NASA TECHNICAL MEMORANDUM

NASA TM X-3370

NASA TM X-3370

ONE-DIMENSIONAL NUMERICAL ANALYSIS OF THE TRANSIENT THERMAL RESPONSE OF MULTILAYER INSULATIVE SYSTEMS

Claud M. Pittman and Kay L. Brinkley

Langley Research Center
Hampton, Va. 23665

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1976

1. Report No. NASA TM X-3370	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ONE-DIMENSIONAL NUMERICAL ANALYSIS OF THE TRANSIENT THERMAL RESPONSE OF MULTILAYER INSULATIVE SYSTEMS		5. Report Date April 1976
		6. Performing Organization Code
7. Author(s) Claud M. Pittman and Kay L. Brinkley		8. Performing Organization Report No. L-10637
		10. Work Unit No. 506-16-43-02
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23665		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

A one-dimensional numerical analysis of the transient thermal response of multilayer insulative systems has been developed. The analysis can determine the temperature distribution through a system consisting of from one to four layers, one of which can be an air gap. Concentrated heat sinks at any interface can be included. The computer program based on the analysis will determine the thickness of a specified layer that will satisfy a temperature limit criterion at any point in the insulative system. The program will also automatically calculate the thickness at several points on a vehicle and determine total system mass.

17. Key Words (Suggested by Author(s))		18. Distribution Statement	
Heat transfer Numerical analysis Insulative systems		Unclassified – Unlimited Subject Category 34	
19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	46	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

ONE-DIMENSIONAL NUMERICAL ANALYSIS OF THE TRANSIENT THERMAL RESPONSE OF MULTILAYER INSULATIVE SYSTEMS

Claud M. Pittman and Kay L. Brinkley
Langley Research Center

SUMMARY

A one-dimensional numerical analysis of the transient thermal response of multilayer insulative systems has been developed. The analysis can determine the temperature distribution through a system consisting of from one to four layers, one of which can be an air gap. Concentrated heat sinks at any interface can be included. The computer program based on the analysis will determine the thickness of a specified layer that will satisfy a temperature limit criterion at any point in the insulative system. The program will also automatically calculate the thickness at several points on a vehicle and determine total system mass.

INTRODUCTION

Some form of analytical tool is necessary for designing thermal protection systems (TPS) for high-speed aerospace vehicles. Numerous analyses have been developed (refs. 1, 2, 3, and 4, for example); however, these analyses are relatively complex and difficult to use and have large computer time and storage requirements. These analyses were developed to analyze materials systems such as ablative or transpirative heat shields. With the advent of projects to develop reusable hypersonic vehicles, such as the hypersonic airplane and the space shuttle, interest has shifted to a much simpler reusable TPS which does not require such complicated analyses.

System synthesis computer programs, which determine optimum aerodynamic and structural configurations for high-speed vehicles, are also being developed. Because the TPS is usually a significant part of the total vehicle mass, an efficient TPS computer program is needed which calculates changes in total heat shield mass with changes in vehicle configuration and which can be used as a subprogram within the vehicle optimization program.

This paper describes the development and use of a one-dimensional, multilayer insulative TPS computer program. Both the differential and finite-difference equations are given. The computer program input quantities are listed and the computer program output for a sample case is presented. The surface environment, initial temperature distribution, and material properties must be specified.

SYMBOLS

Any consistent set of units can be used in this analysis.

C	heat capacity of heat sink, $\left(\rho c_p t\right)_{\text{heat sink}}$, J/m^2-K
c_p	specific heat, J/kg-K
h_e	total enthalpy at edge of boundary layer, J/kg
h_w	local enthalpy of fluid at front surface temperature, J/kg
h^*	effective heat transfer coefficient, W/m^2-K
h_f^*	effective heat transfer coefficient to fluid at back surface, W/m^2-K
h_g^*	effective heat transfer coefficient at air gap, W/m^2-K
k	thermal conductivity, W/m-K
q_C	cold-wall convective heating rate, W/m^2
q_R	radiant heating rate, W/m^2
T	temperature, K
T_b	temperature to which back surface radiates, K
T_f	temperature of fluid at back surface, K
t	heat sink thickness, m
x	coordinate normal to surface
x_j	thickness of jth layer, m

α absorptance of front surface

ϵ effective emittance

ϵ_b effective emittance of back surface

ϵ_g effective emittance at air gap

ϵ_s effective emittance of front surface

ρ density, kg/m^3

σ Stefan-Boltzmann constant, W/m^2-K^4

τ time, sec

Subscripts:

1,2,3 first, second, and third stations or layers

b back surface

g air gap

i interface

j layer

L last station

n integer

s front surface

In the finite-difference equations, primed quantities are evaluated at the end of the time step. Double primed quantities are functions of temperature but are evaluated at the end of the time step and therefore require iteration.

GENERAL DESCRIPTION

This analysis was developed primarily to analyze radiative heat shield systems. It can be used for most systems in which the material surfaces do not move with time and in which two-dimensional effects can be neglected. The sailient features of the analysis are as follows:

(1) From one to four layers of different materials can be considered. More layers can be included by simply changing the dimension statements in the first common block, but this change will increase computer storage requirements.

(2) One layer can be an air gap, but the air gap must have a layer on both sides of it. Heat transfer across the air gap can be by radiation, convection, and conduction.

(3) Perfect thermal contact between layers is assumed. Contact resistance can be simulated by using a separate layer or the air gap option at interfaces which have significant contact resistance.

(4) In each layer, the density is constant, the specific heat can be a function of temperature, and the thermal conductivity can be a function of both temperature and pressure.

(5) If any layer has a thermal conductivity much larger than the conductivity of the other layers, this layer can be input as a concentrated heat sink. Concentrated heat sinks absorb heat according to their material properties and thickness $\left(\rho c_p t\right)$ but do not have a thermal gradient nor occupy space. Concentrated heat sinks can be used at the front surface, at each interface, and at the back surface.

(6) Both convective and radiant heat inputs to the front surface can be used. If cold-wall convective heating is used, a hot-wall correction (based on enthalpy) can be used. If radiant heating is used, the surface absorptance can be specified as a function of temperature.

(7) Radiation from the front surface, across an air gap, and from the back surface can be used. Radiation from the back surface is to a specified temperature which can be a function of time. All emittances can be functions of temperature.

(8) Heat transfer by convection can be used at the back surface. The heat transfer coefficient can be a function of the back surface temperature and the temperature of the fluid can be a function of time.

(9) The temperature at any one location can be specified as a function of time.

(10) The computer program has been extended to permit the calculation of the total vehicle heat shield mass with one set of input data. Details of this option are given in a following section. In brief, the program calculates heat shield mass corresponding to

specified temperature limits at as many as 10 points around the vehicle and integrates over these points to obtain an average heat shield mass per unit area for the entire vehicle.

ANALYSIS

The equations describing the transient temperature response of a one-dimensional, multilayer, insulation system have been developed, put in finite-difference form, and programed for solution on the digital computer. The coordinate system originates at the front surface.

Governing Differential Equation

The governing differential equation for any layer is

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) = \rho c_p\,\frac{\partial T}{\partial \tau} \tag{1}$$

The density ρ within a layer is constant, the specific heat c_p can vary with temperature, and the conductivity k can vary with temperature and pressure. The initial temperature, as a function of position, must be specified.

Boundary Conditions

Energy balances are used at each boundary.

Front surface.- The front surface boundary condition is

$$\overbrace{q_C}^{\substack{\text{Cold-wall convective}\\ \text{heating rate}}}\;\overbrace{\left(1 - \frac{h_w}{h_e}\right)}^{\substack{\text{Hot-wall}\\ \text{correction}}} + \overbrace{\alpha q_R}^{\substack{\text{Radiative}\\ \text{heat input}}} = \overbrace{-k\,\frac{\partial T}{\partial x}}^{\substack{\text{Conduction}\\ \text{to interior}}} + \overbrace{\sigma\epsilon_{s,1}T_1^{\,4}}^{\text{Reradiation}} + \overbrace{C_s\,\frac{\partial T_1}{\partial \tau}}^{\substack{\text{Heat absorbed by}\\ \text{surface heat sink}}} \tag{2}$$

The two quantities in the hot-wall correction (the enthalpy of the fluid at the front surface temperature h_w and the enthalpy at the edge of the boundary layer h_e) should be based on absolute zero. If the base is room temperature, numerical instabilities can result if h_e approaches zero.

Interface.- The boundary condition at the interface between two layers of material is

$$-k_j\,\frac{\partial T}{\partial x}\bigg|_{i,j} = -k_{j+1}\,\frac{\partial T}{\partial x}\bigg|_{i,j+1} + C_i\,\frac{\partial T_i}{\partial \tau} \tag{3}$$

Back surface.- The back surface boundary condition is

$$-k_j \frac{\partial T}{\partial x}\bigg|_{L,j} = C_L \frac{\partial T_L}{\partial \tau} + \sigma\epsilon_b\left(T_L{}^4 - T_b{}^4\right) + h_f^*\left(T_L - T_f\right) \tag{4}$$

where ϵ_b is the effective emittance of the back surface and h_f^* is the effective heat transfer coefficient at the back surface.

Air gap.- The boundary condition at the front side of the air gap is

$$-k_j \frac{\partial T}{\partial x}\bigg|_{i,j} = -k_g \frac{\partial T}{\partial x}\bigg|_{i,g} + \sigma\epsilon_g\left(T_i{}^4 - T_{i+1}^4\right) + h_g^*\left(T_i - T_{i+1}\right) + C_i \frac{\partial T_i}{\partial \tau} \tag{5}$$

where ϵ_g and h_g^* are effective values of the emittance at the air gap and the heat transfer coefficient at the air gap, respectively.

At the back of the air gap, the boundary condition is

$$-k_g \frac{\partial T}{\partial x}\bigg|_{i,g} + \sigma\epsilon_g\left(T_{i-1}^4 - T_i{}^4\right) + h_g^*\left(T_{i-1} - T_i\right) = -k_j \frac{\partial T}{\partial x}\bigg|_{i,j} + C_i \frac{\partial T_i}{\partial \tau} \tag{6}$$

Finite-Difference Equations

The finite-difference stations are spaced at equal intervals throughout a given layer, as shown in figure 1. The first layer includes a station at the front and the back of the layer. All other layers include a station at the back of the layer. The air gap has no stations within the layer.

The differential equations are put in finite-difference form by the use of Taylor's series expansions. Forward, central, and backward differences are used. At the boundaries, the second derivative in the governing differential equation (eq. (1)) is partially expanded to first derivatives and the boundary condition is inserted in place of the first derivative at the boundary. Thus, both the governing equation and the boundary condition are satisfied at the boundary. In the equations, all terms which are not explicit functions of time are averaged over the time step because a Taylor's series expansion of the time derivative at $\Delta\tau/2$ shows that this procedure makes the solution correct to $\Delta\tau^2$. (See ref. 5.)

A summary of the finite-difference equations is given in appendix A. These equations yield an essentially tridiagonal matrix of unknown temperatures. A procedure based on Gauss' elimination method is used to solve the matrix.

Heat Shield Mass Calculation

The computer program has an option which permits iteration of the thickness of a specified layer to satisfy a specified temperature limit criterion. For example, the minimum thickness of the primary insulation layer, which will limit the back surface to a specified temperature, can be determined. Any one layer can be specified for the thickness iteration and any temperature limit can be imposed on any one station.

The program also has an option to determine minimum thicknesses at as many as 10 different locations, which are equivalent to different body points around the vehicle. The surface environments (cold-wall convective heating rate and pressure) at body points 2 to 10 are specified as fractions of the initial cold-wall convective heating rate and pressure tables. These fractions can be functions of time to accommodate transitions to turbulent flow. The enthalpy is assumed to be constant around the body.

The heat shield mass at each body point can also be calculated. In this calculation, the mass of each layer and a specified mass (through which the mass of concentrated heat sinks can be added) are included at each body point.

By assuming that the surface environment at each body point corresponds to an average environment over a specified percentage of the total vehicle area, an average heat shield mass per unit area for an entire vehicle can be computed.

COMPUTER PROGRAM

This computer program (Langley designation A4596)[1] was written in FORTRAN language for the Control Data 6000 series digital computer under the SCOPE 3.0 operating system. The program requires approximately 45000 octal locations of core storage. A flow chart for the program is shown in figure 2. The program input quantities are listed and defined in appendix B.

Computer Program Accuracy

The accuracy of the computer program was determined by comparing the results from the numerical solution with two exact solutions and also comparing results with the established computer program of reference 1.

The first exact solution used was taken from reference 6. Equation (A11) of reference 6 gives the transient temperature response of a constant property slab, subjected to a constant heating rate on one side and perfectly insulated on the other side. For the case selected, the average surface temperature rise rate was 740 K/sec. The maximum

[1]This program is available as LAR-12057 from COSMIC, Computer Center Information Services, 112 Barrow Hall, University of Georgia, Athens, Georgia 30601.

errors for several numerical solutions are given in table I. In all cases the maximum error occurred at the first printout (0.25 sec). (The time step was 0.03125 sec.) The accuracy of the solution was not significantly affected by the number of layers used. The smaller error criterion increased accuracy by a factor of 100 but increased computer time by a factor of about 10.

The exact solution was also computed with the air gap included to check the air-gap equations for correctness and accuracy. When the air-gap equations were used, the error increased slightly and decreasing the error criterion did not increase the accuracy significantly. The increased error was probably caused by the less accurate two station forward and backward differences used at the air gap.

The other exact solution was taken from reference 7. This exact solution was for a constant property slab with a constant surface temperature, losing heat at the back surface to a constant-temperature fluid. For this case, the maximum error in the steady state solution was 0.002 percent.

The computer program was compared with the analysis of reference 1 by using a variable property, transient case. Both programs gave essentially the same results with the temperatures differing in about the eighth place.

Sample Case

The computer program output for a sample case is given in appendix C. The case was selected to demonstrate, fairly simply, the versatility of the program. The environment roughly represented a flight test of a hypersonic research airplane. The heat shield consisted of a metallic outer skin over a metal-foil-encapsulated fibrous insulation with an air gap between the inner foil surface and the main structural skin. This heat shield system was modeled for the computer program as shown in figure 3. A temperature limit criterion for the main structural skin was specified and the insulation layer thickness was iterated until this criterion was satisfied. Calculations were made at 6 body points around the vehicle at nominal heating rate ratios (computer input quantity QRATT) of 1.0, 0.8, 0.6, 0.4, 0.2, and 0.1. The nominal QRATT values of 0.4, 0.2, and 0.1 were increased during the latter part of the heat pulse to simulate transition to turbulent flow. The heat shield unit mass was integrated around the vehicle as a function of percent vehicle area at each heating rate ratio. The total heat shield mass can be found by multiplying the final unit mass value by the total vehicle area.

