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FACE-SEAL LUBRICATION

II - Theory of Response to Angular Misalinement

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16. Abstract

A theoretical analysis was made of a hypothetical seal operating mode. The hypothetical seal model provides for three degrees of primary-ring motion and includes the force and moments induced by primary-ring response to seat angular misalinement. This ring response causes a relative angular misalinement between the faces of the primary seal. Hydrodynamic pressure generation is produced by this misalinement. The analysis is based on the Reynolds equation in short-bearing form and on a balance of forces and moments that arise from hydrodynamic and secondary-seal friction effects. A closed-form solution was obtained that can be solved for film thickness and relative angular misalinement.

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FACE-SEAL LUBRICATION

II - THEORY OF RESPONSE TO ANGULAR MISALINEMENT

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SUMMARY

A theoretical analysis was made that did not impose conditions of constant centerline film thickness or fixed relative face misalinement. Rather, in the hypothetical seal model the primary ring is free to respond to seat-face misalinement, and as a result of secondary-seal friction the primary-seal surfaces take on relative angular misalinement. And this misalinement forms, in effect, a converging and diverging film in the circumferential direction. Positive hydrodynamic pressure is generated in the converging portion. An analysis of this seal model (which is only one of a number of possible seal operating models) resulted in a closed-form solution that yields values of film thickness when secondary-seal friction values are known or assumed. A design example of an actual seal was evaluated by using the derived mathematical expressions. The calculated film thicknesses show a strong dependence on the ratio of spring force to secondary-seal friction force.

INTRODUCTION

Liquid-lubricated, radial face seals are used successfully in many long-life applications. Common examples are water pump seals in automobiles, oil-lubricated aircraft transmission seals, and seals in a host of pumps for various liquids. The long operating life of many of these seals suggests that the primary-seal faces (which are in relative rotational motion) are separated by a lubricating film. Also, there is direct evidence from various experimenters that a lubricating film can exist between the seal faces. For example, reference 1 contains convincing evidence in the form of photographs of the lubricating film taken through a glass seal seat. The light interference patterns allow direct determination of the film thickness. Further evidence is provided by reference 2, which established that seal-gap pressures are higher than the sealed pressure. This is taken as evidence that a lubricating film and hydrodynamic lubrication must exist.

A companion paper (ref. 3) reviews the various hypotheses that have been put forward to explain the mechanisms responsible for the development of the lubricating film pressure that acts to separate the faces of the primary seal. These hypotheses include angular misalinement (ref. 4), surface waviness (ref. 5), surface asperities (ref. 6), fluid-film boiling (ref. 1), axial vibration (ref. 7), and local thermal deformation (ref. 8).

As pointed out in reference 3, the response of the primary ring to the coupling effect of seat angular misalinement, secondary-seal friction, and primary-ring inertia has not been adequately considered. In other words, it is hypothesized that the response of the primary ring is determined by secondary-seal friction and primary-ring inertia and that these induce a relative angular misalinement of the sealing faces. This relative misalinement is then a possible mechanism for liquid lubrication of the face seal.

The objective of this study is to develop a theory of face-seal lubrication based on the response of the primary ring to the combined effect of seat misalinement and secondary-seal friction. Only possible pressure generation mechanisms are considered (e.g., mechanisms regarding leakage, torque, etc., were not evaluated). Also, the transport mechanism by which liquid enters or leaves (leakage) the sealing interface is not covered in this work. Further, the theory proposed (based on dynamic response of the primary ring) is developed from a hydrodynamic viewpoint only; that is, hydrostatic effects due to sealed pressure and seal balance are not included in the study. Thus, the model is most directly applicable to a class of seals that operate with small or no sealed pressure, such as in helicopter and aircraft accessary gearbox transmissions. The hydrodynamic mechanism is, however, operable in high-pressure seals also, but here hydrostatic effects are probably also significant and require consideration for development of a viable model.

BACKGROUND

Primary-Seal Configurations

A companion paper (ref. 3) describes the various possible primary-seal geometries. In this study, only the face angular misalinement geometry (fig. 1) is of interest. The basic idea is that the relative angular misalinement between the faces of a primary ring gives rise to a wedge or inclined-slider-bearing geometry along a circumferential path. This inclined geometry is assumed to be responsible for the pressure generation in this converging portion. In the diverging portion, where fluid striation and cavitation occur, the pressure is assumed to be ambient.

Seal Operating Models

In reference 3, six basic seal operating models were formulated from the primary-seal configurations of waviness and angular misalinement. It is pointed out that the possible combinations proliferate when other operating conditions are introduced (parallel misalinement, sealed pressure, shaft whirl, both faces rotating, etc.). The work described in this report is based on the seal operating model shown in figure 1 (also in fig. 5(a-1) of ref. 3). In this operating model the primary ring rotates; the seat, which is nonrotating, has some angular misalinement with respect to the centerline of rotation. (Seat angular misalinement is very common.) Hydrodynamic pressure exists in the converging portion of the seal gap, but no inertia forces exist since the primary ring has no acceleration. The primary-ring motion consists of a pure rotation about a centerline inclined to the centerline of the rotating shaft.

