NGR 09-015-002

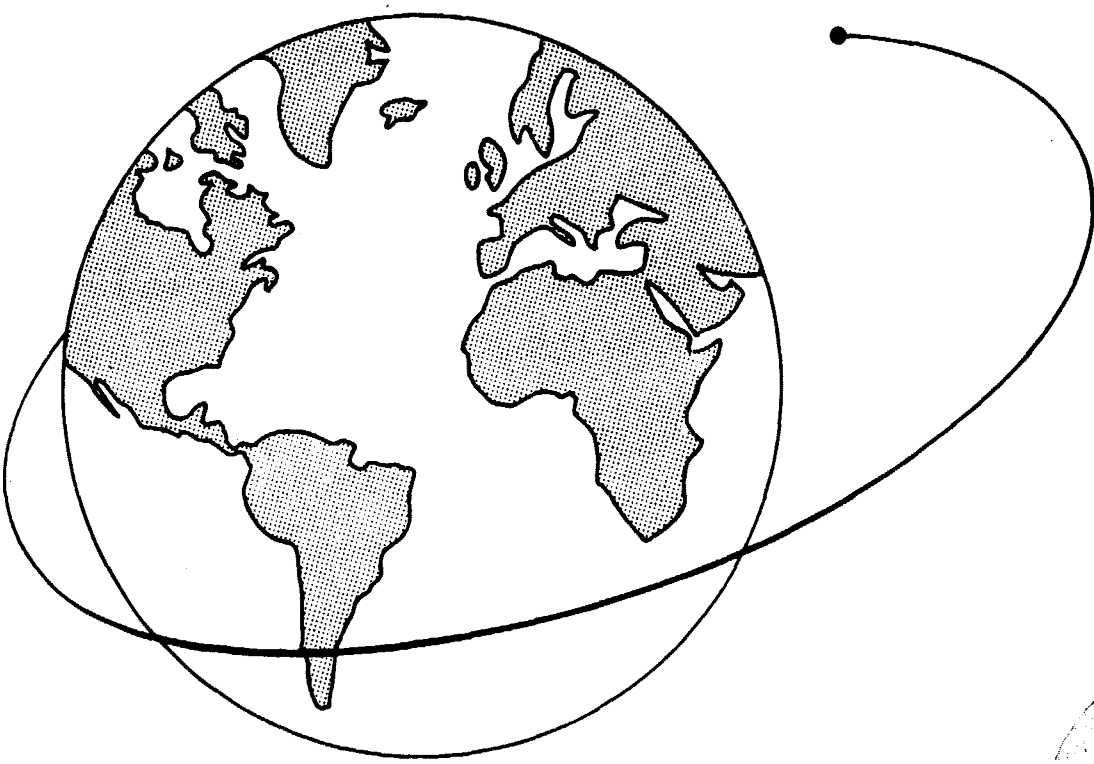
# ASTROGEODETIC GEOID OF JAPAN

Y. GANEKO

(NASA-CR-146855) ASTROGEODETIC GEOID OF JAPAN (Smithsonian Astrophysical Observatory) 38 p HC $4.00 CSCL 08E

N76-21799

Unclas
G3/46 25179





Smithsonian Astrophysical Observatory
SPECIAL REPORT 372

Research in Space Science
SAO Special Report No. 372

# ASTROGEODETIC GEOID OF JAPAN

Yasuhiro Ganeko

March 1, 1976

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

509-053

## TABLE OF CONTENTS

## ILLUSTRATIONS



## TABLES

## ABSTRACT

Three kinds of astrogeodetic geoid maps for Japan are presented: one referred to the global (18, 18) geoid of the 1973 Smithsonian Standard Earth (III) (SE III), one referred to the best-fitting ellipsoid of SE III, and one referred to the reference ellipsoid of the Tokyo datum. Interpolations of the deflection of the vertical are carried out by a least-squares estimation method. The geoid-height differences obtained are compared with solutions of satellite-derived station positions. Good agreement is found in a comparison with doppler-tracking stations.

# ASTROGEODETIC GEOID OF JAPAN*

Yasuhiro Ganeko†

## 1. INTRODUCTION

There are several ways of obtaining the geoid undulation. One is through a satellite-derived geoid, which is limited to long-wavelength components owing to the inherent inaccuracies of satellite tracking. Another is the gravimetric geoid, computed by applying Stoke's formula to surface-gravity data. This method, however, requires gravity values covering the whole surface of the earth, and such dense coverage is currently difficult to obtain.

When a detailed and relative geoid is to be determined in a restricted area and when there are sufficient data in that area, the astrogeodetic geoid is practical to obtain, in that it does not need worldwide data. Even though observations of the deflection of the vertical are time-consuming, the astrogeodetic geoid is valuable because interpolation is possible if suitable amounts of vertical-deflection observations exist.

*This work was supported in part by Grant NGR 09-015-002 from the National Aeronautics and Space Administration.

†Visiting Scientist, Smithsonian Astrophysical Observatory, October 1974 through October 1975. Currently in the Hydrographic Department, Maritime Safety Agency, Tokyo.

## 2. ASTROGEODETIC GEOID

The difference in geoid height between astrogeodetic stations is computed from Helmert's formula

$$N_{i+1} - N_i = -\int_i^{i+1} (\xi \cos A + \eta \sin A)\, ds \quad , \tag{1}$$

where $N_i$ is the geoid height of the ith station; $\xi$ and $\eta$ are the vertical-deflection components in the meridian and prime vertical, respectively; and A is the azimuth of the direction of the ith station to the (i + 1)th station.

The current method of interpolating the deflection of the vertical needs dense gravity measurements around the astrogeodetic stations. Another possibility of interpolation, when the vertical-deflection stations are sufficiently dense, uses least-squares estimation (Heiskanen and Moritz, 1967). Least-squares methods also provide an estimate of accuracy. The number of vertical-deflection observations that have been made in Japan since 1886 is more than 450. This distribution is sufficiently dense for most areas in Japan.

To find the accuracy of an estimate of the deflection, let the vector V be

$$V = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \quad .$$

Assuming that the vertical deflection at point p, $V_p$, can be estimated by the linear combination of observed values, $V_i$, we can write

$$\tilde{V}_p = \sum_i \alpha_{pi} V_i \quad , \tag{2}$$

**PRECEDING PAGE BLANK NOT FILMED**

where $\alpha_{pi}$ are scalar coefficients. This means we can neglect the correlation between $\xi$ and $\eta$. The difference between the correct value at p and the estimated one is the error of estimation:

$$\varepsilon_p = V_p - \tilde{V}_p = V_p - \sum_i \alpha_{pi} V_i \quad .$$

By squaring, we get

$$\varepsilon_p^2 = \left(V_p - \sum_i \alpha_{pi} V_i\right)^T \left(V_p - \sum_i \alpha_{pi} V_i\right)$$

$$= \left(\xi_p^2 + \eta_p^2\right) - 2\sum_i \alpha_{pi} \left(\xi_p \xi_i + \eta_p \eta_i\right) + \sum_{ij} \alpha_{pi} \alpha_{pj} \left(\xi_i \xi_j + \eta_i \eta_j\right) \quad . \tag{3}$$

From equation (3), the root-mean-square error of the estimation, $m_p$, is given by

$$m_p^2 = C_{\xi\xi}^{pp} + C_{\eta\eta}^{pp} - 2\sum_i \alpha_{pi} \left(C_{\xi\xi}^{pi} + C_{\eta\eta}^{pi}\right) + \sum_{ij} \alpha_{pi} \alpha_{pj} \left(C_{\xi\xi}^{ij} + C_{\eta\eta}^{ij}\right) \quad , \tag{4}$$

where $C_{\xi\xi}^{AB}$ and $C_{\eta\eta}^{AB}$ are covariance functions of the vertical-deflection components $\xi$ and $\eta$, respectively. These covariance functions depend on distance and direction. Using the expression

$$G_{AB} = C_{\xi\xi}^{AB} + C_{\eta\eta}^{AB} \quad ,$$

we can write

$$m_p^2 = G_{pp} - 2\sum_i \alpha_{pi} G_{pi} + \sum_{ij} \alpha_{pi} \alpha_{pj} G_{ij} \quad . \tag{5}$$

