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FORMATION OF THE 0.511-MeV LINE IN SOLAR FLARES

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ABSTRACT

The gamma-ray line produced at 0.51-MeV in solar flares is the result either of free annihilation of positrons with electrons or of the decay of positronium by 2-photon emission. Positron annihilation from the bound state of positronium may also proceed by 3-photon emission, resulting in a continuum with energies up to 0.51-MeV. Accurate calculations of the rates of free annihilation and positronium formation in a solar-flare plasma are presented. Estimates of the positronium-formation rates by charge exchange and the rates of dissociation and quenching are also considered. The temperature and density dependence of the ratio of 3-photon to 2-photon emission is obtained. For temperatures less than 10^6K, this ratio is found to depend primarily on the density of the annihilation region. The observability of the 3-photon emission is hindered by the flare-produced continuum radiation and other anticipated line emission but aided by the inherent delay in the production and slowing-down time of the positrons. Asymmetrically-broadened 0.51-MeV line emission could be detected at times late in solar gamma-ray events when the continuum and prompt line emission have essentially disappeared. When the ratio of free electrons to neutral atoms in the plasma is approximately unity or greater, the Doppler width of the 0.51-MeV line is a function of the temperature of the annihilation region. For the small ion densities characteristic of the photosphere, the width is predominantly a function of the density. For temperatures greater than 10^6K, the rate of positron conversion into γ-rays is the free annihilation rate. At lower temperatures this conversion rate is determined by the rates of thermalization and positronium formation, which is faster than free annihilation by at least two order of magnitude.

1. INTRODUCTION

Gamma-ray line emission at an energy of approximately 0.51 MeV was observed by Chupp et al.(1973,75) from the 1972 August 4 and August 7 solar flares. This line is believed to be due to the annihilation of positrons which result from the decay of π^+ mesons and radioactive nuclei produced in nuclear reactions of flare accelerated particles with constituents of the solar atmosphere (Lingenfelter and Ramaty 1967; Ramaty and Lingenfelter 1973; Ramaty, Kozlovsky and Lingenfelter 1975).

The formation of the 0.51-MeV line depends on the sources of the positrons, on the propagation of the positrons in the solar atmosphere, on the density and temperature of the ambient medium in which the positrons slow down and annihilate, and on the mode of positron annihilation since the positrons may annihilate freely or from a ground state of positronium.

Four distinct observable parameters related to the annihilation of positrons may yield information about the ambient medium or the source of positrons. These are the width of the 0.51-MeV line, the strength of this line relative to the intensity of other gamma lines, the strength of the three gamma continuum below 0.51-MeV which comes from triplet positronium decay, and the time dependence of the 0.51-MeV line. The time dependence has already been analyzed by Wang and Ramaty (1975),and the number of positron emitters relative to the number of neutrons has been related to the spectrum of primary particles (Ramaty, Kozlovsky, and Lingenfelter, 1975). In the present paper we investigate in detail the slowing down and annihilation of positrons and the formation of positronium in a solar flare plasma. We then consider how the width of the 0.51-MeV line and its strength relative to the 3γ continuum from positronium decay depend on the temperature and density of the medium in which the positron comes to rest.

Positrons in a solar flare are created with energies of the order of 1 MeV or higher (e.g. Ramaty, Kozlovsky and Lingenfelter 1975). They subsequently lose energy at least as fast as the standard ionization energy loss of charged particles in matter. Approximate calculations indicate that only 10% or fewer of the positrons undergo annihilation in flight before slowing to thermal velocities, if they are in a plasma, or before slowing to velocities at which positronium formation can occur in the neutral gas. In general, the Doppler shift of the gammas from the free annihilation of high energy positrons will be so great that the gammas will not be counted as part of the 0.51-MeV line.

Positronium, denoted by the symbol Ps, is the bound state of a positron and an electron. In a fully ionized plasma it can be formed by the two body radiative recombination reaction with free electrons, or by three body processes only important at very high densities.

The positronium atoms are formed in triplet or singlet spin states according to the statistical ratio of 3:1. If undisturbed for times long compared to their natural decay times the positronium decays via the emission of either 3γ's or 2γ's. $^3Ps \rightarrow 3\gamma$ with a time constant of $\tau_3 = 1.4 \times 10^{-7}$ sec and $^1Ps \rightarrow 2\gamma$ with a time constant of $\tau_1 = 10^{-10}$ sec, for triplet and singlet states respectively. The 3γ decays produce a gamma ray continuum with a maximum energy of 0.51-MeV while the 2γ decays appear as a discrete line. Because the cross section for free annihilation of an electron positron pair in a triplet state is more than two orders of magnitude smaller than for a singlet pair, the observation of a three-gamma continuum is proof of positronium formation and decay.

Positronium once formed in a plasma can be dissociated by collisions with free protons or electrons whenever the energy available in the center-of-mass frame is above 6.8 eV. Since the atomic cross sections are typically of the order of πa_o^2, the critical density for dissociation of the triplet state is about 10^{14} cm^{-3}, while the singlet positronium begins to be broken up at $N \simeq 10^{17}$ cm^{-3}. Another process which depletes positronium in the triplet state is the flipping of the spin from triplet to singlet through elastic or inelastic collisions with free electrons. The density at which this quenching occurs is also of the order of 10^{14} cm^{-3}.

Since temperatures high enough to maintain a fully ionized plasma do not ordinarily occur in regions of the solar atomsphere when densities are as high as 10^{14} cm^{-3}, dissociation and quenching processes in a fully ionized plasma are not considered in the present work.

Another set of processes characterize annihilation of positrons in a neutral gas. While the energy loss of positrons in a plasma is due to many small angle scatterings with electrons in the electron gas, in a neutral medium energy loss takes place through ionization and excitation of atoms or ions. Once the energy of a positron has dropped to about 75 eV, the cross section for positronium formation through charge exchange with atoms becomes significant. In fact it dominates the energy loss cross sections below 30 eV. As we shall show, in a neutral medium nearly all of the positrons form positronium before dropping below 6.8 eV. However, if sufficiently high atomic densities obtain, $N \gtrsim 10^{14} cm^{-3}$, the longer lived triplet positronium will be dissociated by atomic collisions or will be quenched through spin flip collisions with unpaired atomic electrons. The latter process depletes even those ^3Ps atoms which end up with energy below 6.8 eV, the threshold energy for dissociation.

The effect of the dissociation of the triplet state is to increase the number of 2γ annihilations of the positrons either by the formation of ^1Ps subsequent to break-up or by the free annihilation of positrons that have fallen to energies less than 6.8 eV.

The breakup reaction is a very efficient energy loss mechanism since a positron, which goes through the cycle of charge exchange followed by breakup, emerges with less than half of its original energy. The spin flip reaction also enhances the 2γ annihilations through conversion of ^{3}Ps to ^{1}Ps. At densities above 10^{17} cm^{-3} breakup of ^{1}Ps becomes important, as is observed in a typical laboratory situation. Since it is extremely unlikely that solar positrons can penetrate to regions of such high density, singlet breakup is ignored in the present work.

Another quenching effect, which could deplete the relative fraction of ^{3}Ps, is spin flip caused by large magnetic fields. In work reported by Wallace (1960) it was shown that magnetic fields in excess of 7 kilogauss are needed to reduce the population of ^{3}Ps states by approximately 30%, at which level the effect due to the magnetic field saturates. Such fields far exceed those expected in low-density solar regions where other quenching mechanisms are negligible. Since the effects of magnetic quenching in all cases are expected to be small, they have not been considered in the present work.

To summarize, in a fully ionized plasma, positrons thermalize before either free annihilating or forming positronium. Positronium is formed by radiative recombination. In a neutral gas positronium formation through charge exchange takes place well before thermalization can occur. The solar atmosphere, however, is neither fully ionized nor completely neutral. In the present paper, therefore, we investigate the fate of positrons in a partially ionized medium. We limit our detailed calculations to a hydrogen plasma and we subsequently consider the perturbing effects of heavier ions which are found to be negligible.

In Section 2 accurate cross sections for the free annihilation, radiative recombination, and charge exchange processes are presented. In Section 3 we evaluate the rates of positronium formation and the resultant energy distributions of the positronium atoms in a partially ionized medium. The results of this section apply to a medium in which the ambient free electrons have a Maxwell-Boltzmann distribution of finite temperature but the density of the medium is sufficiently low such that positronium atoms decay without further collisions following their formation.

In Section 4 we additionally consider the breakup of positronium as a function of density. The resultant energy distributions of the positronium atoms and the relative numbers of triplet to singlet positronium decays are evaluated for temperatures characteristic of the solar photosphere.

In Section 5 we calculate the width of the 0.51-MeV line and its strength relative to the 3γ continuum as functions of temperature and density in the annihilation region. We also consider the observability of the 3γ continuum in the presence of other solar flare continuum radiations and we summarize our results.

2. FREE ANNIHILATION, RADIATIVE RECOMBINATION, AND CHARGE EXCHANGE

Free Annihilation

The annihilation cross section of positrons with free electrons, under the assumption that the initial particles are plane waves and that the phase shifts introduced by Coulomb interactions are negligible, is given by Heitler (1954).

$$\sigma(\text{plane wave}) = \frac{\pi r_o^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right], \quad (1)$$

where γmc^2 is the total energy of the positron in the electron's rest frame, and $r_o = 2.82 \times 10^{-13}$ cm is the classical radius of the electron. In the nonrelativistic region where most of the annihilations take place, Equation (1) reduces to σ(plane wave, non-relativistic) = $r_o^2 c/u$, where u is the positron's velocity relative to the free electron. The rate of annihilation in this case is independent of energy and given by

$$\lambda_{fa}^{o} = N_e \pi r_o^2 c \quad , \quad sec^{-1} \quad , \quad (2)$$

where N_e is the density of electrons.