The seal operating model of figure 1 is visualized to operate in the following manner: The nonrotating seat has inherent angular misalinement with respect to the centerline of the rotating shaft. Springs try to force the rotating primary ring, which is given axial and angular freedom, into alinement to the face. This alinement attempt introduces a sliding velocity component at the secondary-seal surface because the shaft and primary ring now have different centerlines of rotation. Secondary-seal friction prevents full alinement of the primary ring to the seat face, and a relative angular misalinement then exists between the faces of the primary seal. This relative misalinement acts as an inclined slider geometry and produces hydrodynamic pressure. The model does not take into account the effect of the magnitude of seat-face misalinement. This can be accounted for by making the secondary-seal friction a function of axial sliding velocity or displacement.

Although the analysis of this report applies directly to the model shown in figure 1, the basic procedure and modeling are applicable to other operating models (e.g., configurations B, C, D, E, and F of fig. 5 in ref. 3).

ANALYSIS

Seal-Ring Response Model

Rather than start with an assumption of constant average film thicknesses or constant load, the model for the primary seal is visualized to consist of two plane surfaces which, before seal rotation, are in contact and are parallel. After startup the primary-ring response to seat-face runout or angular misalinement causes a relative angular misalinement between the sealing surfaces. This relative angular misalinement is the source of the load support or hydrodynamic pressure generation.

The seal model is shown in figure 1. The seat is stationary and has an angular misalinement with respect to the centerline of the rotating shaft. The primary ring rotates with the shaft. (A rotating seat configuration introduces primary-ring inertia forces; this configuration is not treated in this report.) The primary ring has axial flexibility and also is free to aline itself with the seat face. These degrees of freedom are duplicated by allowing angular oscillation of the primary ring about the Y- and Z-axes (fig. 1) with their origin at the center of mass. In the third mode of freedom the seal is free to translate along the X-axis. Parallel eccentricity is not considered since it is assumed to have no significant effect on load support.

In order to explain the model (fig. 1), the coordinate system (X, Y, Z) is fixed in space and located at the center of mass of the primary ring. This center of mass is assumed to be on the centerline of the rotating shaft. For the small angular misalinements that occur in seals, this is probably very close to the actual situation. The centerline of the rotating shaft lies along the X-axis and the centerline of the primary ring is inclined at a small angle (β) to the X-axis. A separate cylindrical coordinate system with its origin on the centerline and with the r, θ-plane coincident with the nonrotating surface is used to calculate the hydrodynamic pressures.

Additional details of the mathematical model are shown in figure 2, in which the response of the primary ring to seat angular misalinement α is illustrated. (All symbols are defined in the appendix.) It is postulated that the primary ring has an angular misalinement position between the limiting positions indicated in figure 2. These limiting positions are (1) a primary-ring alinement perpendicular to the shaft centerline ($\beta = 0$) and (2) an alinement parallel to the seat face ($h = C$ and $\beta = \alpha$). In practice the primary-ring face will probably assume a position between the two extremes. (Note that β is equal to or less than α.)

Of interest in this model is the misalinement of the faces in a circumferential direction. The variation of seal-gap height h in the circumferential direction is, in effect, an inclined slider bearing having a converging gap over 180° of arc and a diverging gap in the remaining 180°. This circumferential incline is the source of load support. However, the effect of this angular misalinement on load support in the radial direction is small, and h can be assumed to be constant in the radial direction.

The forces considered in the model of seal operation are (1) the hydrodynamic and shear forces generated in the fluid film, (2) a distributed spring force, and (3) a distributed friction force from the secondary seal acting against the primary ring.

The force generated by the fluid film is determined by the Reynolds equation in the short-bearing form (ref. 3), with the restriction that pressure is positive in the converging portion and ambient in the diverging portion. The spring force may be assumed to be equally distributed with constant magnitude since the seal displacements of interest are only of the order of 0.1 millimeter (0.004 in.) or less. The frictional or damping force that arises because of secondary-seal external friction or internal hysteresis appears in the form of a moment acting about the Z-axis. In other words, the frictional (damping) force on the converging gap side at any one instant is directed in a positive X-direction and the frictional (damping) force on the diverging side is directed in a negative X-direction. Reference to figure 3 helps to clarify the direction in which the forces act. For the rotation direction indicated, any point on the primary ring that is on the converging side is moving in a negative X-direction, and the secondary-seal friction opposes this motion. The opposite is true for the diverging portion.

These frictional forces can be approximated by a distributed force acting along a mean diameter of the secondary ring. Figure 3 illustrates this frictional resistance. (Note that a change in sign occurs when the X, Z-plane passes through the maximum and minimum film thicknesses.) Since the seal radius is usually large compared to the radial dam width, some of the analysis involving the seal dam and the secondary seal can be based on the same characteristic (mean) seal radius R without introducing significant error. This simplifies the calculations.

Hydrodynamic Force and Moments

The hydrodynamic force generated in the converging portion of the seal face is found first. This is a distributed force that can be replaced by a single force (acting through the center of mass) and a force couple. From reference 3 the governing equation applicable to seal geometries is

$$\frac{\partial}{\partial r}\left(h^3 \frac{\partial p}{\partial r}\right) = 6\mu\left(2 \frac{\partial h}{\partial t} + \omega \frac{\partial h}{\partial \theta}\right) \tag{1}$$

This is the short-bearing form of the Reynolds equations. In reference 3 the development of the preceding equation was based on the unwrapped configuration of two nonparallel surfaces that conformed to the Cartesian coordinate system.