To find the coefficients $a_{pi}$ that minimize the estimation error, the necessary conditions are

$$\frac{\partial m_p^2}{\partial a_{pi}} = 0 \quad , \qquad i = 1, 2, \ldots, K \quad ,$$

or

$$\sum_{j=1}^{K} a_{pj} G_{ij} = G_{pi} \quad , \qquad i = 1, 2, \ldots, K \quad , \tag{6}$$

where K is the number of observations used for the estimation. Expression (6) is a system of linear equations with K unknowns, $a_{pj}$. By use of matrix notation, equation (6) can be written as

$$\begin{aligned} \overline{G}\overline{a}_p &= \overline{G}_p \quad , \\ \overline{a}_p &= \overline{G}^{-1}\overline{G}_p \quad , \end{aligned} \tag{7}$$

where

$$\overline{a}_p = \begin{bmatrix} a_{p1} \\ a_{p2} \\ \cdot \\ \cdot \\ \cdot \\ a_{pK} \end{bmatrix} \quad , \qquad \overline{G}_p = \begin{bmatrix} G_{p1} \\ G_{p2} \\ \cdot \\ \cdot \\ \cdot \\ G_{pK} \end{bmatrix} \quad , \qquad \overline{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1K} \\ G_{21} & G_{22} & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ G_{K1} & \cdots & & G_{KK} \end{bmatrix} \quad .$$

The standard error of least-squares estimation is given by

$$m_p^2 = G_{pp} - \left(\overline{G}_p^T \overline{G}^{-1} \overline{G}_p\right) \quad . \tag{8}$$

## 3. COVARIANCE FUNCTION $G_{AB}$

The observed deflection of the vertical includes both the systematic datum shift and the deflection included in the global geoid. To obtain the detailed local geoid by means of a covariance function, these two components should be eliminated.

For the datum shift, we introduce the translation values $\Delta x$, $\Delta y$, and $\Delta z$ of the geodetic datum, determined by satellite tracking. According to the 1973 Smithsonian Standard Earth (III) (SE III) (Gaposchkin, 1973, 1974), these values for the Tokyo datum are

$$\begin{aligned} \Delta x &= -136 \text{ m} \quad , \\ \Delta y &= +521 \text{ m} \quad , \\ \Delta z &= +681 \text{ m} \quad . \end{aligned} \tag{9}$$

By adding these values to the station coordinates on the Tokyo datum and neglecting the geoid height and possible datum rotation, the new three-dimensional geocentric coordinates become

$$\begin{aligned} x' &= (N + h) \cos \phi \cos \lambda + \Delta x \quad , \\ y' &= (N + h) \cos \phi \sin \lambda + \Delta y \quad , \\ z' &= \left[\left(\frac{b}{a}\right)^2 N + h\right] \sin \phi + \Delta z \quad , \end{aligned}$$

where $\phi$ and $\lambda$ are on the Tokyo datum and h is the mean-sea-level height; N is given by

$$N = \frac{a^2}{(a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{1/2}} \quad ,$$

$$b = a(1 - f) \quad ,$$

$$a = 6377397.2 \text{ m} \quad ,$$

$$f = 1/299.1528 \quad ,$$

PRECEDING PAGE BLANK NOT FILMED

in which a and f are the parameters of the reference ellipsoid of the Tokyo datum. The coordinates $x'$, $y'$, and $z'$ can be converted to the geocentric $\phi'$, $\lambda'$, $h'$ system throug 'he inverse transformation of

$$x' = (N' + h') \cos \phi' \cos \lambda' \quad ,$$

$$y' = (N' + h') \cos \phi' \sin \lambda' \quad ,$$

$$z' = \left[\left(\frac{b}{a}\right)^2 N' + h'\right] \sin \phi' \quad ,$$

$$b' = a'(1 - f') \quad ,$$

$$N' = \frac{a'^2}{(a'^2 \cos^2 \phi' + b'^2 \sin^2 \phi')^{1/2}} \quad ,$$

for which the ellipsoid parameters

$$a' = 6378140 \text{ m} \quad ,$$

$$f' = 1/298.256 \quad ,$$

are those of the best-fitting ellipsoid of the SE III solution. We can calculate the components of the deflection caused by datum shifts from the differences between the datum coordinates $(\phi, \lambda)$ and the geocentric ones $(\phi', \lambda')$:

$$\begin{pmatrix} \Delta\xi \\ \Delta\eta \end{pmatrix} = \begin{bmatrix} \phi - \phi' \\ \cos \phi \, (\lambda - \lambda') \end{bmatrix} \quad . \tag{10}$$

The components included in the global geoid are given by using the satellite-derived geopotential of the earth. These are

$$\begin{pmatrix} \xi_s \\ \eta_s \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{Rg_0} \dfrac{\partial T_s}{\partial \phi} \\ -\dfrac{1}{Rg_0 \cos \phi} \dfrac{\partial T_s}{\partial \lambda} \end{pmatrix} \quad , \tag{11}$$

where $T_s$ is the satellite-derived anomalous geopotential, R is the mean radius of the earth, and $g_0$ is the mean value of gravity over the earth. From the SE III geopotential coefficients $C_{nm}$, $S_{nm}$ up through degree and order 18, plus the following earth's constants,

$$a = 6378140 \text{ m} \quad ,$$

$$f = 1/298.256 \quad ,$$

$$GM = 3.986013 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2} \quad ,$$

$$\omega = 7.292115085 \times 10^{-5} \text{ sec}^{-1} \quad ,$$

the anomalous geopotential, from a spherical approximation, is given by

$$T_s = \left(\frac{GM}{R}\right) \sum_{n=2}^{18} \sum_{m=0}^{n} (\delta C_{nm} \cos m\lambda + \delta S_{nm} \sin m\lambda) P_n^m (\sin \phi) \quad ,$$

$$\delta C_{nm} = C_{nm} - C_{nm}^0 \quad ,$$

$$\delta S_{nm} = S_{nm} - S_{nm}^0 = S_{nm} \quad .$$

Following the expressions of Caputo (1967), the normal geopotential coefficients $C_{nm}^0$, $S_{nm}^0$ are

$$S_{nm}^0 = 0 \quad ,$$

$$C_{nm}^0 = 0 \quad , \qquad \text{when } m \neq 0 \text{ or } n \text{ is odd} \quad ,$$

$$C_{2\ell,0}^0 = \frac{(-1)^\ell}{2\ell + 1} \left[1 + \frac{8\ell K_2}{3(2\ell + 3) GM}\right] f^\ell (2 - f)^\ell \quad , \qquad \ell = 1, 2, \ldots,$$

$$K_2 = - \frac{\varepsilon^3 b^3 \omega^2 (1 + \varepsilon^2)}{2[(3 + \varepsilon^2) \tan^{-1} \varepsilon - 3\varepsilon]} \quad ,$$

$$b = a(1 - f) \quad ,$$

and

$$\varepsilon^2 = \frac{a^2 - b^2}{b^2} \quad .$$

The residuals

$$\begin{pmatrix} \xi_R \\ \eta_R \end{pmatrix} = \begin{pmatrix} \xi - \xi_s - \Delta\xi \\ \eta - \eta_s - \Delta\eta \end{pmatrix} \tag{12}$$

express the local variation of the geoid. These residuals were calculated for Japan, by using the SE III geopotential for equation (11), at every station where vertical-deflection observations were carried out from 1947 to 1973; there were 284 such sites.* The means and the mean squares of the residuals are

$$\langle \xi_R \rangle = -0\overset{''}{.}23 \quad , \qquad \langle \xi_R^2 \rangle = (7\overset{''}{.}58)^2 \quad ,$$

$$\langle \eta_R \rangle = +1\overset{''}{.}64 \quad , \qquad \langle \eta_R^2 \rangle = (8\overset{''}{.}16)^2 \quad .$$

