THE EVOLUTION OF COMET ORBITS

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Abstract. This review states and defends seven conclusions on the origin of comets and the evolution of their orbits:

1. There is a $N^{-1/2}$ law of survival of comets against ejection on hyperbolic orbits, where N is the number of perihelion passages.

2. The short-period comets are not created by single close encounters of near-parabolic comets with Jupiter.

3. Observable long-period comets do not evolve into observable short-period comets.

4. Unobservable long-period comets with perihelia near Jupiter can evolve into observable short-period comets.

5. Long-period comets cannot have been formed or created within the planetary region of the solar system. (This conclusion is somewhat qualified because of possible effects of stellar perturbations.)

6. It is possible that some of ths short-period comets could have been formed inside the orbit of Neptune, but it is certain that others have the same distant source as the long-period comets.

7. The circularly-restricted 3-body problem, and its associated Jacobi integral, are not valid approximations to use in studying origin and evolution of comets.

The starting data are the orbits of known comets. We are indebted to the compilations and catalogs of Galle (1894), Porter (1961), and Marsden (1972). Models of comet origin and evolution must produce distributions of periods, inclinations, and other properties that fit these data, taking into account that the data include effects of observational selection.

Analytic methods are often used to study orbit evolution. However, in my opinion treatments that are based on approximating the solar system by the restricted 3-body problem are not valid, and this is discussed in item 7 below. On the other hand, statistical methods, such as those of Shteins (1972) which treat diffusion of orbits, are informative and useful.

The most obvious approach is to start from the known orbits and calculate backwards or forwards in time. One would suppose that upon projecting the orbit back in time and allowing for planetary perturbations one could find the original orbit on which the comet first entered the solar system. This procedure works reasonably well for near-parabolic orbits. The most careful studies of these show no original hyperbolic orbits, but some comets enter the solar system on elliptical orbits so nearly parabolic that their original aphelia are at 25000 to 100000 AU. This latter set correspond to the long-period comets originating at these vast distances within a cloud of comets described by Oort (1950).

Starting with these near-parabolic comets and calculating forward we find that planetary perturbations during their first passage remove half of them so that they then leave the solar system forever on hyperbolic orbits. The other half leave on elliptical orbits and will return. Unfortunately, for those in elongated elliptical orbits all accuracy is lost between the first and second passages. An example: Suppose that a comet after interacting with the planets then moves well outside the orbit of Neptune on an elliptical orbit whose period is exactly 3600 years. When it returns it may pass some distance in front of Jupiter and lose energy such that its period after leaving the planetary region is now 23457 years, and so on. Now take the same comet and

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start again, altering just one of its elements by one part in 10^8. The elliptical orbit might now have a period of 36003 years, and when the comet returns the next time (three years or a quarter of a Jupiter period later than in the first case) it passes close behind Jupiter, gains energy and leaves the planetary region on a hyperbolic orbit never to return. The minute difference between the two cases has caused an entirely different evolution. Whether calculated forwards or backwards, the orbits of very-long-period comets are extraordinarily sensitive to the starting conditions.

The situation is better for the short-period comets, where the periods are more nearly comensurate with planetary periods. (in this paper a short-period comet is one whose period is less than 16 years) Here we recall the work of Kazimirchak-Polonskaya (1973) in projecting the orbits of known short-period comets into the past and into the future. These show the sort of behavior to be expected, but in this case again most of the accuracy is lost after a close approach to Jupiter because the impact parameter at Jupiter and the orbit afterwards is very sensitive to the starting elements.

Probably the most powerful and exact approach is that of numerical experiments with random starting conditions. This is the Monte Carlo method. If one wants to find how the solar system interacts with comets that approach it on parabolic orbits, he can throw a thousand hypothetical parabolic comets at a fairly realistic model of the solar system. No approximations need be made; all 9 planets can be included in their appropriate elliptical orbits. Each comet's orbit is calculated fairly exactly until planetary perturbations remove it on a hyperbolic orbit, even if it makes thousands of revolutions, as it does in some cases. There is the same extreme sensitivity to the starting elements, but the evolution of each orbit is a typical random result, and the distributions of properties are not dependent on the particular set of initial conditions. When the problem is repeated with an independent and new set of random initial conditions, this gives the same overall results within a certain statistical tolerance.

This Monte Carlo approach has already established seven conclusions or facts, and these are enumerated below. I believe they are established beyond reasonable doubt and am prepared to defend them.

1. The effect of planetary perturbations on a parabolic flux of comets is to remove some of these on hyperbolic orbits, the number surviving in elliptical orbits being proportional to $N^{-1/2}$, where N is the number of perihelion passages of each individual comet. The same $N^{-1/2}$ law is reached ultimately when the starting orbits are circular.

The $N^{-1/2}$ law is the result of numerical experiments, Everhart (1972b). The straight lines of slope -1/2 in Figure 1 illustrate the law for two cases. The upper line labeled B follows the survival of 5500 initially parabolic comets of small perihelia, and line A that of 600 such comets with perihelia near Jupiter's orbit.