In addition to the short-bearing approximation the usual thin-film restrictions were applied:

(1) Incompressible fluid
(2) Constant viscosity
(3) Pressure constant through film thickness
(4) No body forces
(5) Inertia forces negligible compared to viscous forces
(6) Velocity gradients with respect to lateral directions negligible compared to the velocity gradient across the film thickness
(7) Newtonian fluid
(8) Laminar flow

The operating model in figure 1 is also based on the following assumptions (some of which are previously stated):

(1) The primary ring rotates in pure spin.

(2) The primary ring exactly tracks the seat faces; that is, the minimum film thickness occurs at the high point of face run-out (infinitely fast response).

(3) Gyroscopic moments of the rotating primary ring are neglected (speed being sufficiently low).

(4) The film thickness is constant in a radial direction.

(5) Hydrostatic pressure is neglected.

In regard to assumption 5, for aircraft transmission seal applications the sealed pressure is very small compared to the hydrodynamic pressure produced between the sealing faces. Therefore, for this class of seal, the sealed-pressure differential can be taken as zero. A near-zero pressure differential does not mean that there will be almost no leakage. In helicopter transmissions, leaky seals with near-zero pressure differential are a common problem (ref. 9).

Since the primary ring has pure spin (assumption 1), the film thickness is not time dependent and equation (1) reduces to

$$\frac{\partial}{\partial r}\left(h^3 \frac{\partial p}{\partial r}\right) = 6\mu\left(\omega \frac{\partial h}{\partial \theta}\right) \tag{2}$$

Since the film thickness was assumed to be constant in a radial direction, equation (2) becomes

$$\frac{d^2 p}{dr^2} = \frac{6\mu\omega}{h^3} \frac{dh}{d\theta} \tag{3}$$

With the zero-pressure-gradient assumption the hydrodynamic fluid pressure in a radial direction has a parabolic shape (fig. 4), and the pressure at any point can be expressed as a function of the pressure along a circumference of mean radius R as follows:

$$p = p_c\left[1 - \frac{4(R - r)^2}{(\Delta R)^2}\right] \tag{4}$$

where

p pressure at any point in converging portion of primary seal

p_c pressure along circumference of mean radius R

R mean radius

r radius

and

$$\Delta R = R_o - R_i$$

Differentiating twice with respect to r gives

$$\frac{d^2p}{dr^2} = \frac{-8p_c}{(\Delta R)^2}$$

and this can be substituted into the Reynolds equation to get

$$\frac{-8p_c}{(\Delta R)^2} = \frac{6\,\mu\omega}{h^3}\frac{dh}{d\theta}$$

$$p_c = -\frac{3}{4}\frac{\mu U(\Delta R)^2}{r}\frac{1}{h^3}\frac{dh}{d\theta} \tag{5}$$

where

$$\omega = \frac{U}{r}$$

The average pressure in terms of the pressure along the circumference of mean radius R is

$$p_a = \frac{2}{3}p_c \tag{6}$$

Thus, the expression for the pressure in the converging gap becomes

$$p_a = -\frac{1}{2}\frac{\mu U(\Delta R)^2}{r}\frac{1}{h^3}\frac{dh}{d\theta} \tag{7}$$

and equation (7) can be integrated over the converging position of the primary seal to get the total hydrodynamic force (fig. 5).

$$F_h = \int_0^{\pi} \int_{R_i}^{R_o} p_a \, r \, dr \, d\theta \tag{8a}$$

$$F_h = \int_0^{\pi} \int_{R_i}^{R_o} -\frac{1}{2} \mu U(\Delta R)^2 \frac{1}{h^3} \frac{dh}{d\theta} \, dr \, d\theta \tag{8b}$$

$$F_h = -\frac{1}{2} \mu U(\Delta R)^3 \int_0^{\pi} \frac{1}{h^3} \frac{dh}{d\theta} \, d\theta \tag{8c}$$

In reference 3 the expression for film thickness of the model in figure 1 is given as

$$h = h_0 + r\gamma \cos \theta \tag{9}$$

where γ is the relative angular misalinement between the faces of the seat and primary ring (fig. 1, $\gamma = \alpha - \beta$). Since the film thickness was taken as constant in a radial direction, equation (9) becomes

$$h = h_0 + R\gamma \cos \theta$$

where R is the mean seal radius. Letting $R\gamma = h_1$ yields an expression for h

$$h = h_0 + h_1 \cos \theta \tag{10}$$

and

$$\frac{dh}{d\theta} = -h_1 \sin \theta \tag{11}$$

Substituting these expressions for h and $dh/d\theta$ into equation (8c) gives

$$F_h = -\frac{1}{2}\mu U(\Delta R)^3 \int_0^{\pi} -\frac{h_1 \sin\theta\, d\theta}{(h_0 + h_1 \cos\theta)^3} \tag{12}$$

The equation (12) is in the form of $\int du/u^3$. Therefore, the integration can be performed directly to give

$$F_h = \frac{1}{4}\mu U(\Delta R)^3 \left[\frac{1}{(h_0 + h_1 \cos\theta)^2}\right]_0^{\pi}$$

$$= \frac{1}{4}\mu U(\Delta R)^3 \left[\frac{1}{(h_0 - h_1)^2} - \frac{1}{(h_0 + h_1)^2}\right]$$

$$F_h = \mu U(\Delta R)^3 \frac{h_0 h_1}{\left(h_0^2 - h_1^2\right)^2} \tag{13}$$

