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FULLY UNSTEADY SUBSONIC AND
SUPersonic Potential Aerodynamics
For Complex Aircraft Configurations
For Flutter Applications

by

Kadin Tseng
and
Luigi Morino

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FULLY UNSTEADY SUBSONIC AND SUPERSONIC POTENTIAL
AERODYNAMICS OF COMPLEX AIRCRAFT CONFIGURATIONS
FOR FLUTTER APPLICATIONS

Kadin Tseng and Luigi Morino
Boston University
Boston, Massachusetts

Abstract

A general theory for steady, oscillatory or fully unsteady potential compressible aerodynamics around complex configurations is presented. Using the finite-element method to discretize the space problem, one obtains a set of differential-delay equations in time relating the potential to its normal derivative (on the surface of the body) which is expressed in terms of the generalized coordinates of the structure. For oscillatory flow, the motion consists of sinusoidal oscillations around a steady, subsonic or supersonic flow. For fully unsteady flow, the motion is assumed to consist of constant subsonic or supersonic speed for time t=0 (steady state) and of small perturbations around the steady state for time t>0; the solution is obtained in Laplace's domain. From the potential, the aerodynamic generalized forces are obtained. Therefore, the final output is the matrix of the aerodynamic influence coefficients, relating the generalized forces to the generalized coordinates, in the form necessary for flutter applications. The theory is embedded in a computer code, SOUSSA (Steady, Oscillatory and Unsteady, Subsonic and Supersonic Aerodynamics), which is briefly described. Numerical results are presented for steady and unsteady, subsonic and supersonic flows and indicate that the code is not only general, flexible, and simple to use but also accurate and fast.

1. Introduction

Presented herein is a general formulation of steady, oscillatory or fully unsteady, subsonic and supersonic potential aerodynamics for an aircraft having arbitrary shape. The objective of this formulation is to describe the time functional relationship between aerodynamic potential and its normal derivative (normal wash) in a form which can be used for computational analysis. The finite-element method is used for space discretization. The matrix

of the aerodynamic influence coefficients, as necessary for flutter calculations, is then obtained. Results obtained with the computer program SOUSSA (Steady, Oscillatory and Unsteady, Subsonic and Supersonic Aerodynamics) are also presented.

The analysis presented herein is based on a new integral formulation, presented in References 1 and 2, which includes completely arbitrary motion. However, the numerical implementation (Refs. 3 and 4) was thus far limited to steady and oscillatory flows. On the other hand, in order to perform a linear-system analysis of the aircraft, it is convenient to use more general aerodynamic formulations, i.e., fully transient response for time-domain analysis and the aerodynamic transfer function (Laplace transform of the fully unsteady operator) for frequency-domain analysis. A general formulation for fully unsteady (initial) aerodynamics was presented in Refs. 5 and 6 where only very preliminary results were given. Consistent with this type of analysis, the unsteady contribution is assumed to start at time t=0, so that for time t>0 the flow is in steady state. Furthermore, consistent with the linear flight dynamics analysis, the motion of the aircraft is assumed to consist of small (infinitesimal) perturbations around the steady-state motion.

It may be noted that within the assumption of potential aerodynamics, there exists other methods to evaluate the aerodynamic loads. Among them, the lifting surface theories, while flexible and efficient, are not sufficiently general. On the other hand, finite-element methods, though sufficiently general for handling complex configurations, are limited to steady flows. In addition, they are usually quite cumbersome to use and invariably require human intervention to define the suitable type of element (source, doublet, etc.) to be used. For oscillatory aerodynamics, the doublet-lattice method is the only other method, besides SOUSSA, which can handle subsonic oscillatory flows around complex configura-

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* Graduate Student, Research Assistant
** Director, Computational Continuum Mechanics Program, Member AIAA

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ations, while SOUSSA is the only program which can analyze oscillatory supersonic aerodynamics.

Finally, for fully unsteady aerodynamics, several problems have been considered since the initial work by Wagner (Ref. 10) on unsteady incompressible two-dimensional flow. Several methods are available for wings in subsonic and supersonic flow (see Refs. 11 and 12), no other code, besides SOUSSA, is available for subsonic and supersonic flows around arbitrary complex configurations for either time or frequency domain analysis.

