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APPLICATIONS OF SATELLITE TECHNOLOGY TO GRAVITY FIELD DETERMINATION

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ABSTRACT

Various techniques for using satellite technology to determine the earth's gravity field are analyzed and compared. A high-low configuration satellite to satellite tracking mission is recommended for the determination of the long wavelength portion of the gravity field. Satellite altimetry and satellite gradiometry experiments are recommended for determination of the short wavelength portion of the gravity field. The recently developed least-squares collocation method for estimating the gravity field from satellite derived data is analyzed and its equivalence to conventional methods is demonstrated in the Appendix.

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INTRODUCTION

The obtaining of an accurate and detailed global gravity field is a major object of NASA's Earth and Ocean Dynamics Applications Program (1). Investigators have proposed many procedures for applying satellite technology to accomplish this object. These procedures fall into the following categories:

1. The classical approach utilized since the late fifties which relies on satellite perturbation data obtained from ground based tracking stations.

2. Satellite to satellite tracking of a high inclination, low altitude satellite using a high altitude relay satellite.

3. Satellite to satellite tracking between two satellites in identical high inclination, low altitude orbits.

4. Spacecraft borne gravity gradiometry.

5. Spacecraft borne radar altimetry.

In this report the authors analyze the advantages and disadvantages of each of these techniques. In doing so we rely on several numerical studies which we and others have performed. We have attempted to interpret and compare the results of these studies and to provide recommendations for future spacecraft missions.

The difficulties encountered in mapping the long wavelength features of the gravity field (say greater than 1,000 km) are different in character from those encountered in determining shorter wavelength features. Hence in this report the recovery of long wavelength and short wavelength features are treated as separate estimation problems in separate sections.

Recently much attention has been devoted to the mathematical procedure of "least squares collocation" as an alternative to conventional least squares techniques for estimating gravity fields from satellite derived data. In the body of the report we have attempted to clarify the nature of this procedure and its relationship to conventional techniques. In the Appendix we examine the application of least squares collocation to the problem of estimating gravity anomalies from altimeter data and we demonstrate that its implementation is equivalent to the implementation of conventional techniques.

This paper was intended for the non-specialist. Consequently, except for the Appendix, the use of mathematical symbols has been kept to a minimum. The reader who prefers more mathematical details can examine the numerous references.

LONG WAVELENGTH GRAVITY FIELD ESTIMATION

A prominent feature of NASA's Earth and Ocean Dynamics Applications Program (EODAP) is the use of satellites as platforms from which highly accurate instruments globally monitor natural phenomena. The accuracy of these instruments has led to demands for commensurate orbit determination accuracy (1). As an example, the altimeter on board the GEOS-3 spacecraft has an altitude resolution of one to two meters. Commensurate orbit altitude determination will be difficult to obtain (2), (3). Another example is the effort to monitor tectonic plate motions by LASER tracking of satellites. Again the major difficulty is the lack of adequate orbit determination accuracy (4). It should be mentioned that other missions not directly related to the EODAP have similar problems. An example is the Earth Observation Satellite (EOS) whose sophisticated imaging equipment cannot be fully exploited without a very accurate orbit determination (5). The major impediment to achieving high orbit determination accuracies is the uncertainty in our estimate of the long wavelength portion of the gravity field. A significant improvement of this estimate is necessary if the EODAP is to achieve its goals.

Another and at least equally powerful argument for the pursuit of this improvement is that with a much more accurate estimate of the gravity field every satellite mission could be performed less expensively since for a given orbit determination accuracy less tracking data acquisition and processing would be required.

Present estimates of the long wavelength portion of the gravity field are based primarily on satellite perturbation data obtained from ground based tracking stations. The usual procedure for obtaining a gravity field from the data is to parameterize the field by means of low degree and order spherical harmonic coefficients and to adjust the coefficients according to a least squares method. It is doubtful if this mathematical procedure can be improved and we agree with Kaula (6) who states, "Because of the characteristics of close satellite orbit dynamics and orbit determination from ground tracking, spherical harmonics will continue to be the most suitable representation of the main part of the gravitational field indefinitely."

It is surprising how much uncertainty remains in satellite derived estimates of the long wavelength gravity field. In (7) the Goddard Earth Model 5, a standard spherical harmonic expansion of the gravity field was calibrated against actual observations of 15° by 15° mean gravity anomalies and nominal standard deviation values were scaled to be consistent with the residuals. The resultant standard deviations are displayed as percentages of Kaula's "rule of thumb" ($10^{-5}/L^2$ where L is the degree of the normalized spherical harmonic coefficient. This is an empirical formula used to approximate the power spectral density function of the gravity field.) in Figure 1. The coefficients to degree 12 are seen to be uncertain to within 5% to 60% of their nominal values. In (4) it is estimated that gravity field estimates must be improved by a factor of 7 for effective satellite monitoring of tectonic plate motions. An analysis of the results in (2) suggests that a gravity field estimate must be improved by a factor between 7 and 8 if altitude resolution of applications satellites commensurate with altimeter accuracy is to be achieved. In (8) it is asserted that a factor of 5 improvement in present gravity models is necessary to obtain the orbit determination accuracy required by the Earth Observation Satellite. We take as a reasonable goal for a satellite mission designed for long wavelength gravity field estimation, a factor of 10 improvement in present gravity field models.

To see what is required for the design of such a mission it is necessary to understand why in spite of the large amount of available satellite perturbation data, gravity field models still exhibit the large errors suggested by Figure 1. In

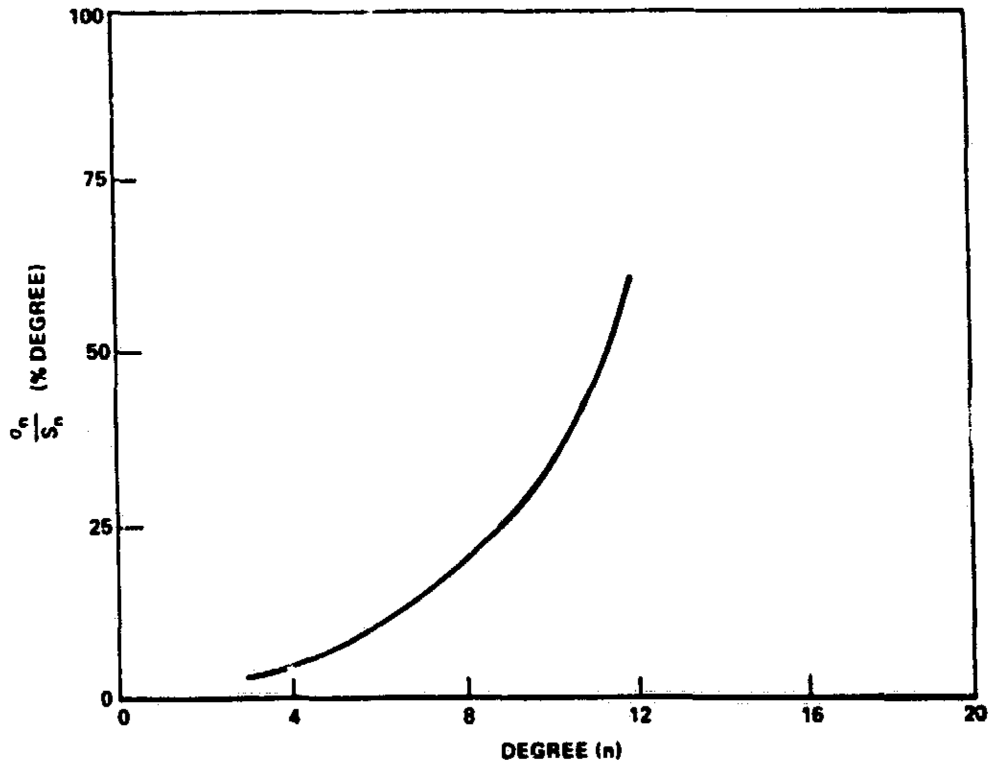


Figure 1. Present Uncertainty of Low Order Geopotential Coefficients

theory the geopotential field is represented by an infinite series of spherical harmonic coefficients. Numerical procedures for estimating the gravity field from satellite perturbation data involves the recovery of coefficients below a certain degree and order. Higher degree and order coefficients are generally set at zero. Since these coefficients are not zero, the assumption introduces an "aliasing" of the resultant estimated gravity field. This effect can be demonstrated in terms of a simple numerical example taken from (9). A natural phenomenon is modeled correctly by the quadratic $Y = x^2 + x + 1$. An investigator, however, assumes a linear model, $Y = ax + b$. This assumed model neglects the second degree term of the correct model, in effect equating it to zero. Next he performs a standard least squares fit to the three data points $Y(0) = 1$, $Y(5) = 31$, and $Y(10) = 111$ using the linear model. The least squares procedure yields as a solution $Y = 11x - 7.33$. The estimated coefficients are $a = 11$ and $b = -7.33$ whereas the correct values are $a = 1$, $b = 1$. Thus neglecting the second degree term in the correct model has seriously degraded the quality of the parameter estimates. The degradation is a simple example of the aliasing phenomenon. In a similar fashion, the neglect of uncertainties in higher degree and order geopotential coefficients aliases the estimates of lower degree and order coefficients. It can be shown that a necessary and sufficient condition for the elimination of this aliasing effect is the possession of a dense and globally distributed data set. Since locations of tracking stations are limited by geographical and political considerations, it is not possible to obtain a global distribution of low altitude satellite perturbation data by conventional methods. Hence estimates of long wavelength gravity fields continue to be plagued by severe aliasing. This is essentially an observability problem and no amount of additional data collected from the same well-covered areas will substantially improve the situation.

