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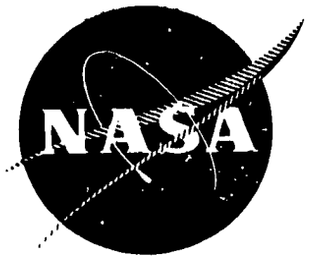

DOUBLE ION PRODUCTION IN MERCURY THRUSTERS

PREPARED FOR
LEWIS RESEARCH CENTER
NATIONAL AERONAUTICS AND SPACE ADMINSTRATION
Grant NGR-06-002-112

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Approved by
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Department of Mechanical Engineering
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16. Abstract

Significant densities of doubly charged ions exist in the discharge chambers of electron bombardment ion thrusters. These ions are undesirable because they are a major plasma constituant effecting the sputtering damage which limits thruster lifetime. It would be desirable to reduce their density while maintaining good thruster performance. The development of a model which predicts the doubly charged ion density is discussed. The accuracy of the model is shown to be good for two different thruster sizes and a total of 11 different cases. The model indicates that in most cases more than 80% of the doubly charged ions are produced from singly charged ions. This result can be used to develop a much simpler model which, along with correlations of the average plasma properties, can be used to determine the doubly charged ion density in ion thrusters with acceptable accuracy. Two different techniques which can be used to reduce the doubly charged ion density, while maintaining good thruster operation, are identified as a result of an examination of the simple model. First, the electron density can be reduced and the thruster size then increased to maintain the same propellant utilization. Second, at a fixed thruster size, the plasma density, temperature and energy can be reduced and then to maintain a constant propellant utilization the open area of the grids to neutral propellant loss can be reduced through the use of a small hole accelerator grid. The reduction in the values of the plasma properties causes a decrease in the doubly charged ion density.

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INTRODUCTION

Electron bombardment ion thrusters are presently being considered for use in deep space probes and for satellite stationkeeping functions. These devices which have the advantage of very high specific impulses also have the attendant disadvantage of low thrust densities. This low thrust characteristic necessitates thruster operation for long periods of time in order to accomplish typical missions (of the order of 10,000 hours). Thruster lifetimes can be limited by the erosion of ion chamber component parts with most of this erosion (or sputtering damage) being caused by doubly charged ions. Long thruster lifetimes therefore require control of doubly charged ion densities.

Many experiments could be performed to determine how the thruster should be operated so that thruster performance would be good and the doubly charged ion density would be reduced to an acceptable level. However, these experiments would have the disadvantage of being time consuming and costly and the results might be applicable to one size and type of thruster only. Instead a theoretical model could be developed which would accurately predict the doubly charged ion density over a wide range of conditions and thruster sizes. This model could be applied at low cost to determine the factors affecting the doubly charged ion density and how they should be adjusted to reduce the double ion density. It should also indicate what effects these changes would have on thruster performance. Such a model has been developed for electron bombardment ion thrusters and has been verified experimentally for thrusters which use mercury propellant. A discussion of the model's development and verification is presented in this paper along with some results and conclusions based upon the model.

THRUSTER OPERATION

Many of the assumptions and approximations used in the development of the model are based upon a knowledge of thruster operation. This section will briefly discuss thruster operation so that the development of the model in the next section will be more easily understood. An ion thruster has two basic tasks to perform:

1) Ionization of the neutral propellant atoms.

2) Acceleration of the ions to high velocities producing thrust.

These two topics will form the basis for the discussion of thruster operation.

Figure 1 shows a schematic for a typical electron bombardment ion thruster. The specific type shown has a strongly divergent magnetic field which is presently the most common type. However, the operation of all types of electron bombardment ion thrusters is very similar[1]. Electrons are emitted from the cathode and are drawn toward the anode which is biased 30-40V positive with respect to the cathode. These electrons (called primary electrons) are injected into the primary electron region with an energy slightly less than that associated with the 30-40V anode voltage. Electrons in this region are kept from going immediately to the anode by a magnetic field set up between the cathode pole piece and the anode pole piece but collisions eventually facilitate electron diffusion across these magnetic field lines so that they can be collected by the anode. As a result of the magnetic field containment the electron density is much higher ($\sim 10^{11} cm^{-3}$) within this region than it is outside of it. The primary electron region's boundary is defined by the surface of revolution of the critical (magnetic) field line and the screen grid. Because the strength of the magnetic field

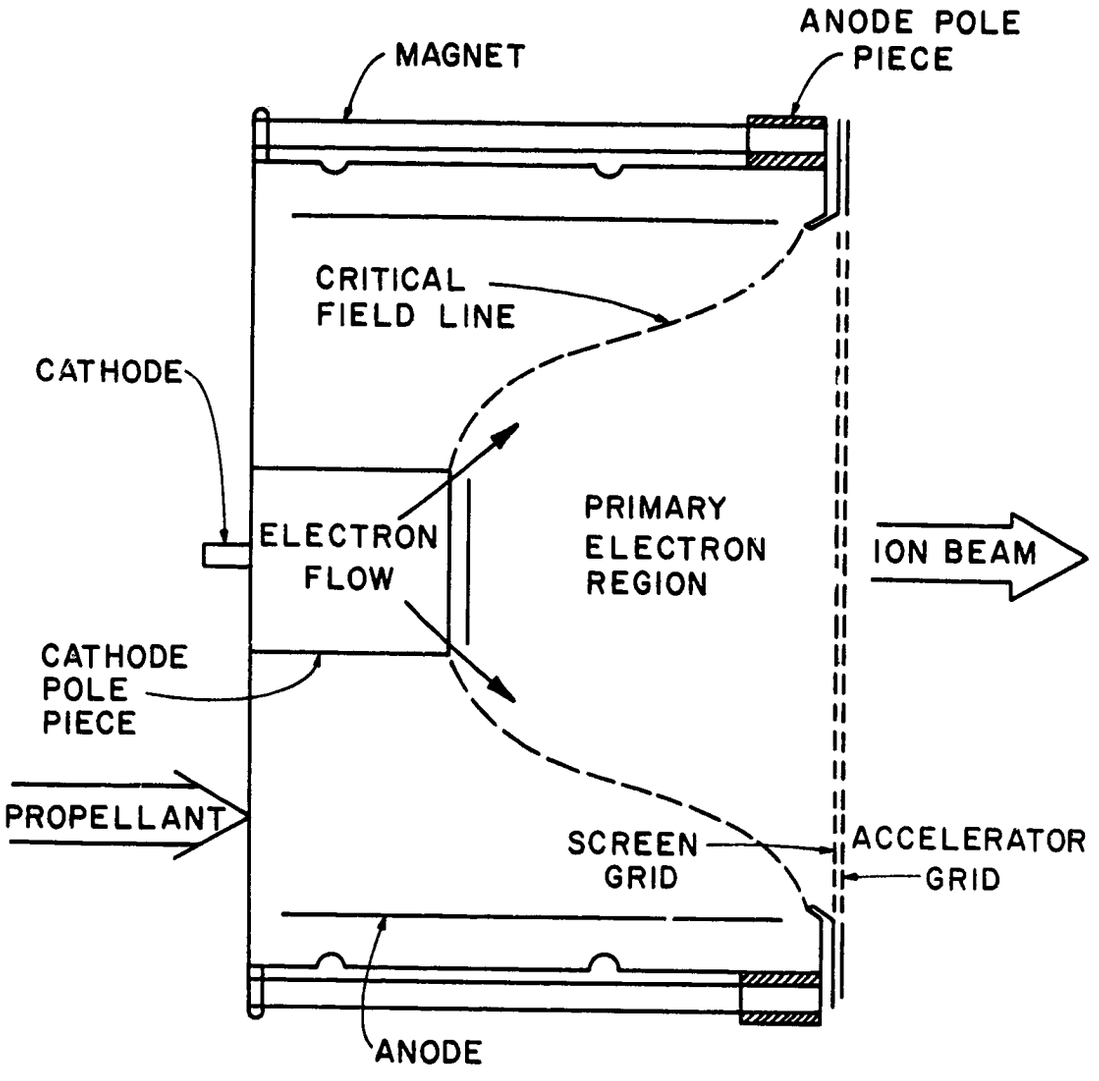


Figure 1 Electron Bombardment Ion Thruster Schematic

is fairly low within it, a fairly uniform electron density exists throughout the entire region.

Neutral propellant (e.g. mercury) is injected into the upstream end of the discharge chamber at low pressure (10^{-4} torr). Most of the interactions between electrons and neutral propellant atoms take place in the primary electron region because higher electron densities and energies exist there. Electrons bombard the neutral atoms occasionally knocking an outer shell electron loose from the atom forming a singly charged ion. The production of a mercury single ion requires more than 10 eV of energy from the incident electron. This electron and the ejected electron then share the remainder of the energy which the incident electron carried originally. This reaction results in the replacement of one high energy electron with two lower energy electrons which rapidly randomize with similarly generated electrons to form a Maxwellian electron group. Ions are extracted from the plasma through holes in the screen and accelerator grids as a result of the large potential difference applied across these two grids. The rate of ion loss through the grids times the ionic charge is called the beam current.

Electron bombardment of atoms and ions also produces doubly charged ions. Many of these ions are extracted from the discharge chamber by the grids, however, some of them go to the walls. As ions near the walls (the cathode pole piece, screen grid, etc.) they are accelerated to high velocities by an electric field that exists at the plasma boundary. When these high velocity ions strike the walls they can knock atoms loose (sputter atoms) from the walls of the discharge chamber. The energy that doubly charged ions possess upon striking the

walls is twice that of singly charged ions, therefore the sputtering damage caused by a double ion is much greater than that caused by a single ion. Double ions are thought to cause most of the sputtering damage even though their density is typically an order of magnitude less than that of the single ions.

THEORETICAL MODEL

Introduction

In order to develop a simple model for determining the double ion density in the discharge chamber only those ionic and atomic species which were considered significant in determining the double ion density were included. The significant species were selected as those which have substantial electron impact cross sections of formation over the electron energy range of interest so that large numbers of these excited atoms or ions will be produced. These states also have sufficiently long effective lifetimes so that they can participate in production processes before they decay. Only those reactions which lead directly or indirectly to the production of double ions were included.

Figure 2 is a discharge chamber reaction schematic showing these dominant species and the reactions in which each specie can participate. The model has been developed for thrusters using mercury propellant but the general procedure is valid for thrusters using other propellants. The symbols used in Figure 2 represent the following species:

Hg^{o} - neutral ground state mercury

Hg^{m} - metastable neutral mercury (6^3P_0 and 6^3P_2 states)

Hg^{r} - resonance state neutral mercury (6^3P_1 and 6^1P_1 states)

Hg^{+} - singly ionized ground state mercury

Hg^{m+} - singly ionized metastable mercury ($6^2D_{3/2}$ and $6^2D_{5/2}$ states)

Hg^{++} - doubly ionized ground state mercury

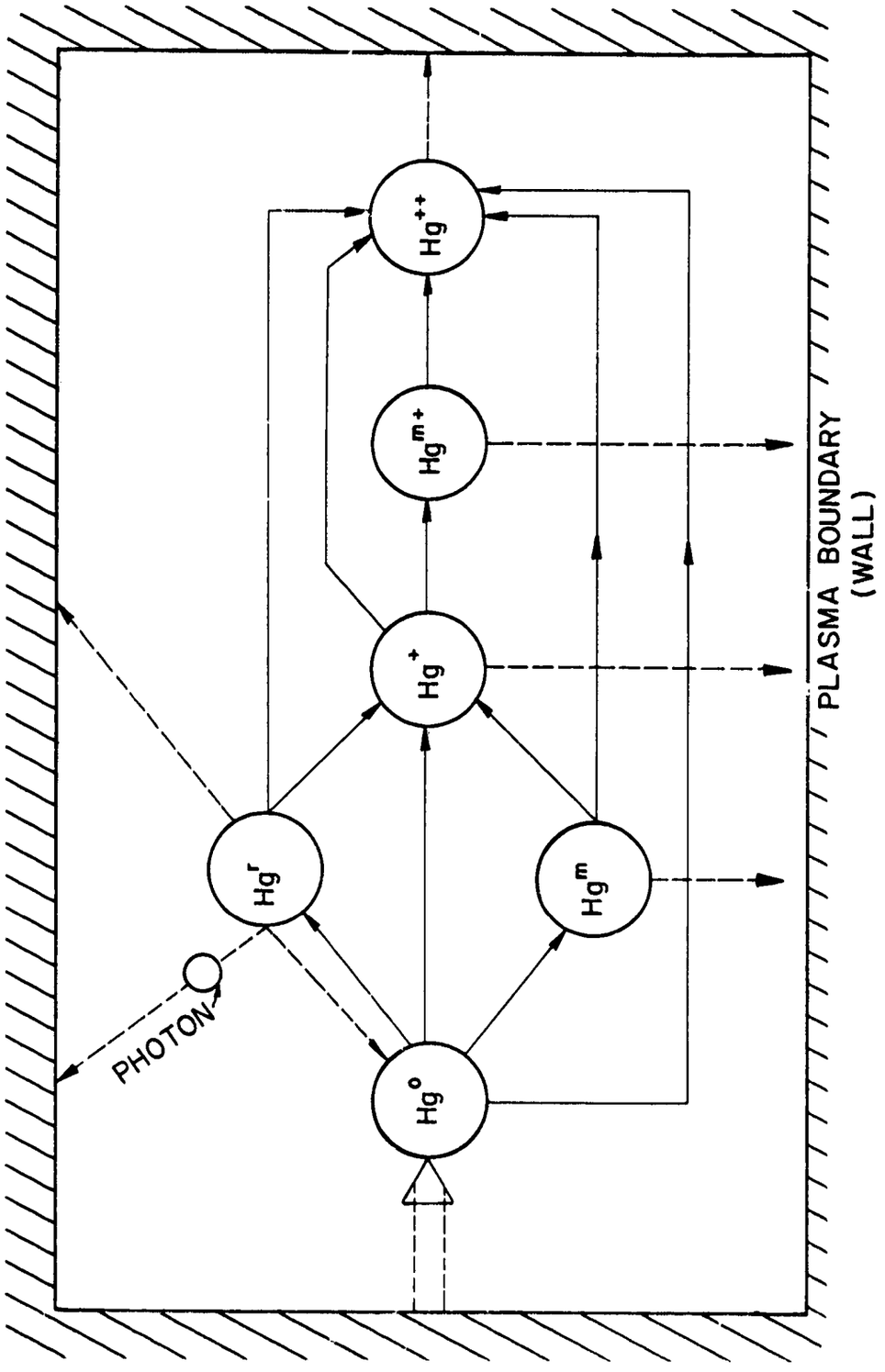


Figure 2 Discharge Chamber Reaction Schematic

The arrows in Figure 2 indicate the various interaction routes considered in this analysis. Three different types of reactions are indicated in this figure. The first type of reaction occurs when an electron interacts with an atom or ion producing a more highly excited specie. This reaction is indicated in Figure 2 by an arrow going from one specie to another more highly excited specie (e.g. the production of double ions from single ions). The production of a highly excited specie also represents a loss mechanism for the less excited specie. The reverse reaction in which, for example, an ion captures an electron is improbable because the reaction requires three bodies to simultaneously collide.

The second type of process is that of an atom or ion going to a plasma boundary. Such a boundary could be either the discharge chamber wall on which the atom or ion would be de-excited or it could be a grid aperture in which case the atom or ion would be extracted from the discharge region. In either case this represents a loss rate for any of the excited states. These losses to the boundary are indicated in Figure 2 by the dotted lines to the wall of the chamber. The large arrow back to the neutral ground state represents the resupply of neutral ground state atoms either from the walls or from the propellant supply system.

A third type of reaction shown in Figure 2 is relevant only to the two resonance states. The resonance states differ from metastable states in that they have a very short lifetime before they de-excite spontaneously by emitting a photon of light. However, the energy of this photon is such that it is readily absorbed by a nearby neutral ground state atom producing another resonance state atom. Since the

transport time of the photon is small compared to the excited state lifetime the excited state can be considered to exist continuously. Eventually the photon can diffuse to a boundary where it will be lost; this is equivalent to the loss of a resonance state atom. This loss mechanism is represented in Figure 2 by a dotted line conveying a photon to the wall and a branching line going from the resonance atom to the neutral ground state atom.

Figure 2 also shows the dominant routes for the production and loss of all of the excited atoms and ions considered. For example, ground state single ions can be produced as a result of electron bombardment of neutral ground state, resonance state, and metastable state atoms and these single ions can be lost as a result of single ion migration to the plasma boundary and the production of metastable single ions and double ions by electron bombardment.

When equilibrium conditions exist in the discharge chamber the rate of production of each specie must equal its loss rate. If, for example, the production rate of single ions increases, the single ion density must also increase to keep the loss rate (which is directly proportional to the single ion density) equal to the higher production rate. This example illustrates the fact that the equilibrium density of any specie is determined by the associated production and loss rates. If equations determining the production and loss rates could be derived, these equations could then be solved for the equilibrium density of any specie under consideration. The remainder of this section is concerned with deriving equations for the production and loss mechanisms indicated in Figure 2 and then solving these equations for the equilibrium densities of the various states.

Electron Bombardment Reactions

The first reaction to be considered is the one which produces excited atoms or ions by electron bombardment from less excited atoms or ions. The total rate of production of any specie γ from specie α (and hence the loss rate of α due to this reaction) is given by:

$$R_\alpha^\gamma = \int_{\text{Plasma Volume}} \int_{E=0}^{E=\infty} n_\alpha \, \sigma_\alpha^\gamma(E) \, v_e(E) \, dn_e d\forall \tag{1}$$

where n_α is the density of specie α at some point $\vec{r}$ in the plasma, $\sigma_\alpha^\gamma(E)$ is the cross section for the production of γ from α at the electron energy E[2-6], v_e is the electron velocity at energy E, dn_e is the density of electrons with energies between E and E + dE at $\vec{r}$, and $d\forall$ is the infinitesimal volume element. The distribution of electrons over the energy spectrum of an ion thruster was assumed to be composed of a Maxwellian electron group which is described by a temperature (T_{mx} -- eV) and a density (n_{mx} -- cm^{-3}) and a monoenergetic group (primary electrons) which is described by an energy (ε_{pr} -- eV) and a density (n_{pr} -- cm^{-3}). This type of electron distribution is generally accepted as appropriate for electron bombardment thruster plasmas.[1,7]

Substituting this electron distribution into Equation (1) and combining terms to form new functions results in the following equation.

$$R_\alpha^\gamma = \int_{\text{Volume}} n_\alpha [n_{pr} \, P_\alpha^\gamma(\varepsilon_{pr}) + n_{mx} \, Q_\alpha^\gamma(T_{mx})] \, d\forall \tag{2}$$

where

$$P_\alpha^\gamma(\varepsilon_{pr}) = v_e(\varepsilon_{pr}) \, \sigma_\alpha^\gamma(\varepsilon_{pr}) \tag{3}$$

and

$$Q_\alpha^\gamma(T_{mx}) = \int_{E=0}^{E=\infty} \sigma_\alpha^\gamma(E)\, v_e(E)\, \frac{dn_{mx}(E)}{n_{mx}} \quad . \tag{4}$$

"$[dn_{mx}(E)/n_{mx}]$" is the Maxwellian distribution function and the other terms are as defined previously.

Where possible the cross sections (σ_α^γ) required for Equations (3) and (4) were selected from experimental data.[2,3,4] If experimental data were not available, theoretical cross sections were either obtained from the literature[5] or calculated using the Gryzinski approximation.[6] The Gryzinski approximation was modified for the cases of the metastable single ion production cross sections to reflect the significant value of the cross sections near the threshold. The cross sections used are presented in Figure 3 along with references indicating their origin.

Using integral equations like Equation (2) in the model would be inconvenient because it would then be very difficult to solve for the density of specie α (n_α) since n_α appears within the integral. For this reason it would be desirable to convert Equation (2) into a simple algebraic equation. Fortunately the plasma is fairly uniform in the primary electron region which is where most of the reactions take place. This suggests using average properties in Equation (2) to obtain the following result.

$$R_\alpha^\gamma = n_i^* \left[n_{pr}^*\, P_\alpha^\gamma(\varepsilon_{pr}^*) + n_{mx}^*\, Q_\alpha^\gamma(T_{mx}^*)\right] \forall \tag{5}$$

The asterisks indicate volume averaged quantities and $\forall$ is the volume of the primary electron region.

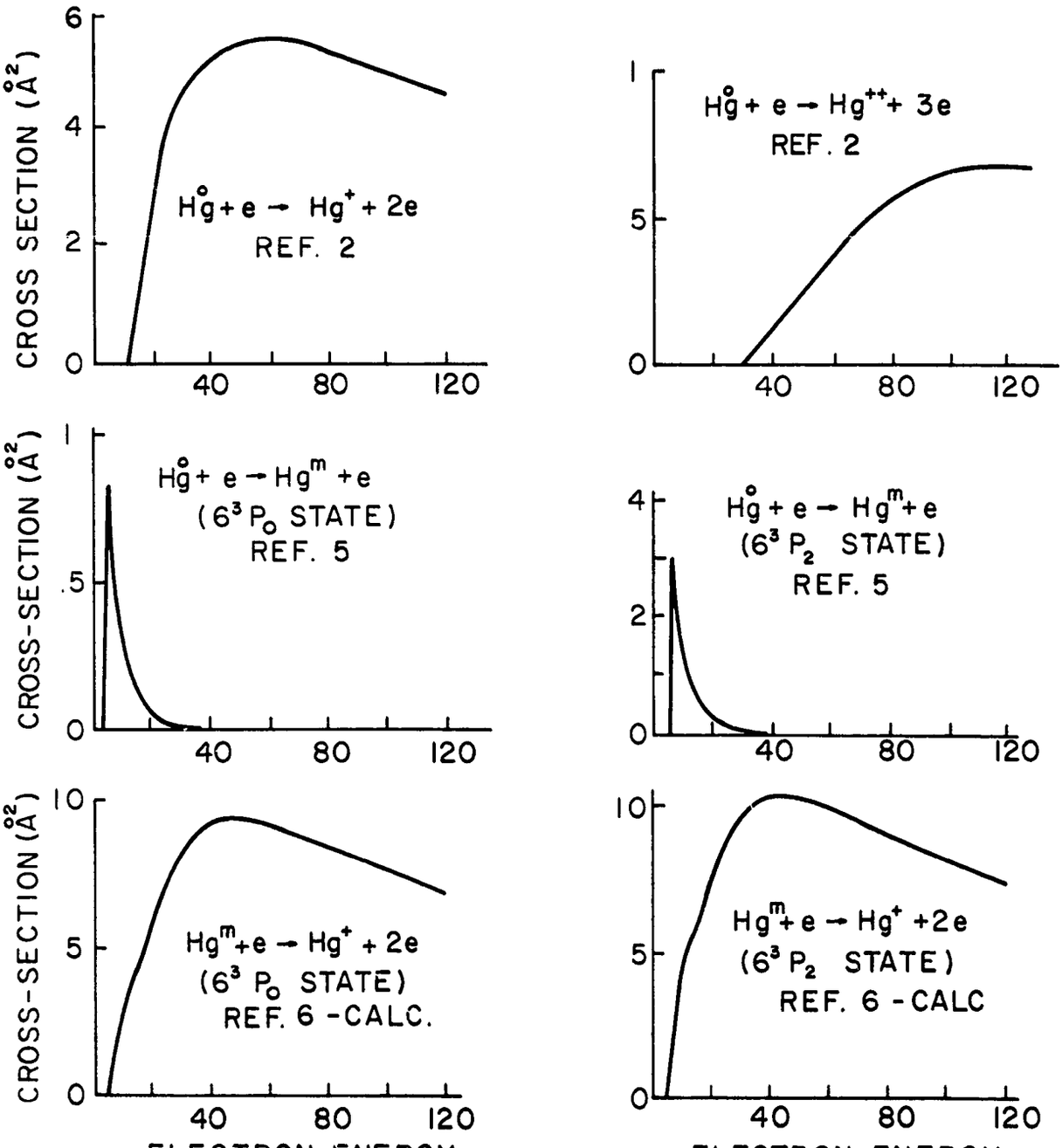


Figure 3 Mercury Cross Sections

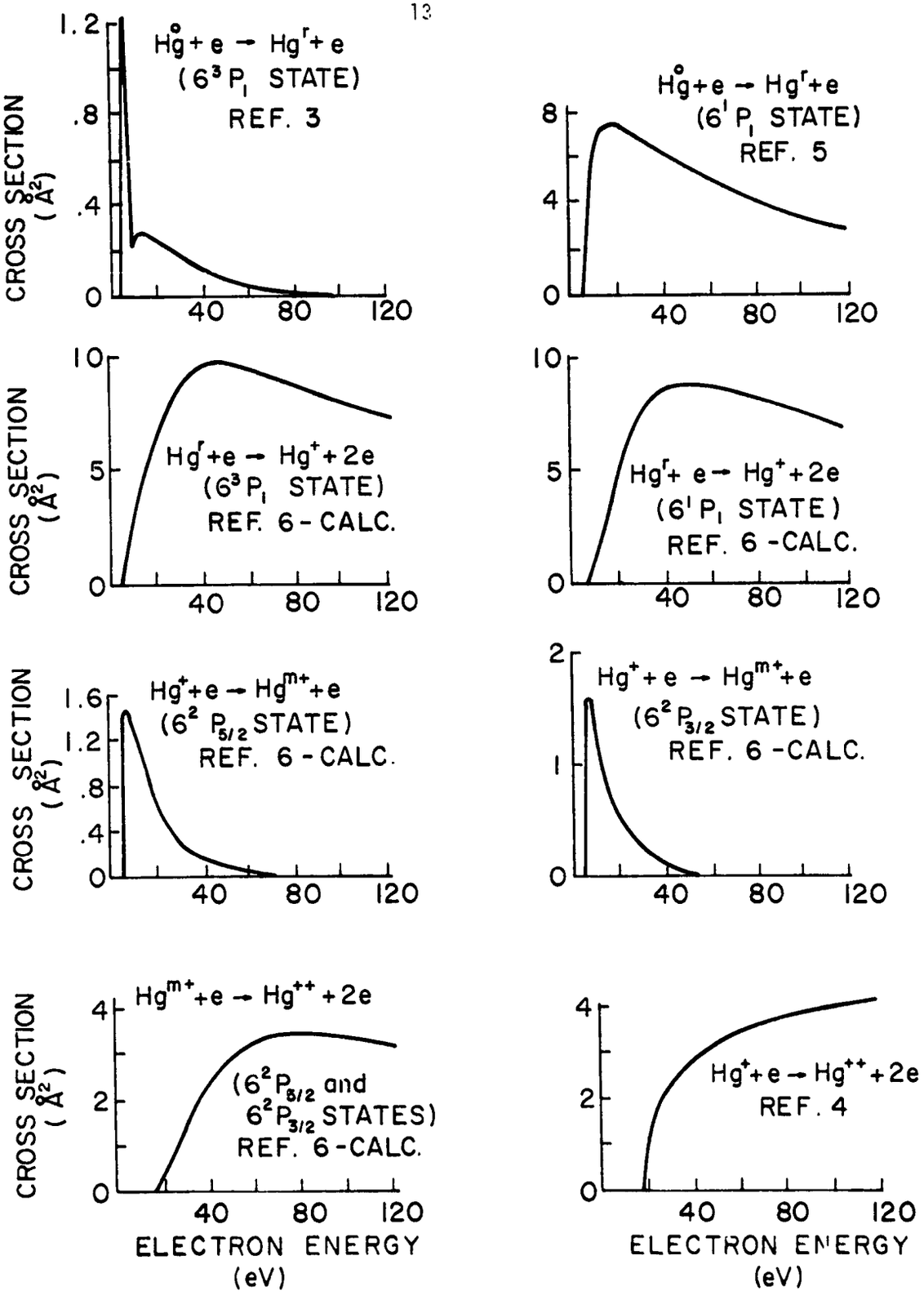


Figure 3 Mercury Cross Sections (continued)