Coulomb corrections are important only in the nonrelativistic region. When they are taken into account (Landau and Lifshitz 1958), the nonrelativistic annihilation cross section can be written as

$$\sigma(u) = \pi r_0^2 \frac{2\pi\alpha \frac{c^2}{u^2}}{1 - \exp\left(-2\pi\alpha \frac{c}{u}\right)} \quad . \tag{3}$$

where α is the fine structure constant. Using this cross section, the annihilation rate of positrons in thermal equilibrium with an isotropic electron plasma of temperature T is given by

$$\lambda_{fa}(T) = N_e \int_0^\infty \int_0^\infty d^3 v_1 \, d^3 v_2 \, f(\vec{v}_1) \, f(\vec{v}_2) \, u\sigma(u) \quad . \tag{4}$$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocity vectors of arbitrary electrons and positrons, and $f(\vec{v}) = (m/2\pi kT)^{3/2} \exp(-mv^2/2kT)$ is the Maxwell-Boltzmann distribution for particles of mass, m, in thermal equilibrium. Equation (4) can be reduced to a single integral by transforming the variables of integration to $\vec{V} = (\frac{1}{2})(\vec{v}_1 + \vec{v}_2)$ and $\vec{u} = \vec{v}_2 - \vec{v}_1$. We obtain (Huang 1963)

$$\lambda_{fa}(T) = \lambda_{fa}^{0} \, 4\pi \left(\frac{m}{4\pi kT}\right)^{3/2} \int_0^\infty u^2 du \, \frac{2\pi\alpha \frac{c}{u} \exp\left(-\frac{mu^2}{4kT}\right)}{1 - \exp\left(-2\pi\alpha \frac{c}{u}\right)} \quad . \tag{5}$$

The rate coefficients, given by $\lambda_{fa}(T)/N_e$, have been evaluated by numerical integration of Equation (5) and are presented in Table 1. As is shown in Figure 1, the effects of Coulomb attraction are most significant at low temperatures. The corrected free-annihilation rate asymptotically approaches the constant approximate value given by Heitler for temperatures $\gtrsim 10^7$K.

TABLE 1. Free-Annihilation Rate Coefficients

T in K	Rate Coefficient in 10^{-14} cm^3/sec
10^4	14.9
2×10^4	10.5
6×10^4	6.1
10^5	4.7
2×10^5	3.4
6×10^5	2.1
10^6	1.8
2×10^6	1.4
4×10^6	1.2
9×10^6	1.0

Positronium Formation by Radiative Recombination

Positronium formation by radiative recombination is essentially the same process as radiative recombination of hydrogen except for the difference in the positron and proton rest masses. Seaton (1959) has calculated the capture rates for hydrogenic ions, based on an asympototic expansion derived by Menzel and Pekeris (1935) and corrected by Burgess (1958). In the present work, the formulation and numerical calculations of Seaton (1959) are used to determine the temperature-dependent positronium-formation rate. From Equations (1) through (7) of Seaton, the total radiative-capture rate coefficient is given by

$$\alpha = \frac{2^5 (2\pi)^{3/2} e^6}{c^3 h \, 3\sqrt{3} \, m} \left(\frac{1}{mkT}\right)^{1/2} \sum_n C_n \left(\frac{mZ^2}{T}\right), \tag{6}$$

for hydrogenic ions where the C_n are the coefficients containing the contributions from all the angular momentum states for each principal quantum number, n.

According to Seaton (1959), the asympototic expansion enables the rates to be calculated with errors not exceeding 2% for temperatures of order 10^4K, but possibly as great as 20% for temperatures of order 10^6K or greater. It should be noted that the widely-used approximation derived by Kaplan and Pikel'ner (1970) is obtained from a similar expression for the radiative recombination rate which is in agreement with that from Seaton (1959). But the expression for the cross-section given by Kaplan and Pikel'ner (1970) in their Equation (I.21) appears to be in error, that is, too low by a factor of two. Since the expression presented by Kaplan and Pikel'ner in Equation (I.22) is in agreement with Seaton, it may be assumed that the discrepancy is only a typographical error.

In order to calculate the radiative capture rate for positronium from Equation (6), the appropriate reduced mass, $\mu = m/2$, must be substituted for each m. This is equivalent to putting a factor of $2^{3/2}$ in the coefficient before the sum, and selecting the $C_n(\lambda)$ from Seaton's tabulation for which the arguments are equal. That is

$$\frac{m/2}{T} = \lambda \,. \tag{7}$$

for positrons in thermal equilibrium.

The results of this calculation are presented in Table 2, and are shown for comparison with the free annihilation rates in Figure 1. Also for comparison the radiative capture rate calculated by Nieminen (1967) is shown as a dashed line. Nieminen's calculation of the radiative capture rate is performed as a function of velocity which he then relates to an equivalent temperature. When Nieminen's values are folded into the Maxwell-Boltzmann velocity distribution, the resulting rate coefficient is in complete agreement with the temperature-dependent, radiative-capture rate presented here. From Figure 1 it can be seen that including the effects due to the Coulomb interaction and the Maxwell-Boltzmann distribution yields a small change in the temperature at which the rates are equal, but yields a large increase in the calculated rates.

TABLE 2. Radiative Recombination Rate Coefficients

T in K	C_n*	Rate Coefficient in cm^3/sec
5×10^3	1.998	1.65×10^{-12}
2.5×10^4	1.324	4.89×10^{-13}
5×10^4	1.055	2.76×10^{-13}
2.5×10^5	0.529	6.10×10^{-14}
5×10^5	0.364	3.00×10^{-14}
2.5×10^6	0.127	4.68×10^{-15}

*In Seaton's notation, $C_n = \frac{x_n}{n} S_n(\lambda)$.

From Figure 1 we conclude that in a fully ionized hydrogen plasma free annihilation and radiative recombination are of comparable size at $T \simeq 8 \times 10^5$K. At higher temperatures free annihilation dominates and at lower temperatures radiative combination becomes the dominant process.

Positronium Formation by Charge Exchange

The alternate reaction producing positronium is the charge exchange of an electron by a positron with atoms or ions in the solar atmosphere. Charge exchange with heavy ions is always suppressed by their low number density relative to hydrogen. At high temperatures, $T \gtrsim 10^6$K, this reaction is additionally suppressed by the high ionization potentials of the residual ions. Estimates indicate that this process cannot compete with the free annihilation rates presented previously. At lower temperatures, $T \lesssim 10^6$K, the only effect of charge exchange with heavy ions is to increase slightly the rate of positronium formation. Therefore, for simplicity, we consider charge exchange with hydrogen only.

Published cross-sections for charge exchange with neutral hydrogen vary greatly, ranging from peak values of 0.065 (Fels and Middleman 1967) to 3.6 (Cheshire 1964) in units of πa_o^2, where a_o is the Bohr radius. A recent calculation by Chan and Fraser (1973) obtains an S-wave contribution with a peak value of $3 \times 10^{-3} \pi a_o^2$, for an energy slightly above the threshold energy of 6.8 eV. This value, obtained with a difficult but presumably accurate variational technique is significantly lower than that expected from partial wave calculations employing the Born approximation. As a best estimate of the charge exchange cross section of positrons with neutral hydrogen, we have used a calculation by Drachman et al. (to be published) which uses the Born approximation for all partial waves besides the S and P. The S-wave result of Chan and Fraser is used and the P-wave is modified by scaling the Born partial wave to agree with that calculated by Chan and McEachran (to be published) at one energy just above threshold. The resultant cross section rises more slowly than the distorted-wave calculation according to Massey and Mohr (1954) but peaks at approximately $3.3 \pi a_o^2$ at about 14 eV positron energy. The values of the cross section for charge exchange with hydrogen as calculated by Drachman et al. and used in the present work are given in Table 3.

TABLE 3. Charge Exchange Cross Sections on Atomic Hydrogen

Positron Energy (eV)	Cross Section (πa_o^2)
7.5	0.50
10.0	2.4
15.0	3.3
25.0	2.0
50.0	0.41
75.0	0.12

Detailed calculations of rate coefficients based on the cross sections given in Table 3 are complicated by the possibility that positrons may form positronium before they thermalize, or that once thermalized, their energies may be below the positronium-formation threshold. These questions are treated in the next section. Nevertheless an estimate of the significance of the charge-exchange process may be obtained by noting that at 10 eV, for example, the charge exchange rate coefficient is approximately 8×10^{-13} $cm^3 sec^{-1}$ for a relative neutral hydrogen abundance at 10^5K of 2×10^{-5} (Gabriel 1971). This rate coefficient is larger by about an order of magnitude than the radiative-recombination rate coefficient shown in Figure 1 at this temperature.

3. POSITRONIUM FORMATION AND ANNIHILATION IN A LOW DENSITY PLASMA

The calculations presented in the previous section demonstrate that the fate of positrons in a fully ionized plasma at densities less than 10^{14} cm^{-3} is completely determined by the ratio of the thermally averaged radiative recombination rate to the free annihilation rate. However, because positrons result from hadronic reactions with relatively long interaction lengths, a more realistic assumption may be that the annihilations occur in the chromosphere or photosphere. Such considerations necessitate the investigation of positron interactions in media with temperatures as low as 10^4K or somewhat less and with a wide range of densities and ion fractions. In this section we re-examine the assumptions implicit in the application of thermally averaged rates to determine the fate of positrons in solar flares. Calculations are presented showing to what extent positrons thermalize before annihilating or forming positronium. For those positrons whose energy falls below 6.8 eV, the threshold for charge exchange with a hydrogen atom, the competing processes of free annihilation and thermalizing up (populating the high energy tail of a thermal distribution) are evaluated. To facilitate these calculations the effects of dissociation and quenching are deferred to the Monte Carlo treatment described in Section 4.

Energetic charged particles, which enter a plasma of lower characteristic thermal energy, slow down due to long range Coulomb interactions. One method of treating the energy loss is by use of a Fokker-Planck equation. The Fokker-Planck equation neglects the generation of plasma waves but includes the Coulomb interaction. Neglect of plasma oscillations in a thermal plasma underestimates the energy loss rate by about 20%. See, for example, Sigmar and Joyce (1971). In order to calculate the relative rates of positronium formation, terms which represent charge exchange with neutral hydrogen and radiative capture must be added to the Fokker-Planck terms. The resulting equation is

$$\frac{\partial f}{\partial t}(\vec{v},t) = FP\left\{f(\vec{v},t)\right\} - v\left(N_e \sigma_{rr} + N_n \sigma_{ce}\right) f(\vec{v},t), \tag{8}$$

where $f(\vec{v},t)$ is the distribution function of the particles entering the plasma; $FP\{\}$ is the Fokker-Planck operator (Rosenbluth et al. 1957, Montgomery and Tidman 1964); N_n is the density of neutral atoms; N_e is the plasma density; σ_{rr} is the radiative capture cross section (Nieminen 1967); and σ_{ce} is the charge exchange cross section from Table 3. Details of this calculation have been presented by Joyce et al (University of Iowa Research Report 76-2, unpublished).

