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ORBITAL ERROR ANALYSIS FOR COMET ENCKE, 1980

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Several recent studies have been undertaken to optimize mission strategies and to select appropriate instrumentation for in situ studies of short period comets (Farquhar et al, 1974; Bender, 1974; Newburn, 1973; Meissinger, 1972; Roberts, 1971). Although some studies have contrasted the physical characteristics of several proposed target comets, few have comprehensively studied the orbital history and ephemeris uncertainties of target comets. In general, the navigational accuracy of cometary flyby probes is almost entirely dependent upon the target comet's position uncertainty at the time of intercept. Although cometary error analyses are necessary for realistic mission planning, such analyses cannot be conducted in the standard fashion. Comets are affected by nongravitational forces (Marsden et al, 1973), they occasionally exhibit slight discontinuities in their orbital motions, and at least one comet (Biela) has completely disintegrated (Marsden and Sekanina, 1971). Each comet is an individual. Comets have steadfastly resisted recent attempts at classification. Hence, it seems clear that, for each comet of interest in mission planning, a separate in-depth error analysis study must be undertaken to realisticly determine the target comet's ephemeris uncertainty at the time of intercept. Such studies should consider a number of criteria in order to assure accurate ephemerides for prospective cometary targets. Using the 1980 apparition of comet Encke as an example, these criteria are outlined below.

CRITERIA FOR ACCURATE COMETARY EPHEMERIDES

1. The target comet should have good observability during the apparition of the proposed intercept.

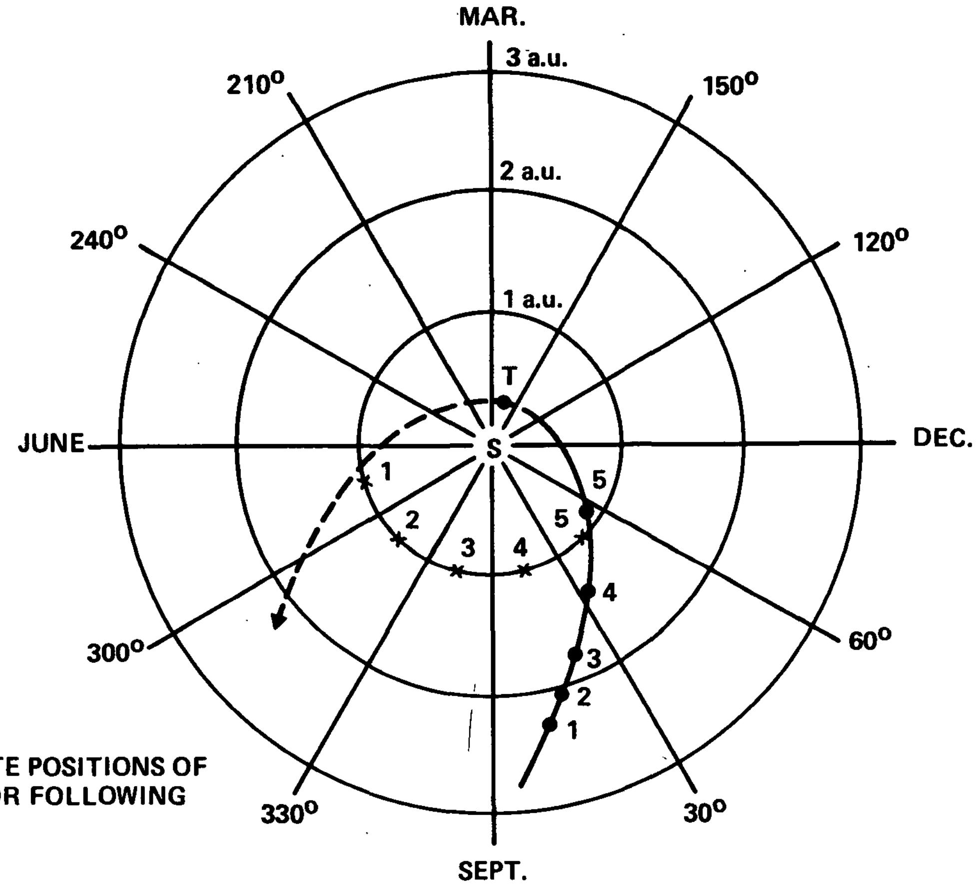
   Ground based observations made prior to an intercept of a comet are critically important for reducing cometary ephemeris uncertainties. However, for many cometary mission opportunities, recovery of the target comet prior to spacecraft launch is not necessary. Provided the target comet is recovered early enough, spacecraft thrusters are fully capable of removing a priori cometary ephemeris errors with midcourse maneuvers. Fortunately, for target comets that are recovered approximately three months prior to intercept, ephemeris corrections up to 0.3 days can be removed with midcourse maneuvers (Farquhar, 1975). For well observed short period comets, modern ephemeris predictions have never required a correction of this size. Naturally, the recovery of a comet, particularly an erratic comet, prior to launch would minimize spacecraft energy expenditures.

