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WALL-WAKE VELOCITY PROFILE FOR
COMPRESSIBLE NON-ADIABATIC FLOWS

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SYMBOLS

a	=	A constant in the expression $\tau = \tau_w(1-\eta^a)$ (see Ref. 5)
A	=	$\{[(\gamma-1)/2] M_e^2/(T_w/T_e)\}^{1/2}$
A_1	=	$(Pr_t)^{1/2} \{[(\gamma-1)/2] M_e^2/(T_w/T_e)\}^{1/2}$
B	=	$\{(1+[(\gamma-1)/2] M_e^2)/(T_w/T_e)\}-1$
B_1	=	$\{(1+(Pr_t)^{1/2} [(\gamma-1)/2] M_e^2)/(T_w/T_e)\}-1$
C	=	a constant in Law of the Wall (usually 5.1)
C_f	=	Skin friction coefficient $\tau_w/(1/2)\rho_e u_e^2$
C_1	=	$5.1 - 0.614/aK + (1/K) \ln (\delta u_\tau/v_w)$
K	=	Constant in mixing length (usually 0.4)
M	=	Mach number
P	=	Pressure
Pr_t	=	Turbulent Prandtl number
T	=	Temperature
u	=	Velocity in streamwise direction
u^*	=	$(u_e/A) \arcsin \{[(2A^2u/u_e) - B]/(B^2+4A^2)^{1/2}\}$
u^{**}	=	$(u_e/A_1) \arcsin \{[(2A_1^2u/u_e) - B_1]/(B_1^2+4A_1^2)^{1/2}\}$
u_τ	=	Friction velocity $(\tau_w/\rho_w)^{1/2}$
U^*	=	$u^* + (u_e/A) \arcsin \{B/(B^2+4A^2)^{1/2}\}$
W	=	Coles' universal wake function
y	=	Coordinate normal to wall
γ	=	Ratio of specific heat
δ	=	Boundary layer thickness
δ^*	=	Displacement thickness

Δ	=	$\int_0^\delta [(u_e^{**}-u^{**})/u_\tau] dy$
θ	=	Momentum thickness
n	=	y/δ
ν	=	Kinematic viscosity
Π	=	Coefficient of Wake Function
ρ	=	Density
τ	=	Shear Stress

Subscripts

e	=	Freestream conditions
o	=	Stagnation conditions
w	=	Conditions at the wall

INTRODUCTION

The wall-wake velocity profile has been used to represent turbulent boundary layer profiles for both adiabatic and non-adiabatic flows and for flows with or without pressure gradients [1-5]. A least squares fit of a wall-wake velocity profile to an experimental velocity profile may be used to determine C_f and δ for the profile. An accurate representation of the mean velocity distribution in the turbulent boundary layer can be very useful in integral analyses of turbulent flow problems; in the analysis of flows in which strong interactions occur (as for example in shock wave-boundary layer interactions) and in which combined viscid-inviscid analyses are required, a profile which provides a good representation of both C_f and δ can be quite important. With most earlier versions of the wall-wake profile the velocity gradient at the boundary layer edge has a non-zero value. The values of δ for these profiles may, as a consequence, correspond to points in the flow where viscous effects are substantial, especially in very high-speed flows. The purpose of this note is to suggest a form of the wall-wake profile which is applicable to flows with heat transfer, and for which $\partial u / \partial y = 0$ at $y = \delta$. The modified profile, which takes into account the effect of turbulent Prandtl number, has been found to provide a good representation of experimental data for a wide range of Mach numbers and heat transfer. The C_f values which are determined by a least squares fit of the profile to the data agree well with values which were measured by the floating element technique. In addition, the values of δ determined by the fit correspond more closely to the outer edge of the viscous flow region than those obtained with earlier versions of the wall-wake profile.

VELOCITY PROFILE

The difference between the profile to be discussed here and some of the earlier versions is due to differences in the law-of-the-wall component.

