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DETERMINATION OF SURFACE HEAT FLUX USING A SINGLE EMBEDDED THERMOCOUPLE

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16. Abstract

An implicit numerical procedure has been developed for predicting the transient heat flux to a material using a single embedded thermocouple. The accuracy of the method has been demonstrated by comparisons with analytically generated test data.

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DETERMINATION OF SURFACE HEAT FLUX USING A SINGLE EMBEDDED THERMOCOUPLE

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CONTENTS

TABLES

FIGURES

DETERMINATION OF SURFACE HEAT FLUX USING A

SINGLE EMBEDDED THERMOCOUPLE

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SUMMARY

A numerical method by which data from a single embedded thermocouple can be used to predict the transient thermal environment for both high- and low-conductivity materials is described. The results of an investigation performed to verify the method clearly demonstrate that accurate transient surface heating conditions can be obtained from a thermocouple 0.762 centimeter from the surface in a low-conductivity material. Space Shuttle Orbiter thermal protection system materials having temperature- and pressure-dependent properties and typical Orbiter entry heating conditions were used to verify the accuracy of the analytical procedure.

INTRODUCTION

The design and development of a reusable thermal protection system (TPS) for the Space Shuttle is dependent on a detailed knowledge of the aerothermodynamic environment to which the TPS will be exposed. The TPS thermal performance is normally obtained from exhaustive plasma arc and radiant heating tests to establish reuse temperature and thermal response characteristics. In a previous study by Curry and Williams (ref. 1), a nonlinear least squares method was developed for the estimation of thermal property values from experimental in-depth temperature data. The current investigation was motivated by the results of the previous analysis and by the opinion that calculations of surface heating rates and temperatures would be of considerable value in the test (flight and ground) and development phases of the Space Shuttle.

The calculation of surface heat flux and surface temperature from an in-depth temperature history measurement is called the inverse heat conduction problem and has been discussed by numerous investigators (refs. 2 to 12). An excellent discussion of previous investigations (refs. 4 to 8) for solving the inverse problem can also be found in reference 3. In particular, Beck and Wolf (ref. 2) presented a method of solution using least squares and future temperatures. In a later publication, Beck (ref. 3) presented a technique using

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nonlinear estimation in the solution of the inverse problem. Howard (ref. 9) developed a numerical method for determining the heat flux to a thermally thick wall with variable thermal properties using a single in-depth thermocouple. His best results were obtained for temperature measurements close to the heated surface in conjunction with a small computing interval.

Cornette (refs. 10 and 11), in analyzing the Project Fire calorimeter data, developed a transient inverse solution that required curve fitting of the basic temperature data. Cornette's solution accounted for variable material properties and yielded a closed-form analytical expression for the local surface heat flux at a given instant of time. The temperature-time data for several thermocouples embedded in a calorimeter plug were smoothed and the data replaced with a polynomial equation for temperature (at a particular thermocouple location) as a function of time. Imber and Khan (ref. 12) developed a closed-form inverse solution for constant properties and heat flux using two in-depth thermocouple readings. The solution was obtained by means of Laplace transform techniques in which the input thermocouple data were approximated by a temporal power series and a second series of error functions.

The purpose of this report is to disseminate the results of an investigation for predicting the surface conditions (heat rate/temperature) for a coated insulative material (low conductivity) and for a high-temperature coated carbon (high conductivity) material. The method, using a single embedded thermocouple, accounts for variable thermal properties (as functions of temperature and pressure) as well as for the effect of radiation losses and in-depth conduction. In addition, the results can be obtained with approximately the same computational time required to solve the thermal model using a known heat rate.

As an aid to the reader, where necessary the original units of measure have been converted to the equivalent value in the Système International d'Unités (SI). The SI units are written first, and the original units are written parenthetically thereafter.

SYMBOLS

A, B, C	quadratic coefficients
a, b, c, d	coefficients of square temperature matrix
C_p	specific heat of material at constant pressure
c	constant defined by equation (11)
f	function defined by equation (10)
i	individual measurements
k	thermal conductivity of material

L	thickness of material
ℓ	material designator
m	node identifier
P	point
q	heat flux
$\dot{q}_{conv}$	convective heating rate
$\dot{q}_i$	calculated convective heating rate
$\dot{q}_i^*$	actual convective heating rate
$\dot{q}_{net}$	heat rate
T	temperature
T'	temperature at end of time step
T_r	calculated value of temperature at node r
TC	thermocouple
t	time
x	distance
z	Newton-Raphson expression
Δt	computing time, interval
Δx	node thickness
δ	convergence tolerance
ε	emissivity
η	arbitrary value
ρ	density of material
σ	Stefan-Boltzmann constant

Subscripts:

i	location
j	thermocouple location
m	node identifier
s	surface

THEORETICAL FORMULATION

Background

The heat conduction within a one-dimensional thermal model is based on the standard heat conduction equation found in numerous heat-transfer textbooks.

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) \tag{1}$$

where ρ is the density of the material, C_p is the specific heat at constant pressure, k is the thermal conductivity, and T is the temperature of the material at location x in the material at time t.

The solution can readily be obtained if the boundary conditions and the initial temperature profile are known. If a heat rate $\dot{q}_{net}$ is imposed on the surface of a material of thickness L and if the backwall boundary is insulated, the boundary conditions are

$$k\frac{\partial T}{\partial x}\bigg|_{x=0} = -\dot{q}_{net} \tag{2a}$$

$$k\frac{\partial T}{\partial x}\bigg|_{x=L} = 0 \tag{2b}$$

with the initial condition given by

$$T(x,t)\bigg|_{t=0} = T_0(x) \tag{2c}$$

If the conducting medium is homogeneous, it can easily be shown that the solution obtained from equations (1) and (2) is unique. Under these conditions, the equations are linear and analytical techniques can be used to solve for the temperatures. When the thermophysical properties are temperature dependent, however, the equations become nonlinear and recourse is usually made to numerical methods. An implicit numerical solution has been used in this investigation.