The next step is to evaluate the covariance function $G_{AB}$ by using the vertical-deflection residuals. When the anomaly correlation distance is small compared to the radius of the earth and when the anomaly is statistically homogeneous, the following relation exists between the gravity-anomaly correlation and the vertical-deflection correlation (Shaw, Paul, and Henricson, 1969; Jordan, 1972):

$$C_{\xi\xi} + C_{\eta\eta} = \frac{1}{g_0^2} C_{gg} \quad , \tag{13}$$

*Bulletin of the Geographical Survey Institute, Japan, vol. II, parts 2 and 3, 1951; vol. III, parts 2 to 4, 1953; vol. IV, parts 3 and 4, 1955; vol. V, part 4, 1958; vol. XVIII, part 1, 1972; and unpublished.new data.

where $C_{gg}$ is the covariance function of the local gravity anomaly. If the gravity anomaly is isotropic, then, from equation (13), $G_{AB}$ is a function of distance only. By averaging $C_{\xi\xi} + C_{\eta\eta}$ in the azimuth, the covariance function G for Japan is well represented by an analytical expression of the form (see Figure 1)

$$G(r) = \sigma^2 e^{-r/D} \quad , \tag{14}$$

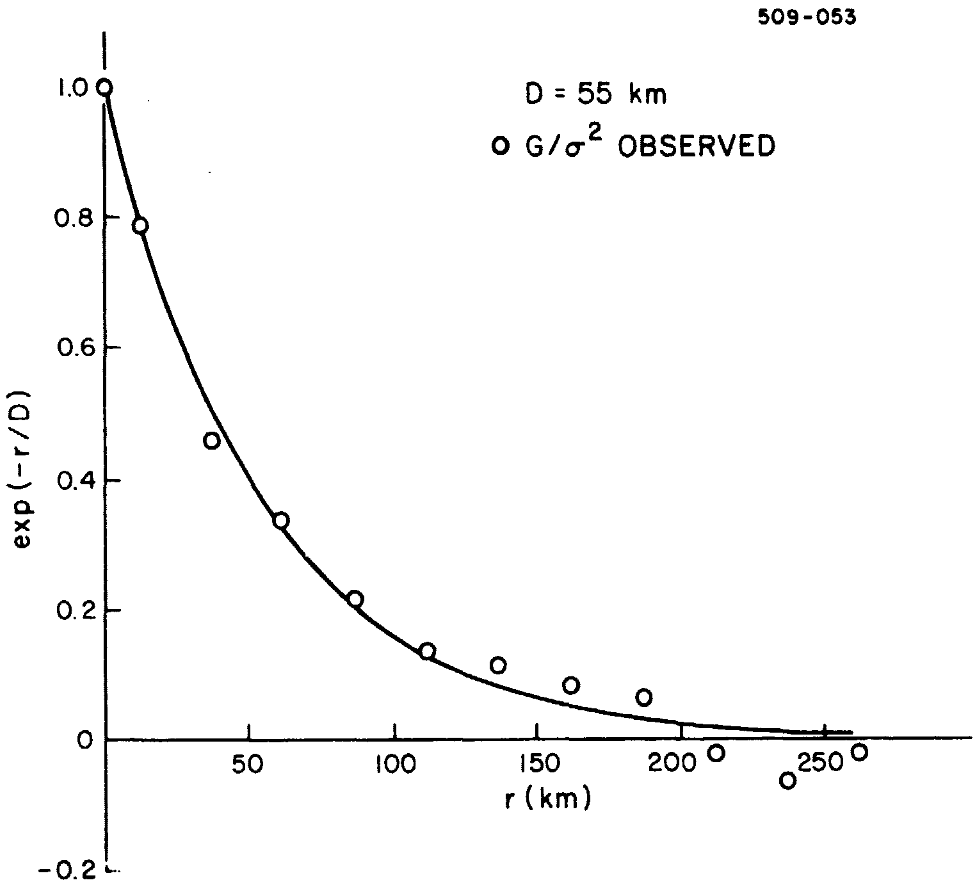



Figure 1. Covariance function $C_{\xi\xi} + C_{\eta\eta} = G$.

where D = 55 km and $\sigma^2 = (11''.1)^2$. From equations (13) and (14), the covariance function of the local gravity anomaly in Japan is obtained:

$$C_{gg}(r) = \sigma_g^2 \, e^{-r/D} \quad ,$$
$$\sigma_g^2 = (53 \text{ mgal})^2 \quad , \tag{15}$$

where $\sigma_g^2$ is the mean-square value of the local gravity anomaly. The above analytical expression for $C_{gg}(r)$ gives the following covariance functions (Shaw et al., 1969):

$$C_{\xi\xi} = \sigma_d^2 \left[ \frac{C_{gg}(r)}{\sigma_g^2} - \cos 2A \, f_c(r) \right] ,$$

$$C_{\eta\eta} = \sigma_d^2 \left[ \frac{C_{gg}(r)}{\sigma_g^2} + \cos 2A \, f_c(r) \right] , \tag{16}$$

$$C_{\xi\eta} = C_{\eta\xi} = - \sigma_d^2 \sin 2A \, f_c(r) ,$$

where A is the azimuth measured clockwise from North,

$$f_c(r) = \frac{2}{r^2} - \left(1 + \frac{2}{r} + \frac{2}{r^2}\right) e^{-r} ,$$

$$\sigma_d^2 = \frac{1}{2g_0^2} \sigma_g^2 ,$$

and r is measured by the unit D. The values obtained from vertical-deflection data and the analytical model are shown in Figures 2a, b, and c.

The absolute value of the computed normalized covariance of $\xi$ and $\eta$ barely exceeds 0.1. Therefore, it is considered suitable to neglect the correlation between $\xi$ and $\eta$.

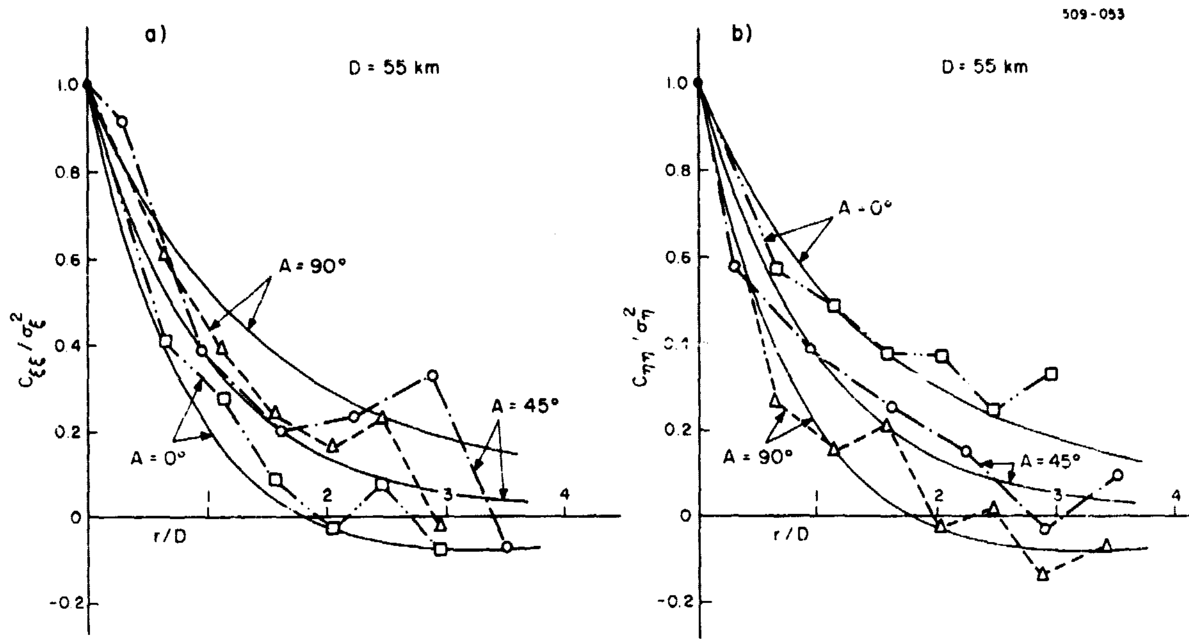



Figure 2. Covariance functions: a) $C_{\xi\xi}/\sigma_\xi^2$. b) $C_{\eta\eta}/\sigma_\eta^2$. Solid lines are from the theoretical model.

