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GRAVIMETRIC INVESTIGATIONS ON THE NORTH AMERICAN DATUM

(1972 - 1973)

R. S. Mather

Geodynamics Branch, Goddard Space Flight Center*

December 1975

*On leave of absence from the University of New South Wales, Sydney, Australia.

GODDARD SPACE FLIGHT CENTER
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SUMMARY

All the available unclassified gravity data on the North American Datum (NAD) and in the surrounding oceans was assembled late in 1972 for the investigation of the gravity field in North America and its relation to North American Datum 1927 (NAD 27). The gravity data in Canada and the United States was compiled on a common datum compatible with the International Gravity Standardization Network 1971 (IGSN 71). The variation in the error of representation in the region is studied. Attempts are also made to study the correlation characteristics of gravity anomalies with elevation.

A free air geoid (FAG 73) was computed from a combination of surface gravity data and Goddard Earth Model (GEM) 4 and this was used as the basis for the computation of the non-Stokesian contributions to the height anomaly. These non-Stokesian contributions as computed from the data sets available at present are found to occur with amplitudes

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less than 3 m and with short wavelength in the Rocky Mountains region. The resulting effects on determinations of the geocentric orientation parameters (geocentric datum shift) for NAD 27 are not found to be of significance at the 30 cm level.

The geocentric orientation parameters obtained by this astro-gravimetric method are compared with those obtained by satellite techniques. The differences are found to be no greater than those between individual satellite solutions. The differences between the astro-gravimetric solution and satellite solutions GSFC 73 and GEM 6 are studied in detail with a view to obtaining a better understanding of these discrepancies.

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GRAVIMETRIC INVESTIGATIONS ON THE NORTH AMERICAN DATUM

(1972 - 1973)

1. INTRODUCTION

1.1 Preamble

Studies made on the Australian Geodetic Datum (AGD) indicated that it appeared feasible to compile a world geodetic system (WGS) by comparing, in effect, the elevations above ellipsoid as determined

a. by gravimetric techniques above a geocentric ellipsoid; and

b. by astro-geodetic methods above an ellipsoid defining the regional geodetic datum

for the evaluation of the geocentric orientation vector $\vec{O}$ (Mather 1970b).

The experience obtained with investigations in Australia showed that comparisons of the free air geoid determinations (N_f) obtained by the use of free air anomalies (Δg_f) in Stokes' integral

$$N_f = \frac{R}{4\pi\gamma} \iint f(\psi)\, \Delta g_f \, d\sigma \tag{1}$$

(symbols and notation not otherwise explained in the text are described in Section 1.4) with astrogeodetic determinations N_a on the regional geodetic datum, given by

$$N_a = -\int_{\text{geoid}}^{P} \zeta_\alpha \, d\ell \tag{2}$$

where

$$\zeta_\alpha = \xi_\kappa \cos A_\kappa \tag{3}$$

and

$$A_1 = \alpha; \; A_2 = \tfrac{1}{2}\pi - \alpha \tag{4}$$

can be used to determine the geocentric orientation parameters $\Delta\xi_i$ defining $\vec{O}$ in the local Laplacian triad at the origin of the regional geodetic system by (Mather 1970a, pp. 62 et seq.)

$$h_o \overrightarrow{\Delta\xi}_o = A^{-1} \, h(\vec{\xi}_g - \vec{\xi}_a) \tag{5}$$

where the elements $h_{io}\Delta\xi_{io}$ of the vector $h_o\overrightarrow{\Delta\xi}_o$ at the regional geodetic origin (ϕ_o, λ_o) are related to the equivalent parameters $h_j\,(\xi_{gj} - \xi_{aj})$ defining the vector $h(\vec{\xi}_g - \vec{\xi}_a)$ at the j-th station (ϕ,λ) on the regional datum by the elements in the matrix

$$A = \begin{vmatrix} \cos\phi_o \cos\phi + \sin\phi_o \sin\phi \cos\Delta\lambda & \sin\phi \sin\Delta\lambda & \sin\phi_o \cos\phi - \cos\phi_o \sin\phi \cos\Delta\lambda \\ -\sin\phi_o \sin\Delta\lambda & \cos\Delta\lambda & -\cos\phi_o \sin\Delta\lambda \\ \cos\phi_o \sin\phi - \sin\phi_o \cos\phi \cos\Delta\lambda & -\cos\phi \sin\Delta\lambda & \sin\phi_o \sin\phi + \cos\phi_o \cos\phi \cos\Delta\lambda \end{vmatrix} \tag{6}$$

$\Delta\lambda$ being given by

$$\Delta\lambda = \lambda_o - \lambda$$

and

$$h_1 = -(\rho + h); \; h_2 = -(\nu + h); \; h_3 = 1 \tag{7}$$

all other notation being described in Section 1.4. Strictly compatible comparisons of geocentric quantities defined from gravimetric considerations (denoted by the subscript $_g$) with equivalent quantities on the regional datum obtained from astro-geodesy (subscript $_a$) are the following:

$$\left.\begin{aligned} &\xi_{g1} = \xi_{f1} + \xi_{c1}; \qquad \xi_{a1} = \xi'_{a1} - 0.17^{(sec)}\, h^{(km)} \sin 2\phi + \Delta\xi_1 \\ &\xi_{g2} = \xi_{f2} + \xi_{c2}; \qquad \xi_{a2} = \xi'_{a2} + \Delta\xi_2 \\ &\xi_{g3}\ (= h) = h_n + \zeta;\ \xi_{a3}\ (= h)\ h_D + \sum_{geoid}^{P} dz - \sum_{geoid}^{P} \zeta_\alpha\, d\ell \end{aligned}\right\} \qquad (8)$$

where $\xi'_{a\alpha}$ are the two components of the astro-geodetic deflection of the vertical on the regional datum, h_D is the height of the levelling datum above the regional reference ellipsoid, ζ_α the component of the astro-geodetic deflection of the vertical in the line of levelling of length $d\ell$ along which the difference of orthometric elevation is dz. It should also be noted that exact equivalence is obtained in the third equation at 8 only when dz and $\zeta_\alpha d\ell$ refer to the same levelling route.

On ignoring these relative subtleties in the definition of $\Delta\xi_3$, a solution was made for the geocentric orientation parameters $\Delta\xi_i$ for the AGD from a direct comparison of the free air geoid with an astro-geodetic determination based on Equation 2 (Mather 1970a) as it was assessed that the indirect effect for a region of limited topographic variation like Australia was less than 60 cm (Fryer 1970).

It was also argued that this procedure could be the basis for the assembly of a world geodetic system (WGS) without (eventually) having recourse to a satellite model of the Earth's gravity field (Mather 1972a). Definite conclusions on the effective practical utilization of such a system on the basis of the Australian study alone were limited by

i. the extent of the AGD (restricted to only 1-1/2% of the global surface area); and

ii. the relatively flat terrain ove.' which the magnitude of the non-Stokesian term tended to be zero.

The study was therefore extended to the North American Datum (NAD) which covered about 2-1/2 times the surface area of the AGD, with a view to investigating

a. the significance of the non-Stokesian contributions to determinations of the geocentric orientation vector; and

b. the stability of appropriate harmonics as determined from "surface fitting," if the astro-geodetic data on NAD was sufficiently dense to warrant such studies.

1.2 Data Available on the North American Datum in 1972

The basic sources of information for the assembly of a gravity data bank on NAD were the following:

- The holdings of the United States National Geodetic Survey, based largely on unclassified material assembled by the Defense Mapping Agency Aerospace Center.

- Data made available by the Gravity Division, Canadian Department of Energy, Mines and Resources, Ottawa.

This information was supplemented by gravity data collected in the surrounding oceans on various surveys and obtained for the Geodynamics Branch, Goddard Space Flight Center, by Computer Sciences Corporation, Silver Spring, Maryland. Other gravity data for these oceanic areas were in the form of 1° x 1° area mean free air anomalies (ACIC 1971; Talwani et al., 1972). The basic gravity data was made compatible with International Gravity Standardization Network 1971 (IGSN 71). This required that a correction of +2.0 mGal be made to the Canadian data to bring it to a common datum with the U.S. gravity data (Tanner , 1972).

The latest low degree surface spherical harmonic model available in the second half of 1972 for the Earth's disturbing potential was that prepared at Goddard Space Flight Center—Goddard Earth Model (GEM) 4—from the combination of data obtained from orbital analysis with surface gravity information. This model was used along with surface gravity data from the sources mentioned above to provide a continuous representation of the global gravity anomaly field as described in Section 3.1.

The effect of the terrain undulations on solutions of the geodetic boundary value problem arise entirely in the non-Stokesian term. An elevation data bank is required for the meaningful evaluation of these effects. The elevation data available for this study in 1972 were parts of the following data sets:

- 5' x 5' mean elevations for the NAD region below parallel 65°N and made available by the Defense Mapping Agency Aerospace Center (Czarnecki, 1970); and

- 1° x 1° mean elevation estimates for the world prepared at the University of California (Kaula et al., 1966).

1.3 Computational Considerations

The computations proceeded on the following lines:

i. The available surface gravity and elevation data was processed to obtain estimates of the area mean free air anomalies and elevations on a tenth degree equi-angular grid covering North America.

ii. Prediction techniques were used to estimate values of the free air anomalies in unsurveyed regions covered by this tenth degree grid in locations where the surface gravity information warranted this approach. For further details, see Section 3.1.

iii. The resulting continuous data set was used to compute consistent 1/2°, 1° and 5° equi-angular area means for all regions of relevance as indicated in Table 1, the surface data being augmented by GEM 4 model values in distant areas and wherever warranted.

iv. This data set was used to compute the free air geoid (N_f) for North America (FAG 73) using the general techniques previously applied to the AGD as described in (Mather, 1970a).

v. FAG 73 was used along with the tenth degree equi-angular elevation mean values to compute the non-Stokesian contribution N_c to the height anomaly ζ given by (Ibid, p. 86)

$$\zeta = N_f + N_c \tag{9}$$

where

$$N_f = \frac{1}{\gamma}\left((W_o - U_o) - R\,M\{\Delta g_f\}\right) + \frac{R}{4\pi\gamma}\iint f(\psi)\,\Delta g_f\,d\sigma \tag{10}$$

and

$$N_c = \frac{R^2}{2\pi\gamma}\iint \frac{1}{r_o}\left(\frac{R}{r_o}\left(\sin\psi\,\frac{dh}{dr} + \frac{h_p - h}{R}\right)\frac{T}{r_o} - \gamma\,\xi_\kappa\,\tan\beta_\kappa\right) d\sigma\ (+\,o\{fN_c\}\text{ if } \tfrac{1}{2}(h_p - h)^2/r_o^2 < f) \tag{11}$$

where

$$\frac{dh}{dr} = \tan\beta_\kappa\,\cos A''_\kappa \tag{12}$$

$$A''_1 = A_\sigma;\ A''_2 = \tfrac{1}{2}\pi - A_\sigma \tag{13}$$

all other quantities being described in Section 1.4.

The first term on the right in Equation 10 is of relevance only if Δg_f is a global sample measured at the surface of the Earth. This term is ignored in the present study as this is not currently the case.

The condition under which the relation for N_c, given as Equation 11, is applicable can be said to be satisfied when the elevation data input is in the form of a tenth degree area mean data bank. As discussed in (Mather, 1972a), the second term on the right of Equation 11 can contribute up to 10% of the Stokesian effect in those mountainous continental regions where the sign of the deflections of the vertical is strongly correlated with that of the topography. The magnitude drops off to zero in predominantly oceanic areas. Its total magnitude under unfavourable conditions is not expected to contribute in excess of the order of the flattening to the magnitude of ζ for $\psi > 5°$ even though a tendency for negative values of $\xi_\kappa \tan \beta_\kappa$ appears likely over the 2-3% of the Earth's surface area where the great mountain ranges occur. This is due to the tendency for the wavelengths of such contributions to be rather small.

The first term on the right of Equation 11 is a function of r_o^{-3} and therefore damps out rapidly (i.e., $< o\{f\}$) for $\psi > 1\text{-}1/2°$. Its magnitude is dominant in mountainous country and the sign of the contribution is variable, depending on the relation of the surrounding terrain to the elevation of the point of computation. The apparent instability of this term as $\psi \to O$ is offset by the fact that $\sin \psi (dh/dr) \to (h_p - h)/R$ as $\psi \to O$. This enables Equation 11 to be evaluated from the limited 10 km mean elevation data bank available without seriously undermining the stability of computations.

It is obvious that T has to be approximated by the free air geoid N_f and the components of the deflections of the vertical (ξ_κ) are computed by using free air anomalies in the Vening Meinesz integrals

$$\xi_\kappa = \frac{1}{4\pi\gamma} \iint \frac{\partial}{\partial\psi} (f(\psi)) \Delta g_f \cos A_\kappa \, d\sigma \tag{14}$$

Further information on the techniques of computation is given in Section 3.

vi. Geocentric orientation parameters $\Delta\xi_i$ are computed for NAD 27 both in the Laplacian triad at the Meades Ranch Origin as well as in relation to a three dimensional Cartesian coordinate system X_i described in Section 1.4. Numerical values for $\Delta\xi_i$ are obtained by comparing either N_f or ζ as obtained above with astro-geodetic values of N_a as defined in Equation 2, using Equations 5 through 8. The results obtained are discussed in Section 4.

1.4 A Guide to Notation

A = parameter associated with azimuth (Equations 4 and 13)

a = equatorial radius of reference ellipsoid

$d\ell$ = distance between terminal bench marks in loops of levelling (Equations 2 and 8)

dz = increment in orthometric elevation between terminal benchmarks in loops of levelling

$d\sigma$ = element of surface area on unit sphere

$\epsilon\{\Delta g\}_{n^\circ}$ = error of representation for a n° x n° equi-angular element of surface area

f = flattening of reference ellipsoid

$f(\psi)$ = Stokes' function = $1 + \operatorname{cosec} \frac{1}{2}\psi - 5 \cos \psi - 6 \sin \frac{1}{2}\psi - 3 \cos \psi \log (\sin \frac{1}{2}\psi + \sin^2 \frac{1}{2}\psi)$ (15)

G = gravitational constant

g = observed gravity at the surface of the Earth

h = elevation above ellipsoid

h_n = normal height = $-\frac{\Delta W}{\gamma}\left(1 - (1 + f + m - 2f \sin^2 \phi) \frac{\Delta W}{a\gamma} + \left(\frac{\Delta W}{a\gamma}\right)^2 + o\{f^3\}\right)$ (16)

$M\{X\}$ = global mean value of X

m = $a\omega^2 / \gamma_e$

N_a = astro-geodetic "geoid"

N_c = non-Stokesian contribution to the height anomaly (Equation 11)

N_f = free air geoid (Equation 10)

$\vec{O}$ = geocentric orientation vector defining the displacement of the origin of the regional geodetic datum from the geocentre (Earth's centre of mass)

$o\{X\}$ = order of magnitude of the largest terms not considered is that of X

R = mean radius of the Earth

r = distance of the element of surface area dS from the point of computation P

r_o = $2R \sin \frac{1}{2}\psi$ (17)

T = disturbing potential

U_o = potential of reference system on the equipotential reference ellipsoid

W_o = potential of the geoid (unknown)

X_i = geocentric three dimensional Cartesian coordinate system with X_3 axis passing through CIO pole and the $X_1 X_3$ plane defining that of zero longitude

α = azimuth from the point of computation

β = ground slope; use with subscripts denote north and east components

γ = global mean value of normal gravity

γ_e = equatorial normal gravity

γ_o = value of normal gravity on equipotential reference ellipsoid

Δg = gravity anomaly = $g - \gamma_o - 2\,\frac{\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma} - 2f\sin^2\phi + o\{f^2\}\right)$ (18)

Δg_f = free air anomaly = $g - \gamma_o + 0.3086^{(mGal)}\,h^{(m)}$ (19)

ΔW = geopotential difference with respect to the geoid $= -\int_{geoid}^{P} g\,dz$ (20)

$\Delta\xi_i$ = components of the geocentric orientation vector $\vec{O}$ in the Laplacian triad at the regional geodetic origin (Equation 26)

ζ = height anomaly

ζ_α = component of the deflection of the vertical in azimuth α

λ = longitude positive east

ν = radius of curvature of reference ellipsoid in prime vertical normal section

ξ_κ = component of deflection of the vertical in the meridian (κ =1) and prime vertical (κ =2) directions, positive if outward vertical is north or east of normal

ρ = radius of curvature of reference ellipsoid in meridian normal section

ϕ = latitude, positive north

ψ = angular distance between the variable element of surface area $d\sigma$ and the point of computation

ω = angular velocity of rotation of the Earth

Significance of Subscripts

$_a$ = astro-geodetic values referred to the regional geodetic datum

$_c$ = non-Stokesian contribution for conversion of free air geoid value to physical surface/telluroid system

$_f$ = free air geoid

$_g$ = geocentric (gravimetrically determined) values

$_o$ = values at the regional geodetic origin

$_p$ = values at the point of computation

$_\sigma$ = values at the element of surface area $d\sigma$

Note:

i. Repeated subscripts in a product indicate summation over all possible values.

ii. Repeated subscripts on both sides of an inequality indicate as many equations as possible values of subscript.

iii. Use of roman letters for variable subscript indicate three values; the number of possible values when greek letters are used is two.

