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SPACE SHUTTLE Ku-BAND
INTEGRATED RENDEZVOUS
RADAR/COMMUNICATIONS SYSTEM STUDY
FINAL REPORT

Contract No. NAS 9-14595

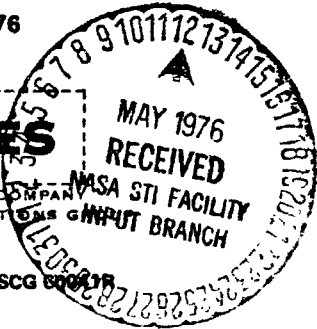
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FOREWORD

This final report documents and summarizes the results of work performed on the Space Shuttle Ku-Band Integrated Rendezvous Radar/Communications System Study under contract NAS 9-14595. The recommendations and conclusions are included in this report, as well as the details explaining the results achieved.

Section 1 presents the recommendations and conclusions of the study effort, including the recommended specifications for the system.

Section 2 defines the requirements upon which the study effort was based and also presents the predicted performance of the recommended system configuration. In addition, shuttle orbiter vehicle constraints (e.g., size, weight, power, stowage space) are discussed.

Section 3 presents the tradeoffs considered and describes the operation of the recommended configuration for an optimized, integrated Ku-band radar/communications system. This section is divided into subsections which discuss basic system tradeoffs, communication design, radar design, antenna tradeoffs, antenna gimbal and drive design, antenna servo design, and deployed assembly packaging design.

Section 4 contains the communications and radar performance analyses necessary to support the system design effort.

Appendices A through D are detailed derivations of the communications thermal noise error, the radar range, range rate, and angle tracking errors, and the communications transmitter distortion parameter effect on crosstalk between the unbalanced quadriphase signals.

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1. STUDY RECOMMENDATIONS AND CONCLUSIONS

1.1 OVERALL SYSTEM DESCRIPTION

Section 1 summarizes the conclusions of the study and describes the highlights of the recommended configuration of the integrated radar and communications system.

Section 3 describes the considerations resulting in the recommended system configuration. The system block diagram, Figure 3-5 of Section 3, shows the transmitter power amplifier and power supply, the receiver first and second IF amplifiers, the search and angle tracking servo, and part of the receiver first mixer local oscillator chain are shared by the radar and communication functions.

1.2 MAJOR SYSTEM COMPONENTS

This section briefly describes the major components of the recommended system configuration depicted in Figure 3-5.

1.2.1 Antenna RF Components

The recommended antenna is a conventional cassegrain type 63.5 cm (25 inch) in diameter with a 15.24 cm (6 inch) diameter subreflector. The subreflector diameter is optimized to achieve a minimum combination of subreflector blockage loss and diffraction loss. A square horn antenna 8.9 cm (3.5 inch) in dimension is provided to illuminate the Tracking and Data Relay Satellite System (TDRSS) during acquisition. It was determined that it was feasible to package the radar and communications receiver front ends and other components in an equipment compartment attached to the antenna. This permits the elimination of one pair of rotary joints and reduces complexity of the antenna support structure. An image enhancement mixer was selected for communications, while a parametric amplifier followed by a conventional mixer are used for radar. Also included in this package are the polarization switch, a circulator providing isolation between the receiver and transmitter, the monopulse comparator network and error modulators, a directional coupler used to combine the error signals with the sum channel, the switch that routes the transmitter output to either the acquisition horn or the main antenna, and a directional coupler used to monitor transmitter power. A layout of this package is shown in Figure 3-41.

1.2.2 Transmitter, Radar Modulator, and Communications Modulator

The transmitter is a high efficiency (50 percent) coupled cavity, grid modulated, TWT with a minimum output power of 50 watts and is based on the 100 watt CW TWT developed by Hughes Electron Dynamics Division for the Japanese Broadcast Satellite. A grid modulated version of this tube is also being developed for NASA Langley. The TWT is grid modulated during radar acquisition and medium range tracking. In the radar short range tracking mode, RF drive pulse modulation in combination with grid modulation, is used to achieve the required short pulse width.

No attempt was made to combine the radar and communications modulator because of the need for radar frequency agility. The radar frequency agility is implemented by using a common synthesizer in the radar exciter and the radar first local oscillator signal. This technique removes the frequency shift in the radar first mixer and allows use of common IF amplifiers for both radar and communications.

The communications modulator provides both the FM and unbalanced quadriphase mode of operation. The only unusual feature is that the signals are modulated at about 1.3 GHz and then upconverted to the 15.008 GHz output frequency. This technique was chosen in preference to direct modulation to minimize carrier frequency drift in the FM mode and to minimize phase unbalance, amplitude unbalance, and relative phase error between the orthogonal carriers in the quadriphase mode.

1.2.3 Communications Receiver

The communications receiver consists of the pseudo-noise (PN) despreader and phase lock demodulator. PN despreading is accomplished in a conventional delay lock synchronizer. Although it is not shown in Section 3 (Figure 3-5 block diagram), it is recommended that threshold detection be accomplished with a differential threshold detector that compares an uncorrelated channel with the output of the synchronized channel. This technique is insensitive to gain variation and has superior detection performance under conditions of low signal-to-noise ratio.

The recommended phase lock demodulator is a simple type that multiplies the second IF frequency by two. This produces a substantial carrier component since the quadriphase input signal is unbalanced and permits the use of a simple tracking loop. A conventional I-Q or Costas demodulator could also be used and has no significant difference in performance, but may be somewhat more complex in implementation. The phase locked loop is closed at the receiver first mixer to achieve the frequency tracking range required for doppler carrier frequency shift. The phase locked loop could also be closed at the second IF frequency by using a fixed first local oscillator and an LC phase locked oscillator. There is no compelling reason to prefer one approach over the other.

1.2.4 Radar Signal Processor

The recommended radar implementation is a coherent type using approximately a 5 kHz PRF during search. Ten frequencies, separated by 25 MHz intervals, are used to ensure a high probability of detection. During search a 60 μ sec pulse is transmitted and a 148 μ sec listening period is used. The duty cycle is high but avoids range ambiguities below the desired detection range of 22.2 km (12 n.mi.). For less critical applications such as Mission I/II targets, a total of 64 pulses are coherently integrated for each of the 10 frequencies during the on-target time. For Mission III-B, which requires a much more rapid scan, eight pulses are integrated during the on-target time.

For search operation, three range gates are employed. Range intervals are determined on the basis of the type of target and the range at which initial acquisition is expected.

Angle search is a shaped raster scan approximating a circular field of view. The search volume varies from 20° to 45° half-cone angles for Missions I/II and III-B, respectively.

Because the antenna must search at a very rapid rate (up to 120 deg/sec), target detection occurrence is followed by a miniscan of 3° by 3° about the coordinates at which detection first occurred.

The recommended range tracker is a conventional early-late gate tracker. Pulse width is varied as a function of target range. Angle tracking is accomplished by a sequential lobing technique identical to that used for communications tracking.

Logarithmic processing is used for range, angle, and range rate processing to enhance signal-to-noise ratio and reduce the effects of target scintillation.

Digital doppler filter banks are used for range rate determination. Range rate ambiguity resulting from the possibility of opening or closing range rates is resolved by using a range rate estimate derived from the range tracker.

Close range tracking is accomplished by using a very short transmit pulse of 200 ns duration or less to obtain the required minimum range accuracy of 30 meters (100 feet).

Angular tracking accuracy, angle rate resolution, and range accuracy requirements at very close ranges pose a very severe problem. Target angular and range extent at short ranges (1 km) are such that the requirements of Table 2-3 of Section 2 can be met only for a point target.

1.2.5 Communications Signal Processor

The communications signal processor performs the functions of bit synchronization and matched filter detection for both the 3 megasymbol/sec convolutionally encoded 1 Mbps data and the conventional 72 kbps data. Convolutional encoding of the 1 Mbps data implies low signal-to-noise ratio such that the bit synchronizer and matched data filter operating at the 3 megasymbol/sec rate may be a problem. This problem can be reduced in magnitude by specifying small tolerances on symbol clock rate for the 1 Mbps data clock. The remaining functions of the signal processor include mode switching logic and routing of the output data streams.

1.2.6 Antenna Gimbal, Drive, and Servo

This subsystem is critical to both radar and communications performance. Figure 3-30 shows the recommended gimbal arrangement. The gimbal angles have been selected to provide a minimum half-cone angle field of view of 60° about the -Z-axis. The trunnion obscuration cone is located such that its blockage coincides with the shuttle vehicle blockage. The resulting coverage of the antenna is shown graphically in Figure 3-35.

A yoke type supporting structure was selected to support the antenna. A single arm support was also considered during the study but discarded because of conflict with the antenna RF components and unbalanced loads on the bearing. The antenna package gimbal supports are located at the center of gravity. Motor torque requirements are 0.579 kg-m (3.19 ft-lb) and 0.521 kg-m (3.77 ft-lb) on the inner and outer gimbal axes, respectively. The torque requirements are determined primarily by the accelerations required during the radar search mode with some contribution from cable loads. The inner gimbal axis travels from 35° down to 85° up, while the outer gimbal axis travels nearly $\pm 180^\circ$.

The recommended drive motors are full rotation brush type dc torque motors. The gimbals are supported in each axis by spring loaded precision angular contact bearings with a light interference fit. This type of bearing is insensitive to temperature gradients causing changes in bearing preload. The bearings are sized to withstand launch load vibration. Dry film lubrication is recommended for the bearings to avoid lubricant contamination.

Redundant solenoid latches for each axis are recommended to meet the requirement to return to the stowed position in the event of a power interruption. The antenna servo uses rate integrating gyros to provide high resolution angle rate sensing and inertial stabilizing during shuttle attitude changes. The bandwidth of the inner or gyro tracking loop is 15 Hz, while the bandwidth of the outer or RF sensor loop is 1 Hz to ensure low thermal noise tracking error.

1.3 RECOMMENDED SPECIFICATIONS

1.3.1 Radar Specifications

The basic radar requirements of Table 2-3 are considered to be adequate and no additional specifications were generated during the study. As shown in subsection 4.2, the requirements are achievable with one exception. That exception is the range and angle tracking requirements at short range when the target angular extent and size is such that the tracking accuracy requirements cannot be met. It is recommended the tracking accuracy specifications be modified at short range to apply only to a point target.

1.3.2 Communications EIRP, G/T, and Acquisition Time

EIRP, G/T, and acquisition time are the critical specifications for the communications subsystem. The recommended values and the minimum requirements used in the study are tabulated below:

	<u>Recommended Value</u>	<u>Minimum Requirement</u>
EIRP, dBW	50	48.6
G/T, dB/°K	4.0	1.6
TDRSS acquisition time	60 sec/max	60 sec/max

A higher value of EIRP is recommended on the basis that it can be achieved and that some margin will be required under the shuttle service conditions. An increase in required G/T is recommended to ensure rapid acquisition of the TDRSS. The minimum required value of G/T is considered adequate for data performance but marginal for TDRSS acquisition, particularly if TDRSS EIRP is reduced during acquisition as it would be if TDRSS points on the basis of S band information during initial acquisition. Although the results of the study showed that for the given acquisition parameters of Table 2-2, TDRSS acquisition could be accomplished in less than 10 seconds, therefore no change is recommended here because it is understood that basic requirements may change.

1.3.3 Other Communications Specifications

Other recommended communications subsystem performance parameters are shown in Table 1-1. In general, these requirements are not critical and many can be varied over a broad range without significant impact on system performance. Many of these parameters should be left to the discretion of the system designer and acceptable values will tend to vary as a function of the system implementation. Table 1-1 is intended to be only a rough guide to specifying communications subsystem parameters and should not be interpreted as definitive requirements.

TABLE 1-1. COMMUNICATIONS SUBSYSTEM SPECIFICATIONS

Parameter	Value or Range	Comments
ANTENNA SIDELOBES	15 to 20 dB down	May be critical depending on TDRSS acquisition scheme
ANTENNA GAIN	35 to 36 dB receive 36 to 37 dB transmit	
TRANSMITTER POWER	50 W minimum	
DESPREADER CHARACTERISTICS		May be critical depending on TDRSS acquisition scheme
Search rate	1 to 4 kHz	
Postdetection bandwidth	2 to 5 kHz	
Acquisition time	1 to 5 sec	Not critical
AGC frequency response	1 to 10 Hz	
ANGLE SERVO CHARACTERISTICS		
Dynamic lag	Negligible	
Sensor error (1 σ)	<0.5 $^{\circ}$ radar <0.1 $^{\circ}$ communications	
Loop bandwidth	15 Hz (gyro loop) 1 Hz (RF sensor loop)	
PHASELOCK DEMODULATOR CHARACTERISTICS		
Noise bandwidth	5 to 30 kHz	
Phase tracking error	5 $^{\circ}$ bias 5 $^{\circ}$ random	
Acquisition time	<0.25 sec	
TRANSMITTER CHARACTERISTICS		
Bandwidth (3 dB)	225 MHz	
Phase nonlinearity	< $\pm 20^{\circ}$ over ± 50 MHz	
Phase unbalance	< $\pm 5^{\circ}$	
Amplitude unbalance	<0.5 dB	
Carrier phase	90 $\pm 5^{\circ}$	
Gain slope and ripple	<1 dB/100 MHz, <1 dB peak	
BIT SYNCHRONIZER CHARACTERISTICS		
Acquisition time	<1 sec	
Loop noise bandwidth	20 Hz	
Clock stability	$\pm 1 \times 10^{-5}$	
TOTAL DATA ACQUISITION TIME (including angle acquisition)	<60 sec	

A specific comment about antenna sidelobe levels is in order. The most significant factor determining this specification is believed to be the desirability of avoiding a false alarm during initial acquisition due to a signal in the antenna sidelobe causing the detection threshold to be exceeded. This is particularly true if the TDRSS pointing is based on S band information. In this case, TDRSS EIRP could conceivably vary between limits of 40 to 49 dBW, which increases the possibility of sidelobe false alarm.

2. CONSTRAINTS, PERFORMANCE REQUIREMENTS, AND SYSTEM PERFORMANCE

2.1 CONSTRAINTS

The most significant physical constraints on system design are:

- Stowed space limitation
- Deployed assembly
- Deployed assembly power dissipation

Figure 2-1 shows three cross sections of the deployed assembly and antenna stowage space. The coordinate axes X, Y, Z are the roll, pitch, and yaw axes, respectively. Station $X_0 = 589$ is the location of the hinge point about which the deployed assembly is rotated for deployment. The largest cross-sectional area is 178 cm (70 in.) from the pedestal. A clearance of 7.6 cm (3 in.) must be allowed with respect to the payload and payload bay door. Thus, the maximum depth of an antenna that is located with the bore-sight axis normal to a line drawn between the two boundaries of Figure 2-1 is 33 cm (13 in.). This tends to limit antenna diameter to the order of 51 cm to perhaps 76 cm (20 to 30 in.).

The deployed assembly weight limit is given by Rockwell International as 43.6 kg (118 lb). The deployed assembly power dissipation, assuming a conventional thermal subsystem design, will be limited to the order of 150 to 200 watts if reasonable temperature extremes are to be maintained.

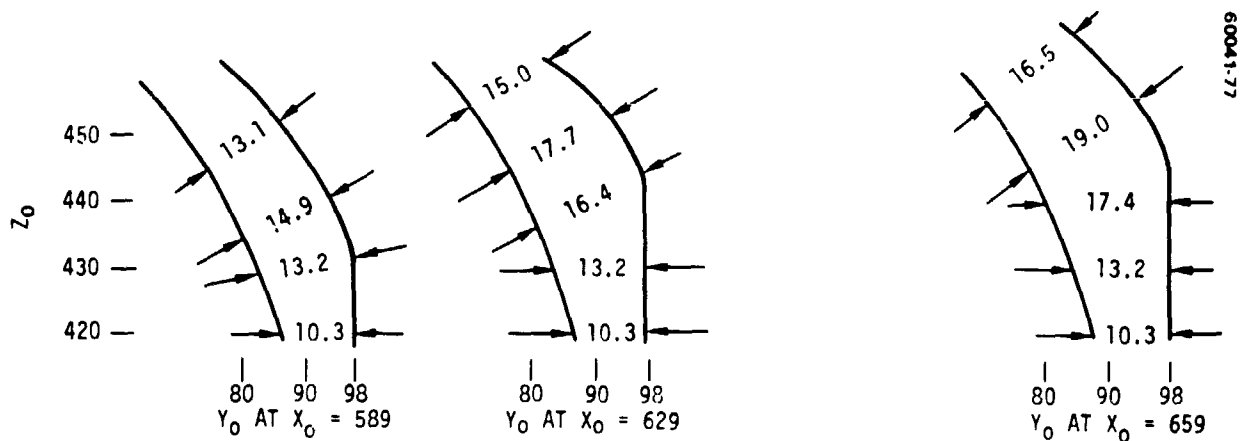


FIGURE 2-1. STOWAGE SPACE CROSS SECTIONS

2.2 PERFORMANCE REQUIREMENTS

Communications and radar performance requirements used as the basis for the study are given in Tables 2-1, 2-2, and 2-3.