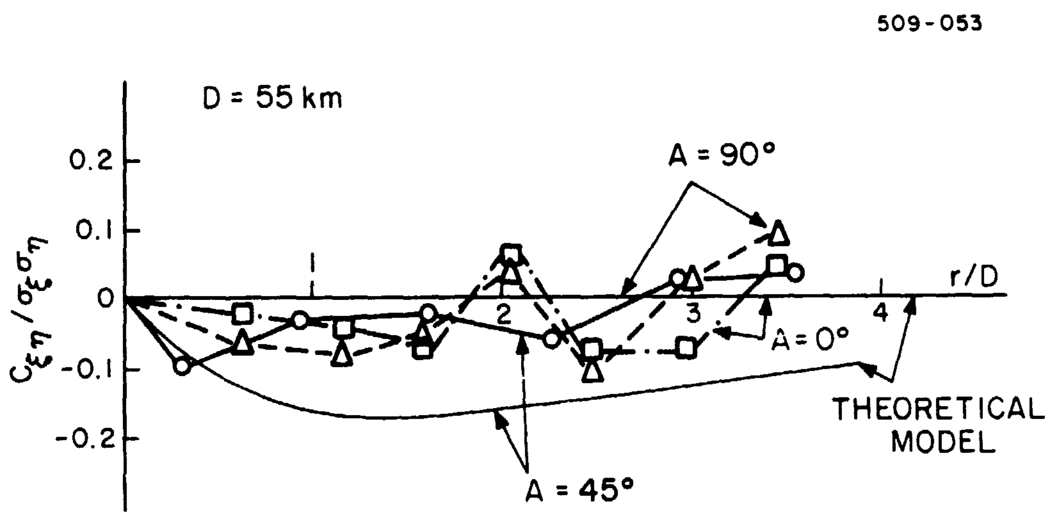



Figure 2c. Correlation between $\xi$ and $\eta$.

## 4. GEOID HEIGHT

Using Helmert's formula [equation (1)] and the least-squares estimation method [equations (2) and (7)] to interpolate the deflection of the vertical, we can calculate the relative local geoid referred to the SE III global geoid. The integration in equation (1) is replaced by the summation

$$N_{i+1} - N_i = -\sum_k \frac{\tilde{\varepsilon}_{k+1} + \tilde{\varepsilon}_k}{2} S \quad , \tag{17}$$

where S is chosen to be 10 km (see Figure 3), $\tilde{\varepsilon}_k$ is obtained from

$$\tilde{\varepsilon}_k = \tilde{\xi}_{R_k} \cos A + \tilde{\eta}_{R_k} \sin A \quad ,$$

and the estimated deflection of the vertical is obtained from equation (2).

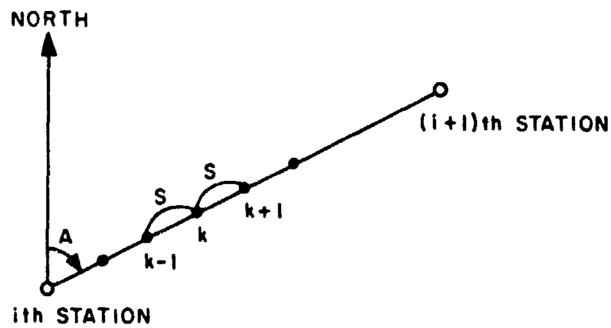



Figure 3. Geoid-height integration steps.

The observed stations covered a square 2° × 2° area around the estimated point. The number of stations in each area varied between 20 and 40. Adding the old data (Torao, 1949) brings to 450 the total number of vertical-deflection stations used to calculate the local geoid for Japan, excluding the old data from the Hokkaido area.

In the Hokkaido area, data before 1947 were not used, because of inconsistencies between the old and the new. Relative weights were not taken into consideration because of the uncertain accuracy of the old data, even though the old data might include large errors. The purpose of this paper, however, is to examine the efficiency of the least-squares estimation method in interpolating vertical deflections.

## 5. COMPARISON OF INTERPOLATION METHODS

The current method of interpolating vertical deflections is a gravimetric one, in which dense gravity observations around astrogeodetic stations are required. A test of gravimetric interpolation was carried out by Ono (1974) in a restricted area in Japan, and geoid-height (quasi-geoid-height) differences were calculated between astrogeodetic stations. Interpolations by means of least-squares estimation were also made along the same route, shown in Figure 4, and the results are presented in Figure 5. The geoid-height differences were transformed to those referred to the best-fitting ellipsoid of SE III. The maximum difference in geoid height due to different interpolation methods is 44 cm, and at the farthest point, numbered 263, the difference is 16 cm. These results would indicate that the least-squares estimation method is one of the most effective ways to interpolate vertical deflections. With this method, we can easily calculate the relative geoid height, without additional data, over that area of Japan that is densely covered with astrogeodetic stations.

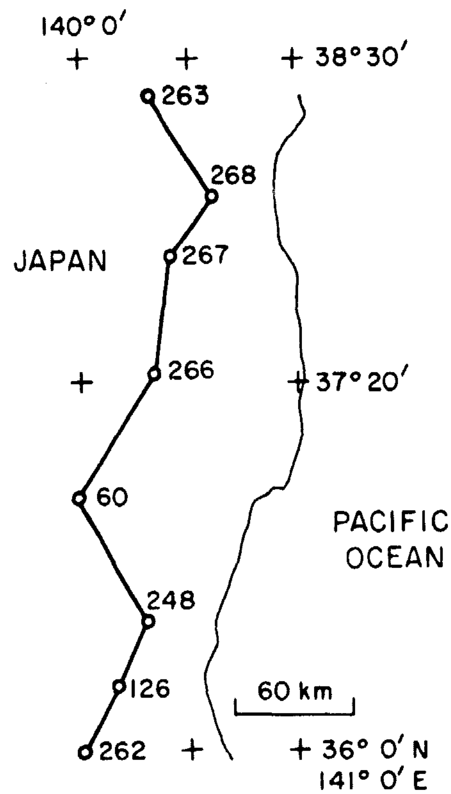



Figure 4. Portions of the route of the two interpolation methods, astrogravimetric and least-squares estimation. The astrogeodetic stations are indicated by circles and are numbered.

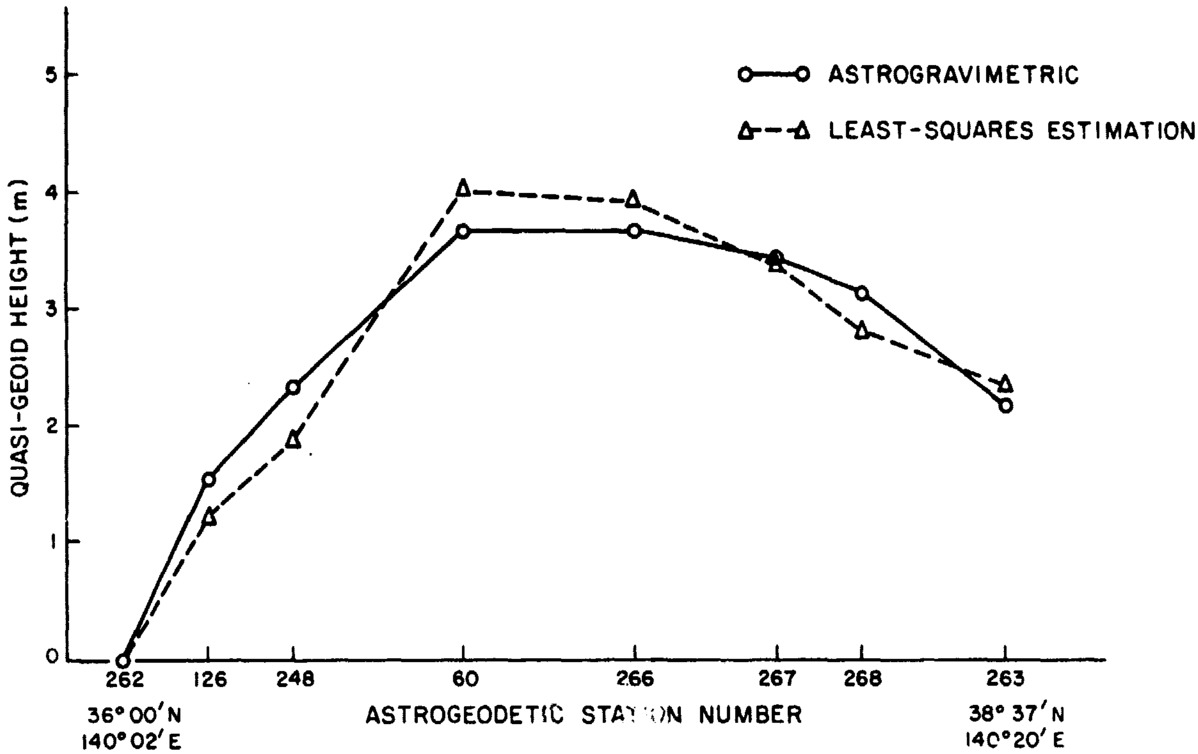



Figure 5. Comparison of the two interpolation methods.

