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PLASMA IRREGULARITIES IN THE COMET'S TAIL

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ABSTRACT

The fluctuations in the intensity of radio source PKS 2025-15 during its occultation by Comet Kohoutek (1973f) on January 5, 1974, can be interpreted as scintillations due to the turbulent plasma in the Comet's tail. It is found that the rms fluctuation of the electron density in the Comet's tail, ΔN, is about 200 electrons cm^{-3}, the inner scale of the fluctuation is about 8×10^7 cm and the largest scale of fluctuation may reach 6×10^{10} cm.

I. INTRODUCTION

Comets are known to develop plasma tails as they approach the Sun. Recently Ananthakrishnan et al. (1975) have reported the fluctuations in the intensity of the extragalactic radio source PKS 2025-15 observed during its occultation by the tail of Comet Kohoutek (1973f) on January 5, 1974. The observations were made at 327 MHz. They found that the scintillation index M_z (defined as(variance of intensity)$^{1/2}$/mean intensity) was $\sim$20% and reached a maximum of $\sim$30%. The average observed time scale of fluctuations is about 10 second. However, they concluded that it is difficult to explain the fluctuations in terms of scintillations produced by the passage of radio waves through the cometary plasma.

The purpose of this note is to point out that the fluctuations of radio intensity observed during occultation can be explained by scintillations caused by the cometary tail if we use the scintillation theory correctly. In Section II, we use a Gaussian spectrum for the plasma irregularities in the cometary tail to analyze the scintillation data, while in Section III, a more realistic spectrum, the Kolmogorov spectrum, is used.

II. Scintillations During Occultation

Since the size of plasma tail of a comet is much smaller than the distance between the comet and the observer on the earth, we will use "thin phase-changing screen approximation" (Salpeter 1967) to analyze the problem. A schematic sketch of this approximation is presented in Figure 1, in which z is the direction of wave propagation and $\underset{\sim}{\rho} = (x,y)$ is the transverse coordinate. The random phase fluctuations of the wave are produced by a layer of irregularity in the plasma tail. After passing the screen, as shown in Figure 1, the phase of the wave is randomized and is characterized by the function $\phi(x,y)$. The intensity fluctuations $\delta I(z,\rho)$ of the electromagnetic wave at position $z > o$ are then built up by diffraction as the wave propagates to the observer.

Let the two-point correlation function of phase be

$$\phi_o^2 \, P_\phi(\underset{\sim}{\rho}) = \langle \phi(\underset{\sim}{\rho}_1) \, \phi \, (\underset{\sim}{\rho}_1 + \underset{\sim}{\rho}) \rangle \tag{1}$$

with $P_\phi(o) = 1$. Here $\langle \; \rangle$ denotes an average over an ensemble of random phases. $P_\phi(\underset{\sim}{\rho})$ is the normalized phase correlation function and ϕ_0 is the rms phase fluctuation. Let the normalized intensity correlation function of the observed wave be

$$P_I \, (\underset{\sim}{\rho}) = \frac{\langle \delta I(\underset{\sim}{\rho}_1) \;\; \delta I(\underset{\sim}{\rho}_1 + \underset{\sim}{\rho}) \rangle}{\langle I \rangle^2} \; -1 \tag{2}$$

where $\langle I \rangle$ is the mean intensity and δI is the fluctuation part. The scintillation index M_z measured at z is defined as

$$M_z{}^2 = P_I(o) \; . \tag{3}$$

Now we consider the case in which the phase correlation function is Gaussian with a characteristic scale a, i. e.

$$P_\phi(\underset{\sim}{\rho}) = \exp(-\rho^2/2a^2) \tag{4}$$

Let the correlation scale of the observed intensity fluctuation be $\bar{\ell}_c$ and let $\hat{P}_\phi(\underset{\sim}{q})$ and $\hat{P}_I(z,\underset{\sim}{q})$ be, respectively, the Fourier transforms of $P_\phi(\underset{\sim}{\rho})$ and $P_I(z,\underset{\sim}{\rho})$ in transverse coordinates.

The results of the "thin phase-changing screen approximation" can be summarized as (Salpeter 1967, Lee and Jokipii 1975b).