The purpose of this paper is to present recent developments on the formulation of Ref. 1. In this paper only the subsonic formulation is presented in detail, the supersonic formulation is only briefly outlined in Appendix A. For conciseness, material previously presented (in particular, the material of Ref. 4) is not repeated herein.

Using the finite-element method to discretize the space problem, one obtains a set of differential-delay equations in time relating the potential to its normal derivative (on the surface of the body), which is expressed in terms of the generalized coordinates of the structure. For subsonic flow, the motion consists of sinusoidal oscillations around a steady, subsonic or supersonic flow. For fully unsteady flow, the motion is assumed to consist of constant subsonic or supersonic speed for time t > 0 (steady state) and of small perturbations around the steady state for time t < 0; the solution is obtained in Laplace's domain. From the potential, the aerodynamic generalized forces are obtained. Therefore, the final output is the matrix of the aerodynamic coefficients, relating the generalized forces to the generalized coordinates. The theory is embedded in a computer code, SOUSSA (Steady Oscillatory and Unsteady Subsonic and Supersonic Aerodynamics), which is briefly described. Numerical results are presented for steady and unsteady, subsonic and supersonic flows and indicate that the code is not only general, flexible, and simple to use but also accurate and fast.

2. Equation for Velocity Potential

The subsonic aerodynamic formulation used in SOUSSA is briefly presented here. The supersonic formulation is given in Appendix A. Assume the flow to be an infinitesimal perturbation from the steady state flow. Then standard use of Green's function method applied to the equation of the velocity yields, after linearization, the following integral equation:

\[ 2\pi \Phi (\vec{R}, t) = - \int_{\Sigma} \left[ \nabla^2 \Psi \right] \frac{1}{R} d\Sigma_{R} + \int_{\Sigma} \left[ \frac{1}{\nabla^2} \left( \frac{1}{R} \right) \right] d\Sigma_{W} \]

where \( \Phi_{R} \) is on the surface of the body, \( \Sigma_{R}, N \) is the outer normal.

\[ \Psi = \nabla \Phi \]

\[ \omega = \frac{\partial \Phi}{\partial t} \]

\[ \alpha = \frac{\partial \Phi}{\partial \alpha} \]

\[ \beta = \frac{\partial \Phi}{\partial \beta} \]

\[ \gamma = \frac{\partial \Phi}{\partial \gamma} \]

\[ \delta = \frac{\partial \Phi}{\partial \delta} \]

\[ \epsilon = \frac{\partial \Phi}{\partial \epsilon} \]

\[ \zeta = \frac{\partial \Phi}{\partial \zeta} \]

\[ \eta = \frac{\partial \Phi}{\partial \eta} \]

\[ \theta = \frac{\partial \Phi}{\partial \theta} \]

\[ \phi = \frac{\partial \Phi}{\partial \phi} \]

\[ \psi = \frac{\partial \Phi}{\partial \psi} \]

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\[ \psi = \frac{\partial \Phi}{\partial \psi} \]

\[ \alpha = \frac{\partial \Phi}{\partial \alpha} \]

\[ \beta = \frac{\partial \Phi}{\partial \beta} \]

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\[ \beta = \frac{\partial \Phi}{\partial \beta} \]

\[ \gamma = \frac{\partial \Phi}{\partial \gamma} \]

\[ \delta = \frac{\partial \Phi}{\partial \delta} \]
lateral elements $E_h$ (which are described in terms of the corner points by use of standard finite-element interpolation technique*) and by assuming $\psi$ and $\phi$ to be constant within each element:

$$
\psi(P, T - \theta) = \phi_h(T - \theta_h)
$$

where $\psi_h(T - \theta_h)$ and $\phi_h(T - \theta_h)$ are time dependent values of $\psi$ and $\phi$ at the centroid $P_h$ of $E_h$ at the time $T - \theta_h$ (where $\theta_h$ is the disturbance-propagation time from $E_o$ to $E_h$).

Next consider the integrals over the wake. In order to facilitate the solution of Eq. (6), it is convenient to divide the wake into strips defined by (steady-state) vortex-lines emanating from the nodes on the trailing edge. The strips are then divided into $N_w$ elements $\Sigma_h(w)$ with nodes along the vortex lines. The potential discontinuity is assumed to be constant within each element

$$
\Delta\phi(P, T - \theta) = \Delta\phi_h(T - \theta_h)
$$

where $\Delta\phi_h(T - \theta_h)$ is the value of $\Delta\phi$ at the centroid $P_h(W)$ of the element $\Sigma_h(n)$ on the wake at time $T - \theta_h$ (where $\theta_h$ is the propagation time from $P_h(W)$ to $E_h$).