In this treatment, the energy loss due to collisions with the neutral gas is neglected which is a reasonable approximation if $N_e/N_n \gtrsim 0.1$. The two cross sections for positronium formation dominate the positron loss rates for the range of temperatures and ion densities considered. We have considered plasmas with the following properties: the temperature of the ambient plasma and neutral gas is 1.16×10^4K corresponding to a kinetic energy of 1 eV. The number density of the neutral gas plus the plasma is 10^{12} cm^{-3} or 10^{14} cm^{-3}. The ratios of plasma to neutral gas are 1 or 0.1. The initial distribution function of the positrons, $f(\vec{v},o)$, is a delta function corresponding to an energy of 50 eV. The results of the integrations of Equation (8) are summarized in Figures 2 through 7.

Figure 2 shows the evolution of the positron velocity distribution, $N(\vec{v},t) = 4\pi v^2 f(\vec{v},t)$ for equal ion and neutral densities. The total number of positrons has been decreased by about 28% by the time the distribution is almost thermal ($t \simeq 3 \times 10^{-7}$ sec). Since the charge exchange cross section drops to zero at a threshold energy of 6.8 eV, positronium formation continues quite slowly after the bulk of the particles have energies characteristic of the plasma thermal energy.

Figure 3 shows the positron evolution for $N_e/N_n = 0.1$ and $N_e + N_n = 10^{14}$. In this case 94% of the positrons form positronium by charge exchange before the distribution thermalizes.

Figure 4 shows the fraction of positronium having formed as a function of time for two ratios of plasma to neutral density. There are two regions of development. First, the positronium fraction rises rapidly until the remaining positrons have thermalized. This is followed by a region of slow increase which is approximately linear in time. By selectively excluding some of the processes in the equation, we have determined that the slow rise is due to charge exchange of positrons in the thermal tail of the distribution which extends to the energy region above the charge exchange threshold. As positrons in the thermal tail are lost through positronium formation, the tail is repopulated by positrons which gain energy.

For $N_e/N_n = 0.1$, most of positronium is formed in the first stage so that the second stage is of no great importance. The duration of the first stage is approximately 10^{-6} sec, consistent with the charge exchange cross section given in Table 3.

For $N_e/N_n = 1.0$, a large fraction of the positronium is formed in the second stage. Since the number of positronium atoms exhibits linear growth, we have extrapolated the curve for $N_e/N_n = 1.0$ to determine the time for which all the positrons can be converted to positronium. The results of this extrapolation are presented in Table 4.

TABLE 4. Time to Convert all Positrons of Initial Energy 50 eV to Positronium in a 1-eV Plasma with $N_e = N_n$.

$N_e + N_n$ in cm^{-3}	Positronium Formation Time in sec
10^{12}	2.3×10^{-2}
10^{13}	2.4×10^{-3}
10^{14}	2.8×10^{-4}

The characteristic times to annihilate by free annihilation or to form positronium by radiative recombination in a medium of any specific density and temperature can be calculated from the corresponding rate coefficients presented in Tables 1 and 2. A comparison of these times for a temperature of 1.16×10^4K and the densities considered in Table 4 shows that the charge exchange process, including the time to thermalize up, dominates over the competing processes of free annihilation and radiative recombination.

Figures 5 and 6 show the distribution of energies at which positronium has formed as a function of time. At large times a sharp peak begins to develop at low energies. This is due to positrons being thermalized upward until they are above the threshold energy for the charge exchange cross section. This effect can be seen more clearly if the plasma temperature is slightly larger. Figure 7 was calculated for a plasma with kT = 4 eV. Since more of the thermal tail overlaps the charge exchange threshold, the development of the peak occurs on a shorter time scale than in the 1 eV plasma. In all of these calculations, the energy loss due to elastic and inelastic collisions with neutrals has been neglected. For the case $N_e = N_n$ this is a good approximation, but for smaller ionization fractions, the neutral particle interactions can cause a significant amount of energy loss. The effect of including energy loss on neutrals and the generation of plasma waves, mentioned previously would result in a faster energy loss so that the mean energy of positronium at formation would be less than predicted from these calculations.

The results of the calculations presented in this section are applicable to positron interactions in media for which the sum of the electron density and the neutral ion density, $N = N_e + N_n$, is bounded by $N \lesssim 10^{14}$ cm^{-3}. For the chromosphere, this density is a reasonable upper limit and justifies the neglect of dissociation of positronium by collisions with electrons, protons, or hydrogen atoms. On the other hand, the observed rise time of the 0.51-MeV line implies densities, $N \gtrsim 10^{12}$ cm^{-3} in the annihilation region (Wang and Ramaty 1975). In the chromosphere, densities in the range 10^{12} to 10^{14} cm^{-3} have been inferred from analyses of white-light flares (Hudson 1972, Machado and Rust 1974). However, because the rise-time observations set only lower limits on the density, annihilation in denser regions such as the photosphere is also possible. In determining the parameters associated with positronium formation in media for which $N \gtrsim 10^{14}$ cm^{-3}, the effects of dissociation and quenching may not be neglected. In the next section these effects as well as approximations for the energy loss processes with neutral atoms and plasma are considered.

4. POSITRONIUM FORMATION, ANNIHILATION, AND DISSOCIATION IN A LOW TEMPERATURE PLASMA

In this section we present a simplified Monte-Carlo calculation of the fate of a positron annihilating in a predominantly neutral gas of temperatures $kT \lesssim 1$ eV and density in the range $10^{12} \lesssim N \lesssim 10^{17}$ cm^{-3}. The density of 10^{12} cm^{-3} is inferred from the rise time of the 0.51 MeV line as discussed in the previous section. Densities up to 10^{17} cm^{-3} correspond to photospheric annihilation regions. Indeed, in the exposition of their model of white light flares, Najita and Orrall (1970) show that energetic ions between 10 MeV and 1 GeV release most of their energy to the ambient gas in the photosphere. As far as positron reactions are concerned, that region is more nearly a neutral atomic gas than a plasma.

The principal mechanism for energy loss by positrons interacting with neutral constituents of the ambient medium is the ionization or excitation of hydrogen atoms. Omidvar (1965) has shown that in the ionization process, the electron carries off a kinetic energy equal, on average, to ¼ its binding energy of 13.6 eV. This means that the positron loses about 17 eV in each ionizing collision. Since the $1S \rightarrow 2P$ excitation costs 10.2 eV, we have taken the average loss to be about 13.6 eV and the energy loss cross section to be the sum of the separate cross sections for ionization and excitation of hydrogen.

In addition to the neutral atoms there are also free electrons and ions in the solar atmosphere. As we saw in Section 3, the scattering of positrons from free electrons is a very efficient energy loss process. To compare it with the atomic processes we can use the expression listed by Green and Wyatt (1965). For the effective cross section for the energy loss ΔE by positrons of energy E in a plasma, we take

$$\sigma = \frac{2\pi e^4}{E\,\Delta E} \ln\left(\lambda_D / b\right). \tag{9}$$

While the effective cross section is written as if it were for a single scattering event, it is actually built up from many small angle scatterings. λ_D= the Debye length, the range of Coulomb force being made finite by shielding, and b is the distance of closest approach. One finds that for 50 eV positrons with $\Delta E = 13.6$ eV, for example, the effective cross section is two order of magnitude larger than the cross sections for ionization and excitation of hydrogen. Thus for $N_e/N \gtrsim 10^{-3}$ the free electrons cannot be ignored. Since the standard solar atmosphere with $10^{12}\mathrm{cm}^{-3} \lesssim N \lesssim 10^{17}\mathrm{cm}^{-3}$ comprises relative electron densities of this magnitude, we took into account loss against the free electrons by treating the cross section of Equation (9) in the same way as the ionization and excitation cross sections.

Our simplfied Monte Carlo calculation has been carried out by assigning each process an average energy loss so that only a small set of positron or positronium kinetic energies are associated with each initial positron energy. Then the probability for either triplet or singlet positronium decay can be written as a finite series of terms, each term being equalto the probability of occurrence for the sequence of reactions it represents. The four processes are: energy loss to neutral hydrogen atoms or free electrons, $\Delta E = 13.6$ eV; positronium formation through charge exchange, $\Delta E = 6.8$ eV; breakup of the positronium through collisions with hydrogen atoms, $\Delta E = (E\,(Ps) + 6.8)/2$ eV; and spin flip of triplet to singlet, $\Delta E = 0$.

From the Monte Carlo calculations we obtain the probabilities that a positron decays via triplet positronium, $P(^3Ps \rightarrow 3\gamma)$, or via singlet positronium, $P(^1Ps \rightarrow 2\gamma)$. A positron that falls below 6.8 eV cannot form positronium so that only free annihilation is possible unless the ion density is sufficient for it to thermalize up (see Section 3). The probability of free annihilation is calculated from

$$P(fa) = 1 - P(^3Ps \rightarrow 3\gamma) - P(^1Ps \rightarrow 2\gamma) . \tag{10}$$

The rates for each process, except the triplet decay rate, depend on density and energy through the relation

$$\lambda = N u \sigma \quad , \tag{11}$$

where N is the number density of target particles, u is the relative velocity of the reactants, and σ is the total cross section for that process. As previously noted, for densities characteristic of the solar atmosphere, only the triplet positronium will be broken up or quenched since the singlet decay rate is large compared to the rates of the alternative atomic processes.

The cross sections used in the calculation are taken either from electron scattering experiments on hydrogen atoms or from theoretical calculations. The cross section for the ionization of hydrogen by positrons is taken to be equal to the electron ionization cross sections reported in Golden and McGuire (1974). The cross section for the 1S→2P excitation of hydrogen by positrons was taken to be the same as t experimental electron cross section to be found in Geltman (1969). The breakup cross section was taken from Massey and Mohr (1954) while the spin flip cross section is a new result of Hara and Fraser (1975). The charge exchange cross section is from Table 3. The temperature is assumed to be so low that the thermal motions in the ambient medium are negligible relative to the kinetic energies of the incident positrons. The relative ion densities are taken from the standard solar atmosphere according to Allen (1973) for each total density considered in these calculations.