In order to use Equation (5) to evaluate all of the production rates the volume averaged plasma properties (n^*_{pr}, n^*_{mx}, ξ^*_{pr}, T^*_{mx}) must be determined. Comparing Equations (2) and (5) the following definitions of the volume averaged properties must apply.

$$n^*_\alpha \, n^*_{pr} \, P^\gamma_\alpha(\xi^*_{pr}) \, \forall = \int_{Volume} n_\alpha \, n_{pr} \, P^\gamma_\alpha(\xi_{pr}) \, d\forall \tag{6}$$

$$n^*_\alpha \, n^*_{mx} \, Q^\gamma_\alpha(T^*_{mx}) \, \forall = \int_{Volume} n_\alpha \, n_{mx} \, Q^\gamma_\alpha(T_{mx}) \, d\forall \tag{7}$$

These two equations show that the volume averaged plasma properties will be weighted in some manner. In order to evaluate the integrals, species α and γ must be chosen. The only specie density (n_α) that can be determined readily is the single ion density because plasma neutrality requires it to be approximately equal to the electron density. Specie γ must also be chosen in order to determine what P^γ_α and Q^γ_α to use. Figure 2 shows only two choices are possible--the singly ionized metastable states and the doubly ionized ground state. Since the whole purpose of the model is to determine the double ion density, specie γ was chosen as the doubly ionized ground state. Using these choices for species α and γ, Equations (6) and (7) were rewritten in the following form where the electron density (n_e) has been used to approximate the single ion density.

$$n^*_e \, n^*_{pr} \, P^{++}_+(\xi^*_{pr}) \, \forall = \int_{Volume} n_e \, n_{pr} \, P^{++}_+(\xi_{pr}) \, d\forall \tag{8}$$

$$n^*_e \, n^*_{mx} \, Q^{++}_+(T^*_{mx}) \, \forall = \int_{Volume} n_e \, n_{mx} \, Q^{++}_+(T_{mx}) \, d\forall \tag{9}$$

The volume averaged values of the primary electron energy (ξ^*_{pr}) and the Maxwellian electron temperature (T^*_{mx}) were defined as shown in Equations (10) and (11). These definitions were chosen because they give reasonable values for the properties involved (i.e. these volume averaged values can't be greater than the peak values, which was possible with some of the other definitions).

$$P^{++}_{+}(\xi^*_{pr}) = \frac{\int_{Volume} n_e \, n_{pr} \, P^{++}_{+}(\xi_{pr}) \, d\forall}{\int_{Volume} n_e \, n_{pr} \, d\forall} \tag{10}$$

$$Q^{++}_{+}(T^*_{mx}) = \frac{\int_{Volume} n_e \, n_{mx} \, Q^{++}_{+}(T_{mx}) \, d\forall}{\int_{Volume} n_e \, n_{mx} \, d\forall} \tag{11}$$

Equations (8) - (11) along with Equation (12), which says the volume averaged electron density is the sum of the volume averaged primary and Maxwellian electron densities, can be combined to obtain the following definitions of the remaining volume averaged plasma properties.

$$n^*_e = n^*_{pr} + n^*_{mx} \tag{12}$$

$$n^*_e = \left[\int_{Volume} n_e^2 \, d\forall\right]^{1/2} / \forall^{1/2} \tag{13}$$

$$n^*_{pr} = \frac{\int_{Volume} n_e \, n_{pr} \, d\forall}{\left[\int_{Volume} n_e^2 \, d\forall\right]^{1/2} \forall^{1/2}} \tag{14}$$

$$n^*_{mx} = \frac{\int_{Volume} n_e \, n_{mx} \, d\forall}{\left[\int_{Volume} n_e^2 \, d\forall\right]^{1/2} \forall^{1/2}} \tag{15}$$

This concludes the mathematical development for electron bombardment reactions.

The volume averaged plasma properties must be evaluated in order to use Equation (5) to calculate the production rates. The plasma properties (n_{pr}, n_{mx}, ε_{pr}, T_{mx}) are measured at many points inside the discharge chamber by a Langmuir probe. This data is then used to evaluate the integrals in Equations (10), (11) and (13)-(15) numerically yielding the needed volume averaged plasma properties.

Migration Losses

The second type of process to be considered is that of an excited atom or ion going to the plasma boundary. The equation for the plasma boundary loss rate of a specie α is given by:

$$R_{\ell\alpha} = \int_{\text{plasma boundary}} n_\alpha v_\alpha \, dA \tag{16}$$

where n_α is the density of specie α at the boundary, v_α is its average velocity toward the plasma boundary, and dA is the infinitesimal area. For neutral particles assumed to have a temperature equal to the discharge chamber wall temperature and having a mass "m_0" the average velocity toward the boundary (v_0) is equal to one-fourth the average thermal speed

$$v_0 = \frac{1}{4}\sqrt{\frac{8k\,T_{wall}}{\pi m_0}} \tag{17}$$

where k is the Boltzmann constant.

For ions this velocity is determined by the Bohm criterion[8,9] and is given by

$$v_q = \sqrt{\frac{T_{mx} q}{m_i} \left(1 + \frac{n_{pr}}{n_{mx}}\right)} \tag{18}$$

where q is the ion charge (coul) and m_i is the ion mass (kg), and T_{mx}, n_{pr} and n_{mx} are as defined previously.

Since the integral equation used to define production rates has been reduced to an algebraic equation in terms of average properties Equation (16) should also be simplified in this manner. Because the migration loss is a surface phenomenon, however, it is necessary to use surface averaged densities and velocities based on surface average properties to obtain

$$R_{\ell\alpha} = n_\alpha^s \, v_\alpha^s \, A \tag{19}$$

where A is the total surface area of the primary electron region and the superscript "s" designates values based on surface averaged properties. Equation (19) could be made more convenient for use in the model if it were based on volume averaged properties as Equation (5) is. Equation (19) was for this reason rewritten in terms of volume averaged densities as follows

$$R_{\ell\alpha} = n_\alpha^* \, v_\alpha^* \, A/F_\alpha \tag{20}$$

where v_α^* is $v_\alpha(T_{mx}^*, n_{pr}^*/n_{mx}^*)$ and F_α is a plasma uniformity factor given by Equation (21) which relates the volume and surface averaged density-velocity product.

$$F_\alpha = \frac{n_\alpha^* v_\alpha^*}{n_\alpha^s v_\alpha^s} = \frac{n_\alpha^* v_\alpha^*}{\left[\int_{\text{plasma boundary}} n_\alpha v_\alpha \, dA\right]/A} \tag{21}$$

This concludes the mathematical development for the migration loss of excited species, but some additional discussion and quantification of the uniformity factor F_α is necessary before it can be used in Equation (20). The evaluation of F_α for neutral excited states is difficult because it is difficult to measure their densities. The migration of excited neutrals to the plasma boundary is however a minor loss mechanism compared to the losses due to the conversion of neutral excited atoms into single ions. Therefore F_α for neutral excited states can be set equal to unity without introducing a significant error into the total loss rate calculation. In the case of ions, however, migration to the boundary is a major loss mechanism and F_α must be evaluated in order to obtain accurate results. For the singly ionized ground state the approximation, $n_+ = n_e$, can again be used in order to evaluate F_+ using Equation (21). The uniformity factor for the singly ionized metastable states was set to unity since these states have a very minor effect on the double ion density. The determination of F_{++} is based on the observation[10] that the volume averaged double ion density (n_{++}^*) is proportional to the volume averaged electron density squared $(n_e^*)^2$. It has been assumed that this proportionality holds locally and this results in the following definition of F_{++}.

$$F_{++} = \frac{(n_e^*)^2 v_{++}^*}{\left[\int_{\text{plasma boundary}} n_e^2 v_{++} \, dA\right]/A} \tag{22}$$

Photon Diffusion Losses

The third type of process to be considered is the loss of resonance state atoms due to photon diffusion to the walls of the discharge chamber. From diffusion theory the rate of photon loss across any plasma boundary and hence the rate of resonance state atom loss by this mechanism is given by the equation

$$R_{\ell r} = \int_{\text{plasma boundary}} D\Delta n_p \, dA = DA[\Delta n_p] \quad . \tag{23}$$

In this equation n_p is the photon density and D is the photon diffusion coefficient which is given by

$$D = \frac{1}{3\tau(n_o^* \sigma_c)^2} \tag{24}$$

"τ" in this equation is the average lifetime of the resonance state atom, n_o^* is the neutral ground state atom density, and σ_c is the cross section for absorption of the photons by neutral ground state atoms.[11] The second equality in Equation (23) reflects the fact that average properties are being used in this analysis. The photon density has been assumed constant up to a point one photon mean free path from the boundary. From this point the density is assumed to decay linearly to zero at the boundary. This assumption yields the following conservative estimate for the photon loss rate

$$R_{\ell r} = DA \, n_p^* / \ell_f \tag{25}$$

where ℓ_f is the mean free path for photon absorption $(\frac{1}{n_o^* \sigma_c})$. This

approximation is valid when the photon mean free path is much less than the characteristic dimension of the plasma, a condition that is readily satisfied for this case where the photon mean free paths are very small ($\ell_f \approx .1$ cm).

Since the neutral density is assumed uniform over the discharge region the photon density profile is similar to the resonance state atom density profile and the following approximation between the photon and resonance state atom density at any location in the plasma applies:

$$n_p = n_r \beta \quad . \tag{26}$$

β is a proportionality constant that can be thought of as the ratio of the probability that a photon will be "free" in the plasma to the probability that it will be "bound" forming a resonance state atom. This ratio of probabilities can also be expressed as the ratio of the average lifetime of a free photon ($\frac{1}{cn_o\sigma_c}$) to the resonance state atom lifetime (τ). Therefore, β is given by:

$$\beta = \frac{1}{cn_o\sigma_c \tau} \tag{27}$$

where c is the speed of light and the other quantities have already been defined. Combining Equations (25), (26 and (27) one obtains the following equation for the loss rate of resonance state atoms due to photon diffusion:

$$R_{.r} = \frac{n_r^*}{3\ell} A \left[\frac{1}{n_o \sigma_c}\right] \quad . \tag{28}$$

Determination of Specie Densities

The equations derived so far in this section can now be combined to determine the equilibrium density of each specie included in the model. Equations of the form of (5) and (20) -- and (28) for the case of resonance state atoms -- along with the values for the volume averaged plasma properties and the plasma uniformity factors can be used to determine the rates of production and loss for each specie in the plasma. The steady state density of these species can then be calculated by equating their total production rates to their total loss rates. For example, the 6^3P_0 metastable atom density is determined by equating the production rate of this metastable atomic state from neutral ground state atoms to the sum of the associated loss rates due to 1) migration to the wall, 2) production of single ions, and 3) the production of double ions, that is

$$n_o^* \left[n_{pr}^* \, P_o^m(\varepsilon_{pr}^*) + n_{mx}^* \, Q_o^m(T_{mx}^*)\right] \Psi = \frac{n_m^* \, v_m^* \, A}{F_m} \tag{29}$$

$$+ n_m^* \left[n_{pr}^* \, P_m^+(\varepsilon_{pr}^*) + n_{mx}^* \, Q_m^+(T_{mx}^*)\right] \Psi + n_m^* \left[n_{pr}^* \, P_m^{++}(\varepsilon_{pr}^*) + n_{mx}^* \, Q_m^{++}(T_{mx}^*)\right] \Psi .$$

Solving this for the metastable atom density ratio one obtains

$$\frac{n_m^*}{n_o^*} = \left[n_{pr}^* \, P_o^m(\varepsilon_{pr}^*) + n_{mx}^* \, Q_o^m(T_{mx}^*)\right] \Big/ \left\{\frac{v_m^*}{\Psi/A \; F_m} + \left[n_{pr}^* \, P_m^+(\varepsilon_{pr}^*) \right.\right. \tag{30}$$

$$\left.\left. + n_{mx}^* \, Q_m^+(T_{mx}^*)\right] + \left[n_{pr}^* \, P_m^{++}(\varepsilon_{pr}^*) + n_{mx}^* \, Q_m^{++}(T_{mx}^*)\right]\right\}$$

where n_m^* is the volume average metastable state density and v_m^* is the average velocity of metastable neutral atoms toward the boundary. A similar type of equation can be derived for each of the other excited states but they are all as complex or more complex than Equation (30).

For example, the equation for the doubly ionized ground state density has eight terms in the numerator. Each of these terms has the same form as the bracketted quantities in the numerator of Equation (30).

The quantity ¥/A in the denominator of Equation (30) has an interesting physical interpretation. It is contained in a term which represents the loss rate per unit volume of metastable state atoms to the plasma boundary. This term shows the manner in which the size and shape of the primary electron region enters into the model. For a large thruster ¥/A will be large and an ion or excited neutral must, on the average, travel great distances to reach the plasma boundary, and so the loss rate per unit volume of these species will be small. For a small thruster ¥/A will be small and on the average the ions and excited species are near the boundary and can reach it readily resulting in a large loss rate of these species per unit volume.

At this point only the relative density of each excited specie (n^*_α/n^*_o) can be calculated. However one additional fact can be added to the model, the requirement that the plasma be neutral (i.e., $n^*_e = n^*_+ + n^*_{m+} + 2n^*_{++}$). This requirement when added to the relative density equations of the ionized states implies unique single, metastable single and double ion densities. These in turn imply a unique neutral atom density and hence a unique density for each specie considered in the analysis. One must however iterate to arrive at these densities because a neutral ground state atom density must be assumed initially to determine photon loss rates from Equation (28). At the conclusion of the analysis then the calculated ground state atom density must agree with the assumed value.

A computer program has been written which calculates the densities of all the species considered in the model. The densities are calculated by using relative density equations similar to Equation (30) and the plasma neutrality condition. The input needed to make these calculations includes the volume averaged plasma properties, the plasma uniformity factors and the volume-to-surface area ratio of the primary electron region. A listing of this computer program entitled "HG" is included as Appendix A.

EXPERIMENTAL PROCEDURES AND RESULTS

The model developed in the previous section will predict the specie densities if the volume averaged plasma properties, uniformity factors and geometric quantities, which are collectively called the input parameters, are known. In order to verify the accuracy of the model, data must be gathered so that the model's input parameters can be determired. These input parameters can be used by the model to predict the specie densities which can then be compared to the measured densities to determine the model's accuracy. The model's accuracy has been determined by comparing the measured and predicted double ion densities since the double ion density is the model's main concern.

In order to test the accuracy of the model over a wide range of conditions data were used from different thrusters and operating conditions. The 15 cm diameter SERT II thruster was operated with two different grid sets and at three different power levels in each of these configurations. Data were collected at each condition allowing the accuracy of the model to be verified at six different points. Data for the 30 cm diameter thruster were also obtained from Hughes Research Laboratories[12] so that the model could be verified over a wider range of thruster sizes, configurations, and operating conditions. Both thrusters have strongly divergent magnetic fields. Their general configuration and manner of operation have been described in the "Thruster Operation" section. More detailed thruster specifications, etc. are available in the literature.[9, 13, 14]

The data gathering procedure for the 15 cm thruster will be used to illustrate the general manner in which the needed data were obtained.

Before the gathering of data could begin thruster operation and flow rates were kept stable for approximately thirty minutes. This insured thruster conditions would change little in the twenty minute period during which the data were obtained. Table I lists the conditions and configurations at which the 15 cm thruster was operated at C.S.U. along with those for the 30 cm thruster as obtained from Hughes Research Laboratories. This table indicates the changes in configuration for both thrusters resulted from using different grid types. The SERT II grids, listed in Table I, are flat grids with hole diameters of ≈.4 cm and in operation are separated by a gap of .23 cm. The high perveance dished grids are dished slightly to prevent the grids from shorting during operation due to their thermal expansion. Their hole diameter is smaller (.25 cm) as is their separation gap (.079). More detailed specifications for the two grid types can be found in Reference 9. The EM (Engineering Model) grids are similar to the high perveance dished grids described above. The SHAG (Small Hole Accelerator Grids) grids have an accelerator hole diameter that is ≈70% of the EM grids' accelerator hole diameter. This smaller hole size reduces the loss of neutral propellant. These two grid types are described in more detail in References 15 and 16. Table I shows, for example, that the 15 cm thruster with the SERT II grids was operated at one condition where the amount of current collected at the anode (I_{arc}) was 1.7A, while the voltage difference between the anode and cathode (V_{arc}) was 37.2 V and the ion current through the grids (I_{beam}) was 0.258 A.

The values of the volume averaged plasma properties and the uniformity factors must be known in order to calculate the theoretical double ion density. In order to determine the values of these average

Table I

Thruster Sizes, Configurations and Conditions

Thruster Diameter (cm)	Grid Type	Anode Current (I_{arc}--A)	Anode Voltage (V_{arc}--V)	Ion Beam Current (I_{beam}--A)	Mass Flow Rate (A)
15.	SERT II	1.00	33.	.180	.310
15.	SERT II	1.70	37.2	.258	.307
15.	SERT II	2.05	42.6	.272	.308
15.	Dished	3.02	32.2	.499	.735
15.	Dished	4.06	37.5	.654	.725
15.	Dished	4.13	40.4	.622	.650
30.	EM	5.0	37.	1.0	1.25
30.	EM	7.5	37.	1.5	1.76
30.	EM	10.0	37.	2.0	2.29
30.	SHAG	9.5	30.	1.5	1.74
30.	SHAG	11.7	30.	2.0	2.30

plasma properties, the primary and Maxwellian electron densities and energies must be determined everywhere in the discharge chamber. The plasma properties at some point in the plasma can be measured using a Langmuir probe and analyzed using the procedure described in Reference 17. The plasma properties at sixteen different points in the plasma were measured, for each 15 cm thruster condition listed in Table I, using the movable Langmuir probe and associated circuitry described in Reference 18. The results of a typical survey (15 cm thruster - SERT II grids -37 V anode voltage) are plotted in Figure 4. This figure shows the spatial variation of the Maxwellian electron temperature, primary electron energy and the primary and Maxwellian electron densities in the discharge chamber. The Maxwellian electron temperature is seen to average approximately 9 eV over the primary electron region defined by the critical field line while the primary electron energy averages about 30 eV. The average Maxwellian electron density is about $10^{11}cm^{-3}$ while the average primary electron density is approximately $10^{10}cm^{-3}$ over the same region. The electron densities and energies are seen to be fairly uniform in the primary electron region but drop off rapidly outside this region. Using data similar to that plotted in Figure 4, Equations (10)-(15) and (21) were evaluated numerically by the computer program "PROP" (listed in Appendix B) to obtain the volume averaged properties and uniformity factors for each case. The results are listed in Table II along with the volume-to-surface area ratio of the primary electron region ($\forall/A$) and the thruster operating specifications which are reproduced from Table I. The average values which resulted from an examination of Figure 4 (T_{mx} 9 eV, $_{pr}$ 30 eV, n_{mx} $10^{11}cm^{-}$, n_{pr} $10^{-}cm^{-}$) are seen to

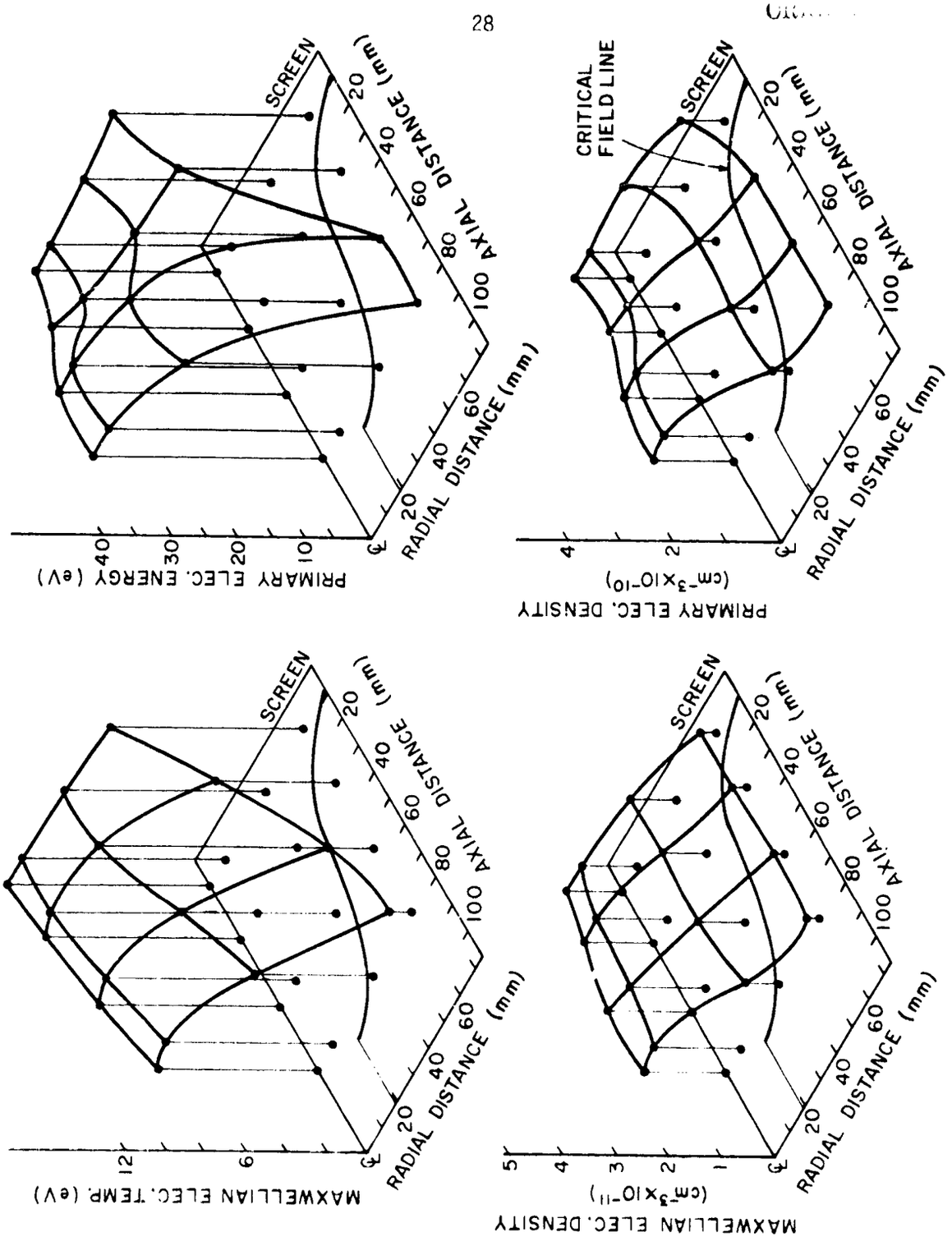


Figure 4 Plasma Property Profiles, 15 cm Thruster - SERT II Grids - 37 V Anode Voltage

Table II

Experimental Results

Operating Variables	Thruster Diameter (cm)	15.	15.	15.	15.	15.	15.	30.	30.	30.	30.	30.
	Grid Type	SERT II			HIGH PERVEANCE DISHED			EM	EM	EM	SHAG	SHAG
	Anode Current (I_{arc}--A)	1.0	1.7	2.05	3.02	4.06	4.13	5.0	7.5	10.0	9.5	11.7
	Anode Voltage (V_{arc}--V)	33.	37.2	42.6	32.2	37.5	40.4	37.	37.	37.	30.	30.
	Beam Current (I_{beam}--A)	.180	.258	.272	.499	.654	.622	1.0	1.5	2.0	1.5	2.0
	Mass Flow Rate (A)	.310	.307	.308	.735	.725	.650	1.25	1.76	2.29	1.74	2.30
Input Parameters	Plasma Volume to Surface Area Ratio (V/A--cm)	1.4	1.4	1.4	1.4	1.4	1.4	2.5	2.5	2.5	2.5	2.5
	Average Maxwellian Electron Temperature (T^*_{mx}--eV)	4.2	9.1	12.2	4.3	7.1	10.2	3.3	3.6	3.8	3.0	2.9
	Average Primary-to-Maxwellian Electron Density Ratio (n^*_{pr}/n^*_{mx})	.034	.033	.166	.017	.042	.134	.50	.35	.25	.19	.22
	Average Primary Electron Energy (V^*_{pr}--eV)	27.5	29.6	38.4	21.5	23.4	31.0	25.4	25.5	27.2	19.6	19.7
	Average Electron Density (n^*_e x 10^{-12} cm^{-3})	9.80	9.10	8.07	36.0	24.3	18.2	7.51	8.97	16.4	8.3	12.8
	Uniformity Factor F_+	2.3	2.1	2.3	2.0	1.9	1.8	1.5	1.5	1.5	1.8	1.7
	Uniformity Factor F_{++}	3.1	2.5	2.6	2.5	2.1	2.0	1.8	1.9	1.8	2.5	2.1
	Measured Double-to-Single Ion Current Ratio (I^{++}/I^+)	.024	.073	.12	.036	.081	.18	.080	.125	.167	.062	.080

agree well with the volume averaged values listed in Table II (T^*_{mx} = 9.1 eV, T^*_{pr} = 29.6 eV, n^*_{mx} = 8.4 x $10^{10}cm^{-3}$, n^*_{pr} = 7.0 x 10^9cm^{-3}). F_+ and F_{++}, which are defined as the ratio of the volume average ion flux to the average flux at the surface of the primary electron region, are seen to have values of 2.1 and 2.5 respectively for the case being discussed. The average plasma properties listed in Table II are observed to cover a wide range in plasma conditions; a situation which is desirable for verification of the model.