Case (i), for $\phi_o \leq 1$

$$\hat{P}_I(z,\underset{\sim}{q}) = 4\,\phi_o^2 \sin^2\left(\frac{z\,q^2}{2k}\right)\ \hat{P}_\phi(\underset{\sim}{q}),\ \ell_c = a; \tag{5}$$

Case (ii), for $\phi_o \geq 1$

(a) $\hat{P}_I(z,\underset{\sim}{q}) = \phi_o^2 (z\,q^2/k)^2\ \hat{P}_\phi(\underset{\sim}{q}),\ \ell_c = a$, for $z \ll \ell_f$ (6a)

(b) $P_I(z,\underset{\sim}{\rho}) = \exp[-2\phi_o^2 (1-P_\phi(\underset{\sim}{\rho}))] - \exp[-2\phi_o^2],\ \ell_c = a/\phi_o$, for $z \gg \ell_f$. (6b)

Here $\ell_f = ka^2/\phi_o$ is the "focal length" and z is the distance from the phase screen. For the region where $z \sim \ell_f$, the correlation scale ℓ_c changes from a to a/ϕ_o smoothly (Lee 1974).

For the occultation of radio source PKS 2025-15 by Comet Kohoutek (1973f) (Ananthakrishnan et al. 1975), we have: $z = 1.3 \times 10^{13}$ cm, M_z = 0.2 - 0.3, and the average time scale of intensity fluctuation (t_c) is $\sim$ 10 sec. With the transverse velocity of the plasma tail to the line of sight, $V_1 \sim 80$ km/sec, we have the spatial correlation scale $\ell_c = V_1\, t_c \simeq$ 800 km. Our problem does not belong to Case (i), where $\phi_o \leq 1$, since $[z/(k\ell_c^2)] \simeq 0.033$ would give us $M_z < 0.1$. The observed value of M_z is between 0.2 and 0.3. It cannot be Case (iib), because $z \ll k\ell_c^2 < \ell_f$. We will see that Case (iia) indeed gives us a consistent explanation.

For $\phi_o \geq 1$ and $z << \ell_f$, we have $\ell_c \simeq a = 800$ km. By Eq. (6a), we have

$$M_z^2 = 8\phi_o^2 \left(\frac{z}{ka^2}\right)^2 \tag{7}$$

With the observed values of $z/ka^2 \simeq 0.033$ and $M_z \simeq 0.2 - 0.3$ Eq. (7) gives us

$$\phi_o \simeq 2 - 3 \tag{8}$$

Note that with the value of ϕ_o in Eq. (8), $\ell_f \simeq 2 \times 10^{14}$ cm and $z << \ell_f$ is satisfied. Thus for $a \simeq 800$ km, and $\phi_o \simeq 2 - 3$, we can indeed obtain a scintillation index M_z of values between 20% and 30%.

The rms phase fluctuation ϕ_o can be related to the rms electron density fluctuation, ΔN, and the thickness of plasma layer, D. We have (Lee and Jokipii 1975b)

$$\phi_o^2 = 8\times 2^{1/2} \pi^{5/2} a\, r_e^2 k^{-2} D \Delta N^2 \tag{9}$$

where r_e is the classical electron radius and k is the wave number. Putting $\phi_o = 2.5$, we obtain from Eq. (9)

$$D \Delta N^2 = 2.4 \times 10^{17} \text{ cm}^{-5} \tag{10}$$

From Figure (2) of Ananthakrishnan et al. (1975), D is about 6×10^{10} cm. Thus we have

$$\Delta N = 20 \text{ cm}^{-3}. \tag{11}$$

The above result must be considered as the lower bound of the rms electron density fluctuation since for the Gaussian spectrum, the value of ΔN corresponds only to the density fluctuation of those inhomogeneities which are important in causing scintillations.

The Gaussian spectrum is rather artificial in interpreting the scintillation data and the power-law spectrum is a more realistic form.

In the earth's atmosphere (Tatarskii 1971), in the solar wind (Jokipii 1973), or in the interstellar medium (Lee and Jokipii 1976), the spectrum of the turbulent medium has been observed to be close to the Kolmogorov power-law spectrum. In the next section, we use the Kolmogorov spectrum to analyze the scintillation data.