Table 2-1 presents the Tracking and Data Relay Satellite System (TDRSS) and space shuttle communication link parameters. It should be noted that all data rates given are maximum values and the basic requirement is the capability to transmit and receive variable rates up to the maximum value. No lower limit has been placed on the data rates. The study emphasized design to achieve required performance at the maximum data rates since the maximum rates correspond to worst case performance.

TABLE 2-1. TDRSS/SHUTTLE LINK PARAMETERS

FORWARD LINK	
Frequency	13.75 to 13.80 GHz
TDRSS EIRP	49 dBW
Shuttle G/T	1.6 dB/K
Polarization	RHCP
Modulation	Unbalanced quadriphase, IMB/S data convolutionally encoded rate 1/3 biphas L, 72 kbps data biphas L modulated on orthogonal carrier; entire spectrum biphas NRZ modulated at 14.24 megachip rate with 2047 bit code.
RETURN LINK	
Frequency	14.896 to 15.121 GHz
Shuttle EIRP	48.6 dBW
TDRSS G/T	24.1 dB/K
Modulation	
Mode 1	Unbalanced quadriphase 50 Mbps data convolutionally encoded rate 1/2 biphas NRZ, 2 Mbps data biphas L modulated on orthogonal carrier.
Mode 2	FM composed of 4.2 MHz baseband TV summed with 8.5 MHz subcarrier unbalanced quadriphase modulated with 2 Mbps data and 192 kbps data, both biphas L.

TABLE 2-2. TDRSS/SHUTTLE ORBITER ACQUISITION REQUIREMENTS

Time to acquire TDRSS	1 min after TDRSS acquires orbiter
Minimum EIRP toward TDRSS	30 dBW
Designation uncertainty	3° (3σ)
Orbiter attitude rates	0.5 to 5.0 deg/sec
Pointing commands	Uncertain, may be every 2 sec
EIRP from TDRSS	49 dBW

TABLE 2-3. RADAR REQUIREMENTS

Parameter	Limits	Random Error (3σ)	Bias Error
Range	18.5 km (10 n.mi.) to 30 m (100 ft) (PASSIVE TGT) 556 km (300 n.mi.) to 30 m (100 ft) (CO-OP TGT)		
R > 9.3 km (5 n.mi.)		91 m (300 ft)	24.4 m (80 ft)
R < 9.3 km (5 n.mi.)		1% of R, or 24.4 m (80 ft)	24.4 m (80 ft)
Range rate	-46 m/sec (-150 fps) to +15.2 m/sec (50 fps) passive mode ± 91.4 m/sec (300 fps) cooperative target	0.3 m/sec (1 fps)	± 0.3 m/sec (1 fps)
Angle	$\pm 40^\circ$	10 mr (0.573 $^\circ$)	< 3 0°
Angle rate	± 0.25 deg/sec (acquire) ± 5.0 deg/sec (track)	0.14 mr/sec (0.008 deg/sec)	0.14 mr/sec (0.008 deg/sec)

Table 2-2 lists the TDRSS acquisition parameters. These parameters are based on the following acquisition scenario.

- TDRSS sends command via S band link for shuttle transmitter turnon.
- Shuttle points at nominal TDRSS position and illuminates TDRSS using a broadbeam antenna
- TDRSS searches for and acquires shuttle signal, then transmits to shuttle.
- Shuttle searches for and acquires TDRSS while continuing transmission on the broadbeam antenna. Upon acquisition, transmission switches to the narrowbeam antenna.

For the purpose of the study, worst case conditions have been assumed, i. e., the TDRSS signal is fully modulated with both data, and the spectrum is spread.

Table 2-3 lists the radar requirements. In this study only the passive or noncooperative target mode of operation was considered. The cooperative target mode was not considered since it was not in the scope of the study.

2.3 PERFORMANCE SUMMARY

Communication system performance parameters are given in Table 2-4. Table 2-5 lists radar performance parameters. It should be noted that the radar tracking performance data are for a point target only and do not include the effects of a target at close range.

TABLE 2-4. COMMUNICATION SYSTEM PERFORMANCE PARAMETERS

Antenna gain	35.3 dB (receive), 36.1 dB (transmit)
System noise temperature	990°K
Transmitter power	50 W
EIRP	50.1 dBW
G/T	5.1 dB
E/N ₀	19.6 dB (72 kbps data), 14.4 dB (1 Mbps data)
Despread acquisition time	<1 sec
TDRSS acquisition time	10 sec
Angle tracking error	0.008°

TABLE 2-5. RADAR PERFORMANCE PARAMETERS

SEARCH	
Average power	14.5 W
Acquisition time	<60 sec Missions I, II, IIIB
Acquisition range	22.2 km (12 n.mi.) Missions I, II 13 km (7 n.mi.) Mission IIIB
Margin	9.3 dB Mission I/II, 10.3 dB Mission IIIB
TRACK	
Angle track error	2 mr (1σ -22 km range point target)
Random range error	46 m (150 ft) 3σ at 22 km (12 n.mi.) range <0.3 m (1 ft) at 30.5 m (100 ft) range, point target
Bias range error	<0.5 m (1.5 ft)
Random range rate error	0.01 m/sec (0.03 fps)
Bias range rate error	0.1 m/sec (0.3 fps)
Angle rate resolution	<0.14 mr/sec

3. STUDY SYSTEM CONFIGURATION DESCRIPTION

This section describes the system configuration and the hardware tradeoffs performed during the course of the study. Descriptions of the radar subsystem communications, antenna subsystem, antenna drive and gimbals, deployed assembly thermal subsystem, and deployed assembly packaging are included.

3.1 INTEGRATED SYSTEM TRADEOFFS

The major factors driving the system design are the antenna and transmitter. The required EIRP is 48.6 dBW. The antenna size is limited by the space available in the stowed position. Other factors that heavily influence the design are the requirements for both linear and circular polarizations and accuracy of radar tracking velocity. The latter requirement tends to make the use of a coherent pulse doppler radar very desirable, if not mandatory. Thermal problems and weight limitations in the deployed assembly (which must be cooled primarily by radiating the heat toward space) also influence the choice of transmitter design since it must be located in the deployed assembly. The objective of the study was to make maximum use of existing components and also to obtain maximum commonality of components for radar and communications.

Several different antenna configurations were considered, including a planar slot array, twist cassegrain antenna, direct fed parabola, and conventional cassegrain. The primary constraint on antenna size is the on-axis depth. The maximum dimension on-axis is limited to 33 cm (13 inches). The planar slot array was considered chiefly because of its small on-axis dimension of about 10.2 cm (4 inches). However, polarization switching can only be accomplished by mechanical rotation of a quarter waveplate on the face of the array. This configuration was considered undesirable because of the added mechanical complexity and was, therefore, dropped from serious consideration. The twist cassegrain configuration was originally considered in an attempt to eliminate subreflector aperture blockage. In this configuration, the primary feed is linear and illuminates a subreflector that reflects primary feed illumination but is transparent to the orthogonal sense of linear polarization. The reflected energy from the subreflector is twisted 90° by the main reflector and passes through the subreflector with very little loss.

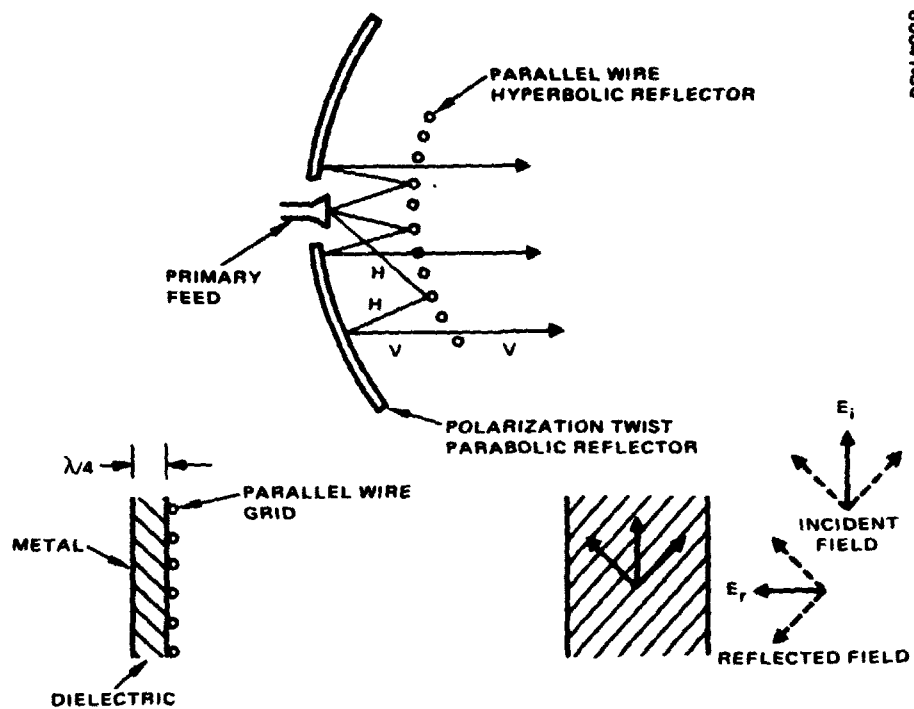


FIGURE 3-1. TWIST CASSEGRAIN

Circular polarization is obtained with a quarter waveplate over the antenna aperture. Figure 3-1 illustrates the operation of the twist cassegrain antenna. This approach also was dropped from further consideration because of the added mechanical complexity required to implement polarization switching. The choice of antennas then narrows down to the direct fed parabola and the conventional cassegrain.

Subsection 3.2 presents the pattern, gain, and tracking scale factor data for both configurations. Although the direct fed parabola is generally superior in performance to the conventional cassegrain, it has a potentially serious mechanical design problem. As shown in subsection 4.2, the search acceleration required of the antenna is approximately 700 deg/sec^2 . Since the direct fed antenna feed consists of a large circularly polarized feedhorn and four difference horns (see subsection 3.4), it is relatively heavy. In addition, the feed is located at the focal point of the parabola and is greater than 23 cm (9 inches) from the gimbal axis. Clearly a very rigid, heavy support structure is required to minimize feed deflection due to the very high acceleration forces encountered during radar search mode. Although this has not been evaluated in detail, the conventional cassegrain configuration has been selected in this study for system sizing purposes.

The feed of the conventional cassegrain antenna has the advantage of being located very near the gimbal axis. (See Figure 3-39.) The subreflector is quite lightweight and does not require a heavy supporting structure to obtain adequate rigidity. The only disadvantage of the cassegrain configuration is the somewhat lower aperture efficiency and less desirable sidelobe performance. The efficiency disadvantage is offset by the fact that a larger diameter main reflector can be used than in the direct fed case. The gain of the two configurations compared in subsection 3.3 are nearly equal.

Major emphasis was given in the study to the problem of optimizing the integrated system by using the maximum number of system components for both radar and communication functions. Table 3-1 lists some of the possible common components other than the antenna.

It would be highly desirable to share the transmitter and the receiver front end. This implies that the radar should transmit and receive in a band

TABLE 3-1. INTEGRATED SYSTEM SHARED COMPONENT POSSIBILITIES

● Shared transmitter and receiver front end ● Shared transmitter ● Shared receiver front end ● Shared IF amplifiers ● Shared servo and angle tracking electronics ● Use of communications biphas transmitter modulator for pulse modulation (RF gating) in radar stationkeeping mode ● Shared frequency multiplier chains ● Shared phase detectors and reference oscillator
--

centered around the communication receive frequency. The transmitter then would require a power amplifier capable of providing adequate power output at both the communications transmit and receive frequencies. A survey of available helix TWTs indicated that the Hughes 856H was the closest to meeting requirements. This TWT has greater than 30 watts output power over the frequency range and could be optimized at the communications frequency to obtain 40 to 50 watts. This tube is qualified for airborne use. Size and weight are substantially less than a coupled cavity tube. However, since the study emphasized use of proven components, the Hughes 294H coupled cavity tube was tentatively selected for the study system configuration. This TWT, developed for the Japanese Broadcast Satellite, is a space qualified TWT, but would still require some modification for this application.

Characteristics of the 294H tube are given in Table 3-2. The tube is a coupled cavity type and has adequate bandwidths for the communications transmitter and the radar transmitter, provided the same center frequency is used. Efficiency of this TWT is high which is desirable in light of the thermal problems of the deployed assembly.

The following modifications are required for this application:

- Scale frequency to 15 GHz
- Reduce power output to 50 watts
- Add modulation grid
- Add modulator to power supply for radar use

These are relatively straightforward modifications. Scaling the frequency is accomplished by scaling cavity dimensions. Reduction of output power is

TABLE 3-2. JAPANESE BROADCAST SATELLITE (294H) TWT CHARACTERISTICS

Weight	7 kg (15.4 lb)
Size	8.26 x 9.53 x 37.08 cm (3.25 x 3.75 x 14.60 in.) excluding collector
	Collector size is 16.00 (length) x 15.24 cm (diameter) (6.3 x 6 in.)
RF power	100 W min, 120 W typical
Frequency	12.0 GHz
Efficiency	48% min, 54% typical
Bandwidth (1 dB)	>200 MHz
Cooling	Radiation for collector, conduction otherwise
Saturated gain	42 to 48 dB
Beam voltage	-8 kV
Collector voltages	0, -3.0, -3.6, -6.15 kV

achieved by reducing beam voltage. Addition of the nonintercepting modulation grid is in progress on a contract with NASA Langley for the development of a 14 GHz, 100 watt power output tube with a nonintercepting grid for modulation. This development is based on the 294H design.

Figure 3-2 is a diagram of a possible system front end configuration where the radar and communications use a common power amplifier. Table 3-3 gives the transmit and receive RF losses for this configuration.

An alternate system front end configuration that was considered is shown in Figure 3-3. In this configuration a parametric preamplifier and first mixer are shared by the communications and radar. Transmit and receive RF losses are shown in Table 3-4. Table 3-5 lists the EIRP and G/T values for radar and communications for both configurations.

Performance of either system is comparable. However, the shared front end configuration requires two transmitters. Since the radar performance margin is adequate, the 856H could be used for the radar function, but the additional TWT weight penalty makes this system unattractive. For this reason, the coupled cavity TWT common transmitter configuration was selected for the baseline system design.

Several other important tradeoffs were considered during the study. The tradeoffs included the 1) method of angle tracking, 2) methods of acquiring the TDRSS signal, 3) implementation of the coherent pulse doppler radar, 4) communications despreaders implementation, and 5) communications demodulator implementation. The last three are discussed in subsections 3.2 and 3.3. The first will be discussed briefly here, and the second is treated in detail in subsections 4.1.4 and 4.1.5.

TABLE 3-3. RF LOSSES FOR SHARED TRANSMITTER CONFIGURATION, dB

Radar Receiver		Communications Receiver		Transmitter	
Circulator (1)	0.3	Circulator switches (2)	0.8	Circulators (2)	0.5
Circulator switches (2)	0.5	Circulator (1)	0.3	Switch (1)	0.2
Directional coupler	0.2	Polarizer	0.1	Circulator switch (1)	0.3
Transmitter switch	0.7	Waveguide	0.2	Bandpass filter	0.5
Waveguide	0.2	Bandpass filter	0.5	Directional coupler	0.1
VSWR loss	0.5	Directional coupler	0.2	Polarizer	0.1*
Waveguide coax adapter	0.1	VSWR loss	0.4	Waveguide	0.3
Pointing loss	0.1	Polarization loss	0.1	Rotary joints (2)	0.4
Total	2.6	Pointing loss	0.1	Polarization loss	0.1
		Waveguide coax adapter	0.1	VSWR loss	0.5
		Total	2.5	Total	3.0 (C) 2.8 (R)

*Communications only.