## 6. GEOID MAPS FOR JAPAN

We examined the closing accuracy over Japan by dividing the country into test loops, labeled A, B, etc. and shown in Figure 6. The absolute values of the closing errors are also given in Figure 6. The closing errors are ~1 m at the most and usually less than 1 m, and the average is 0.71 m.

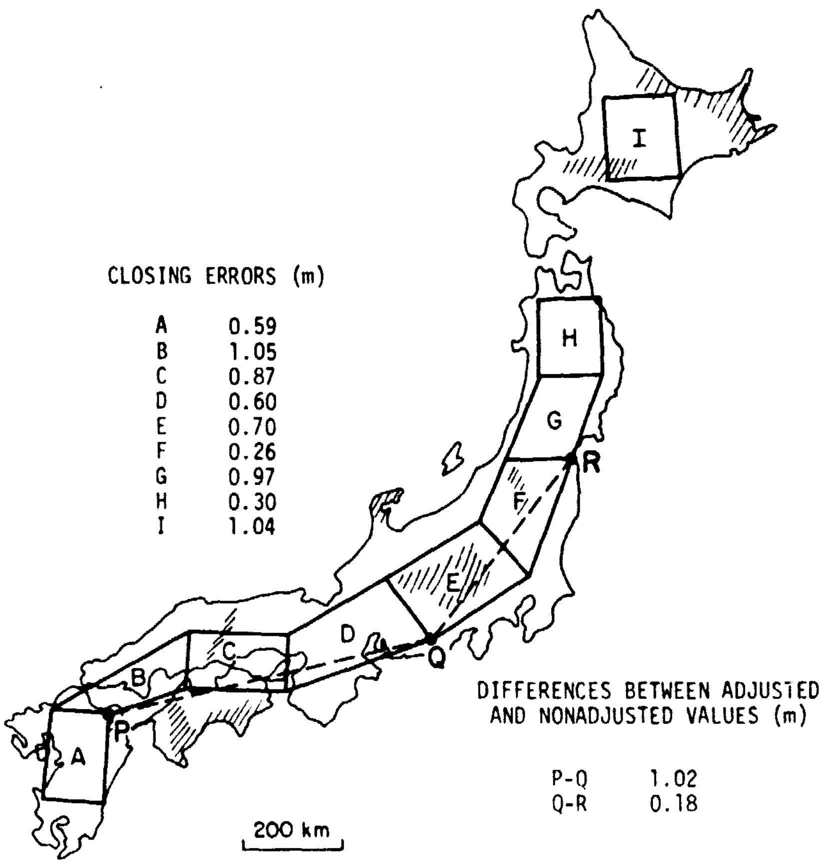



Figure 6. Loops used in the testing of closing accuracy.

To obtain the geoid undulation over Japan, series of loops of geoid profiles were calculated and adjusted so that the loop closures became zero. The adjustments were carried out by hand through trial and error until reasonable corrections could be found. The density of the data and an estimate of the accuracy were taken into consideration in the adjustment procedure. The length of each loop was between 200 and 500 km. Some comparisons between adjusted and directly integrated geoid-height differences [by use of equation (17)] are given in Table 1, and the locations of comparison points and the routes of direct integration are shown in Figure 6 by broken lines. The shaded portions in Figure 6 indicate where large corrections had to be added, in mountainous and sparse-data regions. More vertical-deflection observations are needed in these areas to achieve the same accuracy obtained from geoid-height calculations.

Table 1. Comparison of adjusted and nonadjusted geoid-height differences.

| Beginning point of integration | End point of integration | Direct integration (m) | After net adjustment (m) | Direct minus adjusted (m) |
|---|---|---|---|---|
| P | Q | | | |
| $\phi$ = 33°30'N | 35°00' | -1.98 | -0.96 | -1.02 |
| $\lambda$ = 131°30'E | 138°00' | | | |
| Q | R | | | |
| $\phi$ = 35°00' | 38°00' | | | |
| $\lambda$ = 138°00' | 141°00' | +3.05 | +2.87 | +0.18 |

The local geoid for Japan, calculated from the reduced vertical deflections $\xi_R$ and $\eta_R$ and referred to the SE III global geoid reproduced in Figure 7, is shown in Figure 8. The geoid height at the datum point is taken to be zero. The contour shapes are quite similar to those of the gravimetric geoid by Hagiwara (1967) (see Figure 9), although there are large discrepancies in the contour values themselves. These discrepancies, however, seem reasonable, given the fact that the gravimetric geoid was calculated from gravity data in a restricted area. Hagiwara's gravimetric geoid may be considered representative of local variations of the geoid, perhaps including large truncation errors (Hagiwara, 1970).