A logical solution to the problem is suggested by the possibility of tracking a low altitude high inclination satellite by means of a high relay satellite. The ATS-6 and Nimbus 6 spacecraft System was designed to be the first experiment to evaluate the usefulness of satellite-to-satellite tracking for geodetic studies (10). Numerical studies indicate that with a low satellite at a 300 km altitude there is sufficient sensitivity in the satellite to satellite tracking data to improve present estimates of geopotential coefficients to degree and order 22. We have computed the perturbations of satellite to satellite range rate sum data between a geosynchronous satellite and a satellite in a polar, circular, 300 km orbit caused by geopotential coefficient perturbations. The 276 cosine terms of the spherical harmonic expansion of the geopotential field to degree and order 22 were perturbed by current estimates of term uncertainties as obtained from Figure 1. These individual perturbations were propagated into variations of the range rate sum data over a 24 hour period. Figure 2 is a histogram of the mean absolute values of the range rate sum variations over the data arc. The graph shows that for over 96% of the geopotential coefficients, the data perturbations caused by the

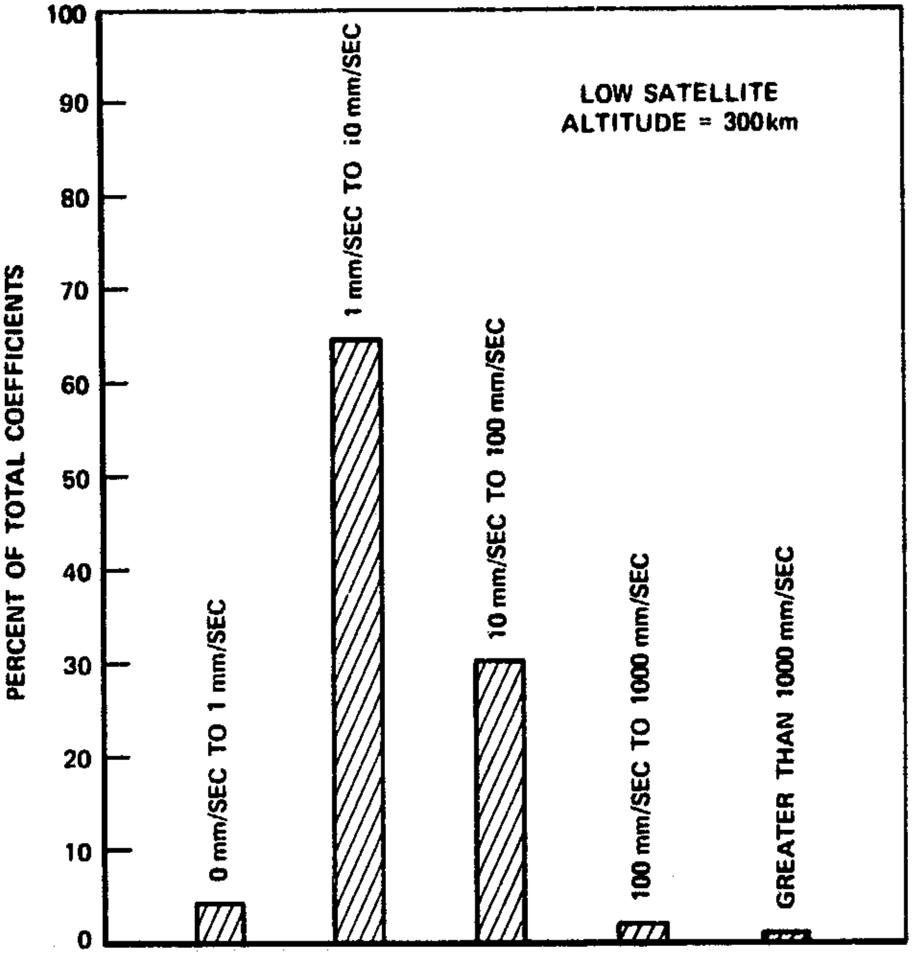


Figure 2. Signal Histogram for Geopotential Terms to Degree and Order 22 in Satellite to Satellite Tracking

difference between nominal and actual values have an average amplitude greater than the present estimate of satellite to satellite tracking accuracy of 1mm/sec.

Figure 2 demonstrates that errors in geopotential coefficients can be sensed in the data. But this does not imply that the coefficients can be decoupled and independently estimated from information supplied by the range rate sum data. To determine the recoverability of the coefficients in this sense generally requires a covariance analysis in which the individual standard deviations and the statistical correlations of the estimates are computed.

In (11) the possibilities of estimating geopotential coefficients from satellite to satellite tracking of GEOS-3 with ATS-6 as a relay satellite are studied. The authors assume that GEOS-3 is continuously tracked for five days by ATS-6 from geosynchronous position 94°W and that at another five day period GEOS-3 is continuously tracked by ATS-6 from geosynchronous position 34°E. The range rate sum data accuracy was assumed to be 0.3 mm/sec for a one minute integration time. The results of the covariance analysis show that if GEOS-3 state and ATS-6 state are simultaneously estimated with coefficients of a geopotential field to degree and order 8, the resultant geopotential coefficient estimates are improved by one to two orders of magnitude over present estimates. The authors do not account for the aliasing effect due to uncertainties in higher degree and order coefficients. Thus the results are no doubt optimistic. Also the correlation coefficients between estimates of GEOS-3 state and ATS-6 state were quite high. This suggests that there may be difficulties with such experiments in obtaining a convergence of the least squares iteration procedure. Preliminary evaluation of ATS-6/GEOS-3 range rate sum data implies that this is the case. An ideal solution to this problem could be provided if an accurate a priori fix on the relay satellite epoch state were extracted from a combination of ranging and trilateration data. This would permit an independent estimate of GEOS-3 state and geopotential coefficients. An accurate estimate of the state of a geosynchronous satellite (say to the 15 or 20 meter level) is difficult to obtain since there is very little motion between the satellite and ground based tracking stations. But the trilateration data type has yielded promising results (12) and research is continuing on this subject.

An alternative configuration for a satellite to satellite tracking experiment, first suggested by Siry (13), is provided by a dual GRAVSAT/GEOPAUSE mission. The GRAVSAT and GEOPAUSE satellites are to be coplaner in orbits perpendicular to both the earth's equator and the ecliptic plane. The high or GEOPAUSE satellite is placed in a circular orbit at about 3.6 earth radii above the earth's surface. The low or GRAVSAT satellite is in a circular orbit about 300 km above the earth's surface. The GRAVSAT is assumed to be equipped with a surface force compensation system. Tracking between the GRAVSAT and GEOPAUSE is relayed from the GEOPAUSE to ground based tracking stations. Six properly chosen tracking stations, three in the Northern Hemisphere and three in the Southern Hemisphere, are adequate to maintain constant ground communication with the GEOPAUSE satellite. It should be mentioned that the GEOPAUSE has other desirable features not related to its functioning as a relay satellite for satellite to satellite tracking data. Siry (14) has shown that the satellite's particular configuration makes it very useful for polar motion and tectonic plate motion monitoring.

In (15) covariance analysis procedures are employed to study the potential of the GRAVSAT/GEOPAUSE mission for determining gravity field coefficients to degree and order 8. Ten days of range rate sum data is assumed. Data accuracy is 0.2 mm/sec for a one minute integration time. Epoch states of the GEOPAUSE and GRAVSAT satellites along with geopotential coefficients to degree and order 8 are assumed to be simultaneously estimated from the data. The results show that the data of the experiment can yield a two orders of magnitude improvement in estimates of geopotential coefficients. The estimates were relatively independent with most of the thirty two hundred correlation coefficients between geopotential estimates of absolute value less than 0.01. Also, there should be little difficulty in obtaining a good a priori fix on the GEOPAUSE epoch state. Hence it should be possible to obtain convergence of the least squares iteration procedure when both satellites are estimated from the data. As is the case with (11), the results of (15) do not account for the aliasing effect due to uncertainties in higher degree and order coefficients.

