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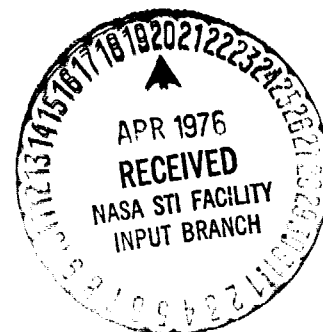
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THE EFFECT OF SUPRATHERMAL PROTONS
ON THE PHYSICAL CONDITIONS IN SEYFERT GALAXY NUCLEI II

by

Ronald Stoner and Roger Ptak
Department of Physics
Bowling Green State University
Bowling Green, Ohio 43403

Received _____



(NASA-CR-146776) THE EFFECT OF SUPRATHERMAL
PROTONS ON THE PHYSICAL CONDITIONS IN
SEYFERT GALAXY NUCLEI II (Bowling Green
State Univ.) 25 p HC \$3.50

CSCI 03B

N76-22122

G3/90 Unclass
24617

NSG-7022

ABSTRACT

The radiative transfer of $\text{Ly}\alpha$, $\text{Ly}\beta$, and $\text{H}\alpha$ in a hydrogen gas containing dust is considered with application to the nuclear gas in Seyfert galaxies. The line radiation originates from the kinetic energy of suprathermal particles, lost initially to a free electron gas, which then delivers it to the radiation through excitation of neutral hydrogen atoms. By neglecting the direct escape of line radiation and by averaging over the gas, we suppress the transfer in space and consider transfer in frequency only. The dust degrades the line radiation via frequency-independent absorption, converting the energy to infrared luminosity. We solve for the source functions in the lines, using appropriate approximations, in order to determine under what conditions the narrow component of the Balmer line radiation from the gas can be self-absorbed and degraded without similar degradation of the broad component, which originates from the suprathermals themselves. The results of the transfer calculation are used to find self-consistent values for the temperature and ionization of the gas for various amounts of dust and various concentrations of suprathermal particles. Finally, we examine the luminosity in the Balmer continuum that would result under these conditions.

I. Introduction

If the broad component of permitted emission lines observed in quasars and Seyfert galaxy nuclei arises from suprathermal ions (Ptak and Stoner 1973, Stoner et al 1974), the ambient gas clouds in which these ions are slowing must be optically thick in the Lyman and Balmer series. This requirement follows from the necessity to trap and ultimately to degrade the emission from ambient, thermal atoms that would otherwise produce very strong central narrow cores in the observed line profiles.

In a previous paper (Ptak and Stoner 1975) we discussed the physical conditions in an ambient gas penetrated by suprathermal protons, assuming all line emission from the ambient atoms is trapped. In order to further investigate the feasibility of the suprathermal protons idea, we deal here with the question of how to produce the necessary large optical thickness in the Balmer lines. Large infrared luminosities have been observed for several Seyfert nuclei and this suggests that the nuclear gas may contain dust. Therefore, in this paper, we consider cases where the bulk of the kinetic energy of the suprathermal particles is ultimately transformed to infrared luminosity by dust.

The presence of dust is related to the problem of producing a sufficient optical thickness in Balmer lines since the $n = 2$ population of the ambient hydrogen atoms depends on the trapped Lyman alpha intensity, and absorption by dust is a mechanism for destroying Lyman alpha. So we consider the radiative transfer of $Ly\alpha$ (in frequency space) when a dust-like absorption mechanism is operating.

We present the results of this calculation as a function of the density and temperature of the electron gas and of a parameter which describes the amount of dust present relative to the amount of gas.

A large optical depth is not sufficient, by itself, to insure that $H\alpha$ photons are degraded before they leak out of the gas. In order to deal with the $H\alpha$ and $Ly\beta$ radiation, we have converted the transfer equations for a three-level atom to an approximately equivalent two-level transfer equation, using the results of the $Ly\alpha$ calculation to fix the $n = 2$ population. This allows us to examine the effect of the dust on the $H\alpha$ source function.

With the $Ly\alpha$ source function determined, we can calculate the temperature and ionization of the gas in a self-consistent way, assuming that the cooling is dominated by infrared radiation from the dust. We have done the self-consistent calculation, considering the gas to be everywhere the same, and the results are presented for various amounts of dust.

