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FACE-SEAL LUBRICATION

I - Proposed and Published Models

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16. Abstract

The numerous published theories on the mechanism of hydrodynamic lubrication of face seals were reviewed. These theories employ either an inclined-slider-bearing macrogeometry (e.g., surface waves) or an inclined-slider-bearing microgeometry (e.g., asperity tips) to produce hydrodynamic pressure that separates the surfaces of the primary seal. Secondary-seal friction and primary-ring inertia effects are not considered. Hypothetical seal operating models were devised to include secondary-seal friction and primary-ring inertia effects. It was hypothesized that these effects induce relative angular misalinement of the primary-seal faces and that this misalinement is, in effect, an inclined-slider macrogeometry. Stable running was postulated for some of these hypothetical operating models. In others, periodic loss of hydrodynamic lubrication was postulated to be possible with certain combinations of waviness and angular misalinement. Application of restrictions that apply to seal operation led to a hydrodynamic governing equation for the new model that is a two-dimensional, time-dependent Reynolds equation with the short-bearing approximation.

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FACE-SEAL LUBRICATION

I - PROPOSED AND PUBLISHED MODELS

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SUMMARY

There is considerable reported evidence that in a large class of radial face seals, hydrodynamic lubrication is responsible for maintaining separation of the seal faces. Many theories to explain this seal lubrication have been developed and reported (angular misalinement, surface waviness, etc.). These theories are based on the inclined-slider-bearing principle and can be divided into two general groups, the inclined-slider macrogeometry and the inclined-slider microgeometry. These published theories have given very valuable insight as to the nature of seal lubrication. However, the response of the primary-seal ring to the combined effect of seal-face angular misalinement, secondary-seal friction, and primary-ring inertia must be considered for construction of a seal model that has general application.

Hypothetical seal operating models were devised having secondary-seal friction, primary-ring inertia, and primary-ring degrees of freedom that simulate actual seal operation. These new seal models employ either angular misalinement, surface waviness, or both, as the mechanism that generates hydrodynamic pressure. Stable operation was hypothesized for the angular-misalinement models and the surface-waviness models. On the other hand, the combined angular-misalinement and surface-waviness models are complex because of changing phase relations between misalinement and waviness. Thus, it was postulated that periodic loss of hydrodynamic lubrication is possible under certain combinations of waviness and misalinement.

The hydrodynamic governing equation for the new seal models was obtained by applying restrictions, applicable to seal operation, to a generalized Reynolds equation in order to obtain a two-dimensional, time-dependent form with the short-bearing approximation. This equation, together with Newton's laws of motion, forms the set of governing equations for the new seal models.

INTRODUCTION

It has been established (refs. 1 and 2) that radial face seals for liquids can operate with a lubricating film between the seal faces. There have been many hypotheses put forth to explain the mechanism (or mechanisms) responsible for the development of the lubricating film pressure that acts to separate the primary-seal faces. These hypotheses include the following: surface angular misalinement (ref. 3), surface waviness (ref. 4), surface asperities (ref. 5), vaporization of the fluid film (ref. 1), axial vibration (ref. 6), and thermal deformation (ref. 7).

Very convincing evidence is put forward in reference 8 that surface waviness is a strong factor in development of lubricating film pressure. It is argued that when one of the faces is carbon-graphite, a surface wave develops in the process of running. Also, surface waves lapped into one of the faces have been used successfully in seals for gases (ref. 9). Thus, there is strong evidence that surface waviness can develop the hydrodynamic pressures necessary to separate the seal faces.

However, other mechanisms (such as those previously cited) can also produce hydrodynamic pressures, and which one dominates may vary from one application to the next. Even within the same application the difference between one seal assembly and the next can be appreciable. For example, seal assembly inspections have revealed that waviness induced by clamping forces, as well as angular misalinement produced in the assembly, may readily vary by a factor of 2 or more. Thus, successful seal operation may depend on one or more factors (waviness, etc.) that are unplanned. Therefore, engineering of a seal for a new application, especially one that operates near the limits of the state of the art, sometimes requires extensive development. In addition, the continuing field problems with face seals suggest that insight into actual seal operation may be lacking.

The objective of this work is to formulate models that represent possible seal operating configurations and to compare these models to the theories of seal lubrication that have been reported in the literature. A further objective is to review the fundamental assumptions that are used in formulating the fluid dynamics of seal lubrication and to arrive at the applicable governing equations. The study is restricted to those seal models and mechanisms that explain fluid-film lubrication of the primary seal. (Leakage, torque, power loss, etc., are not considered.) Further, the study concentrates on seals for low-pressure applications. Therefore, hydrodynamic rather than hydrostatic effects are expected to be of major importance.

BACKGROUND

Arrangements of Seal Components

A conventional arrangement of seal-face components is shown in figures 1 and 2. In some designs the seat rotates and the primary ring is the nonrotating member. The primary ring (nose) has axial flexibility, contains the primary and secondary seal diameters, and also has angular flexibility. The axial flexibility allows accommodation of axial displacements that arise for various reasons (e.g., face runout, thermal growth differences and tolerance variations). The secondary ring can be of various designs, such as elastomeric "O"-rings, bellows, or piston rings.

Nomenclature Applying to Single Parts

Seat	part having a primary seal face and being mechanically constrained with respect to axial motion (e.g., attached to the shaft or housing)
Primary ring	ring having a primary-seal face, flexibility in axial and angular modes, and diameters that define the radial extent of the primary seal
Secondary ring	part (O-ring, piston ring) having secondary-seal surfaces that mate to the secondary-seal surfaces of the primary ring and housing

Nomenclature Applying to an Assembly of Parts

Primary seal	seal formed by the seal faces of the seat and primary ring; relative rotation occurs between these seal faces
Secondary seal	seal formed by the secondary-seal faces of the secondary ring and the primary ring (in the case of a bellows, the secondary seal is the bellows itself)
Film thickness, h	distance between primary-seal faces (h may vary with radial or circumferential position and with time)

Primary-Seal Geometry

Since lubricating films are very thin, in the range of 1 micrometer (40 μin.), very small relative displacements and motions of the primary-seal faces can be significant.

Figure 3 contains some possible primary-seal-face geometries that can occur in operation because of various types of small displacements and motions.

Waviness (geometry a, fig. 3) of the primary-seal faces is considered by some (refs. 10 and 11) to be the principal cause of pressure generation in the fluid film between the seal faces. This surface waviness is reported to come about either in the lapping process or by normal wear during seal operation. Other experience has indicated that thermal deformation (ref. 7) and mechanically induced assembly deformation can also cause waviness.

Angular misalinement (geometry b, fig. 3) of the primary-seal faces, which has been assumed to exist by some (e.g., refs. 3 and 6), causes hydrodynamic fluid-film pressure because of the circumferential converging-diverging seal gap that is created by the angular misalinement. (A basis for the existence of angular misalinement is provided in section Proposed Seal Operating Models.)

Coning (geometry c, fig. 3) commonly arises because of thermal gradients or clamping distortion. This type of deformation affects the seal hydrostatic pressure balance, and therefore is a mechanism that can produce surface separation.

Axial vibration of the seal (geometry d, fig. 3), either externally induced or as a result of seal response, can produce squeeze-film effects. Seal lubrication by the mechanism of axial vibration is considered in reference 6.

Geometries e and f (fig. 3) both induce a radial velocity component. This radial velocity component can affect leakage rate, and perhaps lubrication, since radial velocity can affect the transport of liquid into or out of the primary seal.

RESULTS AND DISCUSSION

Published Seal Models and Governing Equations

Seal models. - As previously mentioned, various mathematical models have been devised to explain the mechanism by which seal faces are separated by a liquid film. The hydrodynamic force (or pressure generation) that occurs in operation of liquid-lubricated face seals is probably an important factor in high-pressure-seal lubrication and is exclusively responsible for lubrication in many classes of low-pressure sealing.

As noted previously, these existing models for hydrodynamic lubrication of face seals are based on the inclined (or wedge)-slider-bearing principle. The inclined slider is formed by one or more of the following: (1) face angular misalinement (refs. 3, 6, 12, and 13); (2) surface waviness (refs. 4, 6, 10, 11, and 12); (3) surface asperity tips or micropads (refs. 5 and 14); and (4) thermal deformation (refs. 7, 15, and 16). These models are summarized in tables I and II and discussed in detail in appendix A. In general, the various hydrodynamic models are based on the same fundamental assumptions that are found in fluid-film bearing theory. These assumptions are (table III):

(1) Incompressible fluid
(2) Constant viscosity
(3) Pressure constant through film thickness (z-direction)
(4) No body forces
(5) Inertia forces negligible compared to viscous forces
(6) Velocity gradients with respect to the x-direction and the y-direction negligible compared to the velocity gradient across the film thickness
(7) Newtonian fluid
(8) Laminar flow

In all analyses cited (tables I and II) the effects of secondary-seal friction and primary-ring inertia are not considered. Also the models generally do not admit angular degrees of freedom that actual seals possess and are based on assumptions of either constant centerline film thickness or constant load (tables I and II). Neither assumption is applicable to the several cases of seal operation (see section Proposed Seal Operating Models). The models do, on the other hand, provide very valuable insight as to possible hydrodynamic mechanisms of seal operation.