The first quantitative study of the aliasing effect in geopotential coefficient determination has been provided in (9). The author assumes a GRAVSAT/GEOPAUSE satellite configuration and he uses the techniques of covariance analysis to obtain quantative measures of the contributions to the uncertainties of geopotential coefficient estimates due to uncertainties of higher degree and order unadjusted coefficients. The results show that even with the excellent data distribution provided by the GRAVSAT/GEOPAUSE combination, aliasing is still a difficult problem. In fact, uncertainties in unadjusted coefficients of degree 12 significantly alias adjusted coefficients of degree as low as 8. This suggests that for a good determination of the field to degree and order 8, a field of degree and order 12 should be estimated from the data and estimates of terms of degree 9 through 12 discarded due to aliasing.

In summary, a global data distribution is necessary for a significant improvement in present estimates of the long wavelength gravity field. The only feasible way to achieve such a distribution is by the satellite to satellite tracking of a low altitude, high inclination satellite using a high relay satellite. Two such configurations have been investigated: the use of a geosynchronous relay satellite; and the use of a high altitude polar satellite (GEOPAUSE) as a relay satellite. Studies suggest that both configurations are capable of providing a data set from which an order of magnitude improvement in estimates of geopotential coefficients to degree and order 8 can be obtained. An analysis of correlations implies that the use of a geosynchronous relay satellite may lead to numerical difficulties in reducing the resultant satellite to satellite tracking data. The use of the GEOPAUSE as a relay satellite avoids such difficulties. Thus, on strictly scientific grounds, the GRAVSAT/GEOPAUSE configuration is preferable.

SHORT WAVELENGTH GRAVITY FIELD RECOVERY

Global knowledge of gravity field fine structure is fundamental to the understanding of solid earth and ocean dynamics (1). A major goal of NASA's applications program is a global gravity field mapping sufficiently detailed to show features as small as three degrees. This is equivalent to estimating spherical harmonic coefficients of the gravity field to degree and order 60.

The ATS-6-Nimbus-6 system was proposed by Von Bun (10) as a test of satellite to satellite tracking for detection of short wavelength gravity anomalies. He found that a mascon of 5×10^{-8} em could produce a change of radial velocity expected in an SST configuration to be up to .2 cm/s. This accuracy is well within the range of tracking systems accuracies for SST data.

The essential difficulty in employing standard parameter estimation techniques to globally determine short wavelength components of the gravity field is that a large number of parameters must be estimated. For instance, the spherical harmonic coefficients of the gravity field to degree and order 60 number over 3,700. It is not possible to simultaneously estimate such large parameter sets. In practice, it is necessary to adjust small subsets of parameters at one time while constraining the rest to a priori values. But unless the data set and the gravity field parameterization bear a certain mathematical relationship to each other, the net effect is that the uncertainties of the unadjusted terms will badly corrupt the estimates of the adjusted terms. This is the aliasing effect discussed at length in the previous section. This so-called orthogonality property is rigorously defined in (16), but in essence it is a relationship between a data set and a parameterization which permits a decomposition of the large dimensional estimation problem into estimation problems of much smaller dimensionality and without serious aliasing. Because of these data reduction considerations, any satellite mission designed to provide a global mapping of gravity field fine structure must generate a data set which has an orthogonality relation with a parameterization of the gravity field.

Several satellite mission configurations and data types have been suggested for accomplishing this end. We divide these proposed missions into four types: satellite altimetry missions; satellite gradiometry missions; satellite to satellite tracking missions involving a high altitude relay satellite and a low altitude satellite; and satellite to satellite tracking missions utilizing two low altitude satellites in identical orbits. Each of these mission types is discussed in a separate section.

Satellite Altimetry

If no dynamic effects such as tides intervened, the mean sea level would cohere to an equipotential surface known as the ocean geoid. After suitable corrections (17), the output of a satellite borne altimeter over an ocean area may be viewed as a direct observation of the height of the ocean geoid at the subsatellite point. With regard to using altimeter data to estimate gravity fields, two limitations are apparent. First, the altimeter output has significance for the gravity field only over the ocean. But, since most of the earth is covered by oceans, this is not a fatal limitation. The second limitation is that errors in the altitude estimate of the satellite project directly onto errors in the altimeter data. Satellite borne altimeters are assumed to be accurate to within one meter. Commensurate altitude resolution of the satellite is difficult to obtain. For instance, in (2) nine well-distributed laser stations were assumed to track the GEOS-3 satellite and the reported altitude resolution of the satellite was between six and eight meters. It may be possible to exploit the spectral properties of altitude errors to remove their effects on the altimeter data. This possibility has not been thoroughly investigated.

At least two different mathematical procedures have been suggested for estimating a gravity field from altimeter data. The first relies on standard least squares estimation procedures. It uses Stokes' formula to parameterize the ocean geoid in terms of mean gravity anomalies. Altimeter data are treated as direct observations of geoid heights and are processed by a least squares filter to yield estimates of mean gravity anomalies. These gravity anomalies in turn define the anomalous gravity field. The second approach uses a model for the second order statistics of the anomalous potential field. This model is translated into the joint covariance matrix of gravity anomalies and altimeter data. The usual regression equation which provides the conditional mean of a random vector (gravity anomalies) given a realization of a correlated random vector (altimeter data) is used to estimate the gravity anomalies from the altimeter measurements. This procedure has been called "least squares collocation." The relative merits of the two approaches are discussed below.

Let N be the anomalous geoid height at a given point. Then the discrete form of Stokes' equation can be written

$$N = \frac{R}{4\pi G} \sum_i \delta g_i \; S(\psi_i) \; d\sigma_i \tag{1}$$

where ψ_i is the spherical distance between the center of the block on which the mean gravity anomaly δg_i is defined, and the observation point $d\sigma_i$ is the area of the i th block and "S" is Stokes' function (18), and R and G are mean values for the radius and gravity of the earth. Equation 1 can be used as an observational equation and a least squares filter can be employed to extract from geoid information represented by the altimeter data an estimate of the gravity anomalies. If all other available information were used to provide an a priori estimate for the filter, the resultant estimate of the gravity anomalies would be optimal in a minimum variance sense. But the summation represented in Equation 1 must extend over the entire earth for the equation to be exactly correct. Formally this implies that a global set of gravity anomalies must be simultaneously estimated in the least squares procedure. But fortunately Stokes' function rapidly attenuates with increasing spherical distance (18). Hence if the blocks on which two mean gravity anomalies are defined are sufficiently separated, the perturbation patterns of the anomalies in altimeter data will be non-overlapping. This is sufficient to insure that the two anomalies are orthogonal in altimeter data and that they can be independently rather than simultaneously estimated and without serious aliasing. Conversely if the block were in close proximity, it would be necessary to simultaneously estimate the gravity anomalies from altimeter data.

It should be clear then, that if a square block of altimeter data is used to estimate a square block of gravity anomalies, the outer layers of the block will contain gravity anomalies whose estimates will be badly aliased by the adjacent unadjusted anomalies. It will be necessary to discard these estimates. But the gravity anomalies in a sufficiently small inner core of the block may be adequately separated from the unadjusted anomalies as to be effectively orthogonal with respect to them. The estimates of these terms presumably will be of sufficient accuracy that they can be accepted. In effect, for every block of gravity anomalies that we intend to estimate it will be necessary to construct a "buffer zone" several layers deep of gravity anomalies which surround the block. The new and larger block of gravity anomalies must be simultaneously estimated and then the estimates of gravity anomalies in the buffer zone must be rejected due to aliasing. With such a procedure local blocks of altimeter data can be reduced to estimate local blocks of gravity anomalies and the data reduction problem implicit in any attempt to obtain a global and detailed gravity field mapping can be reduced to manageable proportions.

Gopalapillai (19) uses numerical simulations to investigate the feasibility of recovering mean gravity anomalies from altimeter data on a non-global basis. He focuses attention on the estimation of gravity anomalies in an area ten degrees on a side. The buffer zone of gravity anomalies which are estimated and then discarded is ten degrees deep. Thus the block of anomalies to be estimated is thirty degrees on a side. The block of altimeter data is also thirty degrees on a side and one altimeter observation per one degree by one degree square is simulated. The effects of unadjusted anomalies outside the block of estimated anomalies are included in the data. Although the simulated data is not corrupted by white noise, the author chooses a weight for the data according to the assumption that a random component of standard deviation one meter is imposed on the data. Perfect a priori estimates are provided but they are weighted as if they were uncertain to within twenty-five mgal. The results of the simulations are that two degree anomalies in the inner core of the block of estimated anomalies are recovered with a root mean square error of one mgal and that one degree anomalies are recovered with a root mean square error of five mgal. Care must be taken in interpreting these results. Since the data was simulated without a white noise component, the actual errors in the recovery of the gravity anomalies reflect only the aliasing effect of unadjusted anomalies, not the effect of data error. Conversely, the a posteriori nominal standard deviations of the recoveries which were nine mgal for two degree anomalies and nineteen mgal for one degree anomalies reflect the assumption that the altimeter data is corrupted by a one meter random error but not the aliasing effect of unadjusted anomalies. Combining the measures of the two effects in a root sum square sense may be the most plausible thing to do. This leads to the conclusion that Gopalapilli's strategy for estimating local blocks of gravity anomalies from local blocks of altimeter data yields a recovery of two degree gravity anomalies accurate to within ten mgals and a recovery of one degree anomalies accurate to within nineteen mgals.