2. GRAVITY DATA PROCESSING

2.1 The Area

All the available gravity data in the area

$$0^\circ \leqslant \phi \leqslant 90^\circ; \; -180^\circ \leqslant \lambda \leqslant 0^\circ$$

was processed along the lines described in this section. In the first instance, the area was divided into 81 10° x 20° regions. The continental area of the North American Datum (NAD) was covered by the twenty nine regions shown in Figure 1. The gravity data in each of these regions was processed separately. Additional input into the basic processing routines was the equivalent equi-angular mean elevation data bank for North America described in Sections 1.2 and 1.3.

After all the available gravity data had been sorted and converted into free air anomalies consistent with IGSN 71 in the appropriate tenth degree equi-angular squares, area mean values of the free air anomaly (Δg_s) and gravity station elevation (h_s) were computed for each square from the available sample. The data was then adjusted to the mean elevation ($\bar{h}$) of the tenth degree square obtained from the continuous elevation data set referred to above, according to the relation

$$\Delta g_f = \Delta g_s - 0.112^{(mGal)}(h_s^{(m)} - \bar{h}^{(m)}) \tag{21}$$

It was believed that the adoption of such a procedure should eliminate any tendency for gravity data samples to be biased as a consequence of data being collected in the more accessible valleys. For further comment on the validity of this assumption, see Section 2.2.

The distribution of gravity data on the North American continent which was available for the present set of calculations is shown in Figure 2. The representation of the free air anomaly data set as contoured from one degree equi-angular area means is shown in Figure 3. The region is one of predominantly negative free air anomalies with positive values occurring primarily in the elevated regions. As mentioned earlier, all the Canadian data is 2 mGal larger than in previous representations which were not on IGSN 71, all data having been adjusted in accordance with Equation 21 in tenth degree equi-angular squares.

2.2 Characteristics of the Free Air Anomaly Data

North America is a region of varying topography with the Rocky Mountains dominating the western part as illustrated in Figure 4. This provided the opportunity for a closer look at the characteristics of free air anomalies in relation to topography. A histogram showing the distribution of one degree area mean elevations for equi-angular squares as a function of elevation is shown in Figure 5.

It is the writer's opinion that the pre-requisite for a successful evaluation of the height anomaly by quadratures methods from irregularly distributed data is an appreciation of the variability of the gravity anomaly field. A study of Figure 3 indicates that the free air anomaly field is questionably holonomic in two dimensions at the surface of the Earth. It also points to some correlation of the magnitude of free air anomalies with elevation though this tendency could well be masked by other factors. In the present investigation, the gravity data was sorted initially into tenth degree equi-angular squares. Area mean free air anomalies for larger half degree, one degree and five degree equi-angular squares were computed from the basic tenth degree values after prediction of estimated free air anomalies in unsurveyed areas using techniques described in (Mather, 1970a, pp. 65 et seq.). In this manner, all data sets were fully represented and were inter-compatible.

The variability of free air anomalies in these various squares can be quantified by the error of representation $E\{\Delta g\}_{m^\circ}$ for a m° x m° square.

$$E\{\Delta g\}_{m^\circ} = \pm\left(\sum_{i=1}^{M} \sum_{j=1}^{N_i} (\Delta g_{ij} - \overline{\Delta g_i})^2 \cos\phi_{ij} \Big/ \sum_{i=1}^{M} \sum_{j=1}^{N_i} \cos\phi_{ij}\right)^{1/2} \tag{22}$$

where N_i is the number of readings in the i-th m° x m° square and M is the total number of such squares.

It was initially hoped to evaluate definitive values of $E\{\Delta g\}_{m^\circ}$ for the square sizes shown in Table 1 for the North American region. In practice, however, it was difficult to achieve this goal for square sizes smaller than 0.5° due to the irregular sample distribution and the general paucity of data at intervals smaller than 10 km. In the case of squares where data was available with a greater density, irregular concentrations resulted in abnormally low values for $E\{\Delta g\}_{0.1^\circ}$ and $E\{\Delta g\}_{0.05^\circ}$. These results are therefore not presented.

$E\{\Delta g\}_{m^\circ}$ as obtained for North America from the data sample available for the present study is given in Table 2 for $m^\circ = 0.5$, 1 and 5. In view of the variation in the surface area of equi-angular squares with latitude, a plot of $E\{\Delta g\}_{0.5^\circ}$ against latitude is illustrated in Figure 6, together with a plot of the mean gravity station elevations and the mean elevations for all the half degree squares along the stated parallel of latitude. There appears to be no significant variation in the magnitude of $E\{\Delta g\}_{0.5^\circ}$ with latitude except in the south where the proportion of mountainous terrain is greater as indicated by the higher values for the mean elevation of the topography. This is principally due to the fact that the maximum possible distance between readings in a single surface element always remains the same (0.5° in latitude).

A more significant effect is the possibility of correlation of $E\{\Delta g\}_{m^\circ}$ with elevation as illustrated by the values for $E\{\Delta g\}_{1^\circ}$ in Table 2. A histogram showing the frequency of occurrence of the various numerical values of $E\{\Delta g\}_{1^\circ}$ in North America is illustrated in Figure 7. This figure illustrates that a wide range of values exists for $E\{\Delta g\}_{1^\circ}$ on NAD with a modal value of around ±10 mGal.

The information illustrated in Figure 6 is made more meaningful if the two factors mentioned above are taken into account. The much less significant latitude effect is easily accounted for by using Equation 22. The more dominant effect of elevation on $E\{\Delta g\}_{m°}$ is studied by classifying the correlations that exist between such values and elevation. Results for m = 1 are illustrated in Figure 8. The data analyzed was restricted to those one degree equi-angular squares containing more than 40 tenth degree equi-angular area means based on observations (i.e., 40% of the total possible representation).

It can be seen that no meaningful trend indicating a correlation of the magnitude of $E\{\Delta g\}_{1°}$ for free air anomalies with elevation can be obtained due to three reasons:

i. There are significantly more squares with lower elevations than higher ones (Fig. 5)

ii. $E\{\Delta g\}$ is correlated not so much with elevation as with ruggedness of topography. (Thus $E\{\Delta g\}$ for a high plateau should be no more susceptible to larger than average values than is the case for a low plain.)

iii. Values of $E\{\Delta g\}$ for coastal areas tend to be disturbingly large, as illustrated in Figure 9. This could be due to the greater variability of terrain/sea bed topography in continental margin areas close to the point of gravity measurement. It is hoped that this feature is not due to the fact that oceanic data is usually supplied by sources different from those providing land data and/or associated incompatibilities in the different data acquisition systems.

Despite the limited samples for the less commonly occurring elevated areas in the NAD region, it is nevertheless useful to study the values of $E\{\Delta g\}_{0.5°}$ as classified by elevation and shown in Table 3. The value for the elevation range $0 \leq h < 1000$ is similar to that obtained for Australia (Mather, 1967, p. 131). In assessing the overall pattern of variation of $E\{\Delta g\}_{0.5°}$ for North America as categorized in Table 3, it would not be unreasonable to expect the variability of the free air anomaly field to be relatively smaller in coastal plains and deep oceans, with $E\{\Delta g\}$ taking greater magnitudes in the vicinity of continental shelf and in mountainous regions. Figures given in Tables 2 and 3 suggest that values of $E\{\Delta g\}$ in the vicinity of shelf areas and in elevated regions could be larger than the average value for coastal plains by a factor of between two and three.

Such considerations have an important bearing on the procedure to be adopted in the prediction of values to represent unsurveyed areas when seeking a model for the fine structure of the free air anomaly field. It is common practice to eliminate height correlation effects by converting free air anomalies to Bouguer anomalies prior to prediction (e.g., Molodenskii et al., 1962, p. 179; Heiskanen & Moritz, 1967, p. 281; Mather, 1967, p. 131). An attempt was therefore made to study the height correlation characteristics of the available free air anomaly data on NAD by linear regression analysis of the data in the three larger square sizes (i.e., half degree, one degree and five degree).

In the case of five degree equi-angular squares, the results indicated that the gradient of a plot of free air anomalies against elevation varied considerably and seldom approached the expected value of $0.11\,\mathrm{mGal\,m^{-1}}$. While the tendency towards correlation of free air anomalies with elevation increased in the case of data confined to a one degree equi-angular square, again, the gradient did not approach the value quoted above with any degree of certainty. On the basis of the data studied, it must be concluded that if there were a positive correlation of free air anomalies with elevation, it is likely to be masked by other factors when the distance involved exceeds 50 km (the minimum distance considered in the present investigation). It can therefore be concluded that any tendency for free air anomalies to be correlated with elevation on a regular basis capable of analytical representation, has not been established from the available data sample on the NAD in the case of distances in excess of 50 km.

Due to the time constraints on this investigation, the statistical analysis of the free air anomaly data was done after the assembly of the data set described in Section 3.1. Consequently, the prediction of values to represent unsurveyed areas was based on the conventional assumptions referred to in the previous paragraph plus one, as controlled using the technique described in (Mather, 1970a). As a result, large positive anomalies tend to occur when predicting in mountainous regions. It is unlikely that this affects the results outside the local area in excess of ±30 cm in view of the short wavelength of such biases (Mather, 1974, p. 103).

The writer plans to implement prediction procedures in future investigations on the basis of findings in this study. Irrespective of the exact prediction procedure adopted, they can be broadly classified as follows:

i. Choose the basic surface area size in which the gravity anomaly data is to be processed (as a percentage of the total surface area of the Earth).

ii. Establish a variability characteristic (e.g., error of representation) for this block size in the case of both gravity anomalies and elevations

   a. for each individual area; and

   b. for the entire sample of such areas.

This characterization of the variational behaviour of the free air anomaly field in each basic area can be embodied in either a statistical measure such as covariance (about zero mean for each area at i.) or the number n_{max} of the harmonic coefficients to be used in the analysis of the available data. In the latter case, the value of n_{max} would also be influenced by the number of readings available in each basic area. In general, the larger the error of representation, the greater the value of n_{max} for a given data distribution. For further discussion on the relation between n_{max} and the distribution of data in a region, see (Mather, 1967, p. 133).

3. COMPUTATIONS

3.1 The Free Air Geoid (FAG 73)

The gravity data referred to in Section 1.2 was used to define a continuous free air anomaly field on the following basis:

In regions where point data was available within a 1° x 1° square

Prediction was preceded by the conversion of all free air anomaly means for tenth degree equi-angular areas to Bouguer anomaly equivalents using the tenth degree elevation data bank. Two dimensional harmonic series were fitted to the residuals from the GEM 4 model free air anomalies using the procedure described in (Mather, 1970a, pp. 65 et seq.). This type of approach was used in the twenty nine regions shown in Figure 1 despite the reservations expressed in Section 2.2 about assuming predominantly height correlated variations of free air anomalies over distances in excess of 50 km. The final set of free air anomalies was generated after the completion of prediction by reversing the Bouguer correction and restoring the GEM 4 model.

In regions where point data was not available within a 1° x 1° square

These regions were largely confined to the surrounding oceanic areas where the required field representation in terms of Table 1 were either in the form of half, one or five degree equi-angular area mean free air anomalies. The following hierarchical structure was adopted for this type of region:

1. Lamont 1° x 1° area means (Talwani et al., 1972).

2. DMAAC 1° x 1° area means (ACIC, 1971).

3. GEM 4 model anomalies (Lerch et al., 1972).

A consistent and continuous representation of the free air anomaly gravity field was constructed on this basis for the area defined in Section 2.1. Thus, the tenth degree equi-angular data set extended only up to 3° from the coastline, while the half degree data set extended up to about 8° from the coast. All data was maintained on Geodetic Reference System 1967 - GRS 67 (IAG, 1971). The fact that this system differs from current estimates of best fitting Earth ellipsoid parameters (e.g., Lerch et al., 1974) was not considered to be of significance for the following reasons:

a. Zero degree terms were not considered in the gravimetric solution for reasons given in Section 1.3(v). Thus the exact value of a and GM used have only marginal effects on the solution smaller than the noise produced by the errors in the input data.

b. The only reference model parameter which has an appreciable non-zero degree effect is the flattening f. As the best fitting value at the present time is held to be around 1/298.255 (e.g., Ibid) instead of the 1/298.247 implicit in GRS 67, the resulting effect is of order 10^{-4} which can be ignored in solutions seeking a precision less than ±5 cm.

While the free air anomalies based on point values were effectively referenced to IGSN 71, other data in the form of one degree area means were implicitly corrected to this system by applying the recommended correction to the Potsdam datum (IAG 1971). The resulting free air geoid for North America is called FAG 73 in the text and was obtained by the use of Equation 1 and the area sub-divisions specified in Table 1. A contour representation of FAG 73 based on a sample plot of values on a two degree grid is illustrated in Figure 10.

3.2 Comments on the Precision of FAG 73

The computational procedures used in the preparation of FAG 73 were designed in order that the precision attainable from an error free data set was ±30 cm. The result illustrated in Figure 10 cannot be expected to have achieved this precision for the following reasons:

- Any errors in GEM 4 which was effectively used to represent the distant zones, would be reflected in the results. This type of error is tentatively estimated as slowly varying at the 1-2 m level.

- The oceanic representation, especially in the Pacific, is not based on any systematic examination of the data by the investigator, the numbers given in the data sets described in the second paragraph of Section 1.2 being accepted at face value. Nevertheless, both these data sets are in reasonable agreement with the data set prepared by the writer in areas of overlap on allowing for the different methods used in the preparation of the latter.

- Prediction errors in scantily surveyed areas (see Fig. 2) could also degrade the quality of the result.

Errors in excess of the norm could therefore be expected for determinations in Alaska, the west coast of North America and Mexico. This expectation appears to be substantiated when comparing FAG 73 with astro-geodetic determinations after allowance for datum translation. Such a comparison is illustrated in Figure 11. Details of how such comparisons are effected could be ascertained by reference to Section 4.

In assessing the nature of the discrepancies between gravimetric and astro-geodetic solutions as illustrated in Figure 11, it should be borne in mind that FAG 73 and the astro-geodetic undulations determined by the application of Equation 2 are not strictly comparable for the reasons set out in Equation 8. The extent of the discrepancy is likely to be exacerbated in elevated regions and this is borne out by the largest non-coastal discrepancy between the solutions occurring in the most elevated region on the datum in Colorado. About 1-1/2 m of the discrepancy is accounted for by the non-Stokesian contribution to the height anomaly (see Fig. 12). Another factor contributing to this discrepancy is the approximation of orthometric elevation in the area by the sum of the orthometric height differences as observed along the line of levelling. The resulting effect in an area with elevations in excess of 2000 m could be as large as 2 m.

The largest discrepancies occur along the north west coast of North America. The quality of neither the gravity data (Fig. 2) nor the astro-geodetic data (Fig. 14) in this area warrants further speculation on the underlying causes of this discrepancy.

FAG 73 shows a less accentuated hump over the Rocky Mountains in Colorado that the Vincent-Marsh determination (Vincent & Marsh, 1973) or—as is more relevant—the GEM 4 geoid (Lerch et al., 1972). This is attributed to the effect of the relatively denser representation of the predominantly negative free air anomaly field afforded by the data set used. The other prominent feature is the geoidal low over Hudson Bay which is usually shown as being lower than 50 m. The low illustrated is at least two meters higher due to the +2 mGal adjustment made to all the Canadian data as described in Section 1.2.

For statistics on tests between astro-geodetic solutions and FAG 73, see Table 4 and Section 4.

3.3 Non-Stokesian Contribution to the Height Anomaly

The non-Stokesian contribution to the height anomaly as estimated for North America is illustrated in Figure 12. As explained in Section 1.3(v), the magnitude of N_c was computed using as input

- FAG 73 computed using Equations 1 and 14; and
- the tenth degree equi-angular mean elevation data bank

in Equation 11. As expected, the largest contributions occur in the Rocky Mountains area, never exceeding ±3 m in magnitude. The unexpected feature was the occurrence of both positive and negative magnitudes, dominant magnitudes of the latter occurring on the oceanic flanks of the Rocky Mountains. As these non-Stokesian contributions have limited amplitude and occur with relatively short wavelength, their effect on determinations of the geocentric orientation parameters are found to be negligible. This is discussed at length in Section 4.