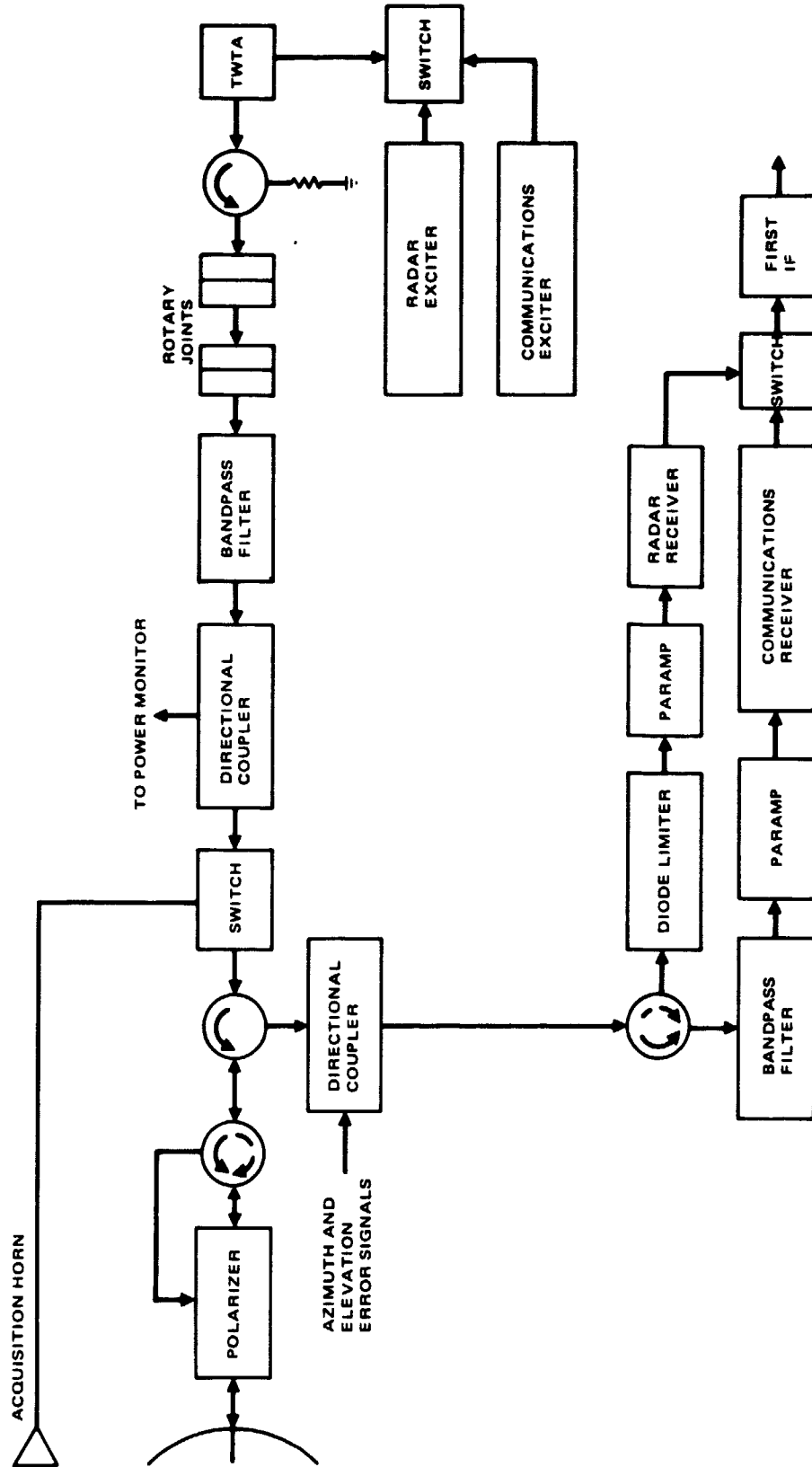


FIGURE 3-2. SHARED TRANSMITTER INTEGRATED FRONT END CONFIGURATION

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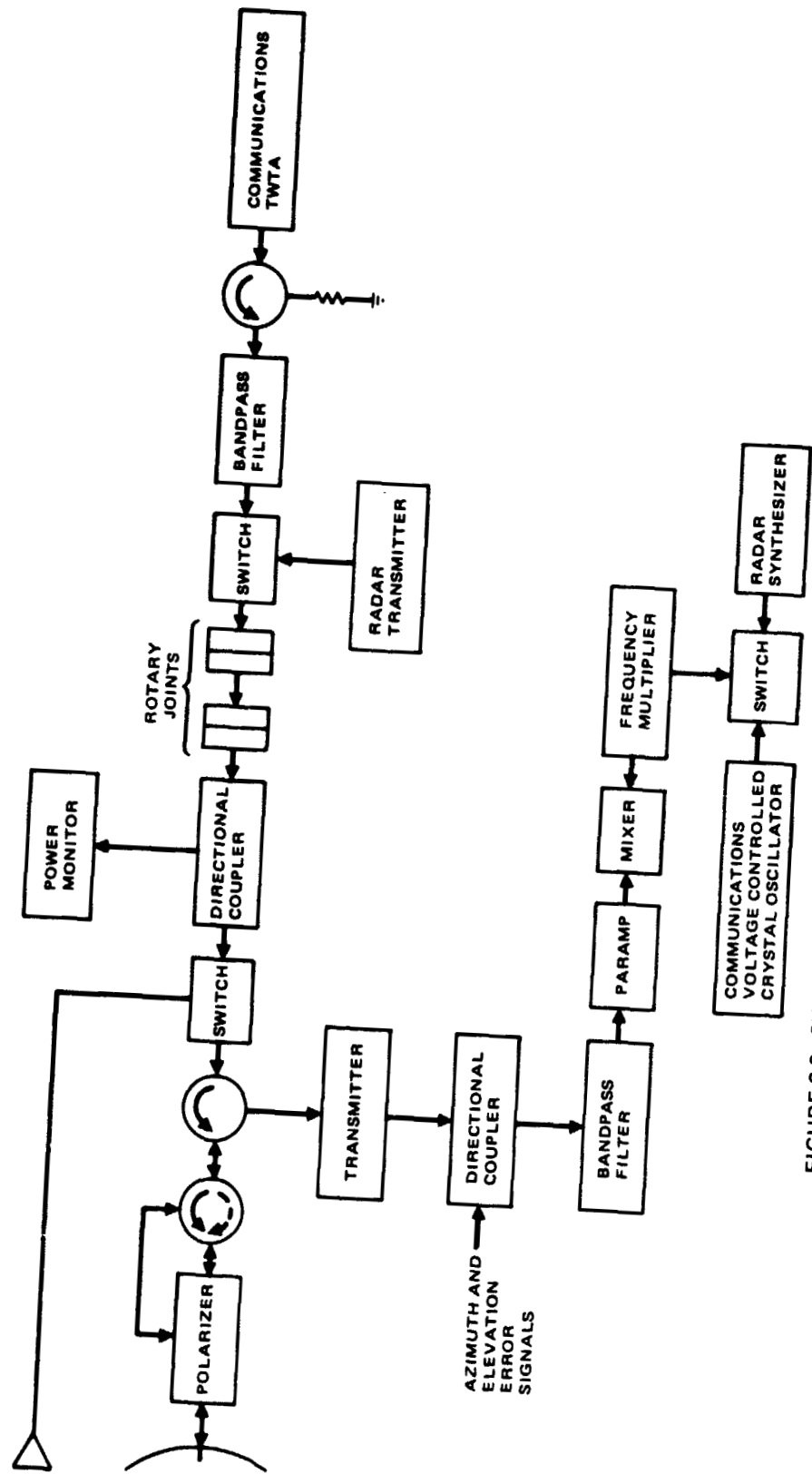


FIGURE 3-3. SHARED PARAM INTEGRATED FRONT END CONFIGURATION

TABLE 3-4. RF LOSSES FOR SHARED RECEIVER FRONT
END CONFIGURATION, dB

Receiver		Radar Transmitter		Communications Transmitter	
Switch (1)	0.15	Circulator	0.3	Circulators (2)	0.5
Circulator	0.25	Switches (2)	0.3	Switches (2)	0.3
Bandpass filter	0.5	Circulator switch (1)	0.3	Circulator switch (1)	0.3
Transmitter switch	0.7	Rotary joints (2)	0.5	Rotary joints (2)	0.5
Polarizer	0.1*	Directional coupler	0.1	Bandpass filter	0.5
Circulator switch	0.3	Waveguide circulator (1)	0.3	Directional coupler	0.1
Directional coupler	0.2	VSWR loss	0.5	Polarizer	0.1
Waveguide to coax adapter	0.1	Pointing loss	0.1	Waveguide	0.3
VSWR loss	0.5	Total	2.4	VSWR loss	0.5
Polarization loss	0.1*			Polarization loss	0.1
Pointing loss	0.1			Pointing loss	0.1
Total	3.0 (C) 2.8 (R)			Total	3.3

*Communications only.

TABLE 3-5. EIRP AND G/T

ASSUMPTIONS: Cassegrain antenna with 36.1 dB transmit gain (G_T) and 35.3 dB receive gain (G_R), paramp noise temperature (T_R) 150°K, transmitter power (P_T) 50 W, mixer noise temperature 440°K, polarization and pointing loss (L_p) 0.2 dB		
$EIRP = G_T + P_T \cdot L_T \cdot L_p$ $T_S = 40 + (L_R \cdot 1) 290 + L_R T_R$ $G/T = G_R \cdot L_p \cdot T_S \text{ (communications), } G_T \cdot L_p \cdot T_S \text{ (radar)}$		
	$L_T =$ transmit RF losses	$L_R =$ receive RF losses
<u>Parameter</u>	<u>Configuration</u>	
Communications EIRP	Shared transmitter 50.1 dBW	Shared receiver front end 49.8 dBW
Communications T_S	990°K (M), 500°K (P)	1140°K (M), 590°K (P)
Communications G/T	5.1 dB (M), 8.1 dB (P)	4.5 dB (M), 7.4 dB (P)
Radar EIRP (peak)	50.3 dBW	50.7 dBW
Radar T_S	1050°K (M), 530°K (P)	1140°K (M), 590°K (P)
Radar G/T	5.8 dB (M), 8.7 dB (P)	5.4 dB (M), 8.1 dB (P)

Several different methods could be used to implement angle tracking. Three methods considered were single channel monopulse, correlation type angle tracking receiver, and conventional three channel monopulse receiver. The single channel monopulse receiver configuration was selected because it minimizes problems of gain and phase matching inherent in the other two implementations.

A simplified block diagram of the single channel monopulse method is shown in Figure 3-4. The error signals are derived in the usual way in a four hybrid network. A five horn feed is used to avoid the compromises between sum channel gain and error channel slope. The output error signals are biphase modulated in the error modulators by square wave signals generated by the modulation oscillator and driver circuits. Phasing of the error signals with respect to the sum channel signal, is adjusted so that when signals are added to the carrier, amplitude modulation of the carrier results. The error signals are added to the received sum channel signal in a 15 dB coupler to minimize attenuation loss in the sum channel. This addition takes place prior to receiver amplification and mixing. The main advantage of this approach is that all the problems of three channel angle tracking receivers, such as systematic error caused by differential phase shift and servo gain variation due to differential gain drift, are minimized. Also, the number of receiver components is greatly reduced. The only disadvantage is the 15 dB directional coupler loss in the error channels, which in some applications, could cause excessive tracking error due to thermal noise. However, in this application the resulting thermal noise tracking error is a small fraction of the antenna 3 dB beamwidth. The thermal noise error is computed in subsection 4.1.7 for communications, and in subsection 4.2.2 for the radar function. Appendices A and B are derivations of the thermal noise error equations for both cases.

Figure 3-5 is a functional block diagram of the system configuration selected in the study. The radar and communications share the transmitter. Selection of transmit mode is made at the input to the TWTA. Separate receiver front ends are employed but the first local oscillator multiplier chain is shared to some extent. The communications first local oscillator signal is upconverted by a frequency synthesizer that removes the radar frequency agility such that the radar first IF is identical to the communications first IF. The synthesizer output is also routed to the radar transmitter exciter. This permits the radar and communications signals to be amplified in common first and second IF amplifiers.

