NASA TECHNICAL NOTE

[redacted]

NASA TN D-8166

ISOTHERMAL ELASTOHYDRODYNAMIC LUBRICATION OF POINT CONTACTS

II - Ellipticity Parameter Results

Bernard J. Hamrock and Duncan Dowson

Lewis Research Center

Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1976

1. Report No.
NASA TN D-8166

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
ISOTHERMAL ELASTOHYDRODYNAMIC LUBRICATION OF POINT CONTACTS. II - ELLIPTICITY PARAMETER RESULTS

5. Report Date
February 1976

6. Performing Organization Code

7. Author(s)
Bernard J. Hamrock, Lewis Research Center; and
Duncan Dowson, Leeds University, Leeds, England

8. Performing Organization Report No.
E-8467

9. Performing Organization Name and Address
Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio 44135

10. Work Unit No.
505-04

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D. C. 20546

13. Type of Report and Period Covered
Technical Note

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A numerical solution of the isothermal elastohydrodynamic problem for point contacts is presented which reproduces all the essential features of the previously reported experimental observations based upon optical interferometry. In particular, the two side lobes, in which minimum film thickness regions occur, emerge in the theoretical solutions. The influence of the ellipticity parameter on solutions to the point contact problem is explored. The ellipticity parameter k was varied from 1 (a ball on a plate) to 8 (a configuration approaching line contact). It is shown that the minimum film thickness can be related to the well known line contact solutions by a remarkably simple expression involving either k or the effective radius of curvature ratio R_y/R_x.

17. Key Words (Suggested by Author(s))
Lubrication
Concentrated contacts
Gears
Bearings

18. Distribution Statement
Unclassified - unlimited
STAR Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	19	$3.25

* For sale by the National Technical Information Service, Springfield, Virginia 22161

ISOTHERMAL ELASTOHYDRODYNAMIC LUBRICATION OF POINT CONTACTS

II - ELLIPTICITY PARAMETER RESULTS

by Bernard J. Hamrock and Duncan Dowson*

Lewis Research Center

SUMMARY

A numerical solution of the isothermal elastohydrodynamic problem for point contacts is presented which reproduces all the essential features of the previously reported experimental observations based upon optical interferometry. In particular, the two side lobes, in which minimum film thickness regions occur, emerge in the theoretical solutions. The influence of the ellipticity parameter on solutions to the point contact problem is explored. The ellipticity parameter k was varied from 1 (a ball on a plate) to 8 (a configuration approaching line contact). It is shown that the minimum film thickness can be related to the well known line contact solutions by a remarkably simple expression involving either k or the effective radius of curvature ratio R_y/R_x.

INTRODUCTION

Most of the work to date in elastohydrodynamic lubrication (EHL) has dealt with line contacts. Grubin (ref. 1) was the first to attempt a solution to the isothermal EHL line contact problem. In his analysis it was assumed that the shape of the elastically deformed surfaces was the same as the shape produced in a dry (Hertzian) contact. This assumption facilitated the solution of the Reynolds' equation in the inlet region to the contact and enabled the film thickness in the central region to be determined with acceptable accuracy. In references 2 and 3 an empirical formula for the solution to the isothermal line contact EHL problem was obtained. This formula shows the effect of speed, load, and material properties on minimum film thickness and is based on their theoretical solutions. The experimental observations concur with the minimum film thickness formula.

The study of point contacts has mostly followed experimental lines (e.g., refs.

*Professor of Mechanical Engineering, Leeds University, Leeds, England.

4 to 6). Before Archard and Cowking (ref. 7) presented their central film thickness formulation for point contact, it was common practice to use line contact minimum film thickness to determine the film thickness in point contact situations such as in ball bearings and gears. Archard and Cowking adopted an approach similar to that used by Grubin (ref. 1) for line contact; that is, a Hertzian deformation is assumed when the shape of the surfaces is identical to that which occurs under the same load in the absence of a lubricant. The film shape resulting from this type of analysis is such that the central region is assumed to be parallel to the contacting plane, and the analysis is primarily concerned with deriving an expression for the central film thickness. Cheng (ref. 8) also used a Grubin approach in his point contact analysis.

In an earlier report (ref. 9) we outlined the complete elastohydrodynamic lubrication (EHL) point contact problem. This called for the simultaneous solution of the elasticity and Reynolds' equations. In the elasticity analysis the contact zone is divided into equal rectangular areas. It was assumed that a uniform pressure was applied over each element. In the numerical analysis of the Reynolds' equation, a ϕ substitution, where ϕ is equal to the pressure times the film thickness to the three halves power, was used to help the relaxation process. Only the theory was presented, with the results to follow in later reports.