Van Driest [6] developed a compressible law-of-the-wall under the assumption of a turbulent Prandtl number of unity and a constant shear stress near the wall.

It may be written as,

$$\frac{u^*}{u_\tau} + \frac{u_e}{u_\tau} \frac{1}{A} \arcsin \frac{B}{(B^2 + 4A^2)^{1/2}} = \frac{1}{K} \ln \frac{yu_\tau}{v_w} + C \quad (1)$$

With the addition of a wake component, Eq. (1) becomes,

$$\frac{u^*}{u_\tau} + \frac{u_e}{u_\tau} \frac{1}{A} \arcsin \frac{B}{(B^2 + 4A^2)^{1/2}} = \frac{1}{K} \ln \frac{yu_\tau}{v_w} + C + \frac{\pi}{K} W\left(\frac{n}{\delta}\right) \quad (2)$$

The left-hand side of this expression is the transformed velocity, U^* , which is used in References 3 and 4.

The use of Eq. (1) in a wall-wake profile results in a non-zero value of the velocity gradient at the boundary layer edge. If we assume, as in Reference 5, that τ may be expressed as $\tau = \tau_w(1-n^a)$, and if for $Pr_t \neq 1$ the temperature distributions may be written as $(T/T_w) = 1 + B_1 (u/u_e) - A_1^2 (u/u_e)^2$, (cf. Schlichting [7]), with $(\rho/\rho_w) = 1/(T/T_w)$, we may follow the procedure used in Reference 5 to obtain a wall-wake boundary layer velocity profile of the following form,

$$\begin{aligned} \frac{u^{**}}{u_\tau} + \frac{u_e}{u_\tau} \frac{1}{A_1} \arcsin \frac{B_1}{(B_1^2 + 4A_1^2)^{1/2}} &= \frac{1}{K} \left\{ \ln n + \frac{2}{a} (1-n^a)^{1/2} \right. \\ &\quad \left. - \frac{2}{a} \ln [1 + (1-n^a)^{1/2}] \right\} + C_1 + \frac{\pi}{K} W(n) \end{aligned} \quad (3)$$

For $a \rightarrow \infty$, which corresponds to the assumption of a constant shear stress distribution in the derivation of the compressible law-of-the-wall, and for $Pr_t \rightarrow 1$, $u^{**} \rightarrow u^*$, as in Eq. (2). Setting $\eta = 1$ in Eq. (3) yields the following expression for Π/K ,

$$\frac{\Pi}{K} = \frac{1}{2} \left\{ \frac{u_e^{**}}{u_\tau} + \frac{u_e^{**}}{u_\tau} \frac{u_e}{u_e^{**}} \frac{1}{A_1} \arcsin \frac{B_1}{(B_1^2 + 4A_1^2)^{1/2}} - \frac{1}{K} \ln \frac{\delta u_\tau}{v_w} - 5.1 + \frac{0.614}{aK} \right\} \quad (4)$$

For mathematical convenience [1-3] we may replace $W(\eta)$ in Eq. (3) by $[1 - \cos \eta\pi]$ and write the wall-wake velocity profile as,

$$\begin{aligned} \frac{u}{u_e} &= \frac{(B_1^2 + 4A_1^2)^{1/2}}{2A_1^2} \sin \left\{ \left[\arcsin \frac{2A_1^2 - B_1}{(B_1^2 + 4A_1^2)^{1/2}} \right] \right. \\ &\times \left[1 + \frac{1}{K} \frac{u_\tau}{u_e^{**}} \left(\ln \eta + \frac{2}{a} (1 - \eta^a)^{1/2} - \frac{2}{a} \ln (1 + (1 - \eta^a)^{1/2}) \right) \right. \\ &\left. \left. - \frac{\Pi}{K} \frac{u_\tau}{u_e^{**}} (1 + \cos \eta\pi) \right] \right\} + \frac{B_1}{2A_1^2} \quad (5) \end{aligned}$$

RESULTS

The method of least squares has been used to fit the wall-wake profile, Eq. (5), to experimental velocity profiles reported by Hopkins and Keener [8], Voisinet and Lee [9], Horstman and Owen [10] and Kilburg [11] for zero pressure gradient flows with heat transfer. The wall-wake velocities were obtained under the assumption of $Pr_t = 0.8$ and $a = 1$, the latter corresponding to the assumption of a linear shear stress distribution in the derivation of the law-of-the-wall. K was taken as 0.4 and C and 5.1. Data in the sublayer were excluded.