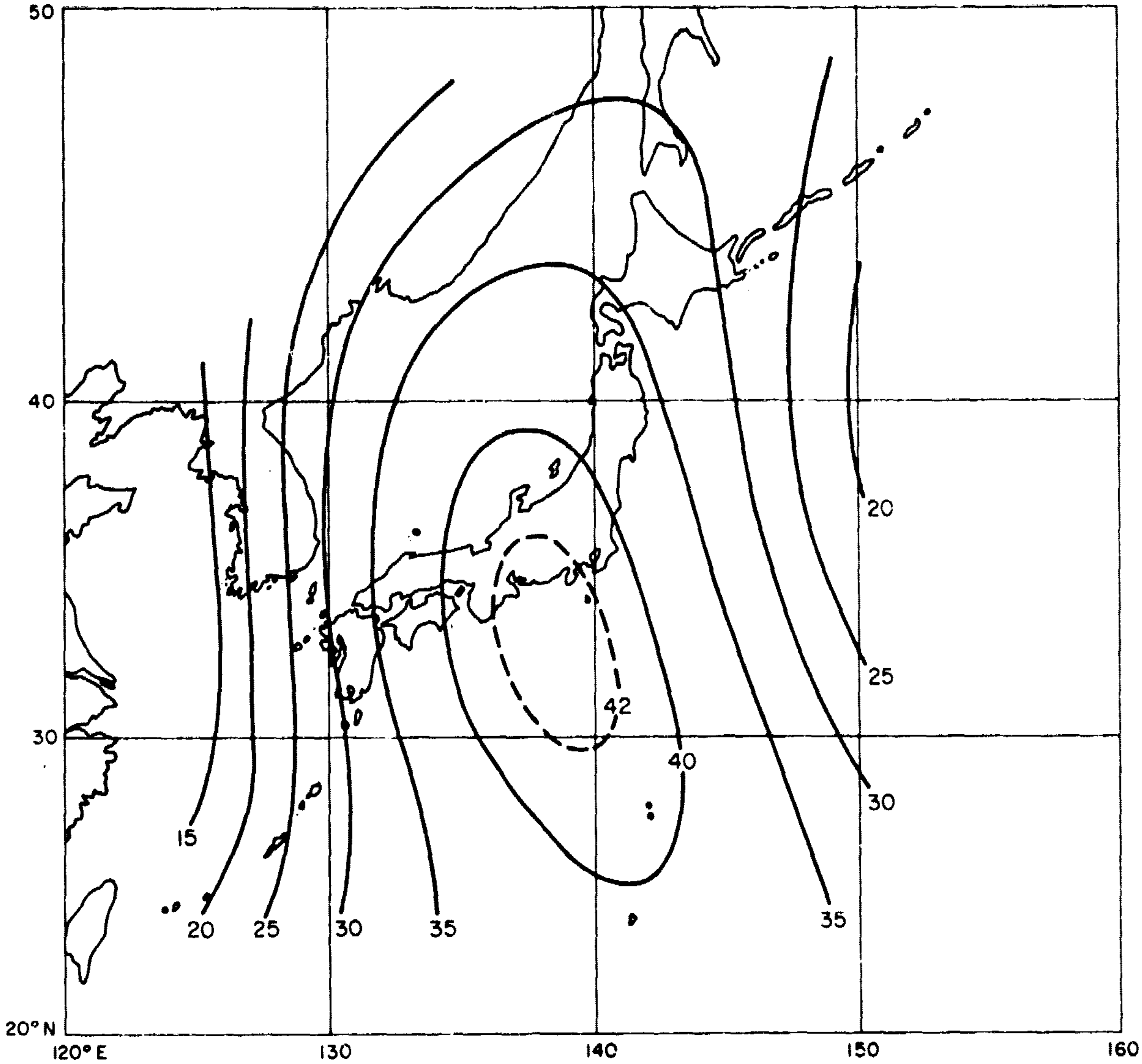



Figure 7. SE III geoid through degree and order 18, f = 1/298.256 (in meters).

Adding the global geoid (Figure 7) to the local geoid (Figure 8), we have the geoid referred to the SE III best-fitting ellipsoid; this is reproduced in Figure 10. The geoid height at the datum point is again taken to be zero. The geoid referred to the reference ellipsoid of the Tokyo datum can also be obtained through transformation by using translation values from equation (9) and ellipsoid parameters. This type of geoid is shown in Figure 11. Fisher (1960) published the same kind of astro-geodetic geoid for all of Japan except for Hokkaido and the southern part of Kyushu. The geoid-height differences between hers and Figure 11 are as much as 3 m in some areas.

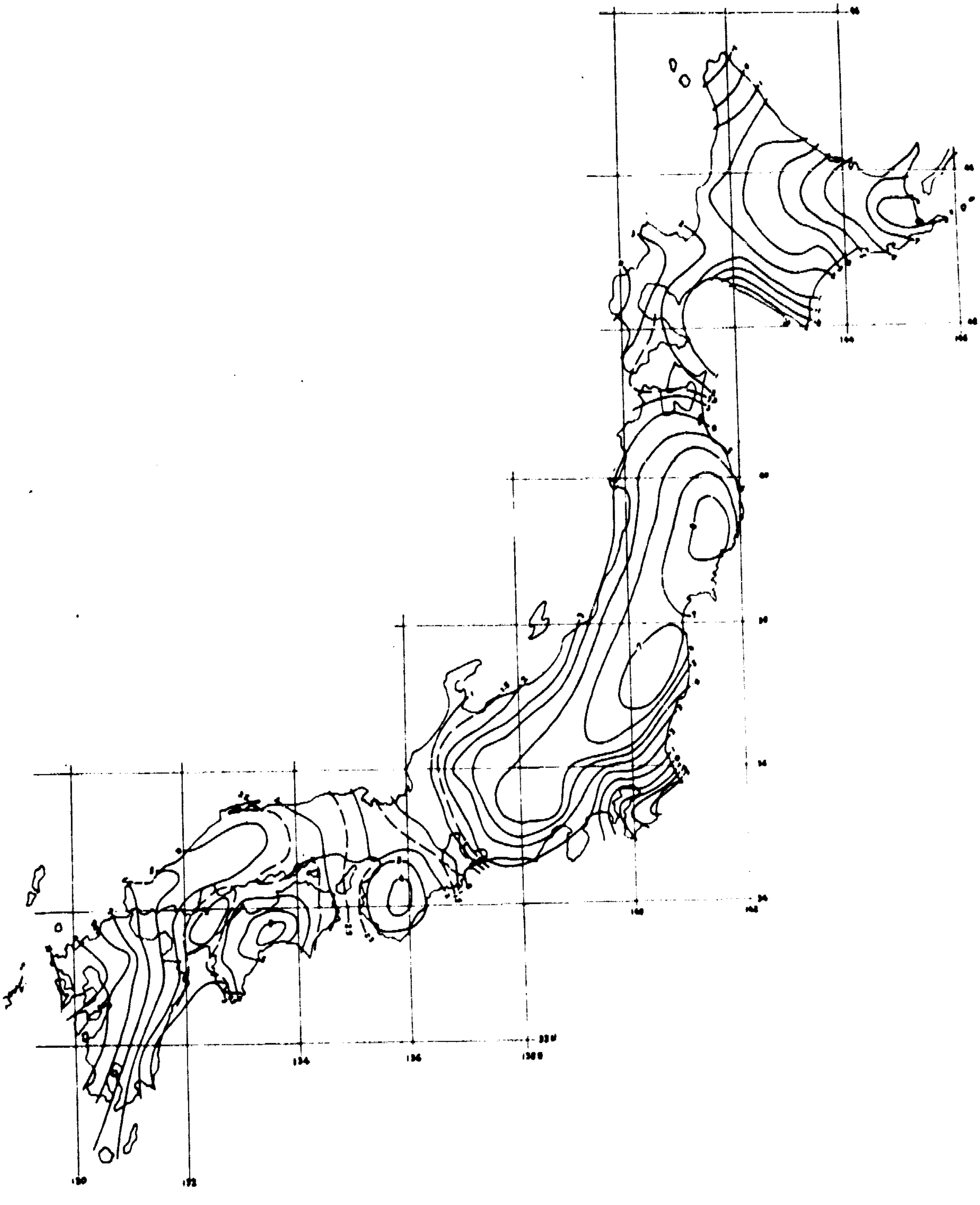


Figure 8. Astrogeodetic geoid for Japan referred to the SE III geoid (in meters).