A detailed description of the sample case printout is given in appendix C. The results from the sample case are given in figures 4, 5, 6, and 7. Figure 4 shows the insulation layer thickness iterations for the specified nominal QRATT values. For the initial QRATT value of 1.0, 3 iterations were required to satisfy the back surface temperature limit criterion, mainly because of the inaccurate initial thickness estimate. The extrapolation equation used to estimate thickness at the lower QRATT values generally gave good estimates. Because the extrapolation equation is based on the nominal QRATT values, the equation tended to underestimate the thickness requirements of the QRATT values (0.4, 0.2, and 0.1) which were increased during the later part of the heat pulse to simulate transition to turbulent flow. The last calculated insulation layer thickness at each QRATT value is plotted in figure 5 as a function of the nominal QRATT values.

The assumed heating rate distribution over the vehicle is given in figure 6, which shows, for example, that 10 percent of the vehicle area has an average heating rate equal to the initial convective heating rate (QRATT = 1.0) and 15 percent of the vehicle has an average heating rate equal to 80 percent of the initial heating rate. Figure 7 shows the cumulative average heat shield unit mass around the vehicle as a function of percent vehicle area. Each mass point includes the unit mass of the layers and the unit mass of the concentrated heat sinks. The average unit mass for the vehicle is 7.7 kg/m^2. This unit mass multiplied by the total vehicle area gives the total heat shield mass.

CONCLUDING REMARKS

A numerical analysis of the transient response of multilayer insulative heat shield systems has been developed. The differential equations, finite-difference equations, computer program input listing, and computer program output for a sample case are given.

The computer program is versatile and efficient and has relatively small computer storage requirements. Within a single calculation (for a specified trajectory and vehicle heating distribution), the program will iteratively determine the heat shield thickness required for a specified temperature limit criterion, determine the thickness requirements at a number of points on the vehicle, and determine the average heat shield unit mass for the vehicle. The program is suitable for use as a subroutine in large computer programs for vehicle system synthesis.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
March 11, 1976

APPENDIX A

FINITE-DIFFERENCE EQUATIONS

The differential equations were put in finite-difference form by the use of Taylor's series expansions. Three station forward, central, and backward differences were used. The terms derived from the second derivative of equation (1) were averaged over the time step making the solution correct to order $\Delta\tau^2$.

At the boundaries, the second derivative in the governing differential equation (eq. (1)) was partially expanded to first derivatives and the boundary condition was inserted in place of the first derivative at the boundary. Thus, both the governing equation and the boundary condition are satisfied at the boundary.

The finite-difference form of equation (1) and combinations of equation (1) and the boundary conditions (eqs. (2) to (6)) are presented in this appendix. In the following equations, unprimed quantities are either evaluated at the beginning of the time step or are constant with time. Primed quantities are evaluated at the end of the time step. Double primed quantities are functions of temperature but are evaluated at the end of the time step and therefore require iteration.

The first subscript on material property values refers to the layer. Further subscripting indicates the station temperatures at which the material property is evaluated. Thermal conductivities are evaluated at the average temperature between two stations.

Layer Equation

The finite-difference equation for any layer is

$$\frac{\left(k_j''\right)_{n,n+1}\left(T'_{n+1} - T'_n\right) + \left(k_j\right)_{n,n+1}\left(T_{n+1} - T_n\right)}{2\,\Delta x_j^2} - \frac{\left(k_j''\right)_{n,n-1}\left(T'_n - T'_{n-1}\right) + \left(k_j\right)_{n,n-1}\left(T_n - T_{n-1}\right)}{2\,\Delta x_j^2}$$

$$- \rho_j \frac{\left(c''_{p,j}\right)_n + \left(c_{p,j}\right)_n}{2} \frac{T'_n - T_n}{\Delta\tau} = 0 \qquad \text{(A1)}$$

APPENDIX A

First Station

The finite-difference equation at the heated surface is

$$\frac{3\left[\left(k_1''\right)_{1,2}\left(T_2' - T_1'\right) + \left(k_1\right)_{1,2}\left(T_2 - T_1\right)\right]}{2\ \Delta x_1^{\ 2}} - \frac{\left(k_1''\right)_{2,3}\left(T_3' - T_2'\right) + \left(k_1\right)_{2,3}\left(T_3 - T_2\right)}{6\ \Delta x_1^{\ 2}}$$

$$+ \frac{4}{3\ \Delta x_1}\left[q_C'\left(1 - \frac{h_w''}{h_e'}\right) + q_C\left(1 - \frac{h_w}{h_e}\right) + \alpha'' q_R' + \alpha q_R - \sigma\epsilon_s''\left(T_1''\right)^3 T_1' - \sigma\epsilon_s T_1^{\ 4}\right]$$

$$- \frac{8C_s}{3\ \Delta x_1}\ \frac{T_1' - T_1}{\Delta\tau} - \rho_1\ \frac{\left(c_{p,1}''\right)_1 + \left(c_{p,1}\right)_1}{2}\ \frac{T_1' - T_1}{\Delta\tau} = 0 \qquad \text{(A2)}$$

Interface Equation

The finite-difference equation at any solid-solid interface is

$$- \frac{3\left[\left(k_j''\right)_{i,i-1}\left(T_i' - T_{i-1}'\right) + \left(k_j\right)_{i,i-1}\left(T_i - T_{i-1}\right)\right]}{2\ \Delta x_j}$$

$$+ \frac{3\left[\left(k_{j+1}''\right)_{i,i+1}\left(T_{i+1}' - T_i'\right) + \left(k_{j+1}\right)_{i,i+1}\left(T_{i+1} - T_i\right)\right]}{2\ \Delta x_{j+1}}$$

$$+ \frac{\left(k_j''\right)_{i-1,i-2}\left(T_{i-1}' - T_{i-2}'\right) + \left(k_j\right)_{i-1,i-2}\left(T_{i-1} - T_{i-2}\right)}{6\ \Delta x_j}$$

$$- \frac{\left(k_{j+1}''\right)_{i+1,i+2}\left(T_{i+2}' - T_{i+1}'\right) + \left(k_{j+1}\right)_{i+1,i+2}\left(T_{i+2} - T_{i+1}\right)}{6\ \Delta x_{j+1}} - \frac{4C_i\left(T_i' - T_i\right)}{3\ \Delta\tau}$$

$$- \rho_j\ \Delta x_j\ \frac{\left[\left(c_{p,j}''\right)_i + \left(c_{p,j}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau} - \rho_{j+1}\ \Delta x_{j+1}\ \frac{\left[\left(c_{p,j+1}''\right)_i + \left(c_{p,j+1}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau} = 0 \qquad \text{(A3)}$$

APPENDIX A

Back Surface

The finite-difference equation at the back surface is

$$
-\frac{3\left[\left(k_j''\right)_{L,L-1}\left(T_L' - T_{L-1}'\right) + \left(k_j\right)_{L,L-1}\left(T_L - T_{L-1}\right)\right]}{2\ \Delta x_j^2}
$$

$$
+\frac{\left(k_j''\right)_{L-1,L-2}\left(T_{L-1}' - T_{L-2}'\right) + \left(k_j\right)_{L-1,L-2}\left(T_{L-1} - T_{L-2}\right)}{6\ \Delta x_j^2} - \frac{4}{3\ \Delta x_j}\left[\sigma\epsilon_b''\left(T_L''\right)^3 T_L' - \sigma\epsilon_b''\left(T_b'\right)^4\right.
$$

$$
\left. + \sigma\epsilon_b\left(T_L^4 - T_b^4\right) + \left(h_f^*\right)''\left(T_L' - T_f'\right) + h_f^*\left(T_L - T_f\right)\right] - \frac{8C_L}{3\ \Delta x_j}\,\frac{T_L' - T_L}{\Delta\tau}
$$

$$
-\rho_j\,\frac{\left[\left(c_{p,j}''\right)_L + \left(c_{p,j}\right)_L\right]\left(T_L' - T_L\right)}{2\ \Delta\tau} = 0 \tag{A4}
$$

Air Gap

The finite-difference equation at the front side of the air gap is

$$
\frac{9\left[\left(k_j''\right)_{i,i-1}\left(T_i' - T_{i-1}'\right) + \left(k_j\right)_{i,i-1}\left(T_i - T_{i-1}\right)\right]}{8\ \Delta x_j}
$$

$$
-\frac{\left(k_j''\right)_{i-1,i-2}\left(T_{i-1}' - T_{i-2}'\right) + \left(k_j\right)_{i-1,i-2}\left(T_{i-1} - T_{i-2}\right)}{8\ \Delta x_j}
$$

$$
-\frac{\left(k_g''\right)_{i,i+1}\left(T_{i+1}' - T_i'\right) + \left(k_g\right)_{i,i+1}\left(T_{i+1} - T_i\right)}{\Delta x_{j+1}} + h_{g,i}''\left(T_i' - T_{i+1}'\right) + h_{g,i}\left(T_i - T_{i+1}\right)
$$

$$
+ \sigma\epsilon_{g,i}''\left[\left(T_i''\right)^3 T_i' - \left(T_{i+1}''\right)^3 T_{i+1}'\right] + \sigma\epsilon_{g,i}\left(T_i^4 - T_{i+1}^4\right) + 2C_i\,\frac{T_i' - T_i}{\Delta\tau}
$$

$$
+ \frac{3\ \Delta x_j\rho_j}{4}\,\frac{\left[\left(c_{p,j}''\right)_i + \left(c_{p,j}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau} + \Delta x_{j+1}\rho_{j+1}\,\frac{\left[\left(c_{p,j+1}''\right)_i + \left(c_{p,j+1}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau} = 0 \tag{A5}
$$

APPENDIX A

At the back side of the air gap, the finite-difference equation is

$$\frac{\left(k_g''\right)_{i,i-1}\left(T_i' - T_{i-1}'\right) + \left(k_g\right)_{i,i-1}\left(T_i - T_{i-1}\right)}{\Delta x_j} - \frac{9\left[\left(k_{j+1}''\right)_{i,i+1}\left(T_{i+1}' - T_i'\right) + \left(k_{j+1}\right)_{i,i+1}\left(T_{i+1} - T_i\right)\right]}{8\ \Delta x_{j+1}}$$

$$+ \frac{\left(k_{j+1}''\right)_{i+1,i+2}\left(T_{i+2}' - T_{i+1}'\right) + \left(k_{j+1}\right)_{i+1,i+2}\left(T_{i+2} - T_{i+1}\right)}{8\ \Delta x_{j+1}}$$

$$- \left(h_{g,i}^*\right)''\left(T_{i-1}' - T_i'\right) - h_{g,i}^*\left(T_{i-1} - T_i\right) - \sigma\epsilon_{g,i}''\left[\left(T_{i-1}''\right)^3 T_{i-1}' - \left(T_i''\right)^3 T_i'\right] - \sigma\epsilon_{g,i}\left(T_{i-1}^4 - T_i^4\right)$$

$$+ 2C_i\,\frac{T_i' - T_i}{\Delta\tau} + \Delta x_j \rho_j\,\frac{\left[\left(c_{p,j}''\right)_i + \left(c_{p,j}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau}$$

$$+ \frac{3\ \Delta x_{j+1}\rho_{j+1}}{4}\,\frac{\left[\left(c_{p,j+1}''\right)_i + \left(c_{p,j+1}\right)_i\right]\left(T_i' - T_i\right)}{2\ \Delta\tau} = 0 \qquad \text{(A6)}$$

APPENDIX B

COMPUTER PROGRAM INPUTS

The input data quantities for the computer program are defined in this appendix. All data storage locations are zeroed out before the data are used. Thus, if any quantity is zero for a specific case, the quantity can simply be left out of the data input for that case.

Input quantities which are functions of time or temperature are used in tables. Each table has four quantities. As an example, consider the table for enthalpy h_e.

(1) The quantity MHE is the order of interpolation. When MHE = 0, h_e is constant. When MHE = 1, linear interpolation is used. When MHE = 2, second-order interpolation is used.

(2) The quantity NHE is the number of values in the h_e table. When NHE = 0, h_e is constant.

(3) TAUHE lists the time values corresponding to each value in the h_e table. When TAUHE = 0, h_e is constant. This independent variable table must be either monotonically increasing or decreasing.

(4) HET lists the h_e values.

From this discussion it follows that when h_e is constant, only the quantity HET need be specified.

HET must always have a nonzero value because it appears in the denominator in equation (2). If the QC table is being used, QRATT must have a value because it multiplies QC.

After each input quantity which can have more than one value, the maximum number of values is given parenthetically. When the letter N appears in the parentheses, either alone or as a second dimension, N refers to the layer in question and cannot exceed 4.

In the following listing, whenever possible, the symbol used in equations (1) to (6) is given after the computer symbol definition. The listing is arranged semialphabetically. The quantities describing a table (usually four) are grouped together and the name of the dependent variable fixes the alphabetical location of the table.