The moment of the hydrodynamic force about the Z-axis is (fig. 6)

$$M_h = \int_0^{\pi}\int_{R_i}^{R_o} r^2 \sin\theta\, p_a\, dr\, d\theta$$

$$= -\frac{1}{2}\mu U(\Delta R)^2 \int_0^{\pi}\int_{R_i}^{R_o} r \sin\theta \frac{1}{h^3}\frac{dh}{d\theta}\, dr\, d\theta \tag{14a}$$

$$M_h = -\frac{1}{4}\mu U(\Delta R)^2\left(R_o^2 - R_i^2\right)\int_0^{\pi} \frac{\sin\theta(-h_1 \sin\theta)}{(h_0 + h_1 \cos\theta)^3}\, d\theta \tag{14b}$$

This integral can be evaluated as follows: let

$$u = \sin\theta \qquad du = \cos\theta\, d\theta$$

$$v = -\frac{1}{2}\,\frac{1}{(h_0 + h_1\cos\theta)^2} \qquad dv = \frac{-h_1 \sin\theta\, d\theta}{(h_0 + h_1\cos\theta)^3}$$

then

$$\int u\, dv = uv - \int v\, du$$

$$= \left[\frac{-\sin\theta}{2(h_0 + h_1\cos\theta)^2}\right]_0^{\pi} + \int_0^{\pi} \frac{\cos\theta\, d\theta}{2(h_0 + h_1\cos\theta)^2}$$

$$= \int_0^{\pi} \frac{\cos\theta\, d\theta}{2(h_0 + h_1\cos\theta)^2}$$

The value of this integral is given in reference 10 as follows:

$$\int_0^{\pi} \frac{\cos\theta\, d\theta}{2(h_0 + h_1\cos\theta)^2} = \frac{1}{2\left(h_0^2 - h_1^2\right)}\left[\left(\frac{h_0 \sin\theta}{h_0 + h_1\cos\theta}\right)_0^{\pi} + \int_0^{\pi} \frac{-h_1\, d\theta}{h_0 + h_1\cos\theta}\right]$$

$$= \frac{-h_1}{2\left(h_0^2 - h_1^2\right)}\left[\frac{2}{\left(h_0^2 - h_1^2\right)^{1/2}} \tan^{-1} \frac{\sqrt{h_0^2 - h_1^2}\tan\frac{\theta}{2}}{h_0 + h_1}\right]_0^{\pi}$$

$$= \frac{-h_1}{\left(h_0^2 - h_1^2\right)^{3/2}}\left[\tan^{-1}(\pm\infty) - \tan^{-1}(0)\right]$$

$$= \frac{-h_1}{\left(h_0^2 - h_1^2\right)^{3/2}}\left(\frac{\pi}{2}\right)$$

$$= \frac{-h_1\pi}{2\left(h_0^2 - h_1^2\right)^{3/2}}$$

Finally, the expression for the moment of the hydrodynamic face about its Z-axis is

$$M_h = \frac{\pi}{8} \mu U(\Delta R)^2 \left(R_o^2 - R_i^2\right) \frac{h_1}{\left(h_0^2 - h_1^2\right)^{3/2}} \tag{14c}$$

Balance of Forces

The spring force can be considered to be an evenly distributed (circumferential) force acting in a positive X-direction. This distributed spring force can be replaced by a single force, the spring force F_s, acting at the origin of the coordinate system (fig. 3).

As previously stated the hydrodynamic force generated in the converging portion is replaced by a displaced force acting along the centerline in the X-direction and a force couple. This displaced hydrodynamic force F_h is balanced by the spring force F_s:

$$F_s = F_h \tag{15a}$$

$$F_s = \mu U(\Delta R)^3 \frac{h_0 h_1}{\left(h_0^2 - h_1^2\right)^2} \qquad \text{(force balance equation)} \tag{15b}$$

Balance of Moments

The hydrodynamic force couple is assumed to be balanced by the frictional moment exerted by the secondary seal. The friction force acting on one half of the secondary ring is known (or assumed) input data and is expressed as a force per unit length of secondary-seal outer diameter

$$F' = \frac{F_d}{\pi R} \tag{16}$$

where

F' friction force per unit length of secondary-seal outer diameter

F_d total friction force acting on one half of the secondary-seal outer diameter

R   characteristic seal radius (≅ mean radius)

The moment (about the Z-axis) of the distributed friction force is

$$M_d = -2 \int_0^{\pi} R \sin(\theta) F' \, dl \tag{17a}$$

where $R \sin \theta$ is the moment arm and dl is the incremental arc over which the frictional force acts

$$M_d = -2 \int_0^{\pi} R \sin(\theta) F' R \, d\theta$$

$$= -2R^2 F' \int_0^{\pi} \sin \theta \, d\theta$$

$$= -2R^2 F' \left(\cos \theta\right)_0^{\pi} = -4R^2 F'$$

$$= -\frac{4RF_d}{\pi} \tag{17b}$$

This moment has a vector sense in a positive Z-direction and, as stated previously, is in equilibrium with the moment produced by the hydrodynamic force

$$M_d = M_h \tag{18a}$$

$$\frac{4RF_d}{\pi} = \frac{\pi}{8} \mu U(\Delta R)^3 (R_o + R_i) \frac{h_1}{\left(h_0^2 - h_1^2\right)^{3/2}} \tag{18b}$$

