

$$\tau = H - 45^\circ + 12^\circ \sin (H + 45^\circ) \qquad (\pm 180^\circ)$$

where

H is the hour angle of the computation point.

2. Mean solar activity correction.

$$T_1 = 362 + 3.60 (\bar{F})$$

where $\bar{F}$ is the input value of mean solar flux.

3. The daily solar activity correction is neglected.

$$T_2 = T_1$$

4. Semi-annual correction

$$T_3 = T_2 + f \bar{F}$$

where

$$f = \left\{ 0.37 + 0.14 \sin \left[2\pi \left(\frac{D-151}{365} \right) \right] \right\} \sin \left[4\pi \left(\frac{D-59}{365} \right) \right]$$

D is day number.

5. Diurnal Correction

$$T_4 = T_3 \left[1 + 0.28 \sin^{2.5} \theta \right] \left[1 + A \cos^{2.5} \left(\frac{\tau}{2} \right) \right]$$

where

$$A = 0.28\left[\frac{\cos^{2.5} W - \sin^{2.5}\,\theta}{1 + 0.28\,\sin^{2.5}\,\theta}\right]$$

$$W = \frac{1}{2}\,(\lambda - \delta)$$

$$\theta = \left|\frac{1}{2}\,(\lambda + \delta)\right|$$

λ = latitude of computation point

δ = declination of the sun

6. Geomagnetic activity correction

$$T_5 = T_4 + La_p + 100\left[1 - \exp\,(-0.08a_p)\right]$$

where

$$L = 1 + 2.85\,(\lambda - 30^o) \text{ for } |\lambda| > 30^o$$

$$L = 1 \qquad \text{for } |\lambda| \leqslant 30^o$$

a_p = input value of geomagnetic index.

B. Temperature at given geometric altitude

$$T_6 = T_5 - \left[T_5 - 355\right]\left[\exp\,(-S\Delta H)\right]$$

where

$$\Delta H = \frac{(Z-120)\ (6476.77)}{6356.77 + Z}$$

Z = geometric altitude, km.

$$S = 1.5 \times 10^{-4} + 0.029\,\exp\,(-X^2/2)$$

$$X = \frac{T_5 - 800}{750 + 1.722 \times 10^{-4}\,(T_5 - 800)^2}$$

C. Number Density Computations

1. Thermal diffusion factor for hydrogen

$$T_D = -10.48947 + 2.844291 \times 10^{-2}\left[T_5\right] - 3.620958 \times 10^{-5}\left[T_5\right]^2 + 2.341193 \times 10^{-8}\left[T_5\right]^3 - 7.577509 \times 10^{-12}\left[T_5\right]^4 + 9.753963 \times 10^{-16}\left[T_5\right]^5$$

2. Hydrogen number density at 500 km altitude

$$N(H)_{500} = \text{anti}\log_{10}\left[73.13 - 39.4\log_{10} T_5 + 5.5(\log_{10} T_5)^2\right]$$

3. Hydrogen number density for altitudes greater than 500 km

$$N(H) = N(H)_{500}\left[B\right]^{(1+T_D+1.008Q)}\left[\exp(-1.008S\,\Delta HQ)\right]$$

where

$$B = \frac{1-P}{1-\rho\exp(-S\Delta H)}$$

$$P = \frac{T_5 - 355}{T_5}$$

$$Q = \frac{1.13619}{T_5 S}$$

4. Helium number density

$$N(HE) = (3.4 \times 10^7)\left[B\right]^{(0.63+4.002Q)}\left[\exp(-4.002S\,\Delta HQ)\right]$$

5. Number density for molecular nitrogen and molecular and atomic oxygen.

$$N(I) = \left[N(I)_{120}\right] \left[B\right]^{\left[1 + QM(I)\right]} \left\{\exp\left[-2\Delta HQM(I)\right]\right\}$$

$$I \rightarrow N_2, O_2, O$$

where

$N(N_2)_{120} = 4.0 \times 10^{11}\ cm^{-3}$

$N(O_2)_{120} = 7.5 \times 10^{10}\ cm^{-3}$

$N(O)_{120} = 7.6 \times 10^{10}\ cm^{-3}$

$M(N_2) = 28.0134$

$M(O_2) = 31.9988$

$M(O) = 15.9990$

D. Mass Density

$$\rho = N(H)\,W(H) + N(HE)\,W(HE) + N(N_2)\,W(N_2) + N(O_2)\,W(O_2) + N(O)\,W(O)\ gm/cm^3$$

where

$W(H) = 1.6731 \times 10^{-24}$ gm/mole

$W(HE) = 6.6435 \times 10^{-24}$ gm/mole

$W(N_2) = 4.6496 \times 10^{-23}$ gm/mole

$W(O_2) = 5.3104 \times 10^{-23}$ gm/mole

$W(O) = 2.6552 \times 10^{-23}$ gm/mole

APPENDIX B

SCIENTIFIC DATA PROCESSING ROUTINE
SUMMARY DOCUMENTATION

IDENTIFICATION

Title Satellite Lifetime Routine

Routine No. 7051 Date Filed 1965 Security Class. U

Responsible Engineer H. U. Everett Unit 2-53012 Telephone Ext. 7694

Date Completed November 1975 Source Language: ☒ FORTRAN Other

Key Words orbit lifetime, orbit element derivatives, atmosphere

RESOURCE REQUIREMENTS

Typical CPU + I/O Time 20 sec Machine(s) CDC 6600 No. Source Cards 2700

Core 115k octal Tape 0 ☐ Plot ☐ Graphics Other

DESCRIPTION (Include: Purpose of routine, input, output, and functional description)

Purpose – The primary purpose of the routine is to calculate time histories of the orbital elements and the orbit lifetime.

Input Description – Input data cards define the initial orbital elements or injection conditions, the ballistic coefficient of the satellite, the perturbing forces to be considered, and parameters of the dynamic atmosphere model.

Output Description – Output consists of columnar time histories of the orbital elements and their time rates of change.

Functional Description – The routine numerically integrates the secular (averaged) time rate of change of the orbital elements to obtain their time history. The math model includes perturbing forces due to the sun, moon, oblate earth gravity, and a dynamic atmosphere. Atmospheric properties are influenced by solar radiation, seasonal variations in atmospheric properties and the "bulge" in the atmosphere that is oriented with respect to the earth-sun line.

DOCUMENTATION

VSD Report No. 2-53010/5R-23029, "Satellite Lifetime Routine User's Manual," dated 1 December 1975.

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