It is assumed that a set of thermocouples TC_j has been placed at known locations x_j in the material. This assumption implies that a set of interior temperature-time histories $T_i(x_j,t)$ exists. If the temperature data are available at x_j, then by solving the boundary condition

$$k\frac{\partial T}{\partial x}\bigg|_{x=x_j} = -\dot{q} \tag{3}$$

for $\dot{q}$ together with the unknown temperatures, the heat rate at location x_i can be found directly (ref. 13).[1] If the temperature-time history data are available at the surface, then the convective heating rate can be found from

$$\dot{q}_{net} = \dot{q}_{conv} - \varepsilon\sigma T_s^{\,4} \tag{4}$$

where $\dot{q}_{net}$ is the total heat rate input, $\dot{q}_{conv}$ is the convective heating rate, ε is the total hemispherical emittance of the surface, σ is the Stefan-Boltzmann constant, and T_s is the temperature at the surface.

Because temperature data generally are not available at the surface, an alternate technique must be used to determine the unknown heat rate by means of thermocouples located within the material. A technique developed by Imber and Khan (ref. 12) was successfully used as a first approximation in the determination of a constant heating rate; however, the method was unsuccessful for a time-dependent heating rate.

The problem in solving for the heating rate at the surface when the temperature at the surface is unknown is that one of the required boundary conditions is unavailable. There is no difficulty in solving for temperatures at

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[1] If the thermocouple is located on the surface, the $\dot{q}_{net}$ value can be calculated directly from the tridiagonal matrix (appendix A).

any location between two thermocouples because both boundary conditions are known. The temperature at $x = x_1$ is given by

$$T(x_1, t) = T_1(t) \tag{5}$$

and equation (2b) can be used at $x = L$.

New Technique

A technique has been developed to solve for the heat rate at the surface at each time step rather than to solve for the entire history. The technique is iterative; initially, a Newton-Raphson technique is used, then a quadratic fit. To derive the technique, it is necessary first to examine the finite difference approximation at the surface.

$$\dot{q}_{net} - \frac{T'_m - T'_{m+1}}{\dfrac{\Delta x_\ell}{2\ {}_\ell k_m} + \dfrac{\Delta x_\ell}{2\ {}_\ell k_{m+1}}} = \frac{{}_\ell\rho_m\ {}_\ell C_{p_m}\ \Delta x_\ell}{2\ \Delta t}\left(T'_m - T_m\right) \tag{6}$$

where Δt is the interval of computing time, $'$ denotes the temperature value at time $t + \Delta t$, m is the node identifier, and Δx is the node thickness. This form of the equation is for a composite material where ℓ is the material designator.

This expression can be rearranged into the tridiagonal form

$$a_m T'_{m+1} + b_m T'_m + c_m T'_{m-1} + d_m = 0 \tag{7}$$

where the coefficients of the temperature matrix are

$$a_m = \frac{2}{\dfrac{\Delta x_\ell}{{}_\ell k_m} + \dfrac{\Delta x_\ell}{{}_\ell k_{m+1}}} \tag{8a}$$

$$b_m = \frac{-2}{\frac{\Delta x_\ell}{{}_\ell k_m} + \frac{\Delta x_\ell}{{}_\ell k_{m+1}}} - \frac{1}{2} \frac{{}_\ell\rho_m \; {}_\ell C_{p_m} \; \Delta x_\ell}{\Delta t} \tag{8b}$$

$$c_m = 0 \tag{8c}$$

$$d_m = \frac{1}{2} \frac{{}_\ell\rho_m \; {}_\ell C_{p_m} \; \Delta x_\ell \; T_m}{\Delta t} + \dot{q}_{net} \tag{8d}$$

The Newton-Raphson expression is

$$z_{i+1} = z_i - c \frac{f(z)}{f'(z)} \tag{9}$$

where the function

$$f(z) = a_m T'_{m+1} + b_m T'_m + c_m T'_{m-1} + d_m = 0 \tag{10}$$

and the constant c is limited to

$$0 < c \leq 1 \tag{11}$$

to prevent excessive corrections.

Assuming equation (4) is valid, then equation (10) may be rearranged as

$$f = \frac{a_m}{b_m} T'_{m+1} + T'_m + \frac{{}_\ell\rho_m \; {}_\ell C_{p_m} \; \Delta x_\ell \; T_m}{2b_m \; \Delta t} + \frac{1}{b_m}\left(\dot{q}_{conv} - \sigma\varepsilon T'^{\,4}_m\right) \tag{12}$$

Obtaining $f' = df/d\dot{q}$ yields

$$\frac{df}{d\dot{q}_{net}} = \frac{\partial f}{\partial T'_m} \frac{\partial T'_m}{\partial \dot{q}_{net}} \tag{13}$$

Then, from equation (12)

$$\frac{\partial f}{\partial T'_m} = 1 - \frac{1}{b_m}\left(4\sigma\varepsilon T'^{\,3}_m\right) \tag{14a}$$

and

$$\frac{\partial T'_m}{\partial \dot{q}_{net}} = \frac{1}{b_m} \tag{14b}$$

Equations (12), (13), and (14) are combined to yield

$$\left(\dot{q}_{net}\right)_{i+1} = \left(\dot{q}_{net}\right)_i - c\,\frac{f\left(\dot{q}_{net}\right)}{f'\left(\dot{q}_{net}\right)} \tag{15}$$

Because the thermocouples generally are located internally rather than at the surface, application of the Newton-Raphson technique will result in a monotonic approach to the desired solution, but convergence is very slow and may require an excessive number of iterations. Therefore, as a practical solution, it is necessary to switch to an alternate technique, such as a quadratic fit. Use of the Newton-Raphson technique will usually guarantee that the newly calculated values of $\dot{q}_{net}$ will differ from the previous iteration; however, if for some reason this difference does not occur, it can be forced by taking $\dot{q}_0 \pm \eta$ for the two additional values required, where η is some arbitrary value such as 0.1. The initial value of $\dot{q}_{net}$ is usually chosen to be the converged value for the previous time step. For the first time step, an arbitrary value such as 1 may be used. The iterative process is halted when

$$\left|T'_r - T^*_n(t)\right| \leq \delta\left|T^*_n(t)\right| \tag{16}$$

where T'_r is the calculated value of the temperature at node r corresponding to the thermocouple location, $T^*_n(t)$ is the temperature given by the thermocouple at that time, and δ is the relative convergence tolerance. It should be noted that both the net heating rate and the surface temperature have unique solutions, whereas the convective heating rate is dependent on the assumed value for emissivity.