The present report presents the first of the results. Herein the ellipticity parameter is varied from 1 (a ball and plate curvature combination) to 8 (approaching line contact) while keeping the other dimensionless parameters fixed. Contour plots of pressure and film thickness are presented. From the results the minimum film thickness formula will be given relating the effect of the elliplicity parameter on film thickness. By using these results, a designer will have at his disposal a complete contour map of film thickness. Comparisons will also be made with Archard and Cowking (ref. 7).

SYMBOLS

a	semimajor axis of contact ellipse
b	semiminor axis of contact ellipse
E	modulus of elasticity
E'	$2\Big/\left\{\left[\left(1 - \nu_A^2\right)\Big/E_A\right] + \left[\left(1 - \nu_B^2\right)\Big/E_B\right]\right\}$
F	normal applied load
G	dimensionless material parameter, $E'/p_{iv, as}$
H	dimensionless film thickness, h/R

H_A	dimensionless central film thickness obtained from ref. 7
H_c	dimensionless central film thickness for point contact
H_{min}	dimensionless minimum film thickness for point contact
$H_{min,L}$	dimensionless minimum film thickness for line contact
h	film thickness
k	ellipticity parameter, a/b
P	dimensionless pressure, p/E'
P_{min}, $\overline{P}_{min}$	defined by eqs. (20) and (22)
p	pressure
$p_{iv,as}$	asymptotic isoviscous pressure
R	effective radius
r	radius of curvature
U	dimensionless speed parameter, $V\eta_0/RE'$
u	surface velocity in x direction
V	$\sqrt{u^2 + v^2}$
v	surface velocity in the y direction
W	dimensionless load parameter, $F/E'R_xR_y$
$x, X, \tilde{X}, y, Y, \tilde{Y}$	coordinate systems used in the report
Z	viscosity pressure index, a dimensionless constant
α, β	constants used to define the density
η_0	atmospheric viscosity
θ	spin to roll ratio defined in eq. (8)
Λ	$1/[1 + (2R_x/3R_y)]$
ν	Poisson's ratio
ρ_0	atmospheric density

Subscripts:

A	solid A
B	solid B
x, y	coordinate system defined in report

DIMENSIONLESS GROUPING

The variables resulting from the isothermal EHL point contact analysis (ref. 9) are

(1) Effective radius in the x direction, R_x, mm

(2) Effective radius in the y direction, R_y, mm

(3) Film thickness, h, mm

(4) Effective elastic modulus, E', N/mm^2

(5) Surface velocity in the x direction, u, mm/sec

(6) Surface velocity in the y direction, v, mm/sec

(7) Atmospheric viscosity, η_0, $N \cdot sec/mm^2$

(8) Viscosity pressure index, Z, dimensionless constant

(9) Normal applied force, F, N

(10), (11) Constants used to define the density of the fluid, α and β, mm^2/N

It has been found (e.g., see ref. 10) that density has very little effect on minimum film thickness for line contact situations; thus, one may assume the same is true for the point contact situation. Even though the compressibility effect is still considered in the EHL theory (ref. 9), the constants (α and β) used to define the fluid in the density equation will not be used in the minimum film thickness formulation. Therefore, the number of variables shown was reduced to nine.

In the EHL theory developed in reference 9, the viscosity of the fluid is defined by Roelands' (ref. 11) formula. Before Roelands' work the effect of pressure on viscosity had been accounted for simply by means of a viscosity-pressure coefficient. Roelands points out that the more general solution would be possible by accounting for the viscosity-pressure dependence of a given oil by means of its asymptotic pressure $p_{iv,as}$ where

$$p_{iv,as} = \eta_0 \int_0^\infty \frac{dp}{\eta}$$

Also, Blok (ref. 12) in 1961 arrived at the very important conclusion that all EHL results achieved hitherto for an exponential viscosity-pressure dependence can, to a fair approximation, be generalized for any given nonexponential dependence simply by substituting the reciprocal of the asymptotic isoviscous pressure $1/p_{iv,as}$ for the viscosity pressure coefficient occurring in those results. From the nine variables the following six dimensionless groupings can be written:

(1) Dimensionless film thickness:

$$H = \frac{h}{R_x} \tag{1}$$

where

$$\frac{1}{R_x} = \frac{1}{r_{Ax}} + \frac{1}{r_{Bx}} \tag{2}$$

The radii of curvature defined in equation (2) are shown in figure 1. It is assumed that convex surfaces (fig. 1) have positive curvatures and that concave surfaces have negative curvatures. Figure 2 shows the various components of the film thickness along with the central and minimum film thicknesses (H_c and H_{min}). As can be seen from this figure, H_c and H_{min} differ, so we will be interested in defining both.