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\Delta\phi(P, T - \theta) = \Delta\phi_h(T - \theta_h)
$$

where $m = m(n)$ identifies the trailing-edge point which is on the same vortex-line as the point $P_h(n)$. Furthermore, $\theta_h$ is the time necessary for the vortex-point to be convected from the trailing-edge point $P_h(TE)$ to the wake-point $P_h(W)$.

It may be worth noting that

$$
\phi_h - \phi_{h+1}
$$

where $h_u$ and $h_{l+1}$ identify the upper and lower trailing-edge nodes on the body corresponding to the $m$th node on the trailing-edge.

In SOUSSA $\psi_h - \phi_h$ approximated with the value evaluated at the centroids of the elements adjacent to the trailing edge. This is reasonable in view of the

* The equation for the elements are of the type $P = P_o + \xi P_1 + \eta P_2 + \xi \eta P_3 (-1 < \xi < 1, -1 < \eta < 1).$ This type of element is called hyperboloidal element and is described in details in Ref. 4.

The Kutta condition is then evaluated from the centroid, as

$$
\pi = 2 \frac{(P(W) - X)}{M}
$$

With the above approximation, it is possible to write

$$
\Delta\phi(T) = \sum S_{nh} \phi_h
$$

where $S_{nh} = 1(S_{nh} = 0)$, if $h$ identifies the upstream point $P_h$ on the body corresponding to the point $P_{n(m)}$ on the trailing edge (i.e., near the point $P_{n(m)}(TE)$ on the trailing edge), and $S_{nh} = 0$ otherwise.

Combining Eqs. (1), (8) and (9) and assuming $\phi = \phi_1$, one obtains

$$
\phi_1(T) = \sum S_{nh} \phi_h(T - \theta_h)
$$

where $S_{nh} = 1(S_{nh} = 0)$, if $h$ identifies the upstream point $P_h$ on the body corresponding to the point $P_{n(m)}$ on the trailing edge (i.e., near the point $P_{n(m)}(TE)$ on the trailing edge), and $S_{nh} = 0$ otherwise.

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Using the above mentioned hyperboloidal quadrilateral elements the coefficients $F_{jhn}$, $C_{jhn}$, $P_{jhn}$ and $G_{jhn}$ are evaluated analytically with the expressions given.
in Ref. 4. The coefficients $D_{i,jh}$ are approximated as $D_{i,jh} = R_{i,jh} C_{i,jh}$ where

Next, taking the Laplace transform with zero initial conditions of Eq. (13) yields

$$
[V_{h}] \{ \tilde{\phi}_h \} = [Z_{h}] \{ \tilde{\psi}_h \}
$$

(16)

where $\tilde{\phi}_h$ and $\tilde{\psi}_h$ are the Laplace transforms of $\phi_h$ and $\psi_h$ while

$$
\tilde{\psi}_h = \psi_h - \left(C_{i,jh} + S D_{i,jh}\right) e^{-S \gamma_h} - \sum_{n} \left(C_{i,jn} + S G_{i,jn}\right) e^{-S \gamma_n} + \left(\varepsilon_{i,n}^{(g)} + \varepsilon_{i,n}^{(f)}\right) S \gamma_{nh}
$$

(17)

and

$$
\tilde{Z}_{i,jh} = b_{i,jh} e^{-S \gamma_h}
$$

(18)

whereas $S$ is the nondimensional Laplace parameter.

4. Pressure and Generalized Forces

In order to complete the formulation, the procedure for the evaluation of the aerodynamic pressure and the generalized forces is presented in this section.