At a particular time, a comet's uncertainty in position can be represented by an error ellipsoid whose semi-major axes (σ_r, σ_n, σ_T) are directed in a radial Sun-comet direction ($\hat{r}$), normal to the orbit plane ($\hat{n} = \hat{r} \times \hat{v}$) and transverse to the orbit plane ($\hat{T} = \hat{n} \times \hat{r}$). In the absence of observations, the error ellipsoid will evolve dynamically. In general, the a priori error ellipsoid component σ_r will reach a maximum value for a true anomaly (ν) of $\pm$ 90 degrees, when the radial velocity is a maximum. The transverse velocity is a maximum at perihelion ($\nu = 0^o$) so that the a priori transverse component (σ_T) is a maximum there. Hence, an ideal observing schedule would include observations made at a phase angle of 90^o when the comet's $\nu = \pm 90^o$, as well as observations made at phase angles of 0^o, 180^o when the comet is at perihelion ($\nu = 0^o$). In a sense, this ideal observing schedule would allow a direct "view" of the largest radial and transverse error components. In addition to observations made at optimum phase angles, the observer-comet distance (range) at the time of the observation is important in reducing a comet's ephemeris uncertainty. For a particular angular position error, the linear position error perpendicular to the line-of-sight is directly proportional to the range. Also, as the range decreases, the relative parallactic displacement increases; in general, the accuracy of a cometary orbit will be enhanced if the relative Earth-comet motion is large.

Figure 1 clearly shows the excellent observability of comet Encke during its 1980 apparition. Comet Encke will be easily visible to Earth based observers for approximately four months prior to perihelion. From July through October (positions 1 - 4 on figure 1), the comet's range decreases from 2.4 to 0.3 A.U. The minimum range is 0.28 A.U. on October 29, 1980 when the relative Earth-comet motion is large. The late October observations are made at a phase angle near 90^{o}. Since the true anomaly at this time is approximately -90^{o}, the radial component of the comet's error ellipsoid is aligned nearly perpendicular to the line-of-sight. Hence the late October and early November observations are critical for minimizing the radial position error of comet Encke in 1980.

2. The target comet should have a good observational history.

Accurate orbit determination is dependent upon the number, quality and distribution of observations. The most accurate orbits are computed using consistent observations spread uniformly over a large range of a comet's true anomaly. An accurate determination of a comet's mean motion and nongravitational parameters requires a linkage of at least three apparitions. The resultant "observed minus computed" residuals in the right ascension and declination provide an indication of an orbit's accuracy. In general, the time intervals used in orbit determination are long enough to accurately determine the nongravitational parameters and short enough so that the unmodeled time dependence in the nongravitational accelerations cannot degrade the residuals. An orbit is considered successful only if there are no systematic trends in the residuals. Although the mean of the absolute values of the residuals is usually somewhat higher for the right ascension, they are close enough so that the measurement errors in right ascension and



NUMBERS 1–5 INDICATE POSITIONS OF EARTH AND COMET FOR FOLLOWING DATES

1. 1980 JULY 9
2. 1980 AUG. 8
3. 1980 SEPT. 7
4. 1980 OCT. 7
5. 1980 NOV. 6

T: PERIHELION OF COMET (1980 DEC. 6)
X: POSITIONS OF EARTH
●: POSITIONS OF COMET

Figure 1. Earth–Comet Encke Relative Geometry in 1980

declination can be considered equal. These residuals are primarily due to position errors in the comparison stars, deviations of the comet's center of light from its center of mass, and modeling errors in the nongravitational accelerations. For twentieth century observations of periodic comets, the means of the absolute values of the residuals range from one to four arc seconds.