4. COMPUTATION OF THE GEOCENTRIC ORIENTATION VECTOR FOR THE NORTH AMERICAN DATUM 1927

4.1 Transformation of Astro-Geodetic Data From Clarke 1866 to Reference Ellipsoid 1967

The use of Equations 5 to 8 for the definition of the geocentric orientation parameters $\Delta\xi_i$ in the local Laplacian triad at the Meades Ranch origin of NAD 27 and hence the geocentric orientation vector $\vec{O}$ is possible only if the astro-geodetic data were referenced to Reference Ellipsoid 1967 (a_G = 6 378 160 m; f^{-1} = 298.247) placed concentric with the Clarke 1866 ellipsoid (a = 6 378 206.4 m; f^{-1} = 294.979) in Earth space. This is achieved by allowing for the corrections (e.g., Mather, 1968, p. 292)

$$\delta\phi = \sin 2\phi \left(df + f\,\frac{da}{a} - df\,\frac{h}{a} + f\,df\cos^2\phi + o\{f^4\} \right) \tag{23}$$

to ϕ and

$$\delta N = da + a(df\sin^2\phi - \delta\phi\tan\phi) + o\{f^3 a\} \tag{24}$$

to the height h above the ellipsoid, where

$$da = a_G - a \text{ and } df = f_G - f \tag{25}$$

The geometry of the system is illustrated in Figure 13 which shows the Clarke 1866 and the 1967 Reference Ellipsoids defining NAD 27 in relation to GRS 67.

4.2 Geocentric Orientation Parameters for NAD 27 from Astro-gravimetric Comparisons

4.2.1 Astro-Geodetic Data

Astro-geodetic data on the NAD is available in one of two forms:

i. Astro-geodetic deflections of the vertical distributed as shown in Figure 14.

ii. Astro-geodetic "geoids" hereafter referred to as astro-geodetic undulation determinations for want of a more accurate description and computed using equations of the type at 2 and 4. Solutions of this type were produced by

   a. the U.S. National Geodetic Survey for selected loops of astro-geodetic stations in the United States (Rice, 1972);

   b. the Canadian Geodetic Survey for south-eastern Canada (Ney, 1952); and

   c. the U.S. Army Map Service as a contour map for the whole of North America (Fischer et al., 1967).

4.2.2 Comparison of Gravimetric and Astro-Geodetic Determinations

Geocentric orientation parameters obtained from the comparison of gravimetric and astro-geodetic determinations of the separation vector can be described as being largely achieved by the process of surface fitting and have already been used to provide a geocentric orientation for the Australian Geodetic Datum (AGD). These comparisons also provide a means of assessing the precision of both the gravimetric and astro-geodetic determinations (Mather, 1970a; Mather, 1972b).

Geocentric orientation parameters for the major geodetic datums are freely available in the form of components on a geocentric Cartesian coordinate system. They are obtained by comparing the coordinates of satellite tracking stations in geocentric solutions with equivalent values assigned to these same stations as part of regional geodetic surveys on the local datum. Results for NAD 27 are given by several investigators (e.g., Anderle, 1974; Gaposchkin, 1973; Lerch et al., 1974; Marsh et al., 1973; Merry & Vanicek, 1974; Mueller, 1974; Schmid, 1974). The technique used in this present study is different from satellite based determinations in several important respects:

a. The geocentric orientation parameters are defined by fitting estimates of the <u>shape</u> of the <u>same</u> surface as determined by different methods—one in relation to the geocentre and the other in relation to NAD 27—on allowing for the difference in shapes of the two <u>reference</u> surfaces (see Section 4.1).

b. The determination is based on comparisons made over the entire extent of the datum rather than at a few selected tracking stations whose geometrical distribution across the datum could well be irregular.

c. The results obtained from satellite solutions and summarized in Table 5, are obtained by comparing coordinates of the same points on the two different datums on the basis of a seven parameter fit (3 translational - ΔX_i, 3 rotational - ω_i and one for scale - σ). The astro-gravimetric method, on the other hand, defines directly a three parameter translational shift as explained in the Appendix.

d. Zero degree effects, if any, in the height anomaly are not included in the solution.

e. The size of the ellipsoid and GM are not critical factors in the solution, provided the differential geometry is correctly allowed for.

Several solutions were made for the geocentric orientation parameters $\Delta\xi_i$ for NAD 27 by comparing gravimetrically determined height anomalies or co-geoid heights for FAG 73 on a geocentric ellipsoid with astro-geodetic undulations on NAD 27. Unfortunately, for the very reason that a re-definition is currently being undertaken for the NAD (see Canadian Surveyor 28(5), 1974, for details) the astro-geodetic data available in 1973 could be expected to be subject to limitations when used in surface shape comparison methods for the definition of the geocentric orientation vector. It has been noted that errors as large as 15 ppm could exist in the Canadian network (Lilly, 1960). The existence of such uncertainties would almost certainly limit the precision which could be achieved by the use of astro-gravimetric methods at the present time.

Nevertheless, it was considered worthwhile to evaluate the geocentric orientation vector for NAD 27 for the following reasons:

- The precision of the gravimetric determinations could be estimated by comparison with astro-geodetic determinations after translation of datums.

- The effect of the non-Stokesian contribution to the height anomaly on the determination of the geocentric orientation vector could be evaluated.

- It is of interest to define the geocentric orientation vector using a technique not sensitive to scale apart from zero degree contributions to the height anomaly. It is not expected that the latter would exceed 3-6 m (Mather, 1970b, p. 98). Any such effect would, however, only influence the determination in the radial direction at Meades Ranch.

- Differences between the scale of satellite solutions and that of the regional geodetic network not dependent on the choice of ellipsoid will, however, be reflected in the results.

4.2.3 Solutions Using the Free Air Geoid (FAG 73)

The different types of comparisons between astro-geodetic and gravimetric solutions made for geocentric orientation parameters are listed in Table 4. Comparisons were effected between gravimetric determinations and two types of astro-geodetic undulation representations:

1. the astro-geodetic undulation map produced by the U.S. Army Map Service (Fisher et al., 1967). The version used was digitized by interpolation on a one degree equi-angular grid and called AMS 67.

2. In this representation, the astro-geodetic data was sampled on a one degree equi-angular grid. These grid corners were represented wherever possible by the nearest astro-geodetic station. Approximately 800 of the available astro-geodetic stations on the NAD were used in this set on this basis. Undulation values at 364 of these stations were obtained either from the astro-geodetic levelling results of the U. S. National Geodetic Survey (NGS 72) prepared by Rice and co-workers (Rice, 1972) or the results for south-eastern Canada reported by Ney (1952). (Note: There are over 3000 deflection stations available in North America but only about 800 were selected to provide an "equal area" coverage of the continent.)

Solutions based solely on undulation comparisons are classified as Class A in Table 4. The geocentric orientation parameters $\Delta\xi$, $\Delta\eta$ and ΔN in the Laplacian triad at Meades Ranch are related to Equation 5 by

$$\Delta\xi_1 = \Delta\xi = -\delta\phi;\ \Delta\xi_2 = \Delta\eta = -\delta\lambda\cos\phi_0;\ \Delta\xi_3 = \Delta N \tag{26}$$

where $\delta\phi$, $\delta\lambda$ and ΔN are the changes in latitude, longitude and normal displacement at the geodetic origin of the NAD required to give equivalent geodetic coordinates on a geocentric ellipsoid. It should, however, be recognized that comparisons between the gravimetrically determined undulations on the one hand and equivalent astro-geodetic quantities on the other, should be preceded by a transformation of the latter from values referred to the Clarke 1966 ellipsoid to corresponding values on the concentric ellipsoid with dimensions equivalent to those used in Geodetic Reference System (GRS) 1967. Details are given in Section 4.1. The values

$$\Delta\xi_0 = -7.5 \text{ arc sec and } \Delta N_0 = 48\,\text{m} \tag{27}$$

should be subtracted from values in columns 5 and 11 in Table 4 to obtain equivalent values for shifting NAD 27.

Class A solutions have been effected on the following basis in the case of comparisons between undulations only of

- FAG 73 on the one hand and AMS 67 (representation on a 1° x 1° grid) on the other (Table 4, Rows 1 & 7);
- FAG 73 and astro-geodetic undulations from solution NGS 72 for USA as supplemented where relevant by Ney's solutions for south-eastern Canada at selected astro-geodetic stations as described above (Table 4, Rows 3 & 6).

The nature of the data set AMS 67 is different from that of NGS 72 and Ney. The values comprising the former are obtained by interpolation from a contour map while the latter solutions are based on values at points in loops of astro-geodetic levelling which are therefore more reliable. This is borne out by comparing the values in column 12, Rows 6 & 7 of Table 4, which are based on comparisons over substantially the same area.

The geocentric orientation parameters obtained by this technique of astro-gravimetric comparisons are, as expected, a function of the area over which the comparisons are made. The parameter least affected is ΔN which has an overall variation of less than 1 m for all Class A solutions, irrespective of the extent of the area over which comparisons are made.

On application of the geocentric orientation parameters obtained from the use of Equations 5 through 8, to transform the astro-geodetic determination to GRS 67, it is possible to compare values of

- the surface undulation; and
- the deflections of the vertical

as obtained by these two <u>independent</u> methods to provide a measure of the success with which the orientation has been achieved.

The goodness of fit can be characterized by the root mean square (rms) residuals $\sigma_{\Delta N}$ in the comparison of the undulations (Table 4, Column 13), $\sigma_{\Delta\xi}$ (Table 4, Column 6) and $\sigma_{\Delta\eta}$ (Table 4, Column 9) in the deflections of the vertical.

It would appear at first glance that over similar areas (e.g., Table 4, Rows 6 & 7), the NGS 72 determination is in significantly better agreement with FAG 73 than AMS 67, the area covered in these two cases being substantially equivalent to the United States. However, the latter solution was based on comparisons at a greater number of points (989 in contrast to 314). In addition, the number of astro-geodetic stations used in preparing both solutions is essentially the same. It is judged that the increased value of $\sigma_{\Delta N}$ for AMS 67 comparisons over that for NGS 72 solutions is due at least in part to the use of the former in comparisons at locations away from astro-geodetic stations and thereby introducing interpolation error. A plot of $\sigma_{\Delta N}$ for the solution given in Row 1 of Table 4 is illustrated in Figure 11.

Further to the discussion in Section 3.2, the quality of agreement in non-coastal areas where adequate astro-geodetic data is available, is on par with the results obtained for Australia (Mather, 1972b, p. 25). The reasons for the discrepancies are self-evident if Figures 1 and 14 are overlaid on Figure 11. They can be summarized as follows:

- Astro-geodetic data north of parallel 60°N is too widely spaced for reliable determinations.
- Gravity data off the west coast of North America is inadequate for reliable gravimetric computations.

As mentioned in Section 3.2, the large discrepancy in the Colorado region is due in part to the non-Stokesian effect (Fig. 12), the incompatibility of the quantities being compared and the inadequate density of astro-geodetic stations in the area.

In view of the doubts that existed about the possibility of computing a reliable astro-geodetic solution in northern Canada and Alaska because of the paucity of data, it was decided to investigate the orientation of NAD 27 using, as input:

- astro-geodetic deflections only, north of parallel 48°N; together with
- both undulations and deflections of the vertical south of parallel 48°N.

Tests on this type of data distribution were carried out on the Australian Geodetic Datum where uniform data coverage was available (Mather, 1972c). These tests confirmed that stable geocentric orientation parameters could be determined using this type of data distribution. Solutions of this type are classified as Class C in Table 4.

4.2.4 Solutions From Astro-Geodetic Deflections Only

A third type of solution classified as B in Table 4 was also made in response to the irregular distribution of astro-geodetic data as a function of latitude (see Fig. 14) and the likelihood of systematic errors in any astro-geodetic undulation representation deduced from such information. These were effected by comparing deflections of the vertical alone against equivalent gravimetric values deduced from Equation 14. The resulting solutions are shown as Class B in Table 4 (Rows 5 and 8).

The following conclusions can be drawn from these results:

- Comparisons over an area approximating to the United States (Row 8) have smaller rms residuals than those made over the entire NAD area.
- The magnitude of both $\sigma_{\Delta\xi}$ (Column 6) and $\sigma_{\Delta\eta}$ (Column 9) are about 2-1/2 arc sec. This is due to three factors:
  a. An inner zone of radius 10 km around the point of computation has been excluded when evaluating FAG 73.
  b. Non-Stokesian contributions have been excluded.
  c. Errors in excess of one arc sec can be expected in the geodetic coordinates of points in the data set used north of parallel 50°N (e.g., Lilly, 1960).

In fact, the values of $\sigma_{\Delta\xi}$ and $\sigma_{\Delta\eta}$ for the United States (Table 4, Row 8) are not significantly different from values obtained during similar computations in Australia and where the subsequent inclusion of inner zone surveys halved the values of $\sigma_{\Delta\xi}$ and $\sigma_{\Delta\eta}$ (Mather, 1970a, p. 72).

Another observations of significance in the case of these Class B solutions based on the comparisons of deflections alone, concerned the changes of determining ΔN from solutions of this type. It has been observed in the case of limited extents like Australia (1-1/2% of the Earth's total surface area) that ΔN was effectively indeterminate (Ibid, p. 72, Type 2 Solutions). This was also observed in the case where determinations were based on comparisons confined to a similar area in North America (Table 4, Row 8). When the area was extended to the entire NAD (Table 4, Row 5), a relatively realistic estimate of ΔN was obtained. This particular solution is therefore totally independent of any astro-geodetic undulation solutions. It is interesting to look at the Cartesian components of the geocentric orientation vector (Table 4, Columns 14 through 16) which are in good agreement with the average satellite solution (Table 5, Row 9) for values of ΔX_2 and ΔX_3 but not ΔX_1, thus apparently pointing to some degree of instability in the determination at right angles to both the rotation axis and the meridian at Meades Ranch. But also see the discussion associated with Table 6 in Section 4.3.2. An important corollary is that the larger the surface area being compared, the more reliable the determination of ΔN from the comparisons of deflections alone.

4.2.5 Non-Stokesian Effects

The effect of non-Stokesian terms is evaluated through Equation 11. These effects were found to have a negligible influence on determinations of the geocentric orientation parameters (see Table 4, Rows 1 & 2). The reason for this becomes apparent on studying Figure 12. While significant magnitudes of the non-Stokesian contribution to the height anomaly occur in the western part of the NAD, ranging from 1-3 m in amplitude, these are largely of short wavelength, the sign of the contribution being controlled by the following factors:

- the sign of the free air geoid height N_f (Fig. 10) which is largely negative in the region; and

- the contribution of [sin ψ (dh/dr) + h_p - h] varying in sign as the point of computation moves across the Rocky Mountains from west to east.

Consequently there is no systematic build-up in the magnitude and wavelength of the non-Stokesian contribution in the western region of NAD to bring about changes of significance in the geocentric orientation parameters for NAD. The corrections obtained are of the order of 30 milliarc sec in $\Delta\xi_\kappa$ and 3 cm in ΔN on the basis of the present calculations. It should be emphasized that Equation 11 is an approximation. A more complete version is given in (Mather, 1974, p. 100), but for reasons given in Section 1.3, it was not warranted to persevere with a more complex version of the expression for N_c in view of the excessive computer requirements. However, there is a possibility that the values of the non-Stokesian effect illustrated in Figure 12 may well underestimate N_c especially in areas of rugged topography where the elevation data available in computer compatible form at the present time is not sensitive enough to model the fine structure of the topography. More research is needed in this area on a regional basis as point computations by themselves give no real information on the significance of the non-Stokesian effects in practical high precision geodesy.

4.2.6 Conclusions

The following conclusions can therefore be drawn from results summarized in Table 4.

i. The free air geoid alone is adequate for determining geocentric orientation parameters for the NAD region to ±0.05 arc sec or its equivalent in each coordinate and non-Stokesian terms need not be considered.

ii. The data available at present gives comparisons between FAG 73 and astro-geodetic undulations at astro-geodetic stations below parallel 48°N which an rms residual $\sigma_{\Delta N}$ (Table 4, Column 12) of ±1.8 m (Table 4, Row 6) compared with ±1.6 m for the 1971 determination for Australia (Mather, 1972b, p. 23). This value drops to ±1.0 m (Table 4, Row 9) if comparisons were restricted to astro-geodetic stations below parallel 40°N. The slightly larger figure than obtained in the Australian study is attributed to the increased ruggedness of the topography on NAD and the resulting (correlated) incompatibility between the data types compared.

iii. The difference between the geocentric orientation parameters obtained for the entire datum (Table 4, Row 1) and the area south of parallel 48°N (Table 4, Row 7) illustrates the necessity for basing determinations of the geocentric orientation parameters on comparisons extending over the entire datum. The significance of the results obtained in this study are, however, not entirely clear-cut in view of the uncertain precision of the geodetic data in northern latitudes when taken as representative of NAD as a whole.

iv. Geocentric orientation parameters determined from deflection comparisons alone (Table 4, Rows 5 & 8) were found to give reasonable values of ΔN when based on comparisons extending over the entire datum, though the ΔN value was for all practical purposes, indeterminate when the comparisons were restricted to the United States alone. Nevertheless, there is a relative indeterminancy in such solutions at right angles to the rotation axis and the plane of the meridian at Meades Ranch.