Seal governing equations. - The studies on liquid seal lubrication are based on the Reynolds equation for lubrication or on simplified Navier-Stokes equations. These studies employ either Cartesian or cylindrical coordinate systems. The argument for use of the Cartesian system is based on the fact that the radial seal faces are narrow in respect to the seal radius and that therefore the circumferential motion, which is the predominant motion, can be approximated by simple rectilinear motion, as indicated in figure 4. Motion is assumed to take place only in the x-direction. As noted previously, motion in the y-direction (e.g., parallel misalinement and whirl) affects leakage and need not be brought into the mathematical model for calculation of hydrodynamic pressure.

The Reynolds equation can be derived from a simplified form of the Navier-Stokes momentum equations in conjunction with the continuity equation (ref. 10). A review (table III) of the assumptions that are made to simplify the Navier-Stokes equation points out the restrictions which apply to the mathematical models for seal analysis and establishes a basis for comparison of the various models in the literature.

With reference to table III, the Cartesian coordinate system is used and the usual assumptions for thin film lubrication are made to simplify the Navier-Stokes equation; the resulting equations are

$$\frac{dp}{dx} = \mu \frac{d^2u}{dz^2} \qquad \text{(x-direction)} \tag{1a}$$

$$\frac{dp}{dy} = \mu \frac{d^2v}{dz^2} \qquad \text{(y-direction)} \tag{1b}$$

$$\frac{dp}{dz} = 0 \qquad \text{(z-direction)} \tag{1c}$$

(Symbols are defined in appendix B.) Two references cited (refs. 3 and 7; see appendix A) essentially employ similar basic equations as a starting point in the analysis of seal models.

In most references cited, some form of the Reynolds equation is used, and these various forms can be obtained from the simplified Navier-Stokes equation (eq. (1)) and the continuity equation. The procedure is outlined in table III. The end result is a two-dimensional Reynolds equation (no axial motion of the seal surface):

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{\partial h}{\partial x} \tag{2}$$

If the restriction of no axial motion is removed, an additional term appears, and the form of the equation is (ref. 17)

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu \left[U \frac{\partial h}{\partial x} + 2(w_1 - w_2)\right] \tag{3}$$

where w_1 and w_2 are the velocities of the seal surfaces in the z-direction.

Some of the reported seal mathematical models in the literature use the short-bearing approximation to obtain a simplified version of the Reynolds equation. As pointed out in reference 18, this short-bearing approximation assumes that the fluid flow in the circumferential direction is due principally to drag flow. The flow due to the pressure gradient $\partial p/\partial x$ is small by comparison. This results in the elimination of the first term in equations (2) and (3). Thus, equation (2) reduces to

$$\frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{\partial h}{\partial x} \tag{4}$$

(Cartesian coordinates, short-bearing form, no axial motion).

The cylindrical coordinate system is natural for seal operation; and the preceding short-bearing form of the Reynolds equation can be transformed into the cylindrical system by noting that y corresponds to the radial coordinate and x corresponds to the circumferential coordinate (i.e., $x = r\theta$). Therefore, equation (4) can be written

$$\frac{\partial}{\partial r}\left(h^3 \frac{\partial p}{\partial r}\right) = \frac{6\mu U}{r} \frac{\partial h}{\partial \theta} \tag{5}$$

(cylindrical coordinates, short-bearing form, no axial motion). In general, the references cited from the literature (tables I and II) use one or the other basic form of equations (1) to (5).

Proposed Seal Operating Models

With reference to the possible primary-seal geometries of figure 3, waviness (geometry a) and angular misalinement (geometry b) are the most likely sources of the fluid-film pressure that is responsible for face-seal lubrication. Coning (geometry c) affects the hydrostatic force balance, but it is not significant in low-pressure seals, which are the subject of this study. Externally imposed axial vibration (geometry d) can produce useful squeeze-film pressures, but it is judged not to be significant because of the high damping that is associated with viscous fluids and secondary-seal friction. (Axial motion that arises from the z-direction velocity component of a rotating seal face does produce useful hydrodynamic pressure and is accounted for by using the appropriate governing equation.)

Parallel misalinement and shaft whirl (geometries e and f) do not produce fluid-film pressures directly but can affect the transport of fluid into and out of the primary seal. In this regard, when the primary ring rotates (nonrotating seat), parallel misalinement does not influence seal operation (leakage, fluid transport, etc.). In rotating, all the points on the surfaces of the primary seal remain within the primary seal. However, when the seat rotates and parallel misalinement is present, some points on the seal surface enter and leave the primary seal during each revolution. This radial velocity component can affect both leakage and lubrication.

Six basic seal operating configurations can be constructed by using the primary-seal geometries of waviness and angular misalinement. These configurations are shown in figure 5. Similarities and differences between these proposed models and the models reported in the literature are discussed in appendix A.

The simplest model from a mathematical standpoint is configuration A of figure 5. Here the primary ring rotates and a relative angular misalinement (i.e., stationary with respect to a laboratory observer) exists between the primary-seal surfaces (also see geometry b, fig. 3). The seal gap does not change with time, and a nonrotating hydrodynamic pressure pattern exists. A positive hydrodynamic pressure exists in the converging portion of the seal gap, and there is no axial component of inertia force since the primary ring has a pure rotation about a centerline inclined to the centerline of the rotating shaft.

Seal configuration A is visualized to operate as described here (also see analysis of misalinement, ref. 19). The nonrotating seat has an angular misalinement with respect to the centerline of the rotating shaft. (The probability of angular misalinement is high.) And the rotating primary ring, which is given axial and angular freedom, is forced by the springs (and sealed pressure) toward alinement to the face. This alinement attempt by the rotating primary ring introduces a friction-force component at the secondary-seal surface because the shaft and primary ring have different centerlines of rotation. In effect, the secondary-seal friction prevents full alinement of the primary ring to the seat face, and a relative angular misalinement then exists between the faces of the primary seal. This relative misalinement acts as an inclined slider geometry and produces hydrodynamic pressure. It is further postulated that a balance of forces and moments (hydrodynamic and friction) will exist and that seal operation can be stable. A mathematical analysis of this model is given in reference 19.

Configuration B is similar to configuration A except that the seat is the rotating member. The primary ring tries to track or follow the rotating misalinement motion of the seat (i.e., aline itself). In effect, the primary ring has a nutating motion with respect to a laboratory observer. This nutation induces inertia forces that act to cause relative face misalinement. The effect is the same as the secondary-seal friction in configuration A. A secondary-seal friction force also exists.

In configuration C the rotating primary ring has two or more surface waves that produce a corresponding hydrodynamic pressure pattern which rotates at shaft speed. Thus, the seal-gap height varies with time, $h = f(t)$. However, there is no induced friction force or inertia force since the primary ring has pure rotation about the shaft centerline.

Configuration D, a rotating seat with a wavy primary-ring surface, is fundamentally the same as configuration C except that the hydrodynamic pressure pattern is nonrotating.

The last two configurations are combinations of waviness and angular misalinement, and the resulting hydrodynamic pressure pattern is now not merely nonrotating or rotating (as in configurations A to D). The pressure patterns are also time dependent because the phase relation between the relative angular misalinement and the waviness is constantly changing. Fundamentally, configurations E and F are equivalent. In both configurations the seal gap changes with time, and primary-ring inertia force and secondary-seal friction exist.

In both configurations E and F the hydrodynamic component due to the relative misalinement sweeps (once per shaft revolution) through the hydrodynamic component due to the surface waviness. This raises the possibility of periodic and partial loss of hydrodynamic pressure. Such partial loss may lead to local wavy wear (as reported in ref. 11) in either or both primary-seal faces.

