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FREQUENCY OF ENCOUNTER OF AIRCRAFT IN A RANDOM HORIZONTAL FIELD

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16. Abstract

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FREQUENCY OF ENCOUNTER OF AIRCRAFT IN A RANDOM HORIZONTAL FIELD

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SUMMARY

Calculations have been made of the frequency of encounter as a function of azimuth of encounter of a passing aircraft with the aircraft in a random planar horizontal field. The field aircraft were all moving at a constant speed but in random directions. These calculations included the total frequency of encounter with the aircraft of the field and the frequency of encounter with those aircraft of the field which were encountered in the fore quadrant, in the lateral quadrants, and in the rear quadrant; the calculations were made for various speed ratios of the field aircraft and the passing aircraft. In the cases studied, the passing aircraft was usually approached in its fore quadrant. Relatively few of the aircraft of the random field were approached from the rear because a speed subtraction effect reduced the encounter velocity in the overtaking situation. The frequency of encounter with the field aircraft which were approached from the lateral quadrant varied little with the speed ratio of the aircraft because in this case there was no subtractive or additive effect of the velocities.

INTRODUCTION

The study of chance encounter of aircraft is important to both civil and military aviation because of collision hazard and is important also to military tactical combat situations. Attention has recently been given to collision probability and to collision-avoidance techniques in air traffic control studies. (See refs. 1 and 2.) Terminal-area collision hazard has been the matter of greatest concern in this work, and both experimentally determined and statistically modeled collision probabilities have been studied.

This investigation provides calculations of the number of aircraft encountered by one passing aircraft traveling at a constant speed and in a constant direction through a uniform and unlimited horizontal planar field of aircraft moving randomly in direction at a second constant speed. The number of encounters per second per radian of the passing aircraft with the aircraft of the field was calculated as a function of the azimuth of encounter relative to the direction of flight of the passing aircraft. No form of air traffic control was considered. The integral of this quantity around the aircraft (the total frequency of encounter) was also determined. The frequency of encounter of field aircraft by the passing aircraft is listed in categories corresponding to (a) those aircraft of the field approached from the fore quadrant, (b) those approached from the lateral quadrants, (c) those approached from the aft quadrant, and (d) the total aircraft encountered. (See fig. 1.) Calculations to determine the effect of relative velocities were made, and the results were normalized with respect to the number density of the aircraft in the random field. Considerations of rarified gas dynamics were used in these calculations, but a distribution function which is random in direction and uniform in magnitude of velocity was employed rather than the Maxwellian distribution common to gas dynamics. This distribution function seems more appropriate than the gas-dynamics distribution function for illustrating the significance of the various parameters of the aircraft-encounter problem. The principal features of these results are discussed in regard to collision hazard and aerial combat tactics. (See ref. 3.)

SYMBOLS

A	aircraft passing at constant speed and direction through field of aircraft moving randomly in direction at another constant speed
B	random field of aircraft
k	constant
N	number of aircraft of field B that enter circle of encounter around aircraft A
$\frac{dN}{dt}$	frequency at which aircraft cross circle of encounter
$\frac{d^2N}{dt\,d\theta}$	frequency at which aircraft cross circle of encounter as function of θ
r	radius of encounter around aircraft A
ds	element of circle of encounter of aircraft A
$d\bar{s}$	vector form of ds
u	speed of aircraft A moving along axis Y relative to axes x' and y'
v'	speed of aircraft of field B relative to axes x' and y'

$\bar{v}$ — vector velocity of aircraft in field B relative to $d\bar{s}$

v_x — speed of aircraft of field B with respect to ds

X,Y — axes fixed to aircraft A with Y alined with inertial axis y'

x,y — axes fixed to aircraft A at angle θ to axes X,Y

x',y' — inertial system of axes

δ — direction of flight of aircraft of field B measured from axis x

δ_o — specific value of δ

θ — angle measured around aircraft A

ρ — density of aircraft in field B

ϕ — direction of flight of one aircraft of field B measured from axis x'

ϕ_ℓ — lower limit of ϕ

ϕ_u — upper limit of ϕ

ϕ_1,ϕ_2 — specific values of ϕ relating to integration limits

ϕ^* — defined value of ϕ

THEORETICAL DEVELOPMENT

Consider an aircraft A which is moving in a given direction at a constant speed u through a uniform and unlimited horizontal planar field of aircraft B which are moving randomly in direction but at a constant speed v'. (See fig. 1.) Calculation of the frequency at which the aircraft of field B reach a circle of encounter of radius r encompassing and centered on aircraft A under various conditions is now desired. Specifically, calculations of the frequency at which the aircraft of field B reach r at various angles about aircraft A, the total frequency at which aircraft of field B are encountered by aircraft A, and the frequency at which aircraft of field B are encountered from the fore, lateral, and rear quadrants at various angles about aircraft A are desired. The aircraft of field B were considered as individual points in the field.

The x- and y-axis system of figure 2 is fixed relative to aircraft A for a given θ and r; the x- and y-axis system moves along the direction Y with speed u relative to the inertial system x',y'. The aircraft B move at speed v' in a direction ϕ relative to x',y' and have a field density $\frac{\rho}{2\pi} d\phi$ where this field density is the number of aircraft B per unit of area in angle $d\phi$.

The velocity $\bar{v}$ of the encountered aircraft of field B relative to the element ds, the vector element $d\bar{s}$, and the field density $\frac{\rho}{2\pi} d\phi$ determine the frequency at which aircraft enter the encounter circle. (See fig. 3.) This frequency is

$$d\left(\frac{dN}{dt}\right) = \frac{\rho}{2\pi} d\phi\ d\bar{s} \cdot \bar{v}$$

or the rate per unit of θ is

$$\frac{d^2N}{dt\ d\theta} = \frac{\rho r}{2\pi} v_x\ d\phi$$

where

$$v_x = v' \cos(\theta - \phi) - u \sin\theta$$

An integration is set up over the angle ϕ for the limits to be considered for all the aircraft of field B which approach the encounter circle from the outside. With certain changes, this formulation gives

$$\frac{d^2N}{dt\ d\theta} = k \frac{\rho r u}{2\pi} \int_{\phi_\ell}^{\phi_u} \left[\frac{v'}{u} \cos(\theta - \phi) - \sin\theta\right] d\phi$$

where

$$\phi_\ell = \theta - \delta_o$$

$$\phi_u = \theta + \delta_o$$

The quantity $\pm\delta_o$ defines an angle enclosing the aircraft of field B that are under consideration for possible encounter with aircraft A. (See fig. 2.) An angle δ_o of 180^o encompasses all the aircraft of field B; an angle δ_o of 45^o encompasses only those aircraft of field B which are approached from the rear. When the integrand is negative, $k = 1$; when the integrand is positive, $k = 0$. This condition insures that only those aircraft in the field B which approach the encounter circle from the outside are included. The completed integration gives

$$\frac{d^2N}{dt\,d\theta} = k\,\frac{\rho r u}{2\pi}\left[\frac{v'}{u}(\cos\theta\sin\phi - \sin\theta\cos\phi) - \phi\sin\theta\right]_{\phi_\ell}^{\phi_u}$$