First, consider an averaging scheme which imposes that the value of the potential $\phi_{k}^{(i)}$ at the node $P_{k}$ is the average of the values of the potential at the centroids of the elements surrounding $P_{k}$. In other words

$$
\{ \phi_{k}^{(i)} \} = [E_{kh}]{\phi}_{h}
$$

(24)

where $[E_{kh}]$ is an averaging matrix defined as

$$
E_{kh} = \frac{1}{N(k)} \text{ if } \bar{P}_{k}^{(i)} \in \Sigma_{h}
$$

(i.e., if the $\bar{P}_{k}^{(i)}$ is one of the corner points of the element $\Sigma_{h}$) and

$$
E_{kh} = 0 \text{ otherwise}
$$

(25)

In Eq. (25), $N(k)$ is the number of elements which have $P_{k}$ as one of their corner points. Having evaluated the values of $\phi$ at the four corner points of each quadrilateral element, the potential is expressed as

$$
\phi = \sum_{h} N_{k} \phi_{h}^{(i)}(\bar{P})
$$

(27)
where $N'_k$ are the first-order global shape functions obtained by assembling local shape functions of the type

$$N'_k = \frac{1}{d_{k} \partial_k^{1}} (x + \xi_k) (y + \eta_k)$$

(28)

where $\xi_k = +1$ and $\eta_k = +1$ are the locations of the corners $\bar{x}_k$ of the element $\Sigma_h$ ($x$ and $y$ are the coordinates over the element) so that

$$\frac{\partial \phi}{\partial x^i} = \sum_{k} \frac{3N'_h}{3x^i} \xi_k \eta_k$$

(29)

where $\xi^1 = \xi$; $\xi^2 = \eta$.

The pressure coefficient may be evaluated from the linearized Bernoulli theorem as

$$c_p = -2 \left( \frac{\sigma \phi + 1}{\rho} \hat{\phi} \cdot \hat{T} \right)$$

(30)

Expressing $\hat{\phi}$ in terms of the tangential derivatives $\delta \phi \delta x^i$ and neglecting the contribution of the normal component, the pressure coefficient is given by

$$c_p = -2 \left( \frac{\sigma \phi + 1}{\rho} \hat{\phi} \cdot \hat{A} \frac{\delta \phi}{\delta x^I} \right)$$

(31)

where $A^\gamma$ are the contravariant base vectors, with $\delta \phi / \delta x^\gamma$ given by Eq. (29).

Next consider the generalized aerodynamic forces

$$Q_n = -\iint q \cdot P \cdot \overline{M}_n \cdot \partial \Sigma$$

(32)

where

$$q = \frac{1}{2} \rho \overline{U}^2_{\infty}$$

(33)

is the dynamic pressure.

By assuming that the pressure coefficient $c_p$ is constant within each element (consistent with the assumption made on $\phi$), Eq. (32) can be expressed as

$$[Q_n] = q [Q_{nh}] \{ \psi_h \}$$

(34)

where

$$Q_{nh} = \iint \bar{n} \cdot \overline{M}_h \cdot \partial \Sigma$$

$$= \iint \bar{n} \cdot \bar{a}_1 \cdot \bar{a}_2 \cdot \overline{M}_n \cdot \partial \Sigma$$

$$= \iint \left( \bar{a}_1 \times \bar{a}_2 \right) \cdot \overline{M}_n \cdot \partial \Sigma$$

(35)

$$= -4 \left( \bar{a}_1 \times \bar{a}_2 \right) \cdot \overline{M}_n \cdot \partial \Sigma$$

(36)

$$\overline{M}_n = \begin{bmatrix} M_{1n} \\ M_{2n} \\ M_{3n} \end{bmatrix}$$

(37)

In Eqs. (36) to (43) compact matrix notations are used. Vectors and matrices are underlined. Tildas indicate Laplace's transform or equivalent operation.
6. Numerical Results