4.3 Comparison of Astro-gravimetric and Satellite Determinations of the Geocentric Orientation Vector

4.3.1 Inter-Relation of Geocentric Orientation Parameters

Several satellite solutions are available for defining the displacement of the origin of NAD from the geocentre. Some of these are summarized in Table 5, along with the average astro-gravimetric determination from Table 4 for ease of comparison. The satellite determinations are obtained by comparing the coordinates of tracking stations as determined in global geodetic networks with equivalent values on the regional geodetic network using a seven parameter fit to allow for translation of origin (three parameters ΔX_i), rotation of axes (three parameters ω_i) and scale (σ_ℓ). Dynamic satellite solutions are generally linked to a Greenwich—Conventional International Origin (CIO) system of reference by incorporating camera (optical) data for provision of orientation. The scale of the system in modern satellite solutions is usually based in part on laser tracking data (e.g., Marsh et al., 1973; Lerch et al., 1974) as well as on the value adopted for GM in the case of dynamic solutions. The scale of purely optical solutions is more dependent on long baselines for scale (e.g., Schmid, 1974). While the parameter σ_ℓ is of importance in attempting to relate regional geodetic datums to the geocentre, the significance of the rotational parameters ω_i need careful consideration when both the regional geodetic datum and the reference system used for the satellite solution are related to Greenwich/CIO using essentially common considerations. For further discussion, see Section 4.3.2 and the Appendix

The quantities Δx_i are simply related to the geocentric orientation parameters $\Delta\xi_i$ defined in Equation 26 by the formulae

$$\left.\begin{aligned} \Delta X_1 &= \rho\,\Delta\xi \sin\phi_o \cos\lambda_o + \nu\,\Delta\eta \sin\lambda_o - \Delta N \cos\phi_o \cos\lambda_o \\ \Delta X_2 &= \rho\,\Delta\xi \sin\phi_o \sin\lambda_o - \nu\,\Delta\eta \cos\lambda_o - \Delta N \cos\phi_o \sin\lambda_o \\ \Delta X_3 &= -\rho\,\Delta\xi \cos\phi_o \qquad\qquad\qquad\qquad - \Delta N \sin\phi_o \end{aligned}\right\} \quad (28)$$

on taking into account that changes $\Delta\xi$ and $\Delta\eta$ are equivalent to changes $\delta\phi$ and $\delta\lambda$ in the latitude and longitude at Meades Ranch in terms of Equation 26. Further, a change ΔN in the ellipsoidal height is obtained by a radial outward displacement of $(-\Delta N)$ of the ellipsoid centre along the normal to the regional geodetic origin. Similarly,

$$\left.\begin{aligned} \rho\,\Delta\xi &= \Delta X_1 \sin\phi_o \cos\lambda_o + \Delta X_2 \sin\phi_o \sin\lambda_o - \Delta X_3 \cos\phi_o \\ \nu\,\Delta\eta &= \Delta X_1 \sin\lambda_o \qquad\qquad - \Delta X_2 \cos\lambda_o \\ \Delta N &= -[\Delta X_1 \cos\phi_o \cos\lambda_o + \Delta X_2 \cos\phi_o \sin\lambda_o + \Delta X_3 \sin\phi_o] \end{aligned}\right\} \quad (29)$$

4.3.2 The Effect of Scale and Axial Rotations

All values of $\Delta\xi_i$ obtained by the use of Equations 26 and 29 from the input values of ΔX_i for global satellite solutions are given in Table 5. The input values into Table 5 in the case of astro-gravimetric solutions are of course, the values $\Delta\xi_i$. Equivalent values on Clarke 1866 ellipsoid are obtained by the application of Equation 27. In assessing the equivalence of values of ΔX_i determined from satellite solutions with the values of $\Delta\xi_i$ determined astro-gravimetrically and related through Equations 28 and 29, the following points should be borne in mind:

- Gravimetric determinations are insensitive to effects of zero degree in ΔN. Effects of a similar type are exlcuded from the satellite determined ΔX_i and could be reflected in the scale σ_ℓ. However, any scale difference between that prevailing on the regional geodetic datum and that defining the satellite solution in the case of astro-gravimetric determinations will be reflected in all $\Delta\xi_i$. The ΔX_i obtained from the astro-gravimetric method could thus be different from the translational parameters ΔX_i obtained from satellite geodesy.

- This factor apart, the astro-gravimetric method is not sensitive to the system of reference adopted or the value of GM provided the geodetic origin of longitudes is the same as the astronomical origin of longitudes and the geodetic network is accordingly oriented in Earth space. For a further discussion see the Appendix.

- Dynamic satellite solutions which are related to CIO/Greenwich using a system similar to the concepts adopted by the Bureau International de l'Heure for the maintenance of the system of reference (Guinot & Feissel, 1969) are implicitly related to the geocentre (Earth's centre of mass) and therefore constitute a determination which is geocentric and oriented by the CIO/Greenwich system of reference to the precision of the observations (±0.2 arc sec or 6 m in each component).

It would therefore appear that it is possible to purposefully study the effect of introducing the rotational parameters ω_i in dynamic solutions when comparing the coordinates of tracking stations as defined on the global satellite system with those on the regional geodetic system. Two non-geometric satellite determinations are catalogued in Table 6, giving parameters deduced for NAD 27. The values of ΔX_i for GEM 6 used in Table 6 (Lerch et al., 1974, p. 84) are slightly different from those in Table 5 which were also reported elsewhere in the same publication (Ibid, p. 88). The other solution considered is GSFC 73 (Marsh et al., 1973, p. 52). In both cases, ω_1 and ω_2 are zero to within the precision of determination, in keeping with the expectations based on the discussion in the Appendix. In both cases, however, ω_3 is significantly larger than zero, being about one arc sec to the order of precision of the determination. This is equivalent to the occurrence of a one second discrepancy between the origin of longitudes (X_1X_3) plane of the satellite solution and that for NAD 27. As discussed in the Appendix, this is not due to any characteristic at the Meades Ranch origin, but rather a function of the network of tracking stations as a whole and represents a relative orientation discrepancy between the satellite and geodetic solutions about the X_3 axis. Such a discrepancy would only occur in astro-gravimetric determinations if there were an error in the orientation of the geodetic network on NAD 27 in relation to the CIO/Greenwich system of reference implied in the astronomical determinations used for computing astro-geodetic deflections of the vertical. If this were the case, an additional effect will be introduced, affecting primarily the quantity $\Delta\eta$ in Tables 4 and 5.

The inter-comparisons of rotated satellite solutions and non-rotated astro-gravimetric solutions could be related as follows. The effect of counter-clockwise rotations ω_i about the X_i axes would produce changes $\delta\Delta X_i$ in the geocentric orientation parameters ΔX_i related to the geocentric Cartesian coordinate system X_i, given by (e.g., Mather, 1973, p. 197)

$$\delta\Delta X_i = \epsilon_{ijk}\,\omega_j X_{ko} \tag{30}$$

where

$$\epsilon_{ijk} = \begin{cases} 0 \text{ if } i = j \text{ or } j = k \text{ or } i = k \\ 1 \text{ if subscripts occur in the order } 123123..... \\ -1 \text{ if subscripts occur in the order } 1321321..... \end{cases}$$

for small rotations ω_i, X_{io} being the geocentric Cartesian coordinates of Meades Ranch (values on the regional geodetic datum for the coordinates of Meades Ranch would be adequate for practical evaluations).

Both GSFC 73 and GEM 6 have significant scale differences in relation to the geodetic network as defined on NAD 27. If the satellite solution has a scale which is greater than that of the regional geodetic network by σ_ℓ parts per million (ppm), the primary effect is on the geocentric orientation parameter ΔN. The difference between ΔN_{GRAV} as obtained from an astro-gravimetric determination and ΔN_{SAT} from a satellite solution on this account is given by the relation

$$\Delta N_{GRAV} = \Delta N_{SAT} - R_m \sigma_\ell \tag{31}$$

where R_m is the mean radius of curvature of the reference ellipsoid at Meades Ranch. On combining Equations 30 and 31 with Equation 28, for all practical purposes,

$$\Delta X_{i_{GRAV}} = \Delta X_{i_{SAT}} + \epsilon_{ijk}\, \omega_j\, X_{ko} - \sigma_\ell\, X_{io} \tag{32}$$

where $\Delta X_{i_{GRAV}}$ would refer to a set of geocentric orientation parameters on a geocentric Cartesian coordinate system based on a solution involving no rotations and allowance for scale, while the value $\Delta X_{i_{SAT}}$ would have been obtained from a seven parameter fit between a global solution and the regional geodetic network.

The application of Equation 32 to GSFC 73 and GEM 6 is illustrated in Table 6 where comparisons are shown in relation to the average astro-gravimetric determination based on FAG 73. It can be seen that the largish (in excess of 2 ppm) discrepancies which exist between the geocentric orientation parameters ΔX_i as determined

a. between the astro-gravimetric solution on the one hand and one of either GSFC 73 or GEM 6 on the other; and

b. between GSFC 73 and GEM 6

decrease by at least a factor of 2 in all cases when allowance is made for the rotations ω_i and the effect of scale σ_ℓ.

It would therefore appear that the goal of determining geocentric orientation parameters for the NAD 27 with the same resolution as satellite methods using astro-gravimetric techniques has largely been achieved. The precision sought of ±0.2 arc sec or its equivalent in each parameter cannot be said to have been achieved with confidence. At face value, the most strongly determined parameter is ΔN. It should be acknowledged however, that the values quoted in Table 4 may still have a bias of up to one part per million (±6 m) because the gravimetric method as used in the present determination is insensitive to scale on a global basis.

On the other hand, the satellite solutions are obtained by substantially independent methods. The main dependence of the gravimetric determination on satellite techniques would be values for the harmonic coefficients in the GEM 4 model with nodal points in excess of 60° apart. Any errors in these values would influence the geocentric orientation parameters obtained by the astro-gravimetric method as used in this study. It should be noted that values obtained from the comparisons of FAG 73 with deflections of the vertical alone

(Table 4, Row 5) over the entire NAD are only in marginal disagreement (less than 1 ppm) with the equivalent GEM 6 in Table 6.

The main conclusion that can be drawn on the basis of the results in Table 6 is that any residual distortion in the network of satellite tracking stations on NAD in the solutions GSFC 73 and GEM 6 must be less than the discrepancies between the equivalent ΔX_i's as obtained for each satellite solution and the astro-gravimetric determination—i.e., at the level of around 1 ppm. This conclusion is subject to the qualification given in the second sentence of the previous paragraph. It would certainly be of interest to repeat these tests using an astro-geodetic network based on the new North American Horizontal Datum (NAHD).

One matter which remains to be dealt with is the possibility of strengthening the values of the low degree harmonic coefficients defining the GEM 4 Earth gravity model. For example, information of interest is contained in Table 4, Rows 5 & 8. The solutions listed in these rows are independent of any errors in computing astro-geodetic undulations. It can be seen that the value of $\Delta\xi$ for the entire datum is smaller than that for the area below parallel 48°N by about 0.5 arc sec. The value of $\Delta\eta$, however, is almost unaffected. Two other pairs of solutions covering the same two areas are also listed in this table—(Rows 1 & 7) and (Rows 3 & 6). The results are summarized on a differential basis in Table 7.

The differences between values for $\Delta\xi$, $\Delta\eta$ and ΔN obtained from solutions extending over the entire NAD (an area subtending an angle of approximately 60° at the geocentre) and those confined to the continental area below 48°N (subtending an angle of approximately 30° at the geocentre) could be attributed to two possible causes provided there were no substantial systematic errors in the global gravity standardization networks—errors of about 1-2 mGal would be necessary to produce an effect of around 0.5 arc sec in the orientation parameters. They are the following:

- systematic errors in the geodetic network on NAD 27 which were variable functions of distance from Meades Ranch; and/or
- errors in the harmonic coefficients in the GEM 4 model in the range $3 < n \leqslant 6$.

The results in Table 7 appear to indicate that the net effect of these errors is, for all practical purposes, zero in the east-west direction while that in the north-south direction is about twice the noise level of the astro-gravimetric determinations in the case of solutions independent of astro-geodetic undulation determinations (Row 1). The equations relating the changes listed in Table 7 to corrections in the harmonic coefficients are given in (Mather, 1970b, pp. 98-99). However, in view of the uncertainty associated with geodetic coordinates in the northern part of the NAD, a conclusive gravity model improvement is unlikely to be achieved using this method on the data available at present. The matter should be reviewed after geodetic coordinates for all astro-geodetic stations used in this investigation are re-establish ed on NAHD.

Meaningful results would only be obtained if such a determination were studied in conjunction with similar investigations for an equivalent area in other parts of the world. An investigation of this type is currently being undertaken for the Australian and Indonesian regions as a whole in the time frame of the next half decade. The combined analysis of such results with those obtained for the North American Region should provide confirmation of the values adopted for the low degree harmonic coefficients in the representation of the Earth's gravity field, independent of satellite values, up to degree 6.

5. CONCLUSION

The primary purpose of this investigation was to study the feasibility of establishing a world geodetic system from the comparisons of gravimetric and astro-geodetic determinations of the separation vector. This could be achieved on a step-by-step basis, considering a single regional datum at a time. The *a priori* model used for the distant zone effects of the free air anomaly field in the present calculations was GEM 4 any any results obtained in this pilot study will be biased by those errors in harmonics of degree less than four. Errors in the values adopted for harmonic coefficients between degrees four and six could, in theory, be separated and identified if a sufficient number of geodetic datums of the same extent were simultaneously investigated. The results obtained in this connection from the present study have an added uncertainty due to widely reported distortions in the geodetic network on NAD 27 in the northern half of the region.

Two sets of independent criteria are available for assessing the precision of the two gravimetric determinations FAG 73 and HTAN 73 (see Table 4) used in this investigation.

- Firstly, the gravimetric undulations can be compared with astro-geodetic undulations after translational correction for the geocentric orientation vector for NAD 27. The root mean square (rms) residual of comparisons in the normal displacement (Table 4, Column 12) are found to vary from ±1.0 m for the region south of parallel 40°N to ±2.9 m for the entire datum, when defined at astro-geodetic stations representing corners of a one degree equi-angular grid wherever such data were available. The rms residual is slightly inferior in the case of comparisons where the astro-geodetic data were obtained by interpolation.

- The second independent criterion is the comparison of geocentric orientation as obtained from satellite geodesy with those computed by the astro-gravimetric method. In the former technique, the geocentric orientation parameters are obtained as three components (ΔX_i) on a geocentric Cartesian coordinate system X_i as part of a seven parameter transformation between coordinates on the regional geodetic system and the global satellite system. On allowing for the fact that any scale errors in the geodetic network and any tendency for the regional geodetic system to be incorrectly oriented in relation to CIO/Greenwich through azimuth control, as discussed in the Appendix, are absorbed in astro-gravimetrically determined geocentric orientation parameters ($\Delta\xi$, $\Delta\eta$, ΔN) in the local Laplacian triad at the regional Meades Ranch origin, it is shown in Section 4.3 that the average value obtained astro-gravimetrically agrees with each of the global satellite solutions GSFC 73 and GEM 6 better than the two satellite solutions agree with each other, as detailed in Table 6.

These tests provide bounds for

a. long wavelength (nodal points > 6000 km apart) errors in GEM 4; and/or

b. systematic error propagation in the present geodetic network on NAD 27.

The discrepancy between "equivalent" satellite solutions and the average astro-gravimetric determination listed in Table 6 also provides an upper limit for any zero degree term in the height anomaly. On the basis of the present study, this does not exceed the precision of the satellite solutions. On the assumption that there is no scale error in the geodetic network on NAD 27 and assuming that no systematic orientation error exists in relation to CIO/Greenwich as implied in both GSFC 73 and GEM 6, the recommended set of geocentric orientation parameters for NAD 1927 on the basis of the astro-gravimetric determinations is $\Delta\xi = 0.1$ arc sec; $\Delta\eta = 1.0$ arc sec; $\Delta N = -34$ m in Laplacian Triad at Meades Ranch or $\Delta X_1 = -7$ m; $\Delta X_2 = 159$ m; $\Delta X_3 = 169$ m on a geocentric Cartesian coordinate system.

Corrections which make allowance for the assumptions mentioned above on the basis of satellite solutions are given in Table 6. The significant parameters are a possible allowance for scale inconsistencies between that provided by the horizontal geodetic network on NAD 27 and the current values of GM (expected discrepancy of 1-2 ppm) and the allowance for a rotation between coordinates on the satellite system and the horizontal geodetic network of about 1 arc sec around the X_3 axis. The only effect in excess of 1 ppm as a consequence of such considerations is a change of about -24 m in ΔX_1 and +0.8 arc sec in $\Delta\eta$ due to the rotation of 1 arc sec about the X_3 axis.

The use of FAG 73 in lieu of HTAN 73 (solution for the height anomaly using Equation 9) is not found to make any non-negligible contribution to the geocentric orientation parameters for the North American Datum. This does not rule out the possibility that these effects could still be of significance when orienting a datum which included the Himalayas. The extent of the datum used has a significant effect on the results (see Table 7). This is tentatively attributed in the present case to the effects of a. and b. above. A redetermination after the establishment of NAHD might possibly reduce the influence of b. While the use of the astro-gravimetric technique may only be of limited interest in future plans for establishing a world geodetic system, it still provides useful information for the study of slowly varying systematic effects and the scale of a regional geodetic system, while at the same time, providing a means for determining the geocentric orientation vector for regional geodetic datums independent of scale as introduced through satellite systems.

