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CENTER FOR RADAR ASTRONOMY

STANFORD ELECTRONICS LABORATORIES

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ABSTRACT

The principal conclusions of this study can be stated as follows: (1) Atmospheric scale heights derived from radio occultation measurements of a turbulent planetary atmosphere would be accurate if the average angle of refraction through the turbulent atmosphere were equal to the refraction angle for the corresponding quiescent atmosphere; (2) These two angles are not equal (although their difference may not be large enough to cause significant error); (3) If necessary, it should be possible to correct for the systematic error introduced by this inequality by using measurements of signal spectra to determine characteristics of the turbulence; and (4) Sensitive dual-frequency measurements could help define the effects of turbulence in future radio occultation experiments. All of these differ directly or in emphasis from conclusions reached in a recent Letter by Hubbard and Jokipii (1975). Reasons for these differences are discussed. It does not appear that the angular offset due to turbulence has been important in past experiments, although it may become significant and require corrective analysis when improved equipment is used to probe deep into turbulent atmospheres with greater measurement precision.

I. INTRODUCTION

The problems encountered in the analysis of the Pioneer 10 and 11 radio occultation measurements of the atmosphere of Jupiter have demonstrated anew the need for careful consideration of all possible sources of error in such experiments (Hubbard, Hunten, and Kliore, 1975; Eshleman, 1975). In a recent Letter, Hubbard and Jokipii (1975) consider atmospheric turbulence as a possible source of error in the derivation of atmospheric scale heights from occultation measurements of doppler frequencies. This is a subject which has not been given the attention it deserves, and their results on wave propagation in a turbulent medium should prove useful in this regard. However, their method of applying such results to the radio occultation technique has led to inconsistent conclusions. The principal problem arises at the start of their derivation where they have introduced a spurious term into the relationship between doppler frequencies and angles of refraction.

The above subjects merit careful discussion because of the importance of the radio occultation technique in planetary exploration, and the recent questions raised concerning the credibility of occultation results. In the treatment that follows, I first use simple geometrical arguments to illustrate the relationship between the average angle of refraction of rays propagated through a turbulent atmosphere, and the experimentally-observed average doppler frequency. I then derive a less general result from a treatment of phase-path lengths, pointing out where Hubbard and Jokipii may have erred in using this more involved approach. Our differences are then discussed further. Finally, I offer comments on how turbulence could affect derived scale heights, and conclude that careful measurement and analysis should make it possible to correct for this effect if it proves to be important. It has been necessary to change some of the notation from that used in Hubbard and Jokipii's treatment in order to identify different parameters for which they use the same symbol.

II. THE BASIC EQUATIONS

From the geometry of Figure 1, with earth and planet fixed and spacecraft velocity $\bar{v}$ in the plane of propagation and normal to the direction to earth,

$$-\lambda f = \frac{\bar{k}_q \cdot \bar{v}}{k_q} = v \sin \alpha_q \cong v\alpha_q \qquad (1)$$

for small angles of refraction α_q in a quiescent atmosphere. Here $\bar{k}_q$ is the vector wave number, λ is the radio wavelength in free space, and f is the doppler frequency due to the refraction. While the problem is reciprocal, it may be helpful to think in terms of the transmitter on earth so that the spacecraft sweeps through the pattern of refracted rays, with the measurements of doppler frequency (received minus transmitted frequency) being made at the spacecraft.

I define a quiescent atmosphere as one in which atmospheric refractivity ν ($\nu = \mu - 1$, where μ is the refractive index) is a function only of height h, so that measured f's yield α_q's from which $\nu(h)$ can be derived (Fjeldbo, Kliore, and Eshleman, 1971). For a corresponding turbulent atmosphere, refraction angles α_t will also depend upon atmospheric structure having horizontal variations, and all of the structure can change with time. Thus a $\nu(h)$ profile derived in the same manner could show spurious small-scale vertical structure and could also depart systematically from the true profile. The concern here is only with possible systematic effects, over height differences comparable to and larger than the atmospheric scale height.