III. Kolmogorof Spectrum

Consider the case in which the turbulent cometary tail has a Kolmogorov spectrum. It can then be shown that the phase correlation function P_ϕ also possesses a Kolgomorov spectrum (Lee and Jokipii 1975a). Thus we write

$$\hat{P}_\phi(q) = A\ (1+q^2L^2)^{-\alpha/2}\ \exp(-q^2\ell^2/2) \qquad (12)$$

with $L \gg \ell$. This spectrum is flat for $q < L^{-1}$, is a power-law with index $-\alpha$ for $L^{-1} < q < \ell^{-1}$, and is cut off for $q > \ell^{-1}$. Here L is the coherence scale (or the largest scale) and ℓ is termed the inner scale. Usually $3 < \alpha < 4$. In this section, we set $\alpha = 11/3$ corresponding to the Kolmogorov spectrum. In Eq.(12), A is a constant and is determined by the condition that $P_\phi(0) = 1$. For $L \gg \ell$, we have

$$A = \left(\frac{\alpha}{2} - 1\right) L^2/\pi\ . \qquad (13)$$

Since $P_I(0) = M_z^2 = 0.04 - 0.09$ from the occultation data, the scintillation is still weak and we have

$$\hat{P}_I(z, \underset{\sim}{q}) = 4\ \phi_o^2\ \sin^2(\ zq^2/2k\ )\ \hat{P}\ (\underset{\sim}{q}\ ) \qquad (14)$$

(Jokipii 1970, Lee and Jokipii 1975b). It can be shown from Eq.(14) that the correlation scale of P_I is

$$\ell_c = \begin{cases} \ell\ , & \text{for}\ \ \ell > (z/k)^{1/2} \\ (z/k)^{1/2}, & \text{for}\ \ \ell < (z/k)^{1/2}. \end{cases} \qquad (15)$$

Noting $\ell_c \neq (z/k)^{1/2}$ in our case, we conclude that $\ell_c = \ell$. Thus the inner scale of the turbulence in the cometary tail is 8×10^7 cm.

Again the rms phase fluctuation ϕ_o is related to ΔN and the thickness D (Lee and Jokipii 1975b) by

$$\phi_o^2 = 76.8 \ \pi^{7/2} L \ r_e^2 \ k^{-2} \ D \ \Delta N^2 \ \Gamma(11/6)/\ \Gamma(1/3). \quad (16)$$

From Eq.(14), we have

$$M_z^2 = P_I(0) = A \ \phi_o^2 \ (z/k)^2 \ 8\pi \ \Gamma(7/6) \ \ell^{-7/3} \ L^{-11/3} \quad (17)$$

by noting that $(z/k)^{1/2} << \ell$. Assuming that the coherence scale is of order of the width of the cometary tail and setting $L = D = 6 \times 10^{10}$ cm, we have from Eq.(17)

$$\Delta N = 200 \ \text{cm}^{-3} \quad (18)$$

for $M_z = 0.25$. Note that from Eqs.(13),(16) and (17), $\Delta N^2 \propto \ell^{7/3} L^{2/3}$. The value of ΔN in Eq.(18) can be considered as the upper bound for the rms electron density fluctuation since L can not be larger than the size of the cometary tail.

IV. Conclusion

From the above analysis, we find that for a Gaussian spectrum, the rms electron density fluctuation ΔN of the plasma irregularities in the cometary tail is about 20 cm^{-3} and the correlation scale a is about 8×10^7cm. However for the Kolmogorov spectrum, $\Delta N \simeq 200\ cm^{-3}$, the inner scale of the turbulence $\ell \simeq 8 \times 10^7$cm, and the coherence scale $L \simeq 6 \times 10^{10}$cm. Since the Kolmogorov spectrum is a more realistic one, we believe that $\Delta N \simeq 200\ cm^{-3}$ is the better value for the rms electron density fluctuation. Our result is consistent with the upper limit of $10^{+4}\ cm^{-3}$ for ΔN determined by the angular broadening of some radio sources during occultation by the comet Arend-Roland (1956f) (Whitfield and Hogbom 1957). It is of interest to compare ΔN obtained here with the mean plasma density $\langle N_e \rangle \simeq 10^3\ cm^{-3}$ estimated from optical data by Wurm(1968). For $\langle N_e \rangle = 10^3\ cm^{-3}$ and $\Delta N = 200\ cm^{-3}$, the ratio ($\Delta N / \langle N_e \rangle$) is about 0.2.

The occultation observations provide us with valuable information about the plasma irregularities in the cometary tail. In order to determine the shape of the power spectrum of the electron density fluctuation, more observations with multiple channels are suggested.