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8. APPENDIX – THE ROLE OF TRANSLATIONS AND ROTATIONS IN ASTRO-GRAVIMETRIC DETERMINATIONS OF THE GEOCENTRIC ORIENTATION VECTOR

The discussion in Section 4 shows that some ambiguity is experienced in practice when comparing determinations of geocentric orientation parameters (datum shifts) as obtained from solutions which specifically exclude rotations (astro-gravimetric solutions) with those which call for seven parameter transformations. The background behind the exclusion of rotational concepts in astro-gravimetric methods is the following.

The orientation of the reference ellipsoid used in gravimetric determinations and its location in Earth space are implicit on the basis of conditions postulated in the derivation of solutions to the geodetic boundary value problem (e.g., Mather, 1970b, p. 84). It can be shown that in a careful derivation of a solution of the geodetic boundary value problem the reference ellipsoid is located in Earth space so that its centre is at the centre of mass of the Earth (or, more specifically, the centre of mass of the solid Earth and oceans which is within ± 5 cm of the geocentre). This is due to the fact that no first degree harmonic can be permitted in the representation of the distrubing potential. The minor axis of the reference ellipsoid is placed in coincidence with the axis of greatest moment of inertia of the Earth if $C_{21} = S_{21} = 0$, in the spherical harmonic expansion of the geopotential. Actual numerical solutions for these values (e.g., Lerch et al., 1972) appear to indicate that their magnitude is about three orders smaller than other coefficients (excluding C_{20}) thus indicating that the effect of uncertainty in this regard is at the 5 cm level in solutions of the geodetic boundary value problem. It can be concluded that the gravimetric solution refers to the CIO/Greenwich/Geocentre system of reference without ambiguity in practical determinations. This system of reference can be fully represented in Earth space by 9 parameters:

- The coordinates X_{gi} of the geocentre on some three dimensional Cartesian coordinate system.
- The direction cosines ℓ_{i1} of the line joining the geocentre to the CIO pole.
- The direction cosines ℓ_{i2} of a line in the meridian of reference for longitude (Greenwich) which is perpendicular to this "axis of rotation."

If the X_i axis system was geocentric, the following conditions hold:

$$X_{gi} = 0 \tag{A-1}$$

$$\sum_{i=1}^{3} \ell_{ij}^2 = 1, \quad j = 1, 2 \tag{A-2}$$

$$\sum_{i=1}^{3} \ell_{i1}\, \ell_{i2} = 0 \tag{A-3}$$

It therefore follows that three parameters (e.g., ℓ_{11}, ℓ_{21}, ℓ_{12}) have to be arbitrarily defined to specify a consistent system in Earth space.

Any point P in Earth space is completely defined in relation to such a system of reference if its three dimensional Cartesian coordinates (X_{io}) were known. Conversely, given the three coordinates (X'_{io}) which are arbitrarily assigned to the point P in Earth space, three degrees of freedom exist when fixing the (unknown) location of the system of reference in Earth space. In the case where the differences

$$\Delta X_i = X_{io} - X'_{io} \qquad \text{(A-4)}$$

are small (of the order of a few hundred meters), the options available can be visualized as follows. Consider the point Q in Earth space whose coordinates with respect to the X_i system defined by Equations A-1 to A-3 are X'_{io}. It can be seen that the assignation of the coordinates X'_{io} to P could be completely described in Earth space by placing the origin of the rectangular Cartesian coordinate system X'_i at a point whose coordinates on the X_i system are ΔX_i <u>without rotation of axes</u>. In this concept, the quantities defined by Equations A-2 and A-3 have been held unaltered by choice when exercising discretionary assignation of the three arbitrarily defined parameters.

Needless to say, the assignation of coordinates X'_{io} to P could also have been achieved by rotating the axes while maintaining Equation A-1. Such a consideration is unnecessarily ambiguous in complex geodetic inter-relations. It is therefore valid to consider the assignation of coordinates <u>at a single point on a geodetic datum</u> to be equivalent to a translation of origin without rotation.

The concept of rotation is introduced to take into account systematic trends <u>in a geodetic network</u>. A classic example where a set of rotations may be of relevance is in instances where the geodetic network is incorrectly oriented in relation to CIO/Greenwich. An example of this type would be a geodetic network where no CIO/Greenwich related Laplace azimuth were available to orientate the net at the origin. Consequently, the question of rotations should not arise in the case of error free observations provided the Laplace azimuths introduced into the network were all properly linked to CIO/Greenwich using appropriate inertial reference and timing systems. In practice, rotations of the order of magnitude of errors in the geodetic network should be expected when orienting geodetic datums to CIO/Geocentre/Greenwich. Comparison of two such networks will define rotations which are also a measure of the average distortion of each network.

It must be concluded that in comparing solutions aspiring to a precision of 1 ppm, only those rotations whose magnitude exceeds 0.2 arc sec can be interpreted as being due to factors other than systems uncertainty. In the case of solutions for the NAD 27, it is only ω_3 which is geodetically significant. Its nature appears to indicate an orientation of the North American geodetic network which is not consistent with CIO/Greenwich by about one arc sec as discussed in Section 5.

Table 1

Area Sub-Divisions Used In Computations

Range of ψ (Degrees)	Free Air Anomaly Data Type	Data Sources
$\psi < 0.1$	Circular Ring Representation	Estimated from $0.05° \times 0.05°$ area means
$0.1 \leq \psi < 0.5$	$0.05° \times 0.05°$ area means	Interpolated from $0.1° \times 0.1°$ area means
$0.5 \leq \psi < 1.5$	$0.1° \times 0.1°$ area means	From data banks and interpolation
$1.5 \leq \psi < 7.5$	$0.5° \times 0.5°$ area means	From $0.1° \times 0.1°$ area means
$7.5 \leq \psi < 22.5$	$1° \times 1°$ area means	i. From $0.5° \times 0.5°$ area means (Regions in Fig. 1) ii. Talwani et al., 1972 or ACIC 1971 [if no i.] iii. GEM 4 [if no i. or ii.]
$\psi \geq 22.5$	$5° \times 5°$ area means	i. From $1° \times 1°$ area means (Regions 1 - 81) ii. From GEM 4 [if no i.]

Table 2

Free Air Anomalies - North America

Error of Representation $E\{\Delta g\}_{m^\circ}$ for Various Equi-Angular Squares of Size m°

Square Size (m°)	$E\{\Delta g\}_{m^\circ}$ ±mGal	Number of Readings $\left(\sum_{i=1}^{M} N_i\right)$	Remarks
0.5	15.9	113,881	For North America (Equation 22)
1	17.8	113,881	-do-
1	16.5	113,881	For zero elevation by linear regression analysis
1	34.6	4,812	For elevations greater than 2000 m
5	24.8	113,881	For North America (Equation 22)

Table 3

Free Air Anomalies - North America

Error of Representation $E\{\Delta g\}_{1/2°}$ for Half Degree Equi-angular Squares as a Function of Elevation

Elevation (h) (m)	$E\{\Delta g\}_{1/2°}$ (±mGal)	Number of Squares
$-5000 < h \leqslant -4000$	9.5	2
$-4000 < h \leqslant -3000$	14.9	7
$-3000 < h \leqslant -2000$	19.1	20
$-2000 < h \leqslant -1000$	24.8	25
$-1000 < h \leqslant 0$	11.5	66
$0 < h \leqslant 1000$	9.6	7137
$1000 < h \leqslant 2000$	21.1	815
$2000 < h \leqslant 3000$	24.1	230
$3000 < h \leqslant 4000$	24.5	114

Table 4

North American Datum 1927

Geocentric Orientation Parameters From the Comparison of

Gravimetric & Astro-Geodetic Data

Soln No.	Grav. Solution	Astro-Geodetic Solution	Class	Laplacian Triad at Meades Ranch									Geocentric Cartesian			Latitude Range (°)		Longitude Range (°)	
				$\Delta\xi_0$ (sec)	$\sigma_{\Delta\xi}$ (±sec)	N_ξ	$\Delta\eta_0$ (sec)	$\sigma_{\Delta\eta}$ (±sec)	N_η	ΔN (m)	$\sigma_{\Delta N}$ (±m)	N_N	ΔX_1 (m)	ΔX_2 (m)	ΔX_3 (m)	Min.	Max.	Min.	Max.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	FAG 73	AMS 67	A	-7.8	2.6	799	1.2	2.8	734	16(.0)	4.6	2526	-13	169	177	15	69	-150	-60
2	HTAN 73	AMS 67	A	-7.8	2.6	799	1.2	2.8	734	16(.0)	4.5	2526	-12	169	177	15	69	-150	-60
3	FAG 73	NGS 72 & NEY	A	-7.2	2.6	799	1.0	2.7	734	16(.7)	2.9	364	-6	156	162	29	63	-140	-60
4	FAG 73	NGS 72 & NEY	C	-7.1	2.6	799	1.0	2.7	734	16(.7)	1.8	316	-7	154	159	29	48	-124	-70
5	FAG 73	DEFLECTIONS	B	-7.6	2.6	799	0.8	2.7	734	7(.6)	-	-	-2	157	178	15	69	-150	-60
6	FAG 73	NGS 72 & NEY	A	-7.1	2.1	570	1.0	2.4	506	16(.7)	1.8	314	-7	153	158	29	48	-124	-70
7	FAG 73	AMS 67	A	-7.1	2.1	570	1.1	2.5	506	16(.2)	2.6	989	-12	154	159	29	48	-124	-70
8	FAG 73	DEFLECTIONS	B	-7.2	2.2	570	0.9	2.5	506	16(.7)*	-	-	-4	148	166	29	48	-124	-70
9	FAG 73	NGS 72	C	-7.2	2.6	799	1.0	2.8	734	16(.8)	1.0	203	-6	155	160	29	40	-124	-70
10	FAG 73	COMPOSITE		-7.7			1.0			11(.0)			-7	162	178	15	69	-150	-60
11	FAG 73	COMPOSITE		-7.1			1.0			15(.1)			-7	154	161	29	48	-124	-70
12	FAG 73	AVERAGE		-7.4			1.0			13(.8)			-7	159	169				

Key to Table 4

Solution Types

FAG 73	-	1973 Free Air Geoid
HTAN 73	-	1973 Height Anomalies
AMS 67		Army Map Service Astro-Geodetic Solution (Fischer et al., 1967)
NGS 72	-	U.S. National Geodetic Survey Astro-Geodetic Levelling (Rice, 1972)
NEY	-	Canadian Astro-Geodetic Levelling (NEY 1952)

Class

A		Undulation Comparisons Only
B	-	Deflection Comparisons Only
C	-	As at B plus undulation comparisons south of Latitude in Column 18
σ	=	rms Residual
N		Number of Comparisons Made
*	-	Value Obtained (116.9) replaced by best Value from Row 6

Table 5

Geocentric Orientation Parameters for the North American Datum 1927

Ref No	Solution Name	Source	In Laplacian Triad At Meades Ranch			Geocentric Cartesian Components		
			$\Delta\xi$ (sec)	$\Delta\eta$ (sec)	ΔN (m)	ΔX_1 (m)	ΔX_2 (m)	ΔX_3 (m)
1	2	3	4	5	6	7	8	9
1	GEM 4	Lerch et al., 1972	-7.6	1.5	0	-24	153	181
2	GEM 6	Lerch et al., 1974	-7.8	1.5	-2	-22	155	187
3	GSFC 73	Marsh et al., 1973	-7.7	2.2	6	-43	162	179
4	DOD WGS 72	Seppelin, 1974	-7.5	1.5	6	-22	157	176
5	SE III	Gaposchkin, 1973	-7.4	1.7	3	-31	154	176
6	WN 14	Mueller, 1974	-7.4	1.1	2	-11	148	175
7	NGS	Schmid, 1974	-7.7	1.8	18	-32	171	173
8	Average	Non-Geometric	-7.6	1.7	3	-28	156	179
9	Average	Geometric	-7.5	1.5	9	-22	159	174
10	Average	Astro-Gravimetric	-7.4	1.0	14	-7	159	169

Table 6

Comparison Between Satellite and Astro-Gravimetric Determinations of Geocentric Orientation Parameters for North American Datum 1927 After Allowance for Rotation and Scale

Description	GSFC 73 Value	GSFC 73 Geocentric Cartesian Components ΔX_1 (m)	ΔX_2 (m)	ΔX_3 (m)	GSFC 73 In Laplacian Triad at Meades Ranch $\Delta\xi$ (sec)	$\Delta\eta$ (sec)	ΔN (m)	GEM 6 Value	GEM 6 Geocentric Cartesian Components ΔX_1 (m)	ΔX_2 (m)	ΔX_3 (m)	GEM 6 In Laplacian Triad at Meades Ranch $\Delta\xi$ (sec)	$\Delta\eta$ (sec)	ΔN (m)
Satellite Value		-43	162	179	-7.7	2.2	6		-24	151	187	-7.7	1.5	-5
Correction for Scale (σ_ξ) ppm	0.9	1	4	-4				1.7	1	8	-7			
Correction for Rotation (ω_1) arc sec	0.05	-	-1	-1				0.2	-	-4	-5			
Correction for Rotation (ω_2) arc sec	0.2	4	-	1				-0.1	-2		-0			
Correction for Rotation (ω_3) arc sec	1.1	26	-4	-				0.8	19	-3	-			
Equivalent Satellite		-12	161	175	-7.6	1.2	11		-6	152	175	-7.4	0.9	5
Average Astro-gravimetric		-7	159	169	-7.4	1.0	14		-7	159	169	-7.4	1.0	14
Astro-grav. Minus Satellite		5	-3	-6	0.2	-0.2	3		-1	7	-6	0.0	0.1	9

Rms Discrepancy

Astro-grav minus GSFC 73 = ±7.3 m

Astro-grav minus GEM 6 = ±8.7 m

GSFC 73 - GEM 6 = ±10.8 m (Equivalent); (±25.0 if uncorrected)

Table 7

The Effect of Area Utilized in Astro-Gravimetric Solutions on the

Geocentric Orientation Parameters for North American Datum 1927

Astro-geodetic Data Type	Class of Solution	Row Reference in Table 4	Change in Parameters on Reduction of Area from Entire Datum to Area South of Parallel 48°N		
			Change in $\Delta\xi$ (sec)	Change in $\Delta\eta$ (sec)	Change in ΔN (m)
Deflections Only	B	5,8	-0.5	0.0	Indeterminate
AMS 67	A	1,7	-0.7	0.1	-0.2
NGS 72 & NEY	A	3,6	-0.2	0.0	0.0

FIGURE CAPTIONS

Figure 1. North America — Area Sub-Divisions for Gravity Data Processing

Figure 2. North America — Distribution of Available Free Air Anomaly Data — n = number of stations per $1^\circ \times 1^\circ$ square

Figure 3. North America — Representation of Free Air Anomaly Data Set ($1^\circ \times 1^\circ$ means) GRS 67 — Contour Interval 20 mGal

Figure 4. North America — Topography From the Available $1^\circ \times 1^\circ$ Mean Elevations — Contour Interval 500 m

Figure 5. Frequency Histogram Showing Occurrence of One Degree Equi-Angular Elevation Means — North America

Figure 6. Free Air Anomaly and Elevation Correlation with Latitude

Figure 7. North America — Frequency Histogram Showing Occurrence of $E\{\Delta g\}$ for One Degree Equi-angular Squares

Figure 8. $E\{\Delta g\}_{1^\circ}$ for North American Datum - Free Air Anomalies Correlation Characteristics of the Available Sample with Elevation

Figure 9. North America — Error of Representation for $1^\circ \times 1^\circ$ Squares $E\{\Delta g\}$ ($\pm$ mGal)

Figure 10. North America — Free Air Geoid 1973 (GRS 67) — Contour Interval 2 m

Figure 11. North America — Discrepancy Between Astro-Geodetic (AMS 67) & FAG 73 Determinations — Contour Interval 5 m

Figure 12. North America — Non-Stokesian Contribution to Height Anomaly ($\psi < 5°$) — Contour Interval 1 m