Numerical difficulties arise in determining the surface heat rate or the surface temperature from data based on interior thermocouples. This difficulty is partly due to the timelag imposed on the system resulting from the finite distance between the surface and the thermocouple location. Another factor is the damping of surface changes at the thermocouple location. Other errors that may arise are due primarily to the method used for approximating the thermal model, the magnitude of $\dot{q}_{net}$, and the magnitude of the thermocouple temperatures. Of course, it is assumed that time steps compatible with the physical system would be used for the thermal model.

As previously discussed, it was necessary to switch from the Newton-Raphson method to a quadratic fit to insure a converged solution. This switch is accomplished by assuming three points: $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, and $P_3(x_3,y_3)$, where $x_i = [T'_r - T^*_n(t)]_i$ and $y_i = (\dot{q}_{net})_i$. A quadratic equation can be formed to include all three points. The equation is

$$y = Ax^2 + Bx + C \tag{17}$$

where A, B, and C are quadratic coefficients. By substituting each of the three points into equation (17), one has a system with three equations and three unknowns (the coefficients A, B, and C).

After the coefficients have been determined, a point on the quadratic curve may be found. The points have been formed such that solution is at $y = C$ or at the point $P_4(0,C)$.

If, in evaluating the original function with C to obtain a new x_4, the solution is not within the desired tolerance as required by equation (16) (i.e., $|x_4| < \varepsilon$), P_4 is substituted for one of the previous points (e.g., one with the largest $|x|$), and the coefficients are determined again. The process is repeated until the desired solution is obtained. Normally, this process requires four iterations since the Newton-Raphson technique was converging to the desired solution.

NUMERICAL EXPERIENCE

The numerical method discussed in the previous section has been evaluated for typical Space Shuttle Orbiter materials and environments. In general, the Space Shuttle Orbiter reusable TPS consists of reusable carbon-carbon (RCC) where surface temperature exceeds 1533 K and reusable surface insulation (RSI) for areas with maximum surface temperatures of less than 1533 K. The thermophysical properties used in the investigation are presented in appendix B. Since the objective was to develop a program to be used in both preflight and postflight analyses of Space Shuttle missions, cases typical of both ground and flight-test environments were considered.

The RSI Thermal Model

The RSI thermal model (fig. 1) consists of 5.08-centimeter-thick primary insulation with embedded thermocouples. The insulation used for this model is an all silica, rigid composite material (LI-900) having a density of 144.18 kg/m^3. The boundary conditions are assumed to be a heat rate on the surface and to be adiabatic on the backwall. An initial temperature of 294 K and an emissivity of 0.8 on the surface with radiation to space were assumed for this model.

Constant-heating rate.- Ground tests are generally characterized by both constant-heating and constant-pressure conditions. Therefore, this investigation was conducted to assess the effects of the single thermocouple location. Four different cases were used for comparison: (1) the lead thermocouple was on the surface, (2) the lead thermocouple was 0.254 centimeter from the surface, (3) the lead thermocouple was 0.508 centimeter from the surface, and (4) the lead thermocouple was 0.762 centimeter from the surface. In each case, the data were generated for the lead thermocouple by using a surface heat rate of 136 188 W/m^2.

Approximately five-place accuracy was maintained at each time step for the first three cases using convergence criteria of 10^{-5}, 10^{-5}, and 10^{-6}, respectively. For case 4, only four-place accuracy was maintained with a convergence criterion of 10^{-6}. This loss of accuracy with a reduction in δ clearly demonstrates the damping effect as the thermocouple is placed farther from the heated surface.

Variable-heating rate.- Flight conditions combine the effects of both a transient heat rate and a changing pressure environment. Therefore, these effects are discussed using the numerical method. The material used for this investigation was LI-900 with and without a surface coating. When the coating was used, it was assumed to have a uniform depth of 0.0381 centimeter. Three trajectories (fig. 2) were applied to the RSI: a short-range trajectory (designated as trajectory A), a medium-range trajectory (designated as trajectory B), and a long-range trajectory (designated as trajectory C).

The primary concern in this investigation was to determine the effects of thermocouple depth x_j and of the convergence criterion δ on the accuracy of surface heating rate calculation. The values used for δ were 10^{-4}, 10^{-5}, and 10^{-6}. Thermocouple depths of 0.254, 0.508, and 0.762 centimeter from the heated surface were used for uncoated materials; for the coated material, the thermocouple was placed 0.0381 centimeter (just behind the coating) and 0.254 centimeter from the heated surface.

The effects of the convergence criterion and thermocouple depth on the average error[2] in the surface heating rate for trajectory A can be seen in table I. The analysis was performed for a constant pressure of 101 325 N/m^2 (1 atmosphere). The effects of the average error for a constant δ are shown in figure 3. Basically, for each additional 0.254 centimeter in depth of the thermocouple, the convergence criterion must be decreased by a factor of 10 to maintain the same relative accuracy of approximately 90 W/m^2.

The heating rate provided by trajectory B was used to show the effects of pressure (used in conductivity calculation) on the accuracy of the surface heating rate calculation. The pressure curve used with trajectory B is shown in figure 4. The summary of the results comparing the accuracy of constant-pressure (101 325 N/m^2 (1 atmosphere)) conductivity with variable-pressure conductivity can be seen in table II. The accuracy for this trajectory was approximately the same as was obtained for trajectory A. The differences between pressure-dependent and constant-pressure results are considered small enough to have almost no effect. In comparing the average error figures, one must also conclude that the differences are of almost no consequence.