The double ion density inside the discharge chamber must also be determined. This can be accomplished indirectly by determining the double-to-single ion density ratio (n_{++}/n_+) in the discharge chamber and the single ion density. The single ion density can be determined with sufficient accuracy by equating the single ion density to the electron density. The value of the double-to-single ion density ratio can be determined from measurements of the ratio of the double ion current to the single ion current in the exhaust beam (I^{++}/I^+), and the equation

$$n_{++}/n_+ = \frac{I^{++}}{I^+ 2\sqrt{2}} \quad . \tag{31}$$

The quantity $2\sqrt{2}$ accounts for charge and Bohm criterion velocity differences between double and single ions.

The quantity "I^{++}/I^+" was measured using a mass spectrometer.[19] The methods used for data acquisition and analysis using such a device are described in Reference 20 for the 15 cm thruster data and in Reference 19 for the 30 cm thruster. The results obtained are listed in the last row of Table II. They show, for example, a double-to-single ion current ratio of 1.3 for the [illegible] = SERT II grid - 37 V

anode voltage case. The general trend observed from these data is that an increase in power input (I_{arc} times V_{arc}) for a certain thruster configuration results in an increase in the ratio I^{++}/I^{+}.

RESULTS AND DISCUSSION

The values of the average plasma properties, listed in Table II, are observed to vary over large ranges. For example, the average Maxwellian electron temperature ranges from a low value of 3.3 eV to a high value of 12.2. Similarly the average primary-to-Maxwellian electron density ratio varies from 0.02 to 0.50. This large variation is considered sufficient to allow a general decision to be made about the accuracy of the model. Comparisons of the experimental and theoretical values of the double-to-single ion density ratio have been used to verify the model's accuracy because this quantity (n_{++}/n_{+}) was determined experimentally. The theoretical and experimental values of the double-to-single ion density ratio are plotted as a function of propellant utilization in Figures 5 and 6. The curves labeled "THEORETICAL" result from predictions made by the model using the "Input Parameters" listed in Table II. The curves labeled "EXPERIMENTAL" result from measurements of the ratio I^{++}/I^{+} made using the mass spectrometer. The trends exhibited by the THEORETICAL an' EXPERIMENTAL curves are very similar and the agreement between the THEORETICAL and EXPERIMENTAL values of the double-to-single ion density ratio is good for plasma physics work with the average error being 3%. The maximum error of 40% is observed at low double-to-single ion density ratios in the 30 cm thruster. These error values indicate the model is accurate over a wide range of plasma conditions and thruster configurations.

Since the model has been shown to be accurate in its predictions of the double-to-single ion density ratio over a wide range of conditions there is a distinct possibility that the specie densities and

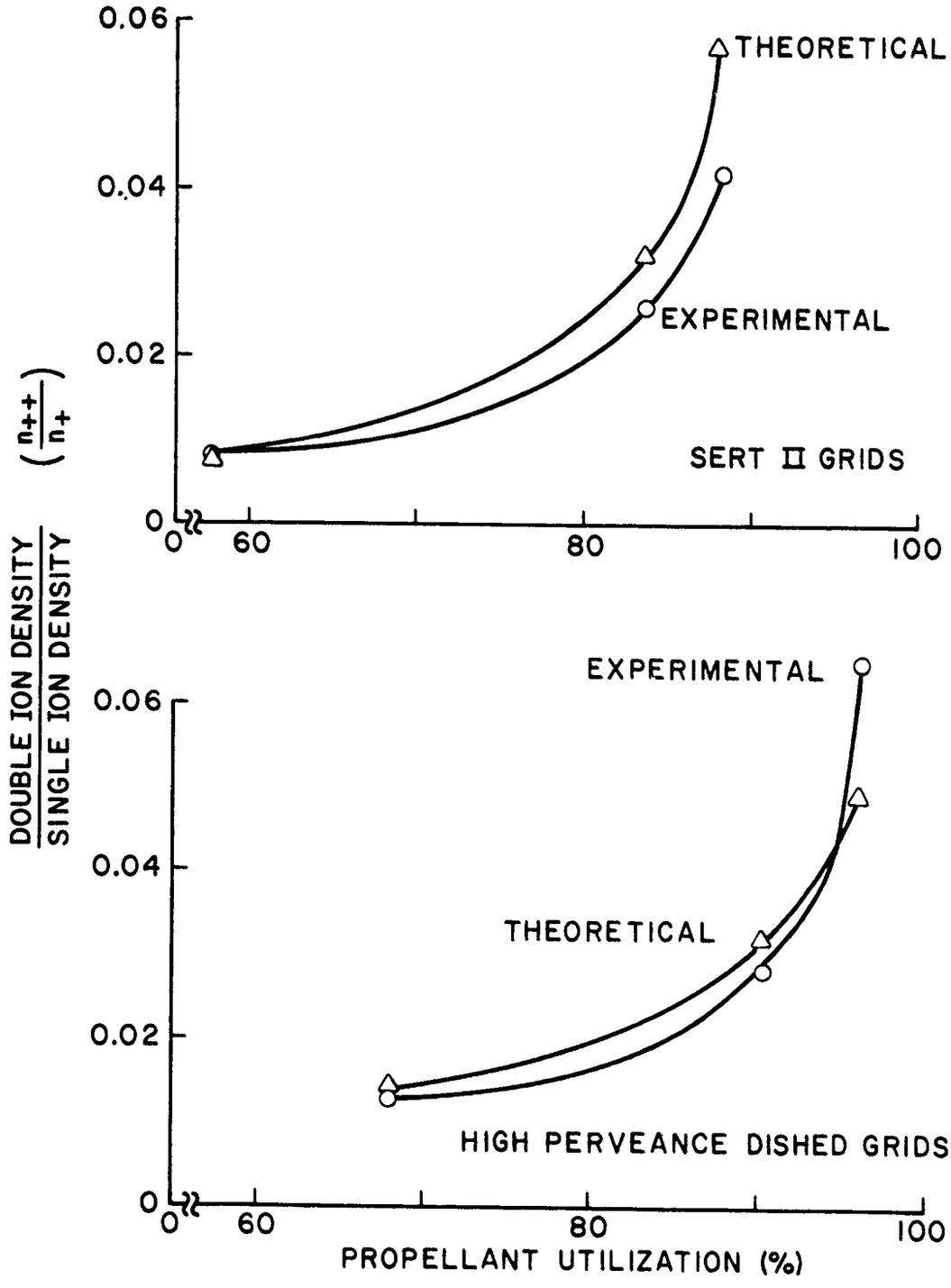


Figure 5 Double-to-Single Ion Density Ratio in a 15 cm Diameter Thruster

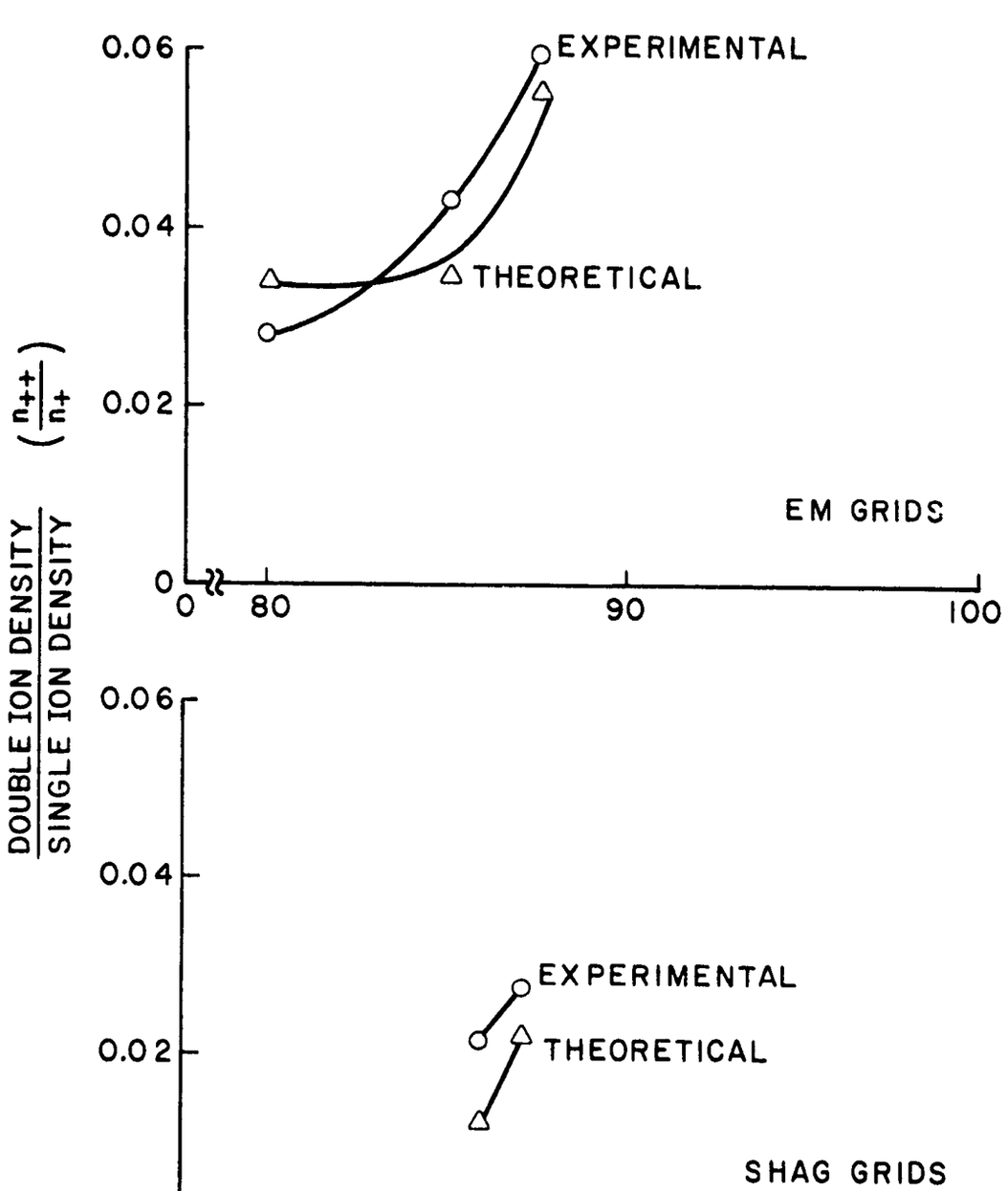


Figure 6 Double-to-Single Ion Density Ratio in a 30 cm Diameter Thruster

reaction rates used to predict the double-to-single ion density ratio are also accurate. The remainder of this section will examine the model's predictions of these specie densities and reaction rates. These quantities are listed in Table III along with the thruster operating variables and the model's input parameters which were reproduced from Table II.

The section in Table III titled "Calculated Normalized Densities" lists the model's predictions of the normalized densities of the states considered in the model where the normalized density of some specie is defined as the specie density divided by the total heavy particle density. The sum of the normalized densities for any thruster condition should therefore equal unity. Table III shows, for example, that the 15 cm diameter thruster operating with SERT II grids at 37 V anode voltage would be predicted to have 68% neutral ground state atoms, 19% neutral resonance state atoms, 6.9% singly charged ground state ions and .2% doubly charged ground state ions. The normalized density of the single ions agrees fairly well in all cases with the 10% value quoted as typical in the literature.[1] As expected the neutral ground state atoms are the most numerous.

These normalized density trends can be explained in terms of variations of plasma properties. For example, the normalized single ion density increases with increasing power input (I_{arc} times V_{arc}) to the thruster in all cases. This occurs because an increase in the values of the volume averaged plasma properties causes the ratio of the production rate of single ions to the total neutral density to increase. The increase in the ratio indicates a smaller total neutral density is

Table III

Predicted Densities and [illegible] Rates

Operating Variables											
Thruster Diameter (cm)	15	15	15	15	15	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Grid Type		SERT II			HIGH PERVEANCE [illegible]		TM	TM	TM	[illegible]	[illegible]
Anode Current (I_{arc}--A)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Anode Voltage (V_{arc}--V)	33	[illegible]	42.6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Beam Current (I_{beam}--A)	186	258	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Mass Flow Rate (A)	313	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible] Parameters											
Plasma Volume to Surface Area Ratio (V/A--cm)	1.4	1.4	1.4	1.4	1.4	1.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Maxwellian Electron Temperature (T^{*}_{mx}--eV)	4.7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Primary-to-Maxwellian Electron Density Ratio (n^{*}_{pr}/n^{*}_{mx})	.034	.083	.166	[illegible]	[illegible]	.134	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Primary Electron Energy (ε^{*}_{pr}--eV)	[illegible]	29.[illegible]	36.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Electron Density, (n^{*}_{e} × 10[illegible] cm[illegible])	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Uniformity Factor F_{+}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Uniformity Factor F_{++}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Measured Double-to-Single Ion Current Ratio (I^{++}/I^{+})	.024	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Calculated Normalized Densities											
Neutral Ground State Atoms	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Neutral Metastable State Atoms	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Neutral Resonance State Atoms	[illegible]	19	14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Singly Charged Ground State Ions	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	098	[illegible]	[illegible]	[illegible]
Singly Charged Metastable Ions	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]
Doubly Charged Ground State Ions	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Calculated Production Rates — [illegible]											
Neutral Ground State	[illegible][a]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Neutral Metastable States	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Neutral Resonance States	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]											
Neutral Ground State	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]
Neutral Metastable States	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Neutral Resonance States	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Singly Charged Ground State	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Singly Charged Metastable Ions States	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[a] Numbers in parentheses are the fraction of the indicated [illegible]

needed to maintain a specified single ion density and so the normalized single ion density increases as previously observed.

The last section in Table III shows the calculated production rates for singly and doubly charged ions through the various intermediate states. These production rates have been normalized by the total production rate of the specie indicated. The fraction of the associated interactions effected by the primary electrons is indicated in parenthesis. For example, at the 15 cm thruster's 37 V, SERT II grid operating point, 59% of the single ions are produced as a result of electron interaction with neutral ground state atoms and 28% resulted from electron bombardment of neutral resonance state atoms. The neutral ground state-to-single ionic state interactions were induced by primary electrons 23% of the time and by Maxwellian electrons the remainder (77%) of the time.

Thruster performance is determined primarily by the mechanism for the production and loss of single ions. The production of these ions is, according to this model, quite dependent on the neutral metastable and neutral resonance states which are ignored in most other analyses. The manner in which single ions are produced however differs a great deal between the two thrusters. In the 15 cm thruster most of the single ions are produced as a result of Maxwellian electron bombardment while primary electrons are unimportant because of their low densities. This indicates that for 15 cm thruster operation the primary electron region is the important reaction region because it is the region where high densities of high energy Maxwellian electrons occur. In the 30 cm thruster, however, relatively high primary electron densities exist and since the Maxwellian electron temperature is low most of the single ion

production results from primary electron bombardment. So for the 30 cm thruster the primary electron region is the important reaction region because it contains high densities of high energy primary electrons.

Table III indicates in all cases a large percentage of the double ions are produced from single ions. This is as one would expect because the minimum energy required to produce a double ion from a single ion is 18.7 eV while 29 eV is required to produce a double ion from a neutral ground state atom. As the power input to the thruster increases the number of electrons with energies greater than 29 eV increases causing the relative importance of the neutral-to-double transition to increase. The least energy is required for the production of double ions via singly ionized metastable states, but the densities of these states are so low that this production mechanism is unimportant.

SIMPLIFIED MODEL

In the previous section it has been shown that most double ions are produced as a result of electron bombardment of single ions. In order to simplify the analysis of the "Theoretical Model" section the other intermediate states for double ion production can therefore be ignored with no significant loss in the accuracy of the double ion density calculations. In the simplified model presented here the approximation is made that the total rate of production of double ions equals the rate of production of double ions from single ions. This production rate is given by:

$$R_p^{++} \simeq R_{p_+}^{++} = n_+^* \left[n_{pr}^* \; P_+^{++}(\xi_{pr}^*) + n_{mx}^* \; Q_+^{++}(T_{mx}^*) \right] \Psi \quad . \tag{32}$$

The total loss rate of double ions is given by the equation

$$R_{\ell_{++}} = \frac{n_{++}^* \, v_{++}^* \, A}{F_{++}} \quad . \tag{33}$$

Equating the loss and production rates and then solving the resultant equation for the double ion density one obtains

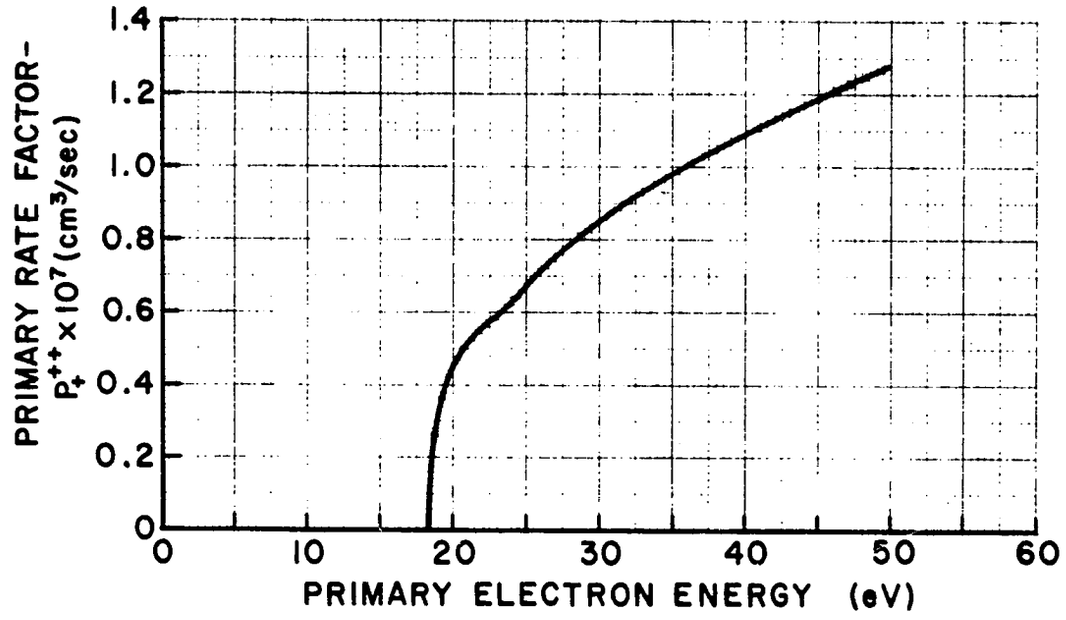
$$n_{++}^* = n_+^* \frac{\left[n_{pr}^* \; P_+^{++}(\xi_{pr}^*) + n_{mx}^* \; Q_+^{++}(T_{mx}^*) \right] \Psi}{\frac{v_{++}^* \, A}{F_{++}}} \quad . \tag{34}$$

The approximation $n_e^* = n_+^*$ can now be used and Equation (18) can be substituted for the double ion velocity to obtain the following equation.

$$n_{++}^{*} = n_e^{*2} \frac{\forall}{A} F_{++} \frac{\left[\frac{n_{pr}^{*}}{n_e^{*}} P_{+}^{++}(\varepsilon_{pr}^{*}) + \frac{n_{mx}^{*}}{n_e^{*}} Q_{+}^{++}(T_{mx}^{*}) \right]}{[T_{mx}^{*} q (1 + n_{pr}^{*}/n_{mx}^{*})/m_i]^{\frac{1}{2}}} \qquad (35)$$

The double ion density can now be determined for a given thruster operating condition using this equation and the plots of $P_{+}^{++}(\varepsilon_{pr})$ and $Q_{+}^{++}(T_{mx})$ found in Figure 7 if the volume averaged plasma properties and the uniformity factor F_{++} are known. This equation will consistently predict lower double ion densities than the complete model since it ignores the production of double ions from neutral states and the singly ionized metastable states, but this error should generally be small. The error will be greatest for plasmas with high energy electrons which can produce double ions directly from neutral states.

The last section of Table III can be used to determine the magnitude of this error for the 11 cases considered in this study. Since the simplified model considers only the single-to-double transition the error associated with this approximation can be determined from the listed value of the percentage of double ions produced from single ions. For example, for the 15 cm-SERT II grid - 37 V anode voltage case the percentage of double ions produced from single ions is 78%. This means that the value of the double ion density predicted by the simplified model would be 78% of that predicted by the complete model. Examination of Table III indicates the double ion densities calculated using the simplified model will agree well with the complete model's predictions for all the 30 cm thruster conditions because in these cases the percentage of double ions produced from single ions is greater than 97%.



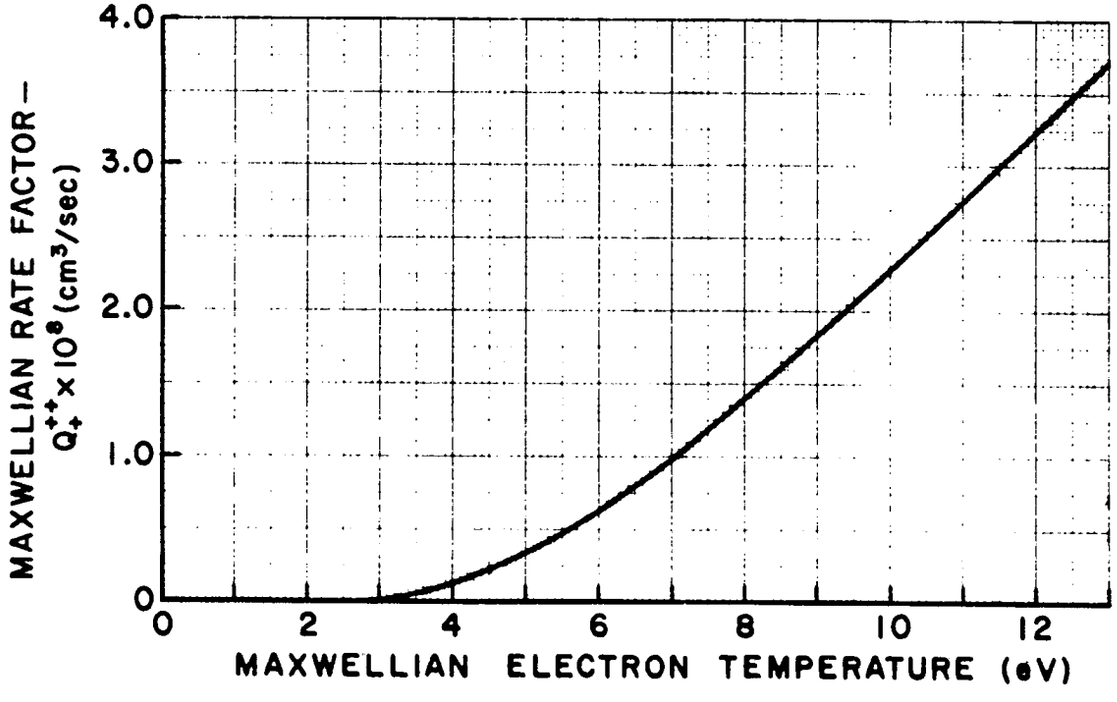


Figure 7 Rate Factors for $Hg^+ \rightarrow Hg^{++}$

In each of these cases few electrons have energies in excess of 29 eV (the minimum energy required for the neutral-to-double transition). The simplified model will however, according to Table III, yield results which are generally low for the 15 cm thruster data (e.g. 30% low for the SERT II grid - 42 V anode voltage condition) because in these cases sufficiently high Maxwellian electron temperatures exist to cause a relatively large percentage of the electrons to have energies in excess of 29 eV.