Since $(R_o + R_i)/2 = R$,

$$\frac{h_1}{\left(h_0^2 - h_1^2\right)^{3/2}} = \frac{16F_d}{\pi^2} \frac{1}{\mu U(\Delta R)^3} \quad \text{(moment balance equation)} \tag{18c}$$

The viscous drag of the liquid between the primary-seal faces would exert a twisting moment on the rotating primary ring if fluid is assumed to exist only in the converging gap and to be nonexistent in the diverging gap because of fluid striation. This twisting moment can be decomposed into components about the X-, Y-, and Z-axes. The X-axis component is resisted by the mechanical antirotational devices of the seal. Its effect on seal dynamics, which is not considered in this model, comes from the introduction of frictional forces at the antirotational devices. In this respect, its influence on seal dynamics is similar to that of the secondary seal.

The vertical component of fluid shear exerts a twisting moment about the y-axis with a lever arm distance defined by the distance x_e between the seal face and the center of mass (fig. 7). For the rotational direction indicated, the moment vector is directed in a positive y-direction and is resisted by the component of the moment of the hydrodynamic force about the y-axis. (Recall that the z-axis component of the hydrodynamic moment is balanced by the secondary-seal friction.)

The film thickness and relative angular misalinement must adjust themselves to satisfy both moment balance equations, but the unknown conditions regarding fluid shear in the diverging portion of the gap make the analysis uncertain. However, an approximate solution for film thickness can be obtained by considering only the z-axis moment balance equation and the force balance equation.

Solution for Film Thickness

Equations (15b) and (18c) can be solved simultaneously to get expression for h_0 and h_1. From equation (15b) (force balance equation),

$$h_1 = \frac{F_s}{\mu U(\Delta R)^3} \frac{\left(h_0^2 - h_1^2\right)^2}{h_0} \tag{15b}$$

and from equation (18c) (moment balance equation),

$$h_1 = -\frac{16 F_d}{\pi^2} \frac{\left(h_0^2 - h_1^2\right)^{3/2}}{\mu U(\Delta R)^3} \tag{18c}$$

Equating these expressions for h_1

$$h_0 = -\frac{\pi^2}{16}\frac{F_s}{F_d}\left(h_0^2 - h_1^2\right)^{1/2} \tag{19}$$

Both sides are then squared to get

$$h_0^2 = \frac{\pi^4}{256}\left(\frac{F_s}{F_d}\right)^2 \left(h_0^2 - h_1^2\right) \tag{20a}$$

$$h_1 = h_0\left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]^{1/2} \tag{20b}$$

where

$$\frac{F_s}{F_d} \geqq 1.62 \qquad \text{for} \quad h_0 \geqq h_1$$

Inspection of this expression reveals that h_1 is always less than h_0 (as it should be). The preceding expression for h_1 is substituted into the force balance equation (15b) to get the expression for h_0:

$$h_0\left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]^{1/2} = \frac{F_s}{\mu U(\Delta R)^3} \frac{\left\{h_0^2 - h_0^2\left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]\right\}^2}{h_0} \tag{21a}$$

$$\left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]^{1/4} = \left[\frac{F_s}{\mu U(\Delta R)^3}\right]^{1/2} h_0\left[\frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]$$

$$h_0 = \left[\frac{\mu U(\Delta R)^3}{F_s}\right]^{1/2} \frac{\pi^4}{256}\left(\frac{F_s}{F_d}\right)^2 \left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]^{1/4} \tag{21b}$$

By rearrangement, equation (21) can be expressed in terms of dimensionless parameters as follows:

$$h_0^2\left[\frac{F_s}{\mu U(\Delta R)^3}\right] = \left(\frac{\pi^4}{256}\right)^2\left(\frac{F_s}{F_d}\right)^4\left[1 - \frac{256}{\pi^4}\left(\frac{F_d}{F_s}\right)^2\right]^{1/2} \tag{22}$$

where

$$h_0^2\left[\frac{F_s}{\mu U(\Delta R)^3}\right]$$

can be defined as a sealing number, and it is a function of only the ratio of spring force to secondary-seal friction force. (This dimensionless number is similar to the bearing number for fluid-film bearings.) The dependence of this sealing number on the ratio F_s/F_d is shown in figure 8, which was obtained by assuming various values of F_s/F_d. Experiments (unreported) indicate that practical values of F_s/F_d range upward to 20. As F_s/F_d increases, the film thickness tends to increase (according to the relation in eq. (22)). On the other hand, as F_s/F_d decreases the seal may stick open (due to the high secondary-seal friction). Thus, high leakage can result from the secondary-seal friction being too high or too low.

DISCUSSION

Inspection of the expressions for h_0 and h_1 reveals that both increase as viscosity, sliding speed, and face width increase. As expected, the spring force causes a decrease in h_0 and h_1. In equations (20b) and (21b) the dominant term in the expression for h_0 and h_1 is the ratio F_s/F_d. Thus, seal performance is a strong function of this ratio.