In (16) the techniques of covariance analysis are used to determine the best strategy for using Stokes' formula to estimate three degree and five degree gravity anomalies. A data density of three observations per each one degree by one degree block is assumed. The model also assumes that the data is corrupted by white noise with a one meter standard deviation. This study also concludes that the optimal buffer zone separating unadjusted anomalies from anomalies whose estimates are accepted is approximately ten degrees. The optimal data block size is slightly smaller than the block of estimated parameters. The resultant accuracies which reflect both the effect of data noise and aliasing from unadjusted anomalies are one mgal for five degree anomalies and five mgal for three degree anomalies. The authors propagated these uncertainties into uncertainties in the ocean geoid and conclude that the reduction of altimeter data yields an estimate of the ocean geoid detailed enough to show five degree features with a resolution of forty centimeters and three degree features with a resolution of 1.2 meters.

The results of (19) and (16) appear to be quite compatible. Both studies conclude that the use of Stokes' formula permits the estimation of local blocks of gravity anomalies in local blocks of altimeter data provided validly estimated gravity anomalies are separated from unadjusted anomalies by at least ten degrees. Gravity anomalies as small as two degrees can be recovered by this procedure with reasonable accuracy and five degree anomalies can be recovered with an accuracy of one mgal. These results are predicated on the assumption that various biasing effects, including those due to orbit determination error, can be removed from the data.

A different procedure for estimating gravity anomalies from altimeter data which relies on different assumptions has been called "least squares collocation" (20). The procedure can be outlined in the following fashion. One assumes a model for the second order statistics of the anomalous potential field. Since mean gravity anomalies and geoid heights are determined by the anomalous potential field, this model can be propagated into a joint covariance matrix for geoid heights at the altimeter measurement points and mean gravity anomalies to be estimated. Computational algorithms for this propagation are provided in (21). Using a linearity assumption one can invoke the regression equation for the conditional mean of a random vector (gravity anomalies) given a realization of a correlated random vector (altimeter measurements) (22). The result is the least squares collocation algorithm for estimating gravity anomalies from altimeter data. An important numerical feature of this procedure is that its implementation involves the inversion of a matrix whose dimension is the size of the data set. This is an undesirable feature since it limits the size of the data set one can use for estimation.

In (23) collocation procedures are used to study the feasibility of estimating mean gravity anomalies from altimeter data. The study assumes one altimeter observation for each one degree block. The significant conclusions are that if altimeter observations are accurate to one meter, 2.5° mean anomalies can be recovered with an accuracy of 5 mgal, and 5° mean anomalies can be recovered with an accuracy of 4 mgal. In (24) similar collocation techniques are used and equivalent results are obtained.

We have found some misconceptions concerning the nature of the least squares collocation technique and its relationship to conventional least squares reduction procedures. In (25) Chovitz states, "The increasing diversity and complexity of data sources and solution parameters have brought to the fore the method of collocation which generalizes conventional least squares adjustment and prediction by considering covariance functions for systematic as well as random error sources."

In no sense is the least squares collocation technique a generalization of conventional estimation procedures. Given the same physical model and the same information, a conventional least squares adjustment procedure will yield the same estimate as will least squares collocation. The equivalence of the two procedures when applied to the problem of estimating gravity anomalies from altimeter data is shown in the Appendix. See Tapley (26) for an equivalence proof in a more general context. There have been other surprising statements. In (27) Moritz asserts: "Least-squares collocation seems to be the only method able to combine heterogeneous data to obtain a consistent and optimal gravity field."

And in the context of the processing of gradiometer data to estimate a gravity field the same author states (28): "Now it is an essential feature of gradiometry that several quantities, that is, various gradients, are measured at the same time. The simultaneous use of these different quantities is necessary if the available information is to be processed in an optimal way, but such a combined use is not possible by the customary methods.

Only recently a method for the simultaneous use and optimum combination of heterogenous data was developed, least-squares collocation."

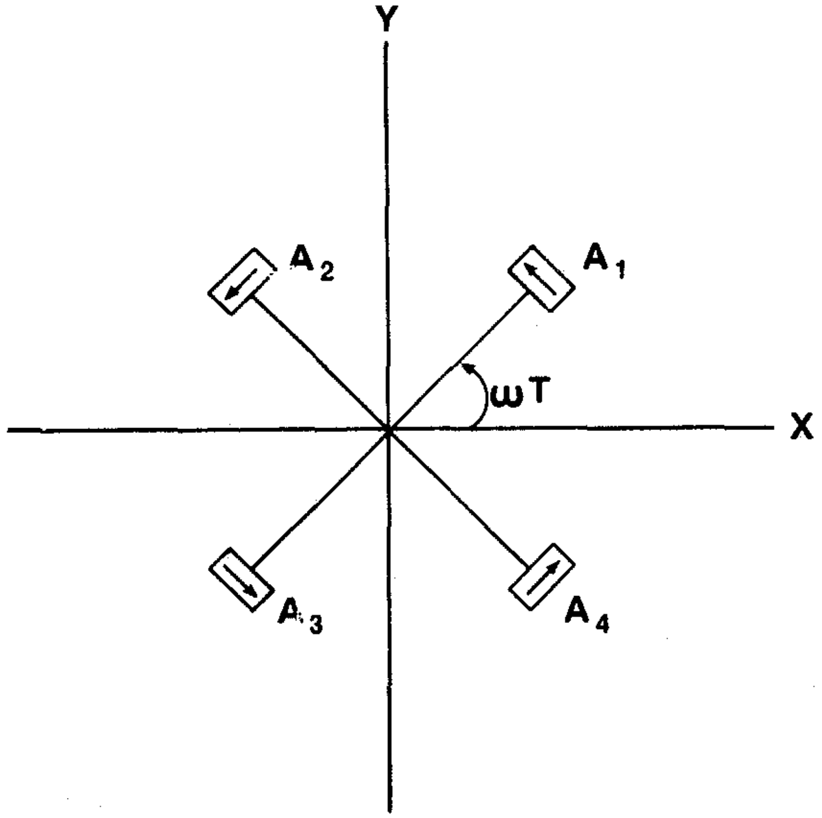
In fact there is not the slightest theoretical difficulty in using conventional least squares adjustment techniques to combine heterogenous data types in a way which yields an optimal solution. And since the beginning of satellite geodesy satellite perturbation data has been optimally combined with surface gravity data to provide gravity field estimates (29), (7).

The derivation in the Appendix shows that the least squares collocation procedure for estimating gravity anomalies from altimeter data is equivalent to a conventional least squares approach using Stokes' formula provided a priori estimates are included. The weights of the a priori estimates must be obtained from a degree variance model for the second order statistics of the anomalous potential field. Hence an evaluation of the two approaches amounts to an estimate of the validity of the degree variance models utilized in (23) and (24). There are many such models (21), and none of them appear to be solidly based on empirical information. One can also object to the assumption that the degree variance model is invariant under rotations. A logical consequence of this assumption is that the spectral properties of the anomalous gravity field are invariant over the surface of the earth. Yet it is well known that the degree of roughness or smoothness of the anomalous field varies considerably over the earth. We believe it is better not to use a priori estimates whose weights are obtained from a model which is so arbitrary and which is known to violate physical reality. Even if such a priori estimates are to be used it is preferable numerically to use them in conjunction with Stokes' formula and the conventional least squares method since this procedure involves the inversion of a matrix whose dimension is the number of estimated parameters. As mentioned previously, the collocation method involves the inversion of a matrix whose dimension is the number of data points.

In conclusion, altimetry appears to be a valuable data type for mapping gravity field fine structure over ocean areas. Using a truncated version of Stokes' formula it should be possible to estimate local blocks of gravity anomalies in local blocks of altimeter data. Thus the computational difficulties implied by any effort to obtain a detailed gravity field will be manageable. The aliasing effects of satellite altitude error may limit the usefulness of altimeter data for gravity field recovery. Further research into the possibility of eliminating this error either by more accurate orbit determinations or by sophisticated filtering techniques is required.

Satellite Gradiometry

We are concerned here with a rotating type gradiometer which appears to be the most likely to be used on a spacecraft mission. Two such instruments are under independent development by the Hughes Research Laboratory (30) and the Bell Aerospace Company (31). The instruments are electro-mechanical analogues of each other and hence their outputs relate to the gravity field in a mathematically identical fashion. Figure 3 is a simplified representation of a rotating gradiometer of the type described in (31). Accelerometers A_1, A_2, A_3, and A_4 rotate in the plane of the figure at angular velocity ω. The outputs of the accelerometers are combined as shown on the figure to form a continuous signal. The measurement



$$\text{SIGNAL} = (A_1 + A_3) - (A_2 + A_4)$$

$$= 2\,(\nabla_{xx} - \nabla_{yy}) \sin 2\omega T - 4\,\nabla_{xy} \cos 2\omega T$$

$$\text{AMP} = 2\left[\,(\nabla_{xx} - V_{yy})^2 + 4\,\nabla^2_{xy}\,\right]^{1/2}$$

Figure 3. Output Signal of Rotating Gradiometer

type of the instrument is taken to be the amplitude of the signal which is a function of second derivatives of the scaler potential field in the sensing plane of the instrument. If such an instrument were mounted on a polar, low altitude satellite, it would provide a global distribution of in situ observations of the gravity field. An equivalent way of expressing this fact is to state that each observation in this globally distributed set would relate to the gravity field only in terms of where the satellite was at the ti e of the observation rather than where the satellite nad been.