In evaluating this expression, the conditions where the integrand v_x/u is positive must be determined to eliminate the unwanted count of aircraft in field B. The following expression is examined:

$$\frac{v_x}{u}(\phi,\theta) = \frac{v'}{u}\cos(\theta - \phi) - \sin\theta$$

where

$$-\frac{\pi}{2} \leqq \theta \leqq \frac{\pi}{2}$$

$$\left(\theta - \delta_o\right) \leqq \phi \leqq \left(\theta + \delta_o\right)$$

$$0 \leqq \delta_o \leqq \pi$$

Thus, v_x/u is seen to be symmetrical about $\phi = \theta$ with a maximum value at $\phi = \theta$, at which point $\frac{v_x}{u} = \frac{v'}{u} - \sin\theta$. At the lower and upper limits of ϕ,

$$\frac{v_x}{u} = \frac{v'}{u}\cos\delta_o - \sin\theta$$

There are three cases relating to the conditions for which v_x/u is positive or negative. (See fig. 4.) These cases are:

(1) When $\frac{v'}{u} - \sin\theta > 0$ and $\frac{v'}{u}\cos\delta_o - \sin\theta > 0$, v_x/u is positive for all ϕ from ϕ_ℓ to ϕ_u and $k = 0$ in the expression for $d^2N/dt\,d\theta$. In this case $d^2N/dt\,d\theta = 0$.

(2) When $\frac{v'}{u} - \sin\theta \geqq 0$ and $\frac{v'}{u}\cos\delta_o - \sin\theta \leqq 0$, v_x/u changes sign twice between ϕ_ℓ and ϕ_u and is negative at ϕ_ℓ and ϕ_u. The points where v_x/u changes

sign are given by equating v_x/u to zero and solving for ϕ_1 and ϕ_2. This process gives

$$\phi_1 = \theta - \phi^*$$

$$\phi_2 = \theta + \phi^*$$

where

$$\phi^* = \cos^{-1}\left(\frac{u}{v'} \sin \theta\right)$$

In this case the expression for $d^2N/dt\, d\theta$ is evaluated from ϕ_ℓ to ϕ_1 and ϕ_2 to ϕ_u with $k = 1$. The constant k is zero from ϕ_1 to ϕ_2.

(3) When $\frac{v'}{u} - \sin \theta < 0$ and $\frac{v'}{u} \cos \delta_o - \sin \theta < 0$, v_x/u is negative for all ϕ from ϕ_ℓ to ϕ_u and the expression for $d^2N/dt\, d\theta$ is evaluated with $k = 1$ from ϕ_ℓ to ϕ_u to obtain the result.

The limits of the function $\sin \theta$ for these three cases are shown in figure 5, and the evaluation of the quantity $d^2N/dt\, d\theta$ for each of these cases is given here.

Case (1):

$$\frac{d^2N}{dt\, d\theta} = 0$$

Case (2):

$$\frac{d^2N}{dt\, d\theta} = 2\left(\frac{\rho r u}{2\pi}\right)\left[\frac{v'}{u}\left(\sin \delta_o - \sin \phi^*\right) - \left(\delta_o - \phi^*\right) \sin \theta\right]$$

Case (3):

$$\frac{d^2N}{dt\, d\theta} = 2\left(\frac{\rho r u}{2\pi}\right)\left(\frac{v'}{u} \sin \delta_o - \delta_o \sin \theta\right)$$

The total rate of encounter of aircraft A with the aircraft of field B dN/dt is obtained by integrating $d^2N/dt\, d\theta$ over the angle θ completely around aircraft A with a numerical quadrature routine. In this evaluation of dN/dt, various possibilities which relate to the exclusion of situations where $\sin \theta$ exceeds its normal limits of ± 1 must be considered. These situations are given in table I.

CALCULATIONS

Calculations of the encounter per second by an aircraft A moving at a constant speed in a field of aircraft B, all of which are moving in random directions but at a constant speed, were made for a field density factor of $\rho ru/2\pi = 1$ per second for speed ratios of the aircraft B to aircraft A of 0, 0.5, 0.8, 1.0, 1.2, and 2.0. The frequency of encounter of aircraft A with the aircraft of field B, in units of encounters per second per radian, was calculated as a function of the azimuth position of encounter relative to the direction of flight of aircraft A. Calculations included the encounters in which aircraft A approached the aircraft of field B from the fore quadrant, from the lateral quadrants, and from the aft quadrant. Calculations were also made for the total frequency of encounter in each of these cases, that is, the integral of the encounters per second over the azimuth angle θ of encounter.

The calculations of frequency of encounter in encounters per second per radian are plotted as a function of azimuth angle θ of encounter in figures 6 to 9 for various speed ratios. The total frequency of encounter is listed in table II as a function of v'/u and is plotted in figure 10.

RESULTS

Figures 6 to 9 show the frequency of encounter $d^2N/dt\,d\theta$ of the aircraft in a random planar field B with a passing aircraft A as a function of the azimuth of encounter θ with aircraft A for various speed ratios v'/u for four cases. These cases correspond to the total frequency of encounter and to the frequency of encounter of those aircraft of field B which are encountered in the fore quadrant, lateral quadrant, and aft quadrant. A general pattern of increasing frequency of encounter with increasing values of v'/u can readily be seen in all the figures. This trend indicates an increased frequency of traverse of a given area in the field as the speed v' is increased. Most of the encounters are made in the fore quadrant of aircraft A; this situation particularly applies in the case of the aircraft of field B which are encountered in the lateral quadrants (fig. 8). Relatively few aircraft of the field B are encountered in the aft quadrant because of the slow rate at which the passing aircraft A overtakes the aircraft of the field B (fig. 9).