Our results are displayed in Figure 8. At low densities, the positrons annihilate through positronium decay with a 3:1 triplet to singlet ratio. As the density increases through 10^{15} cm^{-3} the triplet positronium is either broken up or quenched through spin flip. In either case mostly singlet positronium is produced although the number of free positrons with energies less than 6.8 eV increases. The probability of free annihilation has probably been overestimated since we have ignored those positrons which replenish the high-energy thermal tail above 6.8 eV. However, this error does not affect the estimate of the number of 2γ photons per positron, $N_{2\gamma}$, since

$$N_{2\gamma} = 2\left[P({}^1Ps \rightarrow 2\gamma) + P(f_{\gamma})\right] . \tag{12}$$

$N_{2\gamma}$ varies between 0.5 for $N = 10^{12}\ cm^{-3}$ and 2.0 for $N = 10^{17} cm^{-3}$.

At the lower densities the ^1Ps is formed, on average, at higher kinetic energies since it does not arise from the breakup of ^3Ps and the subsequent ^1Ps formation. Since the decay rate is too rapid for thermalization of the positronium, our Monte Carlo calculation also enables us to find the average ^1Ps kinetic energy before decay. The average energy is shown in Figure 9, from which it is clear that the mean energy decreases with increasing density.

If we assume that positrons which fall below 6.8 eV gradually thermalize, at $T \lesssim 1$ eV, most will free annihilate with the center of mass motion of the $e^+ e^-$ pair being determined by the Fermi motion of the 1S hydrogenic electron. We can assign an approximate kinetic energy of 6.8 eV to this motion. The dotted curve in Figure 9 shows the average kinetic energy for all 2γ sources.

5. DISCUSSION

From the results of the calculations presented in the preceding sections, we can describe the fate of a positron in a solar flare plasma.

For temperatures greater than 10^6K, positrons annihilate via free annihilation at a rate of about $10^{-14} N_e \text{ sec}^{-1}$ as can be seen in Figure 1. For this case the width of the 0.51 MeV line is determined by the center of mass motion of the $e^+ e^-$ pair which is a function of the temperature of the medium. The full-width at half maximum of this line, due to thermal broadening, is given by

$$\Delta E_\gamma = 1.1 \ T_4^{\frac{1}{2}} \ (\text{keV}) \ , \tag{13}$$

where T_4 is the temperature in units of 10^4K. For $T > 10^6$K, $\Delta E_\gamma > 11.0$ keV. But because solar flare plasmas are not expected to be hotter than $\sim 3 \times 10^7$K (Neupert, 1968), $\Delta E_\gamma < 35$ keV.

For temperatures just below 10^6K, radiative recombination dominates over free annihilation. The relative importance of radiative recombination and charge exchange is determined by the residual neutral hydrogen abundance. For the fractional neutral densities characteristic of the quiescent solar atmosphere (Gabriel 1971), charge exchange is expected to be the dominant process.

At temperature below a few times 10^5K, positronium formation through charge exchange is the dominant reaction through which positrons annihilate. The relative rates of the competing processes of slowing down and of forming positronium depend primarily on the fractional ion density of the annihilation region.

When $N_e/N_n \gtrsim 0.5$, most of the positrons first thermalize and then form positronium by charge exchange. This sequence of events could be maintained even if the mean evergy of the ambient electrons were less than the threshold for positronium formation (6.8 eV). This result follows from the fact, demonstrated in Section 3, that the rate of upward thermalization is larger than the rates for both free annihilation or radiative recombination. The annihilation rate is determined by the rate of charge exchange ($\sim 3 \times 10^{-8} N_n \sec^{-1}$, see Table 3) if the mean energy is larger than $\sim$6.8 eV, and by the upward thermalization rate ($\sim 10^{-10}$ N_e $\sec^{-1}$, see Table 4), if this energy is less than approximately 6.8 eV. Both these rates are much larger than the free annihilation and radiative recombination rates shown in Figure 1. When the positrons thermalize before forming positronium and the mean energy of the ambient electrons is above 6.8 eV, the width of the 0.51 MeV line is approximately $\Delta E_\gamma \simeq (T_4 - 7)^{\frac{1}{2}}$. When the mean energy is less than 6.8 eV, most of the positrons form positronium at an energy just above threshold. The resultant energy of the positronium is low, of the order of 1 eV. In this case, $\Delta E_\gamma \simeq 1$ keV.

When the fraction of ions is low, $N_e/N_n \lesssim 0.5$, most of the positrons will form positronium before they are slowed below akinetic energy of about 15 to 20 eV for which the charge exchange cross section is a maximum. The rate of annihilation in this case is determined by charge exchange and is approximately $3 \times 10^{-8} N_n \text{ sec}^{-1}$. Since the 2γ decay occurs well before the positronium atom can thermalize, the Doppler broadening is given by

$$\Delta E_\gamma = 0.97 \text{ (Mean Kinetic Energy in eV)}^{\frac{1}{2}} \text{ (keV)} . \tag{14}$$

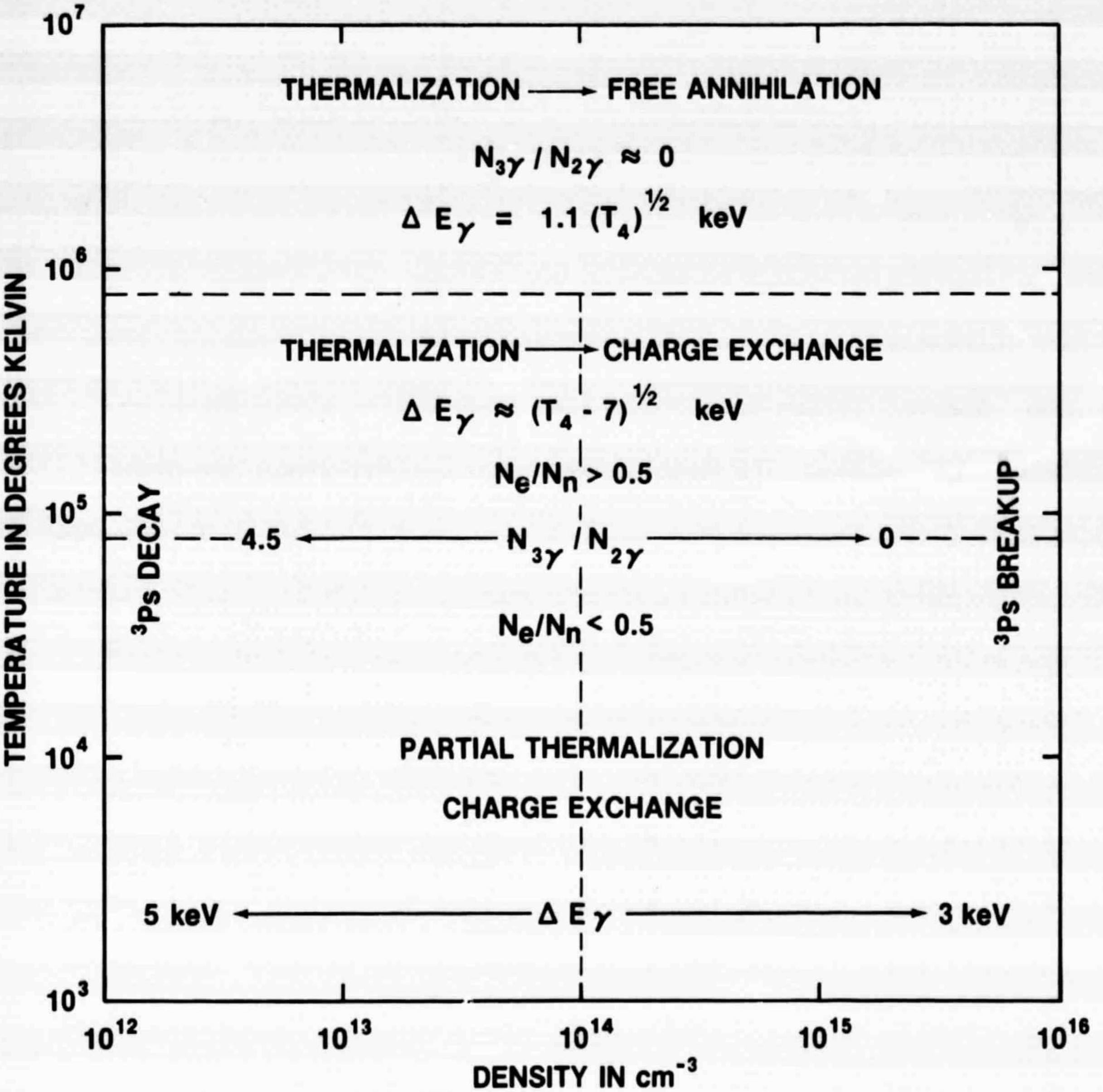
For high densities, $N > 10^{15} \text{ cm}^{-3}$, the mean kinetic energy is approximately 13 eV and hence $\Delta E_\gamma \simeq 3$ eV. Also, breakup of ^{3}Ps results in no 3γ decays. For low densities, $N < 10^{13} \text{cm}^{-3}$, the mean kinetic energy is about 25 eV so that $\Delta E_\gamma \simeq 5$ keV. (See Figure 9.)

The values of the measurable quantities $N_{3\gamma}/N_{2\gamma}$, the ratio of the number of 3γ to the number of 2γ decays, and ΔE_γ that may be expected for various regimes of temperatures and densities are shown in Table 6. For high temperatures, $T > 10^6$K, or high densities, $N > 10^{14} \text{ cm}^{-3}$, $N_{3\gamma}/N_{2\gamma} \rightarrow 0$. Since the temperature in regions of densities greater than 10^{14} cm^{-3}, even during flares, is not expected to be greater than a few times 10^4K, these two cases can be distinguished by the width of the 0.51 MeV line. For high temperatures, $\Delta E_\gamma > 10$ keV; and for high densities, $\Delta E_\gamma \lesssim 3$ keV.

For temperatures lower than 10^6K and densities less than 10^{14}cm^{-3}, all positrons form positronium which annihilates before it dissociates. Thus $N_{3\gamma}/N_{2\gamma} \rightarrow 4.5$. The state of ionization of the medium and its temperature determine the width of the 0.51-MeV line. The largest width, $\Delta E_\gamma \simeq 10$ keV is obtained for $T \simeq 10^6$K, and the smallest width, $\Delta E_\gamma \simeq 1$ keV is obtained for $T \lesssim 7 \times 10^4$K and $N_e/N_n > 0.5$. However, if $N_e/N_n < 0.5$, the width becomes larger, rather than smaller.