Front Surface Absorptance

MAL	order of interpolation for ALT
NAL	number of values in ALT

APPENDIX B

TTAL(10)	temperature table for ALT
ALT(10)	absorptance of front surface α
AREA(10)	percent of total vehicle area to be included in each body point heat shield mass calculation
CF	convergence factor for iterating on layer thickness (if convergence is too slow, make $CF > 1.0$; if thickness iteration diverges, make $CF < 1.0$; if CF is not specified, CF will equal 1.0)
CIT(N,10)	$(\rho c_p t)$ of heat sinks at interfaces C_i (one set of values for each body point

Layer Specific Heats

MCP(N)	order of interpolation
NCP(N)	number of values
TTCP(10,N)	temperature table
CPT(10,N)	specific heat c_p
CST(10)	$(\rho c_p t)$ of heat sink at front surface C_s (one value for each body point)
DTAU1	initial time step $\Delta\tau$ (the time step is increased or decreased automatically during the calculation depending on the number of iterations required for convergence)
ENDTAU	time at which computation stops

Front Surface Emittance

MEPS	order of interpolation
NEPS	number of values
TTEPS(10)	temperature table
EPST(10)	emittance of front surface ϵ_s

Back Surface Emittance

MEPSB	order of interpolation
NEPSB	number of values
TTEPSB(10)	temperature table
EPSBT(10)	emittance of back surface ϵ_b

APPENDIX B

Air-Gap Emittance

MEPSG	order of interpolation
NEPSG	number of values
TTEPSG(10)	temperature table
EPSGT(10)	emittance at air gap ϵ_g
ERRØR	acceptable error in temperature convergence, $(T' - T'')/T'$ (0.001 is usually acceptable)

Total Enthalpy at Edge of Boundary Layer

MHE	order of interpolation
NHE	number of values
TAUHE(20)	time table
HET(20)	total enthalpy at edge of boundary layer h_e

Back Surface Heat Transfer Coefficient

MHF	order of interpolation
NHF	number of values
TTHF(10)	temperature table
HFT(10)	heat transfer coefficient to back surface h_f^*

Air-Gap Heat Transfer Coefficient

MH	order of interpolation
NH	number of values
TTH(10)	temperature table
HT(10)	heat transfer coefficient at air gap h_g^*

Enthalpy of Fluid at Front Surface Temperature

MHW	order of interpolation
NHW	number of values
TTHW(15)	temperature table
HWT(15)	local enthalpy of fluid at front surface temperature h_w

I(N)	number of stations in each layer (first layer must have at least 3 stations; other layers must have at least 2 stations; air gap has only 1 station)
ISTAS	number of station at back surface of air gap, $1 + \sum_{n=1}^{\text{Air gap}} I(N)$

Thermal Conductivity

NP(N)	number of pressure values in each k table (if NP(N) = 1, k = f(T) only)
TABP(5,N)	pressure table
TABT(10,N)	temperature table
NK(N)	number of k values in each table (NK/NP = Number of temperature values in each table)
TABK(50,N)	thermal conductivity values k
LCHAN	layer whose thickness is changed to satisfy a temperature limit criterion
NCHAN	station at which temperature limit criterion is specified
TCHAN	temperature limit at station NCHAN which will be satisfied by iterating on thickness of layer LCHAN
NLYERS	number of layers
NQP	number of body points (points around vehicle) to be analyzed (equal to number of PRATT and number of QRATT values)
PFREQ	print frequency

Body Point Pressure Ratios

MPRAT(10)	order of interpolation
NPRAT(10)	number of values
TAUPRAT(10,10)	time-body point table
PRATT(10,10)	fraction of PT values, used to obtain pressure dependence of thermal conductivity k at different body points

APPENDIX B

Pressure

MPT	order of interpolation
NPT	number of values
TAUP(10)	time table
PT(10)	pressure history used to calculate conductivity values

Cold-Wall Convective Heating Rate

MQC	order of interpolation
NQC	number of values
TAUQC(20)	time table
QCT(20)	cold-wall convective heating rate q_C

Body Point Heating Rate Ratios

MQRAT(10)	order of interpolation
NQRAT(10)	number of values
TAUQRAT(10,10)	time-body point table
QRATT(10,10)	fraction of QCT values as a function of time and body point

Radiant Heating Rate

MQR	order of interpolation
NQR	number of values
TAUQR(20)	time table
QRT(20)	radiant heating rate q_R
RHØ(N)	density ρ
SIGMA	Stefan-Boltzmann constant σ
T(100)	initial temperature distribution T
TAUØ	starting time

Temperature to Which Back Surface Radiates

MTB	order of interpolation
NTB	number of values

TAUTB(10)	time table
TBT(10)	temperature to which back surface radiates T_b

Back Surface Fluid Temperature

MTF	order of interpolation
NTF	number of values
TAUTF(10)	time table
TFT(10)	temperature of fluid at back surface T_f
NTSP	station at which temperature is specified

Station Temperature History Specification

MTS	order of interpolation
NTS	number of values
TAUTS(20)	time table
TST(20)	specified temperature history at station NTSP
X(N)	layer thickness
WTC1(10)	total heat sink unit mass, $\sum \rho t$, to be added to layer unit masses to get total heat shield unit mass at each body point (one value for each body point)

APPENDIX C

SAMPLE CASE

The computer printout for a sample case is given in this appendix. The first information given is a list of the input data. The entire dimensioned field of each input quantity is given. Because the entire field for each quantity is seldom filled, a great many zeros appear in the input listing. The order of the input list is approximately as follows: program control data, material property data, environment data, and data for calculations at several body points.

Following the data input list, the regular printouts begin. The first printout occurs after the solution has converged for the first time step. Subsequent printouts are at the specified print frequency. For the sample case, the print frequency was made much larger than usual to reduce the number of pages of printout. The computer always prints out the last calculation for any normal program stop. For each thickness iteration in this sample case, the last printout is at the time when the back surface temperature starts to decrease.

Each printout starts with the time and the time step. Next, the temperature at each station (starting at the front surface) is given. Each line contains six temperatures. Therefore, if a layer contains more than six stations, the temperatures cover more than one line. However, the temperatures for the next layer always start on a new line. The last temperature in each layer and the first temperature in the next layer are identical because this station is common to both layers.

The rest of the quantities in a printout are quantities which vary with time, thickness iteration, or body point. These quantities are defined as follows:

QC	cold-wall convective heating rate at the beginning of the time step
QCP	cold-wall convective heating rate at the end of the time step
QR	radiant heating rate at the beginning of the time step
QRP	radiant heating rate at the end of the time step
QRR	reradiation from the front surface $\sigma\epsilon_{s,1}T_1^4$ at the beginning of the time step
HE	total enthalpy at the beginning of the time step
HEP	total enthalpy at the end of the time step
HWO	enthalpy of the fluid at the front surface temperature (at the beginning of the time step)

HW	enthalpy of the fluid at the front surface temperature (at the end of the time step)
X	thickness of the layer whose thickness is being iterated
QAERO	net heat input to the front surface at the beginning of the time step $q_C\left(1 - h_w/h_e\right) + \alpha q_R$
QAEROP	net heat input to the front surface at the end of the time step $q'_C\left(1 - h''_w/h'_e\right) + \alpha'' q'_R$
TF	temperature of the fluid at the back surface (at the beginning of the time step)
QRAT	heating rate ratio at the end of the time step
PRAT	pressure ratio at the end of the time step
WTCI	total heat sink unit mass at this body point
AREA	fraction of the total area which has an average heating environment corresponding to this body point
WTBP	sum of the layer unit masses at this body point (this value does not change until the layer thickness iteration has converged)
WTSUM	cumulative average unit mass $\sum$(WTCI + WTBP)AREA (this value does not change until the layer thickness iteration has converged)

This sample case was run with 45000 storage locations and the execution time was 120 sec on a CDC 6400 computer.

Input Data

```
$DATA1
TAUO    =  0.0,
DTAU1   =  0.3125E-01,
ENDTAU  =  0.1E+04,
PFREQ   =  0.2E+03,
ERROR   =  0.1E-02,
SIGMA   =  0.56696 1E-10,
NLYERS  =  3,
I       =  20,   1,   2,   0,
X       =  0.3048E-01,  0.3048E-02,  0.509E-03,  0.0,
ISTAS   =  21,
```

```
RHO     =  0.96E+02,  0.8E+00,  0.27231E+04,  0.0,

MCP     =  1,  1,  1,  0,

NCP     =  2,  1,  1,  0,

TTCP    =  0.275E+03,  0.167E+04,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

CPT     =  0.6276E+00,  0.12552E+01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.1046E+01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.9205E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MAL     =  0,

NAL     =  0,

TTAL    =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

ALT     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MEPS    =  0,

NEPS    =  0,

TTEPS   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

EPST    =  0.8E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MH      =  0,

NH      =  0,

TTH     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

HT      =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MEPSG   =  0,

NEPSG   =  0,

TTEPSG  =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

EPSGT   =  0.8E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MHF     =  0,

NHF     =  0,

TTHF    =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

HFT     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MEPSB   =  0,

NEPSB   =  0,

TTEPSB  =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

EPSBT   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

NP      =  3,  1,  1,  0,
```

```
TABP    =  0.1E-01,  0.1E+00,  0.1E+01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

TABT    =  0.275E+03,  0.167E+04,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

NK      =  6,  1,  1,  0,

TABK    =  0.312E-04,  0.2496E-03,  0.4368E-04,  0.3744E-03,  0.4992E-04,
           0.4368E-03,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.312E-04,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.23088E+00,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MPT     =  1,

NPT     =  4,

TAUP    =  0.0,  0.7E+02,  0.22E+03,  0.1E+04,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,

PT      =  0.6E-01,  0.25E-01,  0.1E+00,  0.1E+01,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,

T       =  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,
           0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.294E+03,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,

MHW     =  1,

NHW     =  10,

TTHW    =  0.278E+03,  0.555E+03,  0.833E+03,  0.1111E+04,  0.1389E+04,
           0.1667E+04,  0.1944E+04,  0.2222E+04,  0.25E+04,  0.2778E+04,  0.0,
           0.0,  0.0,  0.0,  0.0,
```

```
HWT     =  0.267E+03,  0.548E+03,  0.844E+03,  0.116E+04,  0.1489E+04,
           0.1833E+04,  0.2188E+04,  0.2564E+04,  0.3011E+04,  0.3269E+04,
           0.0,  0.0,  0.0,  0.0,  0.0,

MQC     =  1,

NQC     =  15,

TAUQC   =  0.0,  0.26E+02,  0.45E+02,  0.61E+02,  0.66E+02,  0.74E+02,
           0.88E+02,  0.92E+02,  0.103E+03,  0.114E+03,  0.142E+03,  0.17E+03,
           0.197E+03,  0.229E+03,  0.1E+04,  0.0,  0.0,  0.0,  0.0,  0.0,

QCT     =  0.1135E+02,  0.3405E+02,  0.7945E+02,  0.1816E+03,  0.26105E+03,
           0.3405E+03,  0.5675E+03,  0.5675E+03,  0.454E+03,  0.3405E+03,
           0.227E+03,  0.1135E+03,  0.227E+02,  0.1135E+02,  0.454E+01,  0.0,
           0.0,  0.0,  0.0,  0.0,

MQR     =  0,

NQR     =  0,

TAUQR   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

QRT     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MHE     =  1,

NHE     =  15,

TAUHE   =  0.0,  0.26E+02,  0.45E+02,  0.61E+02,  0.66E+02,  0.74E+02,
           0.88E+02,  0.92E+02,  0.103E+03,  0.114E+03,  0.142E+03,  0.17E+03,
           0.197E+03,  0.229E+03,  0.1E+04,  0.0,  0.0,  0.0,  0.0,  0.0,

HET     =  0.397E+03,  0.629E+03,  0.954E+03,  0.1366E+04,  0.158E+04,
           0.1868E+04,  0.246E+04,  0.246E+04,  0.2153E+04,  0.1868E+04,
           0.1357E+04,  0.937E+03,  0.617E+03,  0.395E+03,  0.29E+03,  0.0,
           0.0,  0.0,  0.0,  0.0,

MTB     =  0,

NTB     =  0,

TAUTB   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

TBT     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

MTF     =  0,

NTF     =  0,

TAUTF   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

TFT     =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

NTSP    =  0,

MTS     =  0,

NTS     =  0,

TAUTS   =  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
```

APPENDIX C

```
TST      = 0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

TCHAN    = 0.422E+03,

LCHAN    = 1,

NCHAN    = 23,

NQP      = 6,

CF       = 0.0,

NQRAT    = 1,  1,  1,  4,  5,  6,  0,  0,  0,  0,

MQRAT    = 0,  0,  0,  1,  1,  1,  0,  0,  0,  0,

TAUQRAT  = 0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.114E+03,
           0.12E+03,  0.1E+04,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.103E+03,  0.11E+03,  0.12E+03,  0.1E+04,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.92E+02,  0.1E+03,  0.11E+03,  0.18E+03,  0.1E+04,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

QRATT    = 0.1E+01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.8E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.6E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.4E+00,  0.4E+00,  0.5E+00,  0.4E+00,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.2E+00,  0.2E+00,  0.25E+00,  0.3E+00,  0.2E+00,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.1E+00,  0.1E+00,  0.12F+00,  0.15E+00,
           0.15F+00,  0.1E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,

MPRAT    = 0,  0,  0,  0,  0,  0,  0,  0,  0,  0,

NPRAT    = 0,  0,  0,  0,  0,  0,  0,  0,  0,  0,

TAUPRAT  = 0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,

PRATT    = 0.1F+01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.64E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.36E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.16E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.4E-01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.1E-01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
```

APPENDIX C

```
CIT    =  0.41E+00,  0.0,  0.0,  0.0,  0.41E+00,  0.0,  0.0,  0.0,  0.41E+00,
          0.0,  0.0,  0.0,  0.41E+00,  0.0,  0.0,  0.0,  0.41E+00,  0.0,  0.0,
          0.0,  0.41E+00,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
          0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,

CST    =  0.2E+01,  0.2E+01,  0.163E+01,  0.163E+01,  0.122E+01,  0.122E+01,
          0.0,  0.0,  0.0,  0.0,

WTCI   =  0.488E+01,  0.488E+01,  0.39E+01,  0.39E+01,  0.293E+01,  0.293E+01,
          0.0,  0.0,  0.0,  0.0,

AREA   =  0.1E+00,  0.15E+00,  0.2E+00,  0.2E+00,  0.2E+00,  0.15E+00,  0.0,
          0.0,  0.0,  0.0,

$END
```

Output Data

Body point 1 – first iteration

```
 TAU =      .031250          DTAU =          .06250000
TEMPERATURE TABLE
    2.94044799E+02   2.94000185E+02   2.94000001E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02
    2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02
    2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02   2.94000000E+02
    2.94000000E+02   2.94000000E+02
    2.94000000E+02   2.94000000E+02
    2.94000000E+02   2.94000000E+02   2.94000000E+02

QC    = 1.13500000E+01   QCP   = 1.13772837E+01   QR   = 0.               QRP  = 0.               QRR   = 3.38869510E-01
HE    = 3.97000000E+02   HEP   = 3.97278846E+02   HWO  = 2.83231047E+02   HW   = 2.83231047E+02   X     = 3.04800000E-02
QAERO= 3.25258846E+00    QAEROP= 3.26610434E+00   TF   = 0.               TB   = 0.               QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00   WTCI  = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.

 TAU =   200.000000          DTAU =          .06250000
TEMPERATURE TABLE
    6.74059080E+02   7.03344144E+02   7.24847282E+02   7.39480430E+02   7.47973808E+02   7.50935913E+02
    7.48887144E+02   7.42280886E+02   7.31517647E+02   7.16954785E+02   6.98912998E+02   6.77680220E+02
    6.53513232E+02   6.26637102E+02   5.97242265E+02   5.65478810E+02   5.31447025E+02   4.95182442E+02
    4.56631904E+02   4.15612927E+02
    4.15612927E+02   3.41974721E+02
    3.41974721E+02   3.41973493E+02   3.41973084E+02

QC    = 2.16470215E+01   QCP   = 2.16359375E+01   QR   = 0.               QRP  = 0.               QRR   = 9.37162596E+00
HE    = 5.96404297E+02   HEP   = 5.96187500E+02   HWO  = 6.74924920E+02   HW   = 6.74924920E+02   X     = 3.04800000E-02
QAERO=-2.84997547E+00    QAEROP=-2.85741969E+00   TF   = 0.               TB   = 0.               QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00   WTCI  = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.

 TAU =   400.000000          DTAU =         8.00000000
TEMPERATURE TABLE
    3.88408635E+02   4.01290030E+02   4.12901725E+02   4.23288577E+02   4.32492976E+02   4.40556807E+02
    4.47522913E+02   4.53436185E+02   4.58344438E+02   4.62299156E+02   4.65356145E+02   4.67576101E+02
    4.69025089E+02   4.69774928E+02   4.69903428E+02   4.69494438E+02   4.68637653E+02   4.67428112E+02
    4.65965232E+02   4.64350453E+02
    4.64350453E+02   4.60692872E+02
    4.60692872E+02   4.60692784E+02   4.60692754E+02

QC    = 9.87494163E+00   QCP   = 9.83961089E+00   QR   = 0.               QRP  = 0.               QRR   = 1.04549879E+00
HE    = 3.72256809E+02   HEP   = 3.71712062E+02   HWO  = 3.80258097E+02   HW   = 3.78962797E+02   X     = 3.04800000E-02
QAERO=-2.12251982E-01    QAEROP=-1.91934612E-01   TF   = 0.               TB   = 0.               QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00   WTCI  = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.

 TAU =   460.000000          DTAU =        10.00000000
TEMPERATURE TABLE
    3.73639114E+02   3.84355619E+02   3.94188087E+02   4.03173218E+02   4.11345119E+02   4.18737038E+02
    4.25382380E+02   4.31315423E+02   4.36571844E+02   4.41189097E+02   4.45206673E+02   4.48666250E+02
    4.51611745E+02   4.54089262E+02   4.56146936E+02   4.57834663E+02   4.59203727E+02   4.60306359E+02
    4.61195238E+02   4.61921797E+02
    4.61921797E+02   4.62417803E+02
    4.62417803E+02   4.62417810E+02   4.62417813E+02

QC    = 9.39797665E+00   QCP   = 9.30964981E+00   QR   = 0.               QRP  = 0.               QRR   = 9.03871891E-01
HE    = 3.64902724E+02   HEP   = 3.63540856E+02   HWO  = 3.66132535E+02   HW   = 3.63855311E+02   X     = 3.04800000E-02
QAERO=-3.16734772E-02    QAEROP=-8.05265623E-03   TF   = 0.               TB   = 0.               QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00   WTCI  = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.
```