The functions of figures 6 to 9 were integrated to give the total number of encounters experienced by the passing aircraft A for the various cases. These results are plotted against speed ratio v'/u in figure 10 and are listed in table II. The limiting case where v'/u goes to zero is also included in figures 6 to 10 and table II. The character of the curve of figure 10 for those aircraft of field B which are encountered from the lateral quadrants is somewhat different from the curves for those aircraft approached from the fore or aft quadrant. At a speed ratio v'/u of about 1.2, the curve for those aircraft of the field B which are encountered from the rear goes to zero and remains zero for larger values of v'/u. The curve for those aircraft which are encountered from the sides is less affected by speed ratio than the other curves. The curve for the aircraft which are approached from the front increases rapidly as the ratio v'/u is increased. This effect results from a more rapid sweep of the encounter area by the aircraft of the field as their speed increases. The results presented here may be employed to derive frequency of encounter for specific situations by calculating $\rho ru/2\pi$ and then multiplying by the encounter frequency for the appropriate value of v'/u.

CONCLUDING REMARKS

Calculations were made of the frequency of encounter as a function of azimuth of encounter of a passing aircraft with the aircraft of a random planar horizontal field. All field aircraft were moving at a constant speed but in random directions. These calculations included the determination of the total frequency of encounter by the passing aircraft with the aircraft of the field and the frequency of encounter by the passing aircraft with those aircraft of the field which were encountered in the fore quadrant, the lateral quadrants, and the aft quadrant for various speed ratios of the field of aircraft and the passing aircraft. The results of these calculations are presented in terms of the frequency of encounter as a function of the azimuth of encounter around the passing aircraft for the various speed ratios; the results are also presented as integrated or total frequencies of encounter for the various conditions.

The encounters with the passing aircraft were largely in the fore quadrant of the passing aircraft; relatively few of the aircraft of the field were approached from the rear as a result of a speed subtraction effect which reduced the encounter velocity in the overtaking situation. The frequency of encounter with the aircraft in the field which were approached from the sides varied little with the speed ratio of the aircraft because there was no subtractive or additive effect of the velocities. Basically, the largest number of encounter frequencies tended to occur in the fore quadrant both of the passing aircraft and of the aircraft of the random field.

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January 14, 1976

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2. Research Triangle Inst.: Statistical Evaluation of Aircraft Collision-Hazard Warning System Techniques in the Terminal Area – Phase II. NASA CR-1470, 1969.
3. Tsien, Hsue-Shen: Superaerodynamics, Mechanics of Rarified Gases. J. Aeronaut. Sci., vol. 13, no. 12, Dec. 1946, pp. 653-664.

TABLE I.- POSSIBILITIES TO BE CONSIDERED IN EVALUATION OF TOTAL FREQUENCY OF ENCOUNTER dN/dt

Condition	$\frac{v'}{u} \cos \delta_o \leqq -1$	$-1 < \frac{v'}{u} \cos \delta_o < 1$	$\frac{v'}{u} \cos \delta_o \geqq 1$
$\frac{v'}{u} < 1$	Not possible	All cases are possible	Not possible
$\frac{v'}{u} \geqq 1$	Only case (2) is possible	Only case (1) and case (2) are possible	Only case (1) is possible

TABLE II.- FREQUENCY OF ENCOUNTER BY PASSING AIRCRAFT WITH AIRCRAFT OF UNIFORM PLANAR FIELD

$[\rho ru/2\pi = 1.0]$

Ratio of speed of field to speed of passing aircraft, v'/u	Encounters per second, dN/dt			
	Approached from fore quadrant	Approached from lateral quadrants	Approached from aft quadrant	Total
0	3.14	6.28	3.14	12.56
.5	5.69	6.43	1.25	13.37
.8	7.57	6.66	.45	14.68
1.0	9.00	6.87	.12	15.99
1.2	10.68	7.14	.02	17.84
2.0	17.77	8.96	0	26.73