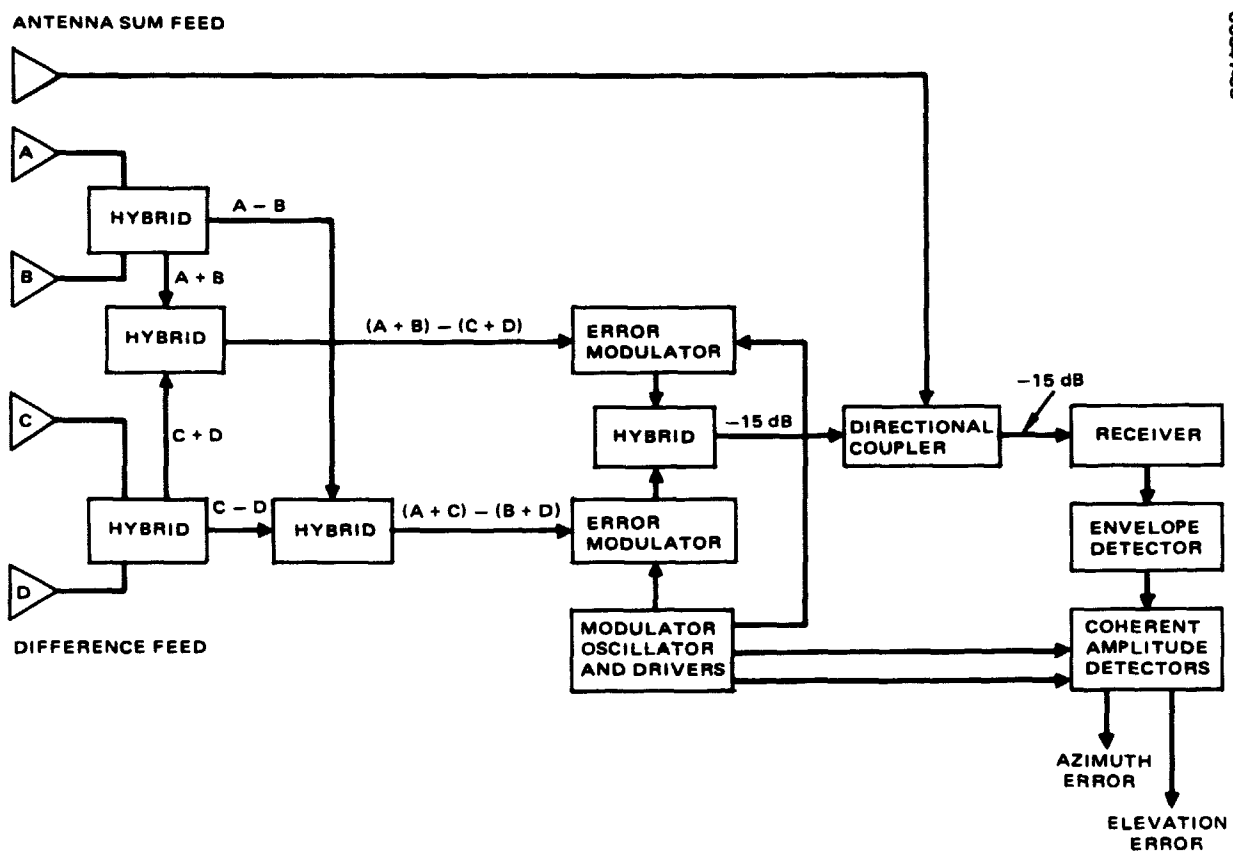


FIGURE 3-4. SINGLE CHANNEL MONOPULSE

FOLDOUT FRAME 1

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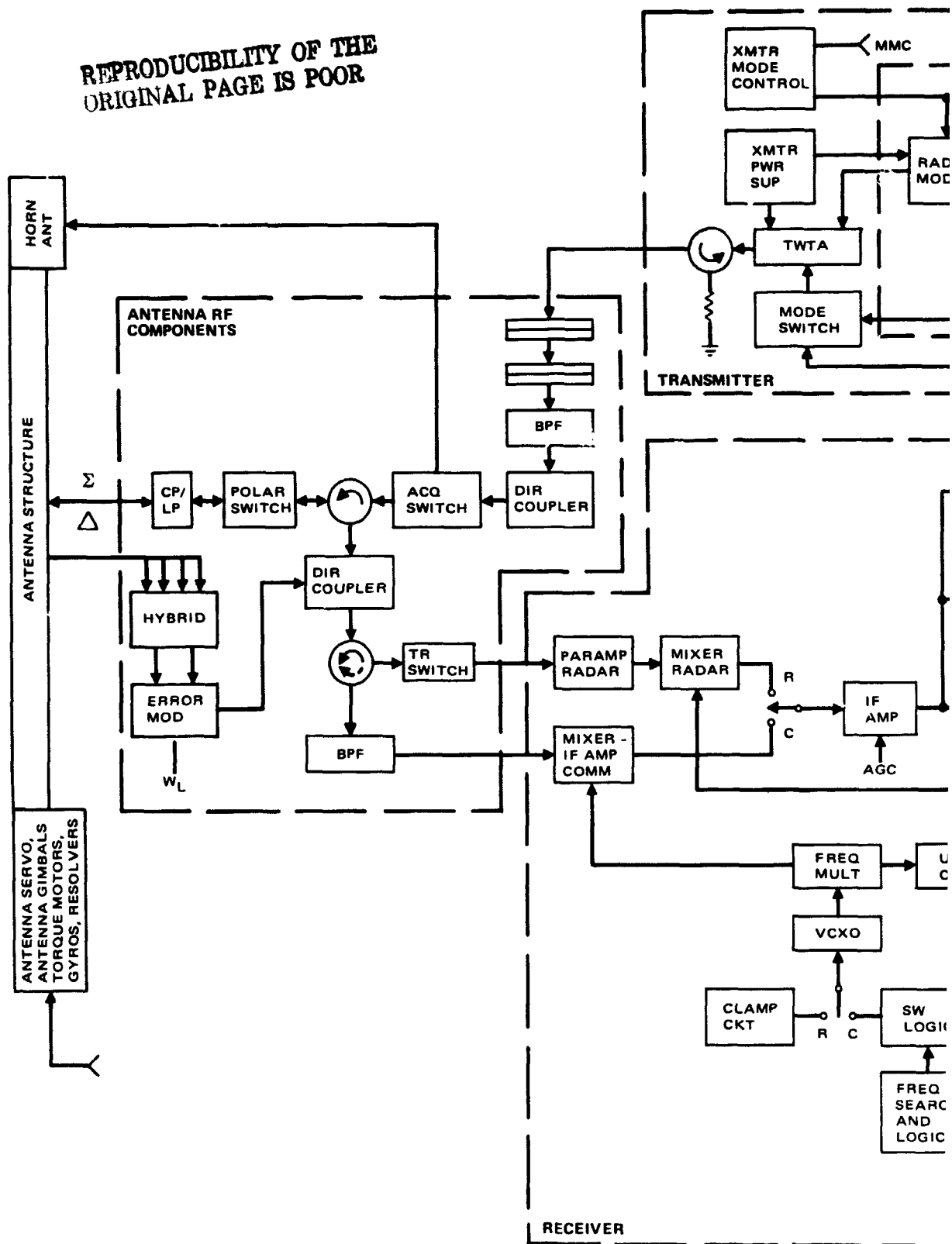
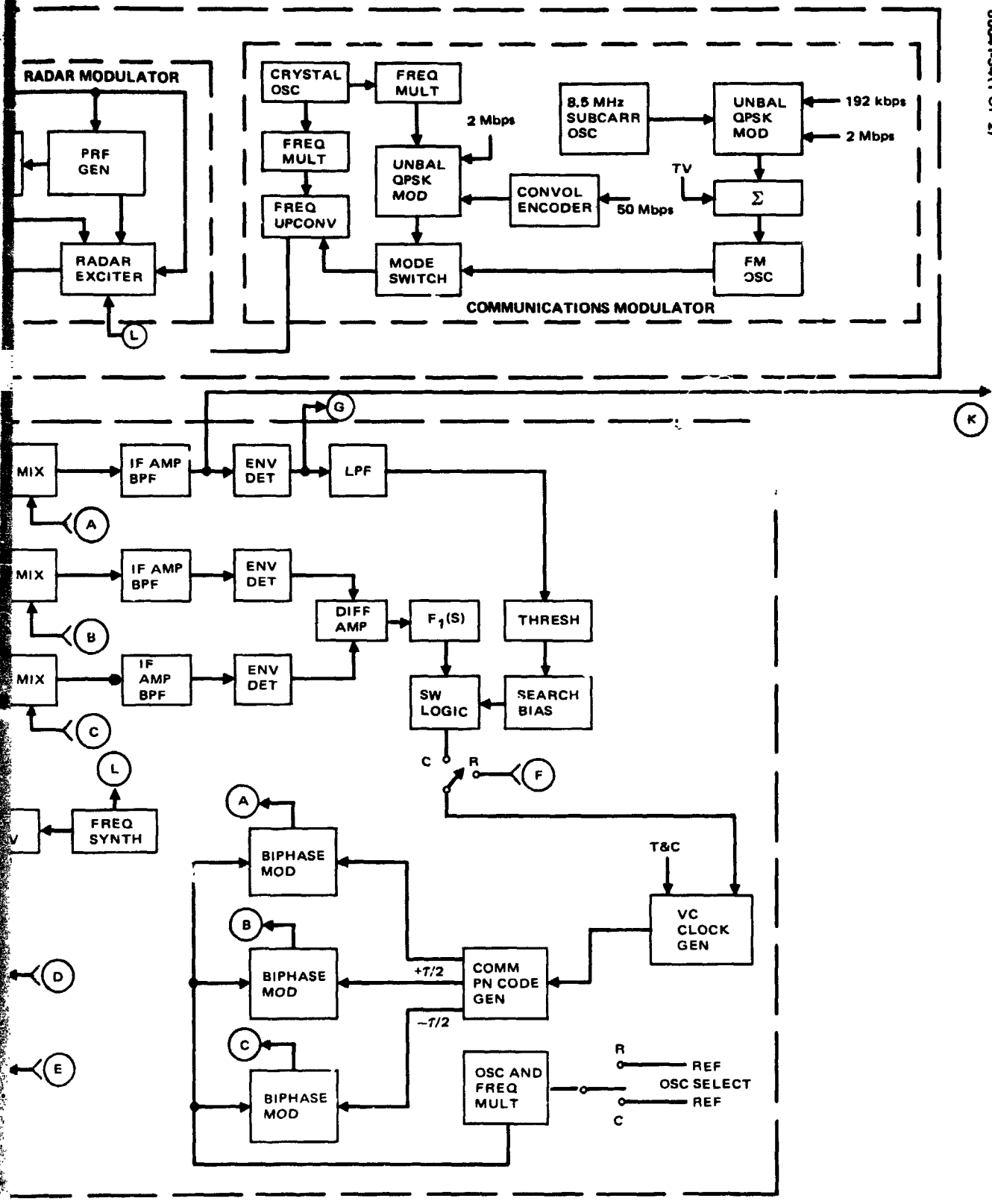


FIGURE 3-5. SPACE SHUTTLE ORBITER K-BAND RENDEZVOUS RADAR/COMMUNICATIONS SUBSYSTEM FUNCTIONAL BLOCK DIAGRAM



FOLDOUT FRAME 1

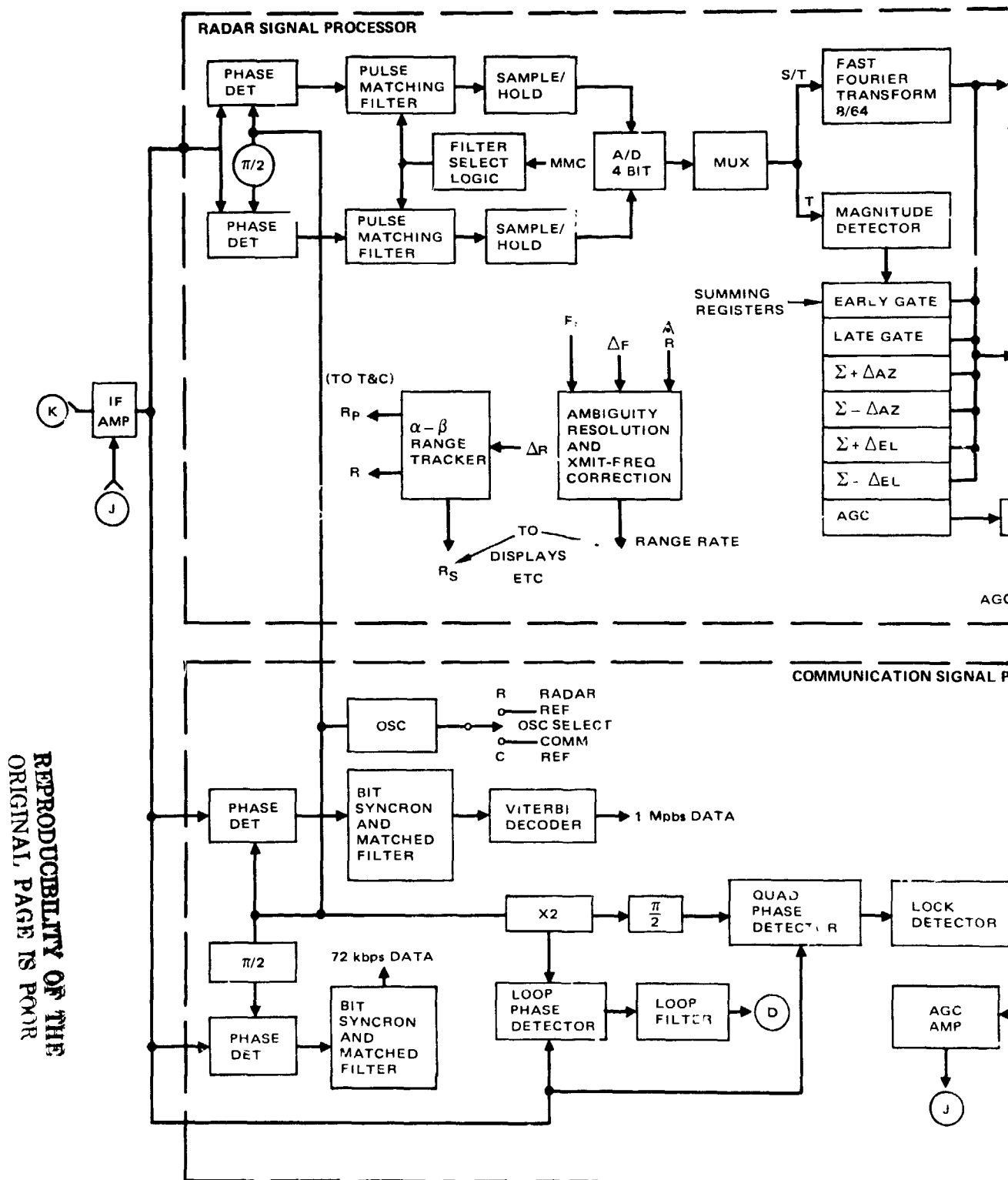
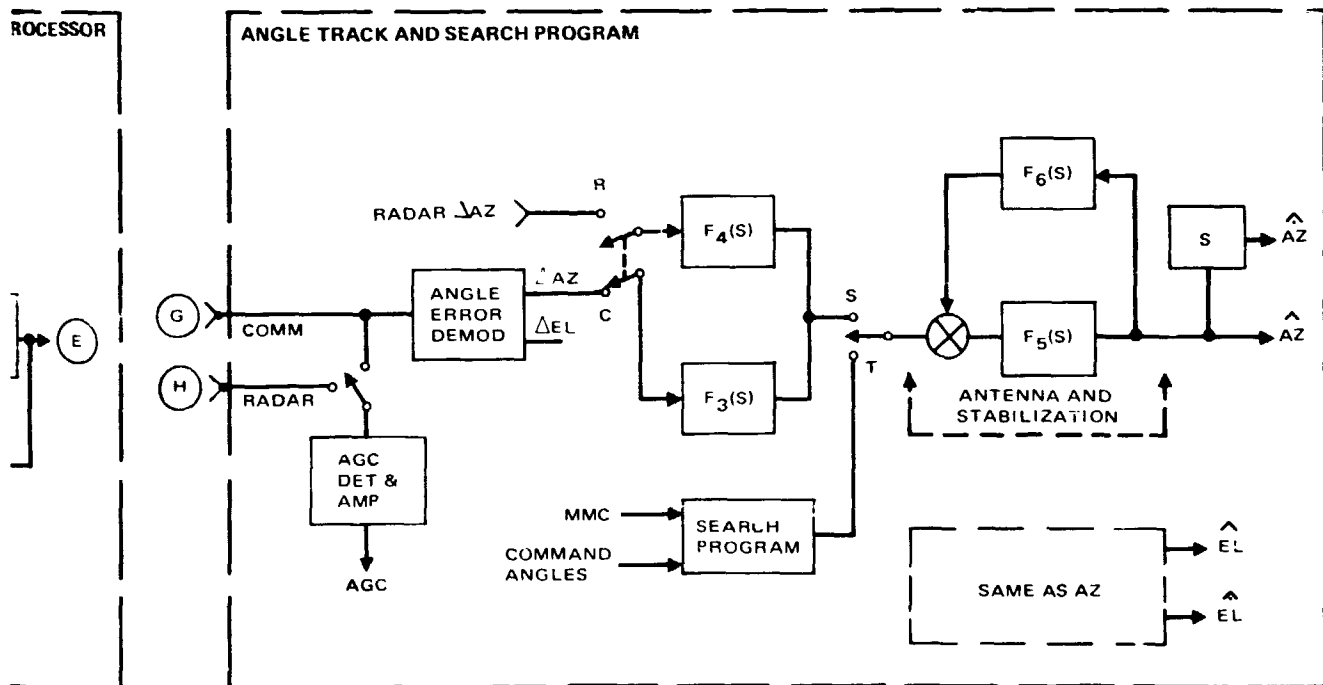
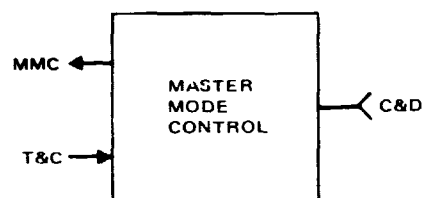
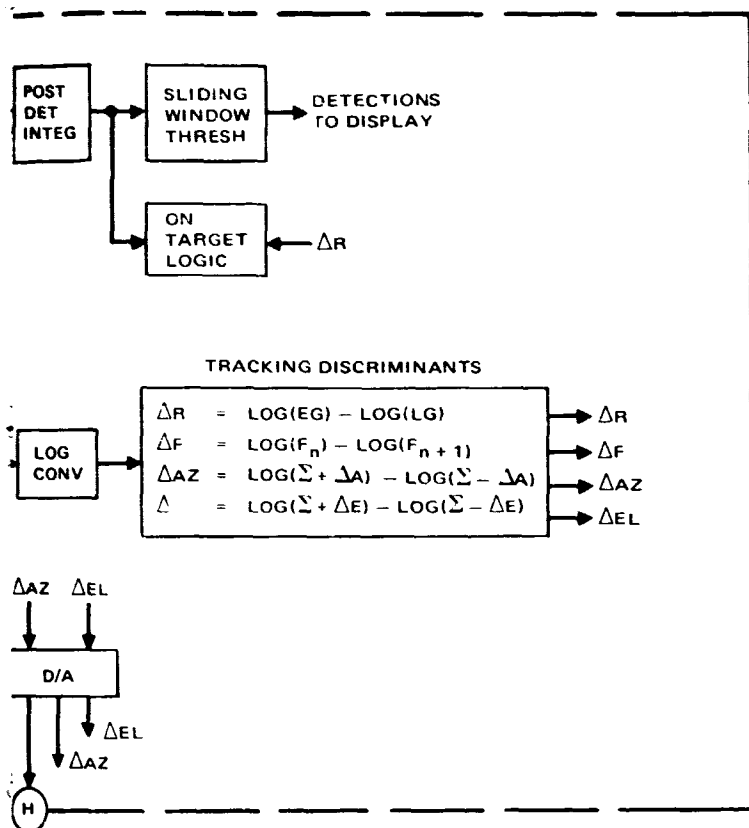


FIGURE 3-5 (CONTINUED). SPACE SHUTTLE ORBITER Ku-BAND RENDEZVOUS RADAR/COMMUNICATIONS SUBSYSTEM FUNCTIONAL BLOCK DIAGRAM



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3.2 COMMUNICATIONS SUBSYSTEM

The communications transmitter requires two modes of operation (see Figure 3-6). In the first mode an unbalanced quadriphase modulated signal is transmitted where the modulation consists of two asynchronous digital data streams, one with data up to 2 Mbps and the other with up to 50 Mbps data. The high data rate stream is NRZ data and is convolutionally encoded in five parallel encoders. The input 50 Mbps data are reclocked and amplified, demultiplexed into five parallel, interleaved data streams, convolutionally encoded by the five parallel encoders at rate one-half, and finally reassembled into a serial 100 Mbps data stream in the parallel to serial converter. Both the 2 and the 100 Mbps data streams biphasically modulate a carrier generated by a crystal oscillator and frequency multiplier. The biphasic modulator output signals are summed in phase quadrature. The required power ratio of 4:1 between the high and low data rate signals is implemented by a variable attenuator in the path of the 2 Mbps signal. The output of the summer goes to a mode selection switch that selects the appropriate input as a function of the communications mode. The switch output drives a frequency upconverter which upconverts the L-band modulated signal to the transmitter output frequency and drives the TWT. The injection frequency to the upconverter is supplied by a crystal oscillator and frequency multiplier chain. A band-pass filter at the TWT output rejects TWT noise at the receive frequency. The output of the TWT can be switched either to the main antenna for normal transmission of data or to a widebeam antenna which is used to ensure illumination of TDRSS after TDRSS has acquired the shuttle signal and while the shuttle angle search is still in progress.