Body point 1 – second iteration

```
 TAU =      .031250           DTAU =            .06250000
TEMPERATURE TABLE
    2.94044547E+02      2.94000107E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02

QC    = 1.13500000E+01    QCP   = 1.13772837E+01    QR   = 0.                 QRP  = 0.                 QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO  = 2.83231047E+02     HW   = 2.83231047E+02     X     = 4.01185699E-02
QAERO= 3.25258846E+00     QAEROP= 3.26610434E+00    TF   = 0.                 TB   = 0.                 QRAT  = 1.00000000E+00
PRAT = 1.00000000E+00     WTCI  = 4.88000000E+00    AREA = 1.00000000E-01     WTBP = 0.                 WTSUM= 0.

 TAU =   200.000000           DTAU =            .06250000
TEMPERATURE TABLE
    6.74171008E+02      7.11435364E+02      7.35747397E+02      7.49048575E+02      7.52846569E+02      7.48384555E+02
    7.36733528E+02      7.18651065E+02      6.95622313E+02      6.67890316E+02      6.36479510E+02      6.02214350E+02
    5.65933353E+02      5.28496815E+02      4.90783333E+02      4.53665903E+02      4.17953383E+02      3.84280992E+02
    3.52941191E+02      3.23669079E+02
    3.23669079E+02      3.00530104E+02
    3.00530104E+02      3.00529803E+02      3.00529703E+02

QC    = 2.16470215E+01    QCP   = 2.16359375E+01    QR   = 0.                 QRP  = 0.                 QRR   = 9.37783834E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO  = 6.75043857E+02     HW   = 6.75043857E+02     X     = 4.01185699E-02
QAERO=-2.85429238E+00     QAEROP=-2.86173595E+00    TF   = 0.                 TB   = 0.                 QRAT  = 1.00000000E+00
PRAT = 1.00000000E+00     WTCI  = 4.88000000E+00    AREA = 1.00000000E-01     WTBP = 0.                 WTSUM= 0.

 TAU =   400.000000           DTAU =           8.00000000
TEMPERATURE TABLE
    3.90414451E+02      4.08627273E+02      4.24445918E+02      4.37963558E+02      4.49260229E+02      4.58408600E+02
    4.65477827E+02      4.70536356E+02      4.73654195E+02      4.74904975E+02      4.74368029E+02      4.72130642E+02
    4.68290667E+02      4.62959651E+02      4.56266652E+02      4.48362896E+02      4.39427359E+02      4.29673239E+02
    4.19354888E+02      4.08773983E+02
    4.08773983E+02      3.93131983E+02
    3.93131983E+02      3.93131699E+02      3.93131604E+02

QC    = 9.87494163E+00    QCP   = 9.83961089E+00    QR   = 0.                 QRP  = 0.                 QRR   = 1.06749600E+00
HE    = 3.72256809E+02    HEP   = 3.71712062E+02    HWO  = 3.82321016E+02     HW   = 3.80996965E+02     X     = 4.01185699E-02
QAERO=-2.66975526E-01     QAEROP=-2.45781177E-01    TF   = 0.                 TB   = 0.                 QRAT  = 1.00000000E+00
PRAT = 1.00000000E+00     WTCI  = 4.88000000E+00    AREA = 1.00000000E-01     WTBP = 0.                 WTSUM= 0.

 TAU =   600.000000           DTAU =          10.00000000
TEMPERATURE TABLE
    3.51253949E+02      3.61458265E+02      3.70673672E+02      3.78944462E+02      3.86309664E+02      3.92805896E+02
    3.98469305E+02      4.03336963E+02      4.07447910E+02      4.10843942E+02      4.13570210E+02      4.15675669E+02
    4.17213357E+02      4.18240535E+02      4.18818645E+02      4.19013066E+02      4.18892646E+02      4.18528965E+02
    4.17995318E+02      4.17365399E+02
    4.17365399E+02      4.16147425E+02
    4.16147425E+02      4.16147401E+02      4.16147393E+02

QC    = 8.16140078E+00    QCP   = 8.07307393E+00    QR   = 0.                 QRP  = 0.                 QRR   = 7.01562201E-01
HE    = 3.45836576E+02    HEP   = 3.44474708E+02    HWO  = 3.42737751E+02     HW   = 3.41211424E+02     X     = 4.01185699E-02
QAERO= 7.31291913E-02     QAEROP= 7.64779829E-02    TF   = 0.                 TB   = 0.                 QRAT  = 1.00000000E+00
PRAT = 1.00000000E+00     WTCI  = 4.88000000E+00    AREA = 1.00000000E-01     WTBP = 0.                 WTSUM= 0.
```

```
 TAU =   660.000000          DTAU =          10.00000000
TEMPERATURE TABLE
     3.43321224E+02     3.52876211E+02     3.61547626E+02     3.69391157E+02     3.76454220E+02     3.82779177E+02
     3.88405520E+02     3.93371431E+02     3.97714896E+02     4.01474509E+02     4.04690043E+02     4.07402825E+02
     4.09655944E+02     4.11494302E+02     4.12964529E+02     4.14114732E+02     4.14994110E+02     4.15652419E+02
     4.16139287E+02     4.16503410E+02
     4.16503410E+02     4.16666603E+02
     4.16666603E+02     4.16666605E+02     4.16666605E+02

QC     = 7.63143969E+00     QCP    = 7.54311284E+00    QR    = 0.                QRP   = 0.                QRR   = 6.39537777E-01
HE     = 3.37665370E+02     HEP    = 3.36303502E+02    HWO   = 3.34554074E+02    HW    = 3.33178087E+02    X     = 4.01185699E-02
QAERO= 7.03171542E-02      QAEROP= 7.01014262E-02      TF    = 0.                TB    = 0.                QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00      WTCI  = 4.88000000E+00      AREA  = 1.00000000E-01    WTBP  = 0.                WTSUM= 0.
```

Body point 1 – final iteration

```
 TAU =      .031250          DTAU =           .06250000
TEMPERATURE TABLE
     2.94044591E+02     2.94000116E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02

QC     = 1.13500000E+01     QCP    = 1.13772837E+01    QR    = 0.                QRP   = 0.                QRR   = 3.38869510E-01
HE     = 3.97000000E+02     HEP    = 3.97278846E+02    HWO   = 2.83231047E+02    HW    = 2.83231047E+02    X     = 3.84511050E-02
QAERO= 3.25258846E+00      QAEROP= 3.26610434E+00      TF    = 0.                TB    = 0.                QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00      WTCI  = 4.88000000E+00      AREA  = 1.00000000E-01    WTBP  = 0.                WTSUM= 0.

 TAU =   200.000000          DTAU =           .06250000
TEMPERATURE TABLE
     6.74170883E+02     7.10155636E+02     7.34162725E+02     7.47927544E+02     7.52804906E+02     7.49914275E+02
     7.40219232E+02     7.24577962E+02     7.03778229E+02     6.78562790E+02     6.49648360E+02     6.17739719E+02
     5.83539295E+02     5.47751232E+02     5.11077099E+02     4.74197990E+02     4.37734846E+02     4.02176548E+02
     3.67765686E+02     3.34336332E+02
     3.34336332E+02     3.03649244E+02
     3.03649244E+02     3.03648835E+02     3.03648698E+02

QC     = 2.16470215E+01     QCP    = 2.16359375E+01    QR    = 0.                QPP   = 0.                QRR   = 9.37783153E+00
HE     = 5.96404297E+02     HEP    = 5.96187500E+02    HWO   = 6.75043726E+02    HW    = 6.75043726E+02    X     = 3.84511050E-02
QAERO=-2.85428765E+00      QAEROP=-2.86173122E+00      TF    = 0.                TB    = 0.                QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00      WTCI  = 4.88000000E+00      AREA  = 1.00000000E-01    WTBP  = 0.                WTSUM= 0.

 TAU =   400.000000          DTAU =          9.00000000
TEMPERATURE TABLE
     3.90170279E+02     4.07411205E+02     4.22471477E+02     4.35428576E+02     4.46350471E+02     4.55300208E+02
     4.62339021E+02     4.67528656E+02     4.70933353E+02     4.72621694E+02     4.72668496E+02     4.71156882E+02
     4.68180641E+02     4.63846977E+02     4.58279753E+02     4.51623258E+02     4.44046479E+02     4.35747691E+02
     4.26958829E+02     4.17948446E+02
     4.17948446E+02     4.04056668E+02
     4.04056668E+02     4.04056405E+02     4.04056317E+02

QC     = 9.87494163E+00     QCP    = 9.83961089E+00    QR    = 0.                QRP   = 0.                QRR   = 1.06486941E+00
HE     = 3.72256809E+02     HEP    = 3.71712062E+02    HWO   = 3.82076378E+02    HW    = 3.80749223E+02    X     = 3.84511050E-02
QAERO=-2.60485948E-01      QAEROP=-2.39223188E-01      TF    = 0.                TB    = 0.                QRAT  = 1.00000000E+00
PRAT  = 1.00000000E+00      WTCI  = 4.88000000E+00      AREA  = 1.00000000E-01    WTBP  = 0.                WTSUM= 0.
```

```
 TAU =  600.000000         DTAU =          10.00000000
TEMPERATURE TABLE
    3.51186652E+02     3.61030335E+02     3.69973107E+02     3.78059468E+02     3.85328555E+02     3.91816725E+02
    3.97559323E+02     4.02591943E+02     4.06951343E+02     4.10676114E+02     4.13807160E+02     4.16388016E+02
    4.18465025E+02     4.20087360E+02     4.21306912E+02     4.22178003E+02     4.22756946E+02     4.23101411E+02
    4.23269619E+02     4.23319352E+02
    4.23319352E+02     4.23000728E+02
    4.23000728E+02     4.23000720E+02     4.23000717E+02

QC    = 8.16140078E+00   QCP   = 8.07307393E+00   QR   = 0.               QRP  = 0.               QRR   = 7.00892988E-01
HE    = 3.45836576E+02   HEP   = 3.44474708E+02   HWO  = 3.42652407E+02   HW   = 3.41144324E+02   X     = 3.84511050E-02
QAERO= 7.51432396E-02   QAEROP= 7.80505307E-02   TF   = 0.               TB   = 0.               QRAT = 1.00000000E+00
PRAT = 1.00000000E+00    WTCI = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.

 TAU =  620.000000         DTAU =          10.00000000
TEMPERATURE TABLE
    3.48488354E+02     3.58109137E+02     3.66865357E+02     3.74805056E+02     3.81969938E+02     3.88398062E+02
    3.94125683E+02     3.99188560E+02     4.03622896E+02     4.07466018E+02     4.10756846E+02     4.13536207E+02
    4.15846986E+02     4.17734146E+02     4.19244604E+02     4.20426972E+02     4.21331148E+02     4.22007763E+02
    4.22507495E+02     4.22880234E+02
    4.22880234E+02     4.23028638E+02
    4.23028638E+02     4.23028639E+02     4.23028640E+02

QC    = 7.98474708E+00   QCP   = 7.89642023E+00   QR   = 0.               QRP  = 0.               QRR   = 6.79276436E-01
HE    = 3.43112840E+02   HEP   = 3.41750973E+02   HWO  = 3.39862185E+02   HW   = 3.38412301E+02   X     = 3.84511050E-02
QAERO= 7.56475920E-02   QAEROP= 7.71425973E-02   TF   = 0.               TB   = 0.               QRAT = 1.00000000E+00
PRAT = 1.00000000E+00    WTCI = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 0.               WTSUM= 0.

 TAU =  620.000000         DTAU =          10.00000000
TEMPERATURE TABLE
    3.48488354E+02     3.58109137E+02     3.66865357E+02     3.74805056E+02     3.81969938E+02     3.88398062E+02
    3.94125683E+02     3.99188560E+02     4.03622396E+02     4.07466018E+02     4.10756846E+02     4.13536207E+02
    4.15846986E+02     4.17734146E+02     4.19244604E+02     4.20426972E+02     4.21331143E+02     4.22007763E+02
    4.22507495E+02     4.22880234E+02
    4.22880234E+02     4.23028638E+02
    4.23028638E+02     4.23028639E+02     4.23028640E+02

QC    = 7.98474708E+00   QCP   = 7.89642023E+00   QR   = 0.               QRP  = 0.               QRR   = 6.79276436E-01
HE    = 3.43112840E+02   HEP   = 3.41750973E+02   HWO  = 3.39862185E+02   HW   = 3.38412301E+02   X     = 3.84511050E-02
QAERO= 7.56475920E-02   QAEROP= 7.71425973E-02   TF   = 0.               TB   = 0.               QRAT = 1.00000000E+00
PRAT = 1.00000000E+00    WTCI = 4.88000000E+00   AREA = 1.00000000E-01   WTBP = 5.07980238E+00   WTSUM= 9.95980238E-01
```