The seal mathematical model (fig. 1) shows that seat-face misalinement is a critical parameter that controls the development of hydrodynamic forces between the primary-seal faces. A condition of zero face misalinement would be undesirable since no hydrodynamic force would be produced. On the other hand, large face runouts would lead to large film thicknesses and attendant high leakage rates. Relatively large face misalinements can probably be tolerated at slower sliding speeds because the primary ring has more time to respond. As mentioned previously this time response is considered (in the model) to be infinitely fast; that is, there is no phase shift or lag between the minimum film thickness and the high point of seat misalinement.

The analysis points up the importance of secondary-seal friction. For elastomeric seals such as "O"-rings, this friction is highly dependent on the cross-sectional squeeze, which can change drastically with just simple tolerance variations. Also the size and physical properties of the O-ring can be drastically changed with exposure to fluid and temperature. These changes in secondary-seal friction with time may account for some seal failures (leakage) that occur after many hours of successful operation.

The seal mathematical model in this report does not cover the case in which the seat rotates and the primary ring is nonrotating. This configuration introduces inertia forces that act in a manner similar to the secondary-seal friction forces and induce an angular misalinement between the primary-seal faces. From the viewpoint of the laboratory observer the film thickness change is time dependent.

The application of the model analyzed in this report can be generalized to cover the case of moderate pressures if the spring force is taken as the total closing force caused by the spring effects plus the hydrostatic effects.

DESIGN EXAMPLE

For the purpose of illustrating the use of equations (21b) and (22) for analysis of seal performance, the following transmission seal design was chosen:

Spring force, F_s, N (lbf) 44.6 (10)
Outer radius, R_o, cm (in.) 7.47 (2.94)
Inner radius, R_i, cm (in.) 7.37 (2.90)
Radial width of primary seal, $\Delta R = R_o - R_i$, cm (in.) 0.102 (0.04)
Mean radius, R, cm (in.) 7.42 (2.92)
Tangential velocity at R, U, cm/sec (in./sec) 932 (367)
Fluid viscosity, μ, N-sec/m^2 (lbf-sec/in.^{2}) . . . . 1.72×10^{-3} (0.25×10^{-6})

A range of operating film thicknesses can be obtained by inserting the appropriate values in equation (22) and letting the F_s/F_d ratio take on values of 2, 5, 10, and 15. The results are

Ratio of spring force to secondary-seal friction force, F_s/F_d	Reference center-line film thickness, h_0		Minimum local film thickness, h_{min}	
	cm	in.	cm	in.
2	0.000071	0.000028	0.000030	0.000012
5	.000569	.000224	↓	↓
10	.002327	.000916		
15	.005253	.002068		

SUMMARY OF RESULTS

The seal operating model analyzed consisted of a rotating primary ring and a nonrotating seat. A theoretical model was formulated in which it was hypothesized that relative angular misalinement occurs between the primary seal faces because of the balance of forces and moments. These forces and moments arise because of seat angular misalinement. The net result is a relative angular misalinement between the primary-seal faces. This relative misalinement produces the hydrodynamic pressure that acts to keep the faces separated. A mathematical analysis of the seal model was made, with the following results:

1. A closed-form solution was obtained that yields values of film thickness and relative angular misalinement of the primary-seal faces when secondary-seal friction values are known or assumed.

2. A dimensionless parameter that characterizes seal operation was found. It was termed the "sealing number" and is a function only of the ratio of spring force F_s and secondary-seal friction force F_d. This sealing number is

$$h_0^2\left[\frac{F_s}{\mu U(\Delta R)^3}\right]$$

where h_0 is the reference local film thickness, μ is the viscosity, U is the sliding velocity in the X-direction, and ΔR is the radial width of the primary seal.

3. The lubricating film thickness was found to be a strong function of a dimensionless parameter, the ratio of spring force to the frictional resistance of the secondary seal.

4. In a numerical example of an oil-lubricated seal for gear transmissions, the calculated values of film thickness were within the expected range.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 17, 1975,
505-04.

APPENDIX - SYMBOLS

C	constant
F	force
h	local film thickness; seal-gap height
l	length
M	moment
p	pressure
R	characteristic (mean) seal radius
ΔR	radial width of primary seal
r	radius; or cylindrical coordinate
t	time
U	sliding velocity
X, Y, Z	coordinates
x, y, z	coordinates
x_e	distance between seal face and center of mass
α	angular misalinement of seat to centerline of rotating shaft
β	angular misalinement of primary ring to centerline of rotating shaft
γ	relative angular misalinement between primary-seal faces
θ	cylindrical coordinate
μ	viscosity
ω	angular velocity

Subscripts:

a	average
c	mean or centerline
d	secondary-seal friction
h	hydrodynamic
i	inner radius of primary seal
max	maximum
min	minimum
o	outer radius of primary seal