The effect shown by coating the surface of the material demonstrates the feasibility of the method for calculating surface heating conditions using actual Space Shuttle materials. The results also indicate the type of accuracy anticipated if the thermocouple is placed very near the surface.

The heating rate for trajectory C is representative of a Space Shuttle Orbiter location characterized by a transition to turbulent flow. No pressure curve was used for this trajectory since it was intended primarily to demonstrate the capability of the program to follow rapid changes in the heating rate associated with a transition from laminar to turbulent flow heating (fig. 2). The embedded thermocouple was located 0.254 centimeter from the heated surface. The average error of 1.3 W/m^2 is an order of magnitude smaller than the average error for trajectories A and B, but this reduction is due to

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[2]The average error equals $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\dot{q}_i - \dot{q}_i^*)^2}$, where $\dot{q}_i$ is the calculated convective heating rate, $\dot{q}_i^*$ is the actual convective heating rate, and n is the total number of individual measurements i taken.

the order of magnitude difference in heating rates. The data presented in table III reveal that the percent error is approximately the same as that achieved for the other trajectories.

The RCC Thermal Model

The RCC thermal model (fig. 5) consists of a 0.635-centimeter-thick layer of RCC, an airgap, and a 5.08-centimeter-thick layer of RSI insulation. Although this model does not properly account for the internal radiation of the actual Space Shuttle configuration, the thermal response is representative and provides for a good analytical experiment using a high-conductivity material. The trajectory (designated as trajectory D in fig. 6) is typical for the fuselage nose area. An emissivity of 0.85 was used for all surfaces for radiation transport properties.

Although the airgap behind the carbon was vented, no pressure curve was used for this trajectory because it was used primarily to demonstrate the capability of the program to follow rapid changes in the heating rate through a high-conductivity material. The thermocouple was installed on the backwall of the RCC (0.635 centimeter from the heated surface). The average error of 30 W/m^2 is of the same order of magnitude as that for trajectories A and B. The percent error is approximately the same as that achieved in the RSI trajectories (table IV).

For the RSI and RCC analytical verification studies already discussed, the same mathematical formulation controls the thermal model for both the generation of data and the prediction of the heat flux[3]. However, under actual test conditions, some of the assumptions used in formulating the mathematical model are violated. Examples of such variations are measurement errors associated with uncertainties in the location of thermocouples, void areas surrounding the thermocouples, and thermophysical property deviations. The effect of any of these errors on the predicted heat flux was not investigated because detailed error analyses of this type can be found in references 9 and 14.

CONCLUDING REMARKS

An inverse solution technique using a single embedded thermocouple has been developed for predicting the transient thermal environment to which the Space Shuttle Orbiter thermal protection system is exposed during entry. The accuracy of the numerical method has been demonstrated for both a high- and a low-conductivity material by comparison with analytically generated data. The

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[3]Comparisons with known values of heating rates indicated a maximum error of 7.8×10^{-2} percent, but most values were within 1×10^{-3} percent.

procedure developed is quite general and has been incorporated into a previously developed program used to compute thermal conductivity values from experimental data. Thus, a capability now exists for computing surface conditions (heat flux and/or temperature) and thermal conductivity values using the data from a single experiment.

In addition to being sensitive to the peak surface-temperature level, the reusable surface insulation material is sensitive to the rate of change of the surface temperature, which induces in-depth thermal gradients and stresses within the reusable surface insulation tiles. The inverse program can be used to define thermal shock test conditions by simply specifying a surface-temperature gradient and then solving for this required transient heat flux.

Lyndon B. Johnson Space Center
National Aeronautics and Space Administration
Houston, Texas, February 16, 1976
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TABLE I.- COMPARISON OF AVERAGE ERRORS IN DETERMINING

HEAT RATES FOR TRAJECTORY A

Material	Lead thermocouple depth, cm	Convergence criterion δ	Average error, W/m^2
Uncoated LI-900	0.254	10^{-4}	95.8
	.254	10^{-5}	15
	.508	10^{-5}	115
	.508	10^{-6}	12
	.762	10^{-5}	317
	.762	10^{-6}	47.7
Coated LI-900	.0381	10^{-5}	10.9
	.254	10^{-5}	13.2

TABLE II.- COMPARISON OF AVERAGE ERRORS IN DETERMINING

HEAT RATES FOR TRAJECTORY B

Material	Lead thermocouple depth, cm	Convergence criterion δ	Average error, W/m^2
Variable pressure			
Uncoated LI-900	0.254	10^{-5}	21.6
	.508	10^{-5}	150
	.762	10^{-6}	104
Coated LI-900	.0381	10^{-5}	20.4
	.254	10^{-5}	31.1
Constant pressure			
Uncoated LI-900	0.254	10^{-5}	17.6
	.508	10^{-5}	149
	.762	10^{-6}	55
Coated LI-900	.0381	10^{-5}	20.2
	.254	10^{-5}	28.9

TABLE III.- COMPARISON OF CALCULATED AND KNOWN

HEAT RATES FOR LI-900 FOR TRAJECTORY C[a]

Time, sec	Known heat rate, W/m^2	Calculated heat rate, W/m^2	Error, W/m^2	Error, percent
0	1 759	1 759.15	-0.15	-0.00306
100	4 857	4 857.4	-.40	-.00061
200	16 569	16 569.8	-.80	-.00170
300	50 843	50 842.9	.10	.00114
350	65 257	65 252	5.0	.00721
400	71 839	71 839.4	-.4	-.00255
480	74 790	74 789.8	.2	.00019
580	73 314	73 313.9	.1	.00075
670	65 711	65 709.7	1.3	.00150
740	53 908	53 906.7	1.3	.00201
830	40 516	40 515.4	.6	.00120
1020	26 557	26 555.9	1.1	.00268
1150	16 116	16 115.6	.4	.00248
1220	25 762	25 762.5	-.5	-.00120
1270	31 550	31 550.5	-.5	-.00158
1310	22 130	22 129.3	.7	.00316
1430	5 777	5 776.7	.3	.0052

[a]The lead thermocouple was located 0.254 centimeter from the surface, and the convergence criterion was 10^{-5}.