(2) Dimensionless load parameter:

$$W = \frac{F}{E'R_x^2} \tag{3}$$

where

$$E' = \frac{2}{\dfrac{1 - \nu_A^2}{E_A} + \dfrac{1 - \nu_B^2}{E_B}} \tag{4}$$

(3) Dimensionless speed parameter:

$$U = \frac{\eta_o V}{E'R_x} \tag{5}$$

where

$$V = \sqrt{u^2 + v^2} \tag{6}$$

(4) Dimensionless material parameter:

$$G = \frac{E'}{p_{iv,as}} \tag{7}$$

where $p_{iv,as}$ is the asymptotic isoviscous pressure obtained from Roelands (ref. 11).

(5) Spin to roll ratio:

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) \tag{8}$$

(6) Ellipticity parameter:

$$k = \frac{a}{b} \tag{9}$$

The ellipticity parameter is determined entirely from the definition of the radii of curvature (r_{Ax}, r_{Bx}, r_{Ay}, r_{By}), and the derivation can be found in reference 9. Therefore, the dimensionless film thickness equation can be written as

$$H = f(W, U, G, \theta, k) \tag{10}$$

The most important practical aspect of the theory developed in reference 9 is the determination of the minimum film thickness in a EHL point contact. Therefore, it is important to observe how the minimum film thickness will vary as these dimensionless groupings are varied.

NUMERICAL SOLUTIONS

The objective of the work presented in this report was to keep W, U, G, and θ constant in equation (10) while varying k and to observe the effect on the minimum film thicknesses. The values of the fixed dimensionless parameters are

$$U = 0.1683\times10^{-11} \tag{11}$$

$$W = 0.3686\times10^{-6} \tag{12}$$

$$G = 4522 \tag{13}$$

$$\theta = 0 \tag{14}$$

The variables and their values that were common to all the results are shown in table I. In this table the values of η_o, Z, and $p_{iv,as}$ correspond to those of a paraffinic mineral oil. Also, the velocity in the y direction is zero; therefore, pure rolling exists.

To keep the dimensionless parameters U, W, G, and θ constant while varying the ellipticity parameter, it was necessary for the radius of curvature r_{By} to vary as shown in table II. Note that the radius of curvature is negative thereby implying the solid to be concave. Also shown in table II is the maximum shear stress.

RESULTS

The theoretical analysis presented by the authors in an earlier report (ref. 9) was programmed on the digital computer along with the input variables. A contour plotting program was also developed.

Figures 3 and 4 show the contour plots of the dimensionless pressure and film thickness for ellipticity parameters k of 2, 4, and 6. The + symbol indicates the center of the Hertzian circle. Note that, because of dimensionless representation of the X and Y coordinates, the actual Hertzian contact ellipse becomes a dimensionless Hertzian circle regardless of the ellipticity parameter. The Hertzian contact circle is denoted on each figure by asterisks. In the upper left corner of each figure the contour label and its corresponding value are given.

In figure 3(a) for $k = 2$ a pressure spike emerges. This is denoted by contour label c at the back end of the contact. In figure 3(b) for $k = 4$ the pressure spike occurs at two locations an equal distance from the axial centerline. In figure 3(c) no pressure spike is evident for the contour values choosen.

As the ellipticity parameter is decreased (going from fig. 4(c) to (b) to (a)), the minimum film thickness contours move away from the axial center, and two minimum areas appear. These results produce all the essential features of the previously reported experimental observations based upon optical interferometry. In particular, the two "side lobes" in which minimum film thickness region occur are shown to emerge in the theoretical solutions.

The effect of ellipticity parameter k on the dimensionless film thickness is shown in figure 5. The physical meanings of the minimum film thicknesses H_c and H_{min} are shown in figure 2. The dimensionless central film thickness H_A was obtained from Archard and Cowking (ref. 7). Writing the Archard-Cowking formula in terms of the dimensionless grouping of this report, one obtains

$$H_A = 2.04(\Lambda GU)^{0.74}W^{-0.074} \tag{15}$$

where

$$\Lambda = \frac{1}{1 + \frac{2R_x}{3R_y}} \tag{16}$$

The Archard-Cowking formula tends to underestimate the central film thickness by up to 25 percent (see fig. 5). It also tends to underestimate the minimum film thickness for values of k greater than three and to greatly overestimate the minimum film thickness for k's less than three. The largest discrepancy occurs for $k = 1$ where the difference between the Archard-Cowking formula and the results of this report is over 400 percent. It should be remembered, however, that the Archard-Cowking formula predicts the central film thickness.