Typical numerical results obtained with SOSSA (Ref. 14) are presented in this section. Figure 1 shows the lift coefficients at various stations of a wing-body in steady, subsonic flow compared against experimental and theoretical results of Ref. 6 and 15. The results were obtained for N=0 and a rectangular wing with chord c=1, and span b=6, thickness t=0.09 and a=6. The body is at zero angle of attack with overall length of 5 chords (forebody with length L=2c, fuselage length L=3). Note that the fuselage is closed at the end by a circular plate. In addition, a flat wake is running from the wing trailing edge and a cylindrical wake from that of the fuselage. Figure 2 shows the convergence analysis of sectional lift of Figure 1 as a function of the number of elements. The computer time used for each case is also indicated. Figures 3 and 4 present the lift and moment coefficients of a rectangular wing in supersonic unsteady flows with aspect ratio AR=2, c=1, b=2, \( \tau =0.001 \) and complex reduced frequencies \( \omega =0.2+1.0, 0.6+1.0 \) and \( 0.4+1.0 \) for Mach number M=2.5 to 2.5. The results are compared with those of Ref. 16. Noted that in contrast to the present method does not require the use of diaphragms. Figure 5 presents a wing-body-tail configuration in fully unsteady flow with specifications of the geometry similar to that of Fig. 1. However, a horizontal tail is added with chord c=1 and b=6 which is stationed 0.5 chords above the center line of the fuselage. The complex reduced frequency was \( \kappa =0.1+10.5 \). No existing result is available for comparison for, as mentioned above, the present method is the only existing one which can analyze fully unsteady flow. The result is presented to demonstrate the generality of the method and its ability to handle fully unsteady flow problems. Figures 6 and 7 present the lift and moment coefficients and their corresponding phase angles for a rectangular wing oscillating in plunging and pitch in subsonic flow with AR=2, \( \tau =0.001 \), Mach number M=0, and 4x7x7 elements on the whole wing. The results are identical with the ones obtained with SOSSA. However, considerable time saving was obtained in the respective frequency and mode calculations by the decomposition of the matrix \( \mathbf{H} \) into frequency and mode dependent matrices \( \mathbf{H}_f \) (see Eq. 43). All the results were obtained in 44 mins. i.e. 82.5 secs. for one aerodynamic coefficient and one frequency (since four coefficients and eight frequencies have been considered). Note that the time for one single coefficient and one single frequency (if evaluated independently) is about 13 minutes.

Tables 1 and 2 contains the generalized forces for an AGARD wing-tail configuration in quasi-steady and oscillatory flow compared with several existing methods (Refs. 17 to 21). While Table 3 included the generalized forces for the same configuration in fully unsteady flow (complex frequency). For all the results the standard AGARD geometry (des-
cribed for instance in Refs. 18 and 19) was used. This consists of two swept tapered lifting surfaces. The first surface has 
\( x_{LE} = 0 \) and \( x_{TE} = 2.25 \) at \( y = 0 \) and \( x_{LE} = 2.75 \) and 
\( x_{TE} = 3.70 \) at \( y = 1 \) and is located at \( z = 0 \). The 
second surface has \( x_{LE} = 2.70 \) and \( x_{TE} = 4.00 \) at \( y = 0 \) and \( x_{LE} = 3.50 \) and \( x_{TE} = 4.25 \) at \( y = 1 \) and is located at \( z = 0 \). All the results 
proscribed here were obtained using \( 4 \times 7 \times 7 \) elements on each surface. Results ob-
tained with \( 4 \times 5 \times 5 \) elements indicate that 
convergence was attained. The results are 
usually in excellent agreement with those 
of Refs. 17 to 21.

7. Conclusions

A general formulation and computer 
program for the analysis of steady, oscil-
latory and unsteady, subsonic and super-
sonic aerodynamic flows around complex 
configurations have been presented. The 
final output of the code is the matrix 
of the aerodynamic influence coeffi-
cients for flutter analysis to be used 
for instance in the program FCAP (Ref. 
22).

It should be noted that, while there 
exists several methods to analyze the 
problem of unsteady compressible flows 
for complex configurations, the present 
method, embedded in the computer program 
SOUSSA, is unique in the following 
aspects:

1. It provides a completely unified 
approach for steady, oscillatory 
and fully unsteady, subsonic and 
supersonic aerodynamic flows.

2. It can be applied to arbitrarily-
complex configurations. Wing-body-
tail configurations in fully un-
steady flows have been presented.

3. It is computationally extremely 
general, flexible, efficient and 
above all, accurate. The elimina-
tion of diaphragms in supersonic 
flow improved considerably the 
simplicity and efficiency of the 
code.

4. SOUSSA is the only existing pro-
gram that can analyze fully unsteady 
complex-configuration potential 
aerodynamics in subsonic or super-
sonic regimes. It is also the only 
program capable of handling oscil-
latory supersonic aerodynamics for 
complex configurations.

5. In contrast to existing methods, 
which in many instances requires 
extensive user's background in 
aerodynamics and familiarity with 
the specific method, the present 
code requires very limited human 
intervention and is extremely easy 
to use.