Figure 13. Relation Between RS 1967 and NAD 27 in Meridian of Meades Ranch

Figure 14. North America — Distribution of Astro-Geodetic Stations

n = Number of Stations — per 1° x 1° Square

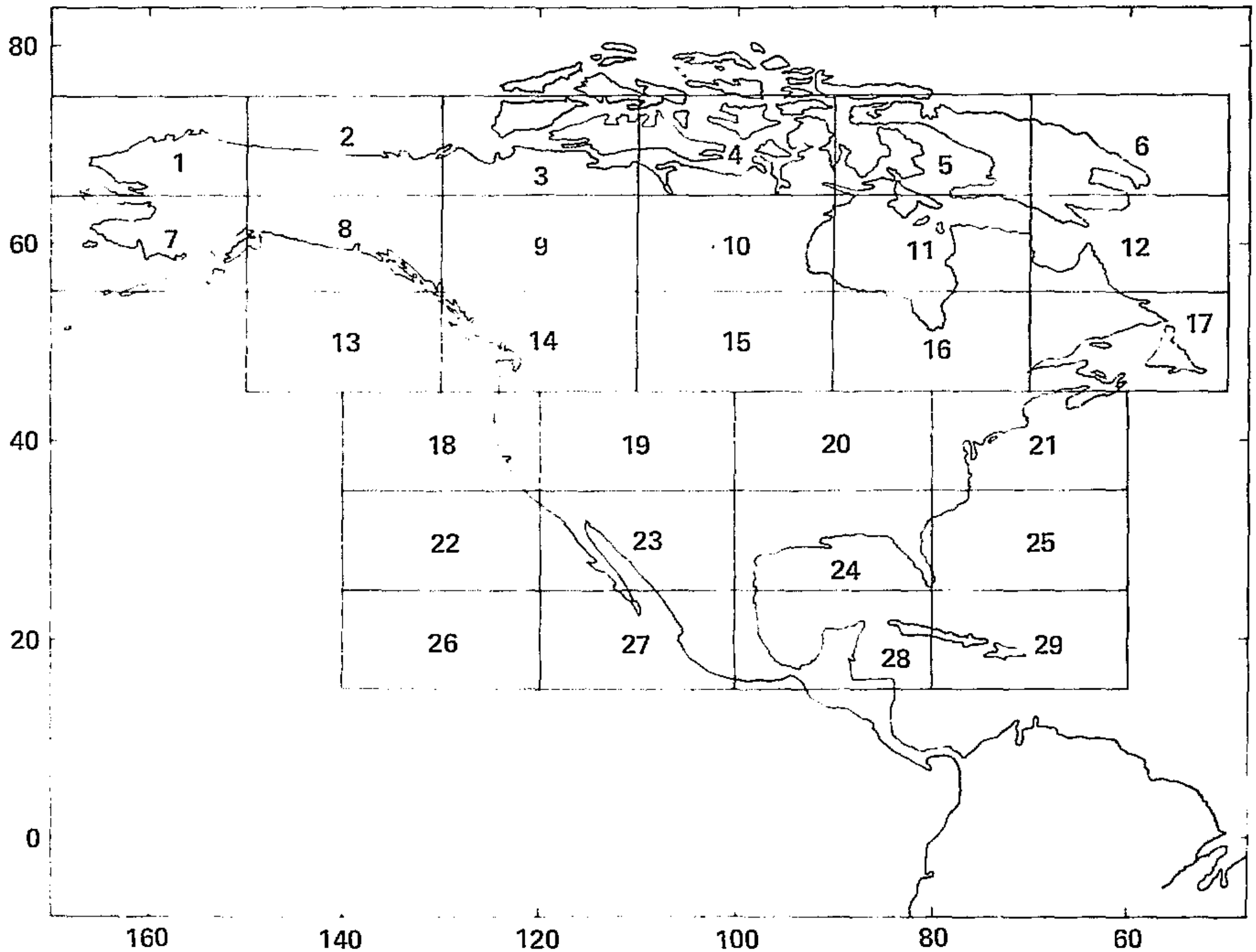


Figure 1. North America — Area Sub-Divisions for Gravity Data Processing

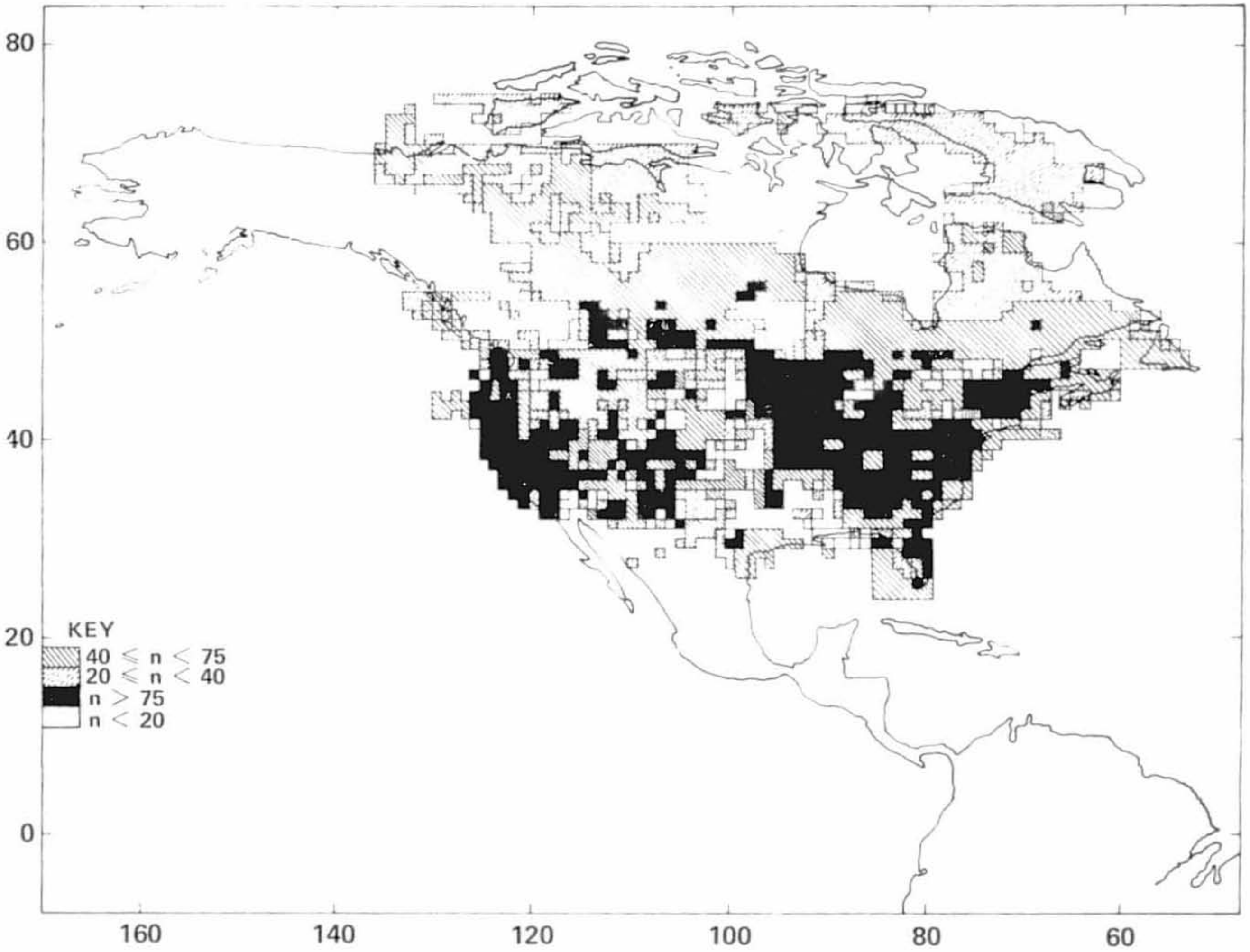


Figure 2. North America — Distribution of Available Free Air Anomaly Data — n = number of stations per 1° x 1° square

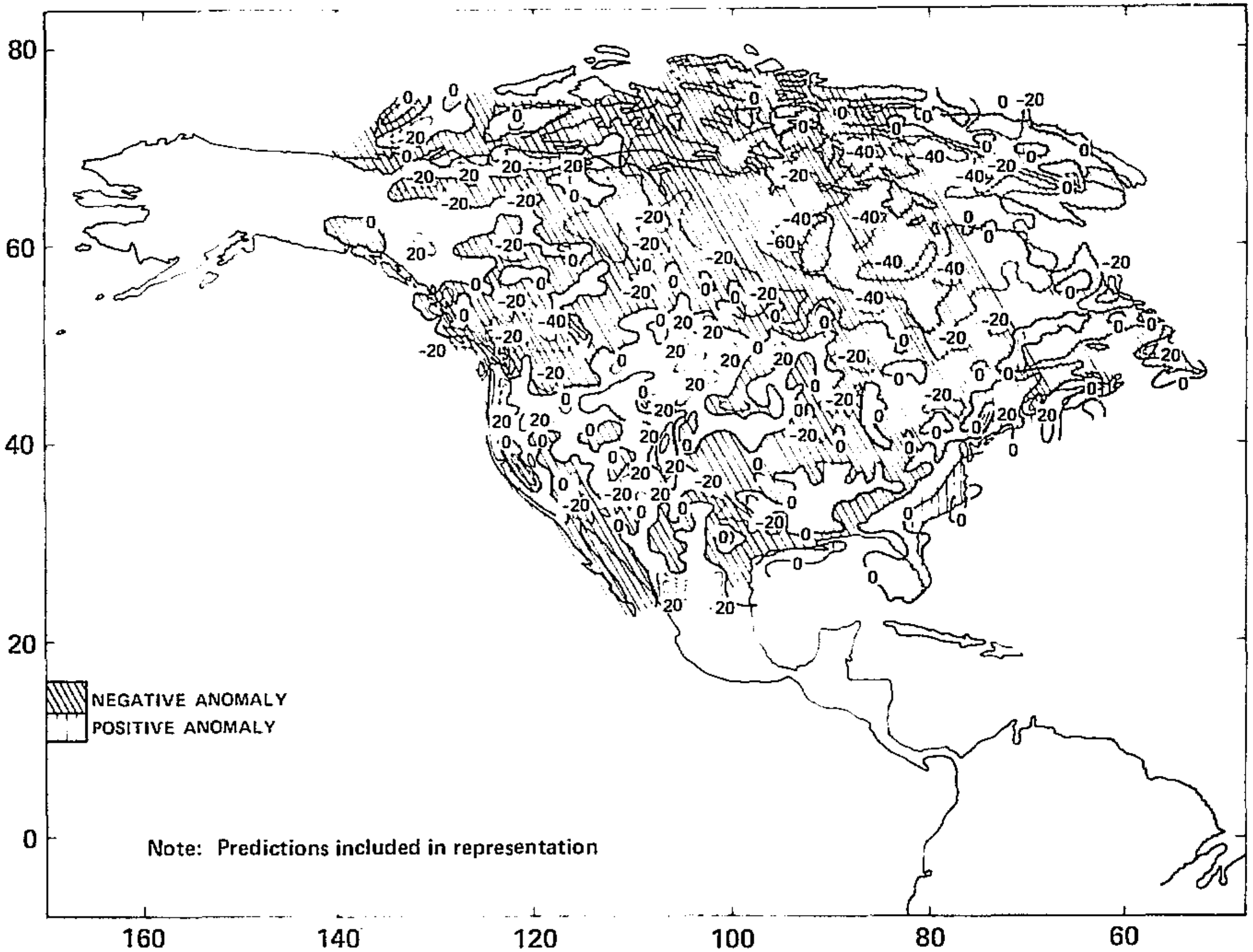


Figure 3. North America — Representation of Free Air Anomaly Data Set (1° x 1° means)
GRS 67 — Contour Interval 20 mGal

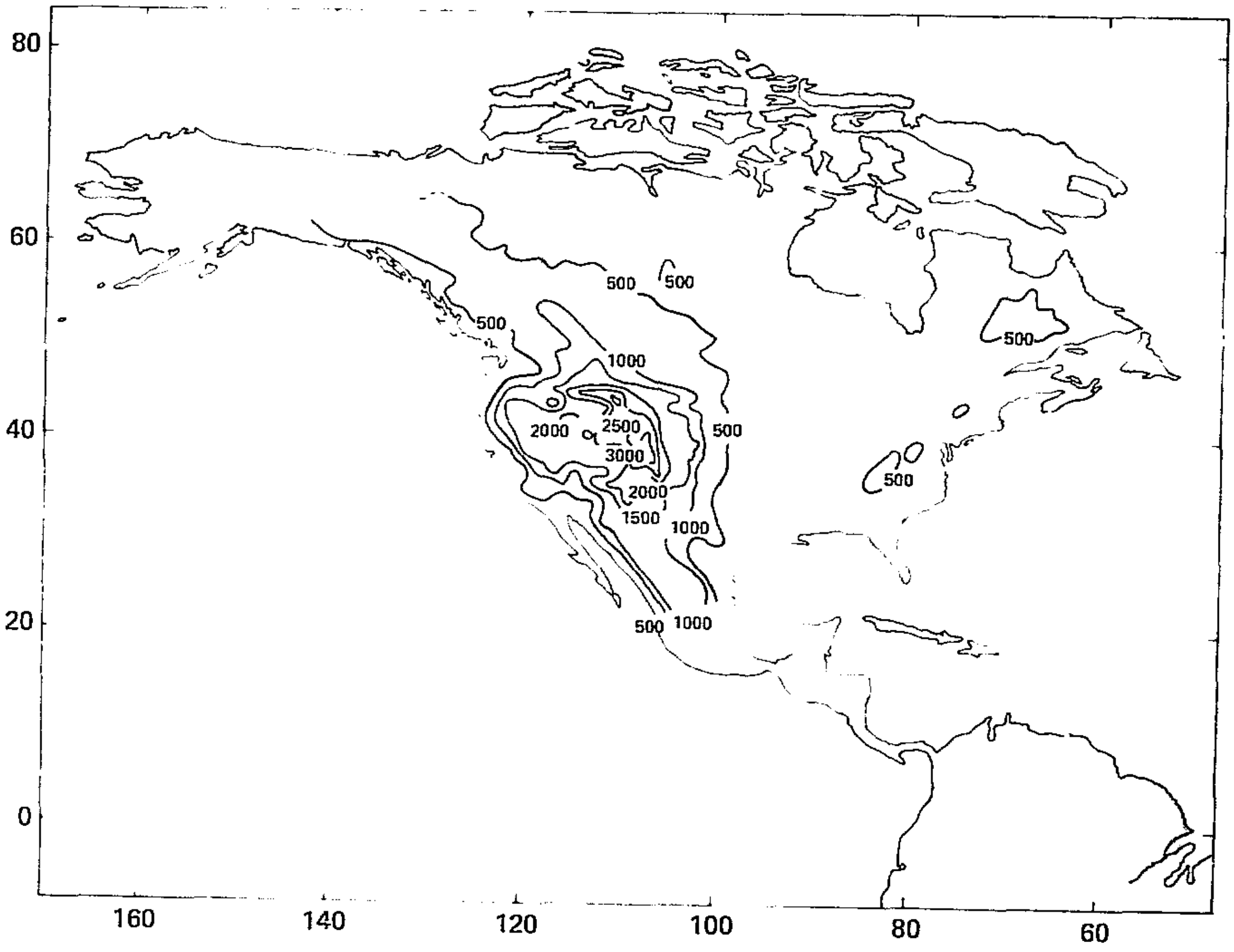


Figure 4. North America — Topography From the Available 1° x 1° Mean Elevations — Contour Interval 500 m

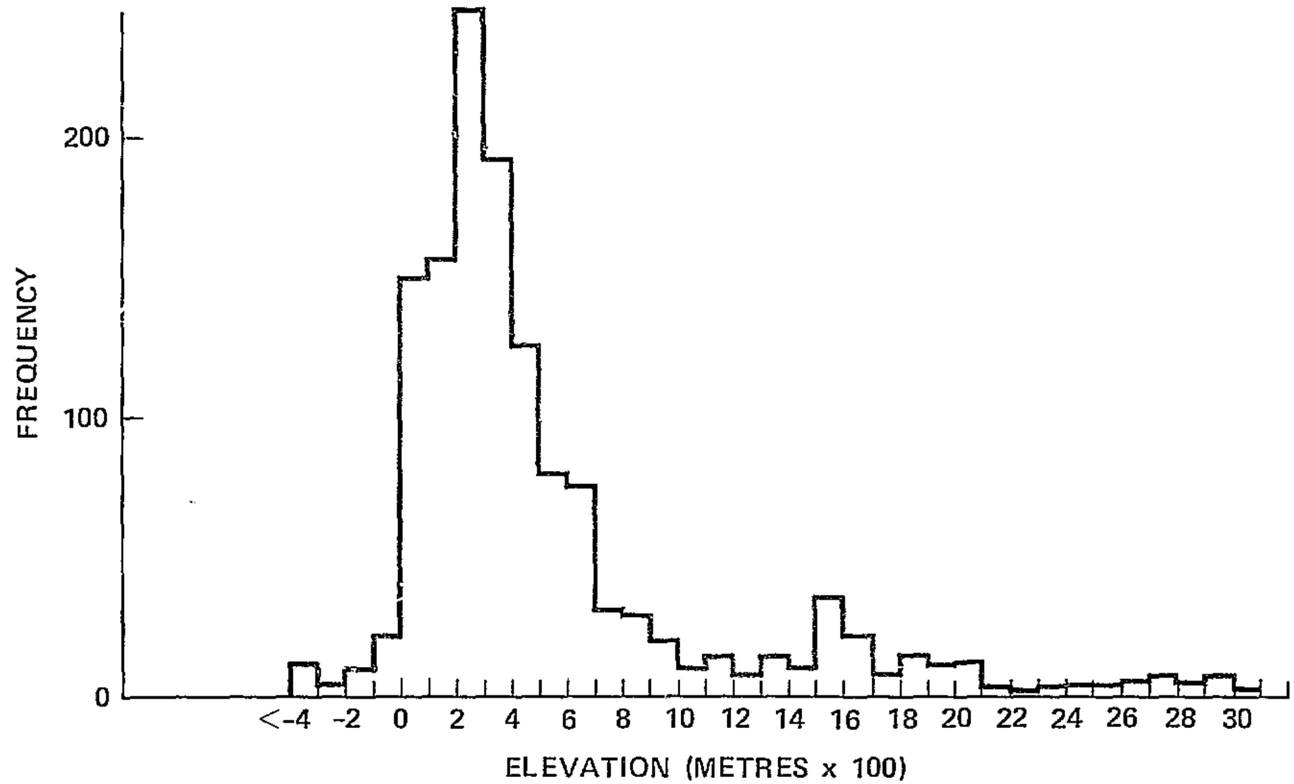


Figure 5. Frequency Histogram Showing Occurrence of One Degree Equi-Angular Elevation Means — North America