Since a small amount of continuum cooling also occurs in the cases we have considered, we discuss what the Balmer continuum emission should be like if the gas is thin to this radiation.

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II. The Ly α Source Function

We wish to calculate the source function which describes the trapped Ly α radiation field in a dense, optically thick gas that is heated by a flux of suprathermal particles. The density and the linear dimensions of the gas cloud are assumed large enough that the overwhelming fraction of the gas is at least several thermalization lengths from the surface. In this limit the source function and the mean intensity of line radiation are equal nearly everywhere, so we can consider the source function to depend only on frequency.

Under these conditions the radiative transfer problem for a two-level atom reduces to the condition for statistical equilibrium of the photons at each point in frequency space near the line center. The photons are assumed injected with a Voigt profile, then to migrate in frequency due to collisions with atoms until they are destroyed in one of three ways: collisional de-excitation, an absorption by an atom leading to the production of a 2-photon continuum, or absorption by dust. The profile for absorption by atoms is the same as the emission profile, while the profile for absorption by dust is assumed frequency independent. The redistribution in scatterings by atoms is assumed coherent in the atom's frame, the situation originally called Case II by Hummer (1962).

There are three parameters which determine the nature of the radiation field. The collisional excitation and de-excitation process is described by ϵ_1 , which is equal to the rate of collisional de-excitation normalized to the rate of Ly α absorption by atoms.

The rate of two photon emission from the 2s state, normalized in the same way, is the parameter ϵ_2 . The effect of the dust is characterized by ϵ_3 , which is the ratio of the coefficient for absorption by dust to the total coefficient for absorption by atoms at the line center, multiplied by $\phi(0)$.

Equating the rate of absorption and emission processes at each frequency and at each point in space, the transfer equation reduces to the form:

$$S(x) \left\{ \phi(x) + \epsilon_3 \right\} = \epsilon_1 \phi(x) + [1 - \epsilon_1 - \epsilon_2] \int_{-\infty}^{\infty} S(x') R(x, x') dx' \quad (1)$$

where

$$x = \frac{\nu - \nu_0}{\Delta \nu_D}$$

$\Delta \nu_D$ being the Doppler width, $S(x)$ is the source function in units of the Planck level at the electron temperature, and $\phi(x)$ is the normalized Voigt profile. For the angle-average redistribution function $R(x, x')$, we choose R_{IIA} (see Adams et al (1971)). The electron temperature and density determine ϵ_1 ; at $T = 10^4 \text{ K}$, $\epsilon_1 \approx 2.1 \times 10^{-17} N_e$. For the densities, we are considering, collisional mixing of the 2s and 2p states proceeds very rapidly, and ϵ_2 is simply the ratio of the two-photon transition probability to that for $2p \rightarrow 1s$, $\epsilon_2 \approx 5 \times 10^{-9}$.

We have solved equation (1) numerically, using the method of cubic splines suggested by Adams et al (1971), choosing various values for N_e , T , and ϵ_3 . The source function is displayed in

figure 1 for four values of ϵ_3 , with the electron density and temperature taken to be $5 \times 10^9 \text{ cm}^{-3}$ and 10^4 K . We find from our results that it is possible to characterize the source function with two simple, semi-empirical formulas for \bar{S}_{12} , source function averaged over the line profile, and x' , the "effective half width". This is the halfwidth of a hypothetical trapezoidal source function with the same total area and altitude as the computed one. To a good approximation, x' is the halfwidth at half maximum of the source function. The semi-empirical formulas are:

$$x' = .15 \epsilon_3^{-1/4} , \quad \text{and} \quad (2)$$

$$\bar{S}_{12} = \frac{\epsilon_1}{\epsilon_1 + \epsilon_2 + 2x'\epsilon_3} . \quad (3)$$

III. The Three Level Atom

If we wish to investigate the trapped H α field, we must include the $n = 3$ states for the hydrogen atoms of the gas, so the Ly β radiation field is immediately involved. Absorption of a Ly β photon by a gas atom may be followed by an H α emission, and likewise an H α absorption may be followed by emission of Ly β . At equilibrium the rates of these two processes will be the same; if we assume that this balance applies for each set of corresponding frequencies, we can simplify the three-level transfer problem.