The possible configurations proliferate as other primary-seal geometries are introduced (fig. 3). Further, the sealed pressure could be at either the outer diameter or the inner diameter of the seal, and both faces could be rotating at different speeds. A group of complex configurations can be formulated by introducing waviness into the seat as well as into the primary ring. These complex configurations - waviness in the seat, plus waviness in the primary ring, plus relative angular misalinement - introduce the same friction and inertia effects as found in configurations E and F. But the pressure patterns are more complex. Again, as with configurations E and F, the net result of the hydrodynamic contributions of waviness and misalinement could be a periodic change of hydrodynamic pressure and an associated undesirable seal performance. Also, in all these models, nonuniform spring loading can cause undesirable seal performance.

Governing Equations for Proposed New Models

The governing equation for hydrodynamic pressure should be expressed in the cylindrical coordinate system for seal-like configurations. The correct forms to fit the proposed models need to be established. This may seem to be a duplication of effort but, as evident in tables I and II, many different governing equations and models are used. This suggests that, compared to bearing theory, seal theory is in a primitive state with little agreement on the predictive model.

One approach is to establish the governing equation for hydrodynamic lubrication in the Cartesian system and then convert to the cylindrical system. With the complex configurations E and F of figure 5 in mind, the basic equation must allow for film-thickness change as a function of time. A good starting point is the three-dimensional Reynolds equation (ref. 20)

$$\frac{\partial}{\partial x_i}\left(h^3 \frac{\partial p}{\partial x_i}\right) = 6\mu \left\{2 \frac{\partial h}{\partial t} + \frac{\partial}{\partial x_i}\left[h(V_i + V_i')\right]\right\} \tag{6a}$$

where repeated index i indicates summation over values of 1 and 2 and where V_i and V_i' are the velocities of the surfaces (in general, each surface (unprimed and primed) will have velocities in the x- and y-directions). Also for the seal model shown in figure 4,

$$x_1 = x$$

$$x_2 = y$$

$$V_1 = U, V_1' = 0 \qquad \text{(only one surface in motion)}$$

$$V_2 = 0, V_2' = 0 \qquad \text{(no motion in y-direction)}$$

Reference 17 states that the short-bearing approximation of the Reynolds equation is accurate in bearings when

$$\frac{L}{D} \leqq \frac{1}{4}$$

where

L bearing length

D bearing diameter

Since seals have equivalent L/D ratios of 1/50 to 1/100, the short-bearing approximation is well within the range of applicability. Thus, use of a more complex form of the equation seems to be needless, and the one term on the left side can be eliminated. Also, as previously noted, in the class of seals considered herein only one surface is in motion ($V_1' = 0$), and radial motion can be neglected for hydrodynamic calculation purposes ($V_2 = V_1' = 0$). Therefore, equation (6a) becomes

$$\frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu\left(2 \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x}\right) \tag{6b}$$

As stated previously, seal-like configurations can be "unwrapped" to fit the Cartesian system since the radial distance across the primary seal is small compared to the seal diameter. But the cylindrical coordinate system is needed for analysis of the proposed models (ref. 19). Noting that

$$y = r$$

$$x = r\theta$$

$$U = r\omega$$

equation (6b) becomes

$$\frac{\partial}{\partial r}\left(h^3 \frac{\partial p}{\partial r}\right) = 6\mu\left(2 \frac{\partial h}{\partial t} + \omega \frac{\partial h}{\partial \theta}\right) \tag{7}$$

The preceding is the two-dimensional, time-dependent, Reynolds equation with the short-bearing approximation. It is proposed as the hydrodynamic governing equation for the proposed seal configurations in figure 5.

The hydrodynamic forces and moments determined from this equation must be balanced by the forces and moments caused by secondary-seal friction, mechanical closing force (spring), and primary-ring inertia. Therefore, equation (7), together with Newton's laws of motion, forms the set of governing equations for the proposed models of figure 5 (e.g., see ref. 19 for analysis of configuration A of fig. 5).

An important consideration is the expression for h needed in each of the configurations of figure 5. Rigid surfaces are assumed. The equations for the surfaces of the primary seal are

$$z = f_2(r, \theta, t) \qquad \text{(primary ring)}$$

$$z = f_1(r, \theta, t) \qquad \text{(seat)}$$

(The coordinate system for calculating hydrodynamic forces is fixed in the ambient fluid at the shaft centerline.) The expression for h is

$$h(r, \theta, t) = f_1(r, \theta, t) - f_2(r, \theta, t)$$

(In ref. 19, it is argued that h can be taken as independent of r without significant error, but in the meantime the dependence is retained.)

With reference to configuration A of figure 5, the primary ring undergoes pure rotation and has relative angular misalinement with respect to the misalined seat face. The contribution of the primary ring to film thickness is $r \tan \beta \cos \theta$ and that of the seat is $r \tan \alpha \cos \theta$. And the film thickness (which is independent of time) has the value

$$h = h_0 + r \tan \alpha \cos \theta - r \tan \beta \cos \theta$$

$$h = h_0 + r(\tan \alpha - \tan \beta) \cos \theta$$

Since, for small angles, $\tan \alpha \cong \alpha$ in radians,

$$h = h_0 + r(\alpha - \beta) \cos \theta$$

$$= h_0 + r\gamma \cos \theta \qquad (8a)$$

where γ is the relative angular misalinement in radians.

Configuration B contains time-dependent terms because the seat has a face run-out motion and the primary ring is responding with a nutating motion. The two motions are assumed to be in phase, and the film thickness is

$$h = h_0 + r\gamma \cos (\theta + \omega t) \qquad (8b)$$

For configuration C

$$h = h_0 + b \sin (n\theta + \omega t) \qquad (8c)$$

where

n number of waves per revolution

b wave amplitude

For configuration D

$$h = h_0 + b \sin n\theta \qquad (8d)$$

For configuration E

$$h = h_0 + r\gamma \cos \theta + b \sin (n\theta + \omega t) \qquad (8e)$$

For configuration F

$$h = h_0 + r \left[\tan \alpha \cos (\theta + \omega t) - \tan \beta \cos \theta\right] + b \sin n\theta \qquad (8f)$$

SUMMARY OF RESULTS

Several new models of seal operating configurations were devised by using primary-seal geometries of waviness and misalinement and by giving the primary-ring three degrees of freedom of motion to simulate actual seal conditions. The assumptions that apply to thin-film lubrication of seal-like configurations were reviewed for the purpose of obtaining governing equations that apply to hydrodynamic lubrication of the proposed new models. Finally, the theories of seal lubrication reported in the literature were reviewed with regard to their mechanisms of lubrication, their governing equations (and corresponding restrictions), and their conformance to models of seal operating configurations proposed in this report as being representative of actual seal operation.

The study confirmed the following:

1. New seal models that represent several possible modes of operation were devised from combinations of relative angular misalinement and surface waviness. The hypothesis was that secondary-seal friction and primary-ring inertia both act to induce a relative misalinement to the seal seat, and this relative angular misalinement produces an inclined-slider-bearing effect.

2. In the new seal models it was postulated that a balance of forces and moments provides stable seal operation for the cases of surface waviness alone and relative angular misalinement alone. The case of combined angular misalinement and surface waviness is seen as potentially complex because of the changing phase relations between the misalinement and the surface waves. Periodic reduction of hydrodynamic pressure is postulated as possible with certain combinations of waviness and angular misalinement.

3. Simplification of the Reynolds equation through use of restrictions applicable to seal operation leads to a governing equation for the new seal models that is a two-dimensional, time-dependent Reynolds equation with the short-bearing approximation. The appropriate expression for the film thickness was determined for each of the new models.

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APPENDIX A

REVIEW OF PUBLISHED MATHEMATICAL MODELS FOR SEALS

Various mathematical models have been devised to explain the mechanism by which seal faces are separated by a liquid film. These models are discussed in this appendix.

Angular Misalinement Models

Reference 3 presents an analysis of a primary seal in which both surfaces are flat but have angular and parallel misalinement (fig. 3). In reference 3 the full Navier-Stokes equations were simplified by a normalizing procedure. The resulting equations are

$$\frac{\partial^2 u}{\partial z^2} = 0 \qquad \theta\text{-direction}$$

$$\frac{\partial p}{\partial r} = \rho \frac{u^2}{r} + \mu \frac{\partial^2 v}{\partial z^2} \qquad r\text{-direction}$$

$$\frac{\partial p}{\partial z} = 0 \qquad z\text{-direction}$$

Reference 3 uses these equations, along with the continuity equation, to obtain explicit solutions for pressure, leakage, torque, and load support (pressure distribution). The expression for pressure distribution (of primary concern herein) is given in table I.