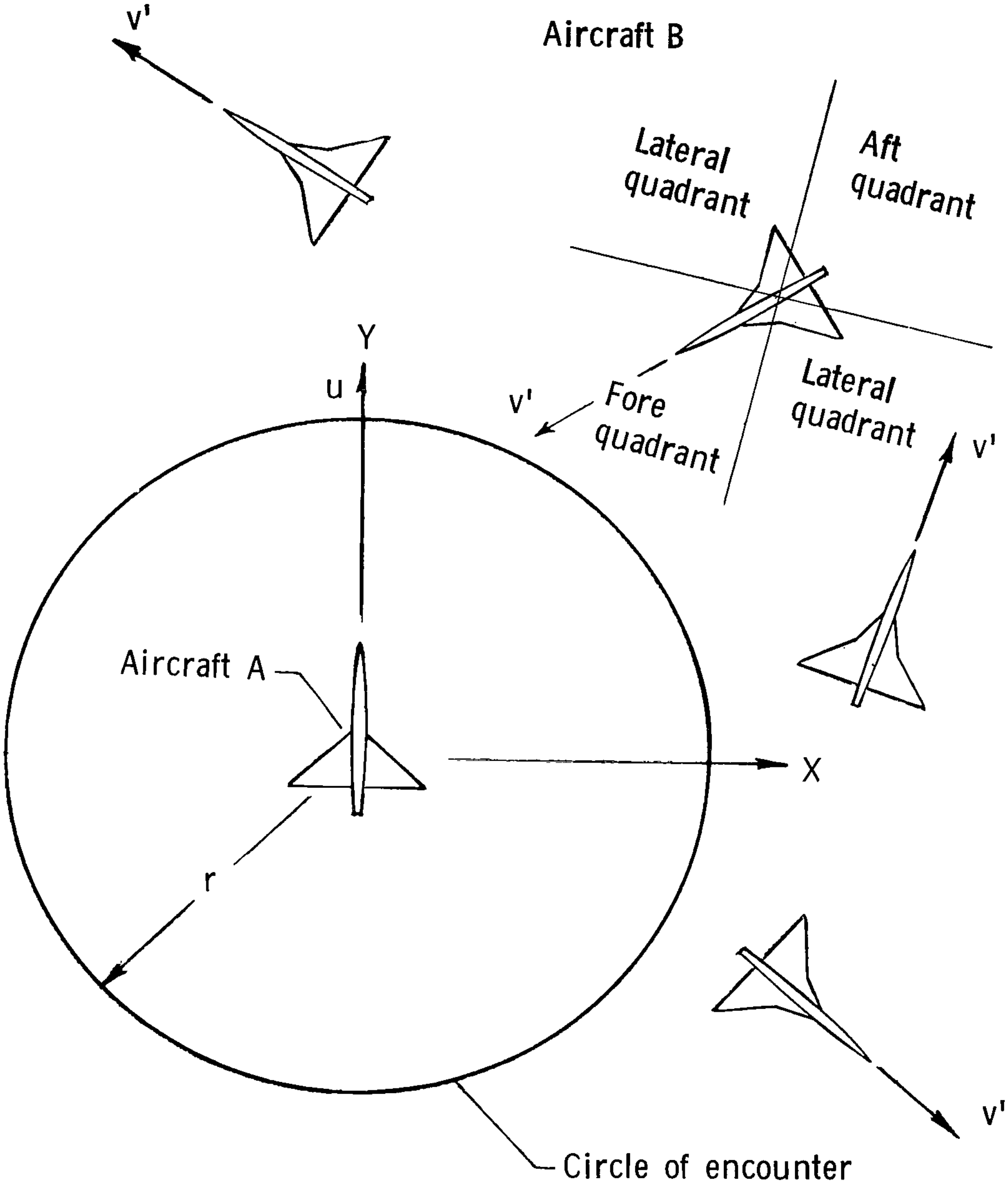


Figure 1.- Aircraft A moving in planar field of aircraft B.

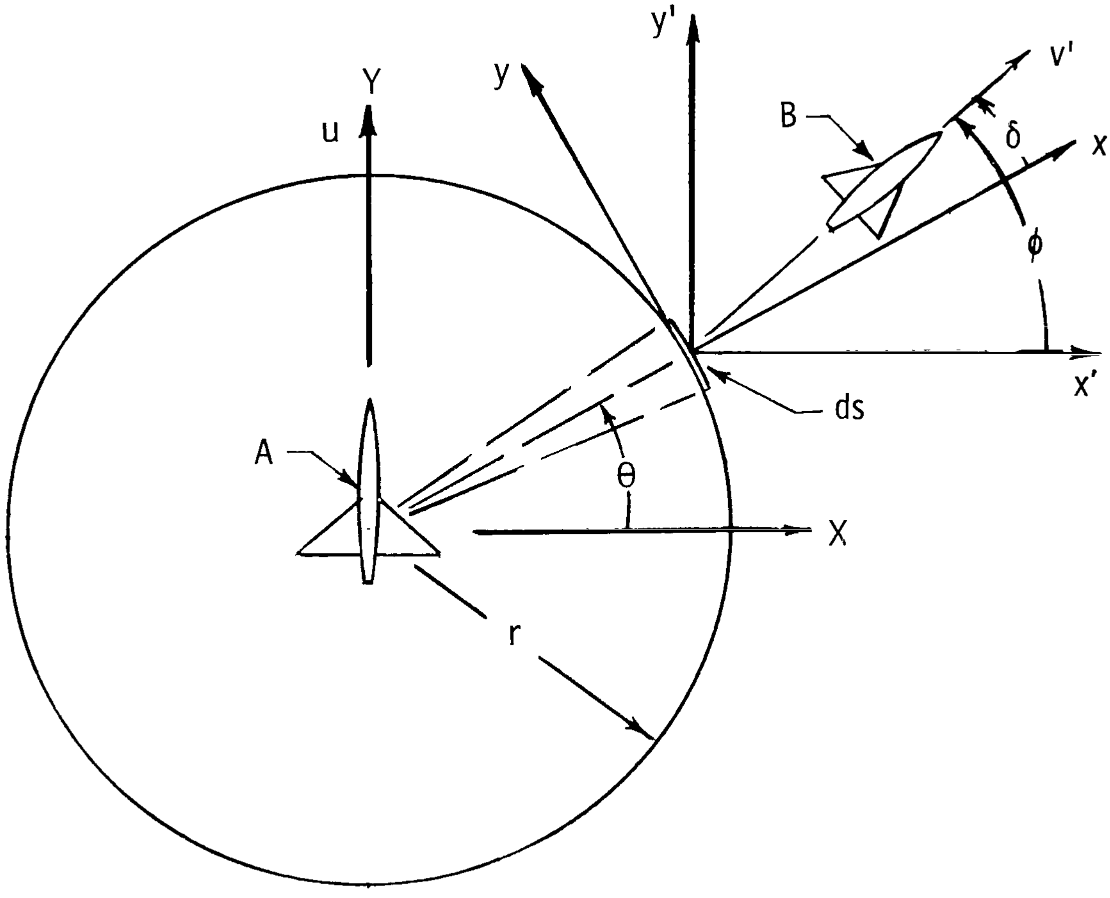


Figure 2.- Axis system for analysis.