As in the case of Ly α , it is convenient to define $S_{13}(x)$ and $S_{23}(x)$, the source functions for H α and Ly β , respectively, each in units of the Planck function at that frequency. The doppler frequency $\Delta\nu_D$ appropriate for each line is proportional to the line center frequency, so that an atom in the $n = 3$ state can emit either a Ly β or an H α photon in a given direction with the same dimensionless difference frequency x . A similar statement holds for absorption processes. Using the approximation that the $n = 2$ population is independent of the speed of the atoms and is determined by the Lyman alpha intensity alone, we find from the balance of H α and Ly β processes at each x that

$$S_{13}(x) = \bar{S}_{12} S_{23}(x) . \quad (4)$$

Balancing the absorption and emission of H α and using equation (4), we obtain the transfer equation which determines the frequency dependence of the H α source function:

$$S_{23}(x) \left\{ \phi(x) + \epsilon_3 \right\} = \epsilon_1 \phi(x) + (1 - \epsilon_1') \int_{-\infty}^{\infty} S_{23}(x) R(x, x') dx' \quad (5)$$

The notation is similar to that used in equation (1). Here $R(x, x')$ is $R_{II,A}$ with an effective value of the parameter a , which is obtained using the lifetimes of both the $n = 2$ and $n = 3$ levels, appropriately weighted according to branching ratios,

$$a_{\text{eff}} = 1.12 \times 10^{-3} .$$

The three parameters in equation (5) can be related to the parameters in the Lyman alpha transfer problem and to the resultant Lyman alpha source function by comparing the rates of corresponding processes. We will write the relationships for these parameters for an electron temperature of 10^4 K , so the slight temperature dependence is not included in these formulas. Comparing the rate at which the electron gas will collisionally excite the $n = 3$ level with the rate of collisional excitation of $n = 2$ and adopting 2.0×10^{-2} for the ratio α_{13}/α_{12} , we find that to two significant figures,

$$\epsilon_1(\text{H}\alpha) = 150 \{1 + 2.6 \times 10^{-3}/\bar{S}_{12}\} \epsilon_1(\text{Ly}\alpha) . \quad (6)$$

Comparing similar collisional de-excitation rates,

$$\epsilon_1'(\text{H}\alpha) = 150 \epsilon_1'(\text{L}\alpha) \quad (7)$$

The dependence on \bar{S}_{12} in equation 6 derives from the direct excitation of $n = 3$ from $n = 1$, which becomes relatively more important

than the excitation from $n = 2$ when the Ly α source function is small.

If the dust particles are relatively large so that they present the same cross sectional areas for absorption of Lyman and Balmer radiation, then a comparison of the absorption rates gives

$$\epsilon_3(\text{H}\alpha) = 4300 \bar{S}_{12}^{-1} \cdot \epsilon_3(\text{L}\alpha) , \quad (8)$$

where the dependence on the Ly α source function results from choosing to measure the source functions in units of the Planck function at the line centers. Equation (8) should be regarded as an upper limit to the ϵ_3 appropriate for equation 5, since if the dust is very small, its cross section for H α relative to Ly α could be reduced by a factor as small as the ratio of wavelengths. We have performed calculations using both the big dust and small dust limits.

The actual calculation proceeds by choosing values for the electron density, the temperature and the dust parameter for Ly α . Equations 2 and 3 then yield the properties of the Ly α source function, which, together with equations 6, 7 and 8, give the parameters in equation 5. Equation 5 is then solved by the method previously used to solve equation 1.

The importance of the results of this calculation is that they can be used to test the feasibility of degrading the "narrow core" radiation without doing the same to the broad line radiation that is produced directly from the suprathermal atoms. We wish to

see if the gas can be thin to dust yet thick to H α at all frequencies where the H α source function is large. So we have computed the ratio of the absorption coefficients of the atoms and the dust, $\tau(x')/\tau_D = \phi(x')/\epsilon_3$ at the "effective half width" frequency for S_{23} . The ratio tends to be slightly greater than one when the effective half width is smaller than about 3.5 Doppler units. When the effective half width is greater than 3.5, a typical photon must undergo several nearly coherent scatterings before reaching this frequency and this results in higher values of the ratio $\tau(x')/\tau_D$, about 40 at $x' = 10$.

Larger values of x' are most easily produced at higher electron densities, as the typical values in Table 1 show. Since $x' = 10$ corresponds to 100 km/sec at 10^4 K and since the HWHM of the "broad component" from suprathermal atoms is about 3500 km/sec, there is a large range of physical conditions where the dust can very effectively remove the "narrow core" without appreciably attenuating the broad component.