For a turbulent atmosphere, there may be a single ray represented by $\bar{k}_t$ and α_t at the spacecraft, with its direction deviating in a random and systematic manner from the ray through the corresponding quiescent atmosphere. For more intense turbulence, the atmospheric structure may cause ray crossovers so that multiple incoherent components are received simultaneously at the spacecraft, as represented in Figure 1. In either case,

$$-\lambda\langle f\rangle = \langle\frac{\bar{k}_t \cdot \bar{v}}{k_t}\rangle = v\langle\sin\alpha_t\rangle \cong v\langle\alpha_t\rangle \equiv v\alpha_{to} \qquad (2)$$

where α_{to} is the average value of the variable or multiple α_t's, all of which are assumed small. The averages of frequencies and angles are assumed to be formed using the same weighting, such as signal intensities, over time intervals comparable to the time for measuring through about one scale height. Additional discussion relative to equation (2) is given in the Appendix.

Hubbard and Jokipii (1975) attempt to determine the effects of turbulence by considering phase-path lengths and their derivatives. Total phase path P is the sum of the physical path and the extra electrical path length in the atmosphere. If both the bending and sinuosity of the rays are small in a turbulent atmosphere, it should be possible to express P in a manner analogous to the case for small bending in a quiescent atmosphere (Fjeldbo et al., 1965; Fjeldbo and Eshleman, 1965). Thus,

$$P \cong L + D\,(1 + \frac{\alpha_t^2}{2}) + \int_{path} \nu ds \qquad (3)$$

where the fixed lengths L and D are illustrated in Figure 1, and sec α_t is approximated by $1 + \alpha_t^2/2$.

To relate phase path lengths to doppler frequencies, note that in general, $-\lambda f = dP/dt$. For the turbulent atmosphere, $-\lambda\langle f\rangle = \langle dP/dt\rangle$ if the continuity of the components is established so that the proper averages can be taken. Hubbard and Jokipii break this down into the product $\langle dP/dh_{to}\rangle\ (dh_{to}/dt)$, where h_{to} is the distance of closest approach of the average ray. From Figure 1, $h_{to} \cong \alpha_{to}D - vt$ where time t is measured from when the spacecraft is opposite the limb. Thus

$$\frac{dh_{to}}{dt} \cong -\frac{v}{1 - D\,\dfrac{d\alpha_{to}}{dh_{to}}} \tag{4}$$

Defining variable parameters as their average values plus their fluctuating components, $\alpha_t = \alpha_{to} + \delta\alpha_t$ and $h_t = h_{to} + \delta h_t$. Using also $\delta h_t \cong D\delta\alpha_t$ and the derivative of the integral of refractivity as discussed below,

$$\begin{aligned}
\left\langle\frac{dP}{dh_{to}}\right\rangle &= \left\langle\frac{D}{2}\,\frac{d\alpha_t^2}{dh_{to}} + \frac{d\left(\int\nu ds\right)}{dh_{to}}\right\rangle \\
&= \left\langle D\alpha_t\,\frac{d\alpha_t}{dh_{to}} + \frac{d\left(\int\nu ds\right)}{dh_t}\,\frac{dh_t}{dh_{to}}\right\rangle \\
&= \left\langle D\alpha_t\left(\frac{d\alpha_{to}}{dh_{to}} + \frac{d(\delta\alpha_t)}{dh_{to}}\right) - \alpha_t\left(1 + D\,\frac{d(\delta\alpha_t)}{dh_{to}}\right)\right\rangle \\
&= -\alpha_{to}\left(1 - D\,\frac{d\alpha_{to}}{dh_{to}}\right)
\end{aligned} \tag{5}$$

so that

$$- \lambda \langle f \rangle = \left\langle \frac{dP}{dt} \right\rangle = \left\langle \frac{dP}{dh_{to}} \right\rangle \frac{dh_{to}}{dt} = v\alpha_{to} \tag{6}$$

as in equation (2), correct to second order in small angles. Since the integral of the refractivity in equation (5) represents the variable ray or one component of the multiple rays, it is differentiated with respect to h_t. This yields $-\alpha_t$, analogous to the case for the quiescent atmosphere (Fjeldbo et al., 1965; Eshleman, 1965 and 1973).