The most accurate way to determine the values of the average plasma properties required in Equation (35) would be to conduct a Langmuir probe survey of the discharge chamber under consideration to determine the plasma properties at many different points and to then use this information in Equations (10) to (15) and (21) to determine average plasma properties. The collection of the plasma property data is however costly and time consuming. For this reason average plasma property correlations were developed. The correlating parameters used are composed of thruster operating parameters (e.g. I_{arc}) and geometric properties (e.g. $\forall/A$). Using the Maxwellian electron temperature data listed in Table III, for example, one obtains the correlation presented in Figure 8. The terms used in the correlating parameter are defined in Table III. The correlating parameter used in Figure 8 was determined by trial and error. The shape of a curve through the resultant data points was picked to match the trends observed in the data points. For example, the slope of the curve in the neighborhood of the low Maxwellian electron temperature points is seen to decrease. This agrees with the trend observed in the data and also agrees with a prediction, based on inelastic collision cross section data, which says a lower

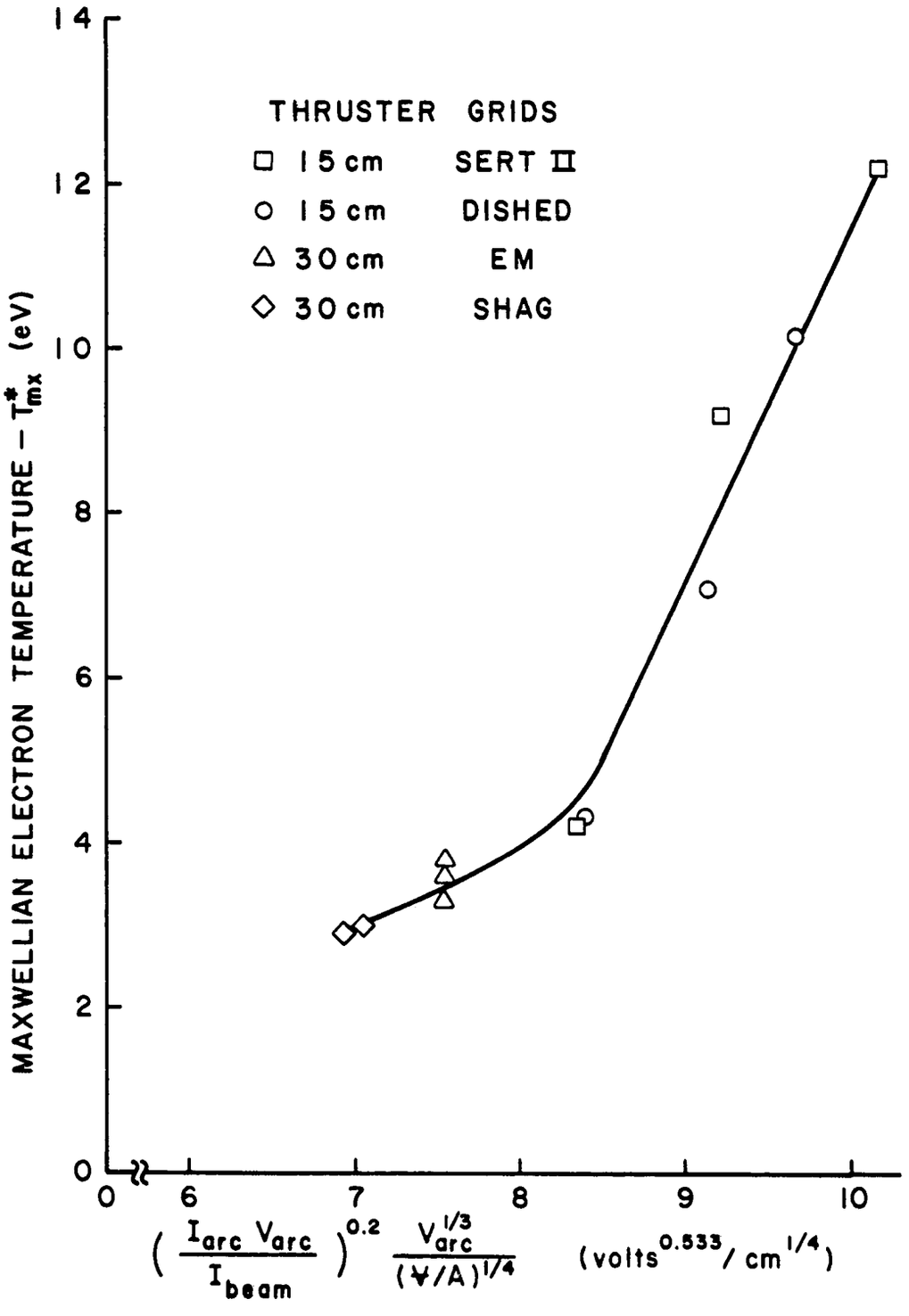


Figure 8 Maxwellian Electron Temperature Correlation

bound on the Maxwellian electron temperature should exist roughly in the neighborhood of 5 eV[1].

The correlation for the primary electron energy is shown in Figure 9. The correlating parameter contains the quantity η_c which is the corrected propellant utilization. The corrected utilization was used in the correlating parameter, instead of the measured propellant utilization, because a better fit of the data points resulted from its use. The propellant utilization (η) of an ion thruster depends upon the plasma properties, the effective open area for the loss of neutral atoms through the grids (A_o) and the effective open area for the loss of ions through the grids (A_+). The propellant utilization is defined by the equation

$$\eta = \frac{n_+ v_+ A_+}{n_+ v_+ A_+ + n_{ot} v_o A_o} \simeq 1 - \frac{n_{ot} v_o A_o}{n_+ v_+ A_+} \qquad (36)$$

where n_{ot} is the total neutral atom density. The primary energy (and other average plasma properties) of a given thruster correlate with the propellant utilization as defined above, but correlation between grid sets having different values of the ratio A_o/A_+ is poor. The problem caused by the utilization's dependence upon grid sets can be corrected by eliminating the ratio A_o/A_+ from Equation (36) and then substituting in its place the value of the ratio A_o/A_+ for some standard grid set. The resultant quantity is the corrected utilization and is defined by the equation

$$\eta_c = 1 - \frac{n_{ot} v_o}{n_+ v_+} \left[\frac{A_o}{A_+}\right]_{\text{standard}} . \qquad (37)$$

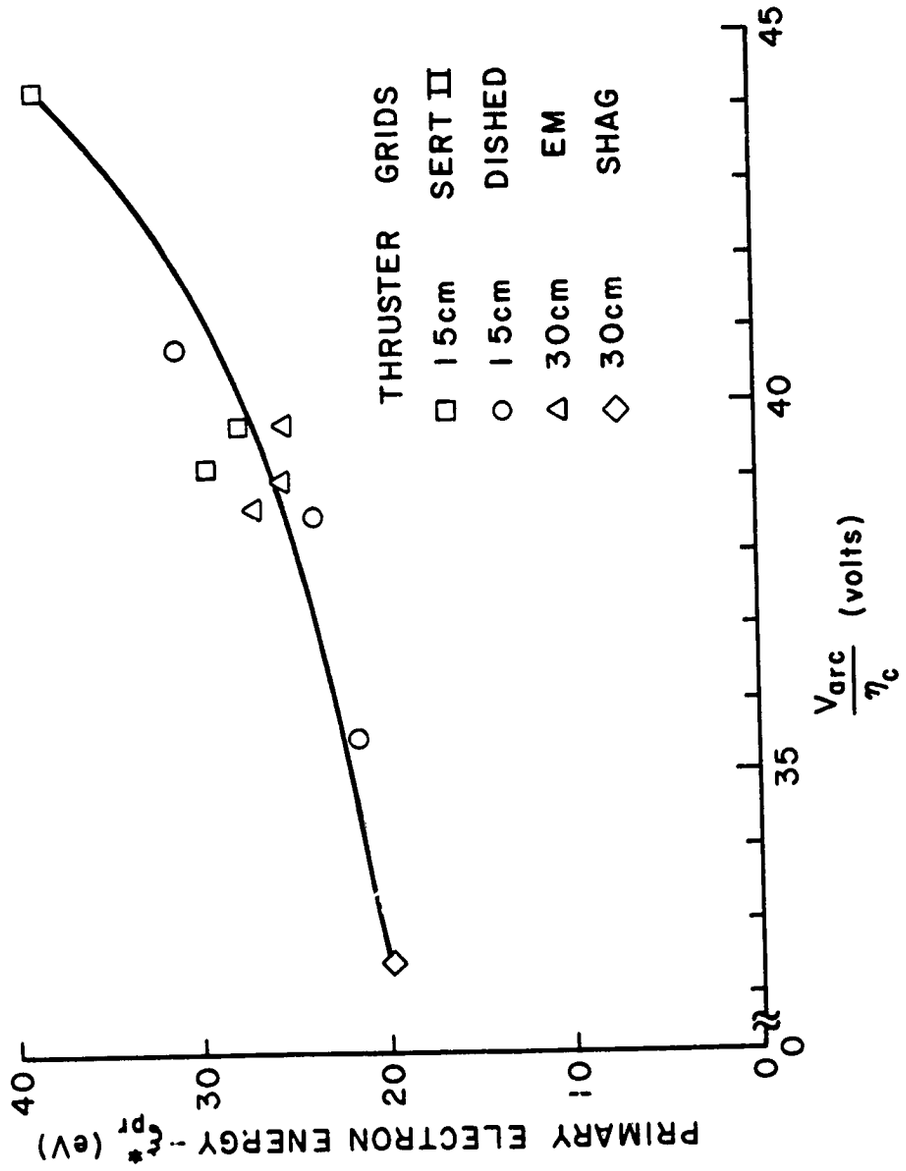


Figure 9 Primary Electron Energy Correlation

The open area for the loss of ions (A_+) from a thruster is proportional to the open area fraction of the screen grid (ϕ_s)[1]. Equilibrium flow theory[1] can be used to determine that the open area for the loss of neutral atoms (A_o) is proportional to the quantity $(\phi_s\ \phi_a)/(\phi_s + \phi_a)$ where ϕ_a is the open area fraction of the accelerator grid. These two approximations can be used to define the ratio A_o/A_+ as follows

$$\frac{A_o}{A_+} = \left(\frac{\phi_s\ \phi_a}{\phi_s + \phi_a}\right) / \phi_s = \frac{\phi_a}{\phi_s + \phi_a} \quad . \tag{38}$$

If Equations (36) - (38) are combined the following result is obtained,

$$\eta_c = 1 - .5(1 - \eta)\ \frac{\phi_s + \phi_a}{\phi_a}\ , \tag{39}$$

where the constant ".5" defines $\phi_a/(\phi_s + \phi_a)$ for the standard grid set.

Figures 10-12 show correlations for the remaining input parameters. These correlations were developed by trial and error in a manner similar to that used to obtain those in Figures 8 and 9. It should be noted that the correlation in Figure 11 is for the quantity $n^*_{pr}[\text{Ŧ}/A]^{-1.5}$ not the primary electron density (n^*_{pr}).

It should be understood that the correlations of Figures 8-12 are based on data obtained from strongly divergent magnetic field thrusters. The average plasma properties predicted using these figures may be inaccurate for other types of thrusters (e.g. multipole or radial field thrusters). Therefore Langmuir probe surveys should be made for these other types in order to obtain good estimates of the average plasma properties and hence accurate predictions of the double ion density.

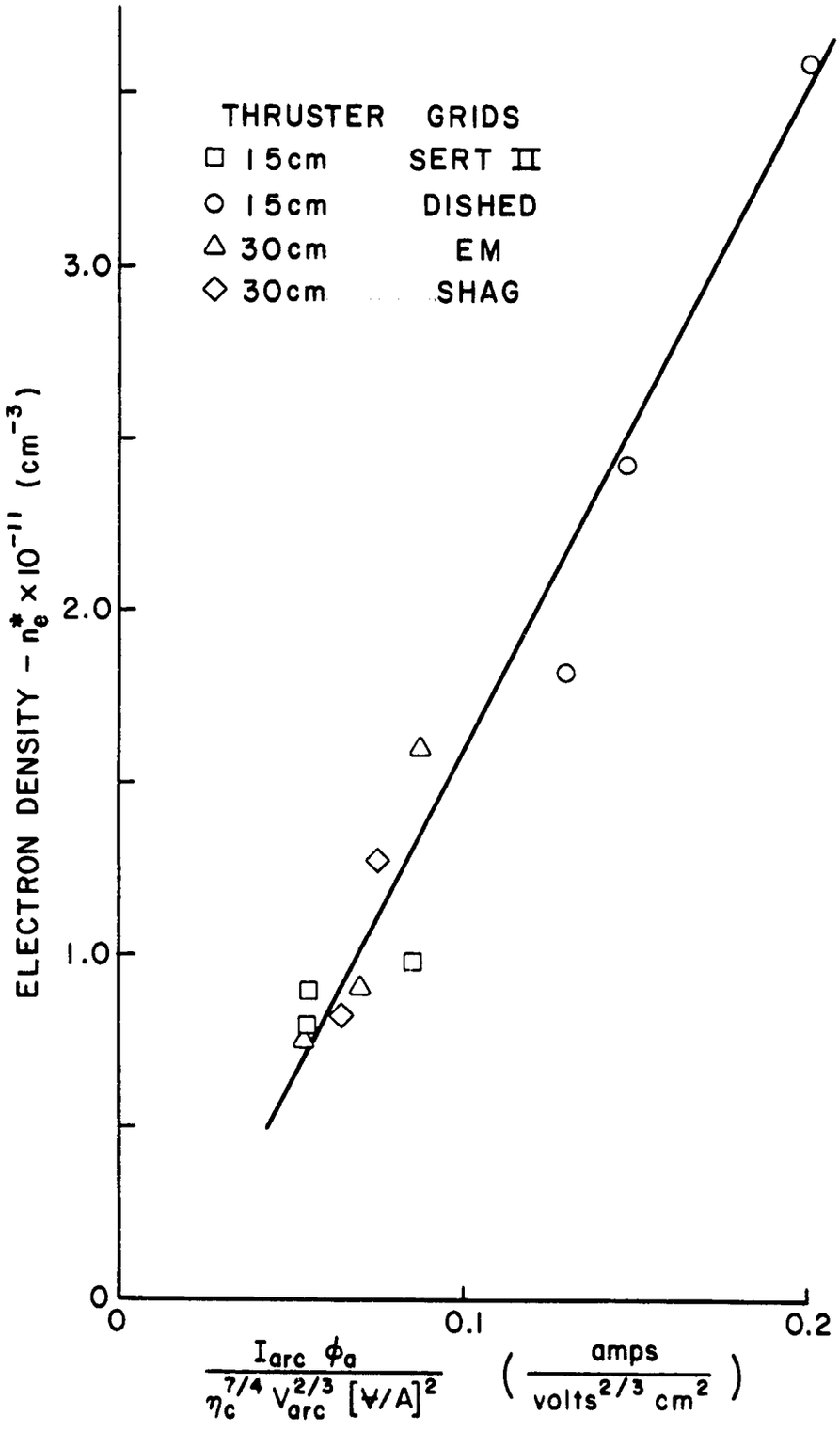


Figure 10 Electron Density Correlation

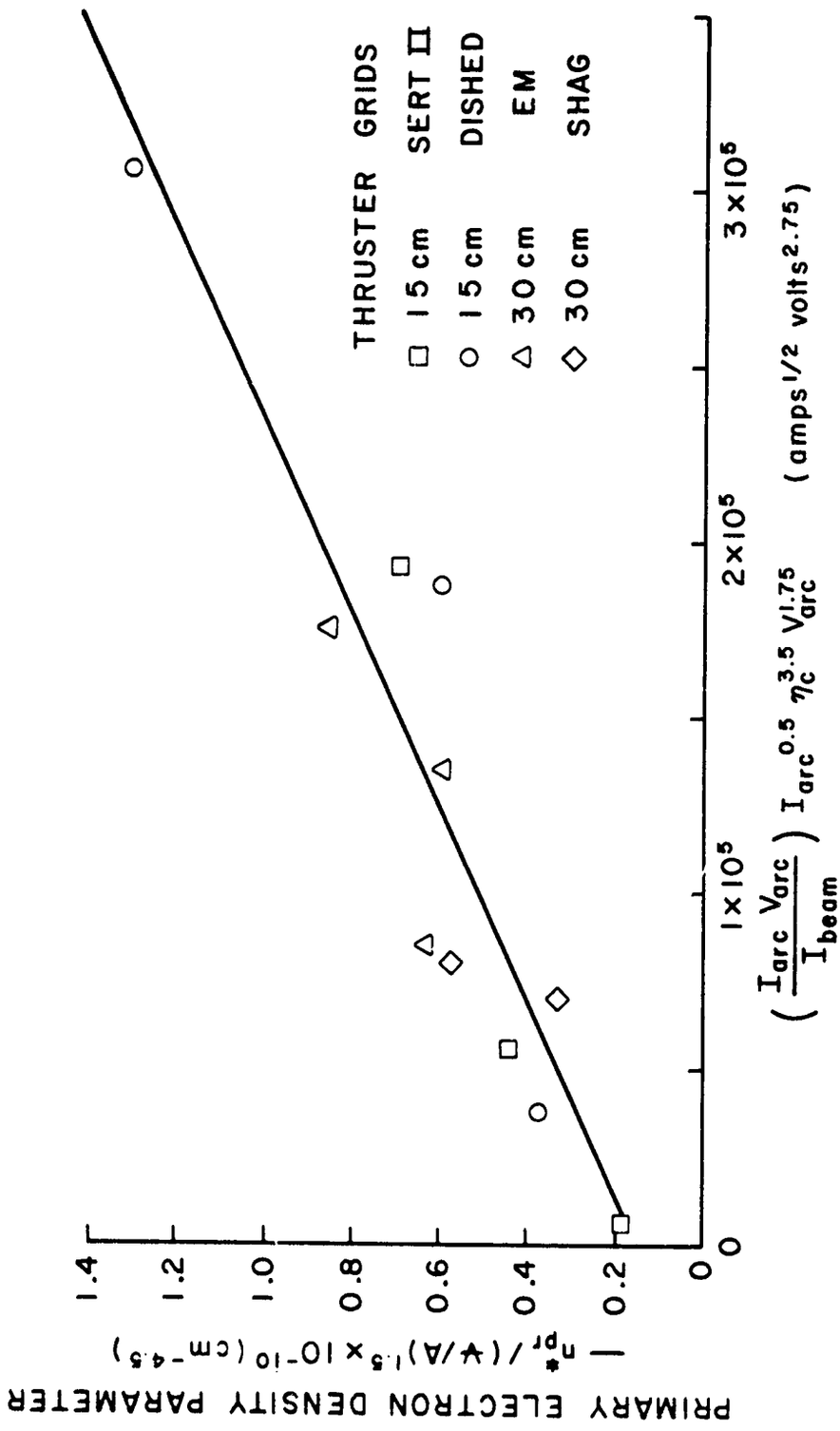


Figure 11 Primary Electron Density Correlation

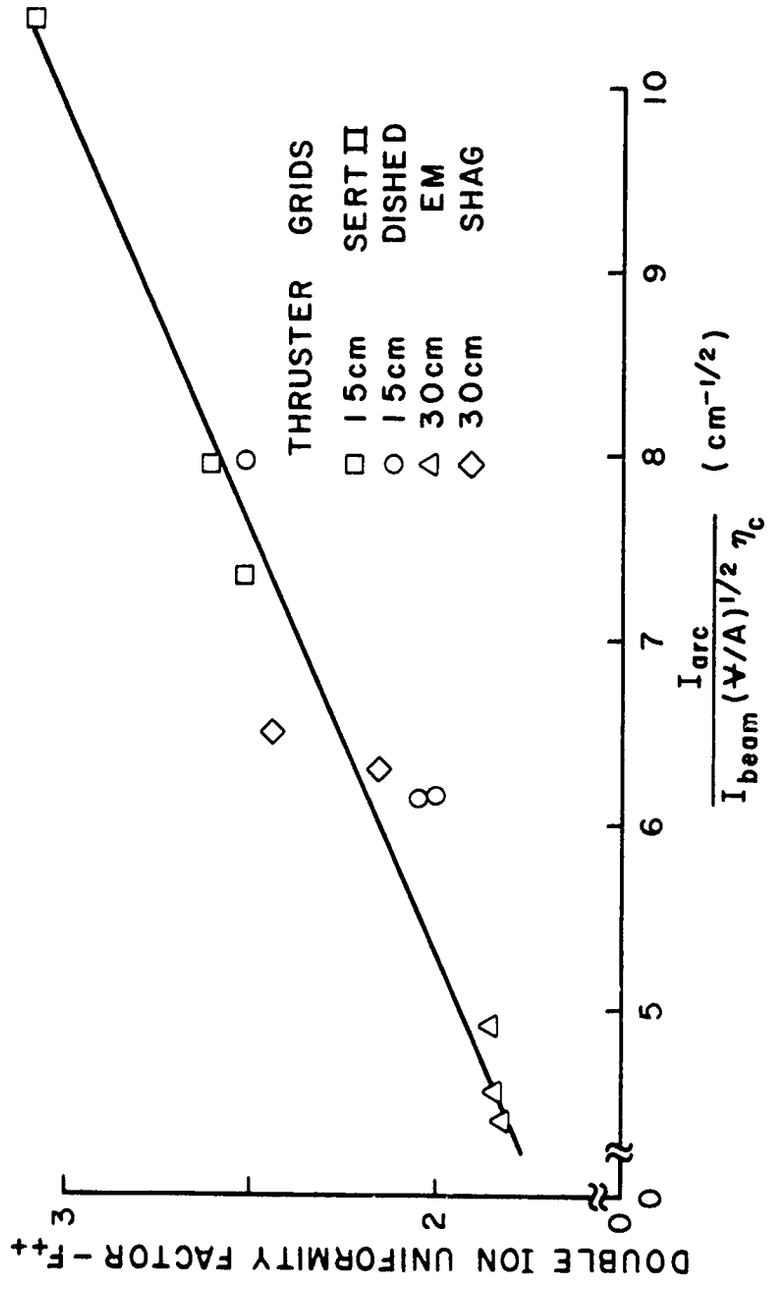


Figure 12 Uniformity Factor Correlation

Application of the simplified model can be best demonstrated through an example. Consider a 15 cm thruster operating at the conditions defined by the first section of Table IV. The corrected utilization (η_c) is first calculated using Equation (39) and a value of 68% is obtained. Next the correlating parameters are calculated. For example, the value of the correlating parameter

$$\left(\frac{I_{arc}\ V_{arc}}{I_{beam}}\right)^{.2}\ V_{arc}^{1/3}\ (\text{¥/A})^{-1/4}$$

used in Figure 8 is $8.3\ \frac{\text{volts}^{.533}}{\text{cm}^{1/4}}$. This value indicates the average Maxwellian electron temperature would be 4.6 eV. The remainder of the average plasma properties were determined in a similar manner. The results obtained are listed in the second section of Table IV. Using the values of the primary electron energy and the Maxwellian electron temperature one can enter Figure 7 and determine P_+^{++}(22 eV) and Q_+^{++}(4.6 eV). These quantities, together with the average densities, the uniformity factor and the volume-to-surface area ratio for this thruster are then substituted into Equation (35) to obtain the double ion density as shown in the last section of Table IV. The double ion density calculated using the simplified model is 6.2 x 10^{9} cm^{-3} while the value calculated using the complete model is 5.2 x 10^{9} cm^{-3}. The major reason for the discrepancy is that the electron temperature in Table IV (4.6 eV) is larger than the value used by the complete model (4.3 eV). This higher electron temperature causes $Q_+^{++}(T_{mx}^*)$ to be too large and results in the over-estimate of the double ion density.

Table IV.

Determination of the Double Ion Density Using the Simplified Model

Measured Thruster Variables

(15 cm Thruster)

I_{arc}	= 3. amps	ϕ_s	= .67
V_{arc}	= 32.2 volt	ϕ_a	= .67
I_{beam}	= .499 amps	$\forall/A$	= 1.4 cm
η	= .68		
η_c	= .68		

Approximate Plasma Properties

T^*_{mx}	= 4.6 eV	n^*_{pr}	= 4.64×10^9 cm^{-3}
ξ^*_{pr}	= 22. eV	n^*_e	= 3.51×10^{11} cm^{-3}
F_{++}	= 2.55	n^*_{mx}	= $n^*_e - n^*_{pr}$ = 3.46×10^{11}cm^{-3}

Calculation of the Double Ion Density

$$n^*_{++} = \frac{(3.51 \times 10^{11}\text{cm}^{-3})^2\ (1.4\text{ cm})\ (2.55)}{\left[9.6 \times 10^9 \frac{\text{cm}^2}{\text{sec}^2\text{-eV}}\ (4.6\text{ eV})\ (1.013)\right]^{\frac{1}{2}}}$$

$$\times\ \left[.013\ \left(.55 \times 10^{-7}\frac{\text{cm}^3}{\text{sec}}\right) + .987\ \left(.23 \times 10^{-8}\frac{\text{cm}^3}{\text{sec}}\right)\right]$$

$$= 6.2 \times 10^9\ \text{cm}^{-3}$$

An examination of Equation (35) will indicate some general trends which should be considered in the design and operation of electron bombardment thrusters. For example, the double ion density varies linearly with the volume-to-surface area ratio. Therefore if two thrusters have the same average plasma properties the larger thruster will have a higher double ion density. Equation (35) suggests it would be desirable to reduce the electron density since the double ion density is proportional to the square of the electron density. However, making arbitrary adjustments in the plasma properties to reduce the double ion density may have an adverse effect on other aspects of thruster performance which must also be considered. An examination of the effect of electron density on propellant utilization will indicate one of the effects such an adjustment would have. The propellant utilization previously defined in Equation (36), is reproduced below.

$$\eta = 1 - \frac{n^*_{ot} v_o A_o/A_+}{n^*_+ v_+} \tag{40}$$

The single ion density (n^*_+) can be approximated by the electron density (n^*_e). The total neutral density (n^*_{ot}) is the sum of the densities of all the neutral species and can be calculated using the equation

$$n^*_{ot} = n^*_o \left(1 + n^*_{mt}/n^*_o + n^*_{rt}/n^*_o\right) \tag{41}$$

where n^*_{mt} and n^*_{rt} are the total metastable and resonance states densities. The values of the ratios in Equation (41) can be calculated using equations similar in form to Equation (30). The neutral ground state density can be calculated using the equation

$$n_o^* = n_+^* \, (n_o^*/n_+^*) \simeq n_e^* \, (n_o^*/n_+^*) \tag{42}$$

where the ratio (n_+^*/n_o^*) again takes a form similar to that of Equation (30). Combining these results into Equation (40) a result of the following form is obtained. "f" is a function of the Maxwellian electron temperature, primary electron energy, primary-to-Maxwellian electron density ratio and the uniformity factor F_+. The dependence of the propellant utilization on the electron density and thruster parameters is explicitly shown.

$$\eta = 1 - \frac{A_o/A_+}{n_e^* \, V/A} \; f(T_{mx}^*, \, \xi_{pr}^*, \, n_{pr}^*/n_{mx}^*, \, F_+) \tag{43}$$

One can see that a reduction in the electron density to reduce the double ion density will also have the undesirable effect of reducing the propellant utilization. However, if some changes in thruster design are made along with a reduction in the electron density the propellant utilization can be held constant while the double ion density is reduced. For example, if a new thruster were being designed one might double the volume-to-surface area ratio by making the thruster larger than its predecessor. It could then be operated at one-half the electron density of the predecessor allowing the propellant utilization to remain constant while exhibiting half the double ion density in accordance with Equation (35).