Figure Caption

Figure 1. A schematic sketch of the thin screen diffraction problem. The random medium is confined to a thin layer of thickness D (from $z = -D$ to $z = 0$). The plane wave $e^{i(kz-\omega t)}$ hits the "thin screen" from the $-z$ direction. After passing the screen, the phase of the wave is randomized and is characterized by the function $\phi(x,y)$.

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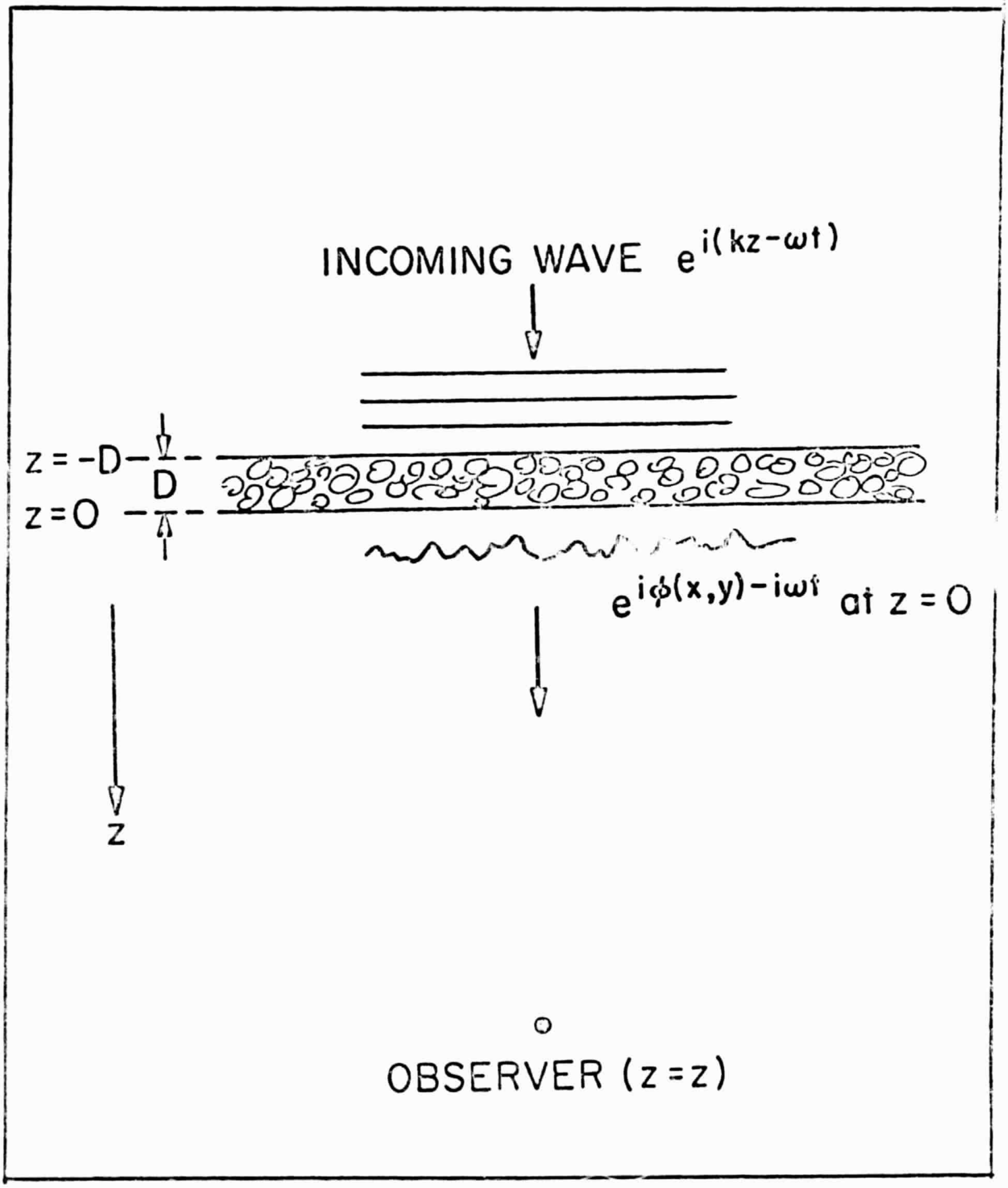
INCOMING WAVE $e^{i(kz-\omega t)}$
z = -D
D
z = 0
$e^{i\phi(x,y)-i\omega t}$ at z = 0
z
OBSERVER (z = z)