Among short period comets, the observational history of comet Encke is unexcelled. Since 1819, comet Encke has only been missed at one apparition (1944). It is the only short period comet passing within the Earth's orbit that has been seen at aphelion (Roemer, 1972). Marsden and Sekanina (1974) have analyzed the orbital motion of comet Encke from its discovery in 1786 until 1971. Differential corrections were generally made over 13 year intervals and, for recent apparitions, the means of the absolute values of the residuals were in the range 1.5-3 arc seconds.

STATISTICAL ERROR ANALYSIS FOR COMET ENCKE (1980)

A statistical covariance error analysis was undertaken to determine the evolution of comet Encke's error ellipsoid during the 1980 apparition. The computer program took into account planetary perturbations and considered the errors inherent in the values for the nongravitational parameters and initial conditions. The partial derivatives utilized in the conditional equations matrices and the state transition matrices were computed numerically.

Marsden and Sekanina (1974) have shown that five apparitions of comet Encke can be linked before the secular decrease in the nongravitational parameters begins to degrade the residuals. For the present analysis, the 5 returns to perihelion (1967-1980) are represented by forty actual observations from August 2, 1967 through October 24, 1973 and by 28 additional, postulated observations from October 24, 1973 through November 16, 1980. One observation was processed at each of the 1978 and 1979 opposition dates and the 1980 recovery of the comet was assumed to occur on July 9. The postulated observation schedule was determined after considering the relative Sun-Earth-comet positions, the available hours of dark observing time as well as the apparent nuclear and total magnitudes for various dates.

The error analysis was initialized in 1967 and the initial a priori 8 x 8 covariance matrix was essentially infinite. Each set of observations was batch processed and the updated covariance was propagated forward in time via the state transition matrix to the date of each observation. The time history of the comet's error ellipsoid is presented in Table 1. The first column represents the dates in 1980

Table 1
Error Ellipse Components for Comet Encke (1980)

Date 1980-81	A priori errors* (in km)			1980 observations Processed** (in km)						
	σ_r	σ_n	σ_T	σ_r	σ_n	σ_T	Δ (a.u.)	r	θ (deg.)	Comments
July 9	4168	2130	3239	3352	1926	2471	2.43	2.33	72	Comet recovered
19	4338	2084	3275	2917	1737	2012	2.21	2.23	78	
29	4521	2036	3327	2572	1567	1683	1.98	2.13	84	
Aug. 8	4718	1985	3399	2271	1406	1426	1.76	2.03	90	
18	4933	1936	3494	1992	1249	1213	1.53	1.92	96	
28	5169	1894	3617	1644	1146	1026	1.31	1.81	101	
Sept. 7	5429	1876	3769	1469	945	836	1.10	1.69	107	
17	5717	1921	3952	1217	799	710	0.89	1.56	111	
27	6040	2117	4146	968	658	564	0.69	1.43	114	
Oct. 7	6403	2604	4301	724	524	427	0.51	1.28	112	
17	6811	3480	4347	504	400	313	0.36	1.13	103	
27	7258	4612	4407	387	308	264	0.28	0.97	77	
Nov. 6	7699	5595	5229	391	269	273	0.32	0.80	45	true anomaly = -90° on Nov. 15
16	7893	5683	8023	416	249	359	0.47	0.62	29	last comet observation
26	6628	4481	12910	401	234	579	0.70	0.44	23	
Dec. 6	399	3477	16833	171	243	874	1.00	0.34	20	perihelion on Dec. 6.6
16	6663	3445	13010	315	289	827	1.30	0.42	15	
26	7880	3452	8789	418	359	688	1.52	0.60	11	true anomaly = +90° on Dec. 27
Jan. 5	7627	2946	6527	433	412	632	1.73	0.78	10	
15	7143	2150	5388	426	445	642	1.90	0.95	11	
25	6663	1432	4706	414	481	677	2.05	1.11	12	

*A priori, one-sigma errors (km) in the radial, normal and transverse directions. Last observation processed was mid-September, 1979.