It might also be desirable to reduce the double ion density of a certain size thruster while maintaining the same propellant utilization. The propellant utilization could be held constant by reducing both the ratio A_o/A_+ (which reduces the relative escape rate of neutrals) and

the electron density in a manner that keeps the ratio $(A_o/A_+)/n_e^*$ constant. According to Equation (35) this would result in a large reduction in the double ion density which varies as the square of the electron density. The data in Table III for the two 30 cm thruster configurations at 1.5 and 2.0 amps beam current can be used to determine if theory and experiment agree for this method of double ion density reduction. The only difference in these two thruster configurations is the open area fraction of the accelerator grid. The EM accelerator grid has an open area fraction (ϕ_a) of 45% while the open area fraction for the SHAG accelerator grid is 23%. Both sets have a 69% open area fraction for the screen grid. The value of the ratio A_o/A_+ can be calculated for both grid sets using Equation (38). For the EM grids the ratio A_o/A_+ has a value of .39 while for the SHAG grids the value of the ratio is .25. The change from EM grids to SHAG grids then allowed operation at a given propellant utilization to occur at a lower arc voltage and hence a lower electron densities and energies and as a result lower double ion densities. In this particular case the double-to-single ion density ratios dropped from 4.4% and 6.0% to 2.2% and 2.8% respectively at two different utilizations when the SHAG grids were used. The theoretical model predicted essentially the same quantitative changes.

CONCLUSIONS

A discharge chamber model for an electron bombardment ion thruster has been developed which considers metastable, resonance and ground state atomic and ionic production and loss mechanisms. The model can be used to predict doubly charged ion densities from plasma property information. These calculated double ion densities agree with measured values to within 40% for low values of the double-to-single ion density ratio ($n_{++}/n_{+} < 2\%$) and to within 20% for the rest of the data. Correlations, which relate average plasma properties to thruster operating variables such as anode current, can be used to estimate the average plasma properties in strongly divergent magentic field thrusters when the properties themselves are not available. Singly charged ions are produced, according to this analysis, in significant numbers in two step processes through intermediate metastable and resonance states in addition to direct ionization from the neutral ground state. Doubly charged ions are produced predominantly via the singly ionized ground state with direct ground state neutral-to-double ion production becoming more significant in plasmas with high Maxwellian electron temperatures and primary electron energies. A simplified model which considers only the singly ionized ground state in double ion production can be used to predict double ion densities that agree with the complete model's predictions to within 5% when primary electron energies and Maxwellian electron temperatures are less than 29 eV and 5 eV respectively. The recent experimental observation[12] that the use of small hole accelerator grids in conjunction with lower anode voltages provides a means for reducing double ion densities in thrusters, without degrading performance, is supported by the model.

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APPENDIX A

The computer program "HG", which can be used to predict the densities of excited atomic and ion states considered in the complete model, is listed below. The input parameters needed by this program can be approximated using the correlations in Figures 8-12. More accurate input parameters can be determined using the computer program "PROP", listed in Appendix B, and data obtained from a Langmuir probe survey of the discharge chamber. The computer program "HG" uses the equations developed in the "Theoretical Model" section and carrys out the calculations in the manner suggested at the end of that section. Comment cards are included in the listing to indicate what calculations, etc. are to be carried in each section.

Values of the functions $P_{\alpha}^{\gamma}(\varepsilon_{pr})$ and $Q_{\alpha}^{\gamma}(T_{mx})$ are listed immediately after the computer program listing. The particular initial state (α) and final state (γ) are indicated in the last twenty columns. For example, the label "HGM-HG+ 3PO" indicates the initial state for the reaction is the 6^3P_0 metastable state and the final state is the singly ionized ground state. The first seven cards listed with a particular identifying label contain the values for $P_{\alpha}^{\gamma}(\varepsilon_{pr})$ while the second seven list values for $Q_{\alpha}^{\gamma}(T_{mx})$.

```
      PROGRAM HG (INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=PUNCHG     10
     1H,FILMPL)                                                                HG     20
      COMMON /A/ NP,NM,TEMP,PRINRG                                             HG     30
      COMMON /B/ UTT(4,25),PD(4,25),IK                                         HG     40
      DIMENSION A(8), B(3), C(3), D(3), F(3), SX00M(2,2,21), SE00M(2,2,2HG     50
     11), SX01(2,21), SE01(2,21), SX02(2,21), SE02(2,21), SX0M1(2,2,21),HG     60
     2 SE0M1(2,2,21), SX11M(2,2,21), SE11M(2,2,21), SX12(2,21), SE12(2,2HG     70
     31), RR(2), TR(2), SE1M2(2,2,21), RA0M0(2), RA1M0(2), SX(2,21), SE(HG     80
     42,21), P2(8), TI(8), X(8), Y(8), G(8), R(5), Z(8), H(8), SX0R(2,2,HG     90
     521), SE0R(2,2,21), SX01(2,2,21), SE01(2,2,21), ER(2), TM(2), W(2),HG    100
     6 Q(2), TK(2), XSM(2), XSMM(2), XPSM(2), XPSMM(2), FREE(2), PHLOSS(HG    110
     72), TAU(2), XLAM(2), OPP(6,6), XNE(6), XL(3,6,10), PNE(6,10), SP(6HG    120
     8,6), P2A(7), HA(7), PYP(30), PNO(4), SD(14,6,10), TT1(14,8), PP(10HG    130
     9), PR(10), EXP(30), SX1M2(2,2,21), TMP(5), QT(8), QR(8)                 HG    140
                                                                              HG    150
      REAL NP,NM                                                              HG    160
      DATA X(1),TTI(1,1),TTI(2,1),TTI(3,1),TTI(4,1),TTI(5,1),TTI(6,1),TTHG    170
     1I(7,1),TTI(8,1),TTI(9,1),TTI(10,1),TTI(11,1),TTI(12,1),TTI(13,1),THG    180
     2TI(14,1),TM,TR,TK,H(1),Z(1),Z(2)/10HPRI ENERGY,10HNEUT DENS.,10HNMHG    190
     3/NO PERC,10HNM/NO PERC,10HN+/NO PERC,10HNM+/NO PER,10HN++/NO PER,1HG    200
     40HN+/NOT    ,10HN++/NOT   ,10HBEAM MA.  ,10HN+ CM-3   ,10HN++ CM-3HG    210
     5  ,10HI ARC AMP.,10HEV/BEAMION,10H          ,10H6 3P0     ,10H6 3PHG    220
     62     ,10H6 3P1     ,10H6 1P1     ,10H6 2D 5/2  ,10H6 2D 3/2  ,10HHG    230
     7N++/N+    ,10HPLOT WITH ,10HM CONSTANT/                                 HG    240
      DATA EP,TAU,XLAM/5.545,6.7,.00000010H,.0000000013,.00002537,.00001HG    250
     1849/                                                                    HG    260
      FR(A,B)=(A(1)*B(1)+A(2)*B(2))/(A(1)+A(2))                               HG    270
C                                                                             HG    280
C     THIS PROGRAM IS SET UP FOR MERCURY ONLY                                 HG    290
C     SET UP OF THE PROGRAM                                                   HG    300
C                                                                             HG    310
      NMZ=10                                                                  HG    320
      NNN=0                                                                   HG    330
      DO 101 I=2,8                                                            HG    340
         B(I)=10H                                                             HG    350
  101 X(I)=10H                                                                HG    360
      DO 102 I=3,8                                                            HG    370
  102 Z(I)=10H                                                                HG    380
      DO 103 J=1,14                                                           HG    390
      DO 103 I=2,8                                                            HG    400
  103 TTI(J,I)=10H                                                            HG    410
      DO 104 I=1,8                                                            HG    420
         QT(I)=TTI(13,I)                                                      HG    430
  104 QR(I)=10H                                                               HG    440
      QR(1)=10HUTL.                                                           HG    450
      NQ=2                                                                    HG    460
      DO 105 I=1,3                                                            HG    470
  105 A(I)=1.                                                                 HG    480
C                                                                             HG    490
C     READ IN THE INTEGRATED CROSS SECTIONS                                   HG    500
C     1 -- INDICATES DATA FOR PRIMARY ELECTRONS                               HG    510
C     2--- INDICATES DATA FOR MAXWELLIAN ELECTRONS                            HG    520
C                                                                             HG    530
C     NEUTRAL TO SINGLE                                                       HG    540
C                                                                             HG    550
      READ (5,168) ((SE01(J,I),SX01(J,I),I=1,21),J=1,2)                       HG    560
C                                                                             HG    570
C     NEUTRAL TO DOUBLE                                                       HG    580
C                                                                             HG    590
      READ (5,168) ((SE02(J,I),SX02(J,I),I=1,21),J=1,2)                       HG    600
C                                                                             HG    610
C     SINGLE TO DOUBLE                                                        HG    620
C                                                                             HG    630
      READ (5,168) ((SE12(J,I),SX12(J,I),I=1,21),J=1,2)                       HG    640
C                                                                             HG    650
C     NEUTRAL TO METASTABLE
```

```
C
      READ (5,168) (((SEOOM(I,J,K),SXOOM(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     NEUTRAL TO RESONANCE
C
      READ (5,168) (((SEOR(I,J,K),SXOR(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     METASTABLE ATOM TO SINGLE
C
      READ (5,168) (((SEOM1(I,J,K),SXOM1(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     SINGLE METASTABLE TO DOUBLE
C
      READ (5,168) (((SE1M2(I,J,K),SX1M2(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     RESONANCE TO SINGLE
C
      READ (5,168) (((SER1(I,J,K),SXR1(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     SINGLE TO SINGLE METASTABLE
C
      READ (5,168) (((SE11M(I,J,K),SX11M(I,J,K),K=1,21),J=1,2),I=1,2)
C
C     READ IN DATA
C     ND=NO. OF DIFFERENT PRIMARY ENERGIES DESIRED FOR EACH ELECTRON DEN
C     DP=(BELOW) DELTA PRIMARY ENERGY
C     NS=NO. OF DIFFERENT ELECTRON DENSITIES USED PER RUN
C     NRR=NO. OF RUNS
C     IFLAG=1  NO INTERPOLATION FOR MASS FLOW RATE
C
      READ (5,142) ND,NS,NRR,IFLAG
C
C     BEGIN THE RUNS
C
      DP=10.
      DO 138 IR=1,NRR
C
C     EQUIVALENT OPEN AREA FOR NEUTRALS AND CHARGED PARTICLES. VOLUME OF
C     PRIMARY ELECTRON REGION. NONUNIFORMITY PARAMETERS.
C
        READ (5,141) OPN,OPC,VOL,F1,F2
C
C     NO. OF DIFFERENT MASS FLOW RATES DESIRED
C
        NC=2
C
C     VALUES OF M
C
        IF (IFLAG.EQ.1) GO TO 106
        READ (5,141) (RMO(I),I=1,NC)
  106   KKK=1
        XNI=.05
        DO 129 JJ=KKK,NS
          RHINRG=15.
          XNI=.05
          XMM=0.
          XE2=0.
C
C     PRIMARY ELECTRON VOLUME TO AREA, MAX. TEMPERATURE, DUMMY, PRIMARY/MA
C     MAXWELLIAN RATIO, ELECTRON DENSITY.
C
            IF ([illegible].1) GO TO 107
            READ (5,143) [illegible],DUMMY,[illegible]
            IF (IFLAG.EQ.1) [illegible]=DUMMY
            IF (IFLAG.EQ.1) GO TO [illegible]
C
```

```
C     ELECTRON DENSITIES FOR ITERATION ON MASS FLOW RATE                    HG  1320
C                                                                           HG  1330
            READ (5,140) (TMP(L),L=1,NS)                                    HG  1340
            GO TO 109                                                       HG  1350
  107       ELDENS=TMP(JJ)                                                  HG  1360
            GO TO 109                                                       HG  1370
  108       WRITE (6,139)                                                   HG  1380
  109       CONTINUE                                                        HG  1390
            XNE(JJ)=ELDENS                                                  HG  1400
            ELD=ELDENS/1.E18                                                HG  1410
            ENCODE (44,144,II) VA,ELD,TEMP,DENR                             HG  1420
            DO 128 II=1,ND                                                  HG  1430
               SEV=0.                                                       HG  1440
               WRITE (6,145) VA,ELDENS,TEMP,PRINPS,DENR                     HG  1450
               WRITE (6,146) VOL,OPN,OPC,F1,F2                              HG  1460
               TOT=1.+1./DENR                                               HG  1470
               NP=ELDENS/TOT                                                HG  1480
               NM=ELDENS/TOT/DENR                                           HG  1490
               EENL=20.*TEMP                                                HG  1500
               CALL YINTEG (SX02,SE02,1.,SUMM,PRSUMM)                       HG  1510
C                                                                           HG  1520
C     CALCULATION OF METASTABLE/ NEUTRAL GROUND STATE   RATIO               HG  1530
C                                                                           HG  1540
               DO 112 J=1,2                                                 HG  1550
                  DO 110 I=1,2                                              HG  1560
                  DO 110 K=1,21                                             HG  1570
                     SX(I,K)=SX00M(J,I,K)                                   HG  1580
  110             SE(I,K)=SE00M(J,I,K)                                      HG  1590
                  CALL YINTEG (SX,SE,1.,SUM,PRSUM)                          HG  1600
                  DO 111 I=1,2                                              HG  1610
                  DO 111 K=1,21                                             HG  1620
                     SX(I,K)=SX0M1(J,I,K)                                   HG  1630
  111             SE(I,K)=SE0M1(J,I,K)                                      HG  1640
                  CALL YINTEG (SX,SE,1.,SUM2,PRSUM2)                        HG  1650
                  WLOSS=236./VA/4.                                          HG  1660
                  WALL=WLOSS                                                HG  1670
                  RADNO(J)=SUM/(WLOSS+SUM2+SUMM)                            HG  1680
                  WRITE (6,147) TM(J),RADNO(J)                              HG  1690
                  WRITE (6,161)                                             HG  1700
                  WRITE (6,163) SUM,PRSUM                                   HG  1710
                  SEV=SEV+SUM*PRSUM                                         HG  1720
                  WRITE (6,162)                                             HG  1730
                  WRITE (6,167) WLOSS,SUM2,PRSUM2,SUMM,PRSUMM               HG  1740
                  TT=WLOSS+SUM2+SUMM                                        HG  1750
                  AA=WLOSS/TT                                               HG  1760
                  AB=SUM2/TT                                                HG  1770
                  AC=SUMM/TT                                                HG  1780
                  WRITE (6,148) AA,AB,AC                                    HG  1790
  112          CONTINUE                                                     HG  1800
               SD(2,JJ,II)=(RADNO(1)+RADNO(2))*100.                         HG  1810
C                                                                           HG  1820
C     CALCULATION AND ITERATION FOR RESONANCE/NEUTRAL, SINGLE/NEUTRAL,      *HG  1830
C     NEUTRAL DENSITY                                                       HG  1840
C                                                                           HG  1850
               JREL=-1                                                      HG  1860
               IFLAG=0                                                      HG  1870
               I2=II-2                                                      HG  1880
               I1=II-1                                                      HG  1890
               IF (II.GT.2) XN1=(2.*SD(4,JJ,I1)-SD(4,JJ,I2))/100.          HG  1900
               XNI=XN1+XNM+XN2                                              HG  1910
               XSM(2)=0.                                                    HG  1920
C                                                                           HG  1930
C     GUESS NEUTRAL DENSITY                                                 HG  1940
C                                                                           HG  1950
  113          XNO=ELDENS/XNI/1000000.                                      HG  1960
C                                                                           HG  1970
```

```
C     CALCULATE RESONANCE/NEUTRAL  RATIO                                  HG  1980
C                                                                         HG  1990
               DO 117 J=1,NQ                                              HG  2000
                  IF (IFLAG.EQ.1) GO TO 116                               HG  2010
                  DO 114 I=1,2                                            HG  2020
                  DO 114 K=1,21                                           HG  2030
                     SX(I,K)=SX0R(J,I,K)                                  HG  2040
  114             SE(I,K)=SE0R(J,I,K)                                     HG  2050
                  CALL YINTEG (SX,SE,1.,XSM(J),XPSM(J))                   HG  2060
                  DO 115 I=1,2                                            HG  2070
                  DO 115 K=1,21                                           HG  2080
                     SX(I,K)=SXR1(J,I,K)                                  HG  2090
  115             SE(I,K)=SER1(J,I,K)                                     HG  2100
                  CALL YINTEG (SX,SE,1.,XSMM(J),XPSMM(J))                 HG  2110
  116             CONTINUE                                                HG  2120
                  DVD=33963./XLAM(J)                                      HG  2130
                  SIGMAC=.112/DVD*XLAM(J)**2/TAU(J)                       HG  2140
                  PREF(J)=1./(XNO*SIGMAC)                                 HG  2150
                  PRLOSS(J)=1./(9.E12*VA*(TAU(J)*XNO*SIGMAC)**2)          HG  2160
  117             RR(J)=XSM(J)/(WALL*PRLOSS(J)+XSMM(J)+SUMM)              HG  2170
                  IF (IFLAG.EQ.1) GO TO 118                               HG  2180
                  CALL YINTEG (SX01,SE01,1.,SUM1,PRSUM1)                  HG  2190
                  CALL SUMIT (SX0M1,SE0M1,2,RA0M0,TSUM1,B,C)              HG  2200
                  A(2)=0.                                                 HG  2210
                  Q(2)=0.                                                 HG  2220
  118             CALL SUMIT (SXR1,SER1,NQ,RR,TSS,W,Q)                    HG  2230
                  IF (IFLAG.EQ.1) GO TO 119                               HG  2240
C                                                                         HG  2250
C     CALCULATION OF SINGLE/NEUTRAL  RATIO                                HG  2260
C                                                                         HG  2270
                  VP=SQRT(TEMP**4.+R03BE9*(1.+NP/NM))/100.                HG  2280
                  WLOSS=VP/VA/F1                                          HG  2290
                  CALL SUMIT (SX11M,SE11M,2,A,TSUM2,D,E)                  HG  2300
                  CALL YINTEG (SX12,SE12,1.,SUM2,PRSUM2)                  HG  2310
  119             RNIO=(SUM1+TSUM1+TSS)/(WLOSS+TSUM2+SUM2)                HG  2320
C                                                                         HG  2330
C     CHECK OF  ERROR IN GUESS OF NEUTRAL DENSITY                         HG  2340
C                                                                         HG  2350
                  RNPO=RNIO+XN2+XNM                                       HG  2360
                  ERROR=ABS((RNPO-XNI)/RNPO)                              HG  2370
                  XNI=RNIO+XN2+XNM                                        HG  2380
                  XN1=RNIO                                                HG  2390
                  JKL=JKL+1                                               HG  2400
                  IFLAG=1                                                 HG  2410
                  IF (ERROR.GT..03) GO TO 113                             HG  2420
C                                                                         HG  2430
C     NEUTRAL GROUND STATE ATOM DENSITY                                   HG  2440
C                                                                         HG  2450
                  XNO=ELDENS/XNI/1000000.                                 HG  2460
                  WRITE (6,149) XNO,JKL                                   HG  2470
                  SO(3,JJ,II)=(RR(1)+RR(2))*100.                          HG  2480
                  SO(1,JJ,II)=XNO                                         HG  2490
                  SO(4,JJ,II)=RNIO*100.                                   HG  2500
                  XMA=100./(100.+SO(3,JJ,II)+SE(2,JJ,II))                HG  2510
                  SO(7,JJ,II)=RNIO*XMA                                    HG  2520
                  XMI=XNO*(1.+W*SQRT(1)*RA0M0(2)+RR(1)+RR(2))             HG  2530
                  XL(1,JJ,II)=XMI*2.75D-                                  HG  2540
C                                                                         HG  2550
C     PRINT OUT RESONANCE ATOM DENSITY RATIO                              HG  2560
C                                                                         HG  2570
                  DO 120 J=1,NQ                                           HG  2580
                     WRITE (6,151) R(J),RR(J),PREF(J)                     HG  2590
                     WRITE (6,151)                                        HG  2600
                     WRITE (6,153) XS(J),XPSM(J)                          HG  2610
                     SEV=SEV+XSM(J)*XPSM(J)                               HG  2620
                     WRITE (6,152)                                        HG  2630
```

```
                WRITE (6,152) WALL,PHLOSS(J),XSMM(J),XPSMM(J),SUMM,PRSHG   2640
     1          UMM                                                        HG  2650
                TT=WALL+PHLOSS(J)+XSMM(J)+SUMM                             HG  2660
                AA=WALL/TT                                                 HG  2670
                AB=XSMM(J)/TT                                              HG  2680
                AC=SUMM/TT                                                 HG  2690
                AD=PHLOSS(J)/TT                                            HG  2700
                WRITE (6,151) AA,AD,AB,AC                                  HG  2710
  120         CONTINUE                                                     HG  2720
C                                                                          HG  2730
C     PRINT OUT SINGLE ION DENSITY RATIO                                   HG  2740
C                                                                          HG  2750
              WRITE (6,153) RNIO                                           HG  2760
              SD(10,JJ,II)=RNIO*XNO                                        HG  2770
              PRIF=FR(B,C)                                                 HG  2780
              WRITE (6,161)                                                HG  2790
              PRIG=FR(A,D)                                                 HG  2800
              PRIH=FR(D,E)                                                 HG  2810
              WRITE (6,164) SUM1,PRSUM1,TSUM1,PRIF,TSS,PRIG                HG  2820
              SEV=SEV+SUM1*PRSUM1+TSUM1*PRIF+TSS*PRIG                      HG  2830
              TT=SUM1+TSUM1+TSS                                            HG  2840
              AA=SUM1/TT                                                   HG  2850
              AB=TSUM1/TT                                                  HG  2860
              AC=TSS/TT                                                    HG  2870
              WRITE (6,154) AA,AB,AC                                       HG  2880
              WRITE (6,162)                                                HG  2890
              WRITE (6,167) WLOSS,SUM2,PRSUM2,TSUM2,PRIH                   HG  2900
              TT=SUM2+WLOSS+TSUM2                                          HG  2910
              AA=WLOSS/TT                                                  HG  2920
              AB=SUM2/TT                                                   HG  2930
              AC=TSUM2/TT                                                  HG  2940
              WRITE (6,148) AA,AB,AC                                       HG  2950
C                                                                          HG  2960
C     CALCULATION OF SINGLE META/NEUTRAL  RATIO                            HG  2970
C                                                                          HG  2980
              DO 123 J=1,2                                                 HG  2990
                 DO 121 I=1,2                                              HG  3000
                 DO 121 K=1,21                                             HG  3010
                    SX(I,K)=SX11M(J,I,K)                                   HG  3020
  121               SE(I,K)=SE11M(J,I,K)                                   HG  3030
                 CALL YINTEG (SX,SE,RNIO,SUM1,PRSUM1)                      HG  3040
                 DO 122 I=1,2                                              HG  3050
                 DO 122 K=1,21                                             HG  3060
                    SX(I,K)=SX1M2(J,I,K)                                   HG  3070
  122               SE(I,K)=SE1M2(J,I,K)                                   HG  3080
                 CALL YINTEG (SX,SE,1.,SUM2,PRSUM2)                        HG  3090
                 WLOSS=VP/VA/F2                                            HG  3100
                 RATMO(J)=SUM1/(SUM2+WLOSS)                                HG  3110
                 WRITE (6,155) TK(J),RATMO(J)                              HG  3120
                 WRITE (6,161)                                             HG  3130
                 WRITE (6,163) SUM1,PRSUM1                                 HG  3140
                 SEV=SEV+SUM1*PRSUM1                                       HG  3150
                 WRITE (6,162)                                             HG  3160
                 WRITE (6,166) WLOSS,SUM2,PRSUM2                           HG  3170
                 TT=WLOSS+SUM2                                             HG  3180
                 AA=WLOSS/TT                                               HG  3190
                 AB=SUM2/TT                                                HG  3200
                 WRITE (6,148) AA,AB                                       HG  3210
  123            CONTINUE                                                  HG  3220
                 XL(2,JJ,II)=(RNIO+RATMO(1)+RATMO(2))*VP*XNO*100.          HG  3230
                 XNM=RATMO(1)+RATMO(2)                                     HG  3240
                 IF (II.GT.1) XNM=2.*XNM-SD(5,JJ,II)/100.                  HG  3250
C                                                                          HG  3260
C     CALCULATION OF DOUBLE/NEUTRAL  RATIO                                 HG  3270
C                                                                          HG  3280
              CALL YINTEG (SX02,SE02,1.,SUM1,PRSUM1)                       HG  3290
```

```
      CALL YINIEG (SX12,SE12,RDIO,SUM2,PRSUM2)
      CALL SUMIT (SX1M2,SE1M2,2,RAIMO,TSUM1,B,C)
      WLOSS=1.4142*VP/VA/F2
      RK=0.
      SD(5,JJ,II)=(RAIMO(1)+RAIMO(2))*100.
      DO 124 I=1,2
  124 RK=RK+RAOMO(I)*SUMM
      RL=0.
      DO 125 I=1,NG
  125 RL=RL+RR(I)*SUMM
      R20=(SUM1+SUM2+TSUM1+RK+RL)/WLOSS
C
C     PREDICTION OF NEXT DOUBLE ION/NEUTRAL FOR USE IN PREDICTION
C     OF NEUTRAL DENSITY
C
      IF (II.EQ.1) GO TO 126
      I1=II-1
      XN2=(2.*R20-SD(6,JJ,I1)*.01)*2.
      GO TO 127
  126 XN2=2.*R20
  127 CONTINUE
      SD(1,JJ,II)=R20*XNO
      SD(2,JJ,II)=R20*XXA
      XL(3,JJ,II)=R20*VP*1.4142*XNO*100.
      SD(6,JJ,II)=R20*100.
C
C     WRITE DOUBLE/NEUTRAL RATIO, ETC.
C
      WRITE (6,156) R20
      G(II)=PRINKG
      PN=FR(B,C)
      WRITE (6,161)
      WRITE (6,165) SUM1,PRSUM1,SUM2,PRSUM2,TSUM1,PN,RK,PRSUMM
      SEV=SEV+SUM1*PRSUM1+SUM2*PRSUM2+TSUM1*PN+(RK+RL)*PRSUMM
      TT=SUM1+SUM2+TSUM1+RK+RL
      AA=SUM1/TT
      AB=SUM2/TT
      AC=TSUM1/TT
      AD=RK/TT
      AE=RL/TT
      WRITE (6,157) AA,AB,AC,AD
      WRITE (6,163) RL,PRSUMM
      WRITE (6,158) AE
      WRITE (6,162)
      WRITE (6,166) WLOSS
C
C     BEAM CURRENT
C
      XXXX=(XL(2,JJ,II)+XL(3,JJ,II)*2.)*1.6E-16*OPC
C
C     MASS LOSS RATE
C
      TOTLOSS=(XL(1,JJ,II)*OPN+(XL(2,JJ,II)+XL(3,JJ,II))*OPC)*1
     1.6E-16
      UTL=XXXX/TOTLOSS
      SD(9,JJ,II)=XXXX
C
C     EV/ION
C
      EV=[illegible]V/(XXXX/1.6E-19)*NO*VOL
C
C     ARC CURRENT
C
      [illegible]
      [illegible]
      [illegible]
```