Appendix A

In this Appendix the formulation for 
the supersonic case is briefly outlined. 
For conciseness, only supersonic trailing 
edges are considered so that the contribu-
tion of the wake can be ignored. (In 
SOUSSA diaphragms are not used and there-
fore the supersonic wake is treated as 
the subsonic one.) Under small-perturba-
tion assumption, the Green theorem for 
potential supersonic flow is given by

\[
2\pi \Phi^0 (r, T) = \int_{\Gamma} \left( \psi_0^0 + \psi^0 \right) \frac{H}{R'} d\Sigma
\]

(A.1)

where \( \psi_0^0 \) is the conormal 
derivative (Ref. 4) and \( \psi^0 \) is the conormal 
wash which is prescribed by the boundary 
conditions, and

\[
X = \sqrt{\beta}, \quad Y = \sqrt{\beta}, \quad Z = \frac{1}{\beta}, \quad T = q_0 \sqrt{\beta}
\]

(A.2)

with \( \beta = \sqrt{M^2 - 1} \). Furthermore,

\[
R = \left( X - X_s \right)^2 - (Y - Y_s)^2 - (Z - Z_s)^2 \right)^{1/2}
\]

(A.3)

whereas

\[
H = \begin{cases} 
1 & \text{for } X_s < X \geq \sqrt{(Y - Y_s)^2 + (Z - Z_s)^2} \\
0 & \text{for } X_s < X \leq \sqrt{(Y - Y_s)^2 + (Z - Z_s)^2} \end{cases}
\]

(A.4)

and

\[
\begin{bmatrix}
\theta^+ \\
\theta^-
\end{bmatrix} = 
\begin{bmatrix}
\theta^+ \\
\theta^-
\end{bmatrix} \frac{1}{1 - \theta^+}
\]

(A.5)

with

\[
\theta^+ = M (X_s - X) \pm R'
\]

(A.6)

Following the same procedure used for 
the subsonic case one obtains
\[ \phi(\Gamma) = \sum_{h} b_{\phi}^{*} \left[ f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{+}) + f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{-}) \right] 
+ \sum_{h} c_{\phi}^{*} \left[ f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{+}) + f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{-}) \right] 
+ \sum_{h} d_{\phi}^{*} \left[ f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{+}) - f_{\phi}^{*}(\Gamma - \Theta_{\phi}^{-}) \right] \]  
(A.7)

where

\[ b_{\phi}^{*} = -\frac{1}{2\pi} \int_{\Gamma_{h}} \frac{H}{R} \frac{d\Gamma_{c}}{d\Gamma_{s}} \bigg|_{P_{h} = P_{s}} \]  
\[ c_{\phi}^{*} = -\frac{1}{2\pi} \int_{\Gamma_{h}} \frac{\partial}{\partial N_{s}} \left( \frac{H}{R} \right) \frac{d\Gamma_{c}}{d\Gamma_{s}} \bigg|_{P_{h} = P_{s}} \]  
\[ d_{\phi}^{*} = -\frac{1}{2\pi} \int_{\Gamma_{h}} \frac{\partial}{\partial N_{s}} \left( \frac{H}{R} \right) \frac{d\Gamma_{c}}{d\Gamma_{s}} \bigg|_{P_{h} = P_{s}} \]  
(A.8)

The definition of \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \) is discussed at the end of this appendix.

Finally, taking the Laplace transform of equation (26) results in

\[ \left[ Y_{h}^{*} \right] = \left[ Z_{h}^{*} \right] \]  
(A.9)

where

\[ Y_{h}^{*} = \left[ f_{\phi}^{*} e^{\Theta_{\phi}^{+}} - S_{\phi}^{+} f_{\phi}^{*} e^{\Theta_{\phi}^{-}} \right] \]  
\[ Z_{h}^{*} = \left[ f_{\phi}^{*} e^{\Theta_{\phi}^{+}} + S_{\phi}^{-} f_{\phi}^{*} e^{\Theta_{\phi}^{-}} \right] \]  
(A.10)

Equation (A.9) yields the matrix \( \mathbf{E}^{(2)} \) to be used in Eq. (39) for supersonic flows. The supersonic matrices \( \mathbf{E}^{(1)} \) and \( \mathbf{H}^{(3)} \) and \( \mathbf{H}^{(4)} \) are equal to the subsonic ones. Therefore the above modifications in the definitions of \( Y_{h}^{*} \) and \( Z_{h}^{*} \) are the only changes necessary for supersonic flows. As mentioned above if the trailing edges are partially subsonic the wake is treated as in the subsonic case.