**Evolution of one-sigma errors (km) if one ground based observation is processed at 10 day intervals from July 9 to November 16. Measurement noise = 3 arc seconds.

on which one simulated ground based observation was made. The next six columns represent the 1-σ position errors (km) for the radial Sun-comet direction ($\hat{r}$), the direction normal to the comet's orbital plane ($\hat{n}$), and the transverse direction defined by the cross product of the first two unit vectors ($\hat{T} = \hat{n} \times \hat{r}$). The columns headed by Δ, r and θ represent the Earth-comet distance in A. U., the Sun-comet distance in A. U. and the Sun-Earth-comet angle in degrees. The a priori errors represent the forward propagation of the covariance matrix obtained by processing all observations from 1967-1979. Columns 5, 6, and 7 reflect the effect of each 1980 observation on the comet's error ellipsoid. The final ground based observation on November 16 reduces the σ_r, σ_n, and σ_T components to 416, 249, and 359 km. In the absence of further observations, the error components evolve dynamically; their magnitudes at any given time are due primarily to the comet's position in its orbit. The exclusion of the first few recovery observations in 1980 or the exclusion of the 1978 and 1979 opposition observations has a negligible effect upon the position errors in 1980. However, by taking 1980 observation at 5-day intervals between July 9 and November 16, the errors on December 6th are reduced to 155, 186, and 660 km (σ_r, σ_n, σ_T). These results underscore the fact that, while observation made during past apparitions define the mean motion and the nongravitational parameters, it is the 1980 observations that contribute most strongly to the reduction of comet Encke's ephemeris uncertainty. The importance of the 1980 observations is due primarily to the proximity of comet Encke and the Earth during October and November, 1980.

The present error analysis of comet Encke assumes a 1-σ observational error of 3 arc seconds for both the right ascension and declination. The 3 arc second value is consistent with the mean residuals obtained from various orbit determinations for past apparitions of comet Encke. Due to comet Encke's relatively high nuclear activity, the appropriate error value is somewhat higher than for most other short period comets. The assumed error for each observation is the same value, and the observations themselves are assumed to be uncorrelated. This being the case, the only nonzero elements ($1/\sigma^2$) of the weighting matrix (W) are equal in value and aligned on the principal diagonal. If F denotes the conditional equation matrix, the normal matrix $F^T WF$ can be reduced to $1/\sigma^2 (F^T F)$ and the simplified covariance matrix becomes $\sigma^2 (F^T F)^{-1}$. Thus the covariance matrix is linear with respect to observational errors. For example, although the current analysis has been undertaken using an observational error of $\sigma = 3$ arc seconds, one only has to multiply the error component entries in Table 1 by 2/3 to obtain the results for $\sigma = 2$ arc seconds.

CHECKS UPON STATISTICAL ERROR ANALYSIS FOR COMET ENCKE (1980)

A less rigorous statistical error analysis of the 1980 apparition of comet Encke has been carried out by Bynes and Boain (1974). For comparable cases, their results agree with the present analysis. However, a statistical error analysis is only as good as its underlying assumptions. For the most part, the statistical error analysis outlined in the preceeding section was based upon simulated or hypothesized observations. In an effort to check the statistical results, it seems prudent to analyze results obtained using actual observations of comet Encke.

The observations of comet Encke from 1937-1973 have been used in five separate differential corrections. Within the given mean errors, comparable orbits agree with those determined by Marsden and Sekanina (1974). The entries in Tables 2 and 3 represent an attempt to compare observed and predicted times of perihelion passage (Table 2) and perihelion distances (Table 3). In each table, the first column represents the observed time interval over which a particular differential correction was made. Columns 2-12 give the times of perihelion passage (Table 2) and perihelion distances (Table 3) for each particular interval. For example, in Table 2, line 1, columns 2-6 give the observed times of perihelion