Assume a representation of the gravity field which has the property that if a given parameter of the representation is perturbed, the representation is perturbed only in a given localized area. The localized parameters which are sufficiently separated should have non-overlapping observability patterns in gradiometer data. This implies that it should be possible to estimate local blocks of parameters in local blocks of gradiometer data. The gravity anomaly parameterization of the gravity field as discussed in the previous section possesses a degree of orthogonality in gradiometer data. This is shown by Figure 4 which displays the perturbation of a gradiometer observation in Eotvos units (1 eotvos unit = 10^{-9} gal/cm) due to a one mgal perturbation of a 3° gravity anomaly. From the figure it can be seen that if two gravity anomalies are separated by approximately 10°, their observability patterns are non-overlapping and they would have an orthogonality relationship in gradiometer data.

Reed (32) reports on the results of numerical simulations designed to show the feasibility of estimating gravity anomalies from gradiometer data. The aliasing effects of unadjusted anomalies were not included in the study. Hence the orthogonality properties of gravity anomalies in gradiometer data was assumed rather than demonstrated. The effects of orbit and attitude errors were also ignored. A grid of gradiometer data of 2° latitude by 1° longitude was generated and a random number generator was used to add white noise of 0.01 E standard deviation to the data. A standard least squares estimator was used in the simulations to recover gravity anomalies from the data. The results are that if the satellite altitude is 300 km, 2° anomalies can be estimated with an accuracy of 1 mgal. With the satellite at an altitude of 250 km, 2° anomalies can be recovered with an accuracy of 0.45 mgal. The recovery of 5° anomalies was accomplished with an accuracy of 0.09 mgal when the satellite altitude was 300 km. The correlation structure of the simulations were good with most correlations between estimates of adjusted anomalies of absolute value less than 0.5.

In (33) covariance analysis procedures are used to investigate the feasibility of estimating gravity anomalies from gradiometer data. The effects of orbit and attitude determination errors are again neglected but the aliasing effect of unadjusted anomalies outside the estimated area are included in the study. The simulations show that gravity anomalies can be accurately estimated from

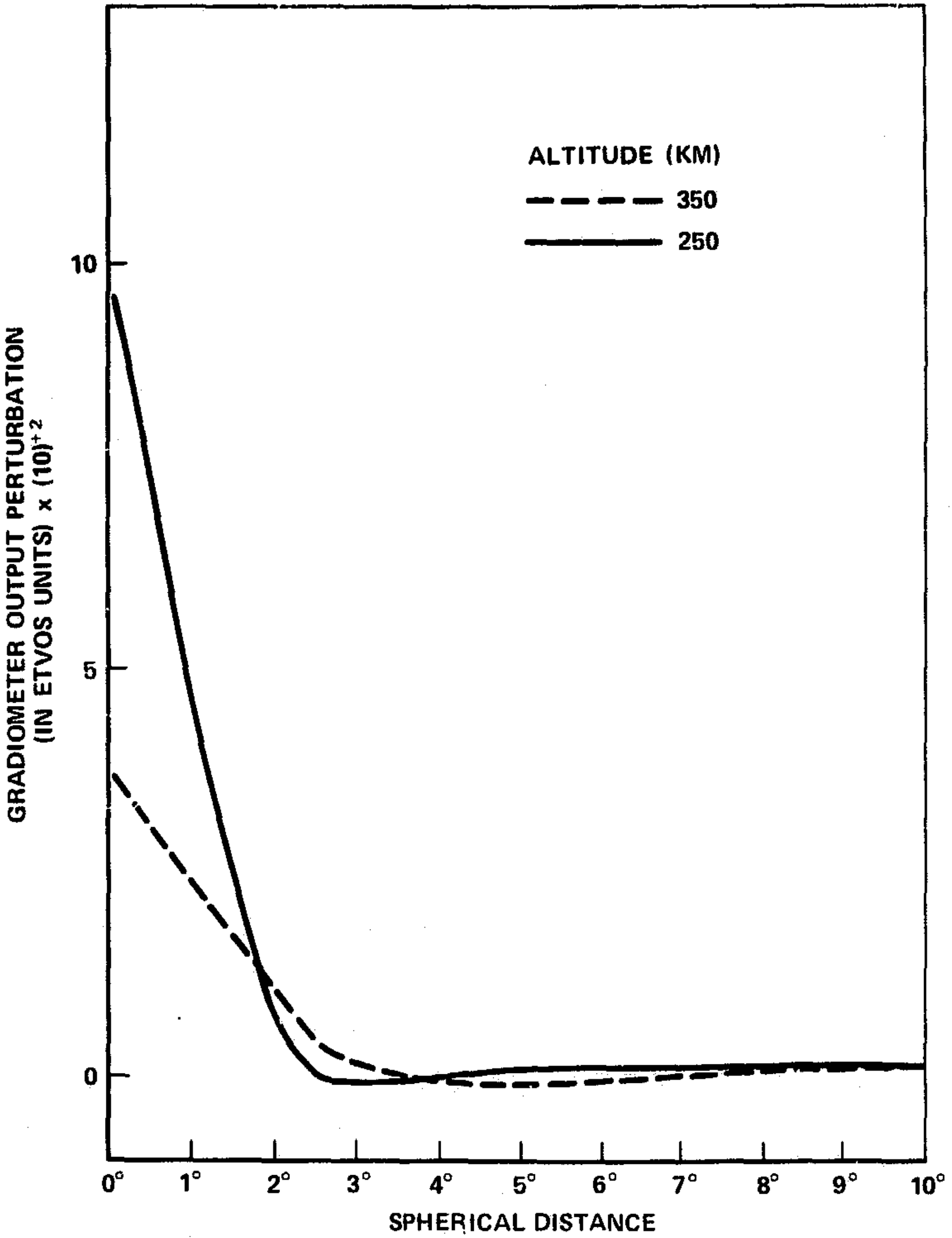


Figure 4. Gradiometer Output Perturbation Due to 1 mgal Perturbation of 3° Gravity Anomaly

gradiometer data provided validly estimated anomalies are separated from unadjusted anomalies by 10°. This is the same distance required for estimating gravity anomalies from altimeter data (16), (19). The study assumes a grid of data 1° latitude by 1.5° longitude. The data is assumed to be corrupted by white noise of 0.1 E standard deviation. The results show that provided the proper distance from unadjusted anomalies is maintained, 3° anomalies can be estimated with an accuracy of 3 mgal when the satellite altitude is 250 km and 7 mgal when the satellite altitude is 350 km. For 5° anomalies the accuracies are 1 mgal for a 250 km altitude and 2.5 mgal for a 350 km altitude.

Although the assumptions are different, the results of (32) and (33) can be compared. The authors of (33) provide equations by which their results which are predicated on a data accuracy of 0.1 E can be scaled to reflect any other accuracy level. Thus, if one assumes a data accuracy of 0.01 E used in (32) the results of (33) imply that 3° anomalies can be recovered to an accuracy of 1.5 mgal when the satellite altitude is 250 km. A gravity anomaly of 3° has 2.25 times the signal strength of a 2° anomaly at any given altitude. This implies that for a satellite altitude of 250 km, 2° anomalies should be recoverable to an accuracy of about 0.67 mgal when the data accuracy is 0.01 E. This is close to the value of 0.45 mgal provided in (32) for a 0.01 E data accuracy and for a 250 km satellite altitude. We concluded that the results of (32) and (33) are in good agreement.

The orbit and attitude determination requirements of a satellite borne gradiometer mission are provided in (34). According to the simulations reported in this study, the orbit determination requirements are 50 m radially and 300 m horizontally. The attitude determination requirements are a 5° resolution for spin vector azimuth and 0.2° resolution for spin vector elevation. The authors conclude that when these requirements are met, 3° gravity anomalies can be recovered from gradiometer data with an accuracy of about 2.5 mgal if the satellite altitude is 300 km.

A rotating gradiometer with a 0.1 E resolution and on board a satellite in a 250 km altitude orbit provides a data set from which gravity anomalies can be estimated with an accuracy equivalent to what is possible from altimeter data. Also the orthogonality properties of gravity anomalies appear to be the same from gradiometer data as from altimeter. Thus there should be no serious computational difficulties in estimating gravity fields from either data type. But gradiometer data is useful for estimating the gravity field all over the earth. Altimetry only has significance for the gravity field over ocean areas. Also the orbit determination requirements for a gradiometer mission appear to be less severe than those implied by an altimeter mission. For these reasons we recommend that the necessary time and resources be devoted to the development of a rotating gradiometer capable of functioning on board a spacecraft with at least a 0.1 E resolution.