The magnitudes of angular misalinement and film thickness were assumed and were used as input data for solving the equations explicitly. A geometric coupling effect between angular misalinement and parallel misalinement (eccentricity) was found. For example, their effect on leakage can be reversed by changing the phase angle between angular misalinement and eccentricity or by reversing the direction of rotation. The separating force is also dependent on the phase relation between misalinement and eccentricity. This model assumes a fixed centerline film thickness (no axial motion) and has a nonrotating hydrodynamic pressure pattern and a time-independent seal gap. Secondary-seal friction or primary-ring inertia effects are not considered. However, valuable insight is provided by the model in that it reveals that angular misalinement is a possible mechanism for the generation of hydrodynamic forces.

Reference 12 presents an analysis of a radial face seal in which it is argued that for stable seal operation a necessary condition is

$$\frac{\text{Change in load capacity}}{\text{Change in film thickness}} = \frac{\partial l}{\partial h} > 0$$

and that $\partial l/\partial h$ will be positive if film thickness changes with respect to r, θ, or t. In other words, one or more of the following terms must exist: $\partial h/\partial r$, $\partial h/\partial \theta$, or $\partial h/\partial t$. The mathematical model of reference 12 admits misalinement and surface waviness and also distortion and axial vibration. It is assumed that the flow is isothermal and isoviscous and that the faces are rigid. However, the face angular misalinements are fixed in one model and in the other the closing load is constant (the case of axial vibration). Thus, tracking response of the nose to the seat runout is not considered. In other words, the relative misalinement is assumed rather than allowed to arise from the natural operation of the seal.

The analysis of reference 12 is based on a two-dimensional Reynolds equation that retains dependence of h upon r and θ in one model (constant centerline film thickness) and on a two-dimensional, time-dependent Reynolds equation that retains the dependence of h upon r, θ, and t for the case of axial vibration. Solution is by numerical techniques. Reference 12 concludes that pressure generation takes place as a result of small misalinement of the faces and that this pressure generation essentially takes place in the θ-direction. Table I contains a summary of the governing equations.

The analysis of reference 13 is based on a seal model which admits angular misalinement of both faces. (Eccentricity is not considered, but this is probably not important for pressure generation calculations.) A key assumption is that the closing load is constant, which is probably not true in actual seal operation. Further, the misalinements, which are assumed a priori, consist of two fixed misalinements, one of which is rotating. Thus, tracking response of the primary ring is not an admissible condition. The analysis is based on the following modified Reynolds equation in cylindrical coordinates:

$$\frac{\partial}{\partial r} r\left[\frac{(h_2 - h_1)^3}{\mu} \frac{\partial p}{\partial r}\right] + \frac{\partial}{\partial \theta}\left[\frac{(h_2 - h_1)^3}{\mu r} \frac{\partial p}{\partial \theta}\right] = 6(h_2 - h_1)\frac{\partial}{\partial r}\left[r(v_1 + v_2)\right] + 6r(v_1 - v_2)\frac{\partial(h_1 + h_2)}{\partial r}$$

$$+ 6(h_2 - h_1)\frac{\partial}{\partial \theta}(u_1 + u_2) + 6(u_1 - u_2)\frac{\partial(h_1 + h_2)}{\partial \theta} + 12r(w_2 - w_1)$$

where subscripts 1 and 2 refer to the stationary and dynamic faces, respectively. And for the stationary face, $v_1 = u_1 = w_1 = 0$. Further, $v_2 = 0$ (no radial flow). With the preceding assumptions, together with the short-bearing approximation, the modified Reynolds equation becomes that shown in table I. The resulting equation is solved by numerical methods.

The analysis of reference 6 is based on the Reynolds equation for two dimensions (table I) and introduces the following equations of state for density and viscosity:

$$\rho = \rho_0 \exp\left(\frac{p}{k}\right)$$

$$\mu = \mu_0 \exp(\alpha p)$$

where k is the bulk modulus and α is the pressure-viscosity coefficient. An expression for pressure is obtained by assuming that the Reynolds equation has a solution of the form

$$p = \left[a\varphi_1(r_1\theta) + a^2\varphi_2(r_1\theta) + O(a^2)\right]$$

where a is the amplitude of waviness or tilt divided by the mean face separation and φ_1 and φ_2 are the first two pressure coefficients.

Circumferential Wave Models

Reference 4 presents an analysis of a radial face seal in which one face is assumed to be flat and the other to be wavy. As mentioned previously, there is evidence that some seals are manufactured with small-amplitude circumferential waves (ref. 10) and that others develop waves during operation (ref. 8). Parallel misalinement of the faces is admissible in the analysis, and the analysis reveals the effect of this misalinement on leakage and separating force. It was found that eccentricity could cause either inward or outward leakage, depending on the phase relation with the waviness. Thus, otherwise identical seals that are assembled differently will have different net leakage rates and separating forces. The analysis is based on simplified momentum equations in cylindrical coordinates. Constant centerline film thickness is assumed, and the hydrodynamic pressure (load support) is produced by the inclined-slider-bearing effect of the wavy surface.

In reference 11 the short-bearing-approximation approach of reference 18 is used, and the Reynolds equation for the short-bearing approximation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(h^3 r \frac{\partial p}{\partial r}\right) = 6\mu\omega \frac{\partial h}{\partial \theta}$$

For sealing applications the short-bearing approach is probably a very good approximation for calculation of pressure generation due to waviness. One face is assumed to be a plane, and the other face is assumed to have two waves that are in a fixed phase relation with the angular misalinement of the primary ring. Thus, in contrast to the proposed models, changing phase relations between waviness and relative angular misalinement is not an admissible mode of operation. The Reynolds equation is solved explicitly to get an equation for pressure (table I).

Reference 21 presents data which rather convincingly show that seals can operate with hydrodynamic lubrication, and reference 21 attributes this hydrodynamic lubrication to surface waviness. The coefficients of friction measured, as low as 0.02, are typical of hydrodynamic lubrication. And this agrees with the data from reference 22, in which it was pointed out that, when the coefficient of friction is plotted as a function of the $\eta U/p$ parameter, the curve has the same general shape as those for journal bearings (fig. 6). Thus, the evidence for hydrodynamic lubrication is very strong.

Reference 21 considers two waves in the circumferential direction, and the load generation capacity is based on this two-wave assumption. In effect, the seal is treated analytically as a plain inclined slider bearing. The simplified Reynolds equation (short-bearing approximation) shown in table I is solved explicitly for the generated pressure.

Pape (ref. 10) concludes that sealing faces generally operate in a fully hydrodynamic manner. He states that the mechanism causing hydrodynamic lubrication or load support is the wedge action and that the minute waviness of the faces, of the order of 0.10 micrometer, that occurs in the circumferential direction can generate a film in the range of 1 micrometer thick. The analysis of reference 10 is based on the Reynolds equation (table I) that is two dimensional and time dependent. Surface waviness, which is assumed, is the source of the load capacity. Considerable attention is given to obtaining correct continuity of flow conditions as affected by cavitation regions. The Reynolds equation in nondimensional form is solved by a finite difference method.

Inclined-Slider-Bearing Microgeometry Models

Reference 14 proposes a micropad theory of load support. It is based on measured nonuniform surface wear of U.S. Navy marine shaft seals in which the surface of operated carbon rings had a pattern of exposed, flat, individual and agglomerated raised pads (average size, 2000 to 6000 μm (0.080 to 0.240 in.)). The recessed area around the pads averaged 60 to 160 micrometers (0.0024 to 0.0064 in.) deep. Analysis indicated that, when they are operating on water, 0.689- to 34.4-N/cm^2 (1- to 50-psi) hydrodynamic pressure can be generated by the micropads at 480 centimeters per second (189 in./sec). The mathematical analysis takes into account the observed statistical distribution of the micropads, and the expression for generated pressure is given in table II.

Another type of load support due to small surface irregularities is proposed in reference 5 and is termed microasperity lubrication. In this case the microasperities were planned and were produced by photoengraving. The microasperities were in the form of circular cylinders 0.3048 centimeter (0.012 in.) in diameter and 2.5 micrometers (0.0001 in.) high. The load support is attributed to small tilts on the tops of the asperities; that is, these tops formed small inclined slider bearings. The two-dimensional Reynolds equation is solved for the average pressure (table II).

Reference 7 puts forward a theory of hydrodynamic load support based on thermal deformation of the metal surface in a seal having a carbon surface mated to a metal surface. The carbon surface is considered to be flat, but the metal surface has a network of scratches several micrometers deep. The distance between scratches is a few tenths of a millimeter. Under the action of a pressure differential the fluid flows with greater velocity along the scratches than in the remaining portions. The surfaces are assumed to be separated by a fluid film. Relative motion causes uneven heating of the fluid film and metal wall, the temperature being lowest at the scratches, where the leakage velocity is highest and film thickness is greatest. Moving away from a scratch in the direction of motion, the fluid progressively heats up and the wall expands unevenly. Therefore, the portion of the metal face between scratches takes on a small slope, and a multiplicity of small converging wall geometries are thus formed. Thus, hydrodynamic forces are generated.