The results shown here are from studies for which directly measured values of wall shear stress were reported [8, 9, 10]. The data reported by Voisinnet and Lee and Hopkins and Keener were obtained on wind tunnel walls. The Horstman-Owen data are for flow along the cylindrical surface of a cone-cylinder model whose axis was aligned with the primary wind tunnel flow direction.

Comparisons of the experimental and wall-wake profiles are given in Figure 1. Also shown on the figures are values of δ^* , θ , C_f and Π/K as determined with the wall-wake profile. Experimental values of δ^* , θ and C_f as obtained from the references are also listed. The wall-wake profile is seen to provide a good representation of the experimental velocity profiles. In addition, the C_f values determined by the fit of the proposed profile agree reasonably well with the directly measured values. For the data of References 8 and 10, the values of δ determined by the fit correspond to points in the flow at which u/u_e , P_o/P_{o_e} and T_o/T_{o_e} are essentially unity. If a is set equal to ∞ in Eq. (5) the values of δ are on the order of five percent lower than for $a = 1$ and the values of u/u_e , P_o/P_{o_e} and δ are correspondingly lower. For reasons which are not known at this time the value of δ determined for the data of Reference 9 corresponds to a point in the flow at which $u/u_e = 0.965$, $P_o/P_{o_e} = 0.56$ and $T_o/T_{o_e} = 0.938$, even though the wall-wake profile and the data agree very closely between the wall and the value of δ determined by the least squares fit. The authors comment on the temperature and history effects in some of their profiles. The rather low value of δ may be associated with these effects. For a $\rightarrow \infty$, δ is about five percent lower than for $a = 1$.

The wall-wake values of δ^* and θ agree well with experimental values for the data of Reference 10. The larger differences for δ^* and θ for the data of Reference 8 are due to differences between the temperature distribution for the experimental profile and the distribution for the wall-wake profile with

$Pr_t = 0.8$. In the case of the profile from Reference 9, the larger differences appear to result from the fact that the velocity, total temperature and total pressure continue to increase well beyond the value of δ determined by the least squares fit.

The values of Π/K shown in Figure 1 were determined from Eq. (4), assuming $Pr_t = 0.8$ and $a = 1$ and with u_τ and δ determined from the wall-wake fit. Note that the expression for Π/K differs from that used in References 2-4 through the inclusion of the Prandtl number effect, through the term $0.614/aK$, and through the differences in u_τ and δ which are obtained with the modified profile. Although the pressure gradient was zero for the three profiles shown, the values of Π/K are quite different. The values varied considerably from profile to profile for all of the data examined and tended generally to be higher than for low-speed or compressible adiabatic flows.

The experimental profiles may also be plotted in the form $(u_e^{**} - u^{**})/u_\tau$ versus y/Δ , where Δ is analogous to the defect thickness used by Gran et al., [4] in their comparison of cold-wall, high-speed data with low-speed data [13]. Such a plot is shown in Figure 2. While the profile from Reference 9 deviates from the low-speed results, the profiles from References 8 and 10 correspond quite closely with the low-speed data. This suggests that the latter two profiles were nearer equilibrium than the former, which, in turn, may account for the fact that the value of δ determined for the profile from Reference 9 does not correspond as closely to the viscid-inviscid flow boundary as was found for the other two profiles.

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Figure 1 Velocity distributions in boundary layer with heat transfer.

Figure 2 Comparison of generalized velocity defect for low-speed flow and compressible flow with heat transfer.