One can think of this survival as the random walk of a population near the edge of a cliff, each member taking steps of a certain distribution of sizes randomly towards or away from the edge. Of course, for the comets the steps are steps in total energy. The elliptical orbits are back from the edge, the parabolic orbit is just at the edge, and the hyperbolic orbit, from which there is no return, corresponds to a step beyond the edge of the cliff. Surely this simple law found here empirically is also derivable from random walk theory.

In the case of initially circular orbits, the population of comets begins its random walk in energy fairly far from the edge of the cliff. There is a delay, and no members are lost for some time. Ultimately, however, some members are lost and the numerical experiments present-

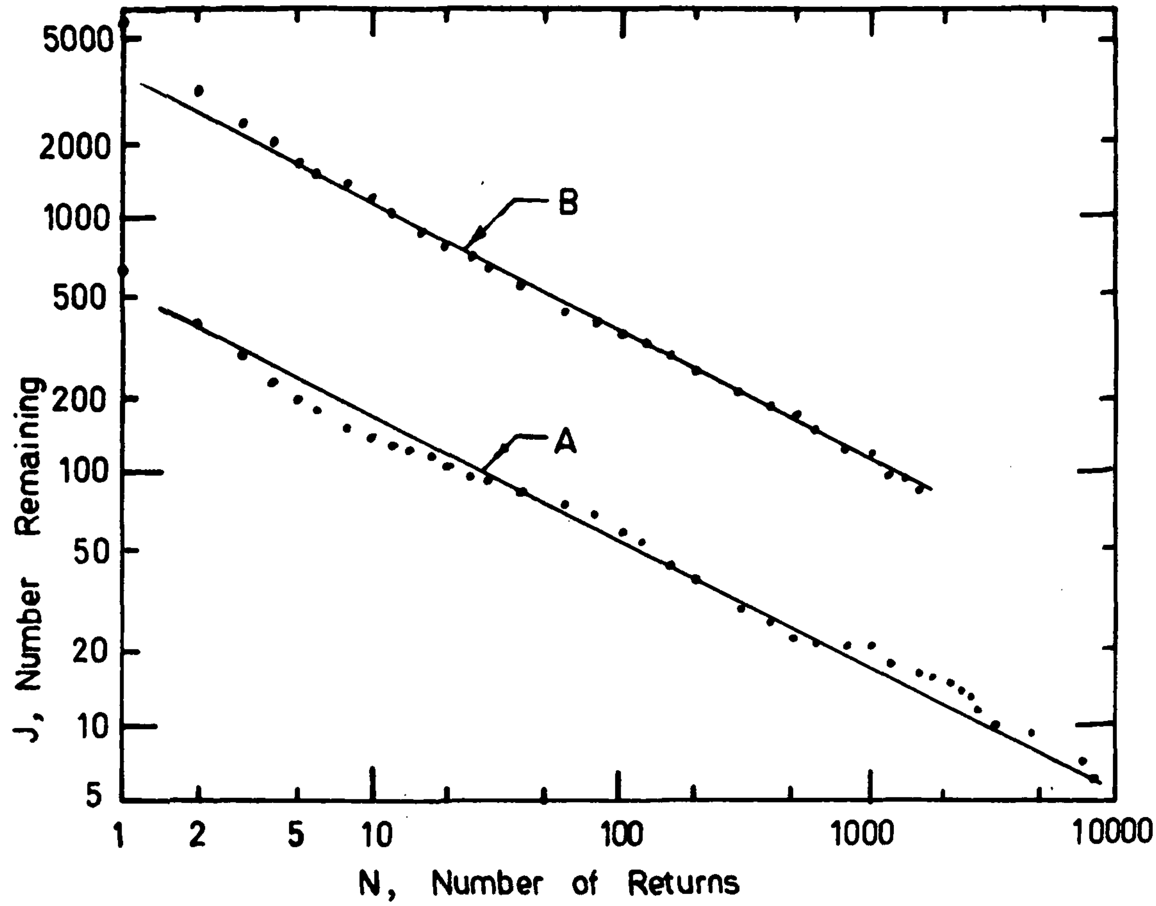

Fig. 1. This concerns survival of originally parabolic comets against being thrown out of the Sun-Jupiter system on hyperbolic orbits. The number remaining in elliptical orbits is plotted vs N, the number of returns. Line A is for hypothetical cases whose original parabolic elements were in the capture region, $i_o < 9^o$ and 4 AU $< q_o <$ 6 AU. Line B is for 5997 cases of all inclination and with $q_o <$ 4 AU. There were 6 left after 8000 returns in case A, and 80 left after 1600 returns in case B. Both lines show a $N^{-1/2}$ dependence.

ly approach the same $N^{-1/2}$ law. See the upper curve in Figure 2., which figure also appears in Everhart (1973b).

Note that this survival vs number of returns is not the same as survival as a function of time. For comets there is no thermal dissipation except when they are near the sun, so in a sense, their lifetime is measured in perihelion passages rather than in years. Lyttleton and Hammersley (1963) have studied the actual time dependence of survival, but this does not appear to have such a simple formula.