III. DISCUSSION

Hubbard and Jokipii (1975) obtain a result which differs from equation (6), and the difference is the principal point of their paper. Their result would be obtained using the above approach if the integral of refractivity in equation (5) were incorrectly assumed to be differentiable with respect to h_{to} to yield either $-\alpha_t$ or $-\alpha_{to}$. Such a procedure would produce a spurious mean-square term from $\langle \alpha_t^2 \rangle = \alpha_{to}^2 + \langle \delta\alpha_t^2 \rangle$, where α_t^2 comes from the expression for the geometrical path length. By this process,

$$\langle \frac{dP}{dt} \rangle = v\alpha_{to} \left[1 - \frac{D d\langle \delta\alpha_t^2 \rangle / dh_{to}}{2\alpha_{to}(1 - D d\alpha_{to}/dh_{to})} \right] \qquad (7)$$

which they would take also to equal

$$\langle \frac{dP}{dt} \rangle = v\alpha_q \left[1 - \frac{D d\langle \delta\alpha_t^2 \rangle / dh_{to}}{2\alpha_q (1 + M_q)} \right] \qquad (8)$$

where $M_q \equiv \alpha_q D/H$ and H is the scale height for an isothermal atmosphere. This last equation is equivalent to Hubbard and Jokipii's equation (3). It follows from the above incorrect equation (7), the assumption of an isothermal atmosphere, and their stated conclusion that the average angle of refraction for a turbulent atmosphere equals the refraction angle for the corresponding quiescent atmosphere, or $\alpha_{to} = \alpha_q$.

There are two separate problem areas relative to equations (7) and (8) and their derivation. First, we have seen how the mean-square term of Hubbard and Jokipii could have been generated by an improper derivative the integral representing the excess electrical phase-path length. One can also see on physical grounds that the extra term introduced into equation (7) by this approach must be spurious since $-\lambda\langle f\rangle$ can differ artibrarily from $\nu\alpha_{to}$ only if the atmosphere can give the illusion of generating or hiding an arbitrary number of wavelengths. Only very artibrary and physically unreasonable model atmospheres could do this.

My second point is that turbulence will affect the average angle of refraction, although possibly at too small a magnitude to be of significance. My differences with Hubbard and Jokipii are now somewhat complex. They say "one may show that" $\alpha_{to} = \alpha_q$, and also conclude that turbulence affects derived scale heights. These conclusions are inconsistent. It is evident from equations (1) and (2) that if $\alpha_{to} = \alpha_q$, turbulence would have no effect on derived scale heights. I now argue that $\alpha_{to} \neq \alpha_q$ so that turbulence will produce errors of some magnitude in scale heights derived by standard techniques.

Let the fractional difference $(\alpha_{to} - \alpha_q)/\alpha_q \equiv A$. The problem of finding A in terms of the characteristics of a turbulent atmosphere is a subject of continuing study. Work done to date convinces me that it is not zero in general, although it may be zero to first order in small quantities. The spacecraft receives the wandering single ray or multiple radio "glints" from a spread of heights. The signal components are affected in both amplitude and direction by characteristics of the turbulence, the average refraction of the atmosphere, and interplay between these phenomena. Different effects push A in both positive and negative directions but they cannot cancel in general since they have different dependencies on factors related to the geometry and the turbulence. (One may note that equation (8), which was derived from (7) in Hubbard and Jokipii's approach by assuming $A = 0$, could conceivably be correct by virtue of $-A$ equalling their term $(D d\langle \delta\alpha^2_t \rangle / dh_{to})/2\alpha_q(1+M_q)$. It does not seem possible to me that the several errors could cancel in this way, or that the total offset would have this functional dependence.)