Table-2
Comet Encke
Observed and Predicted Times of Perihelion Passage

Observed Interval	1947 Nov.	1951 March	1954 July	1957 Oct.	1961 Feb.	1964 June	1967 Sept.	1971 Jan.	1974 Apr.	1977 Aug.	1980 Dec.	1984 Mar.
1947-1961	26.32771	16.20639	2.51788	19.84902	5.59452	3.48639	22.04696	9.95477				
1951-1964		16.20842	2.51807	19.84869	5.59494	3.48951	22.05469	9.96845	28.97037			
1954-1967			2.51998	19.84886	5.59500	3.49099	22.05903	9.97727	28.98501	16.97977		
1957-1971				19.84937	5.59419	3.49046	22.06044	9.98221	28.99491	16.99603	6.55148	
1961-1974					5.59477	3.48950	22.05935	9.98232	28.99754	17.00248	6.56310	27.65721

Table 3
Comet Encke
Observed and Predicted Perihelion Distances (in A.U.)

Observed Interval	1947 Nov.	1951 March	1954 July	1957 Oct.	1961 Feb.	1964 June	1967 Sept.	1971 Jan.	1974 Apr.	1977 Aug.	1980 Dec.	1984 March
1947-1961	0.3410319	0.3380089	0.3384086	0.3381187	0.3390047	0.3392511	0.3381997	0.3388910				
1951-1964		0.3380127	0.3384123	0.3381221	0.3390085	0.3392561	0.3382050	0.3388965	0.3381216			
1954-1967			0.3384116	0.3381211	0.3390075	0.3392548	0.3382038	0.3388953	0.3381204	0.3406577		
1957-1971				0.3381260	0.3390125	0.3392600	0.3382089	0.3389005	0.3381256	0.3406628	0.3399416	
1961-1974					0.3390115	0.3392593	0.3382082	0.3388998	0.3381250	0.3406622	0.3399411	0.3410024

passage for 1947, 1951, 1954, 1957, and 1961 while the remaining times of perihelion passage (columns 7-9) are predicted (extrapolated) beyond the range of observations (1947-1961). Carrying this example further, the first predicted time of perihelion passage (1964 June 3.48639) is compared with the entry directly below it (1964 June 3.48951) which is the observed, or actual, time of perihelion passage in 1964. Strictly speaking, any of the 4 times listed below the 1964 June 3.48639 date is an observed time of perihelion passage in 1964; they all are within their respective observation intervals. By comparing each predicted time of perihelion passage with the observed times of perihelion passage, a systematic correction is noted whereby the predicted and observed times of perihelion passage can be brought into agreement. This empirical correction and its standard deviation is

$$\bar{\Delta}T = +0.00423 \pm 0.00094 \text{ days}$$

This empirical correction ($\bar{\Delta}T$) is required to allow for the decrease in $|A_2|$. In other words, by not mathematically modeling this decrease in $|A_2|$, each predicted time of perihelion passage is underestimated by 0.004 days. In a similar fashion, the empirical corrections to the time of perihelion passage required for predicting 2 and 3 apparitions ahead are +0.013 and +0.03 days respectively. These empirical corrections for predicting 1, 2, and 3 apparitions ahead (+0.004, +0.013, +0.03 days) are similar to the values (+0.005, +0.015, +0.03 days) obtained by Marsden and Sekanina (1974). We can take $\bar{\Delta}T$ as an approximate upper limit to the a priori uncertainty in the transverse position error at perihelion. The comet's velocity at perihelion is approximately 6×10^6 km/day so that, at perihelion, $\bar{\Delta}T$ corresponds to a linear, transverse, position error of 25,380 km. However, the majority of this error is due to the unmodeled secular decrease in the transverse nongravitational acceleration. An empirical $\bar{\Delta}T$ correction can be added to the predicted time of perihelion passage to greatly reduce this error so that the standard deviation of $\bar{\Delta}T$ can be utilized as an approximate lower bound to the a priori, transverse position error at perihelion. At perihelion, this standard deviation (0.00094 days) corresponds to an approximate linear, transverse position error of 5,640 km. In a sense, the upper and lower limits on σ_T at perihelion are "observed" because they are based upon past prediction accuracies of comet Encke's times of perihelion passage (Table 2). From Table 1, the statistical, a priori, transverse, position error at perihelion (1980 December 6) is 16,833 km., a result that is bounded by the aforementioned "observed" upper and lower limits.