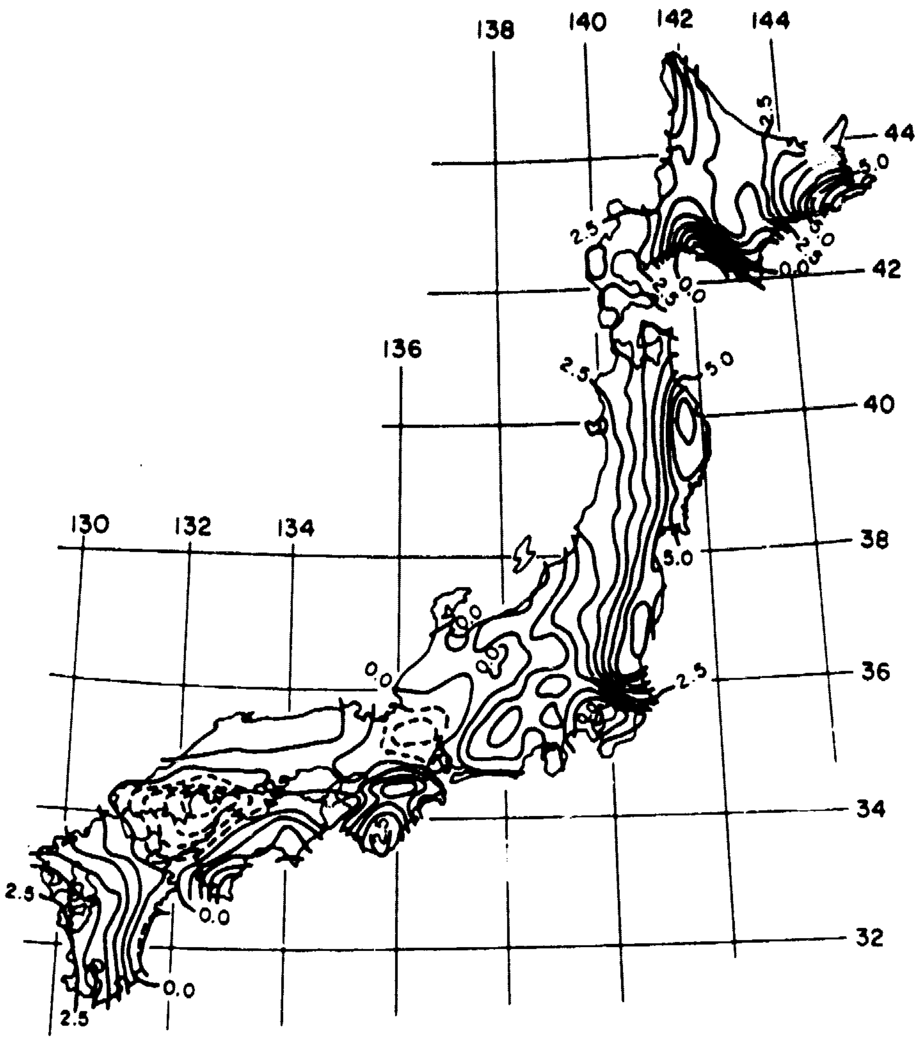



Figure 9. Gravimetric geoid for Japan (Hagiwara, 1967). The contour interval is 0.5 m.

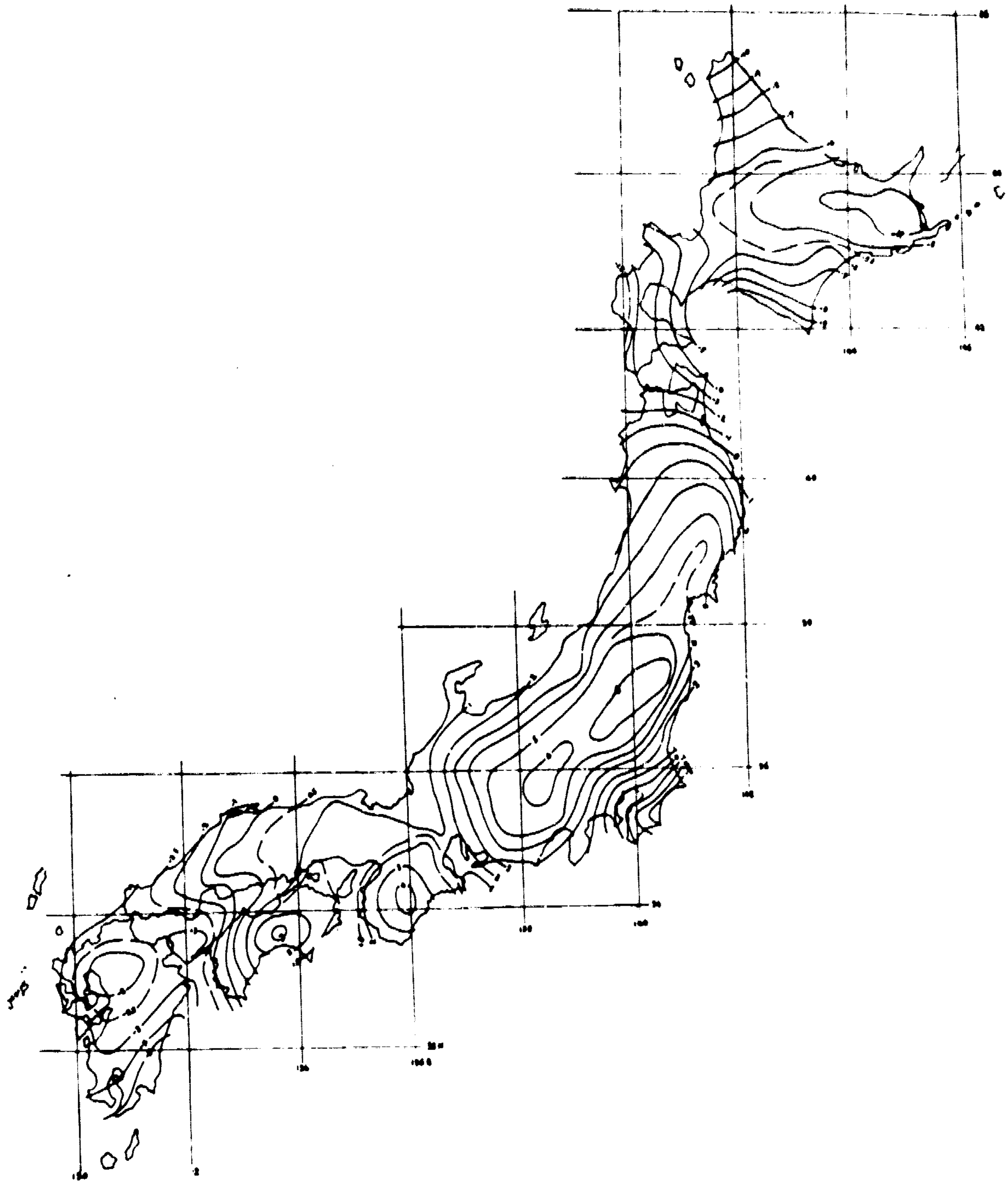


Figure 10. Astrogeodetic geoid for Japan referred to the best-fitting ellipsoid, a = 6378140 m, f = 1/298.256 (in meters).