Finally a few remarks are necessary on the fact that diaphragms are not used in SOUSSA and on how this relates to the correct definition of \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \). It should be noted that \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \) are imaginary if the centroid \( \Gamma_{h} \) of the element \( \Gamma_{h} \) lies outside the Mach forecone of the control point \( P_{j} \). This yields no problem for the elements completely outside or completely inside the Mach forecone. However for the elements partially inside the Mach forecone, a special definition for \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \) must be used: note that \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \) represent the two propagation times for the disturbances emanating from the element \( \Gamma_{h} \) to reach the point \( P_{j} \).

Therefore for elements partially inside the Mach forecone, \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \) are most appropriately defined (from a physical point of view) as the propagation times from the centroid of the portion of the element \( \Gamma_{h} \) intersected by the Mach forecone to the control point \( P_{j} \).

It should be noted that with this definition of \( \Theta_{\phi}^{+} \) and \( \Theta_{\phi}^{-} \), supersonic flows can be treated exactly the same way as subsonic flows. For instance a wing with supersonic leading edge is solved by using both sides of the wing simultaneously. Also for wings with partially supersonic leading edges the use of diaphragms is not necessary. This is a considerable advantage: the use of diaphragms in the program SOUSSA was cumbersome, especially for wing-body-tail and non-coplanar surfaces analyses. In the program SOUSSA there is no difference in the treatment of subsonic and supersonic flows.

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21. Nomenclature

- \( a \) : speed of sound in undisturbed flow
- \( \chi \): reduced frequency, \( \omega/\omega_0 \)
- \( \chi_c \): complex reduced frequency
- \( \chi_n \): reference length
- \( n \): Mach number, \( U_0/a_0 \)
- \( \chi_n \): normal to \( L_n \)
- \( N_w \): number of wake elements in \( x \)-direction
- \( N_x \): number of elements in \( x \)-direction on half wing
- \( N_y \): number of elements in \( y \)-direction on half wing
- \( \rho \): point having coordinates \((x, y, z)\)
- \( \rho_n \): control point, \((x_n, y_n, z_n)\)
- \( \epsilon \): defined by Eq. (5)
- \( \tau \): nondimensional time, Eq. (2)
- \( \epsilon \): velocity of undisturbed flow
- \( \epsilon \): nondimensional space coordinates, Eq. (2)
- \( \Delta \): discontinuity of \( \phi \) across the wake
- \( \Delta t \): time for a disturbance to propagate from \( P \) to \( P' \), Eq. (6)
- \( \tau \): convection time of wake vortices, Eq. (7)
- \( \phi \): surface of body
- \( \phi \): surface of wake
- \( \phi \): nondimensional velocity perturbation potential
- \( \phi \): nondimensional normal wash
- \( \phi \): Laplace Transform of ( )
Fig. 1. Sectional lift coefficient distributions for a wing-body configuration with $a=0$, $a_0=0$, $M=0$ and NELEM=388. Comparison with results of Refs. 6 and 15.

Fig. 2. Convergence study of Fig. 1 with NELEM=200, 264 and 388.

Fig. 3. Lift coefficient, $C_L$, for rectangular wing oscillating in pitch, with $AR=2$, $t=0.001$, $N_x=8$, $N_y=7$. Comparison with results of Ref. 16.

Fig. 4. Moment coefficient, $C_M$, versus $2y/b$ for rectangular wing oscillating in pitch, for $AR=2$, $t=0.001$, $N_x=8$, $N_y=7$. Comparison with results of Ref. 16.

Fig. 5a. Pressure distributions at $2y/b$ =0.78 wing spanwise station of a wing-body-tail configuration in fully unsteady pitching mode with pitch axis at wing mid-chord. Complex frequency $k=0.1+1i.5$, span $b=6$, fuselage radius $r_c=0.5$, thickness ratio $r=0.2$, total number of elements NELEM = 388.
Fig. 5b. Pressure distributions at 2y/b = 0.78 horizontal-tail spanwise station of the same configuration as Fig. 5a.