Also shown in figure 5 is the Dowson-Higginson line contact results where the minimum film thickness is written as

$$H_D = 1.6\, G^{0.6} U^{0.7} W_D^{0.13} \tag{17}$$

where

$$W_D = \frac{\overline{F}}{E' R_x} \tag{18}$$

and

$$\overline{F} = \frac{\text{Force}}{\text{Unit length}} \tag{19a}$$

If the unit length is taken as the major axis of the contact ellipse,

$$\overline{F} = \frac{F}{2a} \tag{19b}$$

From figure 5 we see that the minimum film thickness results of the present paper does approach the Dowson-Higginson asymptotic value.

FILM THICKNESS FORMULAS

Having determined how the dimensionless film thickness varies with the ellipticity parameter for a given U, W, G, and θ, the next task was to determine what a more general formulation for the minimum film thickness might be. Table III shows the effect of the ellipticity parameter on the new minimum film thickness formula (eq. (21)). In this table a film thickness is introduced that has not appeared before: $H_{min, L}$ which is the dimensionless minimum film thickness for line contact obtained from equation (17) and which was found to be 6.955×10^{-6}.

In table III a percent deviation of the minimum film thickness is given where

$$P_{min} = \left(1 - 1.6\ e^{-0.62\ k} - \frac{H_{min}}{H_{min, L}}\right)(100) \bigg/ \left(\frac{H_{min}}{H_{min, L}}\right) \tag{20}$$

The value P_{min} in table III are observed to be small. Therefore, from this table general formulas for the dimensionless minimum film thickness of point contacts can be written as

$$\boxed{H_{min} = H_{min, L}\left(1 - 1.6\ e^{-0.62\ k}\right) \tag{21}}$$

Another approach to determine these formulas would be to use the radius of curvature ratio R_y/R_x instead of the ellipticity parameter k. The results are tabulated in table IV where $\overline{P}_{min}$ is defined by

$$\overline{P}_{min} = \left[1 - 1.6\ e^{-0.62(R_y/R_x)^{2/3}} - \frac{H_{min}}{H_{min, L}}\right](100) \bigg/ \left(\frac{H_{min}}{H_{min, L}}\right) \tag{22}$$

The value of $\overline{P}_{min}$ in table IV is small. It was found that by letting

$$k = \left(\frac{R_y}{R_x}\right)^{2/3} \tag{23}$$

in equation (21) it did not greatly affect the value of $\overline{P}_{min}$ in table IV. Therefore, in terms of the radius of curvature ratio R_y/R_x, the dimensionless minimum film thickness can be written as

$$H_{min} = H_{min, L}\left[1 - 1.6\, e^{-0.62(R_y/R_x)^{2/3}}\right] \qquad (24)$$

In reference 6 Bahadoran and Gohar describe the use of optical interferometry in film thickness experiments. In these experiments the radius of curvature ratio (R_y/R_x) of the contacting solids was varied. These investigators in their minimum film thickness measurements used radius of curvature ratios (R_y/R_x) varying from 6.67 to 22.66. Over this range they make the following conclusion: "the minimum film thickness measured is seen to depend hardly at all on R_y/R_x, which changes by a factor of more than 3." Using equation (24), table V was constructed where the effect of radius of curvature on percent difference in minimum film thickness for line contact relative to that of point contact is shown. From this table it is seen that for $R_y/R_x = 6.67$ and 22.66 the minimum film thickness varied by less than 17 percent. Therefore, for $R_y/R_x \geq 6.67$ the findings of the present work are in complete agreement with reference 6. However, for $R_y/R_x = 1$ the minimum film thickness differs by 86 percent from the line contact solution. One might speculate that, if the investigators of reference 6 had used smaller values of radius of curvature ratios (R_y/R_x), they would have then seen larger differences in the minimum film thickness measured. The same sort of conclusions can be derived from figure 5; that is, for $k \geq 3$ $(R_y/R_x \geq 5.2)$ there is little change in the minimum film thickness. However, for $k < 3$ $(R_y/R_x < 5.2)$ there is a substantial change in minimum film thickness.

SUMMARY OF RESULTS

A numerical solution of the isothermal elastohydrodynamic problem for point contacts presented herein reproduces all the essential features of the previously reported experimental observations based on optical interferometry. In particular, the two side lobes in which minimum film thickness regions occur emerge in the theoretical solutions.

The influence of the ellipticity parameter on solutions to the point contact problem has been explored. The ellipticity parameter k was varied from 1 (a ball on a plate) to 8 (a configuration approaching line contact). It was shown that the minimum film thicknesses can be related to the well known line contact solutions by remarkably simple expressions involving either k or the effective radius of curvature ratio R_y/R_x. If the additional subscript L is used to denote the line-contact solution, it is found that the minimum film thickness H_{min} for a point contact is

$$H_{min} = H_{min, L}\left(1 - 1.6\ e^{-0.62\ k}\right)$$

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, November 17, 1975

505-04.