2. Although it is possible for an orbit of short period to be the result after a parabolic comet makes a single close encounter with Jupiter, this mechanism does not explain the existence of the short-period comets.

This was shown by H. A. Newton (1893). Not wanting to believe his results, and being a little dubious about Newton's procedures, I redid the problem as a numerical experiment and came to exactly the same conclusions, Everhart (1969). The convincing reason that short-period comets were not captured by Jupiter in a single encounter is that, if this were true, then one-fourth of all short-period comets would be retrograde, a result contrary to the data. The predicted distribution of periods also has the wrong shape. The detailed results may be found in Figures 6 and 7 the 1969 paper cited above.

3. There is no evolutionary path for long-period comets of small perihelia to evolve onto orbits of 5- to 13-year periods typical of short-period comets.

Such evolution simply does not happen in the numerical experiments. Some insight is offered by the lower curve, labeled B, in Figure 3. This shows the average period of those comets surviving in elliptical orbits after N returns. Comets that begin on parabolic orbits of small perihelia reach shorter periods very slowly. They cut across Jupiter's

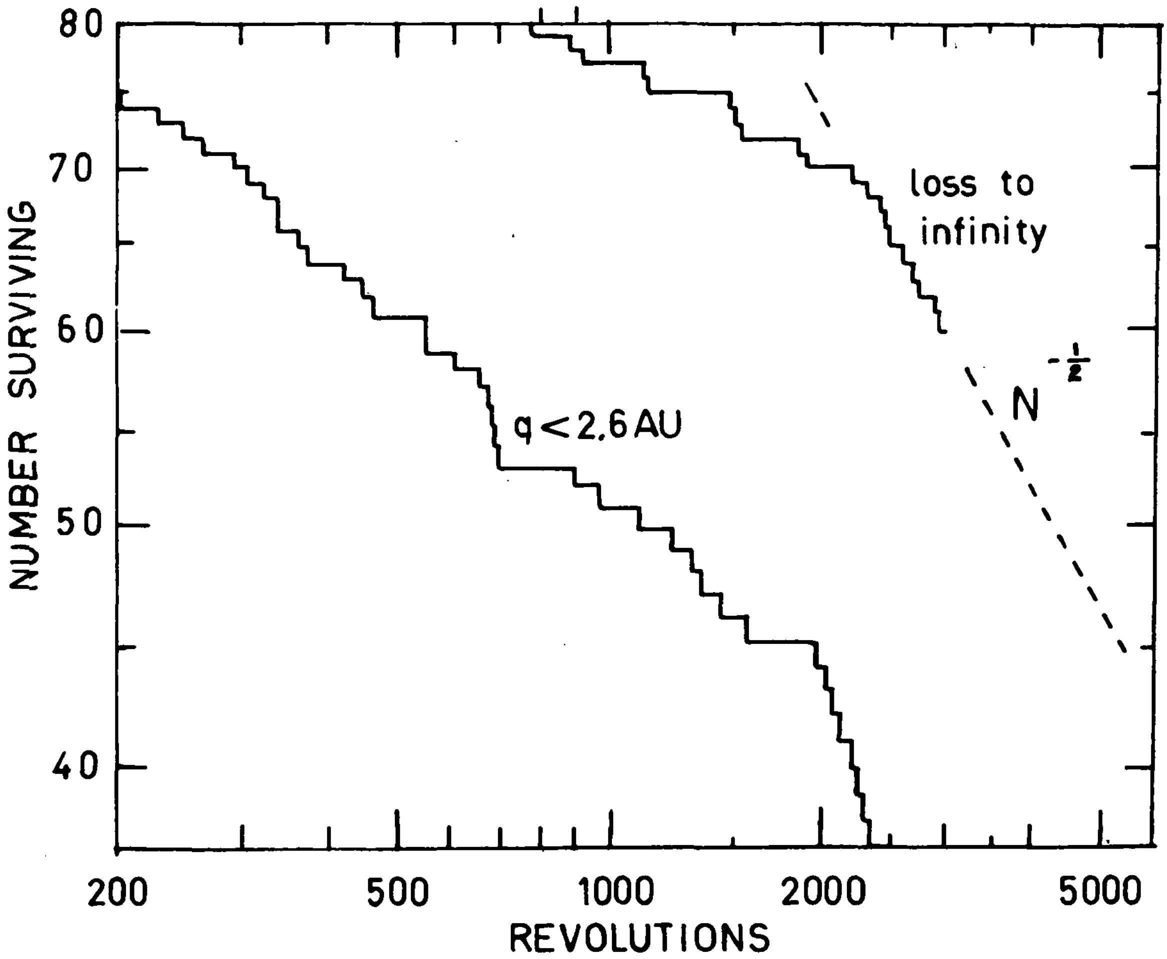

Fig. 2. This concerns the survival of hypothetical comets started in circular orbits in the Jupiter-Saturn region. The curve on the right plots vs revolution number the survival against loss infinity for 80 chaotic orbits. On the left is plotted vs revolution number N the number that have not yet achieved a perihelion value less than 2.6 AU at least once.

orbit at a large angle, the interaction is brief, and the energy perturbations are small. Those that survive the attrition of removal on hyperbolic orbits would not also survive the solar thermal dissipation of hundreds of thousands of returns at small perihelia. These results are from a study of the origins of short-period comets, Everhart (1972a, 1972b), but Figures 1 and 3 here have not previously been published.