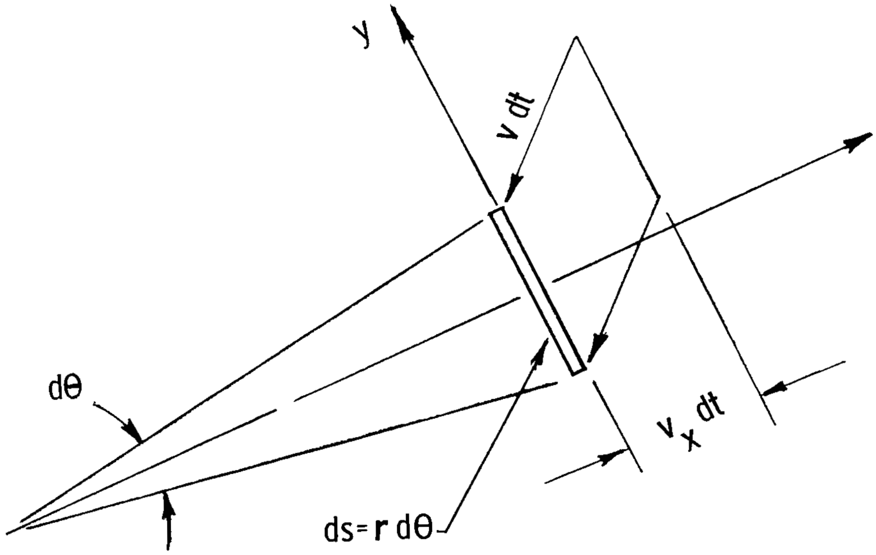
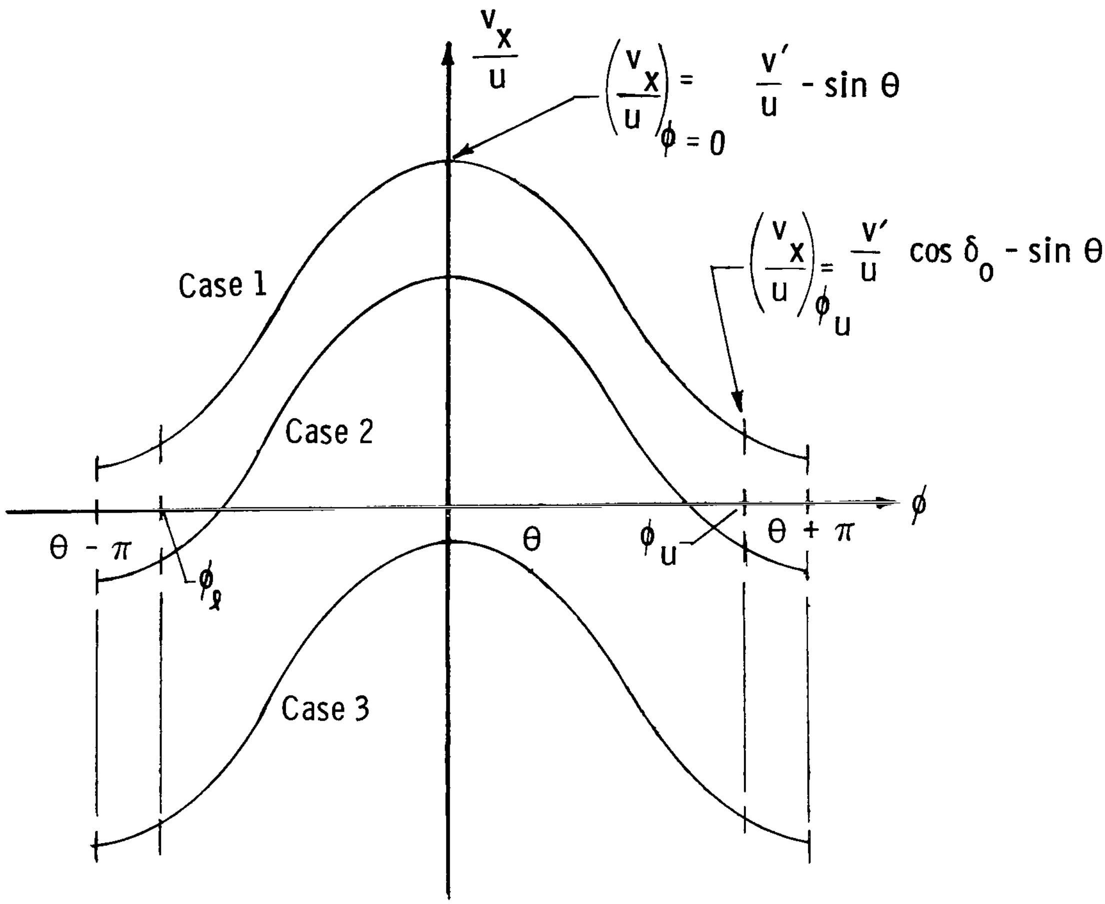
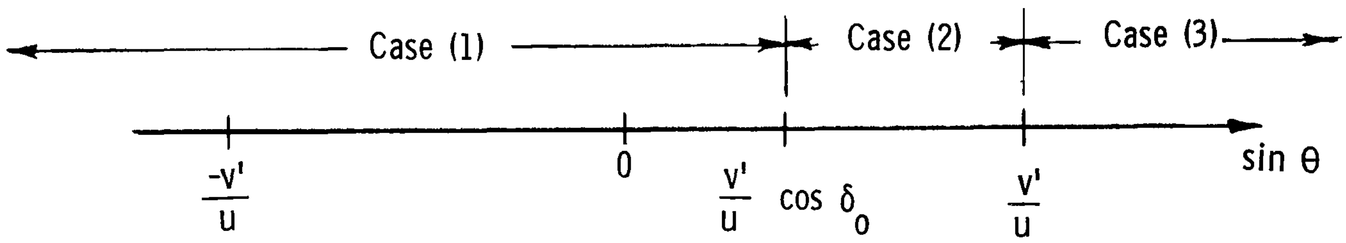


Figure 3.- Encounter geometry of aircraft of field B with circular elements.



Case(1)	Case(2)	Case(3)
$\frac{v'}{u} - \sin\theta > 0$	$\frac{v'}{u} - \sin\theta \geqq 0$	$\frac{v'}{u} - \sin\theta < 0$
$\frac{v'}{u}\cos\delta_0 - \sin\theta > 0$	$\frac{v'}{u}\cos\delta_0 - \sin\theta \leqq 0$	$\frac{v'}{u}\cos\delta_0 - \sin\theta < 0$

Figure 4.- Three cases that relate to sign of v_x/u.



$$\text{Case (1)}: \sin\theta < \frac{v'}{u}\cos\delta_0$$

$$\text{Case (2)}: \frac{v'}{u}\cos\delta_0 \leqq \sin\theta \leqq \frac{v'}{u}$$

$$\text{Case (3)}: \sin\theta > \frac{v'}{u}$$

Figure 5.- Limits of $\sin\theta$ for three cases relating to v_x/u.