s spring

0 reference condition

1 component of angular misalinement

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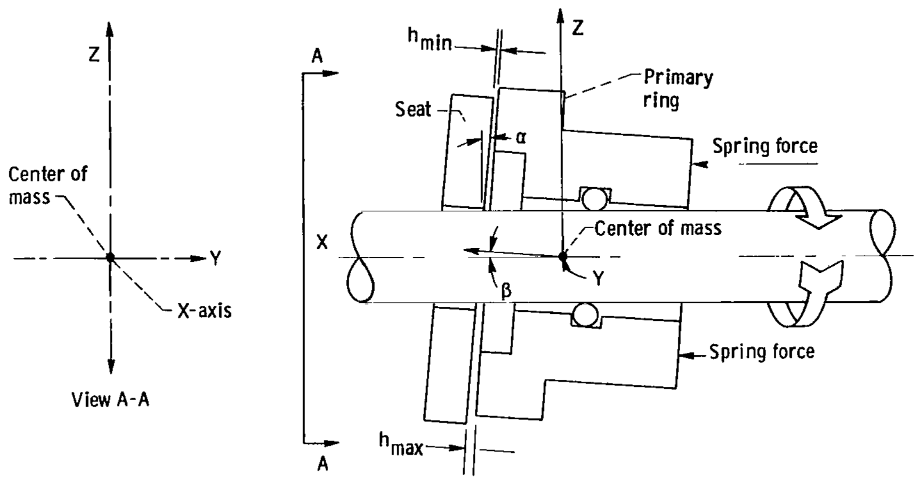

Figure 1. - Mathematical model of radial face seal with rotating primary ring.

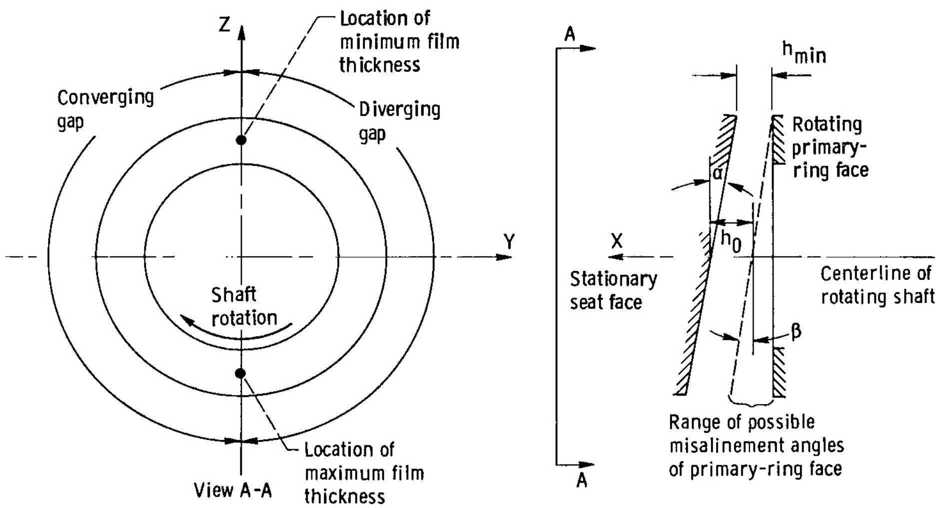

Figure 2. - Converging and diverging gaps formed because of relative angular misalinement of primary-seal faces.

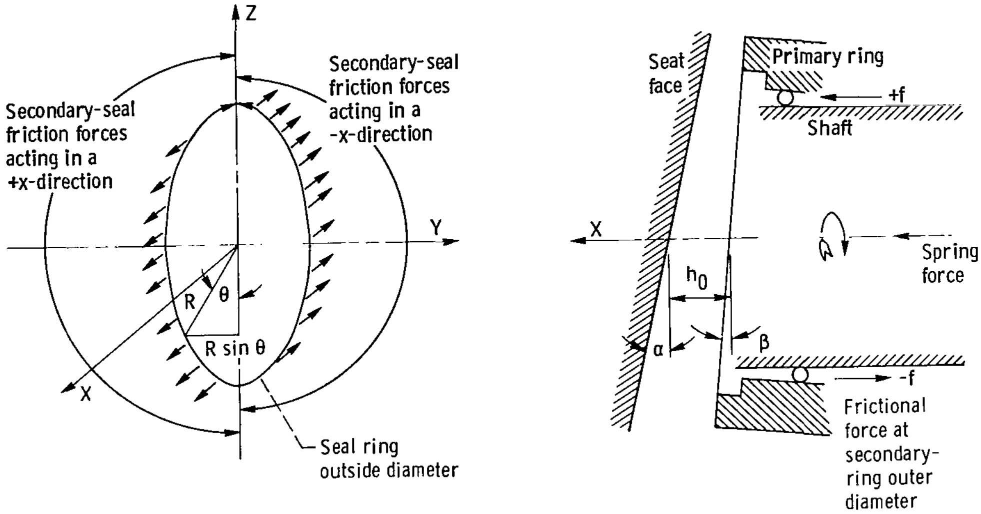

Figure 3. - Friction forces acting on secondary-seal ring.

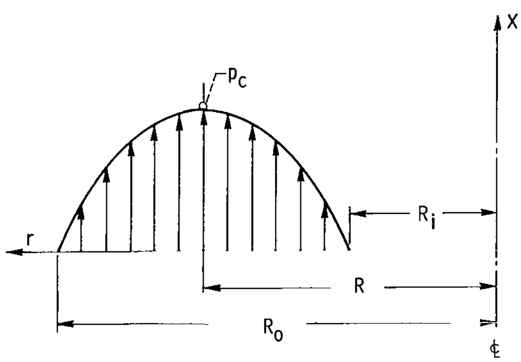

Figure 4. - Hydrodynamic pressure gradient in converging portion of sealing dam.