4. There is a path for long-period comets of small inclination and with original perihelia near Jupiter's orbit to evolve into orbits typical of short-period comets. One phase of the evolution is a near-circular orbit just outside Jupiter's orbit.

This result from Everhart (1972a) may be understood by examining the upper curve of Figure 3. This class of orbits is brought to short periods rather rapidly because they interact strongly with Jupiter. Having their perihelia near Jupiter's orbit, they experience little solar dissipation during most of their evolution. At some stage in the evolution the orbit can become like that of a typical short-period comet. This happens after a rather sudden drop in the perihelion distance. The evolution shown in Figure 4 is accelerated in that each successive revolution as drawn might be the shape reached after intergrating for another 100 revolutions. The circular phase of the orbit outside Jupiter's distance sometimes appears before and sometimes after the small-perihelia phase. It reminds one of the present orbit of Comet Schassmann-Wachmann I.

This evolutionary path is a qualitative result. A paper by Joss (1973) has found the above mechanism to be unable to account for the observed number of short-period comets, but a contrary result by Delsemme (1973) finds the model to be quantitatively acceptable. Joss assumes the existing numbers of long- and intermediate-period comets with perihelia near Jupiter to be random in

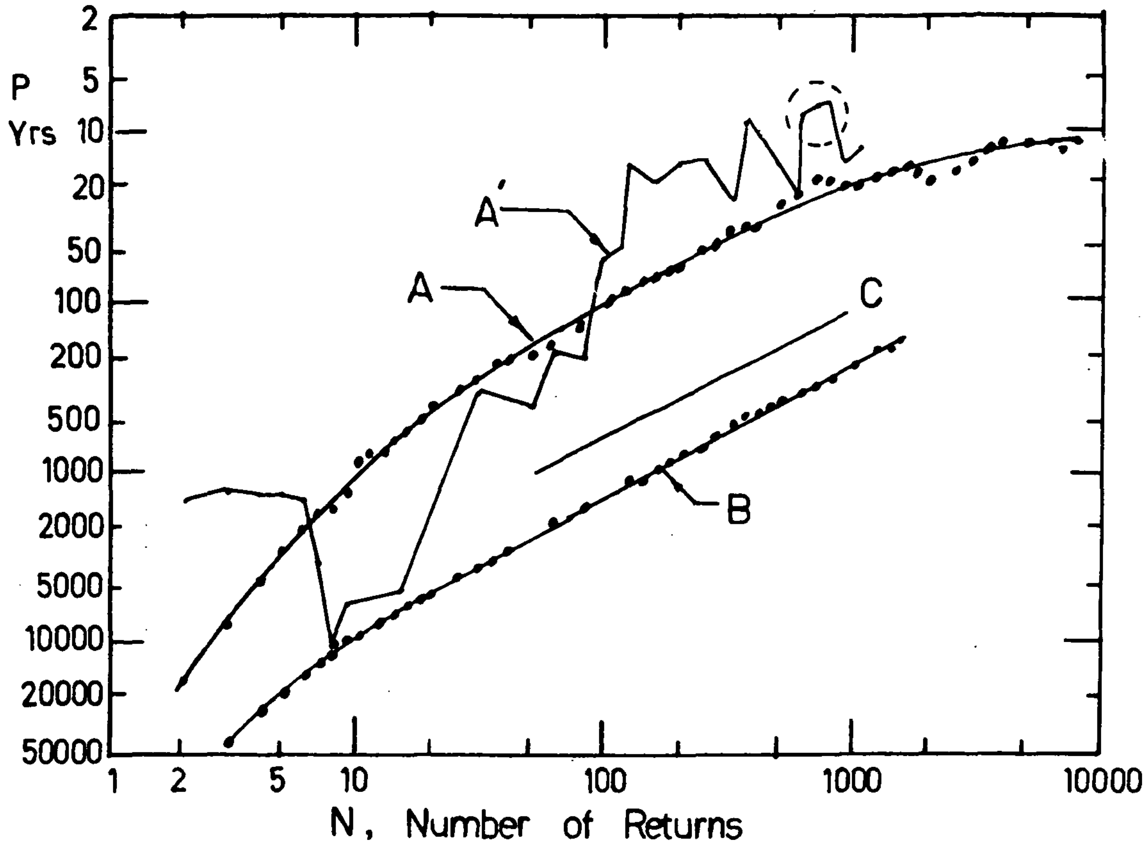

Fig. 3. The average period P is plotted vs the number of returns N. Curve A is for the comets from the capture region, followed up to 8000 returns. The broken line A' is for a particular one of these. It would have been visible as a short period comet of low perihelion distance only between its 784th and 848th return as indicated by the dashed circle. Curve B is for 5997 comets of original perihelion distance $q_o < 4$ AU, followed up to 1600 returns where there were 80 still remaining. The line C indicates the extent to which these curves show a (1/a) dependence on $N^{1/2}$, equivalent to P depending on $N^{-3/4}$.

their orbital parameters, and thus to have an inclination distribution as the sine of the inclination. Accordingly, there would be a very small number of comets with inclination near zero. Delsemme, however, looks at the number of such comets reaching perihelia per unit time, and he concludes, following work by Shteins (1972), that there is a tendency towards a concentration at small inclinations. This is one reason for the different conclusions of the two papers.