Body point 2

```
 TAU =     .031250         DTAU =            .06250000
TEMPERATURE TABLE
    2.94034721E+02     2.94000104E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02

QC    = 9.08000000E+00   QCP   = 9.10182692E+00   QR   = 0.               QRP  = 0.               QRR   = 3.38869510E-01
HE    = 3.97000000E+02   HEP   = 3.97278846E+02   HWO  = 2.83231047E+02   HW   = 2.83231047E+02   X     = 3.43917138E-02
QAERO= 2.60207077E+00   QAEROP= 2.61288347E+00   TF   = 0.               TB   = 0.               QRAT = 8.00000000E-01
PRAT = 6.40000000E-01    WTCI = 4.88000000E+00   AREA = 1.50000000E-01   WTBP = 5.07980238E+00   WTSUM= 9.95980238E-01
```

APPENDIX C

```
 TAU =  200.000000           DTAU =            .06250000
TEMPERATURE TABLE
    6.71358316E+02      7.02973777E+02      7.24232863E+02      7.36604363E+02      7.41185027E+02      7.38918491E+02
    7.30620075E+02      7.17007971E+02      6.98736185E+02      6.76416191E+02      6.50629803E+02      6.21937413E+02
    5.90882675E+02      5.57992146E+02      5.23769100E+02      4.88678089E+02      4.53116480E+02      4.17367200E+02
    3.81524379E+02      3.45379204E+02
    3.45379204E+02      3.08013981E+02
    3.08013981E+02      3.08013469E+02      3.08013299E+02

QC     = 1.73176172E+01     QCP   = 1.73087500E+01     QR   = 0.                 QRP  = 0.                 QRR   = 9.22199669E+00
HE     = 5.96404297E+02     HEP   = 5.96187500E+02     HWO  = 6.72042228E+02     HW   = 6.72042228E+02     X     = 3.43917138E-02
QAERO=-2.19627648E+00      QAEROP=-2.20224429E+00     TF   = 0.                 TB   = 0.                 QRAT  = 8.00000000E-01
PRAT = 6.40000000E-01       WTCI  = 4.88000000E+00     AREA = 1.50000000E-01     WTBP = 5.07980238E+00     WTSUM= 9.95980238E-01

 TAU =  400.000000           DTAU =           10.00000000
TEMPERATURE TABLE
    3.91613065E+02      4.05687569E+02      4.18051830E+02      4.28756499E+02      4.37848815E+02      4.45374632E+02
    4.51380472E+02      4.55914860E+02      4.59029703E+02      4.60781598E+02      4.61233202E+02      4.60454719E+02
    4.58525529E+02      4.55536008E+02      4.51589527E+02      4.46804589E+02      4.41317016E+02      4.35282013E+02
    4.28875677E+02      4.22295483E+02
    4.22295483E+02      4.10956150E+02
    4.10956150E+02      4.10955931E+02      4.10955857E+02

QC     = 7.92821790E+00     QCP   = 7.87168872E+00     QR   = 0.                 QRP  = 0.                 QRR   = 1.09809672E+00
HE     = 3.72801556E+02     HEP   = 3.71712062E+02     HWO  = 3.85138402E+02     HW   = 3.82097278E+02     X     = 3.43917138E-02
QAERO=-2.62362634E-01      QAEROP=-2.19926102E-01     TF   = 0.                 TB   = 0.                 QRAT  = 8.00000000E-01
PRAT = 6.40000000E-01       WTCI  = 4.88000000E+00     AREA = 1.50000000E-01     WTBP = 5.07980238E+00     WTSUM= 9.95980238E-01

 TAU =  580.000000           DTAU =           10.00000000
TEMPERATURE TABLE
    3.52774682E+02      3.61550422E+02      3.69572553E+02      3.76877730E+02      3.83499114E+02      3.89468014E+02
    3.94815040E+02      3.99570928E+02      4.03767142E+02      4.07436305E+02      4.10612491E+02      4.13331413E+02
    4.15630487E+02      4.17548813E+02      4.19127045E+02      4.20407160E+02      4.21432125E+02      4.22245455E+02
    4.22890676E+02      4.23410685E+02
    4.23410685E+02      4.23762702E+02
    4.23762702E+02      4.23762707E+02      4.23762709E+02

QC     = 6.67044358E+00     QCP   = 6.59978210E+00     QR   = 0.                 QRP  = 0.                 QRR   = 7.14349903E-01
HE     = 3.48560311E+02     HEP   = 3.47198444E+02     HWO  = 3.44356954E+02     HW   = 3.42765678E+02     X     = 3.43917138E-02
QAERO= 8.04401837E-02      QAEROP= 8.42609967E-02     TF   = 0.                 TB   = 0.                 QRAT  = 8.00000000E-01
PRAT = 6.40000000E-01       WTCI  = 4.88000000E+00     AREA = 1.50000000E-01     WTBP = 5.07980238E+00     WTSUM= 9.95980238E-01

 TAU =  580.000000           DTAU =           10.00000000
TEMPERATURE TABLE
    3.52774682E+02      3.61550422E+02      3.69572553E+02      3.76877730E+02      3.83499114E+02      3.89468014E+02
    3.94815040E+02      3.99570928E+02      4.03767142E+02      4.07436305E+02      4.10612491E+02      4.13331413E+02
    4.15630487E+02      4.17548813E+02      4.19127045E+02      4.20407160E+02      4.21432125E+02      4.22245455E+02
    4.22890676E+02      4.23410685E+02
    4.23410685E+02      4.23762702E+02
    4.23762702E+02      4.23762707E+02      4.23762709E+02

QC     = 6.67044358E+00     QCP   = 6.59978210E+00     QR   = 0.                 QRP  = 0.                 QRR   = 7.14349903E-01
HE     = 3.48560311E+02     HEP   = 3.47198444E+02     HWO  = 3.44356954E+02     HW   = 3.42765678E+02     X     = 3.43917138E-02
QAERO= 8.04401837E-02      QAEROP= 8.42609967E-02     TF   = 0.                 TB   = 0.                 QRAT  = 8.00000000E-01
PRAT = 6.40000000E-01       WTCI  = 4.88000000E+00     AREA = 1.50000000E-01     WTBP = 4.69010082E+00     WTSUM= 2.43149536E+00
```

Body point 3

```
 TAU =      .031250          DTAU =            .06250000
TEMPERATURE TABLE
     2.94030320E+02     2.94000113E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02

QC    = 6.81000000E+00    QCP   = 6.82637019E+00    QR    = 0.                QRP   = 0.                QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO   = 2.83231047E+02    HW    = 2.83231047E+02    X     = 2.97840978E-02
QAERO= 1.95155307E+00    QAEROP= 1.95966260E+00    TF    = 0.                TB    = 0.                QRAT  = 6.00000000E-01
PRAT  = 3.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.69010082E+00    WTSUM= 2.43149536E+00

 TAU =   200.000000          DTAU =            .06250000
TEMPERATURE TABLE
     6.50269431E+02     6.78657628E+02     6.98363153E+02     7.10604570E+02     7.16271654E+02     7.16122437E+02
     7.10809390E+02     7.00897346E+02     6.86887291E+02     6.69233669E+02     6.48354180E+02     6.24634529E+02
     5.98430083E+02     5.70064530E+02     5.39824528E+02     5.07949795E+02     4.74616670E+02     4.39912061E+02
     4.03792485E+02     3.66015746E+02
     3.66015746E+02     3.17947467E+02
     3.17947467E+02     3.17946772E+02     3.17946540E+02

QC    = 1.29882129E+01    QCP   = 1.29815625E+01    QR    = 0.                QRP   = 0.                QRP   = 8.11675718E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO   = 6.49583855E+02    HW    = 6.49583855E+02    X     = 2.97840978E-02
QAERO=-1.15811947E+00    QAEROP=-1.16266799E+00    TF    = 0.                TB    = 0.                QRAT  = 6.00000000E-01
PRAT  = 3.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.69010082E+00    WTSUM= 2.43149536E+00

 TAU =   400.000000          DTAU =          10.00000000
TEMPERATURE TABLE
     3.85810390E+02     3.97026593E+02     4.07019439E+02     4.15813122E+02     4.23431293E+02     4.29899477E+02
     4.35246398E+02     4.39504465E+02     4.42710573E+02     4.44906845E+02     4.46141372E+02     4.46468976E+02
     4.45951975E+02     4.44660937E+02     4.42675414E+02     4.40084591E+02     4.36987818E+02     4.33494865E+02
     4.29725589E+02     4.25807544E+02
     4.25807544E+02     4.17706922E+02
     4.17706922E+02     4.17706763E+02     4.17706710E+02

QC    = 5.93556420E+00    QCP   = 5.90376654E+00    QR    = 0.                QRP   = 0.                QRR   = 1.02663009E+00
HE    = 3.72529183E+02    HEP   = 3.71712062E+02    HWO   = 3.78462472E+02    HW    = 3.76284938E+02    X     = 2.97840978E-02
QAERO=-9.45360033E-02    QAEROP=-7.26293083E-02    TF    = 0.                TB    = 0.                QRAT  = 6.00000000E-01
PRAT  = 3.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.69010082E+00    WTSUM= 2.43149536E+00

 TAU =   520.000000          DTAU =          10.00000000
TEMPERATURE TABLE
     3.57272392E+02     3.65298763E+02     3.72668068E+02     3.79406515E+02     3.85538921E+02     3.91089745E+02
     3.96083708E+02     4.00546271E+02     4.04503998E+02     4.07984810E+02     4.11018147E+02     4.13635073E+02
     4.15868291E+02     4.17752099E+02     4.19322266E+02     4.20615840E+02     4.21670855E+02     4.22525968E+02
     4.23220119E+02     4.23791769E+02
     4.23791769E+02     4.24193393E+02
     4.24193393E+02     4.24193401E+02     4.24193404E+02

QC    = 5.32080934E+00    QCP   = 5.26781323E+00    QR    = 0.                QRP   = 0.                QRR   = 7.53598169E-01
HE    = 3.56731518E+02    HEP   = 3.55369650E+02    HWO   = 3.49194595E+02    HW    = 3.47311490E+02    X     = 2.97840978E-02
QAERO= 1.12416547E-01    QAEROP= 1.19449926E-01    TF    = 0.                TB    = 0.                QRAT  = 6.00000000E-01
PRAT  = 3.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.69010082E+00    WTSUM= 2.43149536E+00
```

```
 TAU =   520.000000          DTAU =          10.00000000
TEMPERATURE TABLE
     3.57272392E+02     3.65298763E+02     3.72668068E+02     3.79406515E+02     3.85538921E+02     3.91089745E+02
     3.96083708E+02     4.00546271E+02     4.04503998E+02     4.07984810E+02     4.11018147E+02     4.13635073E+02
     4.15868291E+02     4.17752099E+02     4.19322266E+02     4.20615840E+02     4.21670855E+02     4.22525968E+02
     4.23220119E+02     4.23791769E+02
     4.23791769E+02     4.24193393E+02
     4.24193393E+02     4.24193401E+02     4.24193404E+02

QC    = 5.32080934E+00    QCP   = 5.26781323E+00    QR    = 0.                QRP   = 0.                QRR   = 7.53598169E-01
HE    = 3.56731518E+02    HEP   = 3.55369650E+02    HWO   = 3.49194595E+02    HW    = 3.47311490E+02    X     = 2.97840978E-02
QAERO= 1.12416547E-01    QAEROP= 1.19449926E-01    TF    = 0.                TB    = 0.                QRAT  = 6.00000000E-01
PRAT  = 3.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.24776969E+00    WTSUM= 4.06104930E+00
```

Body point 4 – first iteration

```
 TAU =      .031250          DTAU =            .06250000
TEMPERATURE TABLE
     2.94018166E+02     2.94000096E+02     2.94000001E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02
     2.94000000E+02     2.94000000E+02     2.94000000E+02

QC    = 4.54000000E+00    QCP   = 4.55091346E+00    QR    = 0.                QRP   = 0.                QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO   = 2.83231047E+02    HW    = 2.83231047E+02    X     = 2.43186140E-02
QAERO= 1.30103538E+00    QAEROP= 1.30644173E+00    TF    = 0.                TB    = 0.                QRAT  = 4.00000000E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.24776969E+00    WTSUM= 4.06104930E+00

 TAU =   200.000000          DTAU =            .06250000
TEMPERATURE TABLE
     6.43364073E+02     6.64460950E+02     6.79467203E+02     6.89146885E+02     6.94018851E+02     6.94529761E+02
     6.91081737E+02     6.84027646E+02     6.73674684E+02     6.60292673E+02     6.44120750E+02     6.25370712E+02
     6.04227534E+02     5.80848099E+02     5.55358371E+02     5.27848398E+02     4.98364574E+02     4.66897898E+02
     4.33365369E+02     3.97579122E+02
     3.97579122E+02     3.37763154E+02
     3.37763154E+02     3.37762200E+02     3.37761882E+02

QC    = 1.06267965E+01    QCP   = 1.06212784E+01    QR    = 0.                QRP   = 0.                QRR   = 7.77724231E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO   = 6.42225582E+02    HW    = 6.42225582E+02    X     = 2.43186140E-02
QAERO=-8.16448639E-01    QAEROP=-8.20183735E-01    TF    = 0.                TB    = 0.                QRAT  = 4.90909091E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.24776969E+00    WTSUM= 4.06104930E+00

 TAU =   400.000000          DTAU =           8.00000000
TEMPERATURE TABLE
     3.82934634E+02     3.92325743E+02     4.00782723E+02     4.08328442E+02     4.14987528E+02     4.20785083E+02
     4.25748896E+02     4.29909378E+02     4.33300072E+02     4.35958077E+02     4.37924387E+02     4.39244202E+02
     4.39967203E+02     4.40147769E+02     4.39845106E+02     4.39123253E+02     4.38050930E+02     4.36701162E+02
     4.35150569E+02     4.33478007E+02
     4.33478007E+02     4.29556957E+02
     4.29556957E+02     4.29556875E+02     4.29556848E+02

QC    = 4.62775674E+00    QCP   = 4.60672692E+00    QR    = 0.                QRP   = 0.                QRR   = 9.89457607E-01
HE    = 3.72256809E+02    HEP   = 3.71712062E+02    HWO   = 3.74851284E+02    HW    = 3.73418743E+02    X     = 2.43186140E-02
QAERO=-3.22535368E-02    QAEROP=-2.11513472E-02    TF    = 0.                TB    = 0.                QRAT  = 4.68181818E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.24776969E+00    WTSUM= 4.06104930E+00
```

```
 TAU =   470.000000          DTAU =           10.00000000
TEMPERATURE TABLE
    3.64141207E+02      3.71889532E+02      3.79029854E+02      3.85583235E+02      3.91570624E+02      3.97013568E+02
    4.01934185E+02      4.06355710E+02      4.10302633E+02      4.13800836E+02      4.16877682E+02      4.19562042E+02
    4.21884278E+02      4.23876168E+02      4.25570783E+02      4.27002301E+02      4.28205752E+02      4.29216659E+02
    4.30070609E+02      4.30803479E+02
    4.30803479E+02      4.31402337E+02
    4.31402337E+02      4.31402352E+02      4.31402357E+02

QC    = 4.29513389E+00    QCP   = 4.24390432E+00    QR   = 0.                QRP  = 0.                QRR   = 8.16919088E-01
HE    = 3.63540856E+02    HEP   = 3.62178988E+02    HWO  = 3.56615348E+02    HW   = 3.54271318E+02    X     = 2.43186140E-02
QAERO= 8.18229423E-02   QAEROP= 9.26596965E-02    TF   = 0.                TB   = 0.                QRAT  = 4.60227273E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA = 2.00000000E-01    WTBP = 4.24776969E+00    WTSUM= 4.06104930E+00
```