REFERENCES

1. Grubin, A. N.; and Vinogradova, I. E., eds.: Investigation of the Contact of Machine Components. Department of Scientific and Industrial Research (Great Britian), CTS 235 Translation of Tsentral'hyi Nauchno-Issledovatel'skii Institut Tekhnologii i Mashinostroeniua (USSR), Book No. 30, 1949.
2. Dowson, D.; and Higginson, G. R.: Numerical Solution to the Elasto-Hydrodynamic Problem, J. Mech. Eng. Sci., vol. 1, June 1959, pp. 6-15.
3. Dowson, D.; and Higginson, G. R.: New Roller-Bearing Lubrication Formula. Eng. (London), Vol. 192, Aug. 1961, pp. 158-159.
4. Archard, J. F.; and Kirk, M. T.: Film Thickness for a Range of Lubricants Under Severe Stress. J. Mech. Eng. Sci., vol. 6, no. 1, Mar. 1964, pp. 101-102.
5. Wedeven, L. D.; Evans, D.; and Cameron, A.: Optical Analysis of Ball Bearing Starvation. J. Lubri. Tech., Trans. ASME, Ser. F, vol. 93, July 1971, pp. 349-363.
6. Bahadoran, H.; and Gohar, R.: The Oil Film in Elastohydrodynamic Elliptical Contacts. Wear, vol. 29, no. 2, Aug. 1974, pp. 264-270.
7. Archard, J. F.; and Cowking, E. W.: Elastohydrodynamic Lubrication of Point Contacts. Proc. Instru. Mech. Eng., vol. 180, pt. 3B, 1965-1966, pp. 47-56.
8. Cheng, H. S.: A Numerical Solution of the Elastohydrodynamic Film Thickness in an Elliptical Contact. J. Lubr. Tech., Trans. ASME, Ser. F, vol. 92, no. 1, Jan. 1970, pp. 155-162.
9. Hamrock, B. J.; and Dowson, D.: Isothermal Elastohydrodynamic Lubrication of Point Contacts. I - Theoretical Formulation. NASA TN D-8049, 1975.

10. Dowson, D.; and Higginson, G. R.: Elastohydrodynamic Lubrication. Pergamon Press, 1966.

11. Roelands, C. J. A.: Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils. Druk V. R. B.: Groninger, Netherlands, 1966.

12. Blok, H.: Inverse Hydrodynamics. Proc. of the International Symp. on Lubri. and Wear, McCutchan Publishing Corp., 1964, pp. 1-151.

TABLE I. - VARIABLES COMMON TO ALL RESULTS PRESENTED

Modulus of elasticity of solid A, E_A, N/mm^2	2×10^5
Modulus of elasticity of solid B, E_B, N/mm^2	2×10^5
Poisson's ratio of solid A, ν_A, dimensionless	0.3
Poisson's ratio of solid B, ν_B, dimensionless	0.3
Radius of curvature of solid A in x direction, r_{Ax}, mm	11.113
Radius of curvature of solid A in y direction, r_{Ay}, mm	11.113
Radius of curvature of solid B in x direction, r_{Bx}, mm	∞
Surface velocity in y direction, v, mm/sec	0
Lubricant viscosity at atmospheric condition, η_o, $N\cdot sec/mm^2$	0.411×10^{-7}
Viscosity pressure index, z, dimensionless	0.67
Constant used in defining density, α, mm^2/N	5.83×10^{-4}
Constant used in defining density, β, mm^2/N	1.68×10^{-3}
Asymptotic isoviscous pressure, $p_{iv,as}$, N/mm^2	45.22
Normal applied force, F, N	10
Surface velocity in x direction, μ, mm/sec	100

TABLE II. - VALUE OF VARIABLES AS

ELLIPTICITY PARAMETER IS VARIED

[Dimensionless parameters: speed, U, 0.1683×10^{-11}; load, W, 0.3686×10^{-6}; material, G, 4522.]

Ellipticity parameter, k, dimensionless	Radius of curvature of solid B in y direction, r_{By}, mm	Maximum shear stress, σ_{max}, N/mm^2
1	$-\infty$	574.03
2	-17.143	417.32
3	-13.698	353.85
4	-12.634	316.96
6	-11.861	278.31
8	-11.570	246.90

TABLE III. - EFFECT OF ELLIPTICITY PARAMETER ON

MINIMUM FILM THICKNESS FORMULA

[$H_{min, L} = 6.955\times10^{-6}$.]