Evidently the problem needs more study. As pointed out by Kazimirchak-Polonskaya (1973), the outer planets may be effective in capturing comets of very large perihelia, to 30 AU. Her ideas are borne out by further numerical experiments of the writer (yet unpublished). Extending the capture region in perihelia to 30 AU would enlarge by a factor of 5 the number of long-period comets available for eventual capture to short periods by this mechanism.

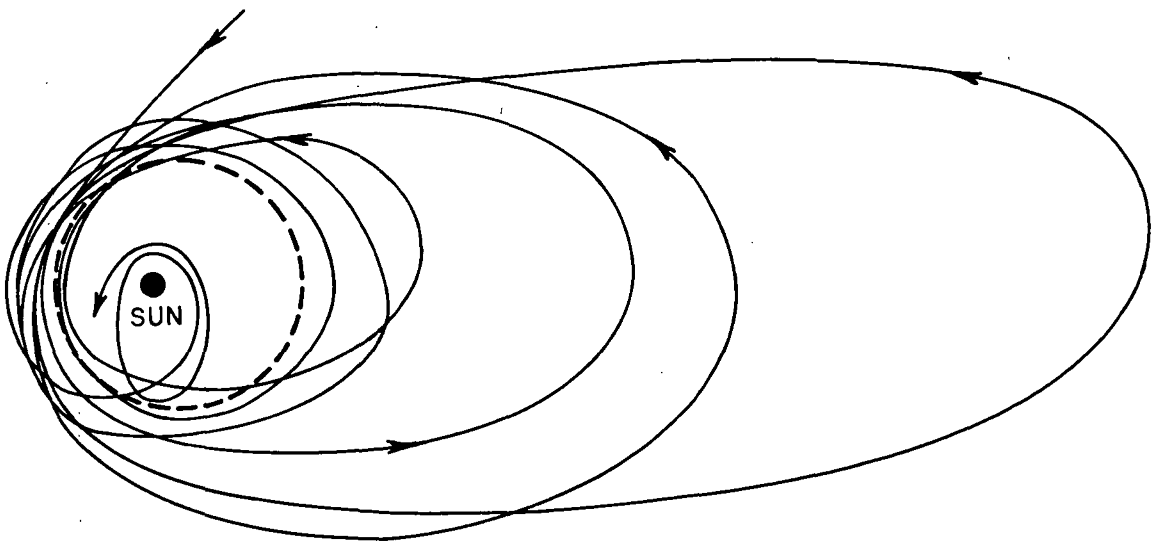

Fig. 4. The dashed line is Jupiter's orbit, and the solid line traces the path of a comet that entered originally on a parabolic path. The evolutionary path is simplified in that such changes would require hundreds of revolutions. The comet has a large perihelion distance except when it is in an orbit like those of the short-period comets.

5. Long-period comets do not originate within the planetary regions of the solar system.

The cloud of comets described by Oort (1950), which is the apparent source of the long-period comets, could not be composed of comets originally created within the orbit of Neptune, if the mechanism for removing them to large distances is that of planetary perturbations. In numerical experiments one can watch the diffusion of 1/a-values for hypothetical comets started within the planetary regions. (Here a is the semimajor axis and 1/a is a measure of the negative energy, positive for ellipses, negative for hyperbolas, and zero for parabolas). Figure 5, reproduced from Everhart (1973b), shows that the number of orbits vs 1/a goes linearly to zero at 1/a = 0. (Any one-dimensional diffusion or random walk problem, such as this one, where there is an absorbing edge, will show a concentration that goes linearly to zero at that edge.) However, this does not agree with the distribution observed for long-period comets, which shows a peak at 1/a = 0. In an experiment, Everhart (1973a, b), starting with circular orbits in the Jupiter-Saturn region, and following many examples for thousands of revolutions, not one orbit typical of an observable long-period comet was found.

It is possible, however, that stellar perturbations on comets very far from the sun would change this conclusion. A study of these effects is in progress by the present author.

6. It is possible that some short-period comets could have originated within the orbit of Neptune.

If one starts a number of hypothetical comets in circular orbits in the Jupiter-Saturn region, a fair number of these are seen to evolve into orbits like those of short-period comets, Everhart (1973b). Thus the distributions of their inclinations and periods are very much like those of the observed comets. However, one also gets the same reasonable-looking distributions if one starts with near-parabolic comets of small inclination with perihelia near Jupiter's orbit.