TABLE IV.- COMPARISON OF CALCULATED HEAT RATES FOR

CARBON-CARBON AND LI-900 FOR TRAJECTORY D[a]

Time, sec	Known heat rate, W/m^2	Calculated heat rate, W/m^2	Error, W/m^2	Error, percent
100	18 158.4	18 158.3149	0.08512	0.00047
200	38 586.6	38 586.6284	-.02837	-.00007
300	90 792.0	90 747.1136	44.89	.04944
400	226 980.0	226 980.7604	.7604	-.00034
500	347 279.4	347 280.0469	-.6469	-.00186
600	379 056.6	379 045.8638	10.74	.00283
700	374 517.0	374 501.3043	15.70	.00419
800	372 247.2	372 267.3445	-20.14	-.00541
900	374 517.0	374 558.3558	-41.36	-.01104
1000	379 056.6	379 068.6072	-12.01	-.00317
1080	381 326.4	381 331.8702	-5.470	-.00143
1100	376 786.8	376 787.2426	-.4426	-.00012
1200	283 725.0	283 726.7024	-1.702	-.00070
1300	192 933.0	192 933.2610	-.2610	-.00014
1400	124 839.0	124 936.5560	-.9756	-.07815

[a]The thermocouple was located on the backwall of the carbon, and the convergence criterion was 10^{-5}.

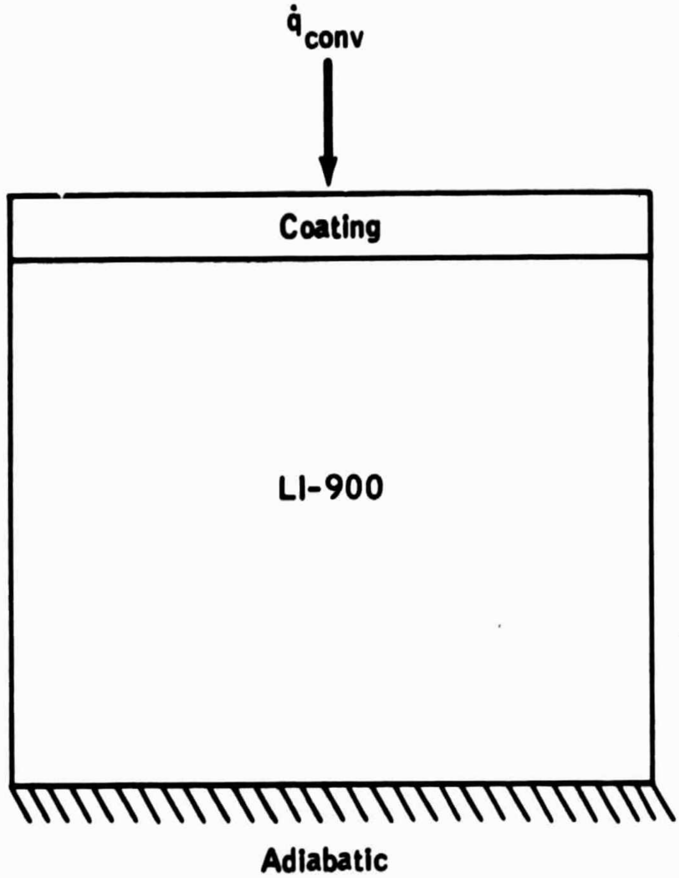

Figure 1.- The RSI thermal model.