Reference 7 bases the analysis on the seven governing equations shown in table II. The first equation is the expression resulting from simplification of the Navier-Stokes equation, and the pressure generation is due to the velocity component in the circumferential direction. The second equation in table II is the continuity equation. The third equation is an expression for heat transfer in a film of incompressible fluid. The fourth equation is the relation between viscosity and fluid temperature. The fifth equation gives the temperature distribution in the metal wall. The sixth equation expresses the change in film thickness as a result of thermal expansion of the wall. The clearance changes only in the x-direction (seventh equation in table II). A method of relaxation is used to solve these seven governing equations; the expression for pressure is given in table II.

Inclined-Slider-Bearing Macrogeometry Models

In references 15 and 16 the importance of maintaining a fluid lubricating film is stressed. For very high pressure (to 2413 N/cm^2 (3000 psi)), reference 15 reveals that radial slots cause thermally induced wedge deformation that promotes hydrodynamic effects. This is referred to as a hydrodynamic type of seal. One hydrodynamic seal described in reference 16 is shown in figure 7. The construction of this hydrodynamic seal is similar to that of a conventional face seal except that the primary-ring face contains circular grooves (fig. 7). Seat rotation causes the sealed liquid to circulate through these grooves, and thus the grooved region is cooler than the other regions. Therefore, thermal deformations produce wedge geometries (inclined slider bearing) that give rise to hydrodynamic forces. Figure 7 also illustrates the hydrodynamic force (pressure profile) that is built up in the region of the circular grooves. This type of seal apparently maintains a more efficient hydrodynamic action (than a conventional face seal) as pressure and speed increase. Thus, it has superior performance. Successful operation at pressures to 2413 N/cm^2 (3000 psi) and 101 m/sec (333 ft/sec) for thousands of hours has been reported (refs. 15 and 16).

Boiling Liquid Model

Reference 1 presents a theory of load support based on visual observations and measurements with a face seal having a rotating glass seat. It was found that the gap between the seal faces contained two distinct regions: a continuous liquid-film region next to the sealed liquid edge, and a region of vapor and liquid from the liquid interface out to the atmospheric edge. Infrared temperature measurements indicated that boiling temperatures existed at the fluid interface. It was postulated that because of the large increase in specific volume accompanying vaporization, the pressure drop across the film would occur mostly in the vapor region (fig. 8). The shaded region in the graph of figure 8 indicates the increased load supported by vaporization, as compared to the linear pressure drop associated with full liquid-film lubrication. Reference 1 states that film vaporization is important because it exerts a stabilizing influence on interfacial temperatures and limits power loss at higher speeds.

Converging Leakage Gap Model

It has been pointed out by many (e.g., ref. 23) that a film converging in the leakage direction provides positive film stiffness. Thus, with proper seal balance, stable operation of the seal with separation of the sealing forces can be attained. The converging film could be generated in a number of ways. For example, the converging shape could be machined; many seals (hydrostatic) operate on this principle. Also, thermal distortion may cause converging films. Load support from converging faces and boiling films is probably a factor in applications that have significant sealed pressure. For low-pressure applications, which are the major concern of this report, other mechanisms of load support must be sought.

APPENDIX B

SYMBOLS

a	wave amplitude divided by mean face separation
b	wave amplitude
c	specific heat
D	bearing diameter
F	force
f	coefficient of friction
h	local film thickness
h_0	centerline film thickness
J	mechanical equivalent of heat
k	bulk modulus
L	bearing length
l	load
n	number of waves per revolution
p	pressure
R	characteristic mean seal radius
R_i	inner radius of sealing dam
R_o	outer radius of sealing dam
r	radius; or cylindrical coordinate
T	temperature
t	time
U	peripheral velocity of seat or primary ring (velocity in x-direction)
u	velocity component in x-direction and θ-direction
V	sliding velocity in y-direction (generalized velocity component)
v	velocity component in y-direction and r-direction
w	velocity component in z-direction (generalized velocity component)
X, Y, Z	coordinates
x, y, z	coordinates

α angular misalinement of seat; coefficient of linear expansion; pressure-viscosity coefficient

β angular misalinement of primary ring

γ relative angular misalinement of seat and primary ring

ϵ seal eccentricity

η kinematic viscosity

θ cylindrical coordinate

λ heat conductivity of liquid

μ absolute viscosity

ρ density

ϕ phase angle between eccentricity line of centers and tilt reference line

φ pressure coefficient

ω angular velocity

Subscripts:

i condition at inner radius of primary seal

i, j vector indices

o condition at outer radius of primary seal

0 reference condition

1 component related to seat face; or stationary face

2 component related to primary ring face; or dynamic face

Superscript:

' nondimensional form; or reference to a mating surface

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TABLE I. - MATHEMATICAL MODELS FOR SURFACE WAVINESS AND MISALINEMENT AS SOURCES OF PRESSURE GENERATION BETWEEN PRIMARY-SEAL SURFACES

Source of pressure generation	Restrictions	Governing equations	Solution technique	Pressure distribution equation	Reference
Angular misalinement of one face	No axial motion; constant centerline film thickness; cylindrical coordinates attached to rotating face	$\frac{\partial p}{\partial r} = \frac{\rho u^2}{r} + \mu \frac{\partial^2 v}{\partial z^2}$ $\frac{\partial p}{\partial z} = \frac{\partial^2 u}{\partial z^2} = 0$ Assumed input data: (1) Film thickness (2) Angular misalinement (3) Eccentricity	Governing equations solved explicitly for pressure, torque, leakage rate, and separating force	$p = 6\mu\left[\epsilon\Omega \sin(\phi - \theta)\right]\int_{R_o}^{r} \frac{dr}{h^2} + 6\mu\left[\omega \cos\gamma \tan^2\gamma \sin\theta \cos\theta\right]\int_{R_o}^{r} \frac{r\,dr}{h^2}$ $+ \frac{\rho r^2}{2}\left[\omega\Omega \cos\gamma + \frac{3}{10}(\omega \cos\gamma - \Omega)^2\right] + 12\mu\Phi(\theta)\int_{R_o}^{r} \frac{dr}{rh^3} + \Psi(\theta)$ $+ 2\mu(\omega \cos\gamma + \Omega) \sin\theta \tan\gamma \int_{R_o}^{r} \frac{r^2\,dr}{h^3}$ where $\Psi(\theta) = p_0$, Ω is angular velocity of the eccentric surface, $h = h_0 - r \cos\theta \tan\gamma$, and $\Phi(\theta) = \frac{1}{12\mu\int_{R_i}^{R_o} \frac{dr}{rh^3}}\left\{p_i - p_0 - 6\mu\left[\epsilon\Omega \sin(\phi - \theta)\right]\int_{R_o}^{R_i} \frac{dr}{h^2}\right.$ $- 6\mu(\omega \cos\gamma \tan^2\gamma \sin\theta \cos\theta)\int_{R_o}^{R_i} \frac{r\,dr}{h^2} + \frac{\rho}{2}\left(R_o^2 - R_i^2\right)$ $\times\left[\Omega\omega \cos\gamma + \frac{3}{10}(\omega \cos\gamma - \Omega)^2\right] - 2\mu(\omega \cos\gamma + \Omega)$ $\left.\times \sin\theta \tan\gamma \int_{R_o}^{R_i} \frac{r^2\,dr}{h^3}\right\}$	3
Angular misalinement (also admits waviness and axial vibration)	Constant centerline film thickness; cylindrical coordinates	$\frac{\partial}{\partial r'}\left(r'h'^3 \frac{\partial p'}{\partial r'}\right) + \frac{\partial}{\partial\theta}\left(\frac{h'^3}{r'} \frac{\partial p'}{\partial\theta}\right) =$ $-r'^2\left[\cos(\theta - \Omega) + x' \cos\theta\right]$ where $\Omega = \omega t$ Nondimensional form of Reynolds equation	Finite difference technique, used for solution of pressure	Nondimensional symbols: $r' = r/R$ $h' = h/h_0$ $p' = \frac{h_0^3}{6\mu R^3 \omega\beta}(p - p_0)$ $x' = \frac{\alpha}{\beta}$	12
	Constant axial load	$\frac{\partial}{\partial r'}\left(r'h'^3 \frac{\partial p'}{\partial r'}\right) + \frac{\partial}{\partial\theta}\left(\frac{h'^3}{r'} \frac{\partial p'}{\partial\theta}\right) =$ $-r'^2\left[\cos(\theta - \Omega) + x' \cos\theta\right]$ $+ \frac{2h_0}{R\omega\beta} \frac{\partial\lambda}{\partial t}$ Nondimensional form of Reynolds equation; $\lambda(t)$ describes squeeze-film motion	Trial and error procedure, used to find specific cases of film-thickness function $\lambda(t)$ which give a constant load		