The annihilation radiation from decay of the triplet state of positronium could possibly be observed in measurements of the gamma-ray spectrum at energies just below 0.511 MeV with detectors having good energy resolution. Such measurements, however, are severely complicated by the existence of strong continuum emission (Crannell, Ramaty and Werntz 1975). A favorable condition for observing positronium annihilation radiation may arise at the late stages of solar gamma-ray events when the continuum is greatly reduced. This follows from the delayed nature of positron annihilation radiation caused by the long half lives of some of the positron emitters (^{11}C and ^{13}N), and possibly also by the long slowing down times of relativistic positrons from π^+ decay (Wang and Ramaty 1975). Thus, when the numer of accelerated particles in the flare region is already diminished and hence no nuclear reactions and bremsstrahlung are produced, positronium annihilation radiation could be more easily observed.

Table 6. Fate of positrons in a Solar Flare and the Resultant Photon Signature.



7. ACKNOWLEDGEMENTS

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Figure 1. The rates for radiative capture and for free annihilation of positrons as a function of temperature. The solid lines are from the results of the present work. The dashed lines indicate previously published results in which significant physical considerations are neglected.

Figure 2. Positron distribution as a function of velocity in units of v/v_e, where v_e is the velocity of an electron with kinetic energy kT, at various times for $N_e = N_n = 0.5 \times 10^{14}$ cm^{-3} and kT = 1 eV.

Figure 3. Positron distribution as a function of velocity in units of v/v_e, where v_e is the velocity of an electron with kinetic energy kT, at various times for $N_e = 0.1\ N_n$, $N_e + N_n = 10^{14}$ cm^{-3}, and kT = 1 eV.

Figure 4. Fraction of positrons which have formed positronium as a function of time for

$N_e = N_n = 0.5 \times 10^{14}$ cm^{-3} and kT = 1 eV;

and for $N_e = 0.1\ N_n$,

$N_e + N_n = 10^{14}$ cm^{-3}, and kT = 1 eV.

Figure 5. Fraction of positrons which have formed positronium as a function of energy at various times for

$N_e = N_n = 0.5 \times 10^{14}$ cm^{-3} and kT = 1 eV.

Figure 6. Fraction of positrons which have formed positronium as a function of energy at various times for

$N_e = 0.1\ N_n$, $N_e + N_n = 10^{14}$ cm^{-3}, and kT = 1 eV.

Figure 7. Fraction of positrons which have formed positronium as a function of energy at various times for

$$N_e = N_n = 0.5 \times 10^{14}\ \mathrm{cm}^{-3} \text{ and } kT = 4\ \mathrm{eV}.$$

Figure 8. Probabilities of 3γ and 2γ annihilation of positronium as a function of density in the standard solar atmosphere. The fraction of free annihilation may be overestimated since thermalization upward is not considered.

Figure 9. Average kinetic energy of the center-of-mass of the two photons resulting from singlet positronium at decay as a function of the density of the ambient medium. The dashed line includes the two photons from free annihilation.

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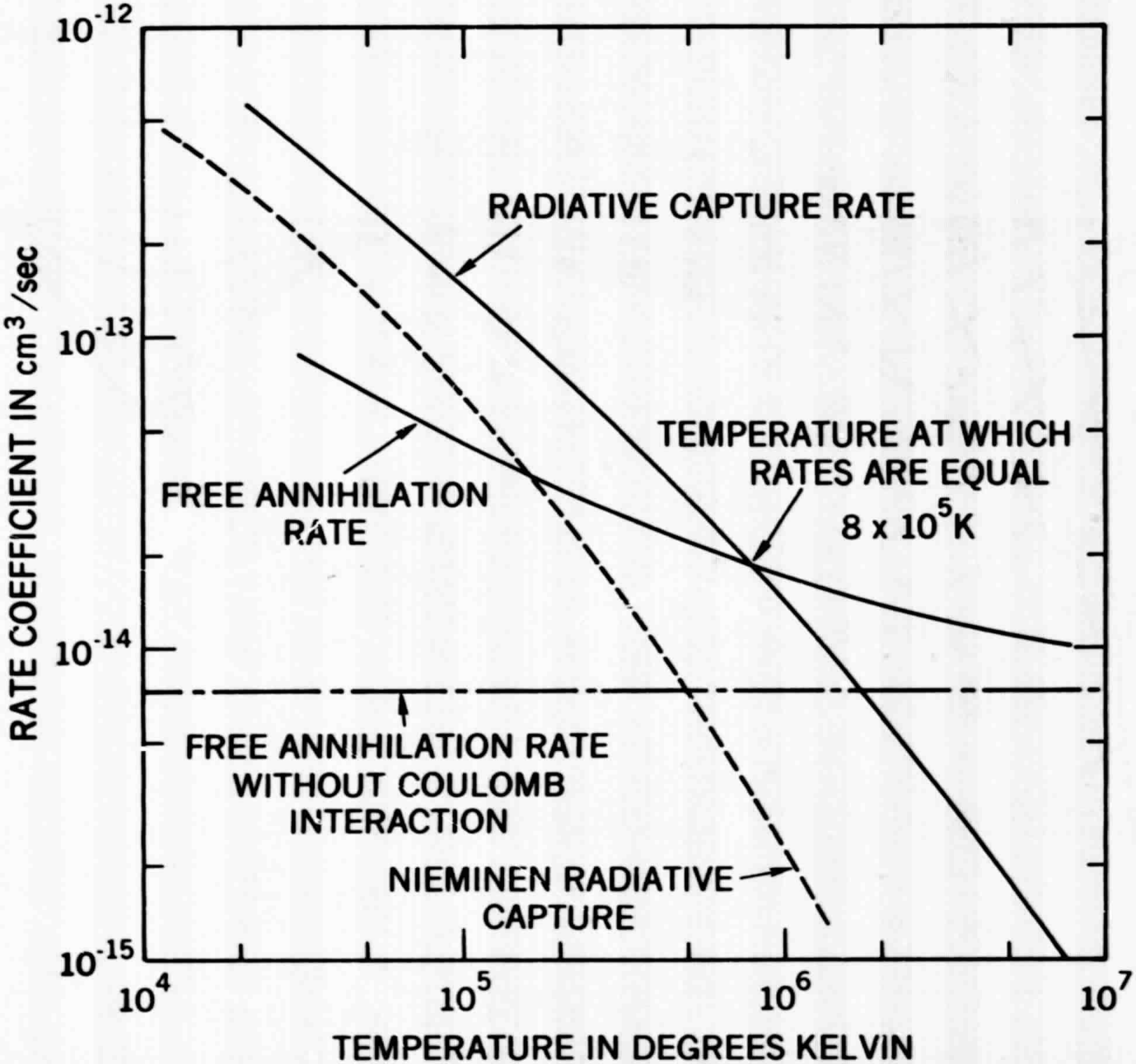


Figure 1.