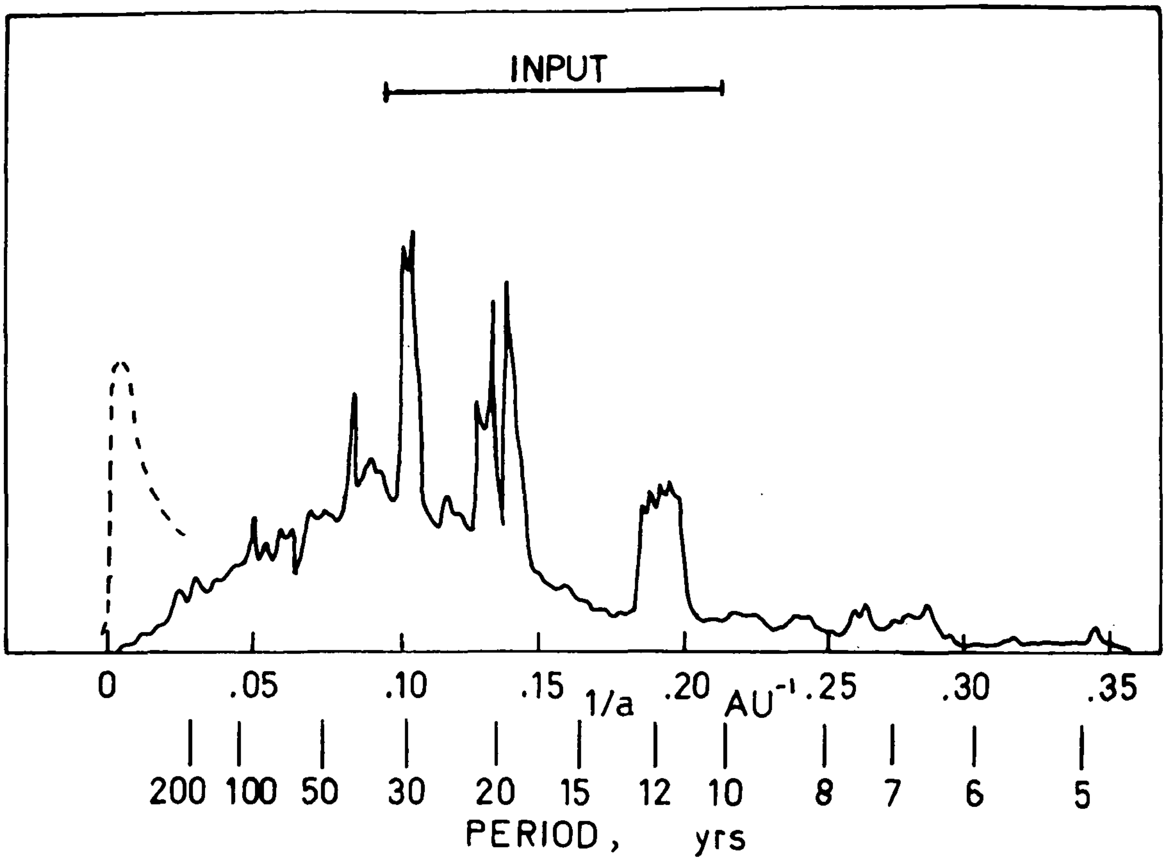

Fig. 5. The distribution of 1/a-values for 100 orbits, each followed for 3000 revolutions. Peaks are seen near Jupiter's period of 11.9 yr, near Saturn's period of 29.5 yr, and between these at the positions of the mid-range orbits. The dashed line near 1/a = 0 is the distribution for known long-period comets.

Whether the fraction of short-period comets originating within the planetary regions is 0% or 99%, we cannot yet say on the basis of orbital evolution studies. We do know that some, if not all, must originate at large distances. Studies such as that of Joss and that Delsemme referred to already, should help decide whether it is necessary to postulate two sources of short-period comets, or whether a single source in a comet cloud at large distances is sufficient

Within the solar system there is a class of orbits that has been called "chaotic orbits", Everhart (1973a,b), as opposed to those in more regular patterns such as Trojans, horseshoes, or librating orbits. When chaotic orbits have small perihelia they resemble orbits of typical short-period comets. The pattern of the chaotic orbits appears to be independent of their previous history or origin.

7. The Jacobi integral (and its approximate forms, such as the Tisserand criterion and the "constant encounter velocity near Jupiter") should not be used in studies of evolution in the solar system.

For the purpose of studying small bodies such as comets, it has been customary to idealize and simplify the solar system retaining only the sun, Jupiter, and the comet in the form of the circularly-restricted problem of 3 bodies. If this were valid then the Jacobi integral could be used in analytic treatments on the origin and evolution of comets. Such papers are easy to write, and there have been dozens of them. Of course, the authors of these papers realize the approximation they are making, but they assume without any proof that it is relatively harmless, and with this approximation they derive simple and far-reaching conclusions. There is a particularly strong incentive to use the Jacobi integral because it is the only conservation equation, and without it an analytic development is difficult, if not impossible.

Unfortunately, numerical experiments with a fairly realistic model of the solar system shows the approximation to be downright wrong.