Angular misalinement of both faces	Cylindrical coordinates; Reynolds equation, time-dependent, constant axial load; short-bearing approximation, fixed misalinement	$\frac{\partial}{\partial r}\left(r \frac{h^3}{\mu} \frac{\partial p}{\partial r}\right) = -6r^2\omega\left[\beta \cos (\theta - \omega t) + \alpha \cos \theta\right] + 12r \frac{\partial h(t)}{\partial t}$	Numerical methods		13
Angular misalinement and surface waves in a circumferential direction	Cylindrical coordinates; Reynolds equation for two dimensions; fixed misalinement	$r \frac{\partial}{\partial r}\left(\frac{r\rho h^3}{12\mu} \frac{\partial p}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \theta}\right) = \frac{\partial}{\partial \theta}\left(\rho \frac{\omega r^2 h}{2}\right)$	Assumed form of pressure solution	$p = \left[a\varphi_1(r, \theta) + a^2\varphi_2(r, \theta) + O(a^2)\right]$ where φ_1, φ_2 first two pressure coefficients	6
Surface waves in a circumferential direction	Cylindrical coordinates; simplified momentum equations; constant centerline film thickness	$\frac{\partial p}{\partial r} = \rho \frac{u^2}{r} + \mu \frac{\partial^2 v}{\partial z^2}$ $\frac{\partial p}{\partial z} = \frac{\partial^2 u}{\partial z^2} = 0$ Continuity equation: $\frac{\partial}{\partial r}\left(r \int_0^h u\, dz\right) + \int_0^h \frac{\partial v}{\partial \theta} dz = 0$	Governing equations, solved explicitly for pressure, torque, leakage rate, and separating force	$p - p_0 = \frac{3}{20}\rho\omega^2\left(r^2 - R_o^2\right) + \frac{6\mu\omega\epsilon}{h^2} \sin (\phi - \theta)(r - R_o) + \frac{3}{2}\frac{\mu\omega}{h^3}\left(r^2 - R_o^2\right)\frac{\partial h}{\partial \theta} + \frac{\ln\left(\frac{r}{R_o}\right)}{\ln\left(\frac{R_i}{R_o}\right)}\left\{P_i - P_o - \frac{3}{20}\rho\omega^2\left(R_i^2 - R_o^2\right) - \frac{3}{2}\frac{\mu\omega}{h^3}\left(R_i^2 - R_o^2\right)\frac{\partial h}{\partial \theta} - \frac{6\mu\epsilon\omega}{h^2} \sin (\phi - \theta)(R_o - R_i)\right\}$	4
Two waves in circumferential direction combined with angular misalinement	Cylindrical coordinates; Reynolds equation with short-bearing approximation	$\frac{1}{r}\frac{\partial}{\partial r}\left(h^3 r \frac{\partial p}{\partial r}\right) = 6\mu\omega \frac{\partial h}{\partial \theta}$	Reynolds equation solved explicitly for pressure	$p = \frac{1}{\ln\left(\frac{R_o}{R_i}\right)}\left\{\gamma p_s + \frac{3\mu\omega}{2h^3}\frac{dh}{d\theta}\left[r^2 \ln\left(\frac{R_o}{R_i}\right) - R_i^2 \ln\left(\frac{r}{R_i}\right) - R_i^2 \ln\left(\frac{R_o}{r}\right)\right]\right\}$ $\gamma = \ln (R_o/r)$ for inside seal, and $\ln(r/R_i)$ for outside seal	11
Circumferential waves	Rectangular coordinates; Reynolds equation; short-bearing approximation	$h^3 \frac{\partial^2 p}{\partial y^2} = 6\mu U \frac{\partial h}{\partial x}$	Reynolds equation solved explicitly for pressure	$p = 3\mu U \frac{1}{h^3}\frac{\partial h}{\partial x}\left[y^2 - \frac{(R_o - R_i)^2}{4}\right]$	21
Circumferential waves	Cylindrical coordinates; Reynolds equation in two dimensions and time dependent	$\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{h^3}{12\mu}\frac{\partial r}{\partial \theta}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial y}\right) = \frac{\omega}{2}\frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t}$	Finite difference method of solution of Reynolds equation in nondimensional form		10

TABLE II. - MATHEMATICAL MODELS FOR LOAD CAPACITY OR PRESSURE GENERATION IN PRIMARY-SEAL SURFACES

Source of pressure generation	Restriction	Governing equations	Solution technique	Pressure distribution equation	Reference
Micropads produced by "run in" wear, which act as small, inclined slider bearings	Assumes each pad is a small, rectangular step bearing		Closed form; observed statistical distribution of pads used	$\bar{p}_b = 2\rho\mu U b \dfrac{\delta}{(h_2 + \delta)^3 + h_2^3}$ where $\rho = \dfrac{96}{\pi^4}\bar{L}^2 \sum_{N=1,3,5,\ldots} N^{-4} \tanh\dfrac{\pi N}{4\bar{L}} \tanh\dfrac{\pi N}{2\bar{L}}$ and N variable of summation $\bar{p}_b$ average hydrodynamic pressure $2b$ circumferential length of micropad δ pad height h_2 minimum clearance L radial width of pad $\bar{L}$ aspect ratio, L/2b	14
Surface asperity tops, which act as small inclined slider bearings	Reynolds equation - two-dimensional form, rectangular coordinates	$\dfrac{\partial}{\partial x}\left(h^3 \dfrac{\partial p}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(h^3 \dfrac{\partial p}{\partial y}\right) = -6\mu \dfrac{U}{R_o}(h_0 - h_m)$	Nondimensional form - expansion of dimensionless pressure in series solution	$\bar{p} = \dfrac{3}{4}\dfrac{\mu U R_o \delta}{h_0^3}$ where $\delta = h_0 - h_2$ and $\bar{p}$ average pressure h_0 average film thickness h_m minimum film thickness R_o asperity radius	5
Small, inclined slider bearings produced by thermal deformation around microleakage channels in primary seal face	Rectangular coordinates; pressure as a function of velocity in the circumferential direction	$\dfrac{\partial p}{\partial x} = \mu \dfrac{\partial^2 u}{\partial z^2}$ (1) $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$ (2) $u \dfrac{\partial T}{\partial x} = \dfrac{\lambda}{c\rho}\dfrac{\partial^2 T}{\partial z^2} + \dfrac{\mu}{c\rho J}\left(\dfrac{\partial u}{\partial z}\right)^2$ (3) $\mu = \mu(T)$ (4) $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial z^2} = 0$ (5) $\delta(h) = -\alpha \int_0^{\Delta} \left[T(x + \delta x, z) - T(x, z)\right] dz$ (6) $h = h(x)$ (7)	Method of relaxation, used to solve the seven governing equations	$\bar{p} = \dfrac{2}{5\pi}\left(\dfrac{\alpha l^3 \mu^2 U^2}{J c \rho h^5}\right)$ where $\bar{p}$ average pressure α coefficient of linear expansion l distribution between scratches J mechanical equivalent of heat c specific heat	7