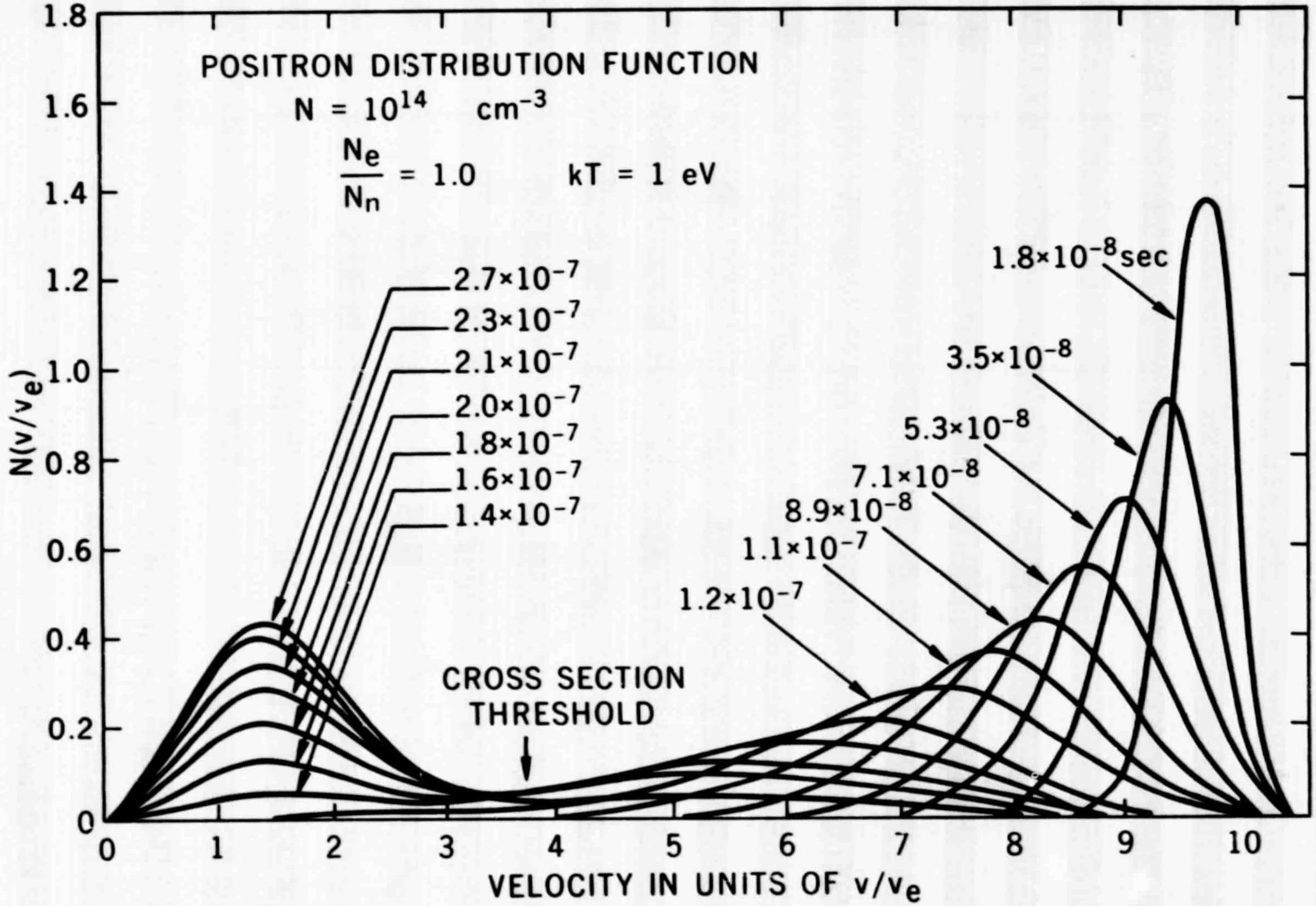
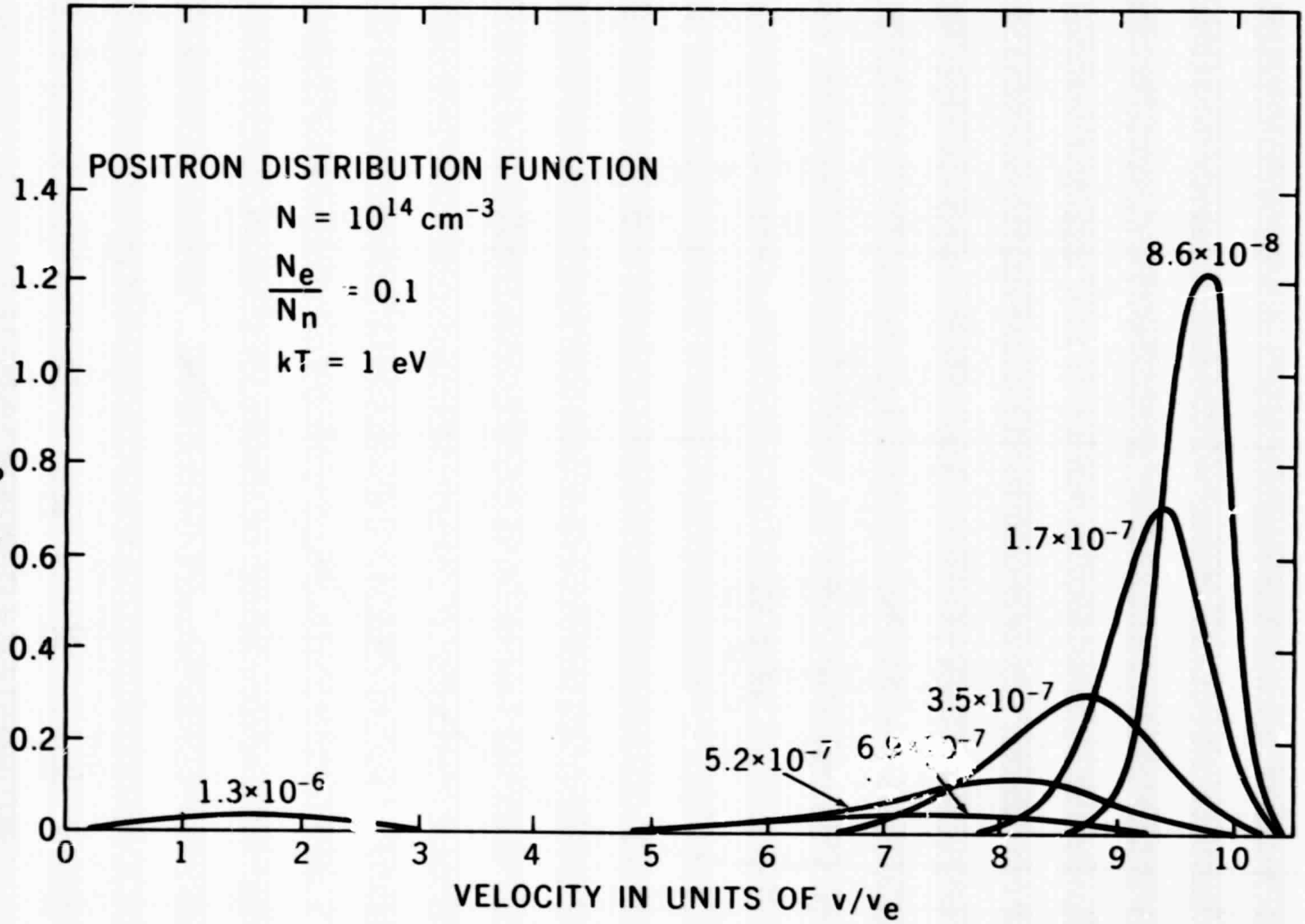

POSITRON DISTRIBUTION FUNCTION
$N = 10^{14}$ cm^{-3}
$\frac{N_e}{N_n} = 1.0$
$kT = 1$ eV
2.7×10^{-7}
2.3×10^{-7}
2.1×10^{-7}
2.0×10^{-7}
1.8×10^{-7}
1.6×10^{-7}
1.4×10^{-7}
1.8×10^{-8}sec
3.5×10^{-8}
5.3×10^{-8}
7.1×10^{-8}
8.9×10^{-8}
1.1×10^{-7}
1.2×10^{-7}
CROSS SECTION THRESHOLD
$N(v/v_e)$
VELOCITY IN UNITS OF v/v_e
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0
1
2
3
4
5
6
7
8
9
10

Figure 2.



POSITRON DISTRIBUTION FUNCTION
N = 10^14 cm^-3
N_e / N_n = 0.1
kT = 1 eV
8.6×10^-8
1.7×10^-7
3.5×10^-7
5.2×10^-7
1.3×10^-6
N(v/v_e)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
VELOCITY IN UNITS OF v/v_e

Figure 3.

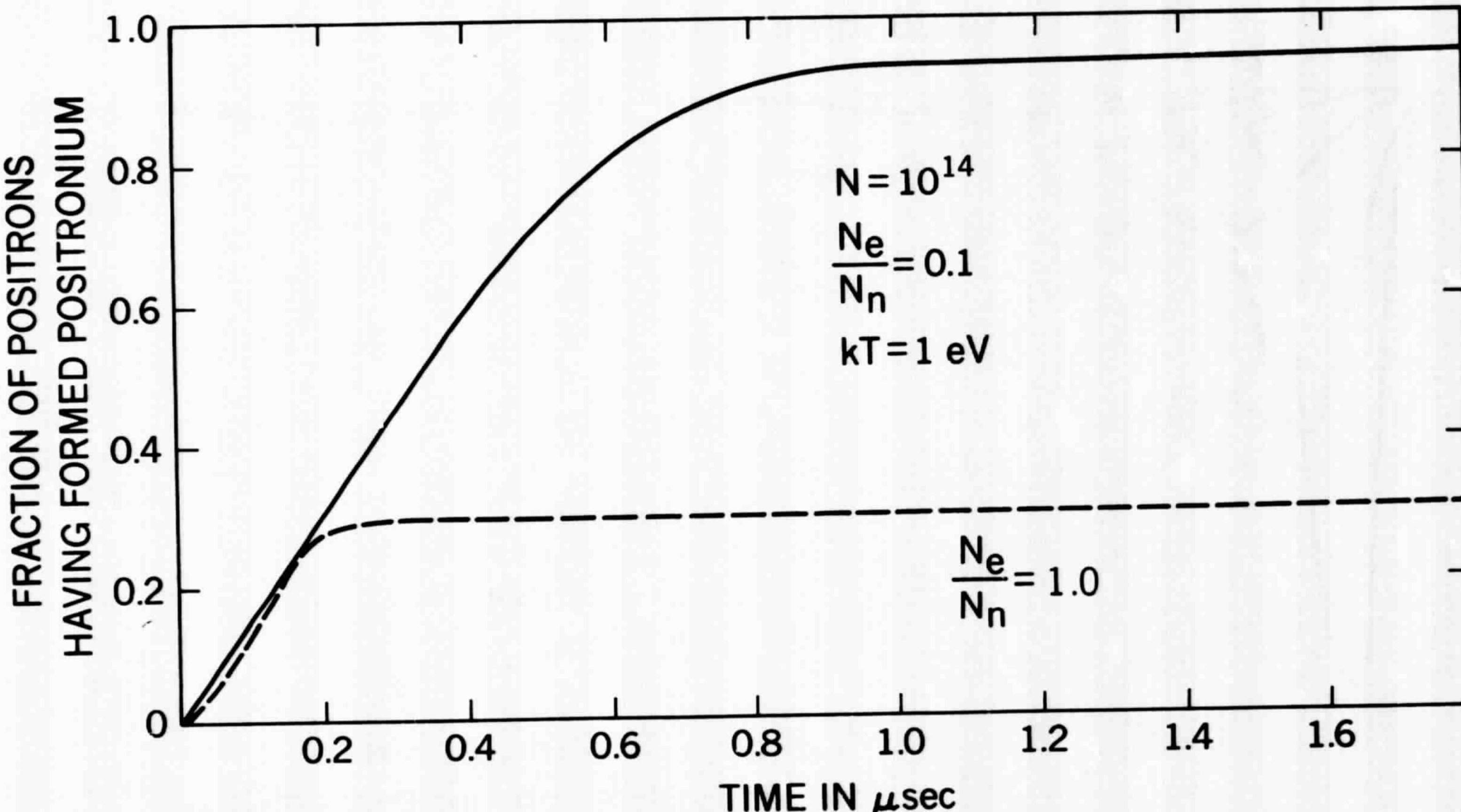

1.0
0.8
0.6
0.4
0.2
0
FRACTION OF POSITRONS HAVING FORMED POSITRONIUM
$N = 10^{14}$
$\frac{N_e}{N_n} = 0.1$
$kT = 1$ eV
$\frac{N_e}{N_n} = 1.0$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
TIME IN μsec

Figure 4.