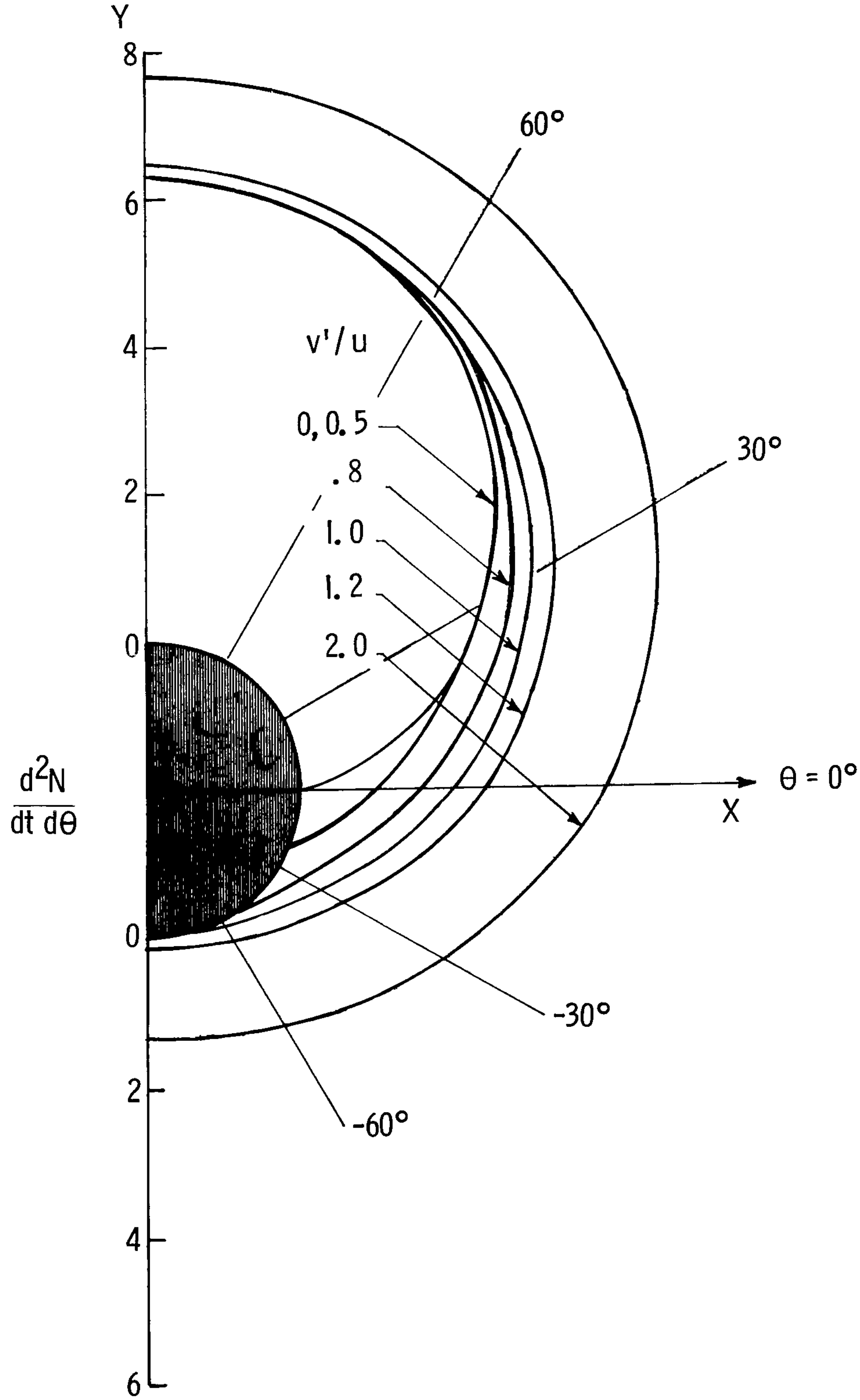


Figure 6.- Total frequency of encounter as function of azimuth of encounter with passing aircraft for various speed ratios, $\rho ru/2\pi = 1.0$.