Ellipticity parameter, k	Minimum film thickness ratio, $H_{min}/H_{min, L}$	$1 - 1.6\ e^{-0.62\ k}$ (a)	Percent deviation of minimum film thickness, P_m, percent
1	0.1366	0.1393	1.968
2	.5320	.5370	.937
3	.7692	.7509	-2.376
4	.8805	.8660	-1.646
6	.9449	.9612	1.728
8	.9777	.9888	1.133

[a]See eq. (20).

TABLE IV. - EFFECT OF RADIUS OF CURVATURE RATIO ON

MINIMUM FILM THICKNESS FORMULA

[$H_{min, L} = 6.955\times10^{-4}$.]

Radius of curvature ratio, R_y/R_x	Minimum film thickness ratio, $H_{min}/H_{min, L}$	$1 - 1.6\ e^{-0.62(R_y/R_x)^{2/3}}$ (a)	Percent deviation of minimum film thickness, $\overline{P}_m$, percent
1.000	0.1366	0.1393	1.968
2.843	.5320	.5390	1.316
5.301	.7692	.7571	-1.573
8.303	.8805	.8741	-.727
15.85	.9449	.9680	2.445
25.29	.9777	.9923	1.493

[a]See eq. (22).

TABLE V. - EFFECT OF RADIUS OF CURVATURE RATIO ON PERCENT DIFFERENCE OF MINIMUM FILM THICKNESS FOR POINT CONTACT RELATIVE TO THAT OF LINE CONTACT

Radius of curvature ratio, R_y/R_x	$\left(1 - \frac{H_{min}}{H_{min, L}}\right)100$, percent	Ellipticity parameter, $k = (R_y/R_x)^{2/3}$
1	86.07	1
6.67	17.78	3.543
22.66	1.117	8.008

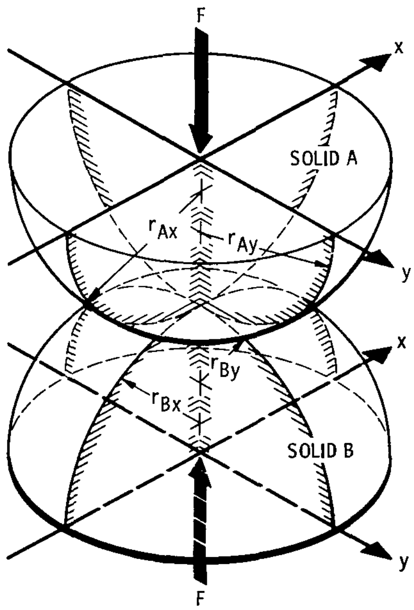


Figure 1. - Geometry of contacting elastic solids.