Since the signal format was given, only one tradeoff was considered. This tradeoff was whether to 1) use direct modulation of the carrier at the output carrier frequency or 2) implement the system as shown in the block diagram (Figure 3-6). The latter method was chosen because of superior performance of the biphasic modulators in phase and amplitude unbalance at lower frequencies. In addition, it is substantially easier to control relative phase which has less variation with temperature at the lower frequency.

In the frequency modulated mode, three channels of data are transmitted. Two are biphasically modulated onto an 8.5 MHz subcarrier oscillator and summed in quadrature to form an unbalanced quadriphase modulated subcarrier. This signal is summed with either a dc to 4 MHz analog signal or 4 Mbps digital data signal, amplified, and used to frequency modulate an FM oscillator. This signal is then routed through the mode selection switch and frequency upconverted as before.

Selection of the oscillator frequency and upconverter injection frequency is of importance. The frequency should be sufficiently high such that the required FM deviation can be achieved with good linearity. On the other hand, it is desirable to keep the frequency relatively low in order to achieve good phase and amplitude balance of the phase modulated signals. A final constraint is that the upconversion ratio should be between the approximate limits of 4:1 to 12:1. If this ratio is too high, the upconverter output filter requirements become difficult. If the ratio is too low, in-band harmonics of the wideband data signal may be produced in the upconverter.

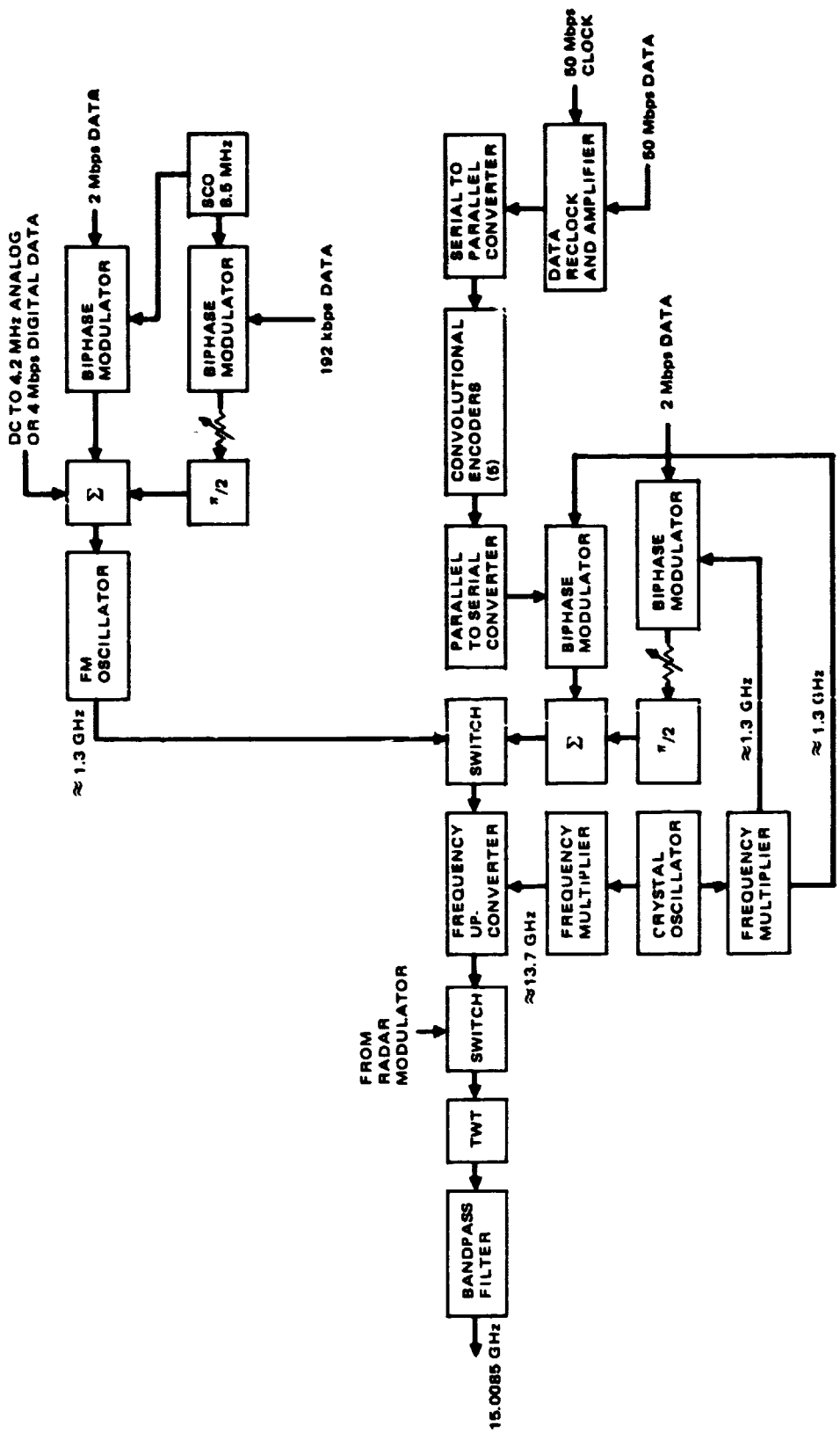


FIGURE 3-6. SHUTTLE KU-BAND TRANSMITTER BLOCK DIAGRAM

A block diagram of the communications receiver is shown in Figure 3-7. For the purpose of clarity, the radar interface and some receiver front end details have been omitted. The receiver consists of three major parts: receiver front end, pseudo-noise (PN) despread section, and phase lock data demodulator. These parts are discussed below.

The receiver sum channel signal passes through a bandpass filter that attenuates the transmitter signal to a level which precludes the generation of undesired spurious responses. The azimuth and elevation error outputs are modulated in biphase modulators, combined in a hybrid, and summed with the receiver sum channel signal in a 15 dB coupler. Relative phasing of the microwave circuits is set so that the error signal modulation appears as amplitude modulation of the carrier. Amplitude rather than phase modulation was chosen for simplicity of detection and to minimize interference with the phase modulated data. The output of the directional coupler is then mixed with the frequency multiplied output of the voltage controlled crystal oscillator (VCXO) to close the phase locked loop.

An image recovery mixer was chosen rather than a parametric amplifier since the resulting system G/T is adequate for this application (see subsection 4.1). The image recovery mixer noise figure is expected to be approximately 4 dB based on less than 3 dB conversion loss and 1 dB noise figure 1F amplifier.

The receiver PN despreader is a delay lock synchronizer. Three local replicas of the PN code, each separated by 1/2 bit period are generated and used to drive three biphase modulators. When the delay lock loop is locked, the receiver data channel is in nearly exact synchronism with the incoming signal and the PN spread spectrum modulation is tracked out. One of the remaining channels is advanced by 1/2 bit and the other is delayed by 1/2 bit. The detected outputs of these two channels are compared in a differential amplifier and an error signal is generated proportional to the amount the loop is out of synchronization and used to control the local clock frequency. The correlation and error functions of the despreader are shown in Figure 3-8.

Acquisition of code synchronization is accomplished by either 1) applying a search bias or 2) periodically deleting a pulse in the pulse train generated by the voltage controlled clock oscillator. The latter method is preferred since the required frequency pulling range of the clock oscillator is less than if a search bias is used. The clock oscillator is crystal controlled since frequency errors result in longer search time or degraded acquisition sensitivity. When code synchronization is approached, signal energy in the data channel increases as the spectrum "collapses" to approximately match the IF bandpass filter bandwidth. This is sensed as a sudden increase in the output of the envelope detector which exceeds a preset threshold voltage in the threshold circuit, causing the search function to stop.

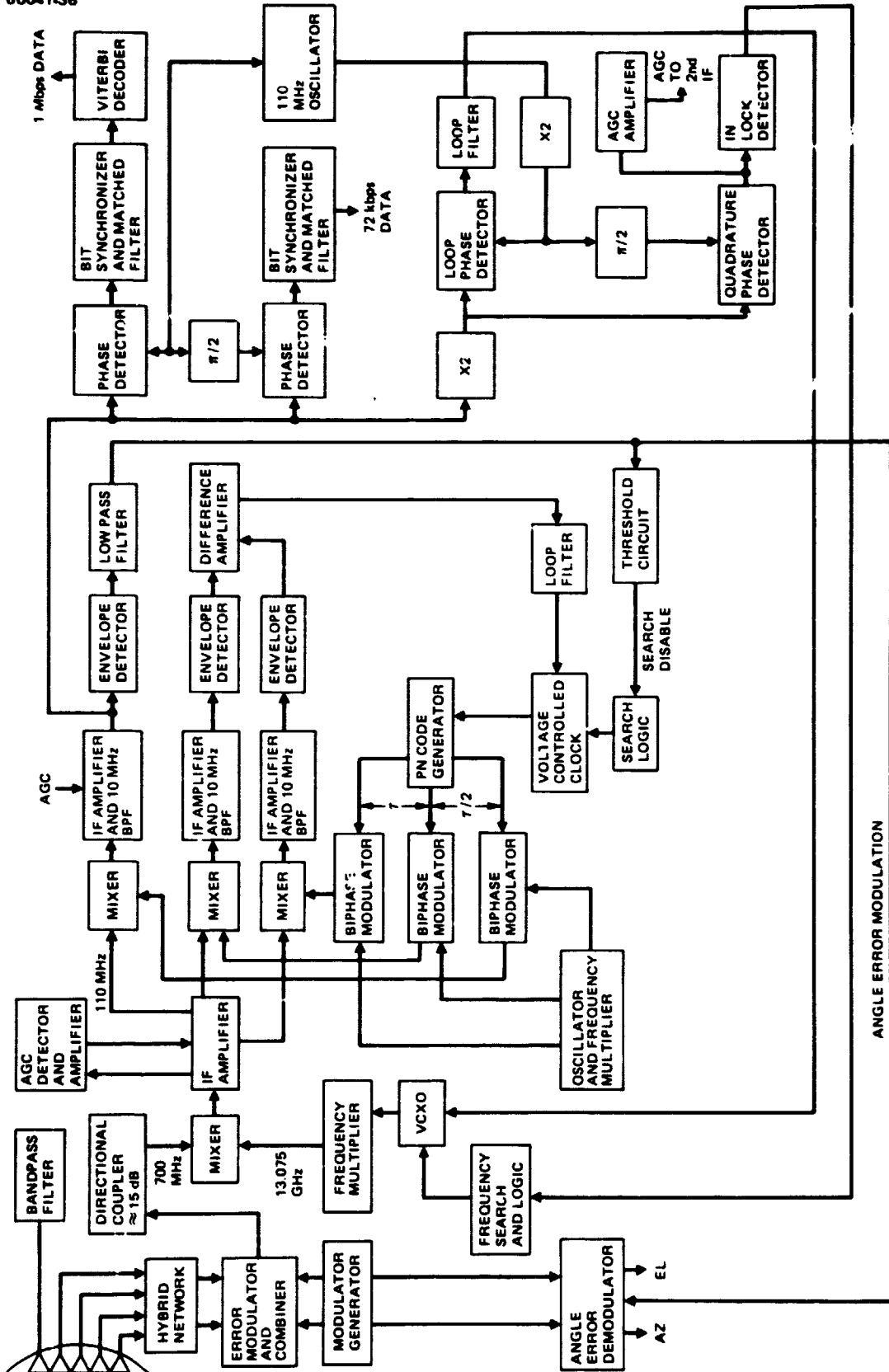


FIGURE 3-7. SHUTTLE KU-BAND RECEIVER BLOCK DIAGRAM

* EXISTING DESIGN AVAILABLE

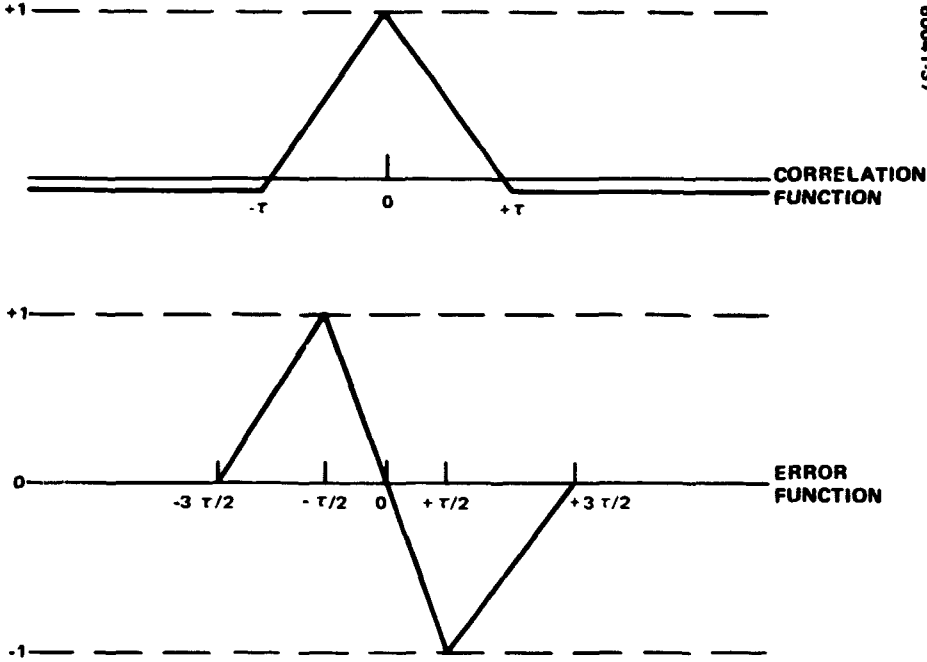


FIGURE 3-8. DESPREADER CORRELATION AND ERROR FUNCTIONS

The angle error amplitude modulation is detected in the data channel and routed to the angle error demodulator when it is synchronously compared with the time shared modulation signal of the modulation generator. Time division was chosen in preference to phase division because of less crosstalk between azimuth and elevation error signals and superior performance in the radar mode.