A High-Low Configuration Satellite to Satellite Tracking Experiment

In an earlier section we discussed the possibility of using a high-low configuration satellite to satellite tracking experiment to determine long wavelength features of the gravity field. This configuration has also been suggested for definition of short wavelength features of the gravity field (1, 10).

Sjogren (35) has simulated the summed doppler data of the ATS-6/GEOS-3 satellite to satellite tracking experiment. He shows that by differentiating spline functions fitted to doppler residuals one can reconstruct the anomalous acceleration profile of the satellite due to short wavelength gravity field features. He does not discuss how such acceleration profiles can be used to uniquely and accurately reconstruct the short wavelength gravity field.

Von Bun et al. (36) have demonstrated that the Apollo-Soyuz satellite to satellite tracking experiment has the ability to detect gravity anomalies along the track of an orbit. The Indian Ocean and Himalayan anomalies were readily visible in the actual data in four different orbital passes of Apollo-Soyuz (tracked by ATS-6). The detection of small local gravity anomalies is important for geologic investigation of the earth's upper crust as well as for studies of the ocean topography.

Hajela (37) has investigated the feasibility of uniquely reconstructing the short wavelength gravity field from satellite to satellite tracking data resulting from a high-low configuration. In a comprehensive set of simulations he examines the possibilities of recovering 10°, 5° and 2.5° equal area gravity anomalies from range rate sum observations when low satellite altitude is 250 km and when low satellite altitude is 900 km. The doppler data accuracy is assumed to be 0.08 cm/sec for a 10 sec integration time. The presence of adjacent unadjusted anomalies are accounted for in the simulation of the data. However, the short data arcs used in the solution only span the gravity anomalies to be estimated and their epoch vectors are assumed to be perfectly known. This assumption artifically eliminates the aliasing effect of distant unadjusted anomalies. Hence Hajela's results are of little use in determining the feasibility of estimating local blocks of gravity anomalies in local blocks of doppler data. The effect of uncertainty in the relay satellite epoch state is also ignored. In fact, this is likely to be a major error source. The results are that when the low satellite is at altitude 900 km, 10° anomalies can be recovered with an accuracy of 2 mgal and 5° anomalies can be recovered with an accuracy of 17 mgal. When the low satellite is at an altitude 250 km, 5° anomalies can be recovered with an accuracy of 6 mgal. The recovery of 2.5° anomalies is not satisfactory at either altitude.

Considered in their entirety, Hajela's results are not encouraging. The assumption of perfect knowledge of low and high satellite epoch states raises serious questions concerning the realism of the results. Yet the results are quantitatively inferior to what is apparently obtainable from satellite altimetry or satellite gradiometry. Also there are a priori reasons for suspecting that the data reduction problem implicit in an effort to recover a detailed gravity field from satellite to satellite tracking data may be insurmountable. The reason that local blocks of gravity anomalies can be successfully estimated in local blocks of gradiometer or altimeter data is that gravity anomalies have localized observability patterns in these two data types. This is shown for gradiometry in Figure 4. Figure 5 shows the magnitude of the vector velocity perturbation of a satellite in a 250 km altitude orbit due to a 1 mgal perturbation of a 5° anomaly directly in its subtrack. If this satellite were tracked by a geosynchronous satellite, the resultant perturbation pattern of the doppler data would be the product of the vector velocity perturbation magnitude and the cosine of the angle between the velocity vector and the line of sight between the low satellite and the relay satellite. In contrast to Figure 4, Figure 5 shows the perturbation of gravity anomalies in satellite to satellite tracking data to be highly non-localized. Thus gravity anomalies are not likely to display any degree of orthogonality in this data type.

In our opinion, a thorough and realistic study of the potentiality of this type of mission for providing a detailed gravity field mapping has not yet been performed. Such a study must address the question of the feasibility of decomposing the implicit large dimensional estimation problem into smaller dimensional estimation problems without a total loss of accuracy. Nevertheless, the results of (37) imply that even if the data reduction problems could be solved the resultant gravity field resolution would not be competitive with what is obtainable from either satellite gradiometry or satellite altimetry.

Low-Low Configuration Satellite to Satellite Tracking Experiment

The low-low configuration satellite to satellite tracking experiment would employ two satellites in the same circular orbit, with one following the other at a distance of a few hundred kilometers. The hypothesis which motivates this configuration is that range rate data between the two satellites is sensitive to local anomalies but not to distant anomalies. Hence it should be possible to estimate local blocks of gravity anomalies in local blocks of range rate data.

In (38), numerical simulations are utilized to study the feasibility of the mission. A range rate observation every thirty seconds is assumed. The satellites are separated by 200 kilometers. The reported results are that at a 700 kilometer altitude, 5° gravity anomalies can be recovered with an accuracy between 1 and 3 mgal. At a 200 km altitude, 2° anomalies can be recovered with the same accuracy.

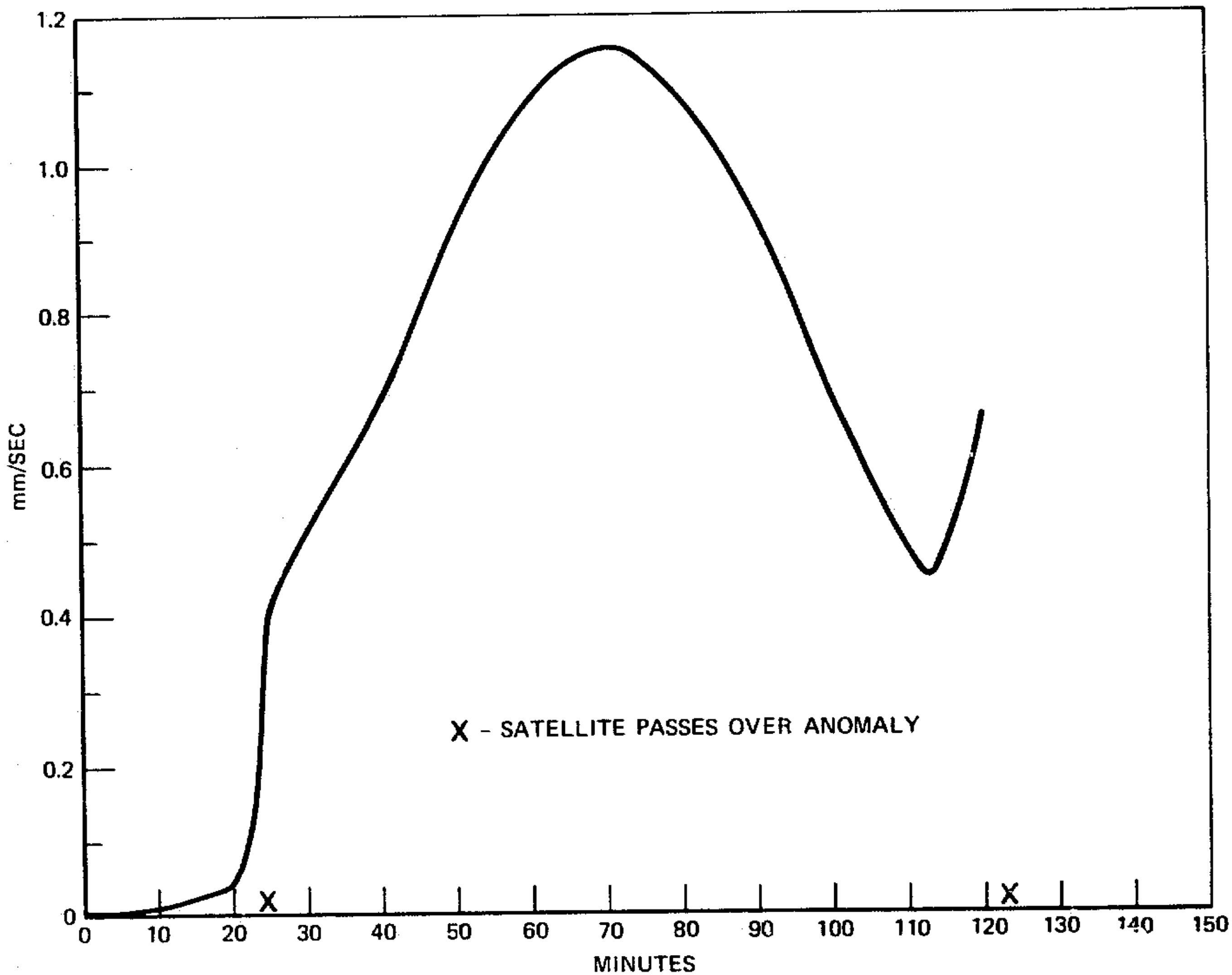


Figure 5. Perturbation on Magnitude of Velocity Vector Due to a 1 mgal Perturbation on a 5° Anomaly

There are several objections to this study. The author assumes a range rate accuracy of 0.05 mm/sec for his simulations. This is at least an order of magnitude superior to what is presently obtainable. Also the simulations postulate that all three position components of each satellite are observable every thirty seconds. This is an artificial and unrealizable assumption. Hence it is impossible to determine from the results of this study what the tracking requirements of this mission actually are. The most serious shortcoming of the study is that it assumes rather than demonstrates that local blocks of anomalies can be accurately estimated from local blocks of range rate data generated by the experiment. If this is not the case then the data reduction problem is unsolvable and the experiment is not feasible. The suitability of this data type for gravity field fine structure determination is not obvious since gravity anomalies do not have a localized perturbation pattern in this data type. This is shown in Figure 6 which displays the perturbation pattern in range rate data between satellites in a 300 km orbit and separated by 300 km due to a perturbation of 8 mgals in a 5° x 5° anomaly block.