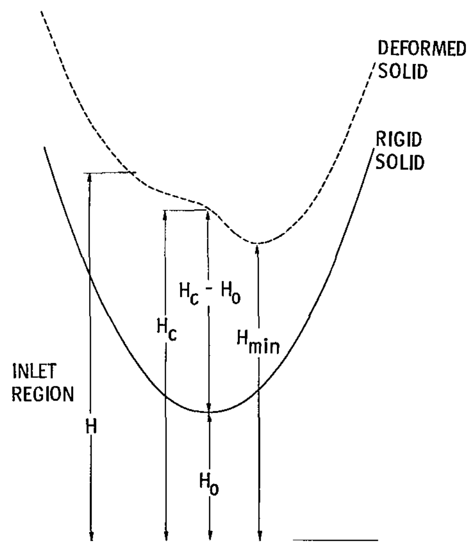
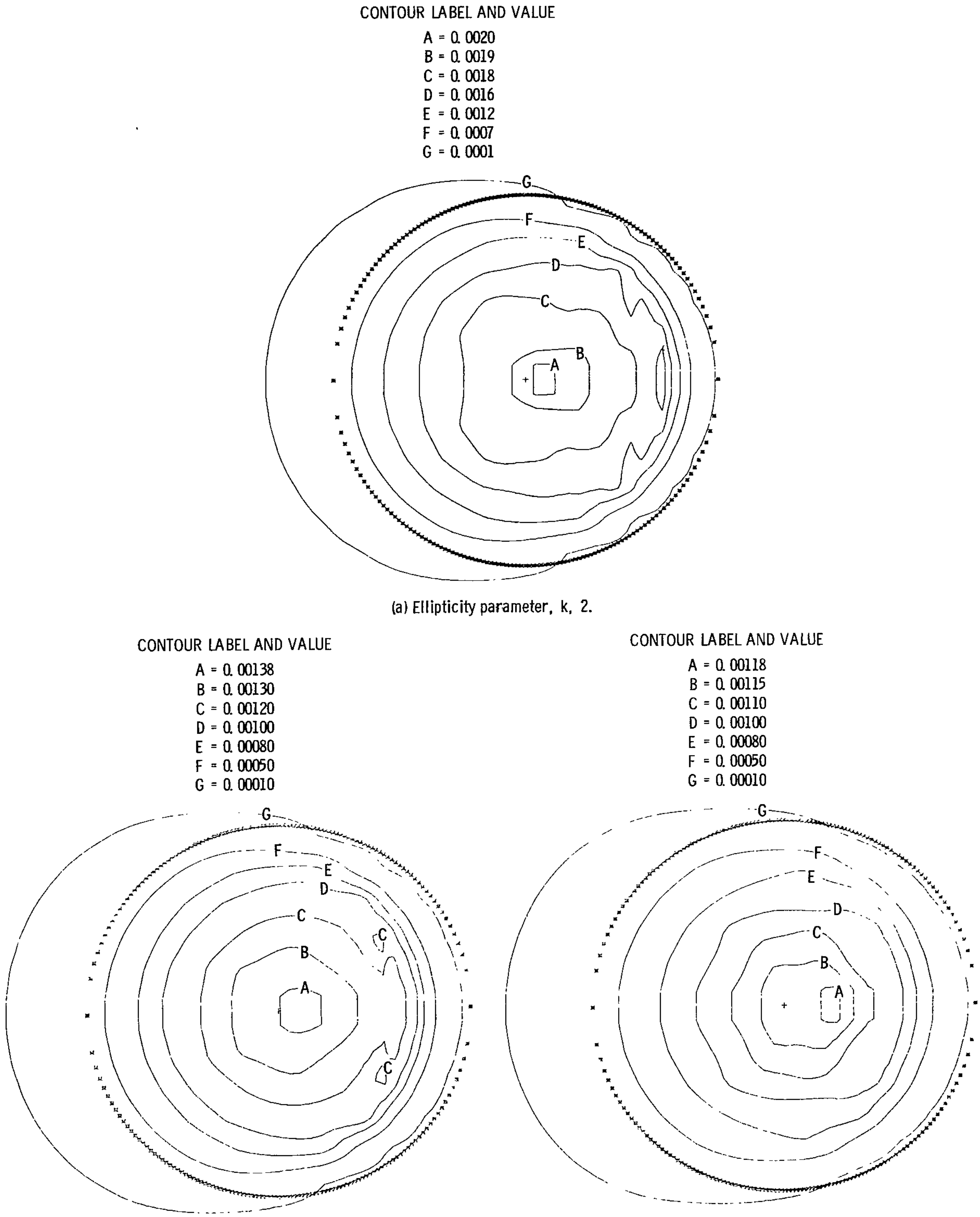


Figure 2. - Components used to define film thickness and central and minimum film thicknesses.