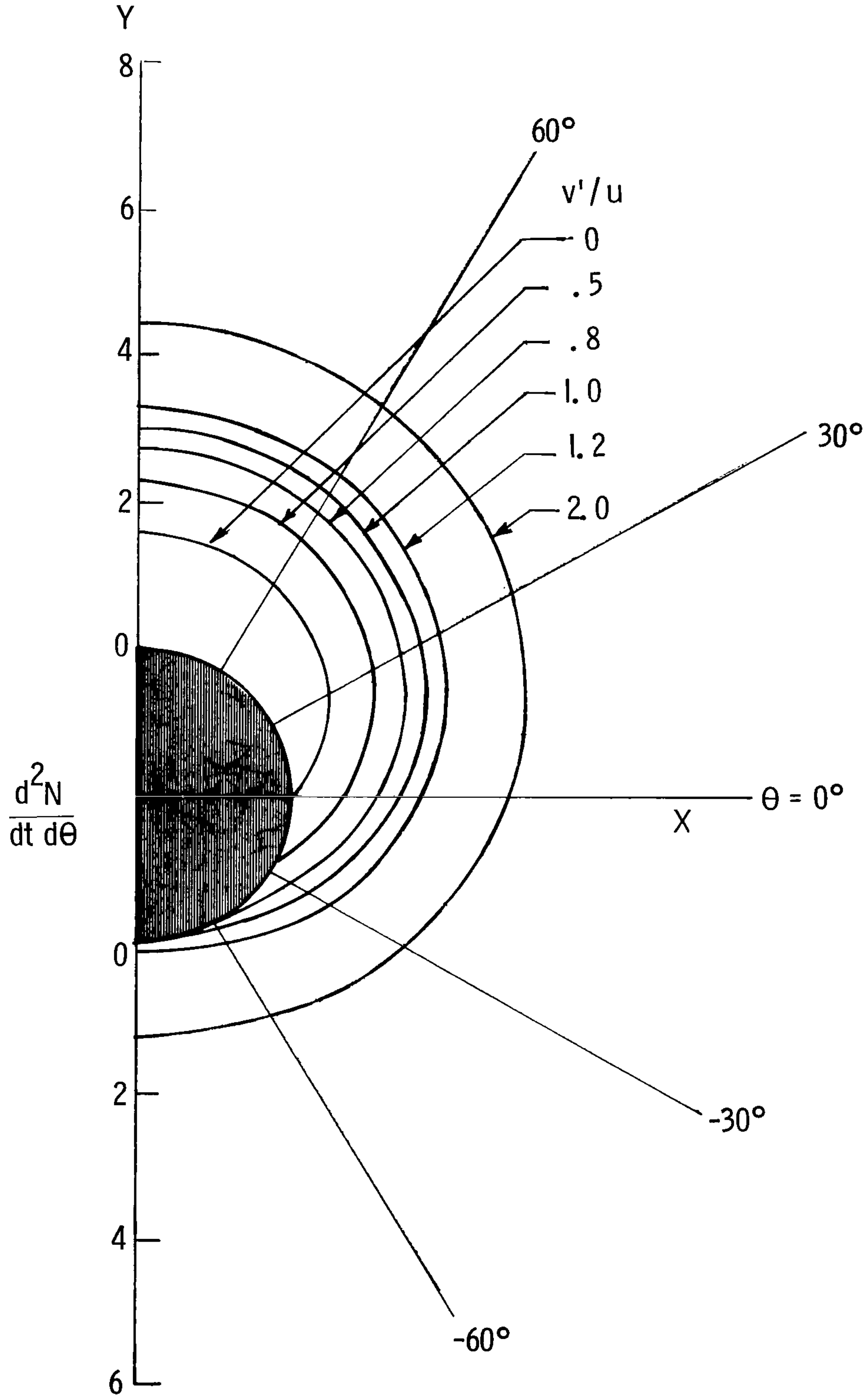


Figure 7.- Frequency of frontal encounter with aircraft of field B as function of azimuth of encounter with passing aircraft for various speed ratios, $\rho ru/2\pi = 1.0$.

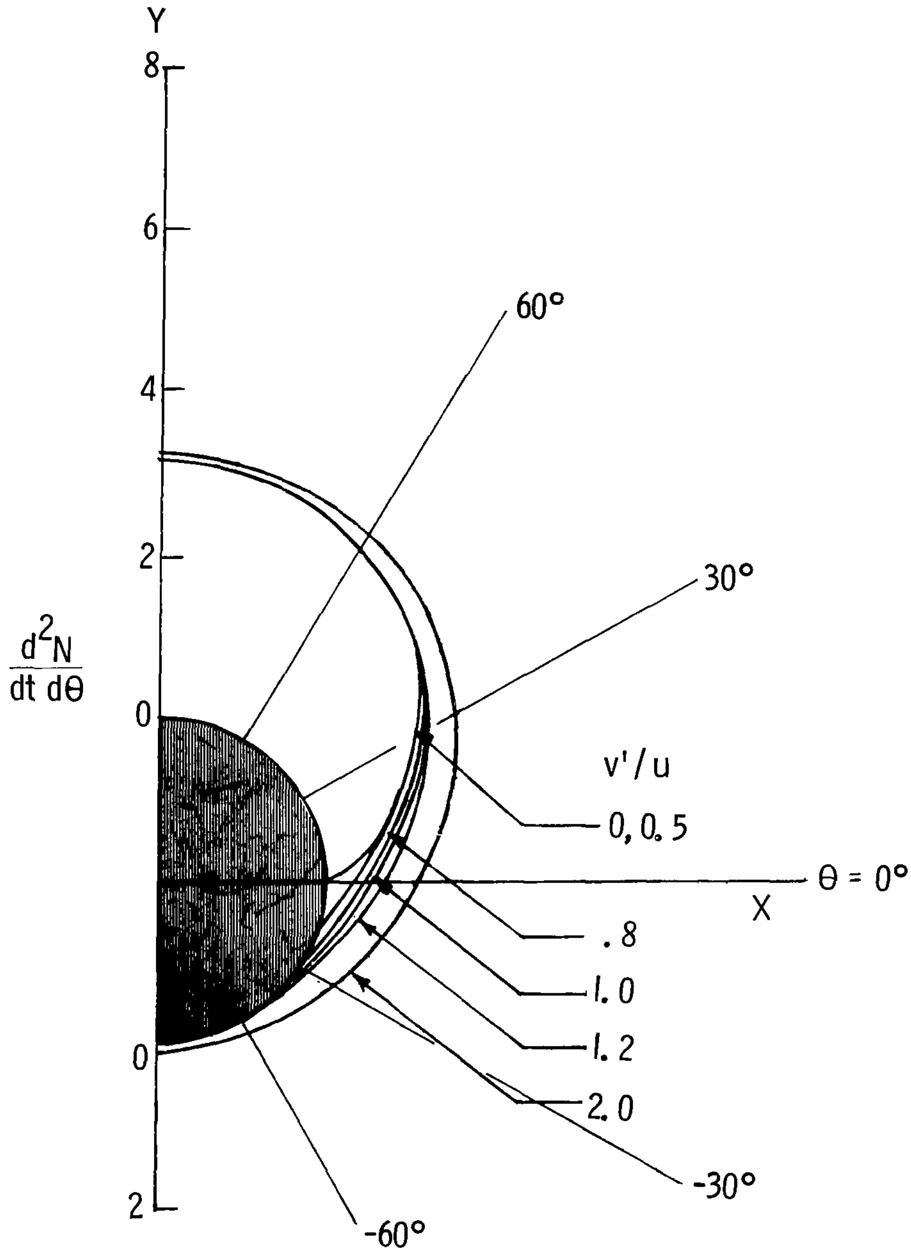


Figure 8.- Frequency of lateral encounter with aircraft of field B as function of azimuth of encounter with passing aircraft for various speed ratios, $\rho ru/2\pi = 1.0$.