The phase locked data demodulator chosen uses a times 2 implementation. The input modulated signal is frequency multiplied by 2. Since the signal is unbalanced quadriphase with an amplitude ratio of 2:1, the resulting output is a signal square wave phase modulated between two phase states. Peak phase modulation θ is given by

$$\theta = 2 \arctan A$$

where A is the amplitude ratio of the low data rate signal to the high data rate signal and is one-half in this case. The resulting peak modulation is 53.1° and the carrier power is 4.4 dB below the total received power. Phase locked loop signal-to-noise ratio (SNR) can then be determined from system parameters. The data threshold occurs at an E_b/N_o of about 9 dB for the 1 MHz data. Total signal power is 1 dB greater for the specified 4:1 power ratio. Signal-to-noise ratio in a 10 MHz predetection bandwidth is, therefore, 0 dB. It can be shown that the loop SNR for this case is

$$\text{Loop SNR} = \alpha \frac{S_i/N_i}{2 \left\{ 2 + \frac{N_i}{S_i} \right\}} \cdot \frac{B_{IF}}{2 B_L}$$

where

α = carrier suppression ratio, 4.4 dB

S_i/N_i = predetection signal-to-noise ratio, 0 dB

B_{IF} = IF bandwidth, 10 MHz

$2 B_L$ = phase locked loop two-sided noise bandwidth, 10 kHz

The resulting loop signal-to-noise is 17.8 dB, more than adequate for this case.

Other implementations of the phase lock demodulator were considered during the study. The I-Q demodulator is capable of performing the function but requires a true multiplier device operating from dc to about

6 MHz which is difficult to implement. Also considered was the modulation "wipe off" demodulator which detects the data, reconstructs the input signal, which is then phase compared with the original signal. This implementation has poor postdetection signal-to-noise ratio in the data streams which are used to regenerate the signal spectrum. This would also cause some degradation in data quality. Since the frequency multiplied by 2 scheme is simple and provides good loop signal-to-noise with virtually no data degradation, it was considered the best choice.

Receiver phase lock and despread parameters are given in Table 3-6.

TABLE 3-6. RECEIVER DESPREAD AND PHASE LOCK PARAMETERS

PHASE LOCK PARAMETERS	
Frequency search range	±520 kHz
Frequency search rate	10 MHz/sec
Acquisition time	0.1 sec
Noise bandwidth	10 kHz
Damping ratio	0.7
Loop gain	33×10^6
DESPREAD PARAMETERS	
Nominal search rate	1.6 kHz
Acquisition time ($C/N_0 = 73 \text{ dB-Hz}$)	1.9 sec
Post detection bandwidth	4 kHz
Tracking loop noise bandwidth	4 kHz

Operation of the receiver AGC is of interest. Two forms of AGC are used. The noncoherent first IF AGC is derived from a broadband first IF and maintains a constant noise level into the despread decision circuit. Noise threshold of this AGC is well above the despread threshold so that the AGC will not track out signal increase in the second IF bandpass due to PN synchronization. The coherent second IF AGC is derived from the demodulator quadrature phase detector and controls only the gain of the second IF amplifier stage which supplies the input signal to the phase lock demodulator. Frequency response of the AGC loops should be kept below about 10 Hz so that low frequency angle error modulation will not be tracked out. The resulting AGC loop noise bandwidths will be approximately an order of magnitude greater depending on AGC loop gain.

3.3 RADAR SUBSYSTEM

3.3.1 Radar Functional Description

The rendezvous radar is a low PRF coherent radar which is integrated with the Ku-band communications subsystem to provide maximum commonality between the radar and communications functions. This design employs the same transmitter, antenna, RF microwave components, receiver front ends, IF amplifier, antenna servo, and angle tracking circuits for both radar and communications. The unique items are separate radar and communications signal processors.

The basic philosophy in the radar design is simplicity consistent with performance requirements which ultimately leads to the lowest cost. The radar signal processor employs digital signal processing that provides maximum performance and reliability with minimum size, weight, power, and cooling requirements. The radar has three basic modes of operation: search, acquisition, and track. These operations are described below. Tradeoffs and analyses leading to this design are discussed in subsection 4.2.2.

3.3.1.1 Search

The radar operates in a low PRF coherent mode of 4800 Hz. Frequency agility is used to enhance the detection of the target, 10 frequencies are cycled over during the on-target time. Three range gates and coherent processing employing multiple (digital) doppler filters are used for detection.

The search waveform consists of a 60 μ sec transmit pulse and a 148 μ sec listening period as shown in Figure 3-9. A long transmit pulse is required to provide the energy for target detection, while the 148 μ sec listening period allows the return from a target at 22.2 km (12 n. mi.) to be received before the next transmission. This provides a PRF of 4.8 kHz and a transmitter duty factor of 0.29, for an average transmitter power of 14.5 watts. For Mission I/II, a total of 64 pulses are coherently integrated for each of the 10 different RF frequencies during the on-target time as shown in Figure 3-10. Adjacent RF frequencies have a minimum separation of 25 MHz for a total bandwidth of 225 MHz. The on-target timing for Mission IIIB is similar to that shown in Figure 3-10 with 8 pulses integrated coherently for each of the 10 RF frequencies (1.67 ms for 8 pulses and 16.7 ms total on-target time). The smaller time on target is dictated by the faster antenna scan rate required to cover the larger Mission IIIB search field of view.

The antenna search pattern employs a shaped raster scan covering the 20° and 45° half-cone angle search volume required for Missions I/II and IIIB, respectively. (See Figure 3-11 which also shows other antenna parameters.)

During search operation, three range gates are formed in the signal processor at the ranges given in Table 3-7. A simplified diagram of the search signal processor is shown in Figure 3-12.



FIGURE 3-9. TRANSMIT WAVEFORM

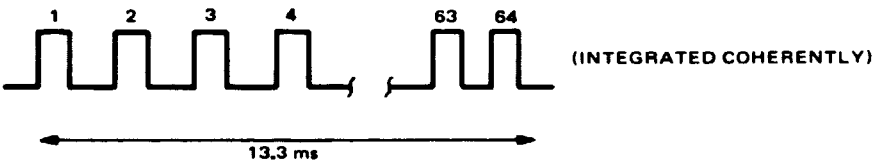
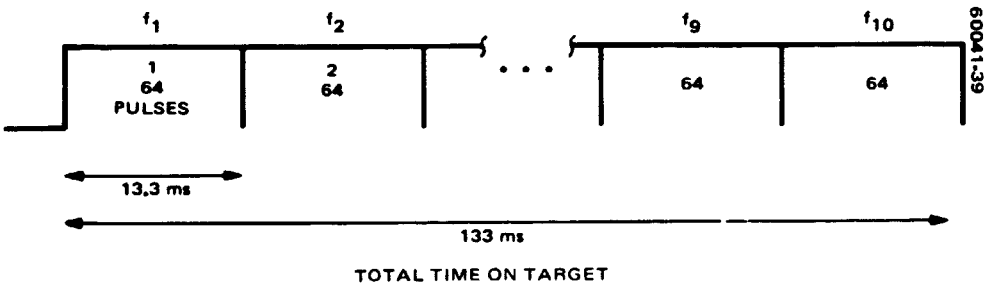


FIGURE 3-10. ON TARGET TIMING MISSION I/II

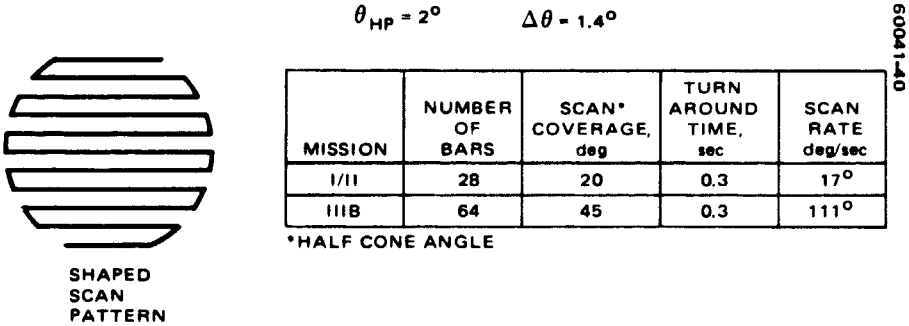


FIGURE 3-11. SHAPED ANTENNA ROSTER SCAN

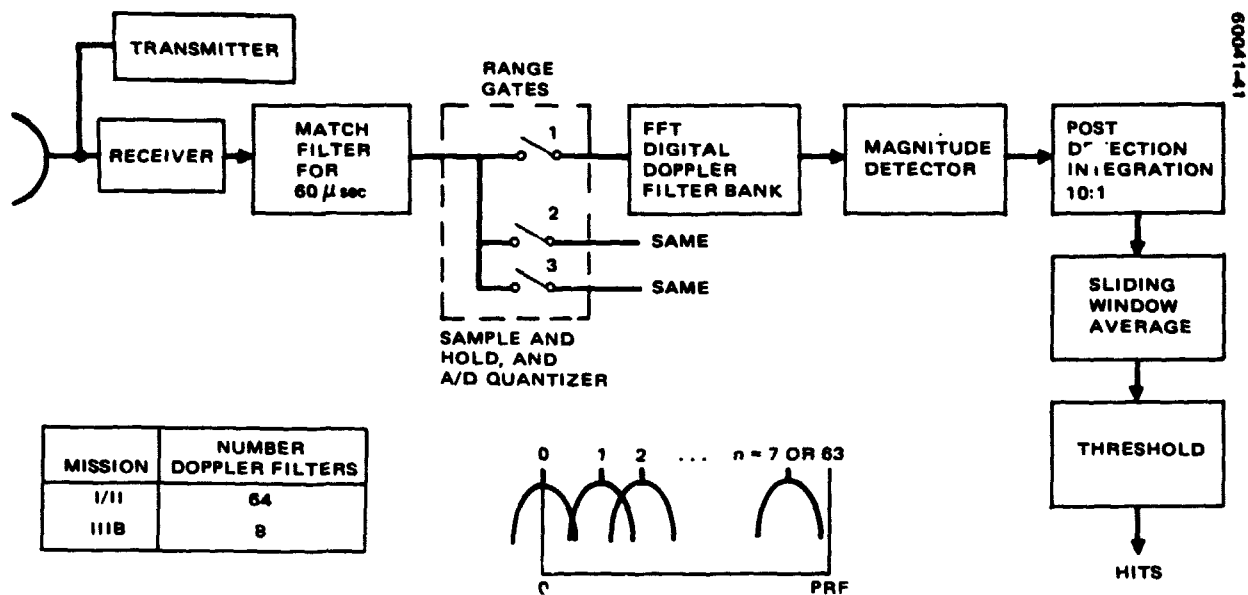


FIGURE 3-12. SEARCH SIGNAL PROCESSOR

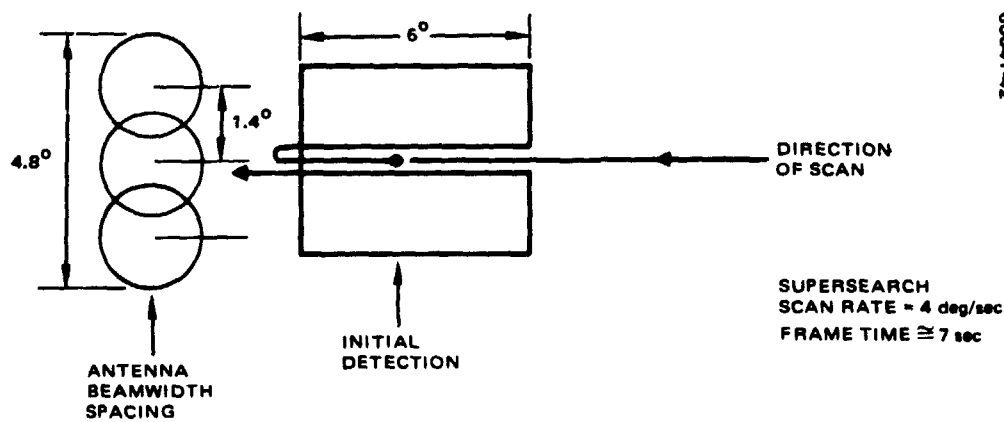


FIGURE 3-13. ACQUISITION SUPER SEARCH

TABLE 3-7. RANGE GATE SAMPLE RANGES

Range Gate	Mission I/II, km (n.mi.)		Mission III B km (n.mi.)	
	km	(n.mi.)	km	(n.mi.)
1	9.3	(5)	9.3	(5)
2	17.6	(9.5)	12.9	(7)
3	22.2	(12)	17.6	(9.5)

The sample and hold will sample the output of the matched filter at the range intervals specified in Table 3-7. Three gates (or samples) are used to be consistent with the tracking (discussed below). The analog-to-digital (A/D) quantizer converts the samples to 4 bit digital quantities for further processing. Either 64 or 8 doppler filters are formed via a fast Fourier transform (FFT). The output of each doppler filter formation is magnitude detected and then added in the PDI (postdetection integration) to other filter outputs obtained for each transmitter frequency. (Note: the FFT will produce an output pulse for every 64 (or 8) input pulses.) A sliding window average over eight doppler filters is used to establish a noise reference level about the filter being examined. A hit (or detection) is declared when the test filter output exceeds this value by a factor of K

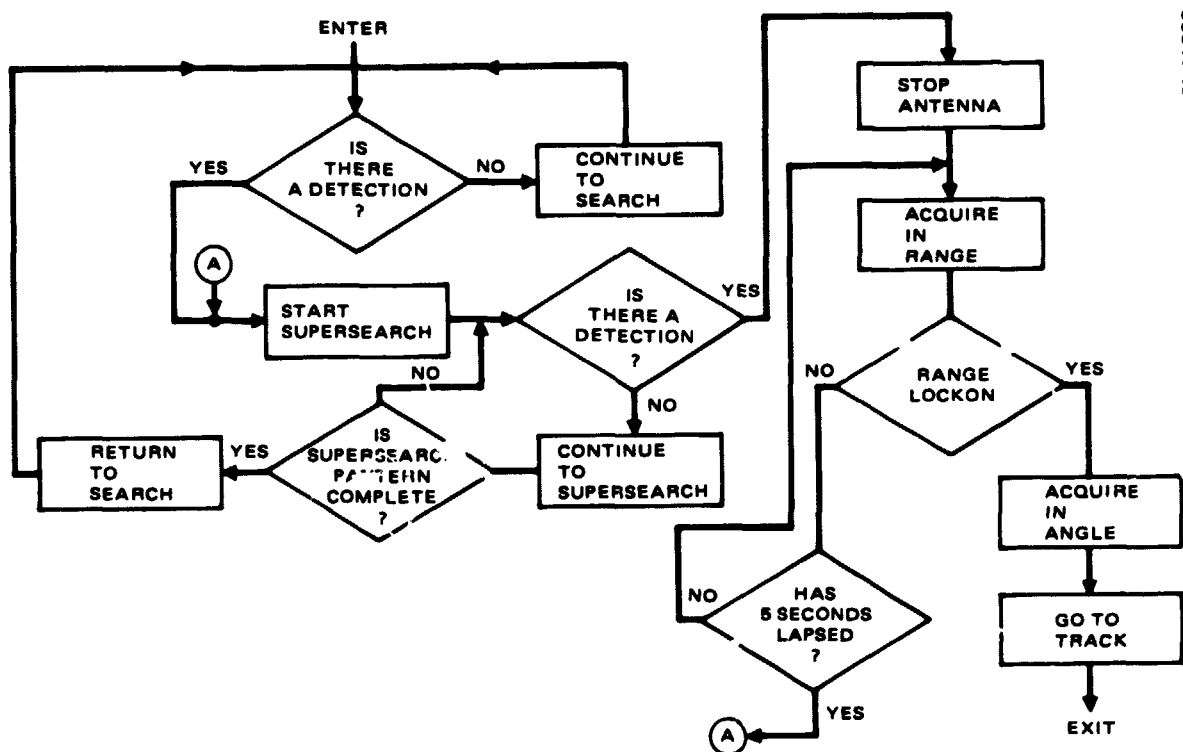
$$F_m \geq \frac{K}{8} \sum_{i=0}^7 F_{m-4+i}$$

where F_m is the output of the m^{th} filter and K is the threshold determined by the false alarm probability required for search ($p_{fa} \cong 10^{-6}$). Once the hit or detection occurs, acquisition then begins.