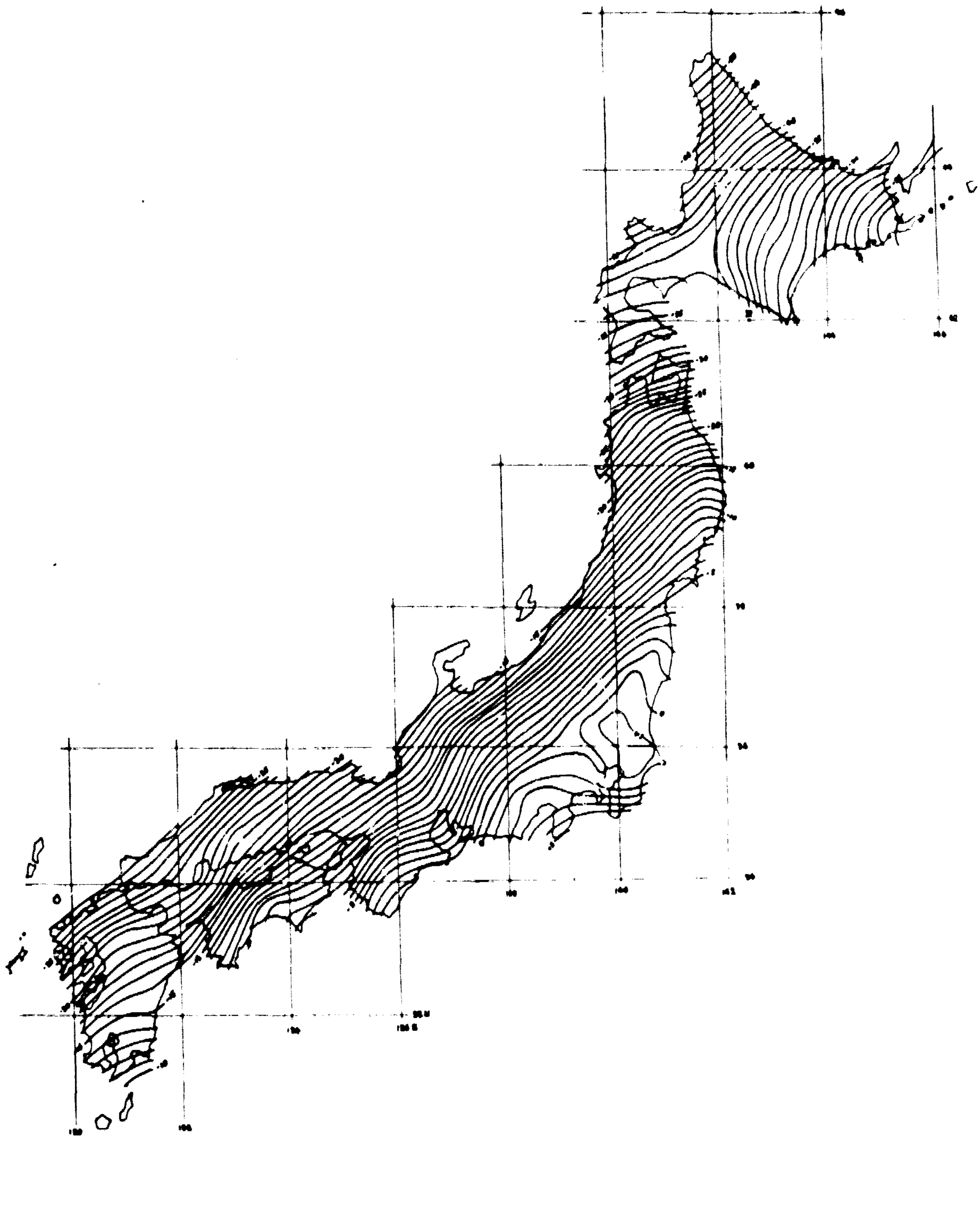


Figure 11. Astrogeodetic geoid for Japan referred to the reference ellipsoid (Bessel ellipsoid) of the Tokyo datum (in meters).

## 7. COMPARISON OF STATION COORDINATES

The geoid-height differences referred to the SE III best-fitting ellipsoid, a = 6378140 m, f = 1/298.256, were compared with solutions derived from SE III and from doppler observations (Anderle, 1974). The results are presented in Table 2. Locations of the satellite-tracking stations are shown in Figure 12.

Table 2. Comparison with satellite-derived geoid heights.

| Stations (geoid height, m) | Distance (km) | Geoid-height differences: Astro-geodetic (m) | SE III (m) | Doppler* (m) | Height discrepancy (m) |
|---|---|---|---|---|---|
| Tokyo – Dodaira (35.2) (38.3) | 48 | -3.1 | -3.1 (constrained) | – | 0.0 |
| Tokyo – Kanoya (35.2) (22.8) | 933 | -6.0 | -12.4 (partly constrained) | – | 6.4 |
| Sasebo – Misawa (30.3) (31.7) | 1334 | -2.3 | – | -1.4 | 0.9 |

*Anderle (1974; private communication, 1975).

The geoid height of Kanoya, which is 22.8 m in the SE III solution, is somewhat too low, considering the local geoid of Japan. The large discrepancy between the SE III and the astrogeodetic-geoid solutions seems to arise partly from the adopted geoid height (-19 m) of Kanoya on the Tokyo datum in SE III, in contrast to the -12 m determined in the astrogeodetic geoid.

PRECEDING PAGE BLANK NOT FILMED

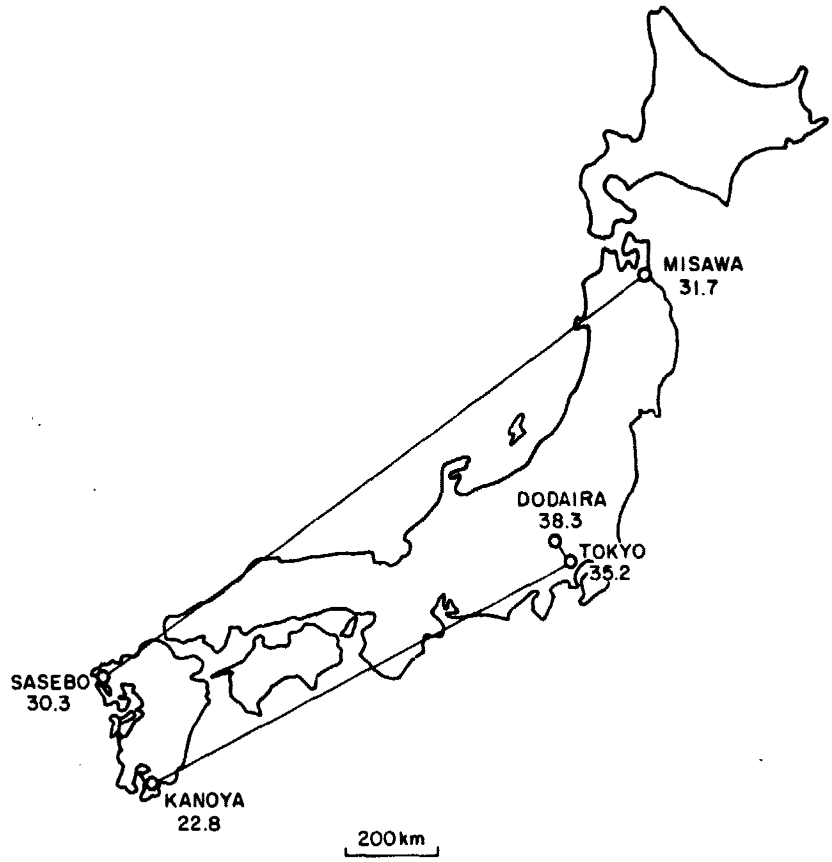


Figure 12. Locations of satellite-tracking stations. The satellite-derived geoid heights (in meters) are shown below the station names.

## 8. CONCLUSIONS

The efficiency of the least-squares estimation method in interpolating deflections of the vertical has been shown. This method requires only vertical-deflection data. When these data are dense enough – roughly speaking, more than 30 in a $2° \times 2°$ area – this method is one of the most effective ways of calculating geoid undulation.

The correlation distance of the local anomaly field in Japan is 55 km, and the root-mean-square value of the deflection of the vertical of the local anomaly field is $11\rlap{.}''1$.

Good agreement is found in the comparison of doppler results with the geoid obtained here.

## 9. ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. G. C. Weiffenbach and Dr. E. M. Gaposchkin for providing support during his stay at SAO. He thanks Dr. Gaposchkin and Ms. C. Wong for reading the manuscript and offering helpful suggestions. He also thanks Dr. R. J. Anderle and the staff of the Geographical Survey Institute and the Hydrographic Department of the Maritime Safety Agency of Japan for their kindness in the distribution of data.

## 10. REFERENCES

ANDERLE, R. J.

1974. Transformation of terrestrial survey data to doppler satellite datum. Journ. Geophys. Res., vol. 79, pp. 5319-5331.

CAPUTO, M.

1967. The Gravity Field of the Earth. International Geophysics Series, vol. 10, Academic Press, New York, 202 pp.

FISHER, I.

1960. An astrogeodetic world datum from geoidal heights based on the flattening f = 1/298.3. Journ. Geophys. Res., vol. 65, pp. 2067-2076.

GAPOSCHKIN, E. M., editor

1973. 1973 Smithsonian Standard Earth (III). Smithsonian Astrophys. Obs. Spec. Rep. No. 353, 388 pp.

1974. Earth's gravity field to the eighteenth degree and geocentric coordinates for 104 stations from satellite and terrestrial data. Journ. Geophys. Res., vol. 79, pp. 5377-5411.

HAGIWARA, Y.

1967. Analyses of gravity values in Japan. Bull. Earthquake Res. Inst., vol. 45, pp. 1091-1228.

1970. Truncation errors in the Stokes formula integration. Journ. Geod. Soc. Japan, vol. 16, pp. 190-198.

HEISKANEN, W., and MORITZ, H.

1967. Physical Geodesy. Freeman and Co., San Francisco, 364 pp.

JORDAN, S. K.

1972. Self-consistent statistical models for the gravity anomaly, vertical deflections and undulation of the geoid. Journ. Geophys. Res., vol. 77, pp. 3660-3670.

ONO, S.

1974. Considerations in obtaining detailed surveys of the form of the quasi-geoid and of the deflection of the vertical (1). Journ. Geod. Soc. Japan, vol. 20, pp. 117-124.

SHAW, L., PAUL, I., and HENRICSON, P.

1969. Statistical models for the vertical deflection from gravity-anomaly models. Journ. Geophys. Res., vol. 74, pp. 4259-4265.

TORAO, M.

1949. Studies on the deflection of vertical (I). Astron. Bull., 2nd ser., no. 17, pp. 129-135.