TABLE III. - SUMMARY OF PROCEDURE FOR SIMPLIFYING THE NAVIER-STOKES AND REYNOLDS EQUATIONS FOR SEAL GEOMETRIES

Restriction, assumption, or boundary condition	Equation or model
Motion is only in x-direction, and only one surface is in motion.	See fig. 4.
Navier-Stokes equation of motion for an incompressible fluid and constant viscosity applies.	$\rho\left(\frac{\partial w_i}{\partial t} + w_j \frac{\partial w_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + F_i + \mu \frac{\partial^2 w_i}{\partial x_j \partial x_j}$
Restrictions are steady state, no body forces, small inertial forces.	$\frac{\partial p}{\partial x_i} = \mu \frac{\partial^2 w_i}{\partial x_j \partial x_j}$
Simplified Navier-Stokes equations for the three directions x, y, and z written out in full.	$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$ (x-direction) $\frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$ (y-direction) $\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$ (z-direction)
Since the seal fluid film is of the order of 10^3 times smaller in the z-direction than in the x- and y-directions, $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2} << \frac{\partial^2 u}{\partial z^2}$ $\frac{\partial^2 v}{\partial x^2}$ and $\frac{\partial^2 v}{\partial y^2} << \frac{\partial^2 v}{\partial z^2}$ the pressure is assumed to be constant across the film.	$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$ (x-direction) $\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2}$ (y-direction) $\frac{\partial p}{\partial z} = 0$ (z-direction)
Solve for fluid velocities u and v by integrating twice and evaluating the constants of integration for the boundary conditions of one surface nonrotating and no axial motion.	$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} z(z - h) + U\frac{(h - z)}{h}$ $v = \frac{1}{2\mu}\frac{\partial p}{\partial y} z(z - h)$
Substitute expressions for u and v into the continuity equation for the condition of no axial motion and integrate over film thickness. $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\right)$	$\int_0^h \frac{\partial}{\partial x}\left[\frac{1}{2\mu}\frac{\partial p}{\partial x} z(z - h) + U\left(\frac{h - z}{h}\right)\right]\partial z + \int_0^h \frac{\partial}{\partial y}\left[\frac{1}{2\mu}\frac{\partial p}{\partial y} z(z - h)\right]\partial z = 0$
Integrate the preceding expression to obtain the Reynolds equation in two dimensions.	$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{\partial h}{\partial x}$
Apply short-bearing approximation, which states that the flow in the direction of motion is due principally to the drag component; that is, the flow due to the pressure gradient in the x-direction is not significant: $\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) \cong 0$	$\frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{\partial h}{\partial x}$

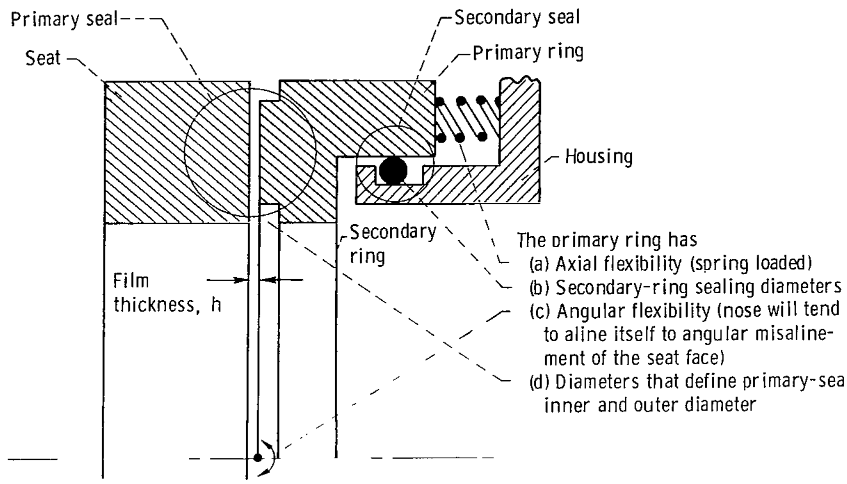


Figure 1. - Arrangement of seal components.

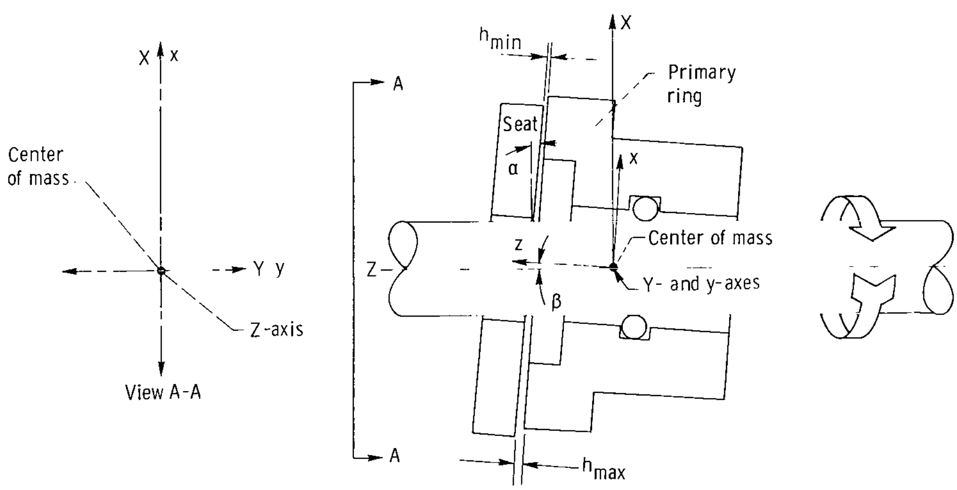
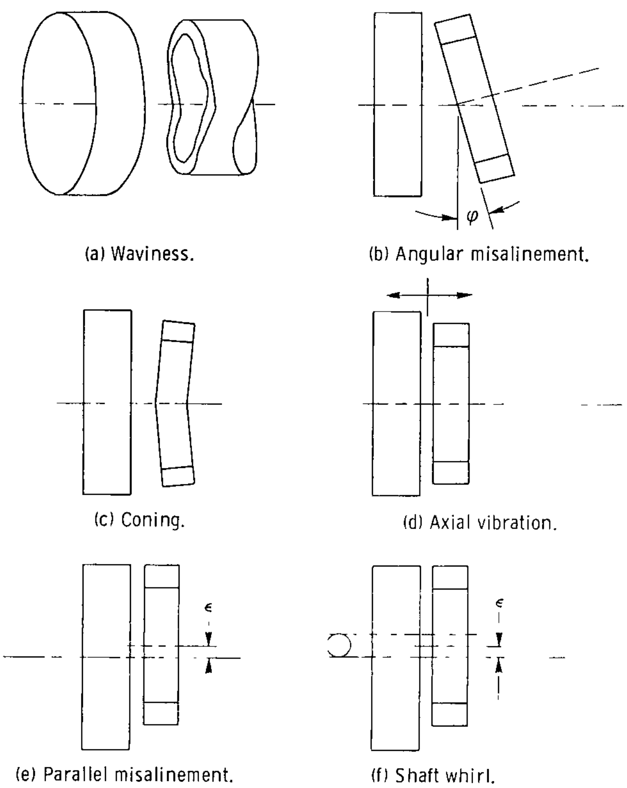


Figure 2. - Mathematical model of radial face seal with rotating primary ring.



(a) Waviness.
(b) Angular misalinement.
(c) Coning.
(d) Axial vibration.
(e) Parallel misalinement.
(f) Shaft whirl.

Figure 3. - Possible primary-seal geometries.

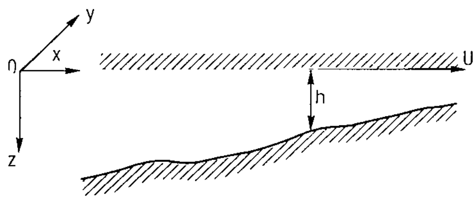
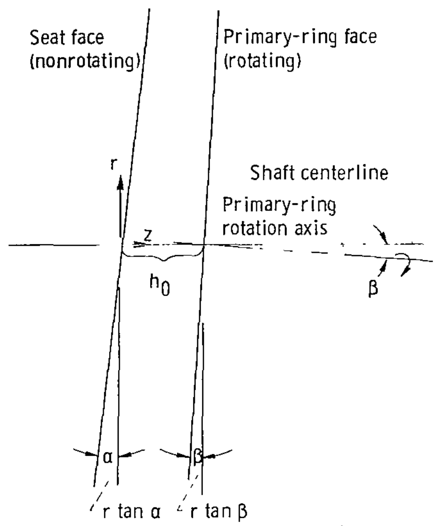
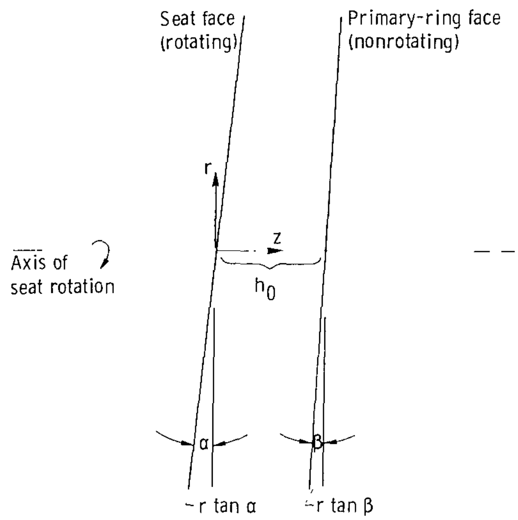


Figure 4. - Rectangular coordinate system for lubricant film.