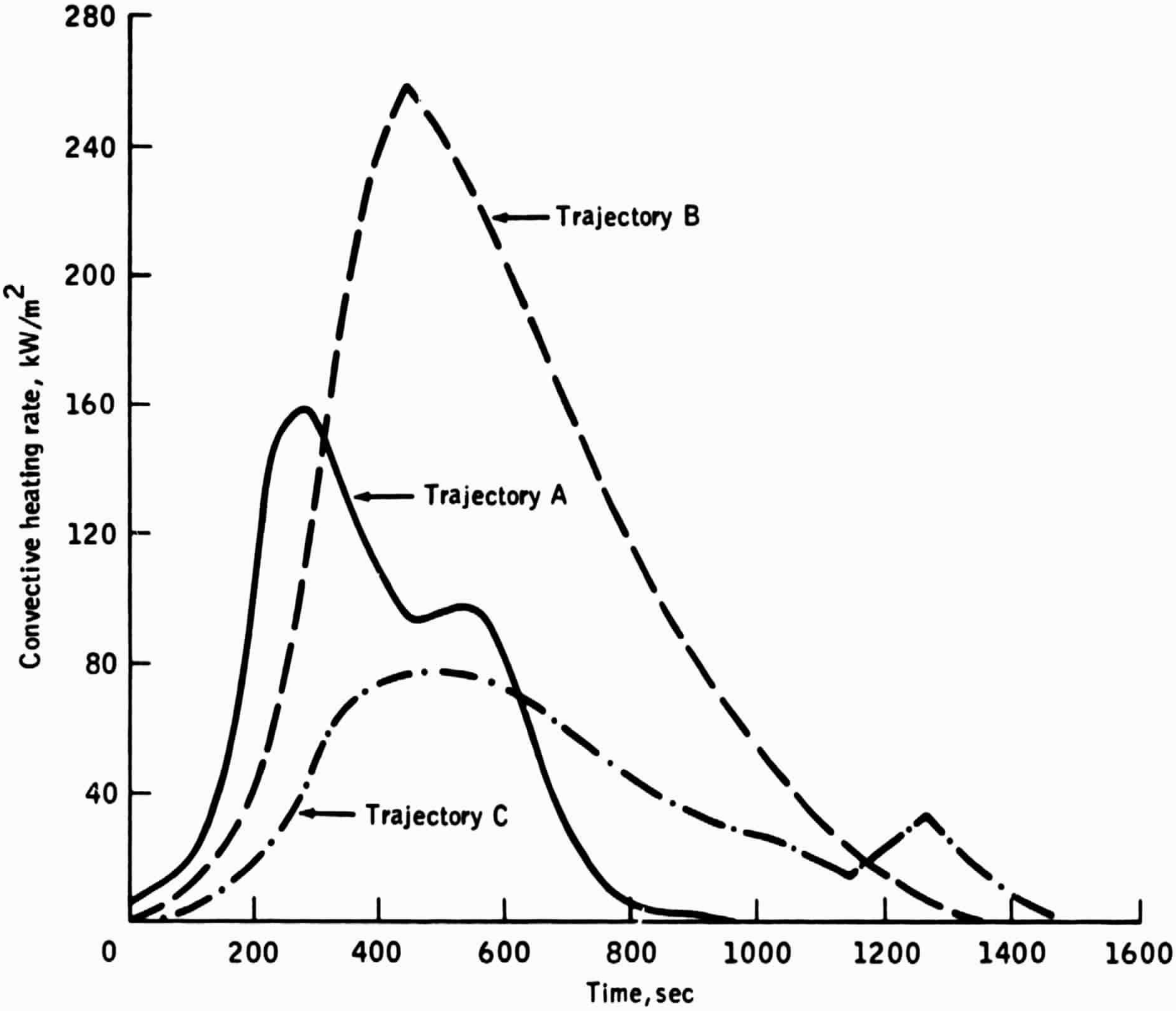

Figure 2.- Heating rat . as a function of time for trajectories A, B, and C.

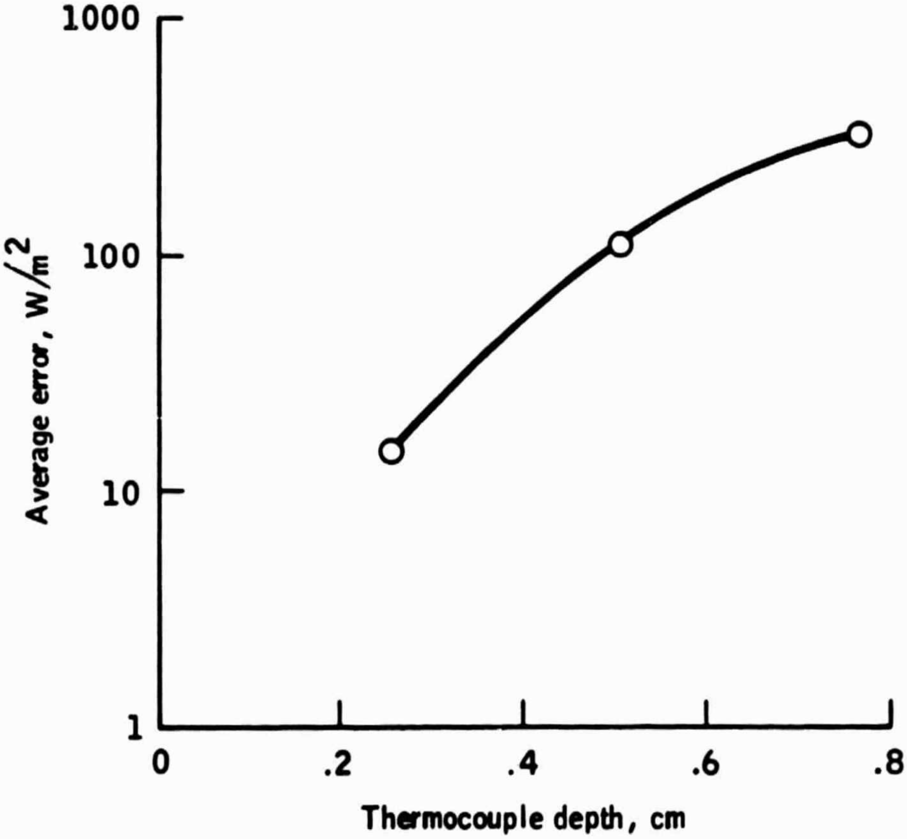

Figure 3.- Effect of average error due to thermocouple depth for a convergence criterion of 10^{-5}.