One example of this: According to the restricted problem there is an absolute barrier such that if the comet's Jacobi quantity is greater than 3.0, then the comet cannot penetrate from a perihelion outside Jupiter's orbit to one inside Jupiter's orbit. However, the exact orbital integrations show that in the course of many hundreds of revolutions a comet can at times have its perihelion well inside Jupiter's orbit and at other times its perihelion outside not only Jupiter's orbit but also outside Saturn's orbit. The corresponding values of the Jacobi quantity range from 2.8 to 3.6. It is just plain wrong to assume that a comet now in a short-period orbit originally entered the solar system with about the same Tisserand constant that it now has. Figure 6, reproduced from Everhart (1973b), shows the large and frequent changes in C_J, the Jacobi quantity referred to Jupiter, in the course of 3000 revolutions. (These changes are not due to inaccuracies in the numerical integration. When the mass of Saturn was set to zero, and Jupiter's orbit was made circular, then C_J was found to be constant to within one part in 10^5 in the course of 1000 revolutions.) Some of the variations in Figure 6 occur because Jupiter's orbit is not circular, but the major and sudden changes in this Jacobi quantity are caused by Saturn. In the paper cited above it is shown that there is an approximate relationship between the changes in C_J and the change in heliocentric energy caused by Saturn.

I hope I have persuaded readers not to write, and not to believe, simple discussions of the evolution of comets based on the restricted problem. Classification of comets according to their Tisserand constant cannot be valid, and developments based on a constant encounter velocity of comets at Jupiter's sphere of influence are not on a good foundation.

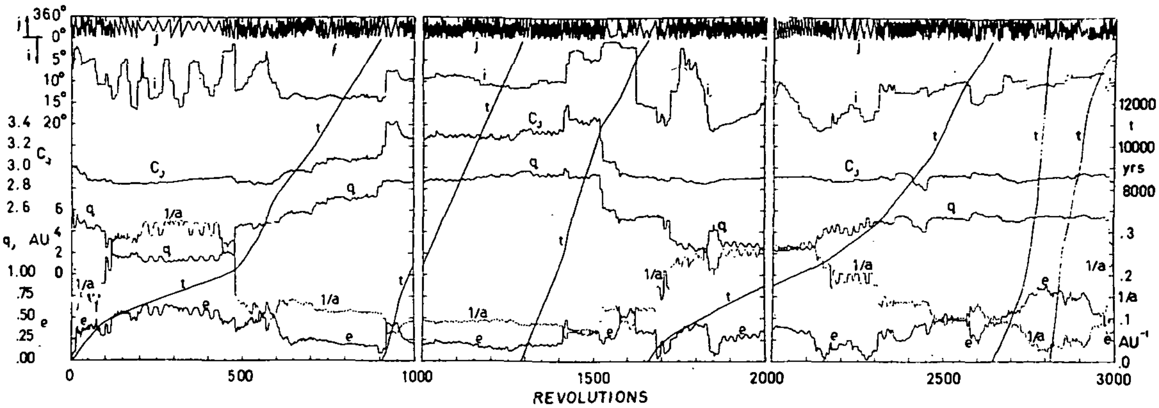

Fig. 6. A detailed history of the orbit of one hypothetical comet integrated for 3000 revolutions. The solar system model included Jupiter and Saturn, both in elliptical orbits. Here i is the inclination, C_J is the Jacobi quantity referred to Jupiter, q is the perihelion distance, e is the eccentricity, and 1/a measures the negative energy. The sloping line labeled t measures time, repeating its traverse every 15000 years. The line j is a comet-sun-planet angle discussed in the paper from which this figure is taken, Everhart (1973b).

Note that C_J varies between 2.8 and 3.6, and that q varies between 1.5 AU and 10 AU.

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DISCUSSION

B. G. Marsden: I am glad that you qualified your original point 5, for while your calculations represent a great step forward in our understanding of the way short-period comets evolve, I don't think we can use them to distinguish between the possibilities that comets originate at the extreme outskirts of the solar system or just beyond the orbit of Neptune. I must agree that the existence of the Oort cloud, particularly now that Sekanina and I have considerably refined the extent of the region from which "new" comets appear to have come, makes the idea of an origin at almost interstellar distances very attractive. But at the same time, when one considers that there are so many really spectacular comets of aphelion distance only a couple of hundred astronomical units—comets like Bennett, Donati, and the Kreutz sungrazers—one does rather wonder if some of them perhaps ejected near Neptune and were never out in the Oort cloud.

B. Lowrey: I feel that comments 2, 3, and 7 are overstated or require modification. While it is true that the Jacobi integral varies in the solar system, it is useful to study it as an evolutionary parameter. In particular, my use of the encounter velocity u (related to Tisserand's constant) in a recent paper (June 1973 A.J.) showed that the short period comets divided into two classes—those of high velocity and those of low velocity. The high velocity short period comets appeared to relate closely to the long period comets, and the low velocity ones did not. This use of the Jacobi integral therefore suggests a more detailed examination of orbit-element distributions to see if the high velocity short period comets compare with the long period comets.