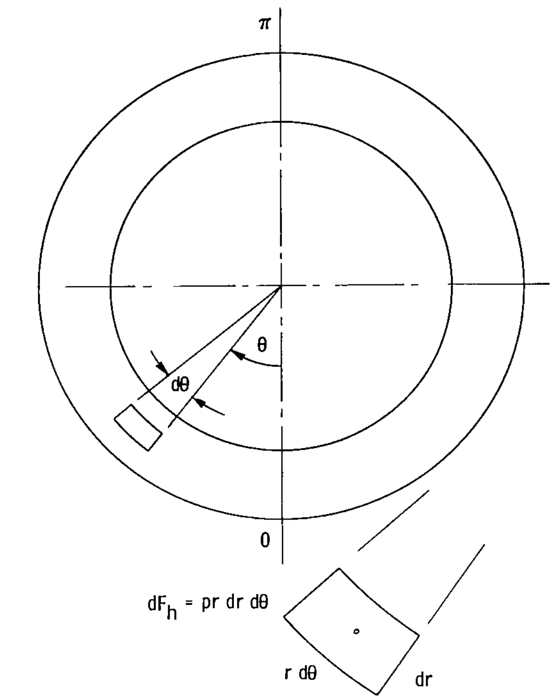

Figure 5. - Model for calculating total hydrodynamic force in converging portion of primary seal.

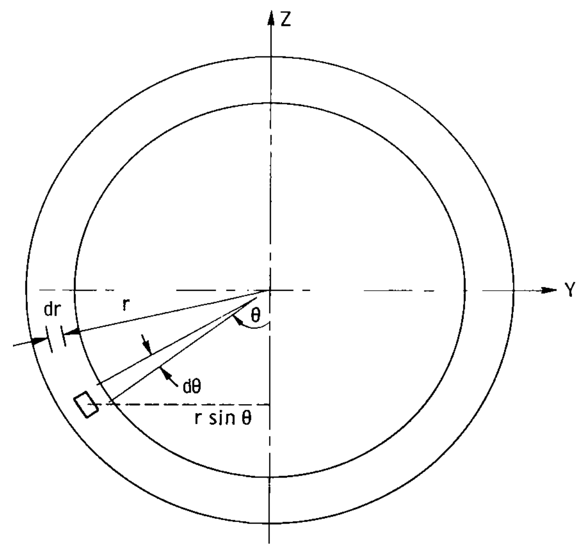

Figure 6. - Model for calculating hydrodynamic force and its moment about Z-axis.

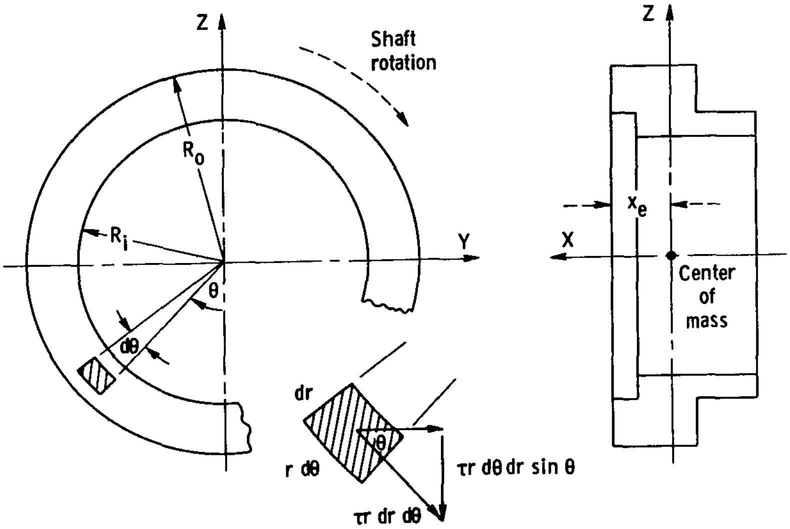

Figure 7. - Fluid shear forces acting on face of rotating primary ring.

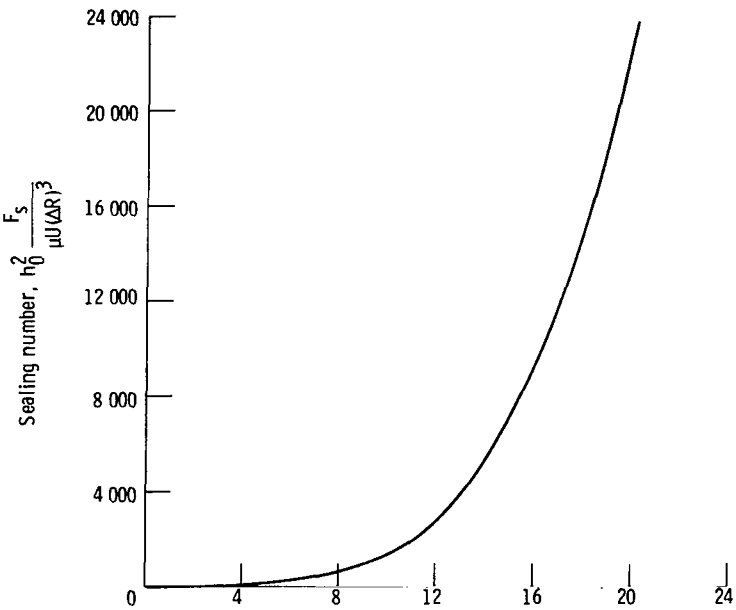

Figure 8. - Theoretical variation of sealing number with ratio of spring force to secondary-seal friction force.

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