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                WRITE (6,159) XXXX,UTL,EV,ARCI                                  HG  3960
C                                                                               HG  3970
C       CHECK FOR PLASMA NEUTRALITY                                             HG  3980
C                                                                               HG  3990
                EP=ABS((XNM+XN2+RNIO)/XNI-1.)                                   HG  4000
                IF (IFLAG.EQ.1.AND.EP.GT..03) GO TO 108                         HG  4010
                PRINRG=PRINRG+DP                                                HG  4020
                IF (IFLAG.NE.0) GO TO 138                                       HG  4030
  128         CONTINUE                                                          HG  4040
  129       CONTINUE                                                          HG  4050
                                                                                HG  4060
C       INTERPOLATION FOR THE ELECTRON DENSITY WHICH YIELDS DESIRED MASS        HG  4070
C       LOSS RATE                                                               HG  4080
C                                                                               HG  4090
           DO 132 IY=1,NS                                                       HG  4100
                                                                                HG  4110
C       LOSS RATE                                                               HG  4120
C                                                                               HG  4130
              DO 131 JZ=1,ND                                                    HG  4140
                 DO 130 JY=1,NS                                                 HG  4150
                    PYP(JY)=(XL(1,JY,JZ)*OPN+(XL(2,JY,JZ)+XL(3,JY,JZ))*OPCHG  4160
     1              )*1.6E-16                                                  HG  4170
  130               CONTINUE                                                    HG  4180
                    WRITE (6,160) (PYP(JK),JK=1,NS)                             HG  4190
                 DO 131 JA=1,NC                                                 HG  4200
                    CALL AITKEN (PYP,XNE,NS,2,PNO(JA),PNE(JA,JZ))               HG  4210
                    IF (PNE(JA,JZ).LT.0.) PNE(JA,JZ)=0.                         HG  4220
  131           CONTINUE                                                        HG  4230
                                                                                HG  4240
C       CALCULATE N++/N+                                                        HG  4250
C                                                                               HG  4260
           DO 132 IZ=1,ND                                                       HG  4270
              OPP(IY,IZ)=SD(6,IY,IZ)/SD(4,IY,IZ)                               HG  4280
  132       CONTINUE                                                          HG  4290
            CALL CRSPLT (PNO,OPP,XNE,PNE,G,X,Y,Z,ND,NS,NC)                      HG  4300
C                                                                               HG  4310
C       INTERPOLATION OF PLASMA PROPERTIES ETC. FOR CONSTANT MASS LOSS RATHG  4320
C                                                                               HG  4330
           DO 135 IK=1,13                                                       HG  4340
              DO 133 IF=1,8                                                     HG  4350
  133         Y(IF)=TTI(IK,IF)                                                  HG  4360
              DO 134 IE=1,NS                                                    HG  4370
              DO 134 IF=1,ND                                                    HG  4380
  134         OPP(IE,IF)=SD(IK,IE,IF)                                           HG  4390
              CALL CRSPLT (PNO,OPP,XNE,PNE,G,X,Y,Z,ND,NS,NC)                    HG  4400
  135       CONTINUE                                                          HG  4410
            DO 137 L=1,NC                                                       HG  4420
               DO 136 K=1,25                                                    HG  4430
                  PXP(K)=UTT(L,K)/PNO(L)                                        HG  4440
  136          PYP(K)=PD(L,K)                                                   HG  4450
               CALL MAPA (5,PXP,PYP,1,25,HL,HH,VL,VH,QR,QT,Z,1)                 HG  4460
  137       CALL MAPA (5,PXP,PYP,1,25,HL,HH,VL,VH,QR,QT,Z,1)                  HG  4470
  138 CONTINUE                                                                  HG  4480
                                                                                HG  4490
C                                                                               HG  4500
  139 FORMAT (2X, 16HITERATION FOR N+)                                          HG  4510
  140 FORMAT (8E10.4)                                                           HG  4520
  141 FORMAT (8E10.4)                                                           HG  4530
  142 FORMAT (16I5)                                                             HG  4540
  143 FORMAT (4E10.4,E10.2)                                                     HG  4550
  144 FORMAT (4HV/A=,F6.3,9X,3HED=,F6.3,3HE1=,9X,2HT=,F4.1,9X,4HP/N=,F5.HG  4560
     12)                                                                        HG  4570
  145 FORMAT (///,3X,14HVOLT/AREA(M+1)=,  ,4,5X,14HELEC DEN(M-3)=,E11.4,5HG  4580
     1X,14HELEC TEMP(EV)=,F7.3,5X,16H  E ENERGY(EV)=,F7.3,5X,6HNP/NM=,FHG  4590
     26.5)                                                                      HG  4600
  146 FORMAT (/,10X,10HVOL  (C43)=,E10.3,10X,7HA-NEUT=,E10.3,10X,10HA-+ (HG  4610
     1CM2)=,E10.3,3X,  3HE1=,F6.2,3X,  3HE2=,F6.2)
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```
  147 FORMAT (///,20X,A10,5X,9HNMETA/NO=,F10.5)                          HG  4620
  148 FORMAT (10X,F10.5,21X,2(F10.5,20X))                                HG  4630
  149 FORMAT (///,10X,23HNEUTRAL DENSITY (CM-3)=,E10.3,10X,15HNO. ITERATIHG  4640
     1ONS=,I4)                                                           HG  4650
  150 FORMAT (//,20X,A10,5X,8HNRFS/NO=,F10.6,10X,20HMEAN FREE PATH (CM)=HG  4660
     1,F8.5)                                                             HG  4670
  151 FORMAT (11X,F10.5,20X,3(F10.5,20X))                                HG  4680
  152 FORMAT (10X,F11.4,20X,F11.4,20X,2(E11.4,1X,E9.2,10X))              HG  4690
  153 FORMAT (///,20X,6HN+/NO=,F10.5)                                    HG  4700
  154 FORMAT (10X,3(F10.5,20X))                                          HG  4710
  155 FORMAT (//,20X,A10,5X,10HN+META/NO=,F10.5)                         HG  4720
  156 FORMAT (///,20X,7HN++/NO=,F10.5)                                   HG  4730
  157 FORMAT (10X,4(F10.5,21X),/)                                        HG  4740
  158 FORMAT (10X,F10.5)                                                 HG  4750
  159 FORMAT (/,5X,5HBEAM=,F10.3,3HMA.,10X,12HUTILIZATION=,F10.6,10X,16HHG  4760
     1EV PER BEAM ION=,F10.5,10X,5HIARC=,F10.3)                          HG  4770
  160 FORMAT (/,2X,3HM/A,8(3X,F10.1))                                    HG  4780
  161 FORMAT (/,20X,49HNUMERATOR TERMS (1/SEC) FOLLOWED BY PRI. FRACTIONHG  4790
     1)                                                                  HG  4800
  162 FORMAT (/,20X,52HDENOMINATOR TERMS FOLLOWED BY PRI. FRACTION (IF AHG  4810
     1NY))                                                               HG  4820
  163 FORMAT (10X,F11.4,1X,E9.2)                                         HG  4830
  164 FORMAT (10X,3(F11.4,1X,E9.2,10X))                                  HG  4840
  165 FORMAT (10X,4(E11.4,1X,E9.2,10X))                                  HG  4850
  166 FORMAT (10X,E11.4,20X,F11.4,1X,E9.2,10X)                           HG  4860
  167 FORMAT (10X,F11.4,20X,2(F11.4,1X,E9.2,10X))                        HG  4870
  168 FORMAT (3(F10.4,F10.3),20X)                                        HG  4880
C                                                                        HG  4890
      END                                                                HG  4900

      SUBROUTINE CRSPLT (PNO,UPP,XNE,PNE,G,X,H,Z,ND,NS,NC)               CRS   10
                                                                         CRS   20
C                                                                        CRS   30
C     THIS SUBROUTINE INTERPOLATES IN THE ARRAY DPP TO GET THE VALUES FOCRS   40
C     CONSTANT MASS FLOW RATE (DET. BY PNE) AND THEN PLOTS THE RESULTS   CRS   50
C                                                                        CRS   60
      COMMON /B/ UTL(4,25),PD(4,25),IK                                   CRS   70
      DIMENSION DPP(6,6), PNO(4), XNE(6), PNE(6,10), G(8), PYP(30), PXP(CRS   80
     130), PP(30), SP(10,30), X(8), Y(8), H(4), Z(4)                     CRS   90
C                                                                        CRS  100
C     INTERPOLATION                                                      CRS  110
C                                                                        CRS  120
      DO 102 KZ=1,ND                                                     CRS  130
         DO 101 KY=1,NS                                                  CRS  140
  101    PYP(KY)=DPP(KY,KZ)                                              CRS  150
      DO 102 KX=1,NC                                                     CRS  160
         CALL AITKEN (XNE,PYP,NS,2,PNE(KX,KZ),SP(KX,KZ))                 CRS  170
         IF (PNE(KX,KZ).GT.1.E18) SP(KX,KZ)=0.                           CRS  180
         IF (PNE(KX,KZ).EQ.0.) SP(KX,KZ)=0.                              CRS  190
  102 CONTINUE                                                           CRS  200
C                                                                        CRS  210
C     FIND MAX AND MIN                                                   CRS  220
C                                                                        CRS  230
      DO 104 JO=1,NC                                                     CRS  240
         DO 103 JP=1,ND                                                  CRS  250
  103    PP(JP)=SP(JO,JP)                                                CRS  260
  104 CALL MAPA (G,PP,1,ND,HL,HH,VL,VH,X,Y,II,1)                         CRS  270
C                                                                        CRS  280
C     PLOT OF DATA                                                       CRS  290
C                                                                        CRS  300
      CALL MAPA (1,PXP,PYP,1,25,HL,HH,VL,VH,X,Y,II,1)                    CRS  310
      CALL MAPM (1,PXP,PYP,1,25,HL,HH,VL,VH,X,Y,II,1)                    CRS  320
      DO 112 IA=1,NC                                                     CRS  330
         DO 105 IB=1,ND                                                  CRS  340
  105    PP(IB)=SP(IA,IB)                                                CRS  350
         CALL XINTERP (G,PP,PXP,PYP,ND,25,3)                             CRS  360
         XX=PNO(IA)
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```
      WRITE (6,113) XX                                                  CRS  370
      WRITE (6,114) X(1),H(1)                                           CRS  380
      DO 106 IX=1,25                                                    CRS  390
106   WRITE (6,115) PXP(IX),PYP(IX)                                     CRS  400
      WRITE (6,116) (PNE(IA,II),II=1,ND)                                CRS  410
      WRITE (6,117) (G(IX),PP(IX),IX=1,ND)                              CRS  420
C                                                                       CRS  430
C     PLOT OF UTILIZATION VERSUS DISCHARGE POWER                        CRS  440
C                                                                       CRS  450
      IF (IK.EQ.9.OR.IK.EQ.13) GO TO 107                                CRS  460
      GO TO 111                                                         CRS  470
107   IF (IK.EQ.13) GO TO 109                                           CRS  480
      DO 108 L=1,25                                                     CRS  490
108   UTL(IA,L)=PYP(L)                                                  CRS  500
      GO TO 111                                                         CRS  510
109   DO 110 L=1,25                                                     CRS  520
110   PD(IA,L)=PYP(L)                                                   CRS  530
111   CONTINUE                                                          CRS  540
      CALL MAPM (2,PXP,PYP,1,25,HL,HH,VL,VH,X,H,Z,1)                   CRS  550
112   CALL MAPA (2,PXP,PYP,1,25,HL,HH,VL,VH,X,H,Z,1)                   CRS  560
      CALL MAPA (4,PXP,PYP,1,25,HL,HH,VL,VH,X,H,Z,1)                   CRS  570
      CALL MAPM (4,PXP,PYP,1,25,HL,HH,VL,VH,X,H,Z,1)                   CRS  580
      RETURN                                                            CRS  590
C                                                                       CRS  600
113   FORMAT (//,20X,13HPICKED VALUE=,2X,E10.3,9HMILLIAMPS)             CRS  610
114   FORMAT (/,10X,A10,20X,A10)                                        CRS  620
115   FORMAT (12X,F8.3,18X,F11.4,10X,E11.4)                             CRS  630
116   FORMAT (/,5X,10HELEC.DENS.,8(2X,F10.3))                           CRS  640
117   FORMAT (8F10.3)                                                   CRS  650
C                                                                       CRS  660
      END                                                               CRS  670

      SUBROUTINE XINTERP (X,Y,XI,YI,NIN,NOUT,INTERP)                    INT   10
C                                                                       INT   20
C     THIS SUBROUTINE RETURNS -NOUT- POINTS WHICH ARE INTERPOLATED TO   INT   30
C     THE -INTERP- DEGREE FROM -X,Y-                                    INT   40
C                                                                       INT   50
      DIMENSION X(NIN), Y(NIN), XI(NOUT), YI(NOUT)                      INT   60
      DX=(X(NIN)-X(1))/FLOAT(NOUT-1)                                    INT   70
      XX=X(1)                                                           INT   80
      DO 101 I=1,NOUT                                                   INT   90
         XI(I)=XX                                                       INT  100
         CALL AITKEN (X,Y,NIN,INTERP,XX,YY)                             INT  110
         IF (YY.LT.0.) YY=0.                                            INT  120
         YI(I)=YY                                                       INT  130
101   XX=XX+DX                                                          INT  140
      RETURN                                                            INT  150
C                                                                       INT  160
      END                                                               INT  170

      SUBROUTINE YINTEG (SIGMA,SIGNRG,PROP,SUM,PRSUM)                   NTG   10
C                                                                       NTG   20
C     EVALUATION OF REACTION RATE                                       NTG   30
C                                                                       NTG   40
      COMMON /A/ NP,NM,T,PRINRG                                         NTG   50
      DIMENSION SIGMA(2,21), SIGNRG(2,21), SX(21), SE(21)               NTG   60
      REAL NP,NM                                                        NTG   70
C                                                                       NTG   80
C     PRIMARY ELECTRONS                                                 NTG   90
C                                                                       NTG  100
      DO 101 I=1,21                                                     NTG  110
         SX(I)=SIGMA(1,I)                                               NTG  120
101   SE(I)=SIGNRG(1,I)                                                 NTG  130
      CALL AITKEN (SE,SX,21,2,PRINRG,RX)                                NTG  140
      PSUM=NP*PROP*RX                                                   NTG  150
C                                                                       NTG  160
```

```
C      MAXWELLIAN ELECTRONS                                                NTG  170
C                                                                          NTG  180
       DO 102 I=1,21                                                       NTG  190
          SX(I)=SIGMA(2,I)                                                 NTG  200
  102  SE(I)=SIGNRG(2,I)                                                   NTG  210
       CALL AITKEN (SE,SX,21,2,T,TR)                                       NTG  220
       TSUM=PROP*NM*TR                                                     NTG  230
       SUM=TSUM+PSUM                                                       NTG  240
       PRSUM=PSUM/SUM                                                      NTG  250
       RETURN                                                              NTG  260
C                                                                          NTG  270
       END                                                                 NTG  280

       SUBROUTINE SUMIT (SIG,SGE,N,PROP,TSUM,SUMS,PRSUMS)                  SUM   10
C                                                                          SUM   20
C      TRANSFER ROUTINE FOR EXCITED STATES                                 SUM   30
C                                                                          SUM   40
       COMMON /A/ NP,NM,TEMP,PRINRG                                        SUM   50
       DIMENSION SIG(N,2,21), SGE(N,2,21), PROP(N), SUMS(N), PRSUMS(N), SSUM   60
      1X(2,25), SE(2,25)                                                   SUM   70
       REAL NP,NM                                                          SUM   80
C                                                                          SUM   90
C      CALCULATES THE SUM OF INTERGALS                                     SUM  100
C                                                                          SUM  110
       TSUM=0.                                                             SUM  120
       DO 102 I=1,N                                                        SUM  130
          DO 101 J=1,2                                                     SUM  140
          DO 101 K=1,21                                                    SUM  150
             SX(J,K)=SIG(I,J,K)                                            SUM  160
  101     SE(J,K)=SGE(I,J,K)                                               SUM  170
          CALL YINTEG (SX,SE,PROP(I),SUMS(I),PRSUMS(I))                    SUM  180
  102  TSUM=TSUM+SUMS(I)                                                   SUM  190
       RETURN                                                              SUM  200
C                                                                          SUM  210
       END                                                                 SUM  220

       SUBROUTINE AITKEN (X,Y,N,K,XR,YR)                                   AIT   10
C                                                                          AIT   20
C         **************************************************************AIT   30
C                                                                          AIT   40
C         AITKEN INTERPOLATION SUBROUTINE                                  AIT   50
C      CALLING SEQUENCE...                                                 AIT   60
C      CALL AITKEN(X,Y,N,K,XR,YR)                                          AIT   70
C         X IS A ONE DIMENSIONAL ARRAY OF INDEPENDENT                      AIT   80
C            VARIABLE(INCREASING OR DECREASING)                            AIT   90
C         Y IS A ONE DIMENSIONAL ARRAY OF DEPENDENT                        AIT  100
C            VARIABLE                                                      AIT  110
C         N IS NO. OF X,Y PAIRS                                            AIT  120
C         K IS DEGREE OF INTERPOLATING POLYNOMIAL  (MAX = 10 )             AIT  130
C         XR IS INDEP. VARIABLE ARGUMENT                                   AIT  140
C         YR IS INTERPOLATED RESULT                                        AIT  150
C                                                                          AIT  160
C         **************************************************************AIT  170
C                                                                          AIT  180
C         TYPE, DIMENSION AND LABELED COMMON STATEMENTS                    AIT  190
C                                                                          AIT  200
       DIMENSION X(N), Y(N), XX(11), YY(11)                                AIT  210
       K1=K+1                                                              AIT  220
       IF (X(N)-X(1)) 110,101,101                                          AIT  230
  101  IF (XR-X(1)) 102,102,103                                            AIT  240
  102  LL=0                                                                AIT  250
       GO TO 119                                                           AIT  260
C                                                                          AIT  270
  103  IF (X(N)-XR) 104,104,105                                            AIT  280
  104  LL=N-K1                                                             AIT  290
       GO TO 119                                                           AIT  300
```