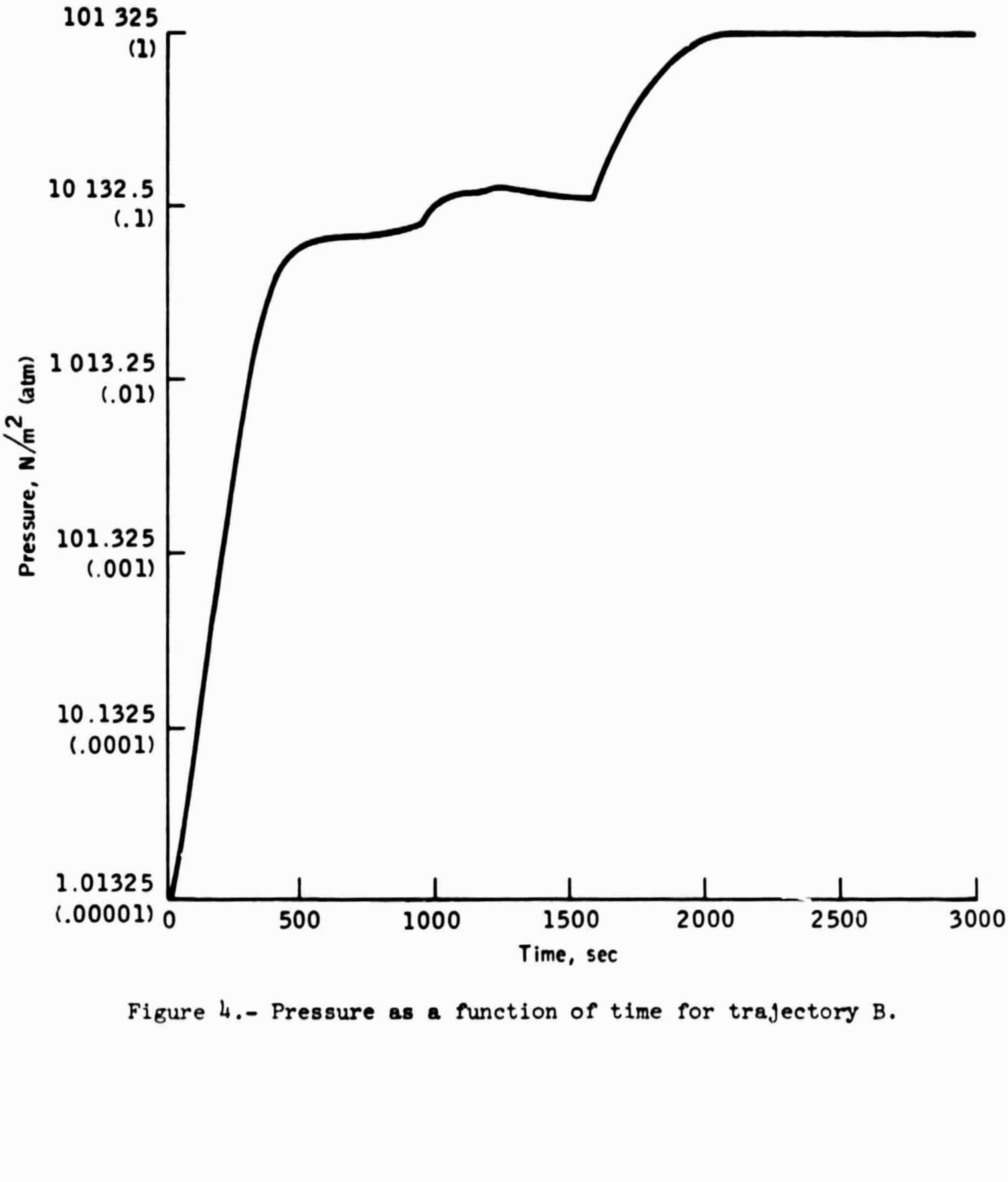

Figure 4.- Pressure as a function of time for trajectory B.

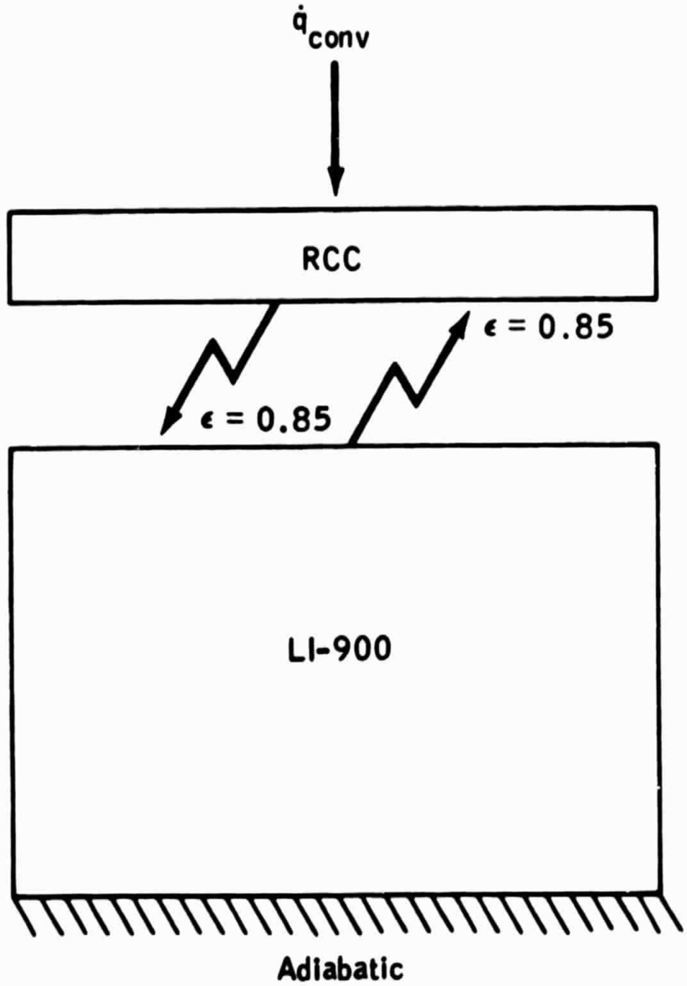

Figure 5.- The RCC thermal model.