3.3.1.2 Acquisition

A hit or detection will signal the antenna programmer to discontinue the search pattern and go into a slow supersearch about the expected target location. A three bar center retrace pattern is used as shown in Figure 3-13. During acquisition, the same pulse length (60 μsec), PRF (4.8 kHz), and frequency agility are used as in search.

The supersearch program will cause the antenna to search $\pm 3^\circ$ about the hit and use a three bar pattern to allow for targets slightly off the center bar. The total time for one frame is 7 seconds. In the event that there is a false alarm, the scan continues on the same bar 7 seconds after the supersearch is initiated. The scan rate during supersearch is 4 deg/sec which allows four separate looks at the target during the one target time. With a



60041-43

FIGURE 3-14. ACQUISITION LOGIC

single look probability of detection of 0.99, the cumulative detection probability over four looks is very close to unity. Therefore, a target will be detected given that it is detected initially in the normal raster scan.

A second detection during supersearch will cause the antenna to stop and range and angle acquisition will begin. First the tracking range gate is slewed to the range corresponding to the detection range gate and the target is locked on in range. Immediately upon range lockon, angle tracking error signals are generated to cause the antenna servo to drive the antenna bore-sight to the target. In the event that lockon does not occur within 5 seconds* after the second detection, the antenna programmer will return to the slow supersearch and repeat the acquisition process. The acquisition logic is shown in Figure 3-14.

3.3.1.3 Track

The range track function is performed by a conventional early-late gate range tracker. Angle tracking is performed by a sequential lobing technique. Angular rates are measured by gyros on the antenna; closing rate is determined from doppler through open loop estimations of the doppler by using the doppler filter bank. The transmitter is operated coherently, however, noncoherent processing (i.e., envelope detection) is used in both the range and angle tracking processing. A functional block diagram of the complete digital processor is shown in Figure 3-15.

The range and angle processing employ three gates or sample periods to form early, on target, and late gates as shown in Figure 3-16. The early and late gates are used to generate the range tracker error while the on-target gate is used for both the angle error generation and the doppler filter generation (for closing rate determination). The same PKF and transmitter frequency cycling are used for both track and search. This is illustrated in Figure 3-17. For each transmitter frequency cycle, a total of 64 pulses are processed where each 8 pulse group is used for the sequential angle error generation (i.e., sum plus difference is cycled)

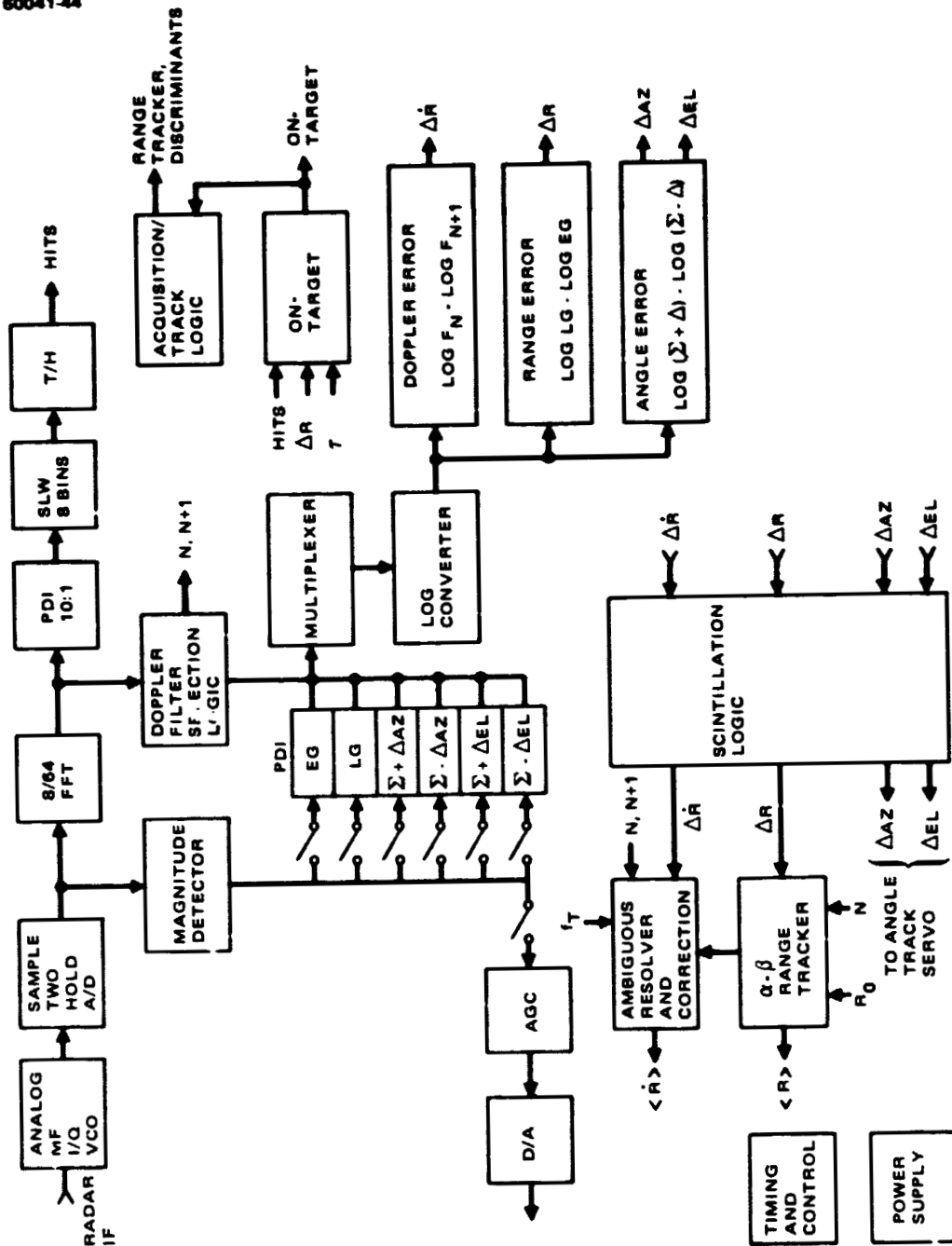
$$\Sigma + \Delta AZ$$

$$\Sigma + \Delta EL$$

$$\Sigma - \Delta AZ$$

$$\Sigma - \Delta EL$$

* Maximum line of sight angular rate at acquisition is 4 mr/sec; and 5 seconds could place the target outside the antenna beamwidth.



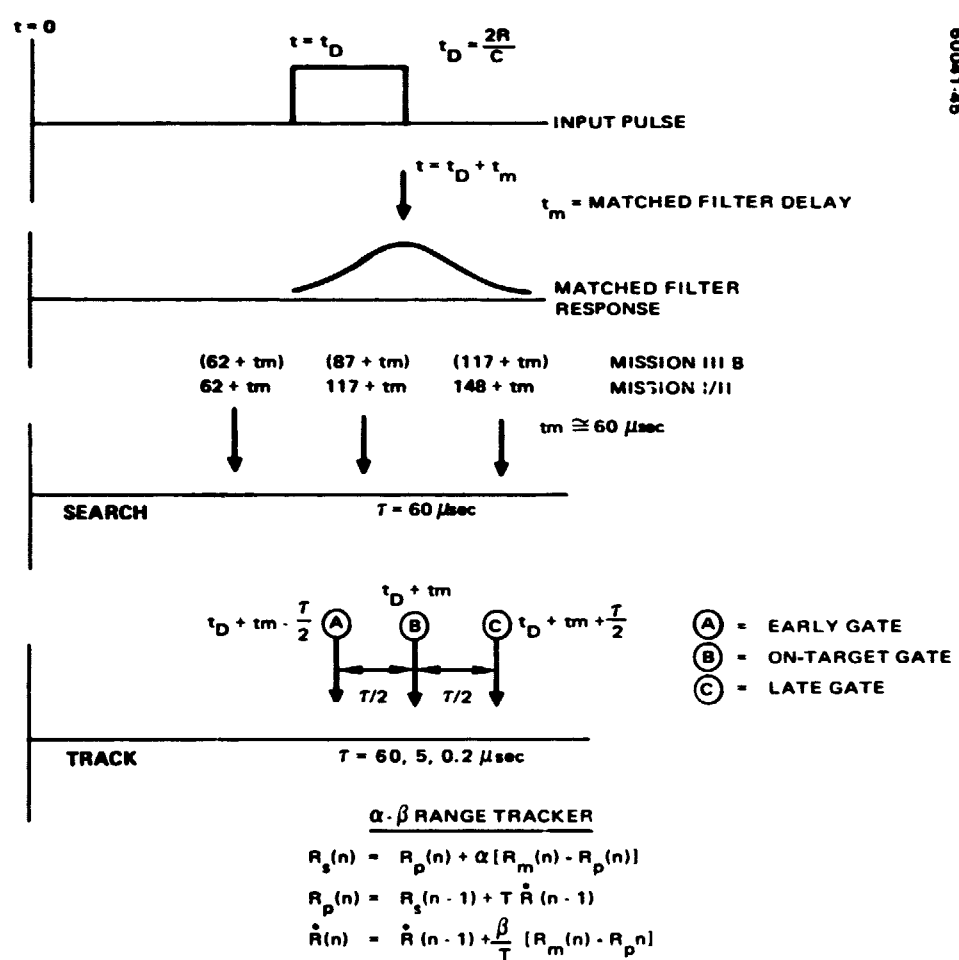


FIGURE 3-16. PROCESSOR TIMING

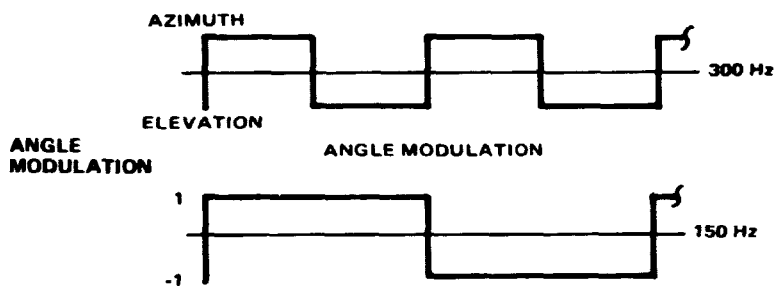
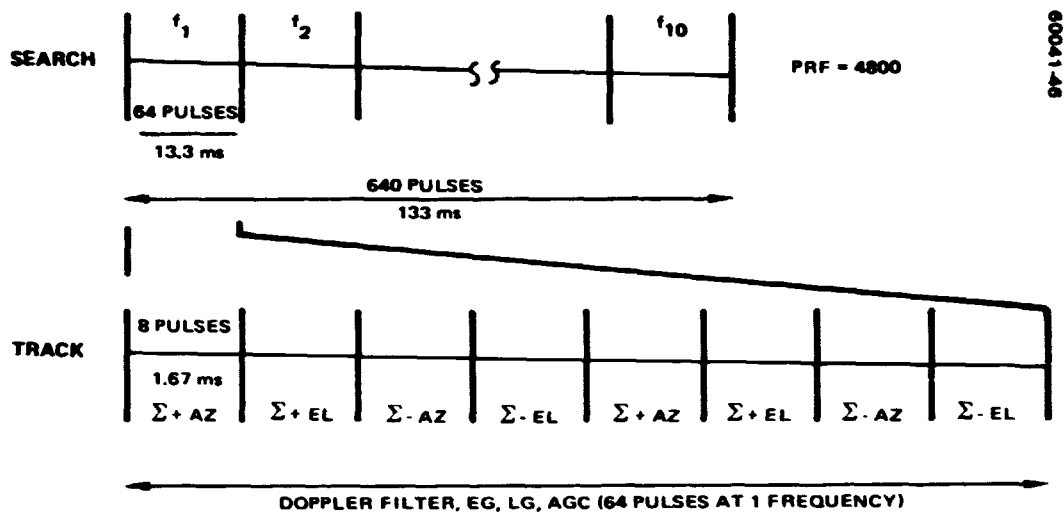


FIGURE 3-17. PRF, RF, AND ANGLE TRACKING TIMING

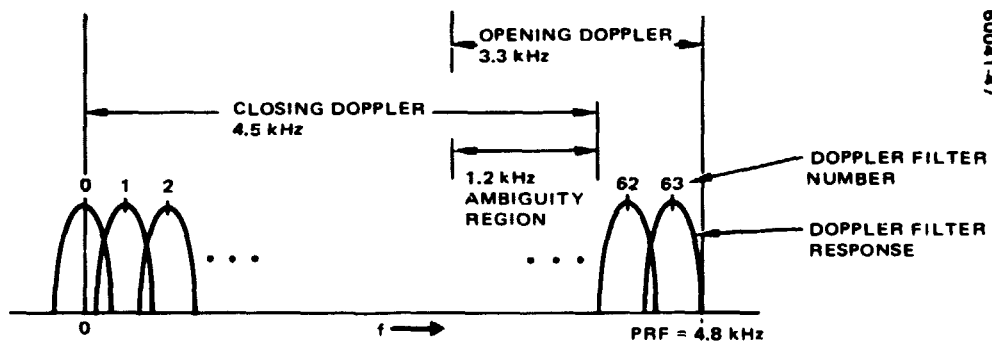


FIGURE 3-18. DOPPLER FILTER BANK CONFIGURATION

at a 300 Hz rate. The above signals are summed over the 16 pulses and then over the 10 transmitter frequencies to increase the effective SNR and at the same time to reduce the effects of scintillation. The error signals are generated by taking logarithms of the summed signals

$$\log (\Sigma + \Delta) - \log (\Sigma - \Delta) = 2 \frac{\Delta}{\Sigma}$$

where

$$\log (\Sigma + \Delta) = \log \Sigma + \log (1 + \Delta/\Sigma)$$

$$\log (\Sigma - \Delta) = \log \Sigma + \log (1 - \Delta/\Sigma)$$

and

$$\log (1 + \Delta/\Sigma) \cong \Delta/\Sigma \text{ for } \Delta/\Sigma < 1$$

This algorithm is also used for range, angle, and doppler processing. An added feature is that it is self-normalizing compared to an alternate algorithm of the form

$$\frac{|\Sigma + \Delta| - |\Sigma - \Delta|}{|\Sigma + \Delta| + |\Sigma - \Delta|} \cong \frac{\Delta}{\Sigma}$$

which requires division to normalize.

The angle error signals AZ and EL go to the angle tracking servo system. The range error signals are applied to an α - β tracker (equations given in Figure 3-17). While this tracker produces both a range and range rate output, only range is outputted. Range rate is used to resolve any ambiguity that may exist in the doppler measurement.