```
C
  105 LL=1
      LU=N
  106 IF (LU-LL-1) 117,117,107
  107 LI=(LL+LU)/2
      IF (X(LI)-XB) 108,108,109
  108 LL=LI
      GO TO 106
C
  109 LU=LI
      GO TO 106
C
  110 IF (XB-X(1)) 111,102,102
  111 IF (X(N)-XB) 112,104,104
  112 LL=1
      LU=N
  113 IF (LU-LL-1) 117,117,114
  114 LI=(LL+LU)/2
      IF (X(LI)-XB) 115,116,116
  115 LU=LI
      GO TO 113
C
  116 LL=LI
      GO TO 113
C
  117 LL=LL-(K1+1)/2
      IF (LL) 102,119,118
  118 IF (LL+K1-N) 119,119,104
  119 DO 120 I=1,K1
         I1=LL+I
         XX(I)=X(I1)-XB
  120 YY(I)=Y(I1)
      DO 121 I=1,K
      DO 121 J=I,K
         IF (XX(J+1).EQ.XX(I)) GO TO 122
  121 YY(J+1)=(1./(XX(J+1)-XX(I)))*(YY(I)*XX(J+1)-YY(J+1)*XX(I))
      YB=YY(K1)
      RETURN
  122 WRITE (6,123) (X(I),I=1,N)
      WRITE (6,123) (Y(I),I=1,N)
      WRITE (6,124)
      A=1./(XX(J+1)-XX(I))
      CCC=3.*A
      RETURN
C
  123 FORMAT (10(2X,F10.3))
  124 FORMAT (10X,17HTROUBLE IN AITKEN)
C
      END
&
     5.0000 0.           6.0000 0.           7.0000 0.         HG-HG+    1
     8.0000 0.           9.0000 0.          10.0000 0.         HG-HG+    2
    11.0000  .624E-14   12.0000  .164E-13   14.0000  .339E-13HG-HG+    3
    16.0000  .506E-13   18.0000  .573E-13   20.0000  .837E-13HG-HG+    4
    23.5000  .111E-12   27.0000  .136E-12   30.0000  .153E-12HG-HG+    5
    33.5000  .172E-12   37.0000  .184E-12   40.0000  .198E-12HG-HG+    6
    42.0000  .205E-12   44.0000  .211E-12   50.0000  .230E-12HG-HG+    7
     3.0000  .225E-14    4.0000  .533E-14    4.3000  .792E-14HG-HG+    1
     4.7000  .103E-13    5.0000  .121E-13    5.3000  .141E-13HG-HG+    2
     5.7000  .169E-13    6.0000  .141E-13    6.3000  .214E-13HG-HG+    3
     6.7000  .244E-13    7.0000  .268E-13    7.5000  .308E-13HG-HG+    4
     8.0000  .348E-13    8.5000  .368E-13    9.0000  .429E-13HG-HG+    5
     9.5000  .469E-13   10.0000  .509E-13   10.5000  .549E-13HG-HG+    6
    11.0000  .589E-13   12.0000  .666E-13   13.0000  .740E-13HG-HG+    7
     5.0000 0.           6.0000 0.           7.0000 0.         HG-HG++   1
     8.0000 0.           9.0000 0.          10.0000 0.         HG-HG++   2
```

	11.0000	0.		12.0000	0.		14.0000	0.	HG-HG++ 3
	16.0000	0.		18.0000	0.		20.0000	0.	HG-HG++ 4
	23.5000	0.		27.0000	0.		30.0000	.617E-15	HG-HG++ 5
	33.5000	.190E-14		37.0000	.306E-14		40.0000	.434E-14	HG-HG++ 6
	42.0000	.536E-14		44.0000	.648E-14		50.0000	.103E-13	HG-HG++ 7
	3.0000	.322E-18		4.0000	.429E-17		4.3000	.743E-17	HG-HG++ 1
	4.7000	.139E-16		5.0000	.210E-16		5.3000	.304E-16	HG-HG++ 2
	5.7000	.470E-16		6.0000	.629E-16		6.3000	.922E-16	HG-HG++ 3
	6.7000	.114E-15		7.0000	.142E-15		7.5000	.197E-15	HG-HG++ 4
	8.0000	.265E-15		8.5000	.346E-15		9.0000	.439E-15	HG-HG++ 5
	9.5000	.546E-15		10.0000	.665E-15		10.5000	.797E-15	HG-HG++ 6
	11.0000	.942E-15		12.0000	.127E-14		13.0000	.164E-14	HG-HG++ 7
5.		-3. E-30	7.		-2.5 E-30	8.5		-2. E-30	HG+ - HG++ 1
10.		-1.5 E-30	12.		-1.25E-30	14.		-1. E-30	HG+ - HG++ 2
16.		-.5 E-30	18.6		0. E-30	19.25		.162 E-13	HG+ - HG++ 3
20.		.443 E-13	20.5		.47 E-13	22.		.529 E-13	HG+ - HG++ 4
25.		.657 E-13	27.5		.766 E-13	30.		.854 E-13	HG+ - HG++ 5
33.		.941 E-13	37.		1.02 E-13	40.		1.1 E-13	HG+ - HG++ 6
42.		1.15 E-13	46.		1.24 E-13	50.		1.33 E-13	HG+ - HG++ 7
	3.0000	.267E-15		4.0000	.127E-14		4.3000	.177E-14	HG+ - HG++ 1
	4.7000	.257E-14		5.0000	.327E-14		5.3000	.405E-14	HG+ - HG++ 2
	5.7000	.521E-14		6.0000	.616E-14		6.3000	.717E-14	HG+ - HG++ 3
	6.7000	.860E-14		7.0000	.973E-14		7.5000	.117E-13	HG+ - HG++ 4
	8.0000	.138E-13		8.5000	.159E-13		9.0000	.181E-13	HG+ - HG++ 5
	9.5000	.203E-13		10.0000	.226E-13		10.5000	.249E-13	HG+ - HG++ 6
	11.0000	.272E-13		12.0000	.318E-13		13.0000	.364E-13	HG+ - HG++ 7
	5.0000	.988E-14		6.0000	.121E-13		7.0000	.107E-13	HG-HGM 3P0 1
	8.0000	.907E-14		9.0000	.744E-14		10.0000	.576E-14	HG-HGM 3P0 2
	11.0000	.441E-14		12.0000	.333E-14		14.0000	.228E-14	HG-HGM 3P0 3
	16.0000	.191E-14		18.0000	.123E-14		20.0000	.103E-14	HG-HGM 3P0 4
	23.5000	.827E-15		27.0000	.720E-15		30.0000	.579E-15	HG-HGM 3P0 5
	33.5000	.423E-15		37.0000	.286E-15		40.0000	.188E-15	HG-HGM 3P0 6
	42.0000	0.		44.0000	0.		50.0000	0.	HG-HGM 3P0 7
	3.0000	.319E-14		4.0000	.378E-14		4.3000	.387E-14	HG-HGM 3P0 1
	4.7000	.395E-14		5.0000	.398E-14		5.3000	.399E-14	HG-HGM 3P0 2
	5.7000	.398E-14		6.0000	.396E-14		6.3000	.393E-14	HG-HGM 3P0 3
	6.7000	.388E-14		7.0000	.384E-14		7.5000	.376E-14	HG-HGM 3P0 4
	8.0000	.367E-14		8.5000	.358E-14		9.0000	.348E-14	HG-HGM 3P0 5
	9.5000	.339E-14		10.0000	.330E-14		10.5000	.321E-14	HG-HGM 3P0 6
	11.0000	.312E-14		12.0000	.294E-14		13.0000	.278E-14	HG-HGM 3P0 7
	5.0000	0.		6.0000	.437E-13		7.0000	.482E-13	HG-HGM 3P2 1
	8.0000	.404E-13		9.0000	.329E-13		10.0000	.260E-13	HG-HGM 3P2 2
	11.0000	.212E-13		12.0000	.185E-13		14.0000	.136E-13	HG-HGM 3P2 3
	16.0000	.105E-13		18.0000	.799E-14		20.0000	.655E-14	HG-HGM 3P2 4
	23.5000	.433E-14		27.0000	.312E-14		30.0000	.227E-14	HG-HGM 3P2 5
	33.5000	.168E-14		37.0000	.144E-14		40.0000	.131E-14	HG-HGM 3P2 6
	42.0000	.113E-14		44.0000	.916E-15		50.0000	.365E-15	HG-HGM 3P2 7
	3.0000	.963E-14		4.0000	.125E-13		4.3000	.130E-13	HG-HGM 3P2 1
	4.7000	.135E-13		5.0000	.138E-13		5.3000	.140E-13	HG-HGM 3P2 2
	5.7000	.142E-13		6.0000	.143E-13		6.3000	.143E-13	HG-HGM 3P2 3
	6.7000	.143E-13		7.0000	.142E-13		7.5000	.141E-13	HG-HGM 3P2 4
	8.0000	.139E-13		8.5000	.137E-13		9.0000	.134E-13	HG-HGM 3P2 5
	9.5000	.132E-13		10.0000	.129E-13		10.5000	.126E-13	HG-HGM 3P2 6
	11.0000	.123E-13		12.0000	.118E-13		13.0000	.112E-13	HG-HGM 3P2 7
	5.0000	.948E-14		6.0000	.361E-14		7.0000	.462E-14	HG-HGM 3P1 1
	8.0000	.410E-14		9.0000	.694E-14		10.0000	.719E-14	HG-HGM 3P1 2
	11.0000	.708E-14		12.0000	.606E-14		14.0000	.528E-14	HG-HGM 3P1 3
	16.0000	.640E-14		18.0000	.641E-14		20.0000	.633E-14	HG-HGM 3P1 4
	23.5000	.595E-14		27.0000	.527E-14		30.0000	.444E-14	HG-HGM 3P1 5
	33.5000	.319E-14		37.0000	.161E-14		40.0000	0.	HG-HGM 3P1 6
	42.0000	0.		44.0000	0.		50.0000	0.	HG-HGM 3P1 7
	3.0000	.234E-14		4.0000	.315E-14		4.3000	.334E-14	HG-HGM 3P1 1
	4.7000	.356E-14		5.0000	.371E-14		5.3000	.384E-14	HG-HGM 3P1 2
	5.7000	.398E-14		6.0000	.408E-14		6.3000	.416E-14	HG-HGM 3P1 3
	6.7000	.426E-14		7.0000	.432E-14		7.5000	.434E-14	HG-HGM 3P1 4
	8.0000	.445E-14		8.5000	.446E-14		9.0000	.451E-14	HG-HGM 3P1 5

9.5000	.452E-14	10.0000	.451E-14	10.5000	.450E-14	HG-HGR 3P1 6
11.0000	.448E-14	12.0000	.441E-14	13.0000	.433E-14	HG-HGR 3P1 7
5.0000	0.	6.0000	0.	7.0000	.132E-13	HG-HGR 1P1 1
8.0000	.349E-13	9.0000	.579E-13	10.0000	.818E-13	HG-HGR 1P1 2
11.0000	.106E-12	12.0000	.130E-12	14.0000	.162E-12	HG-HGR 1P1 3
16.0000	.179E-12	18.0000	.189E-12	20.0000	.194E-12	HG-HGR 1P1 4
23.5000	.209E-12	27.0000	.215E-12	30.0000	.220E-12	HG-HGR 1P1 5
33.5000	.223E-12	37.0000	.224E-12	40.0000	.224E-12	HG-HGR 1P1 6
42.0000	.224E-12	44.0000	.224E-12	50.0000	.222E-12	HG-HGR 1P1 7
3.0000	.166E-13	4.0000	.323E-13	4.3000	.372E-13	HG-HGR 1P1 1
4.7000	.437E-13	5.0000	.485E-13	5.3000	.532E-13	HG-HGR 1P1 2
5.7000	.594E-13	6.0000	.636E-13	6.3000	.682E-13	HG-HGR 1P1 3
6.7000	.737E-13	7.0000	.777E-13	7.5000	.841E-13	HG-HGR 1P1 4
8.0000	.901E-13	8.5000	.957E-13	9.0000	.101E-12	HG-HGR 1P1 5
9.5000	.106E-12	10.0000	.111E-12	10.5000	.115E-12	HG-HGR 1P1 6
11.0000	.119E-12	12.0000	.127E-12	13.0000	.133E-12	HG-HGR 1P1 7
5.0000	0.	6.0000	.951E-15	7.0000	.865E-14	HGM-HG+3P0 1
8.0000	.233E-13	9.0000	.355E-13	10.0000	.477E-13	HGM-HG+3P0 2
11.0000	.590E-13	12.0000	.701E-13	14.0000	.916E-13	HGM-HG+3P0 3
16.0000	.110E-12	18.0000	.130E-12	20.0000	.157E-12	HGM-HG+3P0 4
23.5000	.205E-12	27.0000	.247E-12	30.0000	.278E-12	HGM-HG+3P0 5
33.5000	.309E-12	37.0000	.335E-12	40.0000	.354E-12	HGM-HG+3P0 6
42.0000	.365E-12	44.0000	.376E-12	50.0000	.400E-12	HGM-HG+3P0 7
3.0000	.103E-13	4.0000	.210E-13	4.3000	.247E-13	HGM-HG+3P0 1
4.7000	.294E-13	5.0000	.338E-13	5.3000	.378E-13	HGM-HG+3P0 2
5.7000	.433E-13	6.0000	.476E-13	6.3000	.518E-13	HGM-HG+3P0 3
6.7000	.575E-13	7.0000	.613E-13	7.5000	.690E-13	HGM-HG+3P0 4
8.0000	.762E-13	8.5000	.834E-13	9.0000	.904E-13	HGM-HG+3P0 5
9.5000	.974E-13	10.0000	.104E-12	10.5000	.111E-12	HGM-HG+3P0 6
11.0000	.118E-12	12.0000	.131E-12	13.0000	.143E-12	HGM-HG+3P0 7
5.0000	.682E-15	6.0000	.958E-14	7.0000	.312E-13	HGM-HG+3P2 1
8.0000	.484E-13	9.0000	.652E-13	10.0000	.798E-13	HGM-HG+3P2 2
11.0000	.929E-13	12.0000	.105E-12	14.0000	.129E-12	HGM-HG+3P2 3
16.0000	.149E-12	18.0000	.169E-12	20.0000	.197E-12	HGM-HG+3P2 4
23.5000	.244E-12	27.0000	.285E-12	30.0000	.316E-12	HGM-HG+3P2 5
33.5000	.346E-12	37.0000	.371E-12	40.0000	.390E-12	HGM-HG+3P2 6
42.0000	.400E-12	44.0000	.410E-12	50.0000	.433E-12	HGM-HG+3P2 7
3.0000	.176E-13	4.0000	.327E-13	4.3000	.375E-13	HGM-HG+3P2 1
4.7000	.442E-13	5.0000	.492E-13	5.3000	.542E-13	HGM-HG+3P2 2
5.7000	.610E-13	6.0000	.661E-13	6.3000	.711E-13	HGM-HG+3P2 3
6.7000	.778E-13	7.0000	.828E-13	7.5000	.911E-13	HGM-HG+3P2 4
8.0000	.992E-13	8.5000	.107E-12	9.0000	.115E-12	HGM-HG+3P2 5
9.5000	.123E-12	10.0000	.130E-12	10.5000	.138E-12	HGM-HG+3P2 6
11.0000	.145E-12	12.0000	.159E-12	13.0000	.172E-12	HGM-HG+3P2 7
5.0000	0.	6.0000	0.	7.0000	0.	HGM+ HG++5 1
8.0000	0.	9.0000	0.	10.0000	0.	HGM+ HG++5 2
11.0000	0.	12.0000	0.	14.0000	0.	HGM+ HG++5 3
16.0000	0.	18.0000	0.	20.0000	.625E-15	HGM+ HG++5 4
23.5000	.722E-14	27.0000	.238E-13	30.0000	.396E-13	HGM+ HG++5 5
33.5000	.576E-13	37.0000	.744E-13	40.0000	.877E-13	HGM+ HG++5 6
42.0000	.959E-13	44.0000	.104E-12	50.0000	.124E-12	HGM+ HG++5 7
3.0000	.379E-15	4.0000	.261E-15	4.3000	.397E-15	HGM+ HG++5 1
4.7000	.642E-15	5.0000	.379E-15	5.3000	.116E-14	HGM+ HG++5 2
5.7000	.162E-14	6.0000	.202E-14	6.3000	.247E-14	HGM+ HG++5 3
6.7000	.315E-14	7.0000	.372E-14	7.5000	.477E-14	HGM+ HG++5 4
8.0000	.594E-14	8.5000	.722E-14	9.0000	.860E-14	HGM+ HG++5 5
9.5000	.101E-13	10.0000	.115E-13	10.5000	.134E-13	HGM+ HG++5 6
11.0000	.149E-13	12.0000	.169E-13	14.0000	.221E-13	HGM+ HG++5 7
5.0000	0.	6.0000	0.	7.0000	0.	HGM+-HG++3 1
8.0000	0.	9.0000	0.	10.0000	0.	HGM+-HG++3 2
11.0000	0.	12.0000	0.	14.0000	0.	HGM+-HG++3 3
16.0000	0.	18.0000	0.	20.0000	.625E-15	HGM+-HG++3 4
23.5000	.722E-14	27.0000	.238E-13	30.0000	.396E-13	HGM+-HG++3 5
33.5000	.576E-13	37.0000	.744E-13	40.0000	.877E-13	HGM+-HG++3 6
42.0000	.959E-13	44.0000	.104E-12	50.0000	.124E-12	HGM+-HG++3 7
3.0000	.379E-15	4.0000	.261E-15	4.3000	.397E-15	HGM+-HG++3 1

4.7000	.642E-15	5.0000	.879E-15	5.3000	.116E-14HGM+-HG++3	2
5.7000	.162E-14	6.0000	.202E-14	6.3000	.247E-14HGM+-HG++3	3
6.7000	.315E-14	7.0000	.372E-14	7.5000	.477E-14HGM+-HG++3	4
8.0000	.594E-14	8.5000	.722E-14	9.0000	.860E-14HGM+-HG++3	5
9.5000	.101E-13	10.0000	.116E-13	10.5000	.133E-13HGM+-HG++3	6
11.0000	.149E-13	12.0000	.185E-13	13.0000	.221E-13HGM+-HG++3	7
5.0000	0.	6.0000	.221E-14	7.0000	.129E-13HGR-HG+3P1	1
8.0000	.289E-13	9.0000	.423E-13	10.0000	.553E-13HGR-HG+3P1	2
11.0000	.670E-13	12.0000	.787E-13	14.0000	.101E-12HGR-HG+3P1	3
16.0000	.119E-12	18.0000	.140E-12	20.0000	.167E-12HGR-HG+3P1	4
23.5000	.214E-12	27.0000	.256E-12	30.0000	.288E-12HGR-HG+3P1	5
33.5000	.318E-12	37.0000	.344E-12	40.0000	.363E-12HGR-HG+3P1	6
42.0000	.374E-12	44.0000	.384E-12	50.0000	.409E-12HGR-HG+3P1	7
3.0000	.119E-13	4.0000	.237E-13	4.3000	.276E-13HGR-HG+3P1	1
4.7000	.331E-13	5.0000	.373E-13	5.3000	.416E-13HGR-HG+3P1	2
5.7000	.475E-13	6.0000	.519E-13	6.3000	.563E-13HGR-HG+3P1	3
6.7000	.623E-13	7.0000	.668E-13	7.5000	.743E-13HGR-HG+3P1	4
8.0000	.817E-13	8.5000	.890E-13	9.0000	.963E-13HGR-HG+3P1	5
9.5000	.103E-12	10.0000	.111E-12	10.5000	.117E-12HGR-HG+3P1	6
11.0000	.124E-12	12.0000	.137E-12	13.0000	.150E-12HGR-HG+3P1	7
5.0000	0.	6.0000	0.	7.0000	.793E-15HGR-HG+1P1	1
8.0000	.628E-14	9.0000	.161E-13	10.0000	.246E-13HGR-HG+1P1	2
11.0000	.338E-13	12.0000	.435E-13	14.0000	.625E-13HGR-HG+1P1	3
16.0000	.793E-13	18.0000	.988E-13	20.0000	.126E-12HGR-HG+1P1	4
23.5000	.173E-12	27.0000	.215E-12	30.0000	.247E-12HGR-HG+1P1	5
33.5000	.279E-12	37.0000	.305E-12	40.0000	.325E-12HGR-HG+1P1	6
42.0000	.336E-12	44.0000	.347E-12	50.0000	.372E-12HGR-HG+1P1	7
3.0000	.585E-14	4.0000	.134E-13	4.3000	.162E-13HGR-HG+1P1	1
4.7000	.201E-13	5.0000	.233E-13	5.3000	.265E-13HGR-HG+1P1	2
5.7000	.311E-13	6.0000	.346E-13	6.3000	.382E-13HGR-HG+1P1	3
6.7000	.431E-13	7.0000	.468E-13	7.5000	.531E-13HGR-HG+1P1	4
8.0000	.595E-13	8.5000	.659E-13	9.0000	.722E-13HGR-HG+1P1	5
9.5000	.786E-13	10.0000	.849E-13	10.5000	.912E-13HGR-HG+1P1	6
11.0000	.974E-13	12.0000	.109E-12	13.0000	.121E-12HGR-HG+1P1	7
5.0000	.174E-13	6.0000	.188E-13	7.0000	.200E-13HG+-HGM+ 5	1
8.0000	.210E-13	9.0000	.219E-13	10.0000	.227E-13HG+-HGM+ 5	2
11.0000	.235E-13	12.0000	.241E-13	14.0000	.250E-13HG+-HGM+ 5	3
16.0000	.216E-13	18.0000	.190E-13	20.0000	.168E-13HG+-HGM+ 5	4
23.5000	.137E-13	27.0000	.113E-13	30.0000	.967E-14HG+-HGM+ 5	5
33.5000	.817E-14	37.0000	.677E-14	40.0000	.613E-14HG+-HGM+ 5	6
42.0000	.564E-14	44.0000	.521E-14	50.0000	.416E-14HG+-HGM+ 5	7
3.0000	.806E-14	4.0000	.108E-13	4.3000	.114E-13HG+-HGM+ 5	1
4.7000	.122E-13	5.0000	.126E-13	5.3000	.130E-13HG+-HGM+ 5	2
5.7000	.135E-13	6.0000	.138E-13	6.3000	.140E-13HG+-HGM+ 5	3
6.7000	.143E-13	7.0000	.144E-13	7.5000	.146E-13HG+-HGM+ 5	4
8.0000	.147E-13	8.5000	.148E-13	9.0000	.148E-13HG+-HGM+ 5	5
9.5000	.148E-13	10.0000	.146E-13	10.5000	.147E-13HG+-HGM+ 5	6
11.0000	.146E-13	12.0000	.144E-13	13.0000	.141E-13HG+-HGM+ 5	7
5.0000	0.	6.0000	0.	7.0000	.154E-13HG+-HGM+ 3	1
8.0000	.164E-13	9.0000	.173E-13	10.0000	.181E-13HG+-HGM+ 3	2
11.0000	.188E-13	12.0000	.196E-13	14.0000	[illegible]HG+-HGM+ 3	3
16.0000	.180E-13	18.0000	.157E-13	20.0000	.135E-13HG+-HGM+ 3	4
23.5000	.112E-13	27.0000	.922E-14	30.0000	.789E-14HG+-HGM+ 3	5
33.5000	.664E-14	37.0000	.565E-14	40.0000	.496E-14HG+-HGM+ 3	6
42.0000	.456E-14	44.0000	.421E-14	50.0000	.345E-14HG+-HGM+ 3	7
3.0000	.413E-14	4.0000	[illegible]	4.3000	.688E-14HG+-HGM+ 3	1
4.7000	.754E-14	5.0000	.747E-14	5.3000	[illegible]HG+-HGM+ 3	2
5.7000	.882E-14	6.0000	.911E-14	6.3000	[illegible]HG+-HGM+ 3	3
6.7000	.967E-14	7.0000	[illegible]	7.5000	.101E-13HG+-HGM+ 3	4
8.0000	.103E-13	8.5000	[illegible]	9.0000	.106E-13HG+-HGM+ 3	5
9.5000	.106E-13	10.0000	.107E-13	10.5000	.107E-13HG+-HGM+ 3	6
11.0000	.107E-13	12.0000	[illegible]	13.0000	[illegible]HG+-HGM+ 3	7

1 1 6 1

1. 1. 400. [illegible] [illegible]

.014 4.2 27.5 [illegible] [illegible] [illegible]

APPENDIX B

The computer program "PROP" is listed below. It can be used to determine the values of the volume averaged plasma properties and the uniformity factors needed by the computer program "HG." The data needed to determine these quantities is obtained from a Langmuir probe survey of the discharge chamber in which the plasma properties are determined at many different locations within the chamber. This data is used to numerically evaluate Equations (10) to (15) and (21) yielding the volume averaged plasma properties and the uniformity factors. Comment cards are included in the computer program to indicate the purpose of each section. A CDC 6400 computer will use approximately thirty seconds of Central Processor time to evaluate five sets of data obtained from five Langmuir probe surveys.

```
      PROGRAM PROP (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                PRP   10
C                                                                         PRP   20
C     THIS PROGRAM CALCULATES THE AVERAGE PROPERTIES                      PRP   30
C     PLS-RADIAL POSITION OF LANGMUIR PROBE POINTS (1-CENTERLINE)         PRP   40
C     POS-POS. OF DESIRED DATA POINTS IF NR .NE. 4                        PRP   50
C     POSZ-AXIAL POSITION OF LANGMUIR PROBE POSITIONS (1-UPSTREAM PNT.)   PRP   60
C     NT-NO. OF TRACES PER SET      NR-NO. OF RADIAL POINTS               PRP   70
C     IFLAG=1 IF ONE WANTS TO PRODUCE A SET OF POINTS UPSTREAM            PRP   80
C     IFLAG=2 POINTS PRODUCED AT THE BAFFLE                               PRP   90
C     DIS-DISTANCE FROM SCREEN TO POINT WHERE THE GENERATED SET IS TO     PRP  100
C     BE PLACED                                                           PRP  110
C     RCATH-CATHODE RADIUS                                                PRP  120
C     ENP-LENGTH OF PRIMARY ELECTRON REGION AT CENTERLINE                 PRP  130
C                                                                         PRP  140
      DIMENSION XNP(72), XNM(72), T(70), Z(70), V(50), ZI(21), G(21), F(PRP  150
     121), TF(21), AA(5), BB(5), CC(5), DD(5), EE(5), FF(5), END(11), POPRP  160
     2S(5), W(11), POSZ(11), YT(11), YP(11), YM(11), YZ(11), POR(10), AAPRP  170
     3A(10), VF(51)                                                       PRP  180
      DIMENSION AT(70), AZ(70), ANP(70), ANM(70), A(11), B(11), C(11), DPRP  190
     1(11), E(11), VT(11), PLS(5)                                         PRP  200
      DATA AA,BB,CC,DD,EE/25*0./                                          PRP  230
      DATA POS /0.,1.,1.2,2.2,3.3,4./                                     PRP  210
      DATA PLS /0.,1.,2.,3.,3.5/                                          PRP  220
C                                                                         PRP  240
C     INTEGRATED CROSS SECTIONS FOR + TO ++                               PRP  250
C                                                                         PRP  260
      READ (5,129) (ZI(I),F(I),I=1,21)                                    PRP  270
      READ (5,129) (TF(I),G(I),I=1,21)                                    PRP  280
      WRITE (6,133) (ZI(I),F(I),I=1,21)                                   PRP  290
      WRITE (6,133) (TF(I),G(I),I=1,21)                                   PRP  300
      PI=6.2832                                                           PRP  310
  101 READ (5,126) NR,NT,IFLAG,DIS,RCATH,ENP                              PRP  320
      IF (EOF(5)) 125,102,125                                             PRP  330
  102 ND=NT/NR                                                            PRP  340
      NN=ND                                                               PRP  350
      IF (IFLAG.NE.0) NN=ND+1                                             PRP  360
      READ (5,127) (POSZ(I),I=1,ND)                                       PRP  370
C                                                                         PRP  380
C     RADIAL DISTANCE TO CRITICAL FIELD LINE                              PRP  390
C                                                                         PRP  400
      READ (5,127) (END(I),I=1,NN)                                        PRP  410
C                                                                         PRP  420
C     READ IN THE PROPERTIES                                              PRP  430
C                                                                         PRP  440
      DO 103 I=1,NT                                                       PRP  450
  103 READ (5,130) T(I),Z(I),XNP(I),XNM(I)                                PRP  460
      IF (NR.EQ.4) GO TO 104                                              PRP  470
      IS=0                                                                PRP  480
      DO 106 I=1,NT,NR                                                    PRP  490
         NND=I+NR-1                                                       PRP  500
         DO 104 J=I,NND                                                   PRP  510
            K=J+1-I                                                       PRP  520
            YT(K)=T(J)                                                    PRP  530
            YP(K)=XNP(J)                                                  PRP  540
            YM(K)=XNM(J)                                                  PRP  550
  104       YZ(K)=Z(J)                                                    PRP  560
         DO 105 J=1,4                                                     PRP  570
            L=J+IS                                                        PRP  580
            CALL AITKEN (PLS,YT,5,1,POS(J),AT(L))                         PRP  590
            CALL AITKEN (PLS,YP,5,1,POS(J),ANP(L))                        PRP  600
            CALL AITKEN (PLS,YM,5,1,POS(J),ANM(L))                        PRP  610
  105       CALL AITKEN (PLS,YZ,5,1,POS(J),AZ(L))                         PRP  620
  106 IS=IS+4                                                             PRP  630
      NT=NT*4/NR                                                          PRP  640
      NC=NT/4                                                             PRP  650
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```
      DO 107 J=1,NT                                                    PRP  660
         T(J)=AT(J)                                                    PRP  670
         Z(J)=AZ(J)                                                    PRP  680
         XNP(J)=ANP(J)                                                 PRP  690
  107 XNM(J)=ANM(J)                                                    PRP  700
      WRITE (6,130) (T(N),Z(N),XNP(N),XNM(N),N=1,NT)                   PRP  710
  108 IF (IFLAG.EQ.0) GO TO 114                                        PRP  720
C                                                                      PRP  730
C     CALCULATION OF THE EXTRA SET OF POINTS                           PRP  740
C                                                                      PRP  750
      N=1                                                              PRP  760
      DO 110 I=1,4                                                     PRP  770
         L=0                                                           PRP  780
         DO 109 J=I,NT,4                                               PRP  790
            L=L+1                                                      PRP  800
            YT(L)=T(J)                                                 PRP  810
            YP(L)=XNP(J)                                               PRP  820
            YM(L)=XNM(J)                                               PRP  830
  109       YZ(L)=Z(J)                                                 PRP  840
         K=NT+I                                                        PRP  850
         CALL AITKEN (POSZ,YT,ND,N,DIS,AT(K))                          PRP  860
         K=NT+I                                                        PRP  850
         CALL AITKEN (POSZ,YT,ND,N,DIS,AT(K))                          PRP  860
         CALL AITKEN (POSZ,YZ,ND,N,DIS,AZ(K))                          PRP  870
         CALL AITKEN (POSZ,YP,ND,N,DIS,ANP(K))                         PRP  880
         CALL AITKEN (POSZ,YM,ND,N,DIS,ANM(K))                         PRP  890
         IF (AT(K).LT.0.) AT(K)=0.0                                    PRP  900
         IF (AZ(K).LT.0.) AZ(K)=0.0                                    PRP  910
         IF (ANP(K).LT.0.) ANP(K)=0.0                                  PRP  920
         IF (ANM(K).LT.0.) ANM(K)=0.0                                  PRP  930
         WRITE (6,128) AT(K),AZ(K),ANP(K),ANM(K)                       PRP  940
  110 CONTINUE                                                         PRP  950
C                                                                      PRP  960
C     RESHUFFLING OF POINTS                                            PRP  970
C                                                                      PRP  980
      ND=ND+1                                                          PRP  990
      NNN=ND-1                                                         PRP 1000
      XK=END(ND)                                                       PRP 1010
      DO 111 I=1,NNN                                                   PRP 1020
         J=ND+1-I                                                      PRP 1030
         END(J)=END(J-1)                                               PRP 1040
  111 POSZ(J)=POSZ(J-1)                                                PRP 1050
      END(1)=XK                                                        PRP 1060
      POSZ(1)=DIS                                                      PRP 1070
      NT=NT+4                                                          PRP 1080
      ND=NT/4                                                          PRP 1090
      DO 112 I=5,NT                                                    PRP 1100
         K=NT+5-I                                                      PRP 1110
         J=K-4                                                         PRP 1120
         T(K)=T(J)                                                     PRP 1130
         Z(K)=Z(J)                                                     PRP 1140
         XNP(K)=XNP(J)                                                 PRP 1150
  112 XNM(K)=XNM(J)                                                    PRP 1160
      NTM=NT-3                                                         PRP 1170
      DO 113 I=NTM,NT                                                  PRP 1180
         J=I-NTM+1                                                     PRP 1190
         T(J)=AT(I)                                                    PRP 1200
         Z(J)=AZ(I)                                                    PRP 1210
         XNP(J)=ANP(I)                                                 PRP 1220
  113 XNM(J)=ANM(I)                                                    PRP 1230
C                                                                      PRP 1240
C     START OF THE CALCULATIONS                                        PRP 1250
C                                                                      PRP 1260
  114 DO 1?3 I=1,ND                                                    PRP 1270
         J=4*(I-1)+1                                                   PRP 1280
         K=J+3                                                         PRP 1290
```