In (39) the results of a sensitivity study of a low-low configuration satellite to satellite tracking mission are described. The range rate signal perturbation due to a gravity anomaly is investigated as a function of satellite height and satellite separation. It is shown that an optimal combination of signal strength and resolution is achieved when the satellites are separated by approximately 300 km. The simulations also suggest that at a satellite altitude of 300 km, gravitational features separated by less than 5° cannot be separated by means of the information supplied by the range rate data.

Lancaster and Estes (private communication, 1975) are using the techniques of covariance analysis to study the aliasing problem as it relates to the task of using this mission configuration to resolve gravity field fine structure. Preliminary results indicate that at least in a long arc mode it is not possible to estimate local blocks of gravity anomalies from local blocks of range rate data.

We must conclude that a satisfactory feasibility study of the low-low configuration satellite to satellite tracking experiment has not yet been published. The tracking requirements of the mission are unknown. The possibility of estimating local blocks of gravity anomalies in local blocks of range rate data has not been determined. Hence at the present time there is no basis for recommending this mission configuration for the purpose of short wavelength gravity field mapping.

SUMMARY

This paper has examined and compared procedures for employing satellite technology to determine the earth's gravity field. The problems involved in estimating

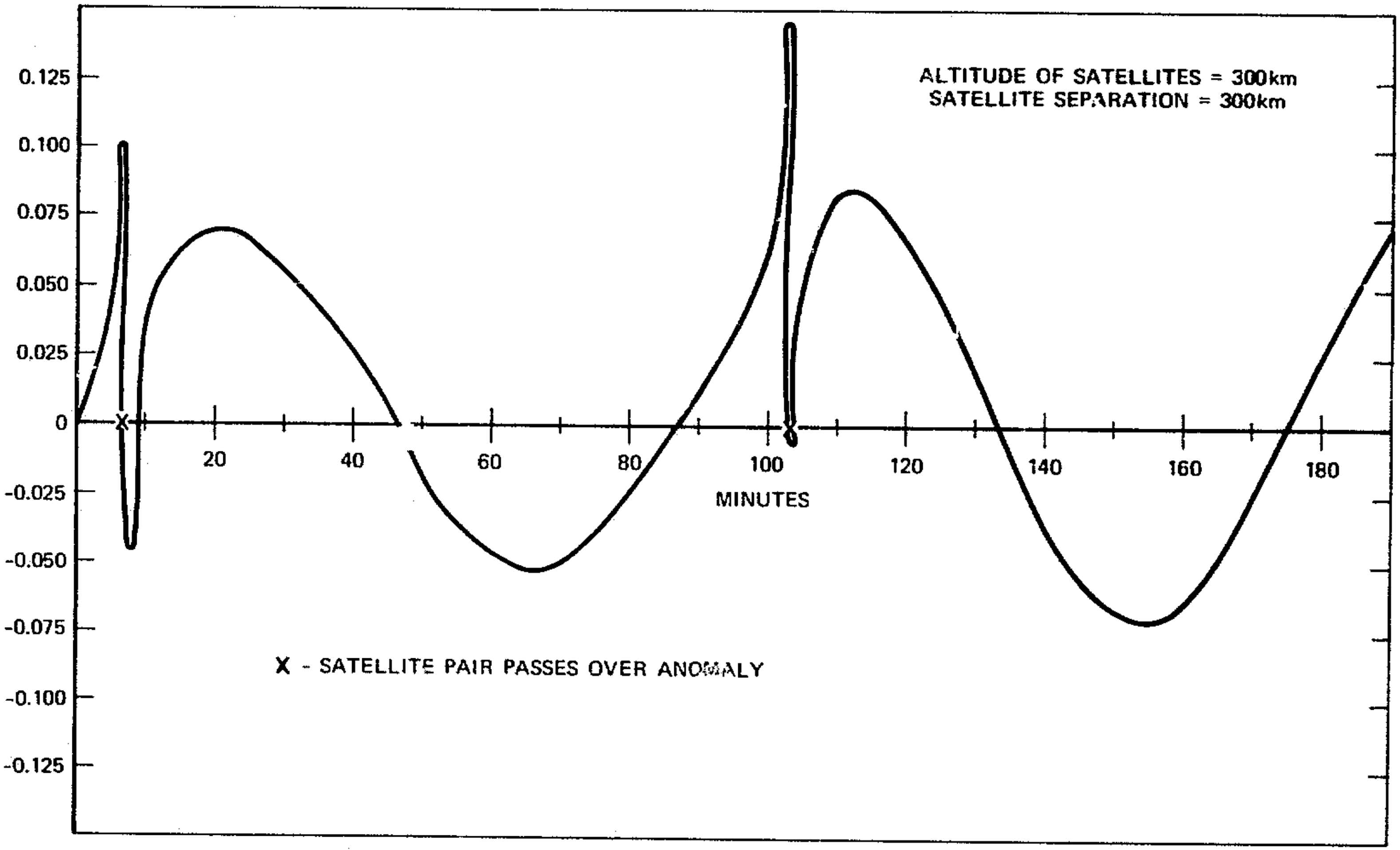


Figure 6. Perturbation on Range-Rate Between Two Satellites Due to a 1 mgal Perturbation of a 5° Anomaly

long wavelength gravity field features are different in character from those involved in estimating short wavelength features. Hence the paper treats the recovery of long wavelength components and short wavelength components of the gravity field as separate estimation problems in separate sections.

Satellite perturbations represent an excellent data type for determining the long wavelength components of the gravity field. The recovery of spherical harmonic coefficients of the earth's gravity field from satellite perturbation data has been standard practice and it is likely the wisest procedure for using satellites to determine the long wavelength gravity field. Present estimates suffer from severe aliasing because of a non global distribution of data. The best satellite configuration for solving this problem is that of a low altitude, polar satellite tracked by a high altitude relay satellite. Separate studies have proposed a geosynchronous orbit for the relay satellite, and a high altitude polar orbit (the GEOPAUSE concept) for the relay satellite. Numerical studies show that both configurations are capable of providing a global data set from which an order of magnitude improvement of estimates of the long wavelength gravity field can be extracted. From a strictly scientific vantage point the GEOPAUSE concept is preferable since the data reduction difficulties involved in its use are more tractible. The GEOPAUSE satellite is also useful for polar motion and crustal motion determination. Economic and operational considerations, however, may dictate the use of a geosynchronous satellite.

The major difficulty in employing parameter estimation techniques to recover the short wavelength gravity field is that a very large number of parameters must be estimated. For this reason it is desirable for a satellite mission to provide a global data set which permits the independent estimation of smaller subsets of the parameters which represent the field. With proper corrections the output of a satellite borne altimeter over an ocean area can be viewed as observations of geoid altitude. This is an in situ data type and studies show that it is possible to accurately estimate local blocks of gravity anomalies in local blocks of altimeter data. Assuming one meter accuracy it has been shown that 2° anomalies can be recovered with an accuracy of 10 mgal, 3° anomalies with an accuracy of 5 mgal, and 5° anomalies with an accuracy of 1 mgal. These results were obtained assuming Stokes' formula and standard least squares adjustment methods. This procedure is equivalent to the least square collocation approach if an a priori estimate weighted according to a degree variance model is used. The Stokes' formula approach is numerically superior since it involves the inversion of a matrix whose dimension is the size of the estimated parameter set. The least square collocation approach involves the inversion of a matrix whose dimension is the size of the data set.

A major limitation of the use of a spacecraft borne altimeter for geodetic purposes is that its output relates to the gravity field only over ocean areas. Another difficulty is that requirements for altitude resolution of the spacecraft are on the order of one meter.

A spacecraft borne rotating gradiometer mission is capable of providing a global distribution of in situ gravity field observations. It has been shown that local blocks of gravity anomalies can be estimated in local blocks of gradiometer data. A rotating gradiometer functioning with an accuracy of 0.1 E and on board a satellite in a 250 km altitude orbit will provide a gravity field estimate equivalent in resolution and accuracy to that obtainable by means of satellite altimetry. The orbit determination requirements for a satellite gradiometer mission are 50 m radially and 300 m horizontally. The altitude determination requirements are 0.2° for satellite spin vector elevation and 5° for satellite spin vector azimuth.