Unlike the $\bar{\Delta T}$ corrections, the corrections (Δq) required to bring predicted perihelion distances into agreement with the observed perihelion distances for comet Encke are not predictable. However an estimate of the "observed" upper and lower limit can be determined from the maximum and minimum values of Δq (determined from Table 3). These values are (8.9-0.6) x 10^{-6} A.U. or 1335-90 km. These "observed" errors bound the statistical, a priori radial, position error at perihelion (399 km from Table 1).

From the statistical error analysis, the radial and transverse position errors after all 1980 observations have been processed are σ_r = 171 km and σ_T = 874 km for 1980 December 6 (see Table 1). These position errors correspond to an error in perihelion distance of 1.1 x 10^{-6} A.U. and an error in perihelion passage time of 1.5 x 10^{-4} days. These results are compatible with the standard deviations associated with the differential corrections to the perihelion distance and perihelion passage time. For example, the orbital solution over the 1961-1973 observations yields a standard deviation of 0.94 x 10^{-6} A.U. and 1.07 x 10^{-4} days for the differential corrections to the perihelion distance and perihelion passage time.

SUMMARY

Since the ephemeris uncertainties of proposed target comets determine the navigational accuracy of cometary flyby space probes, each proposed target comet should be thoroughly investigated to determine its position error at encounter. Before a particular comet is selected as a flyby target, the following criteria should be considered in determining its ephemeris uncertainty:

1. A target comet should have good observability during the apparition of the proposed intercept. The following conditions aid in minimizing a comet's positional uncertainty at encounter:
   a. Perhaps more than any other condition, observations made at small range values substantially reduce the target comet's error ellipsoid.
   b. Ideal observations would include those made at a phase angle of 90°, when a comet's true anomaly is $\pm 90^{\circ}$ and those made at phase angles of 0° and 180°, when the comet's true anomaly is 0°.
   c. If the comet is observable at the proper time, a large parallactic displacement between the Earth and the target comet prior to encounter would allow a reduction in all three error ellipsoid axes.
2. A target comet should have a good observational history. Several well observed and consecutive apparitions allow an accurate determination of a comet's mean motion and nongravitational parameters.

Using these criteria, along with statistical and empirical error analyses, it has been demonstrated that the 1980 apparition of comet Encke is an excellent opportunity for a cometary flyby space probe. For this particular apparition, a flyby to within 1,000 km. of comet Encke seems possible without the use of sophisticated and expensive on-board navigation instrumentation.

REFERENCES

Byrnes, D. V., and Boain, R. J. (1974). "An Integrated Approach to Comet Mission Navigation Utilizing Astronomical Observations of the Target Body," American Institute of Astronautics and Aeronautics (AIAA) paper No. 74-849.

Bender, D. F. (1974). "Encke Ballistic Flybys in 1980," AIAA paper No. 74-782.

Farquhar, R., McCarthy, D. K., Muhonen, D. P., and Yeomans, D. K. (1974). "Mission Design for a Ballistic Slow Flyby of Comet Encke 1980," NASA TN D-7726.

Farquhar, R. (1975). "Mission Strategy for Cometary Exploration in the 1980's," this colloquium.

Marsden, B. G. and Sekanina, Z. (1971). Astron. J. 76, 1135.

Marsden, B. G. and Sekanina, Z. (1974). Astron. J. 79, 413.

Marsden, B. G., Sekanina, Z., and Yeomans, D. K. (1973). Astron. J. 78, 211.

Meissinger, H. F. (1972). "Study of a Comet Rendezvous Mission," TRW Technical Report 20513-6006-R0-00.

Newburn, R. L. (ed.) (1973). "Science Rationale and Instrument Package for a Slow Flyby of Comet Encke", J. P. L. Document 760-90.

Roberts, D. L. (ed.) (1971). "Proceedings of the Cometary Science Working Group. Yerkes Observatory, June, 1971," IIT Research Institute Document.

Roemer, E. (1972). Mercury Nov./Dec., 1972, p. 19.

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