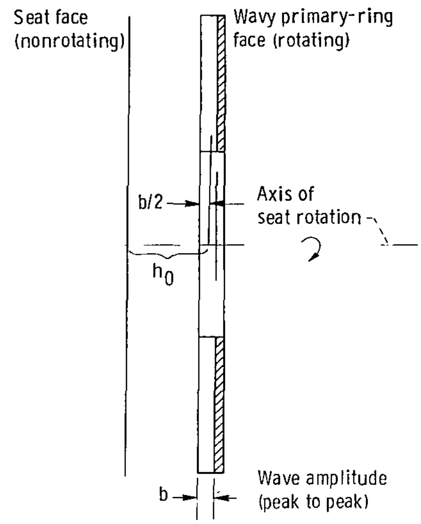
(a-1) Rotating primary ring (configuration A): nonrotating hydrodynamic pressure pattern; time-independent seal gap; secondary-seal friction (force couple exists); no primary-ring inertia.



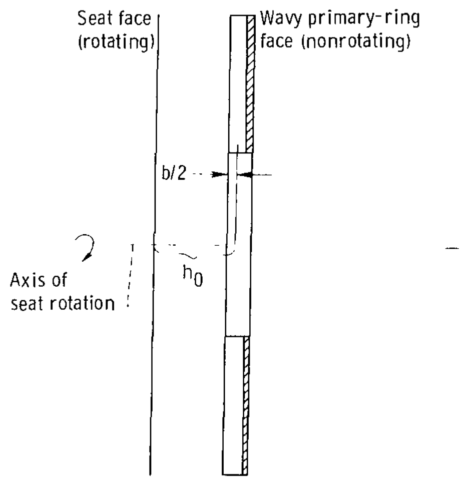
(a-2) Rotating seat (configuration B): rotating hydrodynamic pressure pattern; time dependent seal gap; secondary-seal friction (force couple exists); primary-ring inertia.

(a) Relative angular misalinement between primary ring and seal.

Figure 5. - Proposed models of face-seal operation.



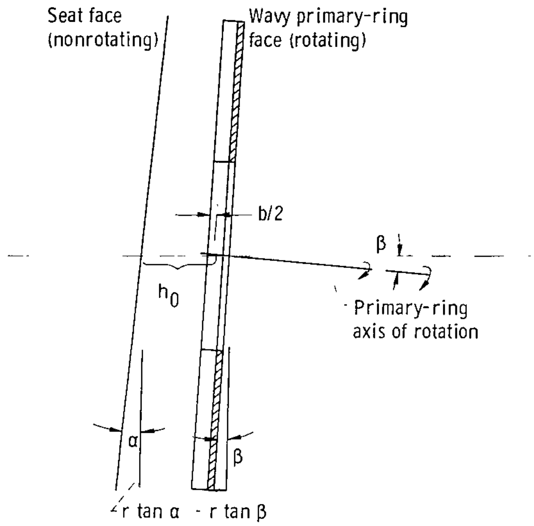
(b-1) Rotating primary ring (configuration C): rotating hydrodynamic pressure pattern; time-dependent seal gap; no secondary-seal friction; no primary-ring inertia.



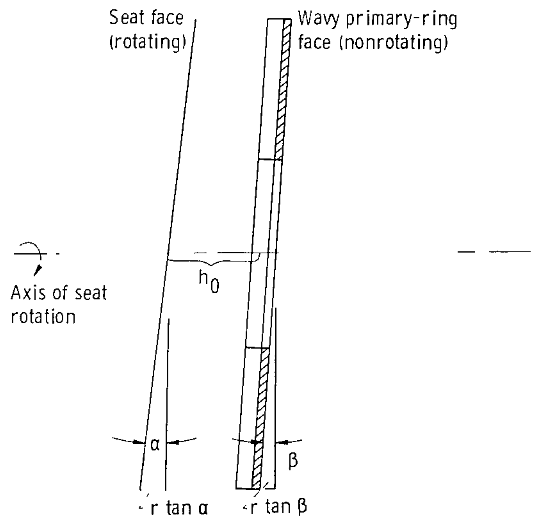
(b-2) Rotating seat (configuration D): non-rotating hydrodynamic pressure pattern; time-independent seal gap; no secondary-seal friction; no primary-ring inertia.

(b) Primary-ring waviness.

Figure 5. - Continued.



(c-1) Rotating primary ring (configuration E): hydrodynamic pressure pattern with time-dependent and combined rotating and nonrotating components; time-dependent seal gap; secondary-seal friction (force couple exists); primary-ring inertia.



(c-2) Rotating seat (configuration F): hydrodynamic pressure pattern with time-dependent and combined rotating and non-rotating components; time-dependent seal gap; secondary-seal friction (force couple exists); primary-ring inertia.

(c) Combined relative angular misalinement and primary-ring waviness.

Figure 5. - Concluded.

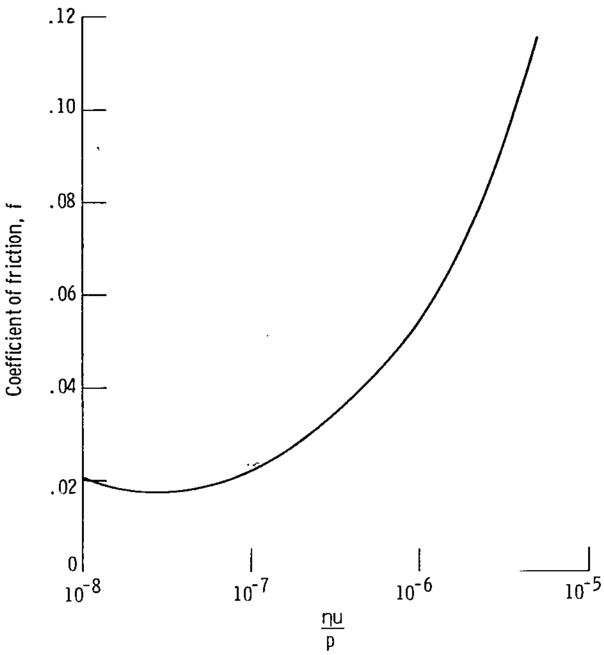


Figure 6. - Variation of seal coefficient of friction with $\eta u/p$. (From ref. 22.)

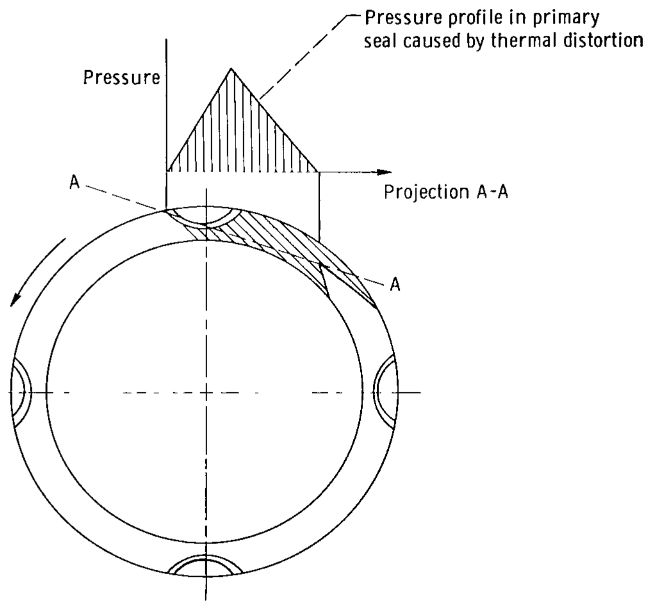
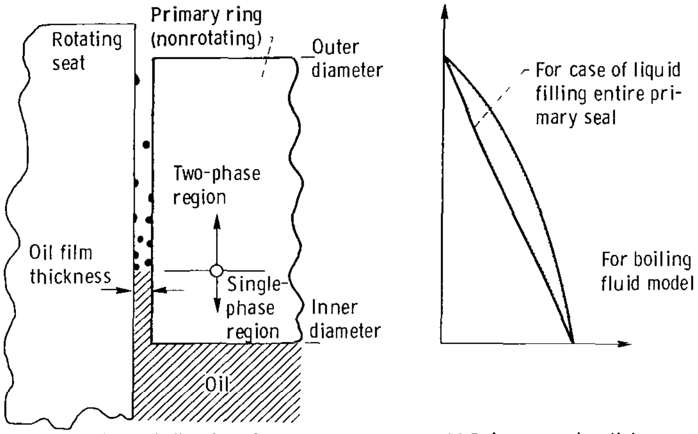


Figure 7. - Circular grooves in face of primary ring. (From ref. 16.)



(a) Schematic of oil film in primary seal.

(b) Primary-seal radial pressure profile.

Figure 8. - Conventional oil-lubricated radial face seal. Seal sliding velocity, 11.3 m/sec (37 ft/sec). (From ref. 1).

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