We summarize the results of the calculations in table 1, where we display values of: the effective half-width of the H α source function, the optical depth in H α at this frequency, and the optical depths in the Balmer and Paschen continuums. These are all expressed relative to the optical depth in dust, and the gas temperature is taken to be 10^4 K.

If the size of the region is just the distance needed for the suprathermal protons to stop in the gas, the neutral hydrogen column density can be simply related to the ionization of the gas and the initial energy of the most energetic suprathermals. From

Ptak and Stoner (1973), we have $N_H L = 10^{20} \frac{E_0^2}{F_1}$, and so the Ly α optical depth at the line center is

$$\tau_0(\text{Ly}\alpha) = 10^7 \frac{E_0^2}{F_1}.$$

Taking as a typical value $\tau_0(\text{Ly}\alpha) = 5 \times 10^8$, the values for ϵ_3 can be converted into optical depths for the dust. We could then convert the optical depths in table 1 into absolute values. For this choice of $\tau_0(\text{Ly}\alpha)$, an ϵ_3 of 10^{-10} corresponds to a τ_D of about 0.1, $\epsilon_3 = 10^{-9}$ corresponds to $\tau_D = 1.0$, and so it goes.

IV. Self-Consistent Physical Conditions

Having obtained the dependence of the Ly α source function on the electron temperature and density and on the dust parameter ϵ_3 , we can now determine self-consistent values for the temperature and ionization of the gas by solving the appropriate balance equations. In doing this, we assume that the gas is heated by the suprathermal particles which are thermalized in it, and that the radiation field is determined by the suprathermals and by the gas; there are no external sources.

In photo-ionization models (MacAlpine 1972), a large flux of ionizing photons is present in the line-emitting region of quasars and Seyfert galaxies. The existence of a hard uv flux does not follow directly from observations, but it is assumed to exist from extrapolation of observed optical continua and in analogy with other, more familiar objects such as planetary nebulae. If there is such a photon flux in any of these objects, then we must consider what effect it would have on the calculation we are doing here. Let us consider a dense cloud of gas at some distance from a central continuum source. Incident on one side of the cloud is the ionizing photon flux and the flux of suprathermal particles. If the gas density is large enough, the particles will penetrate further than the photons, and it is in the region shielded from the photons that our calculation applies.

Let us make a numerical estimate of the density required. The photo-ionized layer will have a thickness d determined by:

$$F = N_e^2 \alpha_T d , \quad (9)$$

where F is the number of ionizing photons incident per cm^2 per sec., N_e is the electron number density, and α_T is the total recombination coefficient. The suprathermal protons will penetrate a distance d' given by

$$d' \cong 10^{20} \frac{E_0^2}{N_e}, \quad (10)$$

where E_0 is the initial kinetic energy of the protons (see Ptak and Stoner 1973). Let us suppose that the cloud is one light year from the source, then, for an ionizing photon luminosity of 10^{56}sec^{-1} , typical of a bright quasar, $F \approx 10^{19} \text{cm}^{-2} \text{sec}^{-1}$. If we take $E_0 \sim 10 \text{ MeV}$, then d' exceeds d if $N_e > 10^{10} \text{cm}^{-3}$. For a typical Seyfert galaxy luminosity, $F \approx 10^{18} \text{cm}^{-2} \text{sec}^{-1}$, and we need $N_e > 10^9 \text{cm}^{-3}$. These densities are within the range of 10^8cm^{-3} to 10^{11}cm^{-3} which we are considering in this model, and so the assumption of no external source is a reasonable one. Of course, a more complicated geometry could reduce the required density by a large amount. Also, it is possible that the optical continuum arises from a hot gas which is energized by the suprathermal particles themselves, and which is reddened by the dust.

We find the temperature and ionization of the gas by simultaneously satisfying two balance equations. The ionization balance is described by equation (1) of Ptak and Stoner (1975), except now the Ly α number density is calculated using the correct source function. The energy balance equation is obtained by equating the rate at which the gas is heated by the suprathermals with the rate at which it is cooled by the dust:

$$\frac{N_S E_0}{\tau} = U C N_D \sigma_D \quad (11)$$

Here N_S is the number density of suprathermals, τ is the time for a suprathermal to stop in the gas, U is the Ly α energy density, N_D is the dust number density, and σ_D is the dust absorption cross section. If we take the initial energy to be about 1 MeV, this equation becomes

$$N_S f_i (2 \times 10^{-17}) = (.031) (2 \cdot x' \cdot \bar{S}_{12}) (1 - f_i) \quad (12)$$

$$\times \epsilon_3 \exp(-E_{12}/kT) ,$$

where f_i is the ionized fraction of the gas, and E_{12} is 10.2 eV.