Body point 4 – final iteration

```
 TAU =      .031250          DTAU =            .06250000
TEMPERATURE TABLE
    2.94018143E+02      2.94000083E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02
    2.94000000E+02      2.94000000E+02      2.94000000E+02

QC    = 4.54000000E+00    QCP   = 4.55091346E+00    QR   = 0.                QRP  = 0.                QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO  = 2.83231047E+02    HW   = 2.83231047E+02    X     = 2.61192039E-02
QAERO= 1.30103538E+00   QAEROP= 1.30644173E+00    TF   = 0.                TB   = 0.                QRAT  = 4.00000000E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA = 2.00000000E-01    WTBP = 4.24776969E+00    WTSUM= 4.06104930E+00

 TAU =   200.000000          DTAU =            .06250000
TEMPERATURE TABLE
    6.43501218E+02      6.66185599E+02      6.81927031E+02      6.91642084E+02      6.95970483E+02      6.95470084E+02
    6.90637573E+02      6.81907285E+02      6.69661328E+02      6.54241577E+02      6.35957447E+02      6.15089160E+02
    5.91888035E+02      5.66574750E+02      5.39335121E+02      5.10312918E+02      4.79598793E+02      4.47213004E+02
    4.13078297E+02      3.76974935E+02
    3.76974935E+02      3.24943689E+02
    3.24943689E+02      3.24942910E+02      3.24942650E+02

QC    = 1.06267965E+01    QCP   = 1.06212784E+01    QR   = 0.                QRP  = 0.                QRR   = 7.78386582E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO  = 6.42371416E+02    HW   = 6.42371416E+02    X     = 2.61192039E-02
QAERO=-8.19047115E-01   QAEROP=-8.22781805E-01    TF   = 0.                TB   = 0.                QRAT  = 4.90909091E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA = 2.00000000E-01    WTBP = 4.24776969E+00    WTSUM= 4.06104930E+00

 TAU =   400.000000          DTAU =           8.00000000
TEMPERATURE TABLE
    3.83747485E+02      3.93981313E+02      4.03108383E+02      4.11150592E+02      4.18131485E+02      4.24076238E+02
    4.29013255E+02      4.32974387E+02      4.35995571E+02      4.38117389E+02      4.39385608E+02      4.39851758E+02
    4.39573713E+02      4.38616260E+02      4.37051613E+02      4.34959831E+02      4.32429056E+02      4.29555449E+02
    4.26442638E+02      4.23200308E+02
    4.23200308E+02      4.16831113E+02
    4.16831113E+02      4.16830987E+02      4.16830945E+02

QC    = 4.62775674E+00    QCP   = 4.60672692E+00    QR   = 0.                QRP  = 0.                QRR   = 9.98081146E-01
HE    = 3.72256809E+02    HEP   = 3.71712062E+02    HWO  = 3.75697983E+02    HW   = 3.74242844E+02    X     = 2.61192039E-02
QAERO=-4.27793774E-02   QAEROP=-3.13646580E-02    TF   = 0.                TB   = 0.                QRAT  = 4.68181818E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA = 2.00000000E-01    WTBP = 4.24776969E+00    WTSUM= 4.06104930E+00
```

```
 TAU =   500.000000         DTAU =          10.00000000
TEMPERATURE TABLE
    3.58326932E+02    3.65964060E+02    3.72984429E+02    3.79408214E+02    3.85255518E+02    3.90547030E+02
    3.95304478E+02    3.99550863E+02    4.03310709E+02    4.06610214E+02    4.09477346E+02    4.11941882E+02
    4.14035392E+02    4.15791167E+02    4.17244074E+02    4.18430354E+02    4.19387328E+02    4.20153064E+02
    4.20766048E+02    4.21264304E+02
    4.21264304E+02    4.21490279E+02
    4.21490279E+02    4.21490282E+02    4.21490283E+02

QC    = 4.14204740E+00    QCP   = 4.09142006E+00    QR    = 0.                QRP   = 0.                QRR   = 7.64114673E-01
HE    = 3.59455253E+02    HEP   = 3.58093385E+02    HWO   = 3.50458639E+02    HW    = 3.48389578E+02    X     = 2.61192039E-02
QAERO= 1.03669097E-01     QAEROP= 1.10871507E-01    TF    = 0.                TB    = 0.                QRAT  = 4.56818182E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 4.24776969E+00    WTSUM= 4.06104930E+00

 TAU =   500.000000         DTAU =          10.00000000
TEMPERATURE TABLE
    3.58326932E+02    3.65964060E+02    3.72984429E+02    3.79408214E+02    3.85255518E+02    3.90547030E+02
    3.95304478E+02    3.99550863E+02    4.03310709E+02    4.06610214E+02    4.09477346E+02    4.11941882E+02
    4.14035392E+02    4.15791167E+02    4.17244074E+02    4.18430354E+02    4.19387328E+02    4.20153064E+02
    4.20766048E+02    4.21264304E+02
    4.21264304E+02    4.21490279E+02
    4.21490279E+02    4.21490282E+02    4.21490283E+02

QC    = 4.14204740E+00    QCP   = 4.09142006E+00    QR    = 0.                QRP   = 0.                QRR   = 7.64114673E-01
HE    = 3.59455253E+02    HEP   = 3.58093385E+02    HWO   = 3.50458639E+02    HW    = 3.48389578E+02    X     = 2.61192039E-02
QAERO= 1.03669097E-01     QAEROP= 1.10871507E-01    TF    = 0.                TB    = 0.                QRAT  = 4.56818182E-01
PRAT  = 1.60000000E-01    WTCI  = 3.90000000E+00    AREA  = 2.00000000E-01    WTBP  = 3.89593988E+00    WTSUM= 5.62023727E+00
```

Body point 5 – first iteration

```
 TAU =      .031250         DTAU =           .06250000
TEMPERATURE TABLE
    2.94007870E+02    2.94000069E+02    2.94000001E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02
    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02
    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02    2.94000000E+02
    2.94000000E+02    2.94000000E+02
    2.94000000E+02    2.94000000E+02
    2.94000000E+02    2.94000000E+02    2.94000000E+02

QC    = 2.27000000E+00    QCP   = 2.27545673E+00    QR    = 0.                QRP   = 0.                QRP   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278946E+02    HWO   = 2.83231047E+02    HW    = 2.83231047E+02    X     = 1.84690662E-02
QAERO= 6.50517691E-01     QAEROP= 6.53220867E-01    TF    = 0.                TB    = 0.                QRAT  = 2.00000000E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 2.00000000E-01    WTBP  = 3.89593988E+00    WTSUM= 5.62023727E+00

 TAU =   200.000000         DTAU =           .06250000
TEMPERATURE TABLE
    6.02251772E+02    6.15266672E+02    6.24734057E+02    6.30972119E+02    6.34233911E+02    6.34732938E+02
    6.32658036E+02    6.28176629E+02    6.21435819E+02    6.12564224E+02    6.01674245E+02    5.88864057E+02
    5.74219004E+02    5.57812429E+02    5.39706128E+02    5.19950653E+02    4.98585576E+02    4.75639818E+02
    4.51132394E+02    4.25074843E+02
    4.25074843E+02    3.65329697E+02
    3.65329697E+02    3.65328636E+02    3.65328282E+02

QC    = 6.29739221E+00    QCP   = 6.29409091E+00    QR    = 0.                QRP   = 0.                QRR   = 5.97183967E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO   = 5.98441416E+02    HW    = 5.98441416E+02    X     = 1.84690662E-02
QAERO=-2.15097994E-02     QAEROP=-2.37951163E-02    TF    = 0.                TB    = 0.                QRAT  = 2.90909091E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 2.00000000E-01    WTBP  = 3.89593988E+00    WTSUM= 5.62023727E+00
```

```
 TAU =   398.000000           DTAU =          10.00000000
TEMPERATURE TABLE
    3.67546076E+02     3.74703877E+02     3.81294194E+02     3.87341958E+02     3.92869771E+02     3.97900893E+02
    4.02458539E+02     4.06565208E+02     4.10248018E+02     4.13528836E+02     4.16434360E+02     4.18991180E+02
    4.21226799E+02     4.23169614E+02     4.24848850E+02     4.26294452E+02     4.27526945E+02     4.28607273E+02
    4.29536556E+02     4.30355411E+02
    4.30355411E+02     4.30996637E+02
    4.30996637E+02     4.30996649E+02     4.30996653E+02

QC    = 2.68079211E+00    QCP   = 2.64578256E+00    QR   = 0.                QRP  = 0.                QRR   = 8.56720130E-01
HE    = 3.73346304E+02    HEP   = 3.71984436E+02    HWO  = 3.61061457E+02    HW   = 3.57677870E+02    X     = 1.84690662E-02
QAERO= 8.82106490E-02    QAEROP= 1.01757111E-01    TF   = 0.                TB   = 0.                QRAT  = 2.68409091E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA = 2.00000000E-01    WTBP = 3.89593988E+00    WTSUM= 5.62023727E+00
```

Body point 5 – final iteration

```
 TAU =      .031250           DTAU =            .06250000
TEMPERATURE TABLE
    2.94007860E+02     2.94000060E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02

QC    = 2.27000000E+00    QCP   = 2.27545673E+00    QR   = 0.                QRP  = 0.                QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO  = 2.83231047E+02    HW   = 2.83231047E+02    X     = 1.97771316E-02
QAERO= 6.50517691E-01    QAEROP= 6.53220867E-01    TF   = 0.                TB   = 0.                QRAT  = 2.00000000E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA = 2.00000000E-01    WTBP = 3.89593988E+00    WTSUM= 5.62023727E+00

 TAU =   200.000000           DTAU =            .06250000
TEMPERATURE TABLE
    6.02814889E+02     6.17200566E+02     6.27545695E+02     6.34245675E+02     6.37613409E+02     6.37914441E+02
    6.35382808E+02     6.30224436E+02     6.22619384E+02     6.12724987E+02     6.00678929E+02     5.86601480E+02
    5.70596740E+02     5.52753075E+02     5.33143015E+02     5.11822665E+02     4.88830585E+02     4.64186186E+02
    4.37887734E+02     4.09910397E+02
    4.09910397E+02     3.51607458E+02
    3.51607458E+02     3.51606480E+02     3.51606154E+02

QC    = 6.29739221E+00    QCP   = 6.29409091E+00    QR   = 0.                QRP  = 0.                QRR   = 5.99418326E+00
HE    = 5.96404297E+02    HEP   = 5.96187500E+02    HWO  = 5.99040502E+02    HW   = 5.99040502E+02    X     = 1.97771316E-02
QAERO=-2.78355116E-02    QAEROP=-3.01198114E-02    TF   = 0.                TB   = 0.                QRAT  = 2.90909091E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA = 2.00000000E-01    WTBP = 3.89593988E+00    WTSUM= 5.62023727E+00

 TAU =   400.000000           DTAU =           4.00000000
TEMPERATURE TABLE
    3.67301492E+02     3.74743859E+02     3.81532855E+02     3.87692509E+02     3.93246795E+02     3.98219533E+02
    4.02635307E+02     4.06519666E+02     4.09899379E+02     4.12802662E+02     4.15259333E+02     4.17300910E+02
    4.18960677E+02     4.20273686E+02     4.21276713E+02     4.22009140E+02     4.22507775E+02     4.22816555E+02
    4.22976090E+02     4.23028191E+02
    4.23028191E+02     4.22200894E+02
    4.22200894E+02     4.22200879E+02     4.22200874E+02

QC    = 2.64578256E+00    QCP   = 2.63880474E+00    QR   = 0.                QRP  = 0.                QRR   = 8.31195610E-01
HE    = 3.71984436E+02    HEP   = 3.71712062E+02    HWO  = 3.58228471E+02    HW   = 3.57584605E+02    X     = 1.97771316E-02
QAERO= 9.78409029E-02    QAEROP= 1.00291609E-01    TF   = 0.                TB   = 0.                QRAT  = 2.68181818E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA = 2.00000000E-01    WTBP = 3.89593988E+00    WTSUM= 5.62023727E+00
```

```
 TAU =  420.000000          DTAU =          10.00000000
TEMPERATURE TABLE
    3.61519314E+02     3.68681248E+02     3.75261145E+02     3.81282349E+02     3.86767721E+02     3.91739796E+02
    3.96221558E+02     4.00236611E+02     4.03809385E+02     4.06965300E+02     4.09730878E+02     4.12133793E+02
    4.14202896E+02     4.15968189E+02     4.17460756E+02     4.18712649E+02     4.19756716E+02     4.20626364E+02
    4.21355147E+02     4.21976207E+02
    4.21976207E+02     4.22285368E+02
    4.22285368E+02     4.22285375E+02     4.22285377E+02

QC    = 2.60403608E+00    QCP   = 2.56946816E+00    QR    = 0.               QRP   = 0.               QRR   = 7.98921841E-01
HE    = 3.70350195E+02    HEP   = 3.68988327E+02    HWO   = 3.54551408E+02   HW    = 3.51588707E+02   X     = 1.97771316E-02
QAERO= 1.11085700E-01     QAEPOP= 1.21163102E-01    TF    = 0.               TB    = 0.               QRAT  = 2.65909091E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 2.00000000E-01   WTBP  = 3.89593988E+00   WTSUM= 5.62023727E+00

 TAU =  420.000000          DTAU =          10.00000000
TEMPERATURE TABLE
    3.61519314E+02     3.68681248E+02     3.75261145E+02     3.81282349E+02     3.86767721E+02     3.91739796E+02
    3.96221558E+02     4.00236611E+02     4.03809385E+02     4.06965300E+02     4.09730878E+02     4.12133793E+02
    4.14202896E+02     4.15968189E+02     4.17460756E+02     4.18712649E+02     4.19756716E+02     4.20626364E+02
    4.21355147E+02     4.21976207E+02
    4.21976207E+02     4.22285368E+02
    4.22285368E+02     4.22285375E+02     4.22285377E+02

QC    = 2.60403608E+00    QCP   = 2.56946816E+00    QR    = 0.               QRP   = 0.               QRR   = 7.98921841E-01
HE    = 3.70350195E+02    HEP   = 3.68988327E+02    HWO   = 3.54551408E+02   HW    = 3.51588707E+02   X     = 1.97771316E-02
QAERO= 1.11085700E-01     QAEPOP= 1.21163102E-01    TF    = 0.               TB    = 0.               QRAT  = 2.65909091E-01
PRAT  = 4.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 2.00000000E-01   WTBP  = 3.28710094E+00   WTSUM= 6.86365746E+00
```