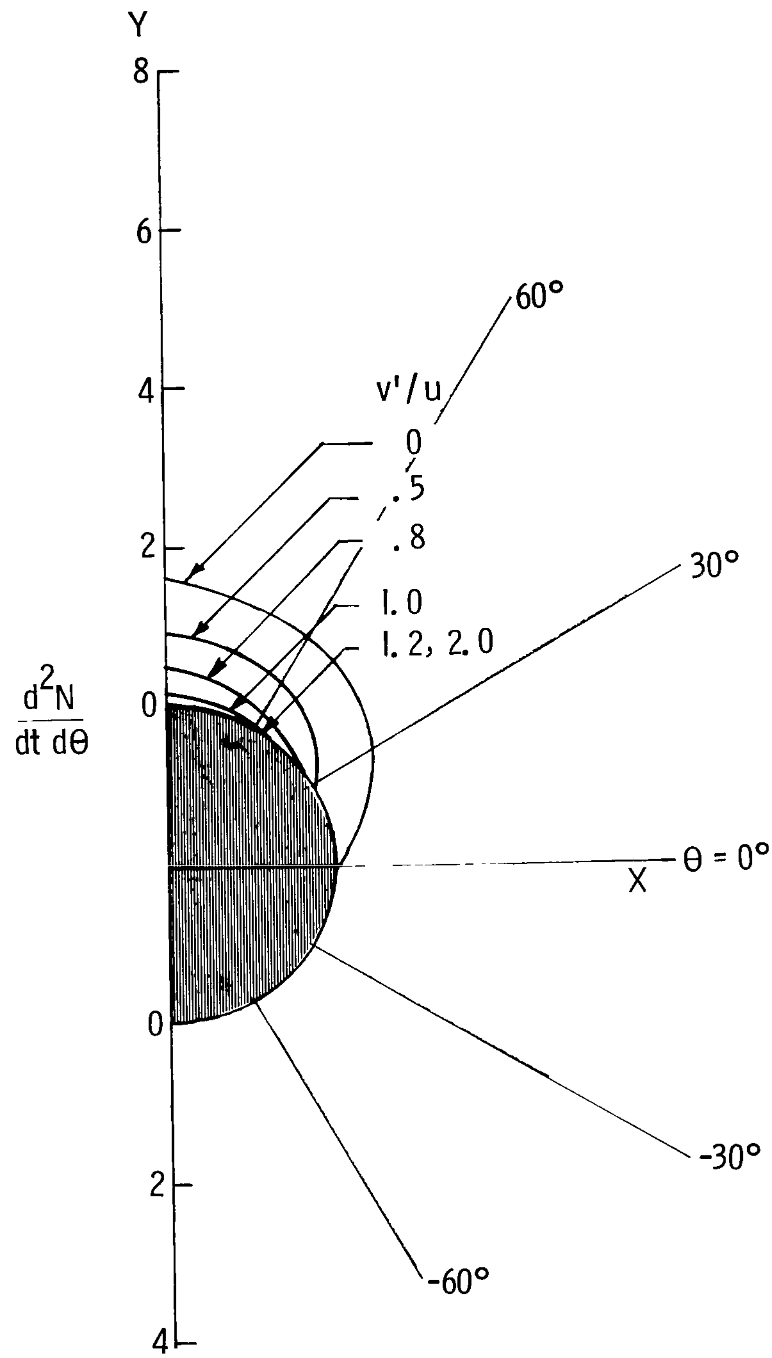


Figure 9.- Frequency of rear encounter with aircraft of field B as function of azimuth of encounter with passing aircraft for various speed ratios, $\rho ru/2\pi = 1.0$.

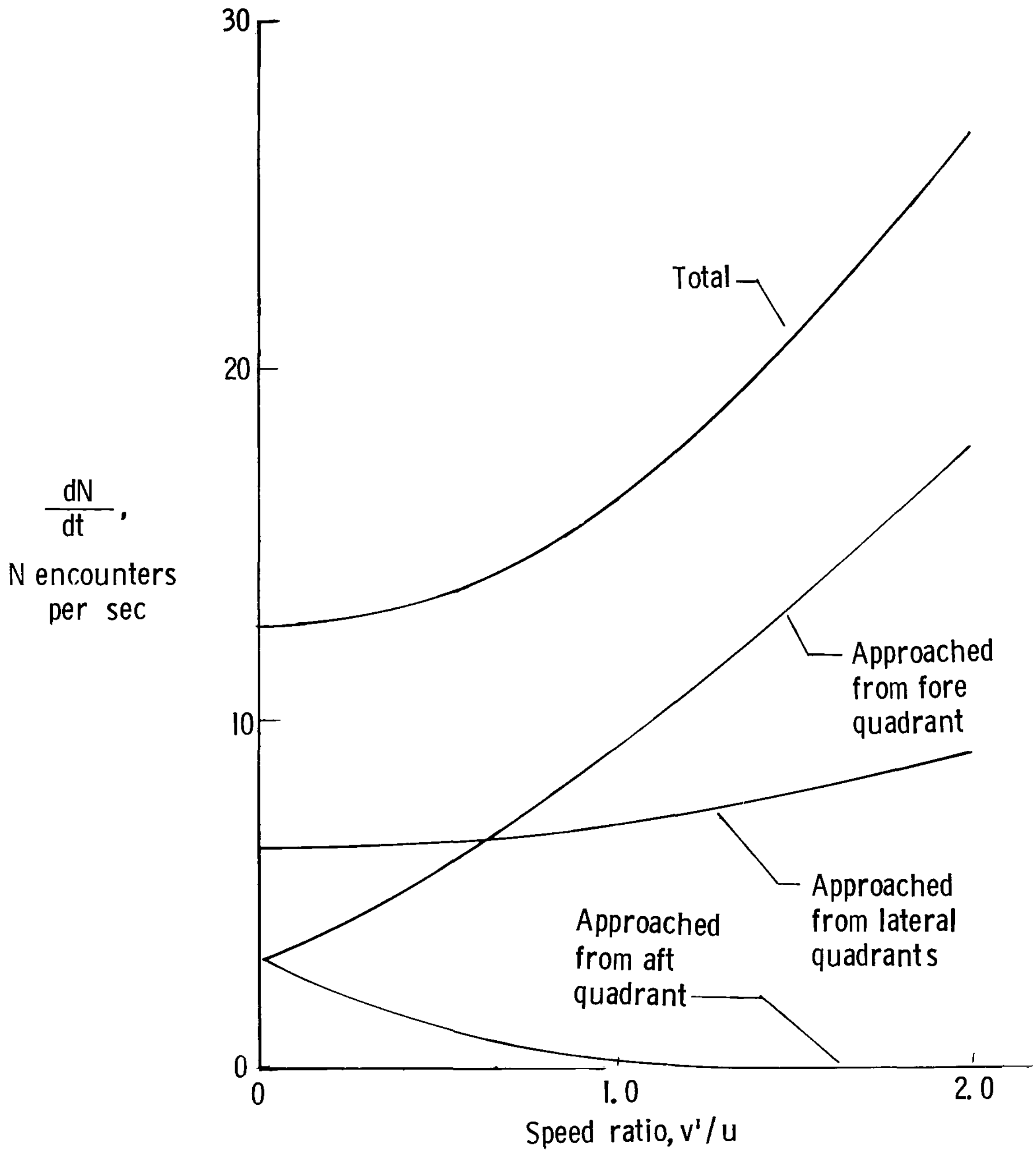


Figure 10.- Frequency of encounter by passing aircraft with aircraft of uniform planar field, $\rho ru/2\pi = 1.0$.

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