Solving these balance equations together with the use of equations (4) and (5) gives us the ionization and temperature of the gas for given values of N_S , N_H , and ϵ_3 . Representative results are presented in table 2.

For those cases which are displayed, a small amount of the cooling is accomplished by continuum processes (a few percent of the total). Therefore, if the gas is thin to the Balmer continuum, an appreciable amount of energy comes out in this way, otherwise it goes into the Paschen continuum. We can estimate the relative strength of the Balmer continuum to that of H β fairly simply. The emission rate of Balmer continuum photons is about the rate of recombination to the $n = 2$ state: $N_e^2 \alpha_2$. In the context of our model, the H β emission is due to the suprathermal protons and the rate is $\frac{N_S R_\beta}{\tau}$, where R_β is the number of H β photons produced by

each suprathermal which stops in the gas. For the small ionization fractions indicated in table 2, $R_B \approx 10$. If we divide these two rates, we find that the number of Balmer continuum photons produced per H β should be

$$\begin{aligned} \frac{L(\text{BAC})}{L(\text{H}\beta)} &\approx \frac{N_e^2 \alpha_2 \tau}{10 N_S} \\ &\approx \frac{N_H}{N_S} f_i (10^{-3}) \end{aligned} \quad (13)$$

for $E_0 \approx 1$ MeV. And since

$$\frac{N_H}{N_S} > 10^6,$$

this ratio must be on the order of 100 or more when $f_i = 0.1$.

A large value for this ratio is apparently observed in some low redshift quasars by Baldwin (1975), but the profiles observed for these objects are not fit by profiles we have calculated so far. We know of no published values for this ratio for those Seyfert galaxies and quasars for which we have obtained a compelling fit (for these fits, see Ptak and Stoner 1973 and Stoner et al 1974).

V. Discussion of the Results

We have investigated the conditions under which a dense gas heated by suprathermal particles and cooled by dust will be optically thick in Lyman and Balmer lines. At the same time, we have determined values of the temperature and ionization of the gas that are consistent with such a picture. These two aspects are intimately interconnected; a large value for the dust parameter will produce a fairly neutral gas for a wide range of number densities for the gas and the suprathermal particles. On the other hand, a large ϵ_3 lowers the optical depth in H α relative to that of the dust.

In order for the cooling of the gas to be accomplished by infrared radiation from dust, the parameter ϵ_3 cannot be much less than 10^{-11} . The need to trap H α photons until they can be degraded requires that ϵ_3 be about 10^{-8} or smaller.

The calculations reported here appear to establish the feasibility of producing broad emission lines like the ones observed in the nuclei of some Seyfert galaxies and in some quasars via the interaction of suprathermal atoms with a dense gas if an appropriate amount of dust is present to eliminate the "narrow core" photons. While these calculations have taken detailed account of the migration of the line radiation in frequency due to the interaction with a thermal, partially ionized hydrogen gas, some other important features of a more realistic radiative transfer calculation are missing.

Since the broad line emitting regions of these objects are unresolved and since quite different geometries can give the same

profiles, we have no strong clues to guide us in extending the calculation to include transfer in space. A full radiative transfer calculation, including transfer in space, seems uninteresting in the absence of stronger evidence for a particular geometry.

Further, the transfer in frequency itself may be somewhat different from what we have assumed. The neglect of leakage of narrow core photons from deep inside the gas cloud is justified by the results of the calculation, but a large fraction of those produced near to the gas surface must escape easily. More important effects may come from deviations from a Maxwell-Boltzmann velocity distribution due to the presence of suprathermal atoms. For example, the enhancement in the "tail" of the distribution represented by suprathermals could change the rate at which photons migrate from the line center to the wings of the line. Also, the atoms in the high velocity tail could directly produce a contribution to the broad component via the process we called "optical reverberation" in a previous paper (Ptak and Stoner 1975). The effect of the latter on the source function would be similar to the effect of dust we have included in the present calculation.