Body point 6

```
 TAU =     .031250          DTAU =           .06250000
TEMPERATURE TABLE
    2.93999673E+02     2.93999995E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02
    2.94000000E+02     2.94000000E+02     2.94000000E+02

QC    = 1.13500000E+00    QCP   = 1.13772837E+00    QR    = 0.               QRP   = 0.               QRR   = 3.38869510E-01
HE    = 3.97000000E+02    HEP   = 3.97278846E+02    HWO   = 2.83231047E+02   HW    = 2.83231047E+02   X     = 1.39845439E-02
QAERO= 3.25258846E-01     QAEPOP= 3.26610434E-01    TF    = 0.               TB    = 0.               QRAT  = 1.00000000E-01
PRAT  = 1.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 1.50000000E-01   WTBP  = 3.28710094E+00   WTSUM= 6.86365746E+00

 TAU =  200.000000          DTAU =          4.03125000
TEMPERATURE TABLE
    5.66783138E+02     5.71562992E+02     5.74596040E+02     5.76007563E+02     5.75888049E+02     5.74310104E+02
    5.71238678E+02     5.67034889E+02     5.61456585E+02     5.54657968E+02     5.46689375E+02     5.37597499E+02
    5.27425994E+02     5.16216304E+02     5.04008675E+02     4.90843336E+02     4.76761924E+02     4.61809285E+02
    4.46035742E+02     4.29499733E+02
    4.29499733E+02     3.77913015E+02
    3.77913015E+02     3.77912069E+02     3.77911753E+02

QC    = 3.89972839E+00    QCP   = 3.21900534E+00    QR    = 0.               QRP   = 0.               QRP   = 5.14429200E+00
HE    = 6.29222222E+02    HEP   = 5.96187500E+02    HWO   = 5.74963794E+02   HW    = 5.60217459E+02   X     = 1.39845439E-02
QAERO= 3.36277270E-01     QAEROP= 1.94213658E-01    TF    = 0.               TB    = 0.               QRAT  = 1.48780488E-01
PRAT  = 1.00000000E-02    WTCI  = 2.93000000E+00    AREA  = 1.50000000E-01   WTBP  = 3.28710094E+00   WTSUM= 6.86365746E+00
```

```
 TAU =  360.000000          DTAU =          10.00000000
TEMPERATURE TABLE
    3.72990923E+02      3.78186924E+02      3.82980739E+02      3.87405650E+02      3.91475740E+02      3.95208044E+02
    3.98621079E+02      4.01733104E+02      4.04561951E+02      4.07125319E+02      4.09440998E+02      4.11526949E+02
    4.13401305E+02      4.15082356E+02      4.16588523E+02      4.17938322E+02      4.19150305E+02      4.20243026E+02
    4.21235067E+02      4.22144883E+02
    4.22144883E+02      4.23173902E+02
    4.23173902E+02      4.23173925E+02      4.23173932E+02

QC    = 1.43561289E+00    QCP  = 1.41706425E+00    QR   = 0.                QRP  = 0.                QRR  = 9.18671000E-01
HE    = 3.78521401E+02    HEP  = 3.77159533E+02    HWO  = 3.67683178E+02    HW   = 3.63196422E+02    X    = 1.39845439E-02
QAERO= 4.11059781E-02    QAEROP= 5.24622194E-02   TF   = 0.                TB   = 0.                QRAT = 1.39024390E-01
PRAT  = 1.00000000E-02    WTCI = 2.93000000E+00    AREA = 1.50000000E-01    WTBP = 3.28710094E+00    WTSUM= 6.86365746E+00

 TAU =  360.000000          DTAU =          10.00000000
TEMPERATURE TABLE
    3.72990923E+02      3.78186924E+02      3.82980739E+02      3.87405650E+02      3.91475740E+02      3.95208044E+02
    3.98621079E+02      4.01733104E+02      4.04561951E+02      4.07125319E+02      4.09440998E+02      4.11526949E+02
    4.13401305E+02      4.15082356E+02      4.16588523E+02      4.17938322E+02      4.19150305E+02      4.20243026E+02
    4.21235067E+02      4.22144883E+02
    4.22144883E+02      4.23173902E+02
    4.23173902E+02      4.23173925E+02      4.23173932E+02

QC    = 1.43561289E+00    QCP  = 1.41706425E+00    QR   = 0.                QRP  = 0.                QRR  = 9.18671000E-01
HE    = 3.78521401E+02    HEP  = 3.77159533E+02    HWO  = 3.67683178E+02    HW   = 3.63196422E+02    X    = 1.39845439E-02
QAERO= 4.11059781E-02    QAEROP= 5.24622194E-02   TF   = 0.                TB   = 0.                QRAT = 1.39024390E-01
PRAT  = 1.00000000E-02    WTCI = 2.93000000E+00    AREA = 1.50000000E-01    WTBP = 2.73101251E+00    WTSUM= 7.71280934E+00
```

REFERENCES

1. Swann, Robert T.; Pittman, Claud M.; and Smith, James C.: One-Dimensional Numerical Analysis of the Transient Response of Thermal Protection Systems. NASA TN D-2976, 1965.

2. Curry, Donald M.: An Analysis of a Charring Ablation Thermal Protection System. NASA TN D-3150, 1965.

3. Clark, Ronald K.: An Analysis of a Charring Ablator With Thermal Nonequilibrium, Chemical Kinetics, and Mass Transfer. NASA TN D-7180, 1973.

4. Williams, S. D.; and Curry, D. M.: Parameter Optimization : An Aid to Thermal Protection Design. J. Spacecr. & Rockets, vol. 9, no. 1, Jan. 1972, pp. 33-38.

5. Hovanessian, Shahen A.; and Pipes, Louis A.: Digital Computer Methods in Engineering. McGraw-Hill Book Co., Inc., c.1969.

6. Brooks, William A., Jr.: Temperature and Thermal-Stress Distributions in Some Structural Elements Heated at a Constant Rate. NACA TN 4306, 1958.

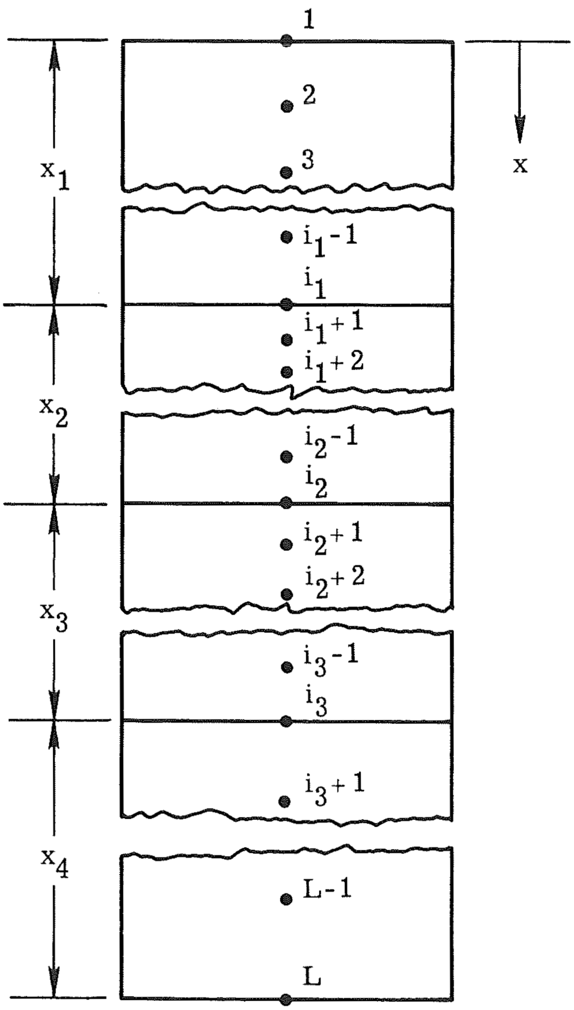
7. Giedt, Warren H.: Principles of Engineering Heat Transfer. D. Van Nostrand Co., Inc., c.1957.

TABLE I.- COMPARISON OF EXACT AND CALCULATED TEMPERATURES FOR CONSTANT PROPERTY SLAB[a]

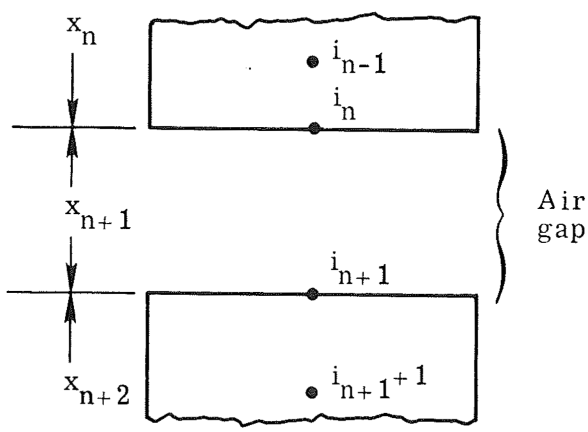
Number of layers	Error criterion $\left(\text{ERROR} < \frac{T' - T''}{T'}\right)$	Maximum error $\left(\frac{\left(T_{exact} - T_{calc}\right) \times 100}{T_{exact}}\right)$, percent
1	0.001	0.5
1	.0001	.005
4	.001	.6
4	.0001	.005
[b]4	.001	.76
	.0001	.64

[a]The total thickness is the same for all cases. In the first two cases, 41 stations were used. For the remaining cases, the first layer had 11 stations and all other layers had 10 stations except that when the air-gap equations were used, the third layer had only 1 station.

[b]The air-gap equations are used in the third layer.



(a) Grid point arrangement.



(b) Air-gap configuration.

Figure 1.- Locations of finite-difference stations.

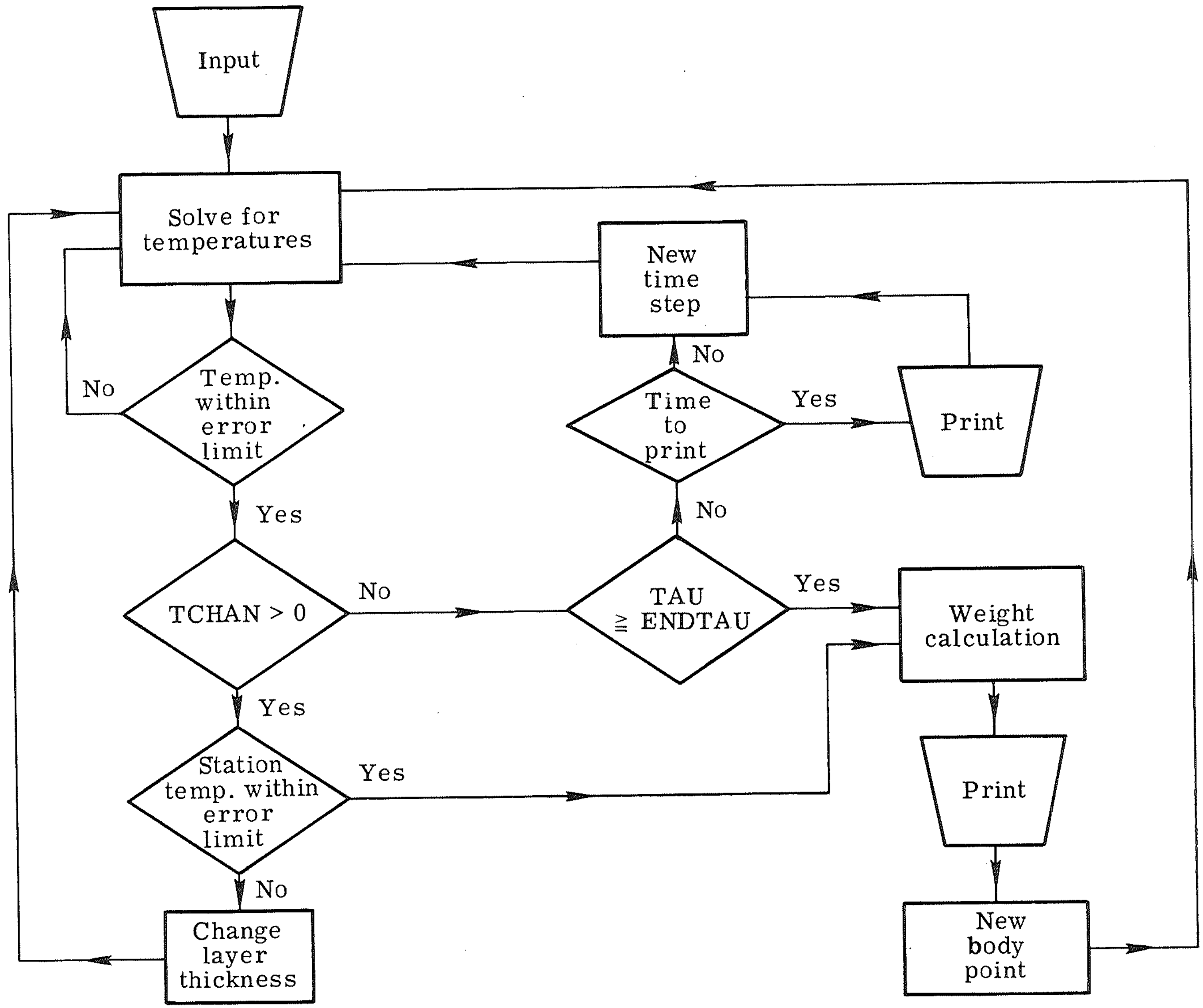


Figure 2.- Simplified program flow chart.