Fig. 7. Moment coefficient, $C_M$, versus $k$, for a rectangular wing oscillating in plunge, with AR=2, $\tau=0.001$, $M=0$, $N_x=7$, $N_y=7$, $N_z=20$, wake length $L_w=2c$. Comparison with results of Ref. 16.

Fig. 6. Lift coefficient, $C_L$, versus $k$, for rectangular wing oscillating in plunge, with AR=2, $\tau=0.001$, $M=0$, $N_x=7$, $N_y=7$, $N_z=20$, wake length $L_w=2c$. Comparison with results of Ref. 16.
<table>
<thead>
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<th>Generalized Force in</th>
<th>Caused by Pressure in</th>
<th>( C_{14} )</th>
<th>( C_{13} )</th>
<th>( C_{14} )</th>
<th>( C_{15} )</th>
<th>Method</th>
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<td>Wing twist</td>
<td>Wing twist</td>
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<td>Wing bending</td>
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<td>Tail Pitch</td>
<td>Tail Pitch</td>
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<td>0.2016</td>
<td>0.7766</td>
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<td>0.1425</td>
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<td>0.5308</td>
<td>0.1262</td>
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<td>0.0431</td>
<td>0.5175</td>
<td>0.1332</td>
<td>0.7243</td>
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<tr>
<td>Wing twist &amp; Tail Roll</td>
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<td>0.5101</td>
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<td>0.0515</td>
<td>0.5055</td>
<td>0.1612</td>
<td>0.7078</td>
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**Table 1**
Generalized Aerodynamic Force Coefficients for ASABO Wing-Tail Interference N=0.6,25/\( L=0.6 **
### TABLE 3a
Generalized Aerodynamic Force Coefficients for AGARD Wing-Tail Interference M=3.0, z/l=0.6

<table>
<thead>
<tr>
<th>Generalized Force in</th>
<th>Caused by Pressure in</th>
<th>$C_{ij}$</th>
<th>$C_{ij}$</th>
<th>$C_{ij}$</th>
<th>$C_{ij}$</th>
<th>Method</th>
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<tbody>
<tr>
<td>Wing twist</td>
<td>Wing twist</td>
<td>1.1</td>
<td>0.0213</td>
<td>0.1462</td>
<td>Ref. 20</td>
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<tr>
<td>Wing bending</td>
<td>Wing twist</td>
<td>2.1</td>
<td>0.3601</td>
<td>0.0090</td>
<td>Ref. 20</td>
<td></td>
</tr>
<tr>
<td>Tail roll</td>
<td>Wing twist</td>
<td>3.1</td>
<td>0.1252</td>
<td>0.0551</td>
<td>Ref. 20</td>
<td></td>
</tr>
<tr>
<td>Tail pitch</td>
<td>Wing twist</td>
<td>4.1</td>
<td>0.0856</td>
<td>0.0243</td>
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<tr>
<td>Wing twist</td>
<td>Wing bending</td>
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<td>0.0760</td>
<td>0.0201</td>
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</tr>
<tr>
<td>Wing bending</td>
<td>Wing bending</td>
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<td>0.0733</td>
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<tr>
<td>Tail roll</td>
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<td>0.0218</td>
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<td>Wing bending</td>
<td>4.2</td>
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<td>Tail roll</td>
<td>5.2</td>
<td>0.0282</td>
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</tr>
<tr>
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<td>Tail roll</td>
<td>6.3</td>
<td>0.0072</td>
<td>0.0018</td>
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<td>Tail pitch</td>
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<td>0.0006</td>
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<tr>
<td>Tail pitch</td>
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<td>0.0002</td>
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### TABLE 3b
Generalized Aerodynamic Force Coefficients for AGARD Wing-Tail Interference M=0.6, z/l=0.6

<table>
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<tr>
<th>Generalized Force in</th>
<th>Caused by Pressure in</th>
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<th>$C_{ij}$</th>
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<td>Wing bending</td>
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<td>0.0122</td>
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<td>-0.0003</td>
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<tr>
<td>Wing bendig &amp; Tail pitch</td>
<td>Wing twist &amp; Tail roll</td>
<td>6.3</td>
<td>0.0427</td>
<td>0.0122</td>
<td>-0.0085</td>
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