The fractional angular offset A can be used with formulas from Eshleman (1975) to determine fractional errors in scale height or temperature T and pressure p for an isothermal atmosphere. Thus for small errors,

$$\frac{\delta H}{H} = \frac{\delta T}{T} \cong A(1+M_q)\left(\frac{H}{A}\frac{dA}{dh} - 1\right) \tag{9}$$

$$\frac{\delta p}{p} \cong \frac{3}{2}\left(\frac{\delta T}{T}\right) \tag{10}$$

Since the magnitude factor M_q may reach thousands in future experiments at Venus and the major planets (Eshleman, 1975), it is apparent that a very small turbulence-induced change in the average angle of refraction could introduce significant errors in derived temperatures and pressures if no attempt is made in the analysis of the data to correct for the effects of turbulence. However, M_q has been as high as 200 in past measurements at Venus, where strong turbulence is known to occur (Woo, 1975). Yet in this case the terms of equation (9) evidently were not large enough to produce noticeable error, since the results based on the standard analysis of doppler frequency measurements check within experimental error both in-situ measurements by the Soviet Venera spacecraft and independent occultation measurements based on signal intensities (Fjeldbo et al., 1971), where the magnification-or-error effect of equation (9) does not occur (Eshleman, 1975). There is no evidence that turbulence has caused significant errors in any past radio occultation experiment. This includes Pioneer 10 and 11 at Jupiter, where M_q was not as large as for Venus, and where it appears that other explanations of the initial erroneous results are at hand (Hubbard, Hunten, and Kliore, 1975; Eshleman, 1975).

In any event, there is no reason to depend only on measurements of f in future studies of atmospheric scale heights, since detailed spectra should also be available to provide information on characteristics of the turbulence. The scale and intensity of the turbulence affect both the angular offset and such spectra. I believe it will be possible to correct for effects of turbulence should it prove important to do so, using theory and the measurements of turbulence to find approximate values of A. Thus I am more optimistic than Hubbard and Jokipii, who are concerned that the effects of the average atmosphere will be difficult to separate from those of turbulence.

Finally, I note several additional problems associated with points raised by Hubbard and Jokipii.

a) The example they give to explain the Pioneer results would not do so even if their theory were correct. It would require a region of extreme cold (or even negative temperature) to match the high atmosphere to space, and this was not seen. A valid explanation which involves turbulence would require the fractional angular offset A to be approximately constant with height, and to have a value of about -0.1 for the Pioneer 10 entry example. For Pioneer 11 exit, on the other hand, turbulence would have had to create a value of A of the opposite sign if it were to be the cause of the original problem.

b) Hubbard and Jokipii conclude that measurements at different radio frequencies will not be helpful in resolving the errors caused by turbulence. However, the long history of efforts to obtain dual-frequency measurement capability in occultation experiments includes considerations of turbulence. Such measurement capability combined with sensitive radio instrumentation and careful analysis would help separate effects of ionospheric and atmospheric turbulence, would help determine characteristics of the atmospheric turbulence from the different signal properties at the two frequencies, would help in efforts to separate signal effects due to layers and turbulent structure, and would help determine the relative contributions of turbulent scattering, absorption, and refraction to measured changes in signal intensities.

c) While they state that additional detailed calculations will be required to estimate the doppler spread of the signal, this spread is actually due primarily to the mean-square fluctuating angle that they have already derived in terms of the fluctuations of refractivity in the turbulent atmosphere. If the radio rays sweep through the turbulent structure with a speed which is large compared to the characteristic speeds of the effective eddies, the root-mean-square doppler frequency spread δf, relative to the trend of the average frequency, is found simply from

$$\lambda \delta f \cong v \langle \delta\alpha_t^2 \rangle^{1/2} \tag{11}$$

Turbulent motions, if sufficiently rapid, could cause some additional spread.