```
C                                                                       PRP 1300
C      THIS PRODUCES AN ARRAY OF F(Z) AT CONSTANT Z POINTS              PRP 1310
C                                                                       PRP 1320
       DO 117 J=L,K                                                     PRP 1330
          M=J-L+1                                                       PRP 1340
          R=POS(M)*PI                                                   PRP 1350
          CALL ATTREN (TF,G,21,2,T(J),GI)                               PRP 1360
          CALL ATTREN (ZI,F,21,2,Z(J),FI)                               PRP 1370
          XT=XNP(J)+XNM(J)                                              PRP 1380
          IF (XNM(J).EQ.0.0) GO TO 115                                  PRP 1390
          TERM=1.0+XNP(J)/XNM(J)                                        PRP 1400
          GO TO 116                                                     PRP 1410
  115     TERM=1.0                                                      PRP 1420
  116     VF(J)=SQRT(T(J)*4.ROBE9*TERM)                                 PRP 1430
          AA(M)=XNP(J)*XT*FI*R                                          PRP 1440
          BB(M)=XT*R                                                    PRP 1450
          CC(M)=XT*R*XNP(J)                                             PRP 1460
          DD(M)=XNM(J)*XT*GI*R                                          PRP 1470
          EE(M)=XNM(J)*XT*R                                             PRP 1480
  117     FF(M)=R                                                       PRP 1490
C                                                                       PRP 1500
C      INTEGRATION OF F YIELDING G(Z)                                   PRP 1510
C                                                                       PRP 1520
          M=4                                                           PRP 1530
          NM=19                                                         PRP 1540
          CALL INTR (POS,AA,M,NM,2,END(I),H)                            PRP 1550
          A(I)=H                                                        PRP 1560
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 118                           PRP 1570
          CALL INTR (POS,AA,M,NM,2,RCATH,HM)                            PRP 1580
          A(I)=A(I)+HM                                                  PRP 1590
  118     CALL INTR (POS,FF,M,NM,2,END(I),H)                            PRP 1600
          B(I)=H                                                        PRP 1610
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 119                           PRP 1620
          CALL INTR (POS,BB,M,NM,2,RCATH,HM)                            PRP 1630
          B(I)=B(I)+HM                                                  PRP 1640
  119     CALL INTR (POS,CC,M,NM,2,END(I),H)                            PRP 1650
          C(I)=H                                                        PRP 1660
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 120                           PRP 1670
          CALL INTR (POS,CC,M,NM,2,RCATH,HM)                            PRP 1680
          C(I)=C(I)+HM                                                  PRP 1690
  120     CALL INTR (POS,DD,M,NM,2,END(I),H)                            PRP 1700
          D(I)=H                                                        PRP 1710
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 121                           PRP 1720
          CALL INTR (POS,DD,M,NM,2,RCATH,HM)                            PRP 1730
          D(I)=D(I)+HM                                                  PRP 1740
  121     CALL INTR (POS,EE,M,NM,2,END(I),H)                            PRP 1750
          E(I)=H                                                        PRP 1760
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 122                           PRP 1770
          CALL INTR (POS,EE,M,NM,2,RCATH,HM)                            PRP 1780
          E(I)=E(I)+HM                                                  PRP 1790
  122     CALL INTR (POS,FF,M,NM,2,END(I),H)                            PRP 1800
          VT(I)=H                                                       PRP 1810
          IF (IFLAG.NE.1.OR.I.NE.1) GO TO 123                           PRP 1820
          CALL INTR (POS,FF,M,NM,2,RCATH,HM)                            PRP 1830
          VT(I)=VT(I)+HM                                                PRP 1840
  123 CONTINUE                                                          PRP 1850
C                                                                       PRP 1860
C      INTEGRATION TO Z DIRECTION                                       PRP 1870
C                                                                       PRP 1880
          M=NP                                                          PRP 1890
          CALL [illegible]                                              PRP 1900
          CALL ATTREN [illegible]                                       PRP 1910
          CALL ATTREN [illegible]                                       PRP 1920
          AA=A+A                                                        PRP 1930
          CALL [illegible]                                              PRP 1940
          CALL ATTREN [illegible]                                       PRP 1950
```

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      CALL AITKEN (POSZ,V,ND,2,0.0,RR)                                  PRP 1960
      QR=QR-RR                                                          PRP 1970
      CALL QTEG (POSZ,C,V,ND)                                           PRP 1980
      CALL AITKEN (POSZ,V,ND,2,ENP,QC)                                  PRP 1990
      CALL AITKEN (POSZ,V,ND,2,0.0,RC)                                  PRP 2000
      QC=QC-RC                                                          PRP 2010
      CALL QTEG (POSZ,D,V,ND)                                           PRP 2020
      CALL AITKEN (POSZ,V,ND,2,ENP,QD)                                  PRP 2030
      CALL AITKEN (POSZ,V,ND,2,0.0,RD)                                  PRP 2040
      QD=QD-RD                                                          PRP 2050
      CALL QTEG (POSZ,E,V,ND)                                           PRP 2060
      CALL AITKEN (POSZ,V,ND,2,ENP,QE)                                  PRP 2070
      CALL AITKEN (POSZ,V,ND,2,0.0,RE)                                  PRP 2080
      QE=QE-RE                                                          PRP 2090
      CALL QTEG (POSZ,VT,V,ND)                                          PRP 2100
      CALL AITKEN (POSZ,V,ND,2,ENP,QVT)                                 PRP 2110
      CALL AITKEN (POSZ,V,ND,2,0.0,RVT)                                 PRP 2120
      QVT=QVT-RVT                                                       PRP 2130
C                                                                       PRP 2140
C     CALCULATION OF PROPERTIES                                         PRP 2150
C                                                                       PRP 2160
      FZI=QA/QC                                                         PRP 2170
      GTE=QD/QE                                                         PRP 2180
      ANE=QR/QVT                                                        PRP 2190
      AP=QC/QR                                                          PRP 2200
      AM=QE/QR                                                          PRP 2210
      G=QC/QE                                                           PRP 2220
      XP=SQRT((AP*ANE)/(1.+1./G))                                       PRP 2230
      XM=SQRT((AM*ANE)/(1.+G))                                          PRP 2240
      XNE=XP+XM                                                         PRP 2250
      CALL AITKEN (G,TE,21,1,GTE,TES)                                   PRP 2260
      CALL AITKEN (E,ZI,21,1,EZI,ZIS)                                   PRP 2270
      R=XP/XM                                                           PRP 2280
      WRITE (6,133) QA,QR,QC,QD,QE                                      PRP 2290
      WRITE (6,131) XP,XM,R,TES,ZIS                                     PRP 2300
      WRITE (6,132) XNE,QVT                                             PRP 2310
      VEA=SQRT(TES*4.803E9*(1.+R))                                      PRP 2320
C                                                                       PRP 2330
C     FIND NONUNIFORMITY FACTORS                                        PRP 2340
C                                                                       PRP 2350
      CALL AREA (XNP,XNM,XNE,POS,POSZ,ND,RCATH,VE,VEA)                  PRP 2360
      IF (IFLAG.EQ.0) GO TO 124                                         PRP 2370
      NT=NT-4                                                           PRP 2380
      ND=NT/4                                                           PRP 2390
  124 CONTINUE                                                          PRP 2400
      GO TO 101                                                         PRP 2410
  125 STOP                                                              PRP 2420
C                                                                       PRP 2430
  126 FORMAT (3I5,4E10.4)                                               PRP 2440
  127 FORMAT (8E10.6)                                                   PRP 2450
  128 FORMAT (10X,2E10.4,2E10.3)                                        PRP 2460
  129 FORMAT (3(E10.4,E10.3),20X)                                       PRP 2470
  130 FORMAT (10X,2E10.4,2E10.3)                                        PRP 2480
  131 FORMAT (2X, 11HNP (CM-3) =,E11.4,3X, 11HNM (CM-3) =,E11.4,3X,  6HNE/NP 2490
     1P/NM=,E10.4,3X,  8HT (EV) =,E10.4,3X, 13HP/I, EN,(EV)=,E10.4)     PRP 2500
  132 FORMAT (10X, 13HEFFECT, ENS,=,E11.4,5X,  9HVT (CM3)=,E10.4/)      PRP 2510
  133 FORMAT (/8(E11.4,2X))                                             PRP 2520
C                                                                       PRP 2530
      END                                                               PRP 2540

      SUBROUTINE INTG (X,Y,SIN,NUSED,INT,XP,YP)                         INTG   10
C                                                                       INTG   20
C     THIS SUBROUTINE INTEGRATES Y(X) AND THEN FINDS THE VALUE OF THE   INTG   30
C     INTEGRAL AT XP                                                    INTG   40
C                                                                       INTG   50
      DIMENSION X(SIN), Y(SIN), A(100), B(100), C(100)                  INTG   60
```

```
      CALL XINTERP (X,Y,A,R,NIN,NUSED,INT)
      CALL GTEG (A,B,C,NUSED)
      CALL AITKEN (A,C,NUSED,INT,XP,YP)
      RETURN
C
      END
      SUBROUTINE AREA (XNP,XNM,XNEE,DTR,DTY,NLN,R,VE,VEA)
C
C     THIS SUBROUTINE CALCULATES THE PLASMA NONUNIFORMITY FACTORS
C     PRY=POINTS (Z) DEFINING PRIMARY FIELD LINE (FROM SCREEN GRID)
C     PRR=POINTS (R) DEFINING PRIMARY FIELD LINE (FROM CENTER LINE)
C     NPRF=NO. OF PRIMARY FIELD LINE POINTS
C     IFLAG=1 IF LOSSES ARE ALLOWED TO THE CATHODE POLE PIECE
C     DI= DISTANCE TO BAFFLE FROM SCREEN
C     CZU=POINT WHERE PRIMARY FIELD LINE INTERSECTS CATHODE POLE PIECE
C     (IF IFLAG=1)
C
      DIMENSION DTR(5), DTY(19), XI1(11), XI2(11), XI3(11), XI4(11), PRR
     1(10), PRY(19), R(5), Z(5), XNP(51), XNM(51), Q(5), VE(51), V(5), VA
     2E1(11), VE2(11), VE3(11), VE4(11)
      REAL I1,I2
      NFLAG=0
      MFLAG=0
      READ (5,107) NPRF,IFLAG,DI,CZU
      CZD=DI
      READ (5,111) (PRR(I),PRY(I),I=1,NPRF)
      NR=NLN*4
C
C     BEGIN CALCULATIONS
C     SHUFFLE NO. DENSITIES INTO CORRECT ARRAYS
C
      DO 101 I=1,NR,4
         J=I+1
         K=I+2
         L=I+3
         M=I/4+1
         VE1(M)=VE(I)
         VE2(M)=VE(J)
         VE3(M)=VE(K)
         VE4(M)=VE(L)
         XI2(M)=XNP(J)+XNM(J)
         XI3(M)=XNP(K)+XNM(K)
         XI4(M)=XNP(L)+XNM(L)
  101 XI1(M)=XNP(I)+XNM(I)
C
C     WRITE OUT THESE ARRAYS
C
      WRITE (6,110) (XI1(I),I=1,NLN)
      WRITE (6,110) (XI2(I),I=1,NLN)
      WRITE (6,110) (XI3(I),I=1,NLN)
      WRITE (6,110) (XI4(I),I=1,NLN)
      AZ=0.
      DY=PRY(NPRF)/200.
      AN=0.
      AR=0.
      YZ=DY/2.
C
C     BEGIN CALCULATION OF THE SURFACE INTEGRAL
C     PRIMARY FIELD LINE SECTION
C
      DO 13[illegible] I=1,200
         IF (I.EQ.1) [illegible]
         Y[illegible]=YZ+DY/2.
         YU=YZ-DY/2.
         CALL AITKEN ([illegible])
```

```
      CALL AITKEN (PRY,PRR,NPRF,1,TP,DP)                                ARA  600
      CALL AITKEN (PRY,PRR,NPRF,1,YM,DM)                                ARA  610
      DX=DP-DM                                                          ARA  620
      N=1                                                               ARA  630
      IF (I.GT.225) N=1                                                 ARA  640
      CALL AITKEN (DTY,XI1,NLN,N,YZ,P(1))                               ARA  650
      CALL AITKEN (DTY,XI2,NLN,N,YZ,P(2))                               ARA  660
      CALL AITKEN (DTY,XI3,NLN,N,YZ,P(3))                               ARA  670
      CALL AITKEN (DTY,XI4,NLN,N,YZ,P(4))                               ARA  680
      CALL CHECK (P,4,NFLAG)                                            ARA  690
      J=4                                                               ARA  700
      P(5)=P(4)/5.                                                      ARA  710
      CALL AITKEN (DTY,VE1,NLN,N,YZ,V(1))                               ARA  720
      CALL AITKEN (DTY,VE2,NLN,N,YZ,V(2))                               ARA  730
      CALL AITKEN (DTY,VE3,NLN,N,YZ,V(3))                               ARA  740
      CALL AITKEN (DTY,VE4,NLN,N,YZ,V(4))                               ARA  750
      CALL CHECK (V,4,NFLAG)                                            ARA  760
      V(5)=V(4)/5.                                                      ARA  770
      CALL AITKEN (DTR,P,J,N,XP,XNE)                                    ARA  780
      CALL AITKEN (DTR,V,J,N,XP,VEE)                                    ARA  790
      YZ=YZ+DY                                                          ARA  800
      AR=AR+SQRT(DX**2+DY**2)*6.2832*XP                                 ARA  810
      AZ=AZ+SQRT(DX**2+DY**2)*6.2832*XP*XNE**2*VEE                      ARA  820
  102 AN=AN+SQRT(DX**2+DY**2)*6.2832*XP*XNE*VEE                         ARA  830
C                                                                       ARA  840
C     SECTION FOR GRIDS                                                 ARA  850
C                                                                       ARA  860
      P(1)=6.2832*DTR(1)*XI1(NLN)                                       ARA  870
      P(2)=6.2832*DTR(2)*XI2(NLN)                                       ARA  880
      P(3)=6.2832*DTR(3)*XI3(NLN)                                       ARA  890
      P(4)=6.2832*DTR(4)*XI4(NLN)                                       ARA  900
      CALL CHECK (P,4,NFLAG)                                            ARA  910
      P(5)=0.                                                           ARA  920
      V(1)=VE1(NLN)                                                     ARA  930
      V(2)=VE2(NLN)                                                     ARA  940
      V(3)=VE3(NLN)                                                     ARA  950
      V(4)=VE4(NLN)                                                     ARA  960
      V(5)=0.                                                           ARA  970
      DO 103 IK=1,5                                                     ARA  980
  103 Q(IK)=V(IK)*P(IK)                                                 ARA  990
      CALL QTEG (DTR,Q,Z,5)                                             ARA 1000
      WRITE (6,108) Z(5)                                                ARA 1010
      I1=Z(5)                                                           ARA 1020
      AN=AN*Z(5)                                                        ARA 1030
      DO 104 IK=2,5                                                     ARA 1040
  104 Q(IK)=Q(IK)*P(IK)/(6.2832*DTR(IK))                                ARA 1050
      CALL QTEG (DTR,Q,Z,5)                                             ARA 1060
      I2=Z(5)                                                           ARA 1070
      AZ=AZ+Z(5)                                                        ARA 1080
      Z(5)=Z(5)*2.2828                                                  ARA 1090
      WRITE (6,108) Z(5)                                                ARA 1100
      AR=AR*3.14159*DTR(5)**2                                           ARA 1110
      Q(5)=0.                                                           ARA 1120
C                                                                       ARA 1130
C     SECTION FOR BAFFLE                                                ARA 1140
C                                                                       ARA 1150
      N=1                                                               ARA 1160
      CALL AITKEN (DTY,XI1,NLN,N,DI,P(1))                               ARA 1170
      CALL AITKEN (DTY,XI2,NLN,N,DI,P(2))                               ARA 1180
      CALL AITKEN (DTY,XI3,NLN,N,DI,P(3))                               ARA 1190
      CALL AITKEN (DTY,XI4,NLN,N,DI,P(4))                               ARA 1200
      CALL CHECK (P,4,NFLAG)                                            ARA 1210
      CALL AITKEN (DTY,VE1,NLN,N,DI,V(1))                               ARA 1220
      CALL AITKEN (DTY,VE2,NLN,N,DI,V(2))                               ARA 1230
      CALL AITKEN (DTY,VE3,NLN,N,DI,V(3))                               ARA 1240
      CALL AITKEN (DTY,VE4,NLN,N,DI,V(4))                               ARA 1250
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```
      CALL CHECK (V,4,MFLAG)                                          ARA 1260
      Q(1)=6.2832*DTR(1)*P(1)*V(1)                                    ARA 1270
      Q(2)=6.2832*DTR(2)*P(2)*V(2)                                    ARA 1280
      Q(3)=6.2832*DTR(3)*P(3)*V(3)                                    ARA 1290
      Q(4)=6.2832*DTR(4)*P(4)*V(4)                                    ARA 1300
      CALL QTEG (DTR,Q,Z,S)                                           ARA 1310
      CALL AITKEN (DTR,Z,S,1,R,EC)                                    ARA 1320
      DO 105 IJ=1,4                                                   ARA 1330
  105 Q(IJ)=Q(IJ)*P(IJ)                                               ARA 1340
      CALL QTEG (DTR,Q,Z,S)                                           ARA 1350
      CALL AITKEN (DTR,Z,S,1,R,ED)                                    ARA 1360
      AR=AR+3.1416*R**2                                               ARA 1370
      AN=AN+EC                                                        ARA 1380
      AZ=AZ+ED                                                        ARA 1390
      IF (IFLAG.NE.1) GO TO 106                                       ARA 1400
C                                                                     ARA 1410
C     CATHODE POLE PIECE SECTION                                      ARA 1420
C                                                                     ARA 1430
      CALL AITKEN (DTY,XI1,NLN,N,CZU,P(1))                            ARA 1440
      CALL AITKEN (DTY,XI2,NLN,N,CZU,P(2))                            ARA 1450
      CALL AITKEN (DTY,XI3,NLN,N,CZU,P(3))                            ARA 1460
      CALL AITKEN (DTY,XI4,NLN,N,CZU,P(4))                            ARA 1470
      CALL CHECK (P,4,NFLAG)                                          ARA 1480
      P(5)=P(4)/5.                                                    ARA 1490
      CALL AITKEN (DTR,P,S,N,R,XN1)                                   ARA 1500
      CALL AITKEN (DTY,VE1,NLN,N,CZU,V(1))                            ARA 1510
      CALL AITKEN (DTY,VE2,NLN,N,CZU,V(2))                            ARA 1520
      CALL AITKEN (DTY,VE3,NLN,N,CZU,V(3))                            ARA 1530
      CALL AITKEN (DTY,VE4,NLN,N,CZU,V(4))                            ARA 1540
      CALL CHECK (V,4,MFLAG)                                          ARA 1550
      V(5)=V(4)/5.                                                    ARA 1560
      CALL AITKEN (DTR,V,S,N,R,VN1)                                   ARA 1570
      CALL AITKEN (DTY,XI1,NLN,N,CZD,P(1))                            ARA 1580
      CALL AITKEN (DTY,XI2,NLN,N,CZD,P(2))                            ARA 1590
      CALL AITKEN (DTY,XI3,NLN,N,CZD,P(3))                            ARA 1600
      CALL AITKEN (DTY,XI4,NLN,N,CZD,P(4))                            ARA 1610
      CALL CHECK (P,4,NFLAG)                                          ARA 1620
      P(5)=P(4)/5.                                                    ARA 1630
      CALL AITKEN (DTR,P,S,N,R,XN2)                                   ARA 1640
      CALL AITKEN (DTY,VE1,NLN,N,CZD,V(1))                            ARA 1650
      CALL AITKEN (DTY,VE2,NLN,N,CZD,V(2))                            ARA 1660
      CALL AITKEN (DTY,VE3,NLN,N,CZD,V(3))                            ARA 1670
      CALL AITKEN (DTY,VE4,NLN,N,CZD,V(4))                            ARA 1680
      CALL CHECK (V,4,MFLAG)                                          ARA 1690
      V(5)=V(4)/5.                                                    ARA 1700
      CALL AITKEN (DTR,V,S,N,R,VN2)                                   ARA 1710
      XL=CZU-CZD                                                      ARA 1720
      AR=AR+XL*6.2832*R                                               ARA 1730
      XN=(XN1+XN2)/2.                                                 ARA 1740
      VN=(VN1+VN2)/2.                                                 ARA 1750
      AN=AN+6.2832*R*XL*XN*VN                                         ARA 1760
      AZ=AZ+6.2832*R*XL*XN**2*VN                                      ARA 1770
C                                                                     ARA 1780
C     WRITE OUT RESULTS                                               ARA 1790
C                                                                     ARA 1800
  106 WRITE (6,113) AR                                                ARA 1810
      WRITE (6,112) AN                                                ARA 1820
      WRITE (6,114) AZ                                                ARA 1830
      F1=AR*XNEE/AN**2*EA                                             ARA 1840
      F2=AR*XNEE**2/AZ*EA                                             ARA 1850
      BER=2.92*R*I**2/(T1*XNEE)                                       ARA 1860
      WRITE (6,115) F1,F2,BER                                         ARA 1870
      WRITE (6,116) NFLAG                                             ARA 1880
      WRITE (6,109) MFLAG                                             ARA 1890
      RETURN                                                          ARA 1900
C                                                                     ARA 1910
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  107 FORMAT (2I5,4F10.4)                                                    ARA 1920
  108 FORMAT (2X,  9HI (CM-1)=,E11.4)                                        ARA 1930
  109 FORMAT (2X, 40HNEGATIVE VELOCITY EXTRAPOLATION OCCURRED,I5,  7H   TARA 1940
     1IMES)
  110 FORMAT (10X,8(F10.3,2X))                                               ARA 1950
  111 FORMAT (8F10.5)                                                        ARA 1960
  112 FORMAT (3X, 13HAN (CM2/CM3)=,F11.4)                                    ARA 1970
  113 FORMAT (3X,  9HAR (CM2)=,E11.4)                                        ARA 1980
  114 FORMAT (3X, 13HAZ (CM2/CM5)=,F11.4)                                    ARA 1990
  115 FORMAT (2X,  3HF1=,F10.4,2X,  3HF2=,F10.4,2X, 13HI++/I+ FACTOR,F10ARA 2000
     1.4)                                                                    ARA 2010
  116 FORMAT (2X, 39HNEGATIVE DENSITY EXTRAPOLATION OCCURRED,I5,  8H   TARA 2020
     1IMES)                                                                  ARA 2030
C                                                                            ARA 2040
      END                                                                    ARA 2050
                                                                             ARA 2060
      SUBROUTINE CHECK (P,N,NFLAG)
      DIMENSION P(N)                                                         CK    10
      DO 101 I=1,N                                                           CK    20
         IF (P(I).GE.0.0) GO TO 101                                          CK    30
         P(I)=0.0                                                            CK    40
         NFLAG=NFLAG+1                                                       CK    50
  101 CONTINUE                                                               CK    60
      RETURN                                                                 CK    70
C                                                                            CK    80
      END                                                                    CK    90
#                                                                            CK   100
```