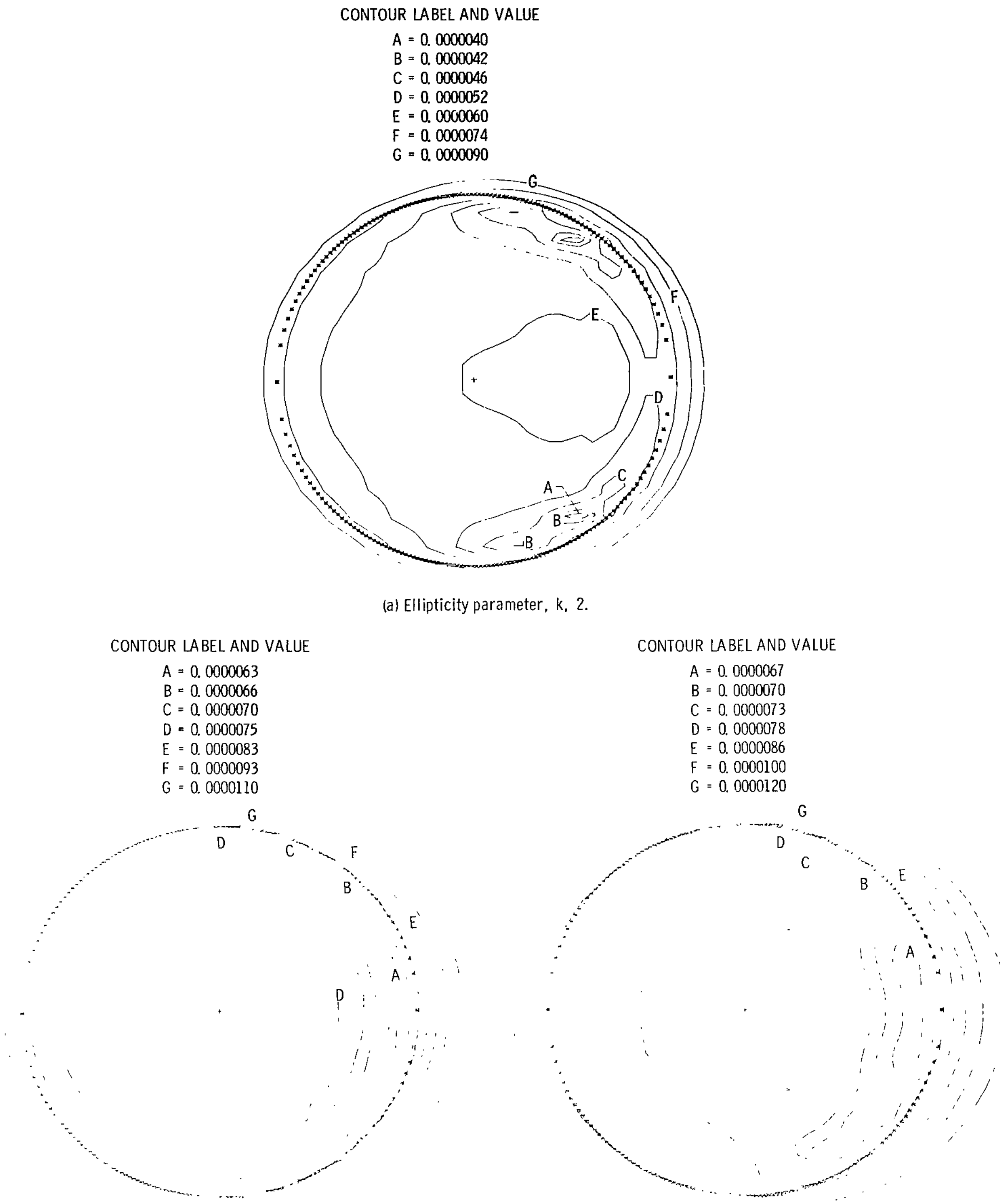


(a) Ellipticity parameter, k, 2.

(b) Ellipticity parameter, k, 4.

(c) Ellipticity parameter, k, 6.

Figure 3. - Contour plot of dimensionless pressure (P = p/E').



(a) Ellipticity parameter, k, 2.

(b) Ellipticity parameter, k, 4.

(c) Ellipticity parameter, k, 6.

Figure 4. - Contour plot of dimensionless film thickness ($H = h/R_x$).

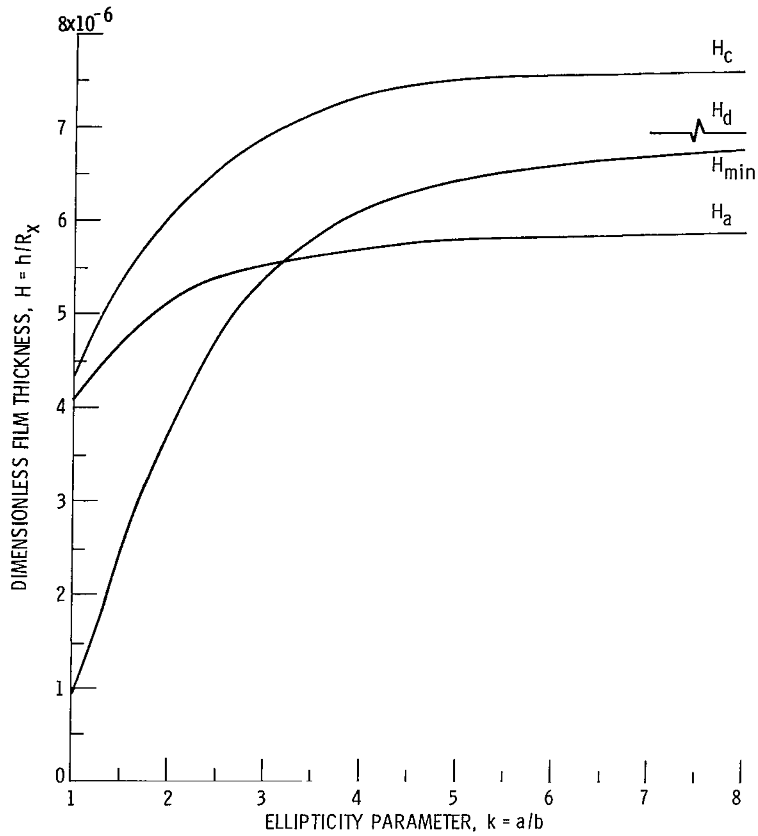


Figure 5. - Effect of ellipticity parameter on dimensionless minimum film thickness.

NASA-Langley, 1976 E-8467

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546