D. Yeomanns: As you know, the aphelion distance of comet Encke is 4.1 A.U. The non-gravitational acceleration of Comet Encke's mean motion has been suggested as a possible mechanism for the evolution of Encke's aphelion distance within Jupiter's orbit. In your investigation of the evolution of parabolic orbits with high perihelia into short period comets of low perihelia, did you find any examples of short period comets whose aphelia were inside Jupiter's orbit?

E. Everhart: No, not at all; I found that did not happen. On the other hand, I am a little bit dubious about the non-gravitational effects having that effect on Comet Encke, because Marsden has calculated this thing for a number of apparitions backward in time, and I could detect no systematic change in its energy or its aphelia over a period of time. If I had to guess what caused Comet Encke, I would say it was an encounter with Earth or Venus, simply because that would and could bring it in closer.

B. G. Marsden: The smallest aphelion distance I know of produced by entirely gravitational means from a comet that was originally on the outside is 4.5 astronomical units, and this was in the case of Comet Oterma. That happened to be perturbed by Jupiter into a nice resonance orbit, at 3 to 2 resonance with Jupiter, although it's hard to see that the 3 to 2 resonance had anything to do with where the comet was thrown out. It was at least thrown into that 4.5 A. U. aphelion; I suppose this is rather close to the limits that one can do by gravitational means.

B. Jambor: Given a comet in a near parabolic orbit and releasing many small particles, all in hyperbolic orbits, is it possible that these hyperbolic orbits could be thrown into elliptical orbits by one encounter with Jupiter (or another planet)?

E. Everhart: I think it's possible, but it is very unlikely. The reason why we don't make comets in a single encounter is that the scale of influence around Jupiter that would do it is so small it has to be a cumulative group of small encounters.

D. A. Mendis: While it is clear that with the restricted 3-body problem one cannot get comets with initial low relative velocity thrown out of the solar system, repeated encounters with an elliptic, precessing and changing orbit could energize the comet towards equipartition (as in the Fermi process) and ultimately throw it out. This was first pointed out, I believe, by Opik, and the earliest numerical calculations in support were done in 1965 by Arnold.

E. Everhart: Yes, in fact, I find this without using the restricted problem at all, by simply doing an exact calculation. A particle which is well bound to the solar system with not nearly enough energy to leave, sooner or later will be thrown out by repeated encounters with Jupiter. That was the second figure, showing 80 objects in circular orbits, and by 3000 revolutions some 20 of them had already been thrown out of the solar system.

S. Vaghi: I understand that for your question concerning the Jacobi integral, your conclusions were derived from an experiment, a very special experiment, concerning only Jupiter and Saturn, which is very far from the approximation of the 3-body problem. You know, perhaps, that in '72 a paper was published by Kresak, concerning the use of the Jacobi integral as a classificational and evolutionary parameter for comets and asteroids. I would like to know where he was mistaken.

E. Everhart: That's one of the papers I am objecting to. I didn't mention any names, but it's simply that, for something like an asteroid, which doesn't get very far away from its home base, this might be all right, but for something like a comet, I don't think so. The comet can range all over its classifications at various times in its orbital evolution. That's one of the papers I would say is not valid.

G. Wetherill: I think you overstated the case against the use of the Jacobi integral in discussions of orbital evolution. If you recall your own figure, in which the evolution of the various orbital parameters is shown, you will find that the fluctuation from the mean value of the Jacobi integral is only 10-20 percent, whereas other quantities, such as the semi-major axis and the eccentricity, change by large factors. It would be better if you were to say that the Jacobi integral should not be misused rather than saying that it should not be used. Actually it is quite useful to follow the random walk of the Jacobi integral as well as that of other quantities, such as 1/a and e.

One way in which this is useful in is distinguishing phenomena which are essentially dependent on the eccentricity and inclinations of the planetary orbits from those which still would occur to about the same extent in a more simple solar system and therefore are not critically dependent on assumptions concerning the constancy of the present values of the eccentricity and inclination of the planets. When this point of view is taken, it turns out most of your conclusions would also be valid in a much simpler solar system. In contrast, one phenomenon which is essentially dependent on failure of the restricted 3-body problem is that of the evolution of nearly circular orbits into hyperbolic escape orbits. This was understood and discussed by Arnold in his Monte Carlo work published in the Astrophysical Journal in 1965. For this reason, as well as others, a discussion of the possible evolution into the Oort cloud of a comet initially in a near-circular orbit in the Jupiter-Saturn region does not say very much about the equivalent problem in the region of Uranus and Neptune.

The other comment I would like to make concerns the previous work regarding Comet Encke. I have carried out Monte Carlo calculations for short period comets in which the perturbation of Earth and Venus as well as those of Jupiter are included. It turns out that it is very difficult to reduce the aphelion to that of Encke by Earth or Venus perturbations on a time scale of 10^3 years. Such changes are found on a time scale of 10^6 years but are very improbable for a comet young enough to still be active. It is much more likely that Encke's present orbit is a consequence of non-gravitational forces.

E. Everhart: A comment about the Jacobi integral, I was stating that if you regard it as a time-varying quantity whose instantaneous value tells you something about the current orbit, then I will agree it's properly used, but that's not the way many people have used it.