The full range of radiative transfer effects due directly or indirectly to the presence of suprathermal particles are probably capable of explaining the large Balmer decrement observed in most broad-line emission objects. Such effects might also produce line profiles of quite different character from the asymmetric, flat-topped variety that our original simple models gave and thus be consistent with some of the more recent profile observations, so long as the widths of the observed profiles are not too large.

On the other hand, it is difficult at present to see how to produce hydrogen emission lines as broad as 15,000 km/sec or greater by the actions of suprathreshold particles without invoking some kind of mass motion of the ambient gas (rotation, high velocity expansion, etc.). This seems to be an undesirable increase in complexity in the model. However, if the detailed profiles can be fit by assuming a single motion for all of the ambient gas or double sources with relative motion, then the number of parameters would only be increased by one.

We are in the process of including the effect on the radiative transfer problem of the suprathreshold atoms in a self-consistent way. This can be important both for the energy balance of the gas and for the radiative transfer problem. It is possible that for those gas densities at which the suprathreshold atoms act as the major cooling agent (the optical reverberation process), the required flux of suprathreshold particles and the line profiles produced will both be substantially affected. The results of this self-consistent calculation will be presented in a subsequent paper.

Acknowledgements

We are grateful to Dr. Thomas Adams for sending us information that was useful in checking our computing codes and to Mr. Brian Davis for assistance with some of the computations. Conversations with Drs. Adams, Eugene Capriotti, Lewis Fulcher, Edward Kimmer, John Mathis and Gordin MacAlpine are gratefully acknowledged. This work was supported by NASA through grant number NSG-7022.

References

- Adams, T. F., Hummer, D. G., and Rybicki, G. B. 1971, JQSRT, 11, 1365.
- Baldwin, J. A. 1975, Ap. J., 201, 26.
- Hummer, D. G. 1962, MNRAS, 125, 21.
- MacAlpine, G. M. 1972, Ap. J., 175, 11.
- Ptak, R. and Stoner, R. 1973, Ap. J., 185, 121.
- Ptak, R. and Stoner, R. 1975, Ap. J., 200, 558.
- Stoner, R., Ptak, R., and Ellis, D. 1974, Ap. J., 191, 291.

FIGURE CAPTION

The frequency dependence of the Lyman α source function for four different values of the parameter ϵ_3 , using $\epsilon_1 = 10^{-7}$ and $\epsilon_2 = 5 \times 10^{-9}$. The source function is in units of the Planck level so $S = 1$ corresponds to thermal equilibrium between the radiation and the free electron gas.