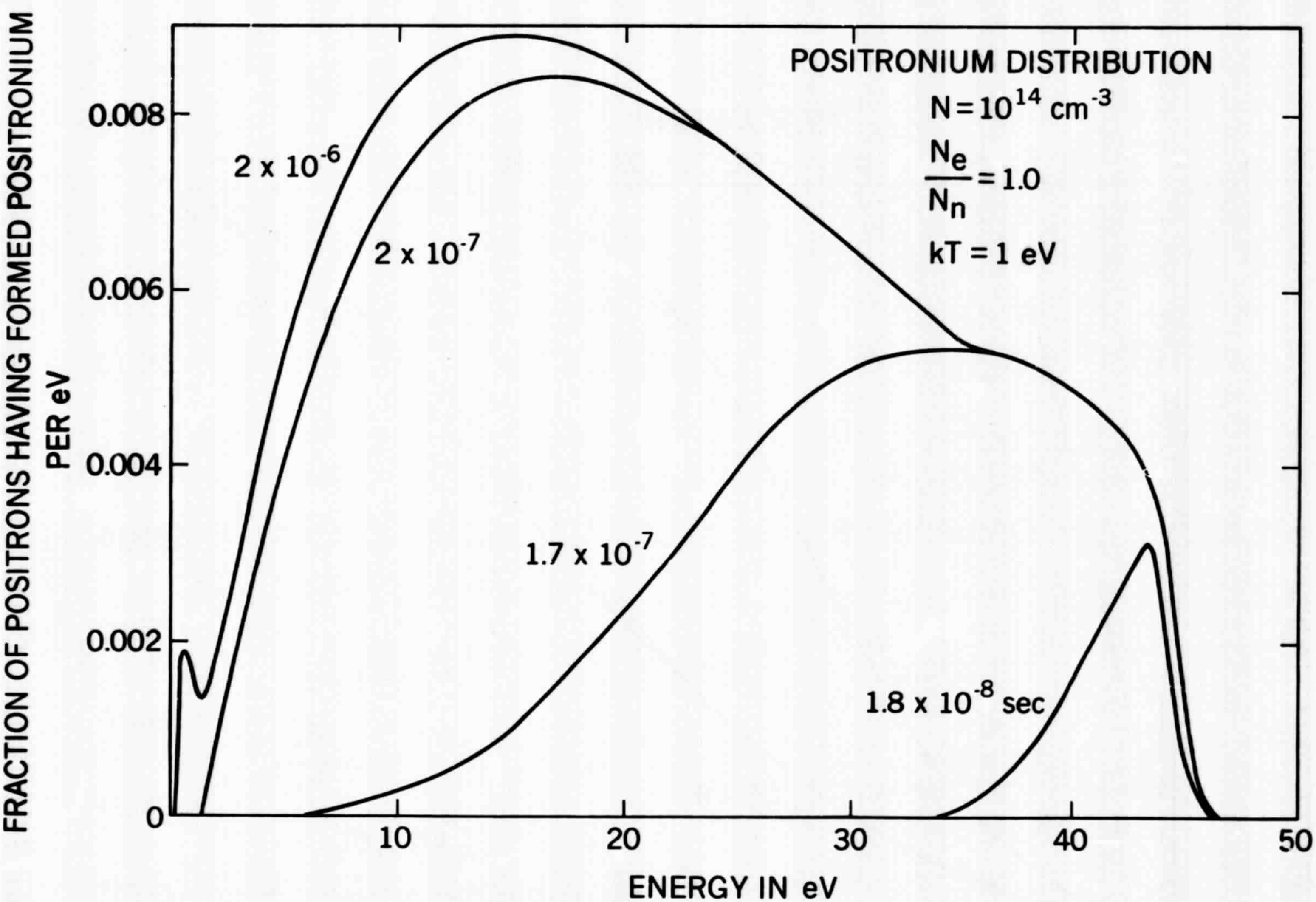

POSITRONIUM DISTRIBUTION
$N = 10^{14}\ cm^{-3}$
$\frac{N_e}{N_n} = 1.0$
$kT = 1$ eV
2×10^{-6}
2×10^{-7}
1.7×10^{-7}
1.8×10^{-8} sec
FRACTION OF POSITRONS HAVING FORMED POSITRONIUM PER eV
ENERGY IN eV
0.008
0.006
0.004
0.002
0
10
20
30
40
50

Figure 5.

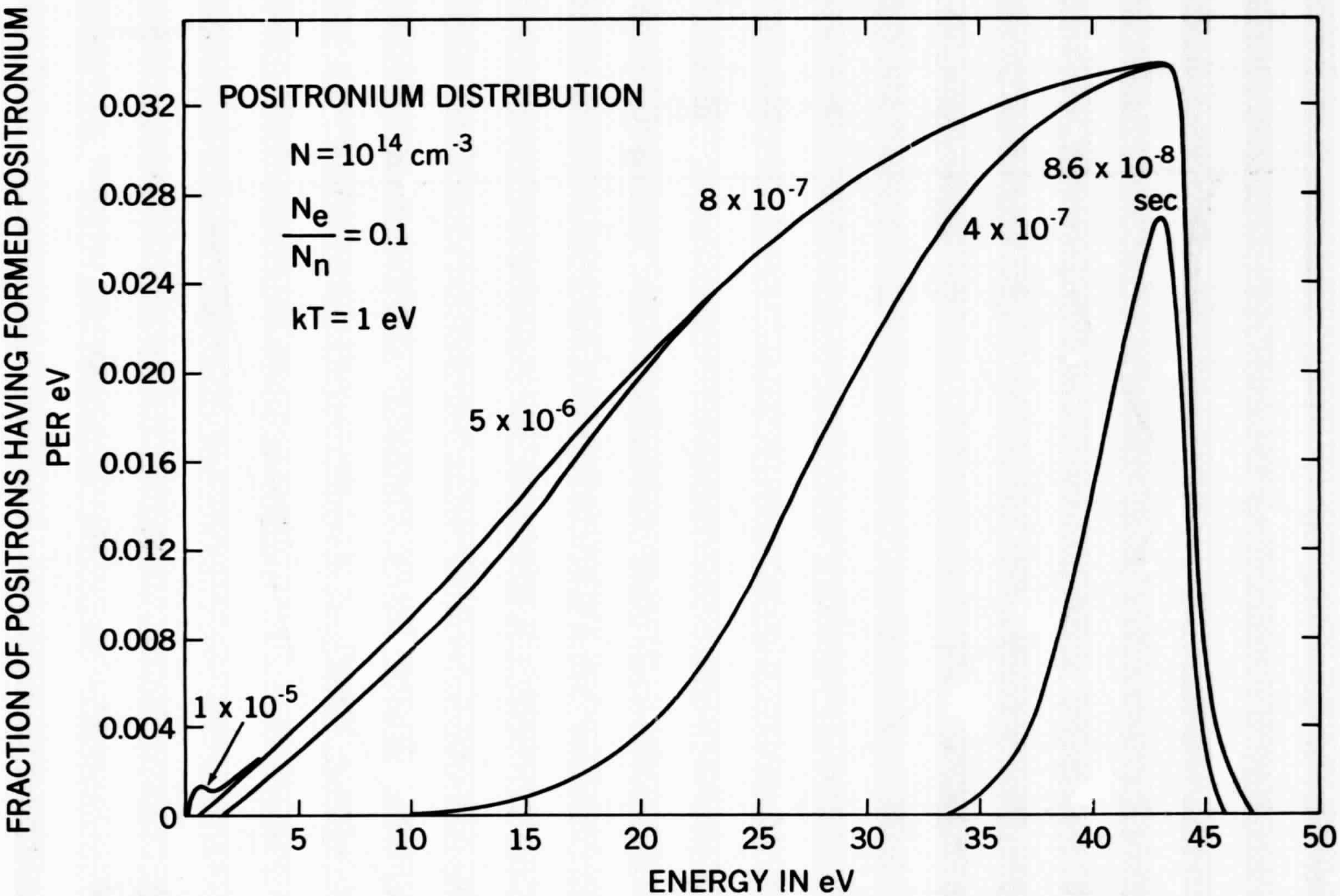

POSITRONIUM DISTRIBUTION
$N = 10^{14}\ cm^{-3}$
$\frac{N_e}{N_n} = 0.1$
$kT = 1\ eV$
8.6×10^{-8} sec
4×10^{-7}
8×10^{-7}
5×10^{-6}
1×10^{-5}
FRACTION OF POSITRONS HAVING FORMED POSITRONIUM PER eV
ENERGY IN eV
0.032
0.028
0.024
0.020
0.016
0.012
0.008
0.004
0
5
10
15
20
25
30
35
40
45
50

Figure 6.