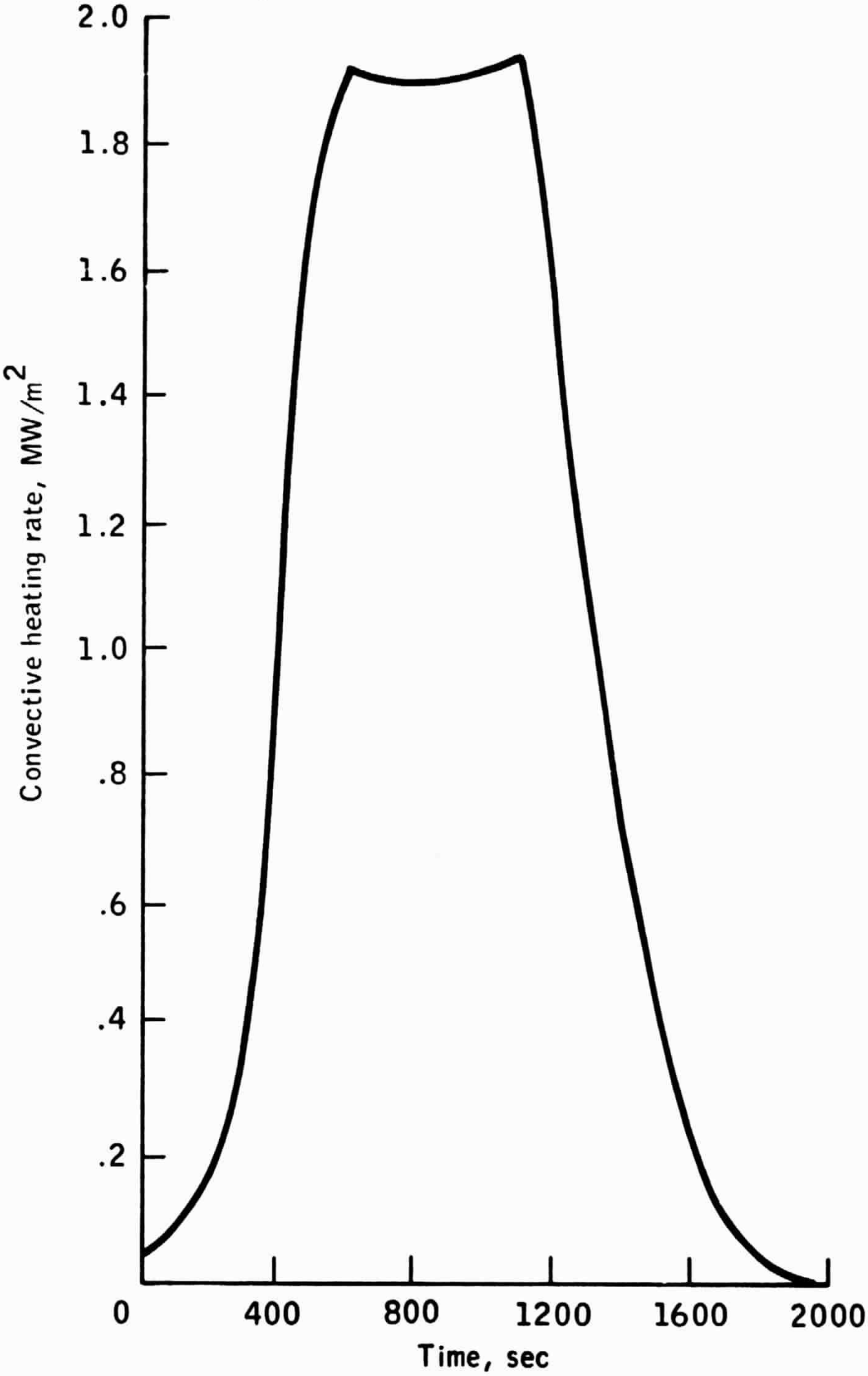

Figure 6.- Heating rate as a function of time for trajectory D.

APPENDIX A

CALCULATING SURFACE HEAT FLUX FROM KNOWN SURFACE TEMPERATURES

To predict the heating rate at a surface node, the boundary condition

$$k\frac{\partial T}{\partial x}\bigg|_{x=0} = -\dot{q}_{net} \tag{2a}$$

is used. Using an implicit formulation, the finite difference equations can be rearranged into a tridiagonal system of the form

$$a_m T'_{m+1} + b_m T'_m + c_m T'_{m-1} + d_m = 0 \tag{7}$$

To visualize the relationship of the coefficients used to solve for $\dot{q}_{net}$ together with the other unknown temperatures, the matrix formulation is examined.

$$\begin{bmatrix} b_1 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & a_2 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & c_3 & b_3 & a_3 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & c_4 & b_4 & a_4 & 0 & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & c_{n-1} & b_{n-1} & a_{n-1} \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & c_n & b_n \end{bmatrix} \begin{bmatrix} \dot{q}_{net} \\ T'_2 \\ T'_3 \\ T'_4 \\ \vdots \\ T'_{n-1} \\ T'_n \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ \vdots \\ d_{n-1} \\ d_n \end{bmatrix} = 0 \tag{A1}$$

The Gauss elimination method, discussed in reference 15, is applied to solve the system of equations. The solution of this matrix gives the temperature of each node in the system for the next future time step.

APPENDIX B

THERMOPHYSICAL PROPERTIES OF MATERIALS

This appendix contains supportive data (tables B-I to B-III) concerning the thermophysical properties of reusable surface insulation (RSI) and reusable carbon-carbon (RCC) materials used in this investigation.

TABLE B-I.- THERMOPHYSICAL PROPERTIES OF RSI

[Density 144 kg/m^3]

Temperature, K	Transverse thermal conductivity, W/(m·K), at a pressure, N/m^2, of -					Specific heat, J/(kg·K)
	10.13	101.3	1013	10 132	101 325	
117	0.009	0.013	0.026	0.038	0.040	293
172	.010	.014	.029	.040	.043	439
256	.013	.017	.032	.043	.048	628
394	.016	.022	.039	.055	.059	879
533	.022	.029	.048	.069	.075	1054
672	.030	.038	.056	.085	.092	1151
811	.040	.048	.068	.104	.114	1205
950	.053	.060	.085	.125	.135	1238
1089	.072	.079	.107	.151	.163	1256
1200	.092	.100	.127	.176	.189	1264
1228	.098	.105	.133	.183	.196	1268
1339	.121	.130	.156	.212	.226	1268
1367	.127	.135	.163	.219	.235	1268
1422	.140	.148	.174	.235	.252	1268
1450	.147	.156	.182	.244	.261	1268
1533	.167	.177	.200	.268	.288	1268

TABLE B-II.- THERMOPHYSICAL PROPERTIES OF RSI COATING

[Density 1666 kg/m^3]

Temperature, K	Thermal conductivity, W/(m·K)	Specific heat, J/(kg·K)
117	0.735	628
172	.788	711
256	.843	795
394	.951	900
533	1.045	1004
672	1.131	1088
811	1.218	1192
950	1.297	1355
1089	1.376	1318
1200	1.434	1360
1228	1.449	1381
1339	1.506	1423
1367	1.528	1444
1422	1.549	1464
1450	1.564	1477
1533	1.614	1506

TABLE B-III.- THERMOPHYSICAL PROPERTIES OF RCC

[Density 1361.7 kg/m^3]

Temperature, K	Thermal conductivity normal to ply, W/m^2	Specific heat, J/(kg·K)
117	2.106	--
144	--	439.60
256	4.327	--
367	--	837.34
439	5.770	--
533	6.347	1088.54
672	6.952	--
811	7.241	1348.12
1089	7.255	1549.08
1333	7.212	--
1367	--	1653.75
1644	--	1737.48
1922	6.433	1783.53
2144	6.433	--

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