$$\epsilon_1 = 10^{-7}$$
$$\epsilon_2 = 5 \times 10^{-9}$$

LOG(S) VS X

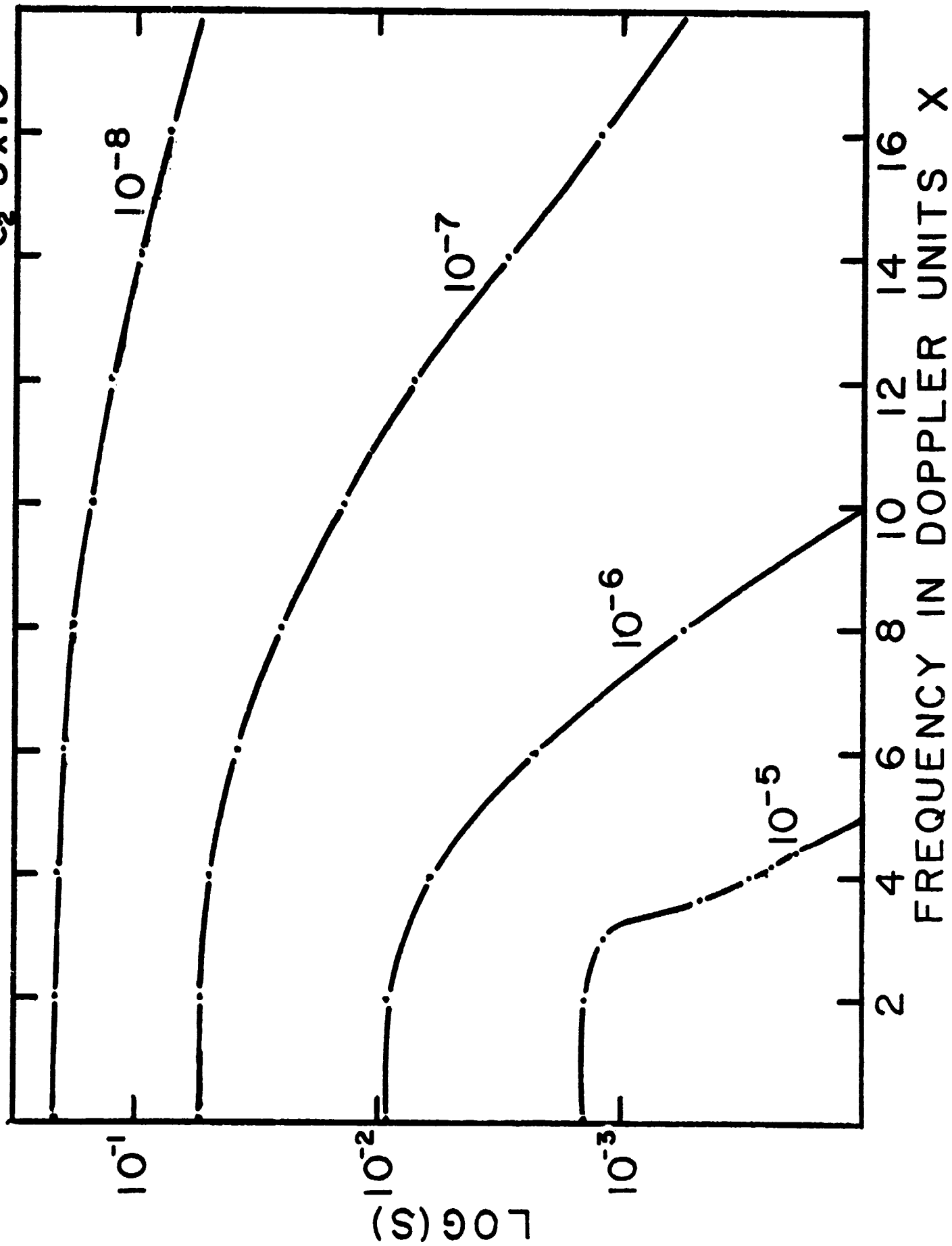


TABLE 1
Optical Depths Relative To Dust

N_e	ϵ_3	x'	$\tau(x')$	$\tau(\text{BAC})$	$\tau(\text{PAC})$
10^8	10^{-10}	6.5	14.0	5.10	.06
	5×10^{-10}	4.1	3.2	.43	7×10^{-4}
	10^{-9}	3.5	1.6	.14	9×10^{-5}
5×10^8	10^{-10}	8.4	26.0	16.00	1.80
	5×10^{-10}	5.2	7.6	1.80	.04
	10^{-9}	4.3	4.0	.60	6×10^{-3}
10^9	10^{-10}	9.1	31.0	23.00	4.50
	10^{-9}	4.7	5.6	1.00	.03
10^{10}	10^{-10}	10.0	39.0	35.00	13.00
	10^{-9}	5.8	10.0	3.00	.79
5×10^{10}	10^{-10}	10.0	40.0	37.00	14.00
	10^{-9}	6.0	12.0	3.60	1.30

TABLE 2

Temperature and Ionization of the Ambient Gas

N_H	N_S	ϵ_3	T	α_1
10^7	10	10^{-9}	12,650	.31
10^8	10	10^{-9} or 10^{-10}	10,040	.09
10^9	10^2	10^{-9}	10,060	.09
10^9	10^2	10^{-10}	10,010	.13
10^{10}	10^3	10^{-9}	10,310	.13
10^9	10	10^{-9} or 10^{-10}	8,740	.03
10^{10}	10^2	10^{-9} or 10^{-10}	8,850	.03
10^{11}	10^3	10^{-9}	9,250	.03
10^{11}	10^3	10^{-10}	9,940	.07
10^9	1	10^{-9} or 10^{-10}	7,930	.008
10^{10}	10	10^{-9} or 10^{-10}	7,950	.008
10^{11}	10^2	10^{-9}	8,100	.008
10^{11}	10^2	10^{-10}	8,390	.01
10^{12}	10^3	10^{-9}	8,830	.009
10^{12}	10^3	10^{-10}	9,520	.02