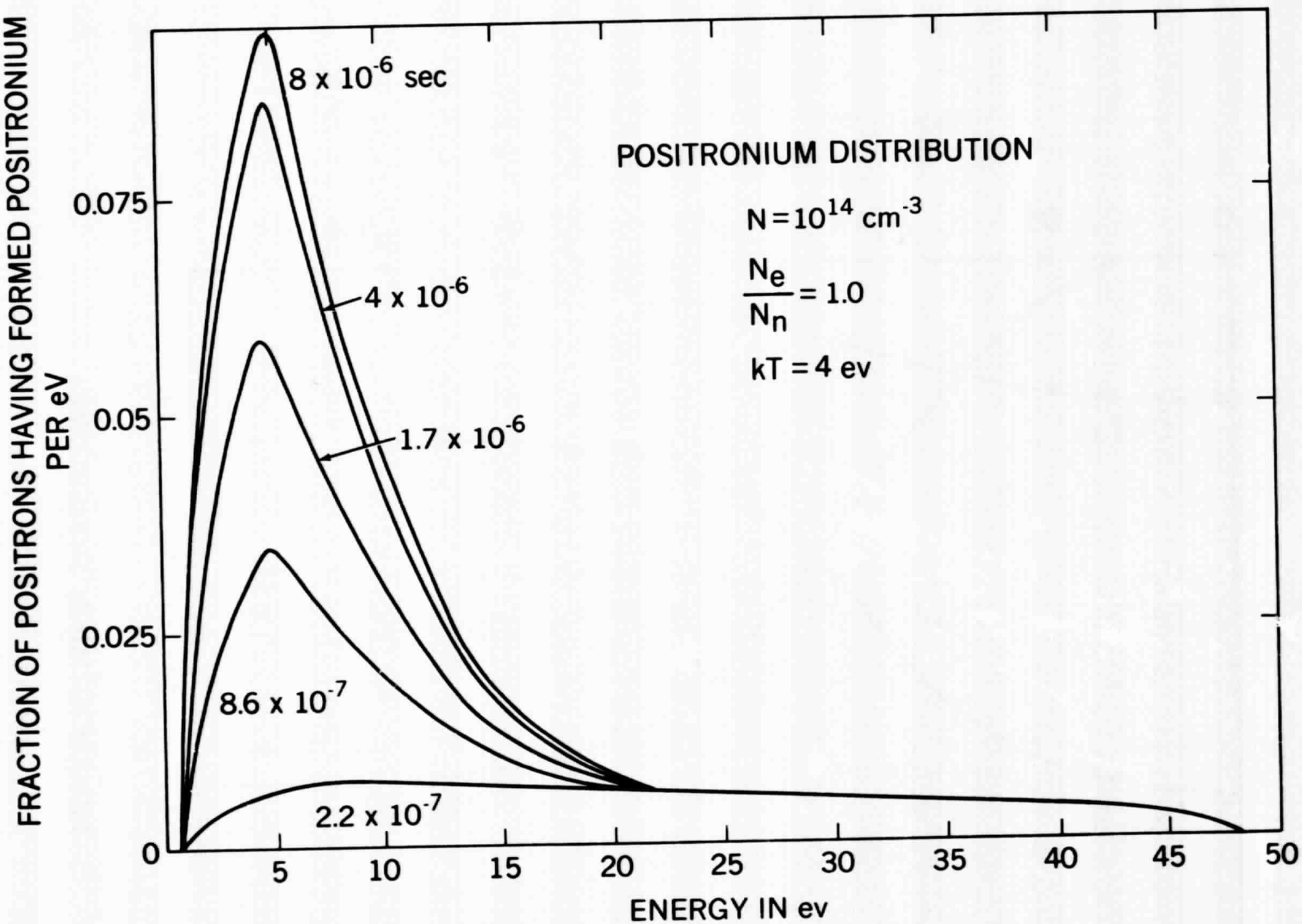

FRACTION OF POSITRONS HAVING FORMED POSITRONIUM PER eV
0.075
0.05
0.025
0
5
10
15
20
25
30
35
40
45
50
ENERGY IN ev
POSITRONIUM DISTRIBUTION
$N = 10^{14}$ cm^{-3}
$\frac{N_e}{N_n} = 1.0$
kT = 4 ev
8×10^{-6} sec
4×10^{-6}
1.7×10^{-6}
8.6×10^{-7}
2.2×10^{-7}

Figure 7.

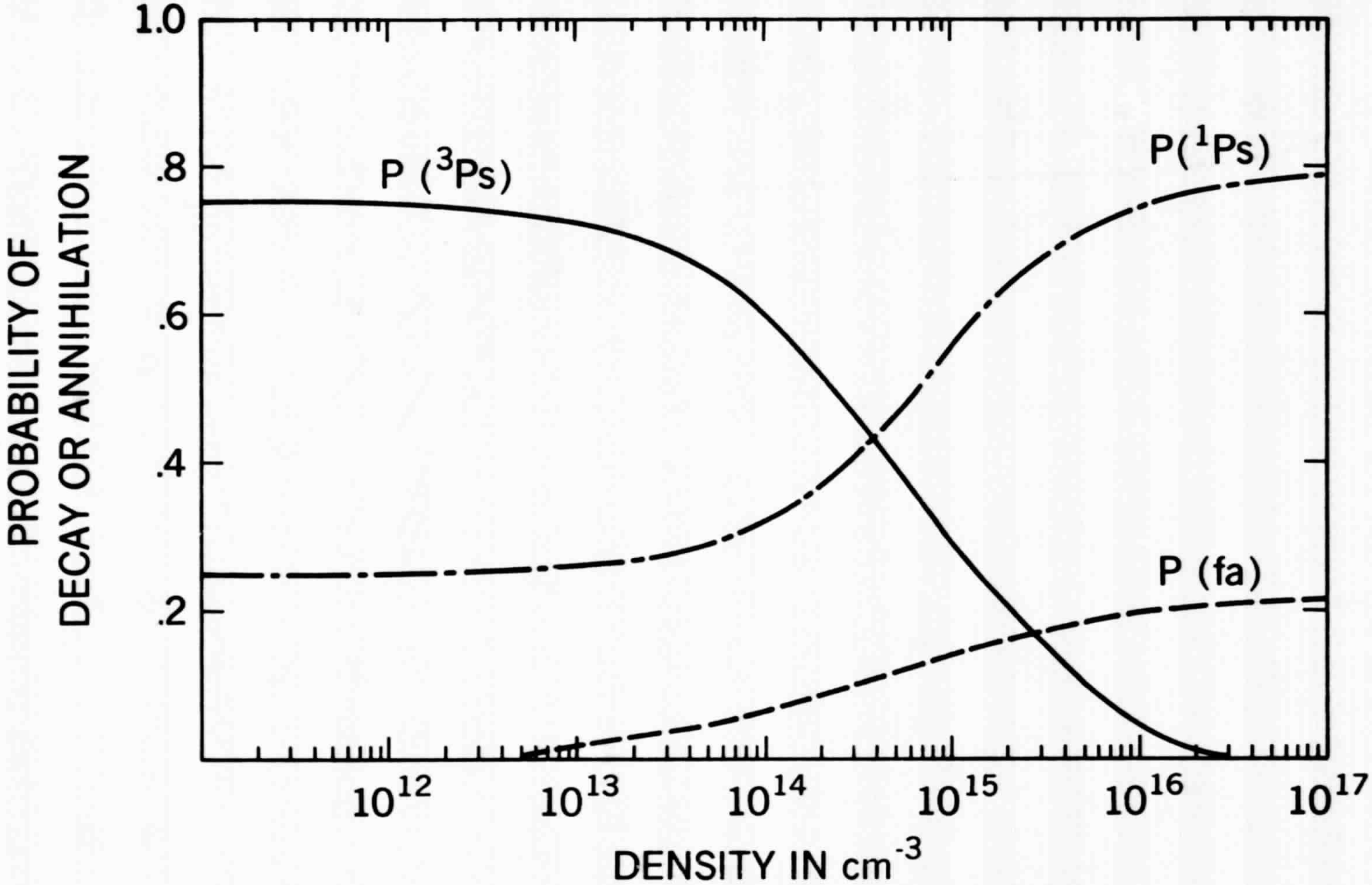
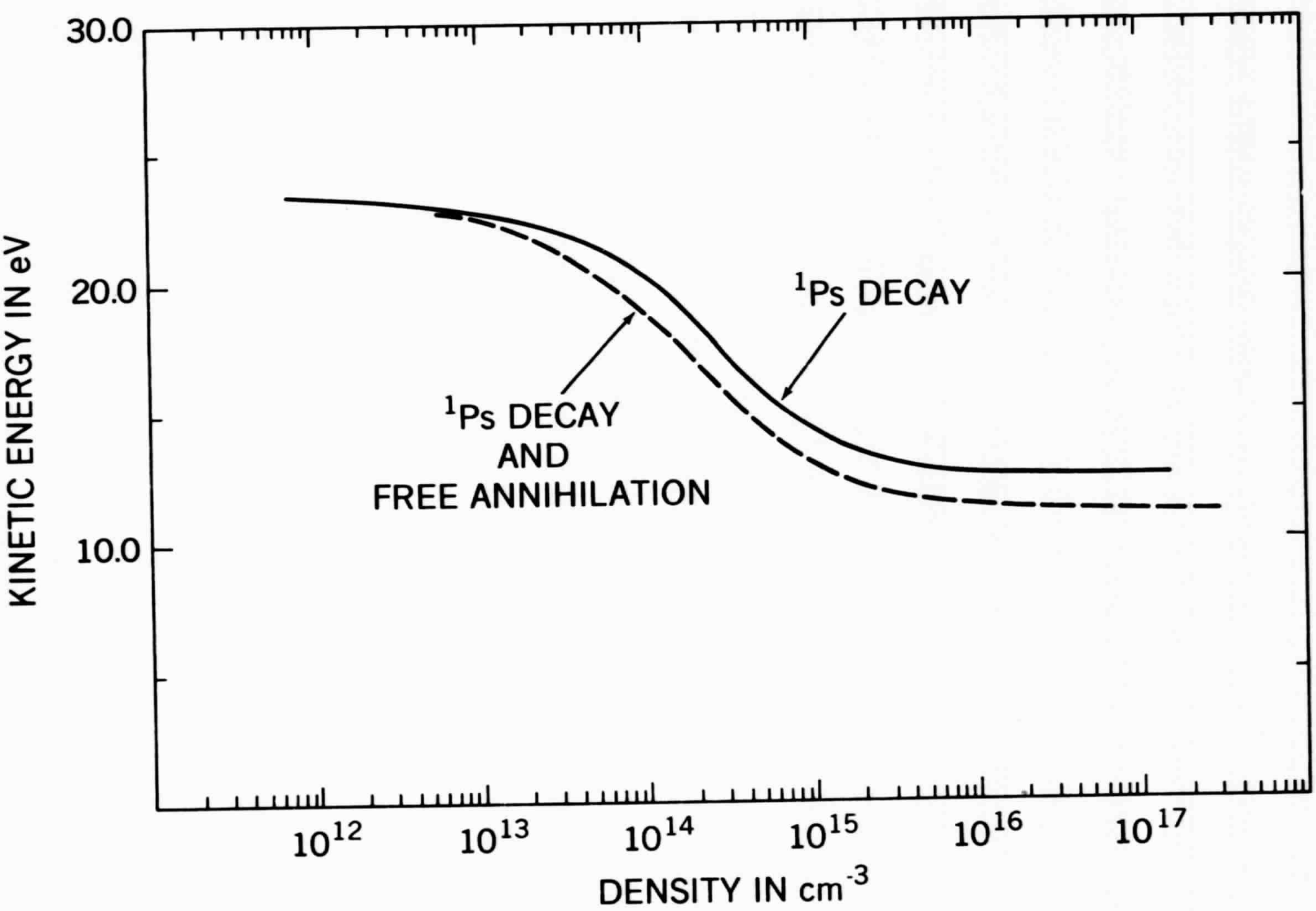

PROBABILITY OF
DECAY OR ANNIHILATION
1.0
.8
.6
.4
.2
P (^{3}Ps)
P(^{1}Ps)
P (fa)
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}
DENSITY IN cm^{-3}

Figure 8.



30.0
20.0
10.0
KINETIC ENERGY IN eV
¹Ps DECAY
¹Ps DECAY
AND
FREE ANNIHILATION
10¹²
10¹³
10¹⁴
10¹⁵
10¹⁶
10¹⁷
DENSITY IN cm⁻³

Figure 9.