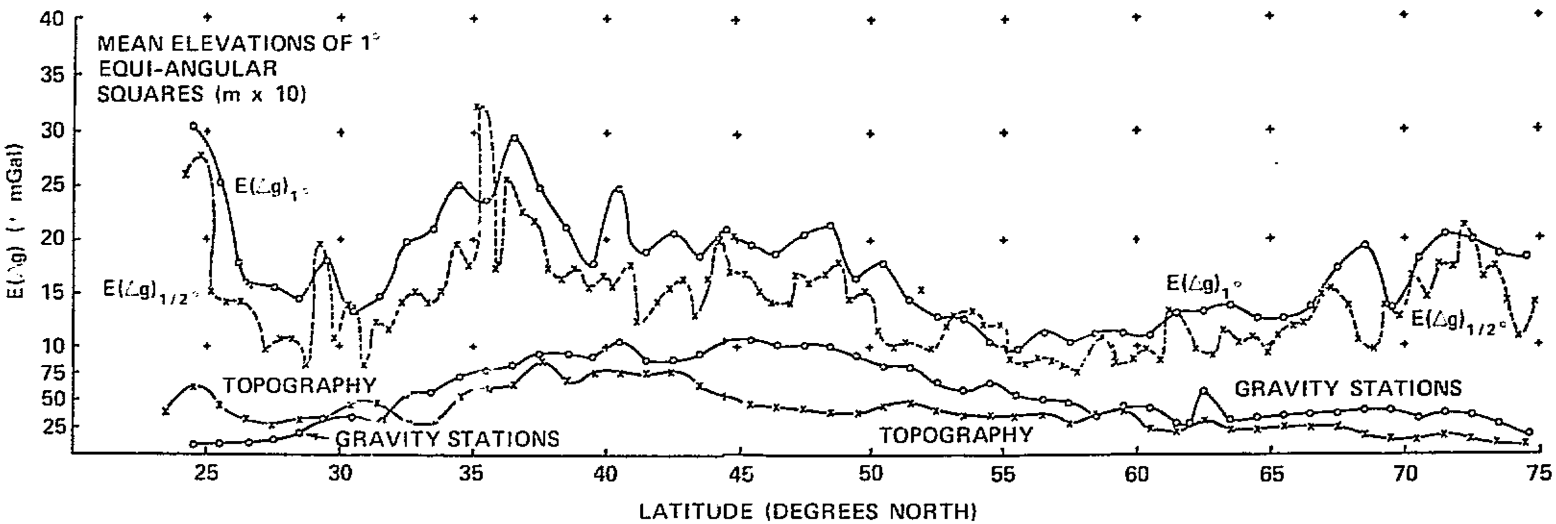


Figure 6. Free Air Anomaly and Elevation Correlation with Latitude

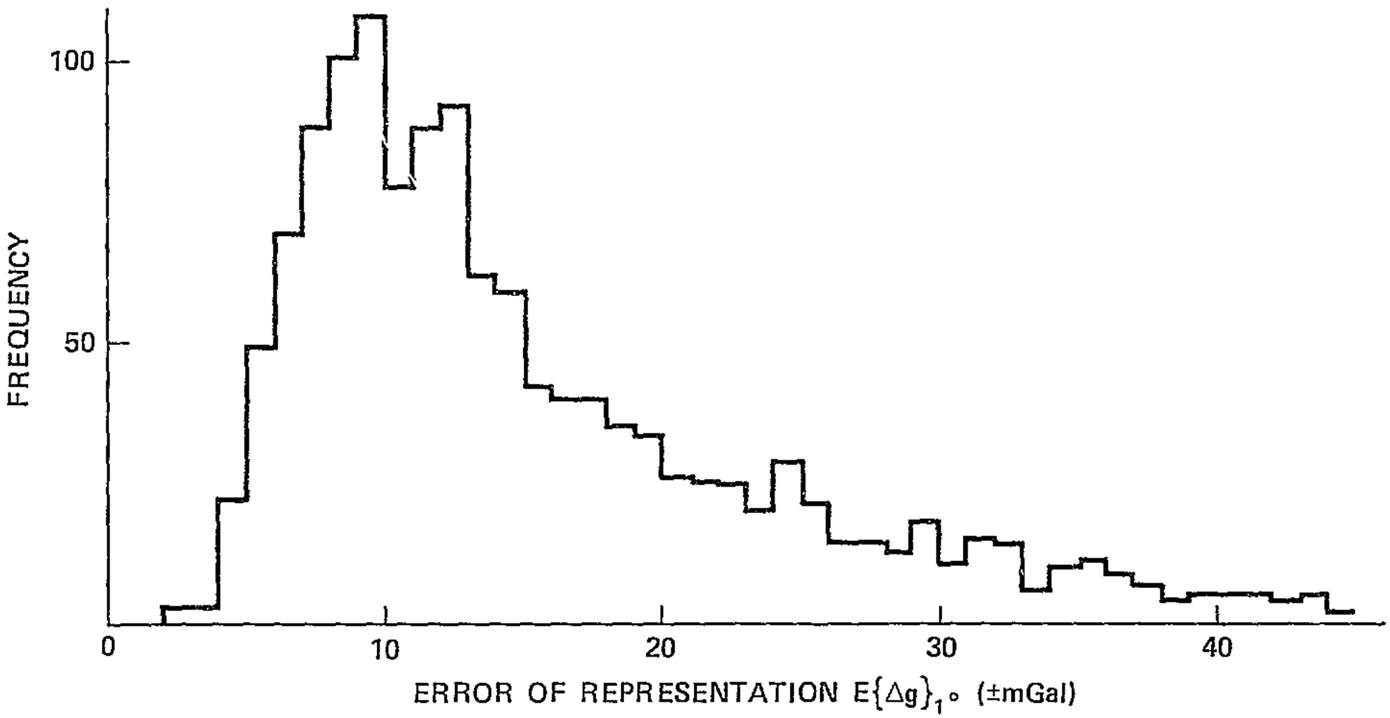
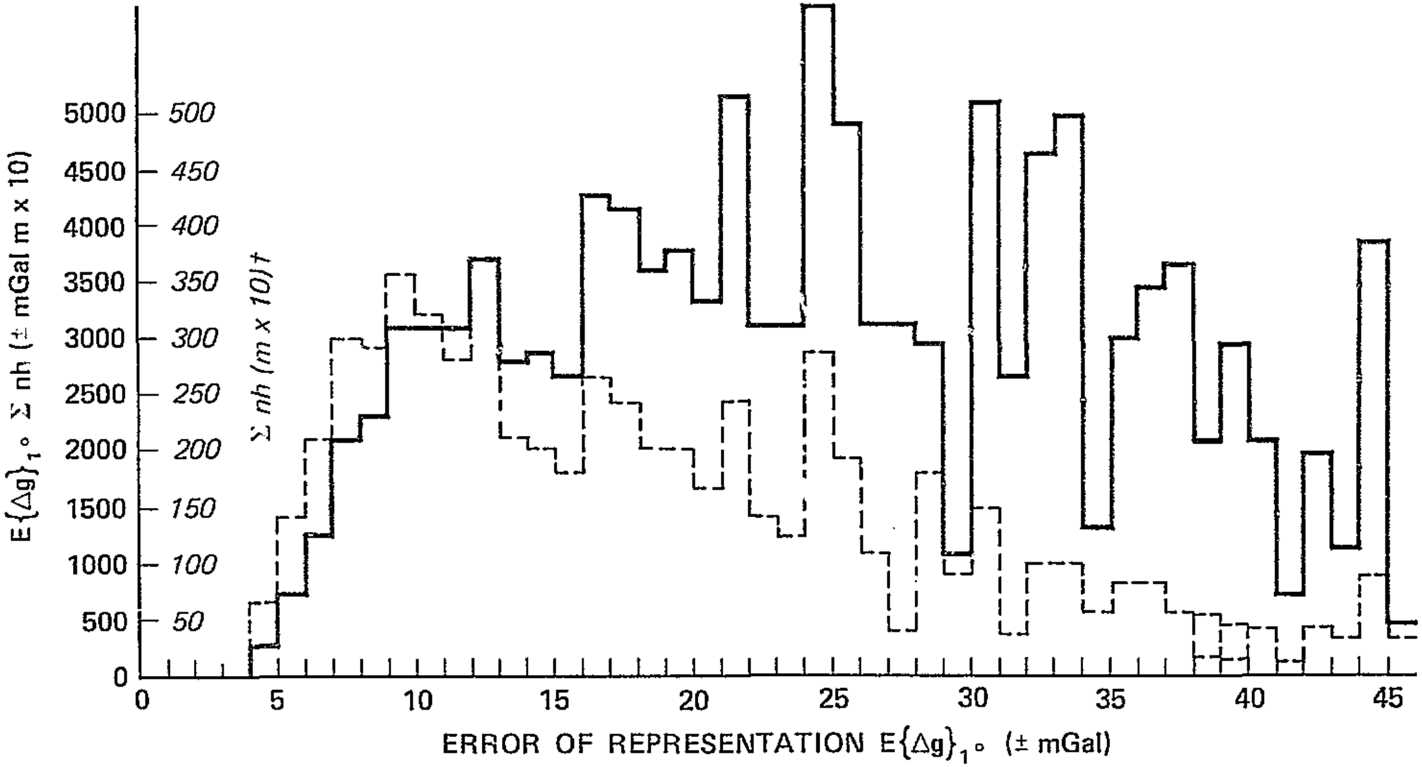


Figure 7. North America — Frequency Histogram Showing Occurrence of E$\{\Delta g\}$ for One Degree Equi-angular Squares



KEY:

n = SAMPLE SIZE

h = ELEVATION

† SCALE IN ITALICS REFERS TO BROKEN LINE HISTOGRAM

Figure 8. $E\{\Delta g\}_{1^\circ}$ for North American Datum - Free Air Anomalies Correlation Characteristics of the Available Sample with Elevation

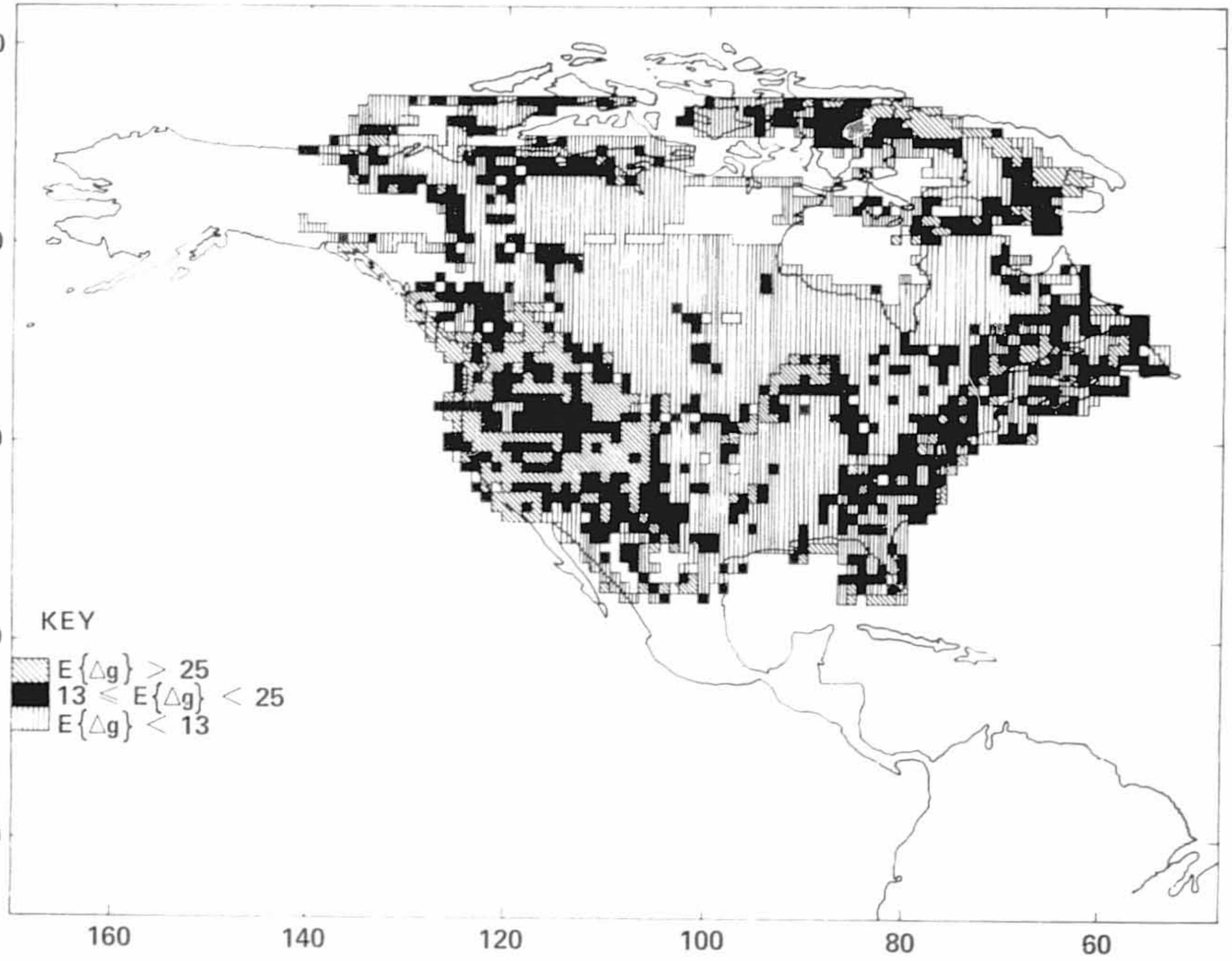


Figure 9. North America — Error of Representation for 1° x 1° Squares $E\{\Delta g\}$ ($\pm$ mGal)