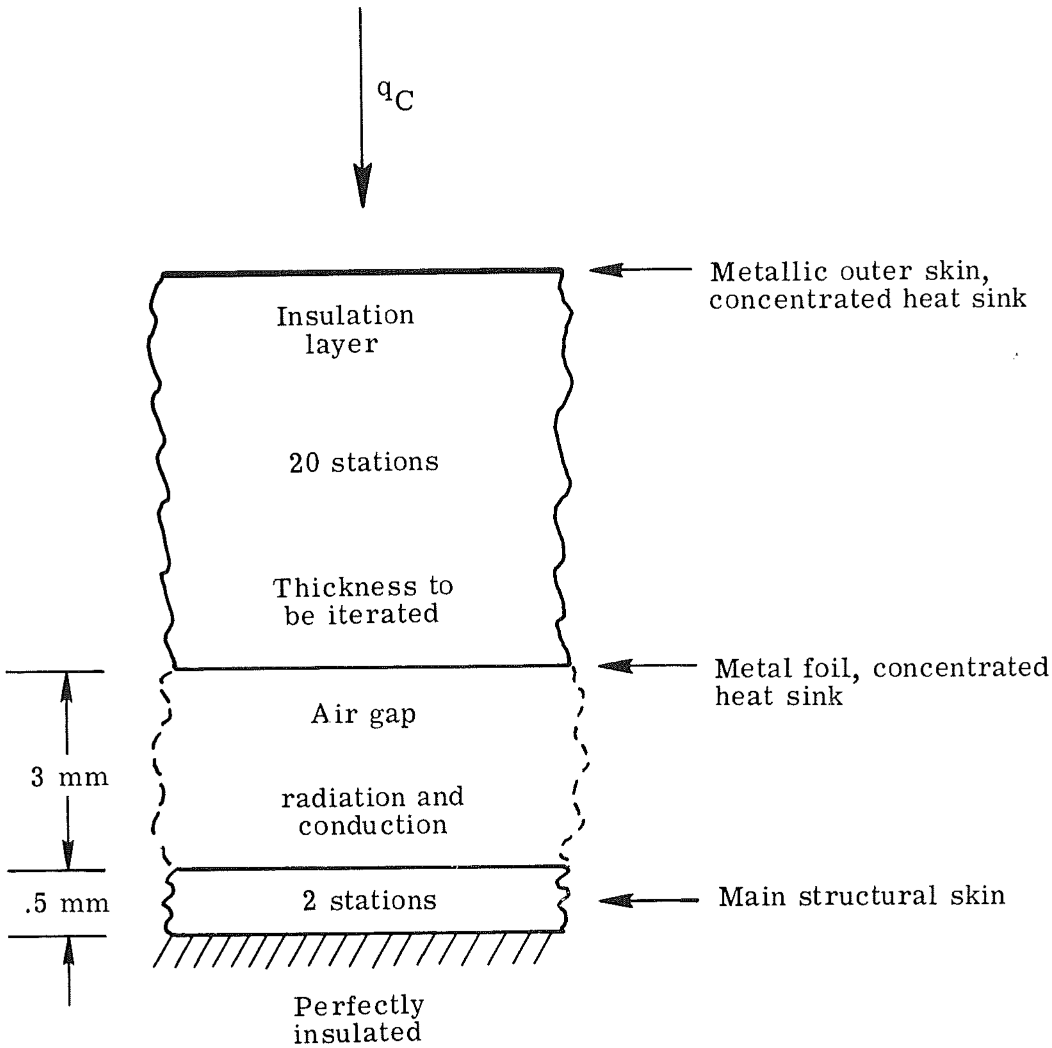


Figure 3.- Computer program model of sample case heat shield system.

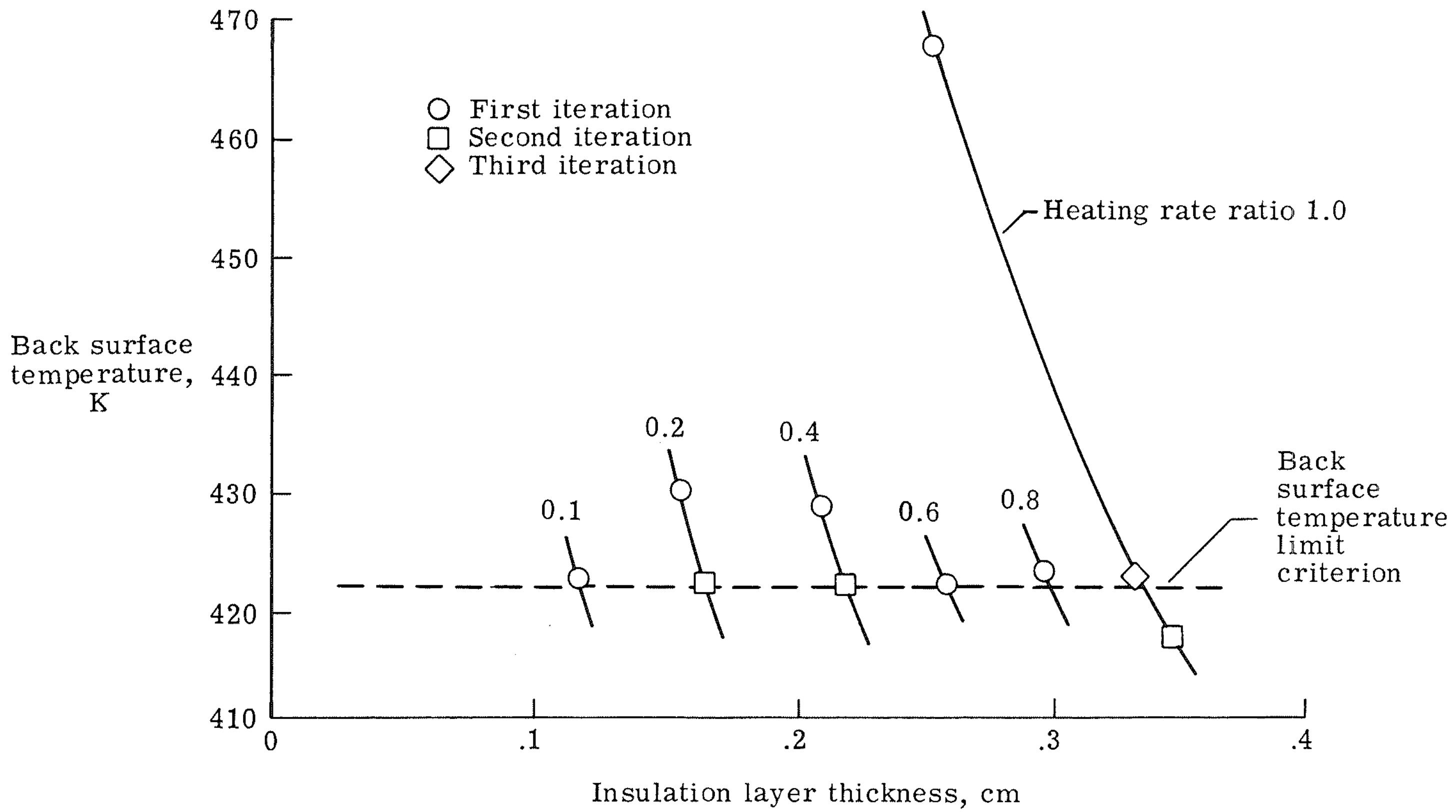


Figure 4.- Insulation layer thickness iterations for sample case.

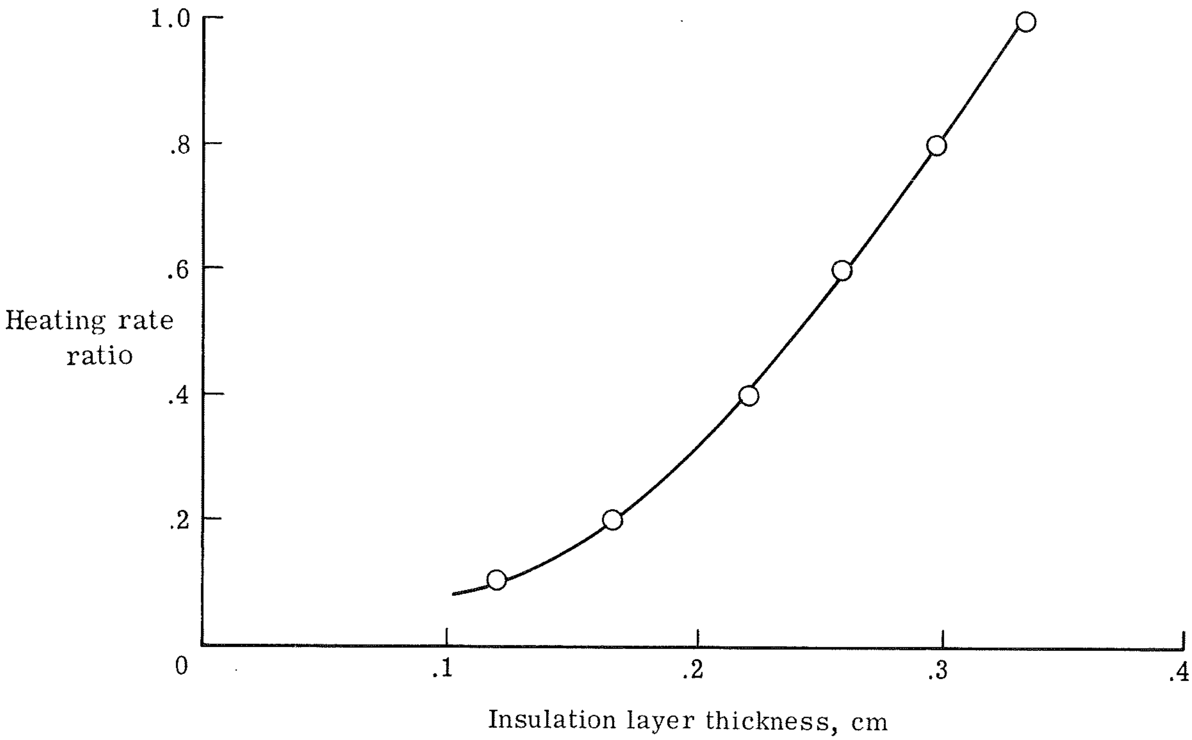


Figure 5.- Final insulation layer thickness for each heating rate ratio.

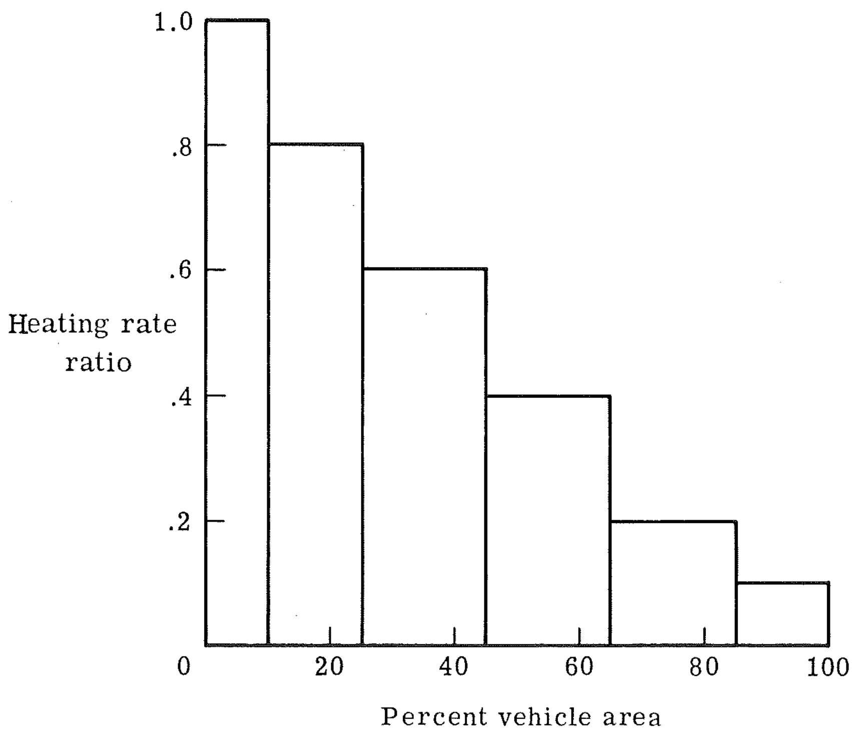


Figure 6.- Assumed heating rate distribution over vehicle.

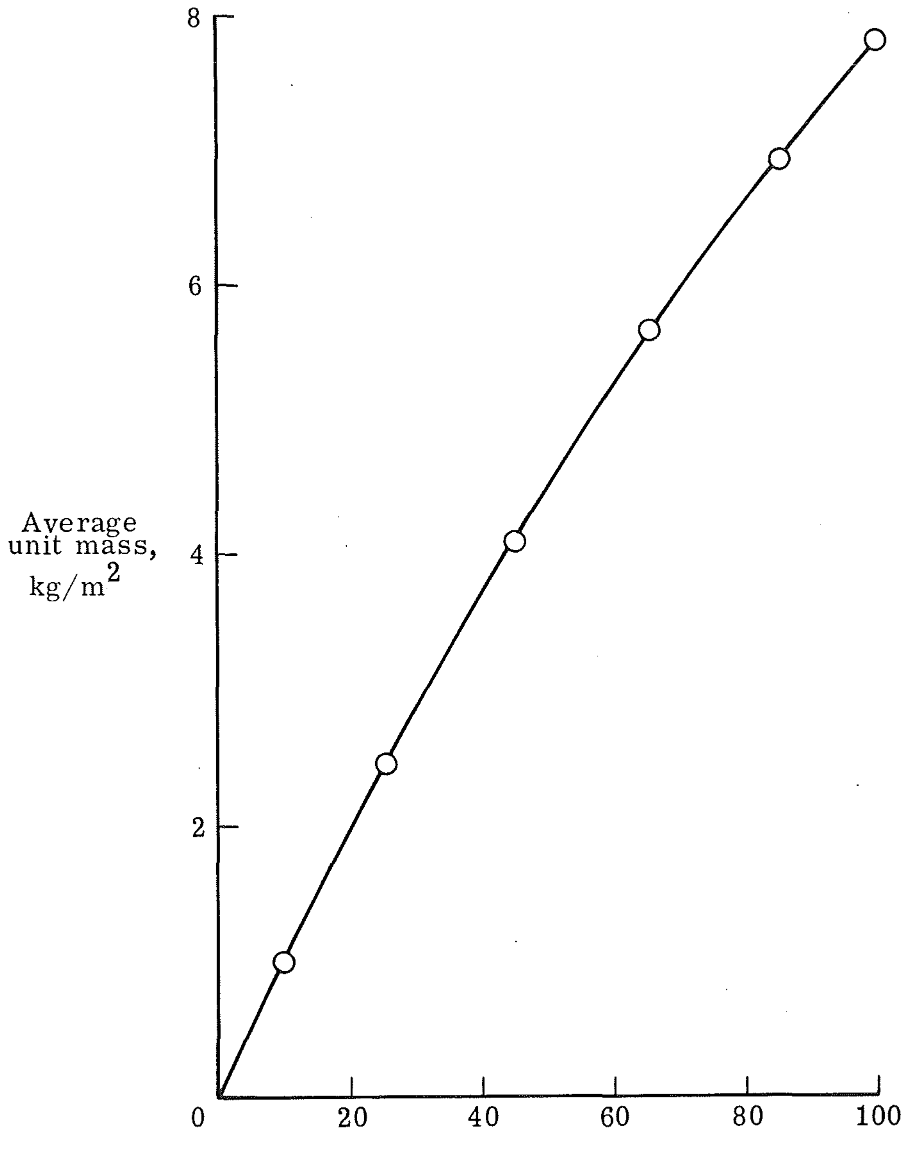


Figure 7.- Cumulative average heat shield unit mass for assumed heating rate distribution.

NASA-Langley, 1976 L-10637

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE $300

SPECIAL FOURTH-CLASS RATE
BOOK

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION
451

U.S.MAIL

POSTMASTER : If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546