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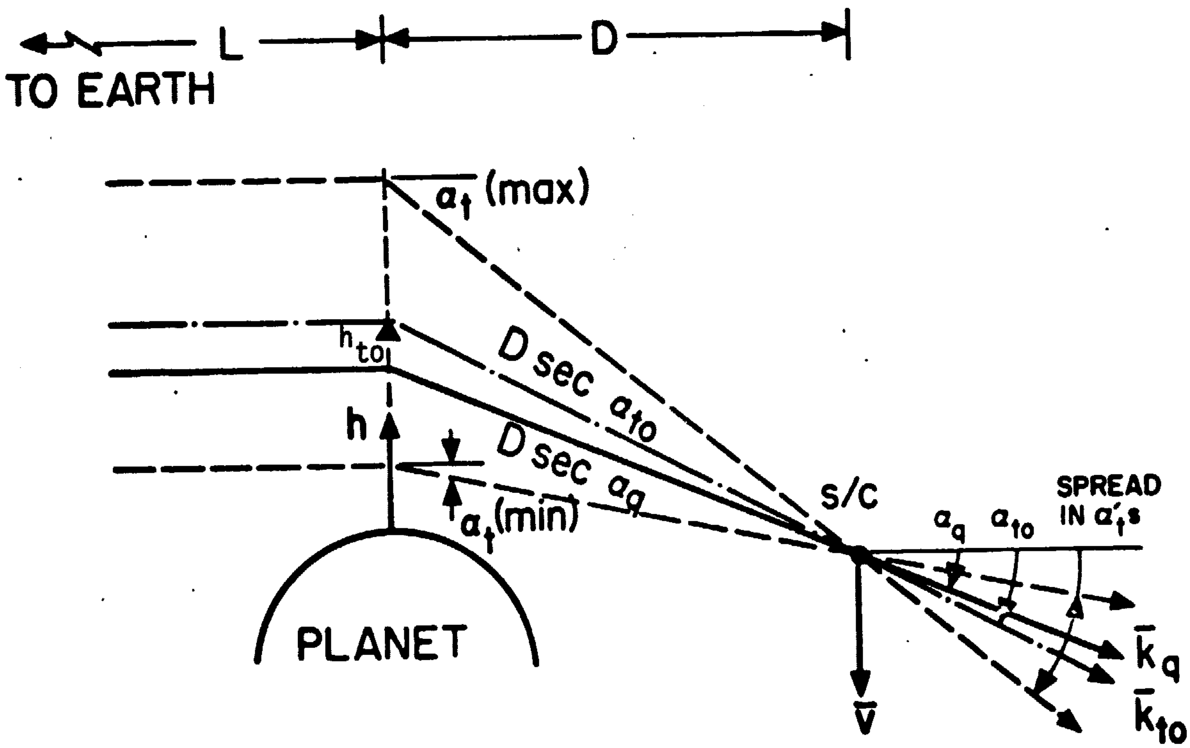

APPENDIX: Further Discussion of Equation (2)

Since the turbulent atmosphere is dynamic, the signals received at the spacecraft may be spread in frequency both by the changes in angles of the $\bar{k}_t$'s, and by true changes in frequency in the planet's frame of reference - that is, by changes in the lengths of the $\bar{k}_t$ vectors. For a stationary spacecraft only the spread due to the differences in lengths would be observed. This would be expected to result in a zero-mean doppler frequency with fluctuations which are independent of the angular changes, so that equation (2) is applicable to the sum of the two processes.

It has been tacitly assumed throughout this discussion that there is a sufficient number of signal components and sufficient time per scale height to form meaningful averages for the terms in the formulas for turbulent atmospheres. Fjeldbo (1975) has indicated that this may not always be the case in practice. Even though there may be practical limits, the present work is at least potentially applicable to the problems related to turbulence since one could use very small values of v, or a sequence of spacecraft, to obtain the needed data. Also, since past experiments have been very limited in signal-to-noise ratio, more components may in fact be observable when more sensitive radio instruments are employed. Finally, the mean ray is moving down only $(1+M_{to})^{-1}$ times as fast deep in the atmosphere as at the top, so more time can be used to form the required signal averages when signals are weak.

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