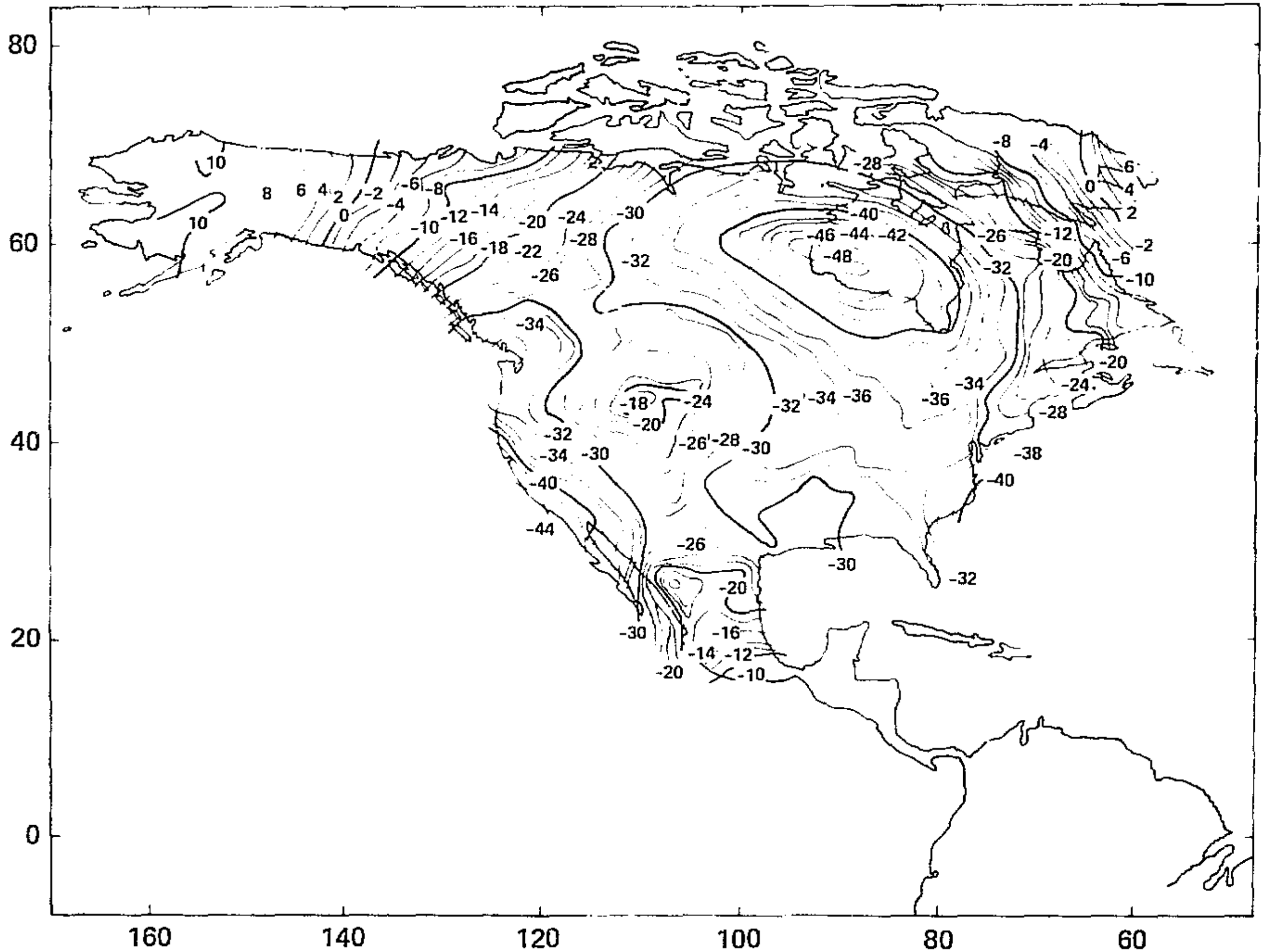


Figure 10. North America — Free Air Geoid 1973 (GRS 67) — Contour Interval 2 m

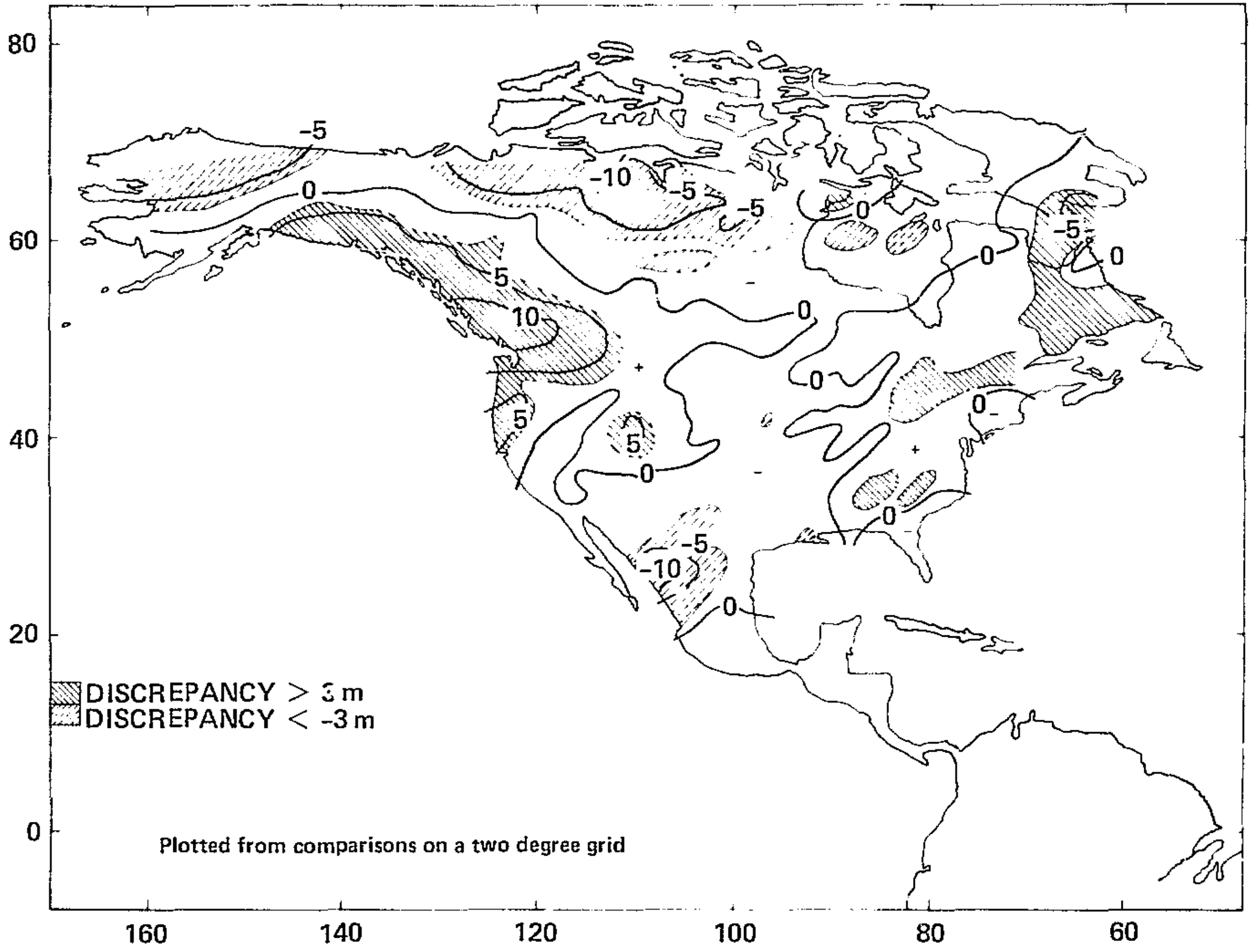


Figure 11. North America — Discrepancy Between Astro-Geodetic (AMS 67) & FAG 73 Determinations † — Contour Interval 5 m

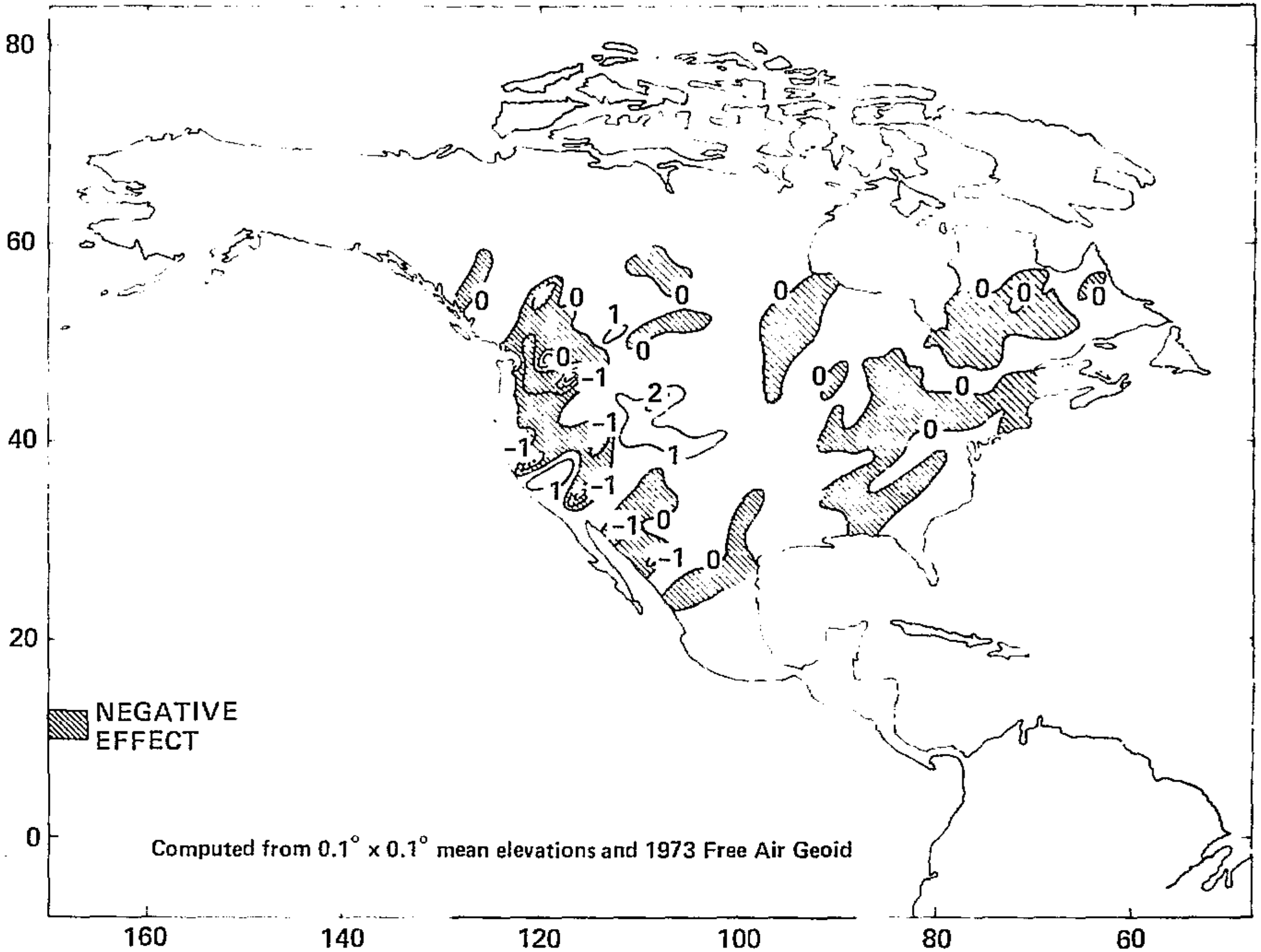


Figure 12. North America — Non-Stokesian Contribution to Height Anomaly (ψ<5°)† — Contour Interval 1 m

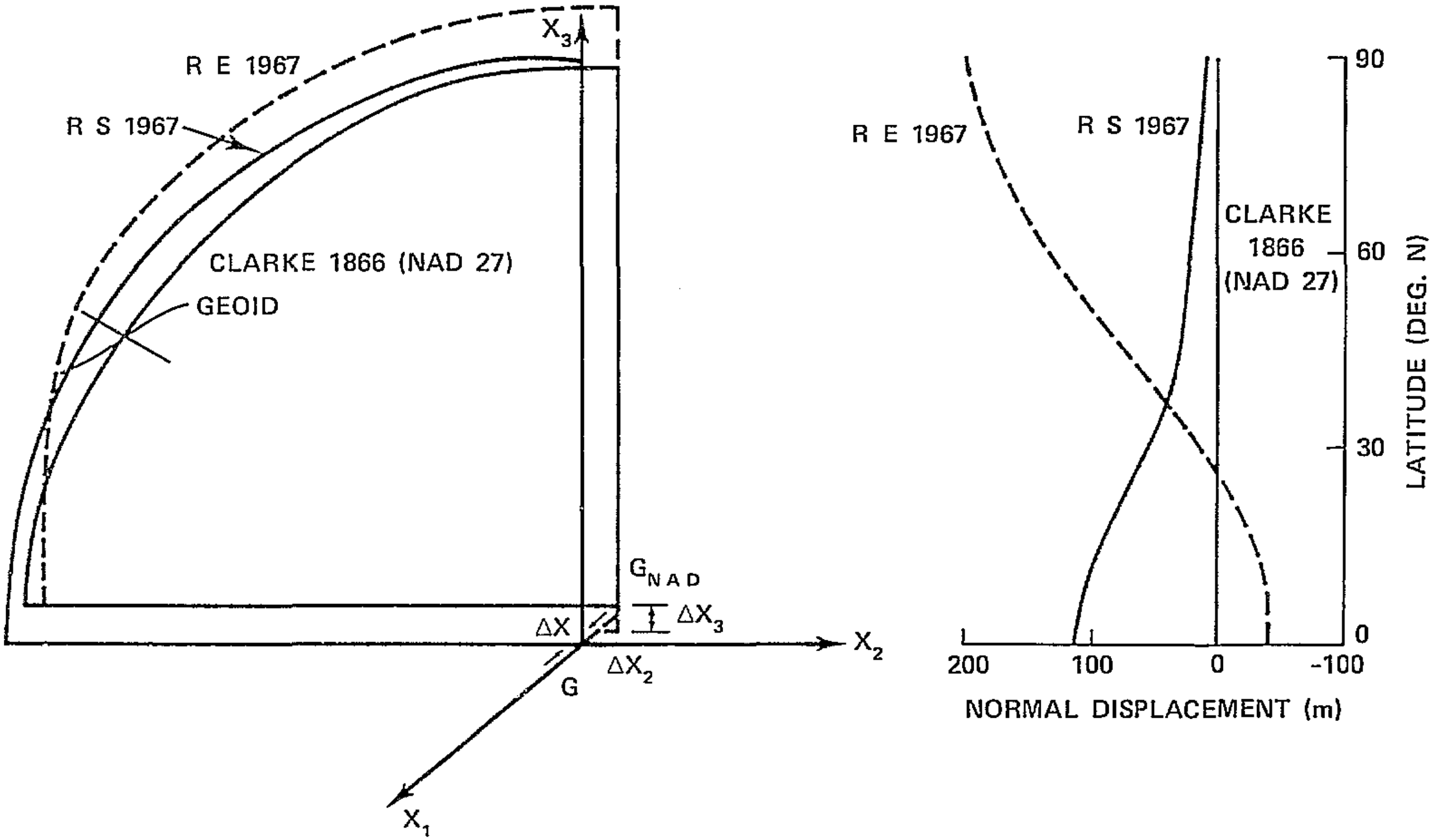


Figure 13. Relation Between RS 1967 and NAD 27 in Meridian of Meades Ranch

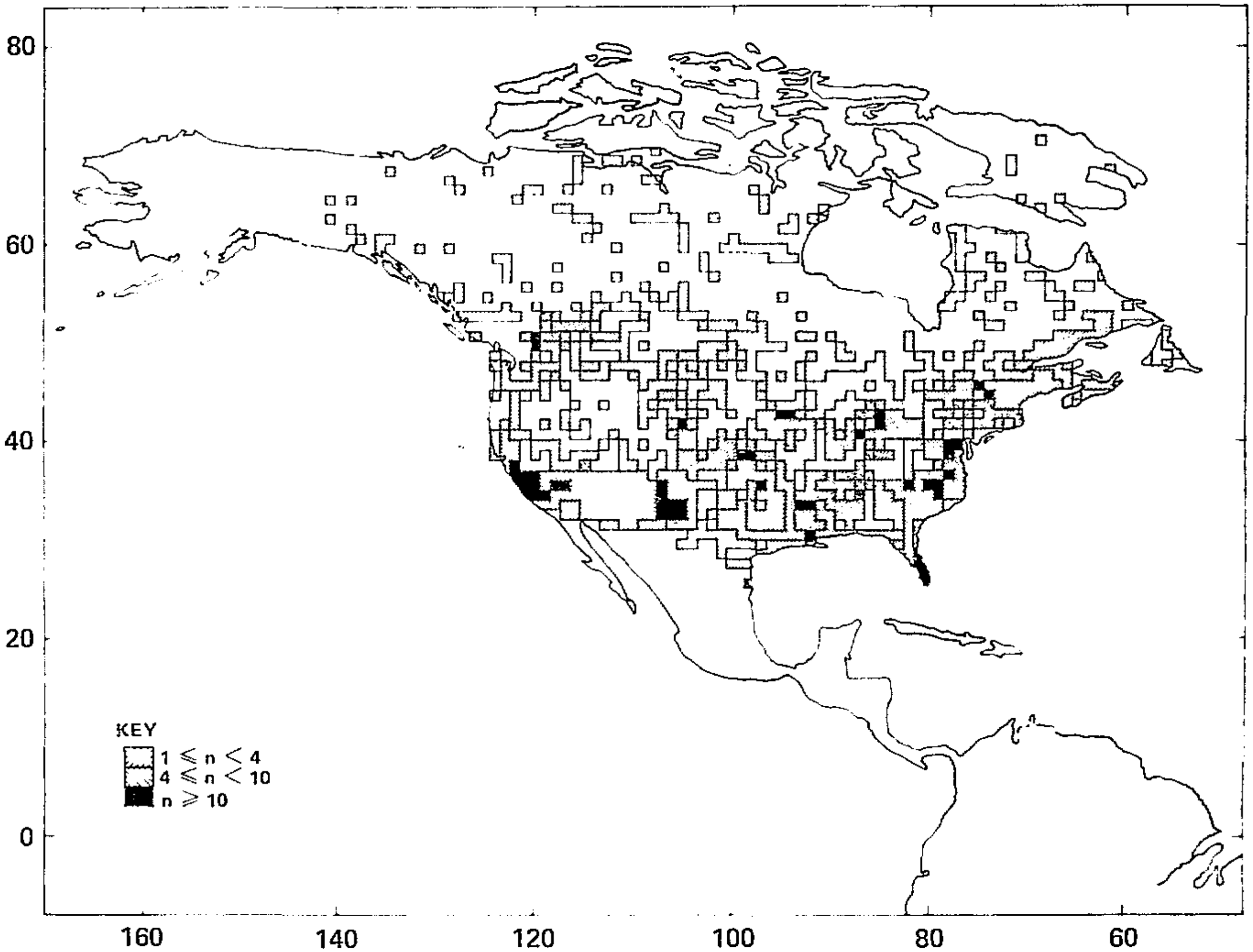


Figure 14. North America — Distribution of Astro-Geodetic Stations — n = Number of Stations per 1° x 1° Square