The possibilities of using satellite to satellite tracking data either from a high-low, or a low-low configuration to map gravity field fine structure have not been adequately investigated. A prerequisite for the serious consideration of such missions is a convincing demonstration that local blocks of satellite to satellite tracking data can be used to accurately estimate local blocks of gravity field parameters. Unless such a demonstration can be provided we believe it is preferable to concentrate on the development of satellite gradiometry and satellite altimetry mission concepts since for these data types it is known that data reduction problems are solvable.

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APPENDIX

PROCEDURES FOR ESTIMATING GRAVITY ANOMALIES FROM ALTIMETER DATA — A COMPARISON OF LEAST SQUARES COLLOCATION WITH CONVENTIONAL LEAST SQUARES TECHNIQUES

Our object is to define the circumstances under which the least squares collocation procedure for estimating ravity anomalies from altimeter data is equivalent to a conventional least squares approach to the problem utilizing Stokes' formula. In the next section we define the problem, introduce the concept of the degree variance model, and derive the least squares collocation solution. The succeeding section uses the same model and the same information and approaches the problem with a conventional least squares technique. Finally an equivalence between the two procedures is demonstrated.

The Least Squares Collocation Solution

After suitable corrections the output of a spacecraft borne altimeter over an ocean area can be considered as a measurement of the deviation of the ocean geoid from a reference geoid. The problem is to obtain from a set of such observations $\{\delta N'\}$ a "best" estimate of mean gravity anomalies $\{\delta g\}$. We will define the "best" estimate to be the conditional expectation of $\{\delta g\}$ given a realization of the observations $\{\delta N'\}$. Since the smallest second moment of a random variable is the second moment about the mean, this is equivalent to applying a minimum variance criterion.

The starting point of the least squares collocation approach to obtaining the best estimate of $\{\delta g\}$ is the assumption that one has full knowledge of the second order statistics of the anomalous potential field everywhere on and outside the reference geoid. (The first order statistics of the anomalous potential field are assumed to be zero.) Let $P(x_1)$ and $P(x_2)$ be the anomalous potentials at points x_1 and x_2 on or outside the reference geoid. We assume the possession of a function $K(x_1, x_2)$ such that

$$E\,(P(x_1)\,P(x_2)) = K\,(x_1, x_2) \tag{1}$$

The function $K(x_1, x_2)$ is the so-called degree variance model and it is generally defined to be invariant under rotations. Hence the second order statistics of the anomalous potential field are assumed to be independent of location.

Let $\{\delta N\}$ be the set of deviations of the ocean geoid from the reference geoid at the observation points of a spacecraft borne altimeter. Also let $\{\delta g\}$ be a set of globally distributed gravity anomalies. Since both $\{\delta N\}$ and $\{\delta g\}$ are determined by the anomalous potential field all second order statistics relating to the two random vectors can be readily derived from the degree variance model. Hence define

$$\text{a) } E(\delta N \delta N^T) = A, \quad \text{b) } E(\delta g \delta N') = B, \quad \text{c) } E(\delta g \delta g^T) = C \tag{2}$$

Computational algorithms for obtaining matrices A, B, and C from a degree variance model can be found in (19) or (20). The actual observations $\{\delta N'\}$ obtained from the instrument are, of course, corrupted by noise. Hence

$$\delta N' = \delta N + \nu, \quad E(\nu) = \overline{O}, \quad E(\nu\nu^T) = Q \tag{3}$$

Equations 2 and 3 permit us to write the joint covariance matrix of the random vectors $\{\delta N'\}$ and $\{\delta g\}$ as

$$\mathrm{COV}\begin{bmatrix}\delta g \\ \delta N'\end{bmatrix} = \begin{bmatrix} C & B \\ B^T & A + Q \end{bmatrix} \tag{4}$$

A realization of the random vector $\{\delta N'\}$ is obtained by means of the actual measurements. Symbolically we do not distinguish between this random vector and its realization. We desire the conditional expectation and the conditional covariance of $\{\delta g\}$ given a realization of the correlated random vector $\{\delta N'\}$. By assuming either that the random vectors are normally distributed or that the conditional expectation of $\{\delta g\}$ is a linear function of the measurements we can resort to the familiar regression equations for the conditional mean and the conditional covariance of a random vector given a realization of a correlated random vector (21). The results are

$$\delta \hat{g} = B\,(A + Q)^{-1}\,\delta N' \tag{5}$$

$$\mathrm{COV}\,[\delta \hat{g}] = C - B\,(A + Q)^{-1}\,B^T \tag{6}$$

The solution represented by equation 5 is the least squares collocation estimate of a global set of gravity anomalies given a degree variance model and given the measurement set $\{\delta N'\}$.

In actuality one would not attempt to estimate a global set of anomalies from a set of altimeter measurements obtained from a certain area. It is only possible to significantly improve knowledge of gravity anomalies in the area covered by the observations. Decompose $\{\delta g\}$ as follows:

$$\delta g = \begin{bmatrix} \delta g_1 \\ \delta g_2 \end{bmatrix} \tag{7}$$

where δg_1 is the set of anomalies covering the region where the measurements are available and where δg_2 is the set of anomalies outside of this region. Then the matrix B can be decomposed:

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{8}$$

where

$$B_1 = E\,(\delta g_1\ \delta N^T),\quad B_2 = E\,(\delta g_2\ \delta N^T) \tag{9}$$

The least squares collocation estimate for $\{\delta g_1\}$ becomes

$$\delta \hat{g}_1 = B_1\ (A + Q)^{-1}\ \delta N' \tag{10}$$

The Conventional Least Squares Solution Employing Stokes' Formula

Stokes' formula provides a linear relation between $\{\delta N\}$ and $\{\delta g\}$ which can be written as

$$\delta N = S\ \delta g \tag{11}$$

The elements of the matrix S are obtained by evaluating Stokes' function at the required computation points. Equation 11 can be used as an equation of condition for a least squares estimate of $\{\delta g\}$. But the resultant solution would not be optimal unless all information were used. Consequently, if one accepts the validity of the degree variance model it would be proper to utilize the zero vector as an a priori estimate of $\{\delta g\}$ with a weight provided by the inverse of the covariance matrix of equation 2.c. The resultant loss function to be minimized has the form

$$L(\delta\hat{g}) = (\delta N' - S\delta\hat{g})^T Q^{-1} (\delta N^T - S\delta\hat{g}) + \delta\hat{g}^T C^{-1} \delta\hat{g} \tag{12}$$

The estimator which minimizes the right side of equation 12 is

$$\delta\hat{g} = (S^T Q^{-1} S + C^{-1})^{-1} S^T Q^{-1} \delta N' \tag{13}$$

Equation 13 provides the standard least squares solution for $\{\delta g\}$ using equation 11 as an equation of condition and using the zero vector weighted according to a degree variance model as an a priori estimate.

Derivation of an Equivalence Relation

The conventional least squares estimate of $\{\delta g\}$ as defined by equation 13 and the least squares collocation estimate of $\{\delta g\}$ as defined by equation 5 are different in appearance. We will show that they are, in fact, equivalent.

Equation 11 defines the zero expectation random vector $\{\delta N\}$ in terms of the zero expectation random vector $\{\delta g\}$. Thus the covariance matrix of $\{\delta N\}$ and the joint covariance matrix of $\{\delta N\}$ and $\{\delta g\}$ can be obtained in terms of the covariance matrix of $\{\delta g\}$. Equation 2.c provides the covariance matrix of $\{\delta g\}$ as derived from the degree variance model of equation 1. Equation 11 along with equations 2.c and 3 permit us to write:

$$\mathrm{COV}\begin{bmatrix}\delta g\\ \delta N'\end{bmatrix} = \begin{bmatrix} C & CS^T \\ SC & SCS^T + Q\end{bmatrix} \tag{14}$$

The regression equation can again be used to obtain the conditional expectation of $\{\delta g\}$ as:

$$\delta\hat{g} = CS^T\ (SCS^T + Q)^{-1}\ \delta N' \tag{15}$$

A comparison of equation 4 with equation 14 yields:

$$B = CS^T\ ,\quad A = SCS^T \tag{16}$$

Hence the estimate of $\{\delta g\}$ provided by equation 15 is equivalent to the least squares collocation estimate of equation 5. We can use the well known Shure matrix identity to translate equation 15 into the alternative form:

$$\delta\hat{g} = (S^T\ Q^{-1}\ S + C^{-1})^{-1}\ S^T\ Q^{-1}\ \delta N' \tag{17}$$

Equation 17 is identical to equation 13. This demonstrates that a standard least squares approach to estimating gravity anomalies from altimeter data which utilizes an a priori estimate weighted according to a degree variance model yields a solution identical to what is obtained through least squares collocation.