Closing rate is determined by using the same digital doppler filter bank as in search. This is illustrated in Figure 3-18. Since the expected target range rate will vary from 45 m/sec (150 fps) closing to 15 m/sec (50 fps) opening, the doppler will lie between +4.5 and -1.5 kHz. And because of the aliasing, an opening doppler will fold over as illustrated in Figure 3-18. From this figure there is an ambiguity region of 1.2 kHz (12 m/sec (40 fps)) as to whether a target has a high closing or opening rate. This ambiguity will be resolved by the range tracker using its range

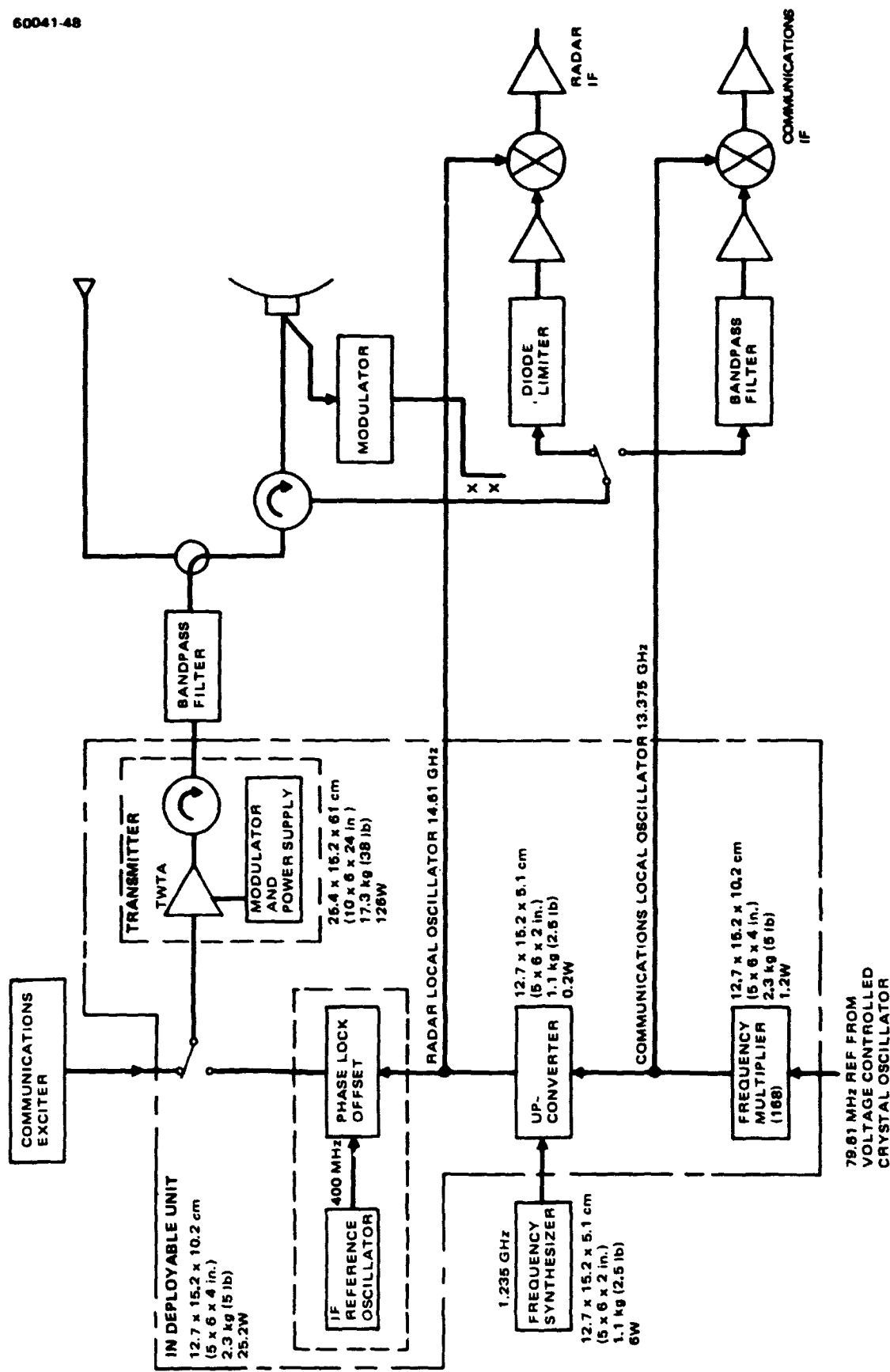


FIGURE 3-19. RADAR ELECTRONICS BLOCK DIAGRAM

rate estimate (which has an accuracy adequate enough to resolve the ambiguity but not meet the system accuracy.)*

Since 64 doppler filters are formed digitally, each doppler filter will have a bandwidth of approximately 75 Hz. This corresponds to a closing rate of 0.76 m/sec (2.5 fps). To get a finer estimate on the closing rate, the output of two adjacent doppler filters are used to form a discriminant by taking the difference of the logarithms of the output. This will give an output to a fraction of a meter per second. Since the transmitter frequency agility bandwidth is 225 MHz (± 122 MHz with regards to center frequency), a correction must be made that corresponds to the transmitter frequency and the filter number where target is detected. If uncorrected, the closing rate error could be as large as ± 0.3 m/sec (± 1 fps) for high closing rate targets.

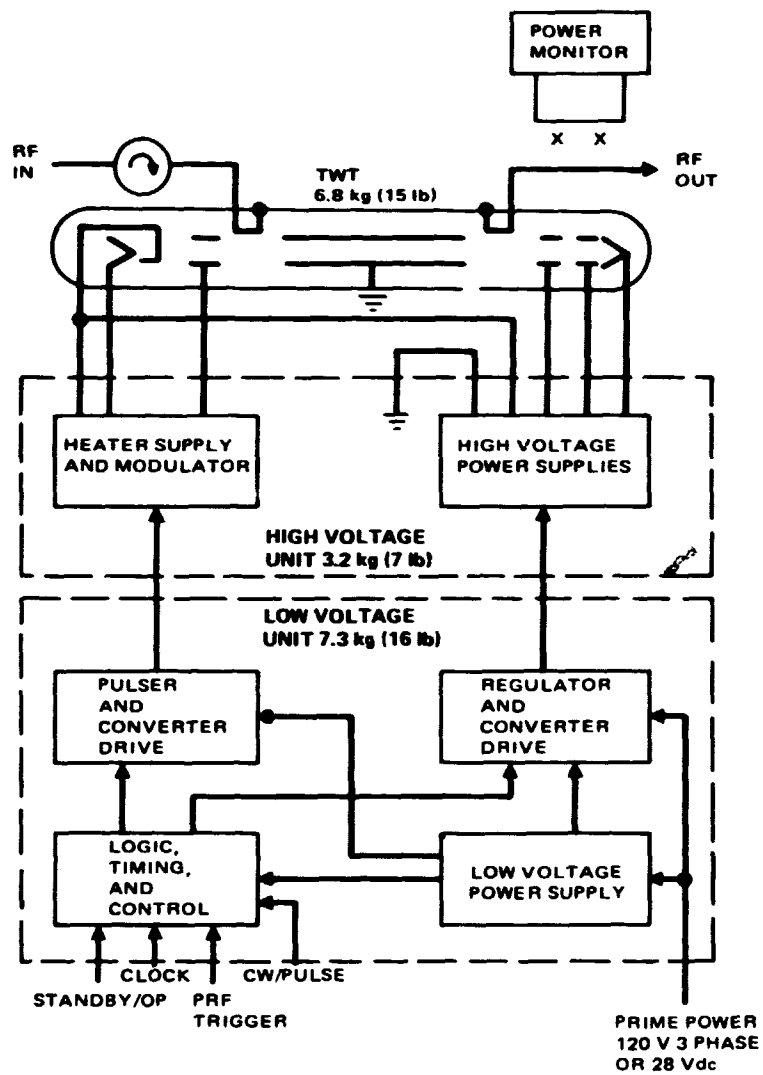
The effects of scintillation on both range and angle tracking are handled by means of the frequency averaging over the 10 transmitter frequencies. Both the range and angle tracker will have closed loop bandwidth of less than 1 Hz and the input error data will be supplied to the tracker at a 7.5 Hz rate.

The automatic gain control functions to hold the input to the A/D converter relatively constant. Since the range variation is from 27 km (12 n. mi.) down to 30 meters (100 feet), the required dynamic range is about 90 dB, considering the change in pulse length at the shorter ranges. Target size variation can be 20 dB or more. Therefore, the total adjustable dynamic range of the receiver must be on the order of 110 dB. The instantaneous dynamic range can be much less however.

3.3.2 Radar Transmitter/Receiver Electronics

The electronics required for radar operation and its relationship to that required for communication operation is outlined in Figure 3-19. Functionally, the 79.61 MHz reference frequency received from the VCXO is frequency multiplied by a factor of 168 to provide the required 13.375 GHz communication local oscillator (LO) signal. The radar LO is generated by upconverting the communication LO with a variable offset frequency provided by a frequency synthesizer. The synthesizer generates 10 frequency agile channels with a nominal 25 MHz channel spacing. As shown in Figure 3-19, the radar LO signal is offset by the 400 MHz IF frequency in a phase lock loop to generate the RF transmit frequency. The narrow band tracking characteristics of the loop and the attendant ease of achieving the required transmitter drive power with low spurious response levels led to the selection of this IF offset approach. Once generated, this RF signal is then amplified in the transmitter before being sent to the antenna.

* Note that doppler resolution is not required in search since one is only interested in the presence absence of a target and not in the closing rate.



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FIGURE 3-20. TWTA GRID MODULATED TRANSMITTER

The transmitter, frequency multiplier, upconverter, and IF offset/biphase modulator are all part of the deployable unit. Size, weight, and dissipation of the functional elements are also shown in Figure 3-19.

The transmitter is a grid modulated TWT design using a modified Hughes 294H tube. The unit design is functionally outlined in Figure 3-20. The tube will provide 60 watts of CW power (50 watts minimum) with an efficiency of 45 percent; the transmitter will be used for both radar and communication operation. The modulator will dissipate approximately 20 watts in addition to the 106 watts of dissipation in the tube and power supply (based on CW operation and 85 percent efficiency of the supply). Since reflected power from the antenna during radar operation occurs at about the same frequency as the echo return, a passive diode limiter will be provided at the radar receiver input to reduce the power levels and to protect the front end components.

3.3.3 Radar Signal Processor

The signal processor receives the IF signal and LO reference signal from the receiver. The processor converts the IF signal to in phase and quadrature (I/Q) video which is then sampled each pulse repetition interval at three precise ranges. The separation of these range gates is equal to one-half the transmitted pulse width. The sampled video is then quantized to 16 levels and encoded into 4 bit digital words. The digital data corresponding to each range gate is filtered and thresholded. The threshold crossing or hit data are sent to the display. When a hit occurs the processor automatically enters an acquisition and track mode. In the track mode, the processor tracks the target in range and provides angle error data to the antenna servo system. The processor also generates an automatic gain control (AGC) voltage which is sent to the receiver. This AGC is required to maintain the target signal within the dynamic range of the A/D converter. A self-contained power supply operating from the 28 volts provides regulated voltages for the signal processing functions. A block diagram of the radar signal processor is shown in Figure 3-21. The following paragraphs describe the various functions of the signal processor.

3.3.3.1 Analog-to-Digital Converter

The IF signal is mixed with the reference signal to generate the I/Q video. The video is filtered by selectable bandwidth filters matched to the transmitter pulse width. The filtered video is sampled at three ranges and converted to 4 bit data which is sent to the filter processor.

3.3.3.2 Filter Processor

The filter processor performs a 64 point fast Fourier transform on the I/Q data. The filter outputs for each of the three range bins are individually summed and accumulated for the 10 different transmitter frequencies. The data for each range bin is then averaged across the 64 filters. This

sliding window (SLW) average generates a threshold voltage which is then compared to the signal. The SLW automatically tracks variations in the noise level. A threshold crossing or hit is obtained whenever the signal exceeds the SLW threshold. When a hit occurs in the search mode, the antenna is commanded to a supersearch mode with a reduced search volume. When a second or confirming hit occurs, the antenna scan is stopped and the processor enters the acquisition mode. A timing and control function then positions the three range samples onto the target. The gates will sample the leading and trailing edges of the target and the third sample will be positioned onto the center of the target. The data corresponding to these early, late, and on-target gates are next processed by the track processor.

3.3.3.3 Track Processor

During the track mode, the sum and difference signals are combined at the antenna to produce the $\Sigma + \Delta$ signals for elevation and azimuth. The processor timing and control generates the lobing command which sequences the antenna switching every 8 pulse repetition intervals. The video for each range sample is integrated for the 8 pulses and then averaged over the 10 transmitter frequencies. The output of the track processor is the integrated signal corresponding to the $\Sigma + \Delta$ data of antenna azimuth and elevation for each of the three range samples. These data are sent directly to the range tracker and also log converted and sent to the angle tracker.

3.3.3.4 Range Tracker

The range tracker first sums the data for the lobing sequence. The early and late range gate data are then subtracted and this range error signal is sent to the range and range rate tracker. The tracker is an α - β tracking system where the error signal is multiplied by constants representing α and β . This signal is then used to update the estimated values of range and range rate. The target range is further corrected by adding an estimated range charge derived from the range rate. Initial values of range are obtained from the timing and control which determine the time to the on-target gate. Initialization of range rate is obtained from the doppler frequency of the target as determined from the doppler filter number and the transmitter frequency. The range tracker is a closed loop system with the gain and bandwidth determined by the constants α and β .

3.3.3.5 Angle Tracker

The log converted data from the on-target gate is the $\Sigma + \Delta$ signal for the azimuth and elevation channels. These log data are subtracted which are equivalent to division and produces the Σ/Δ error signal. Signal is then converted to an analog voltage and sent to the antenna servo system. During track, the on-target data are also sent to the AGC function. The AGC maintains the target amplitude within the dynamic range of the A/D converter. The gain is held constant during a complete angle track cycle. During search

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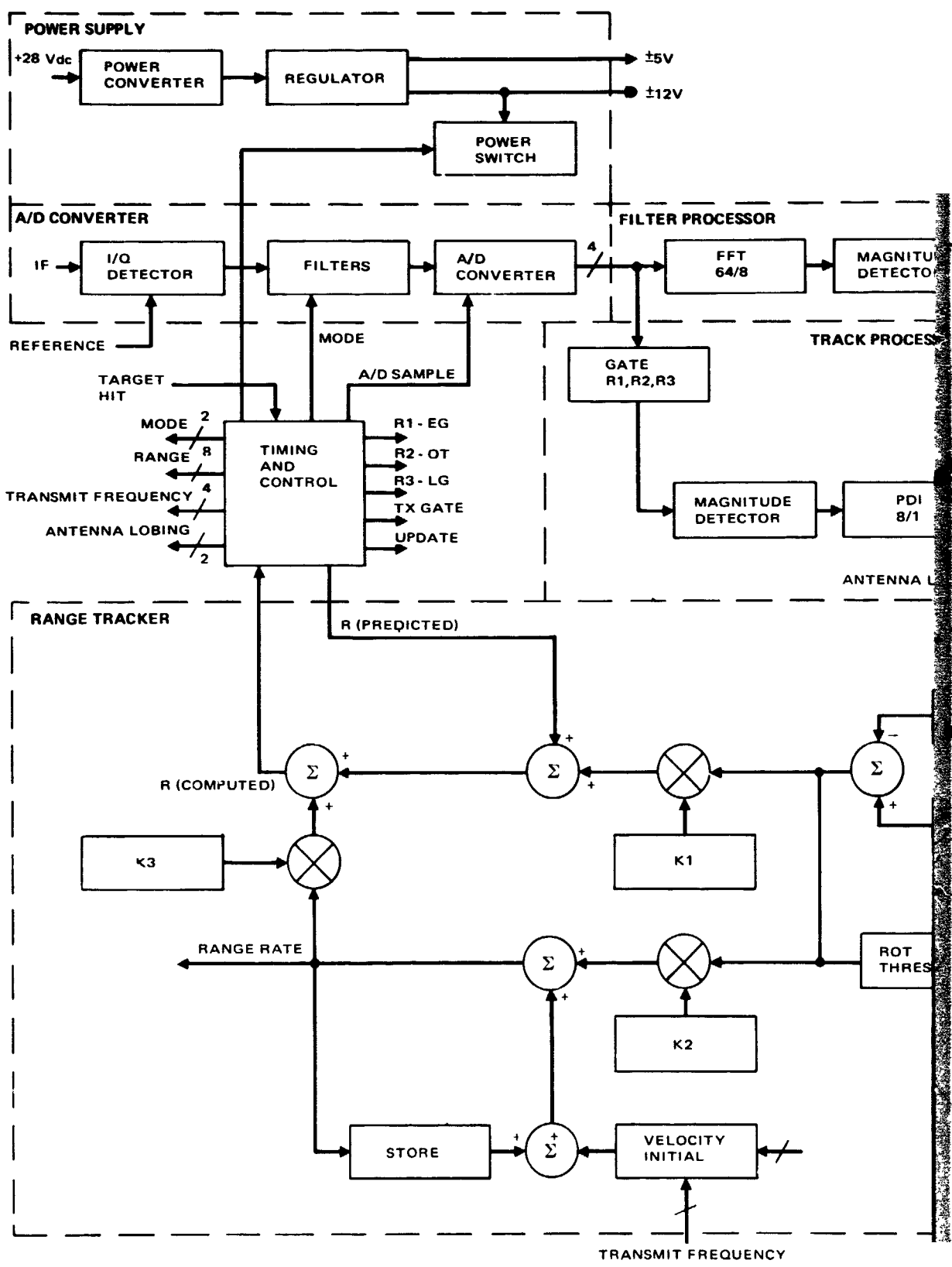
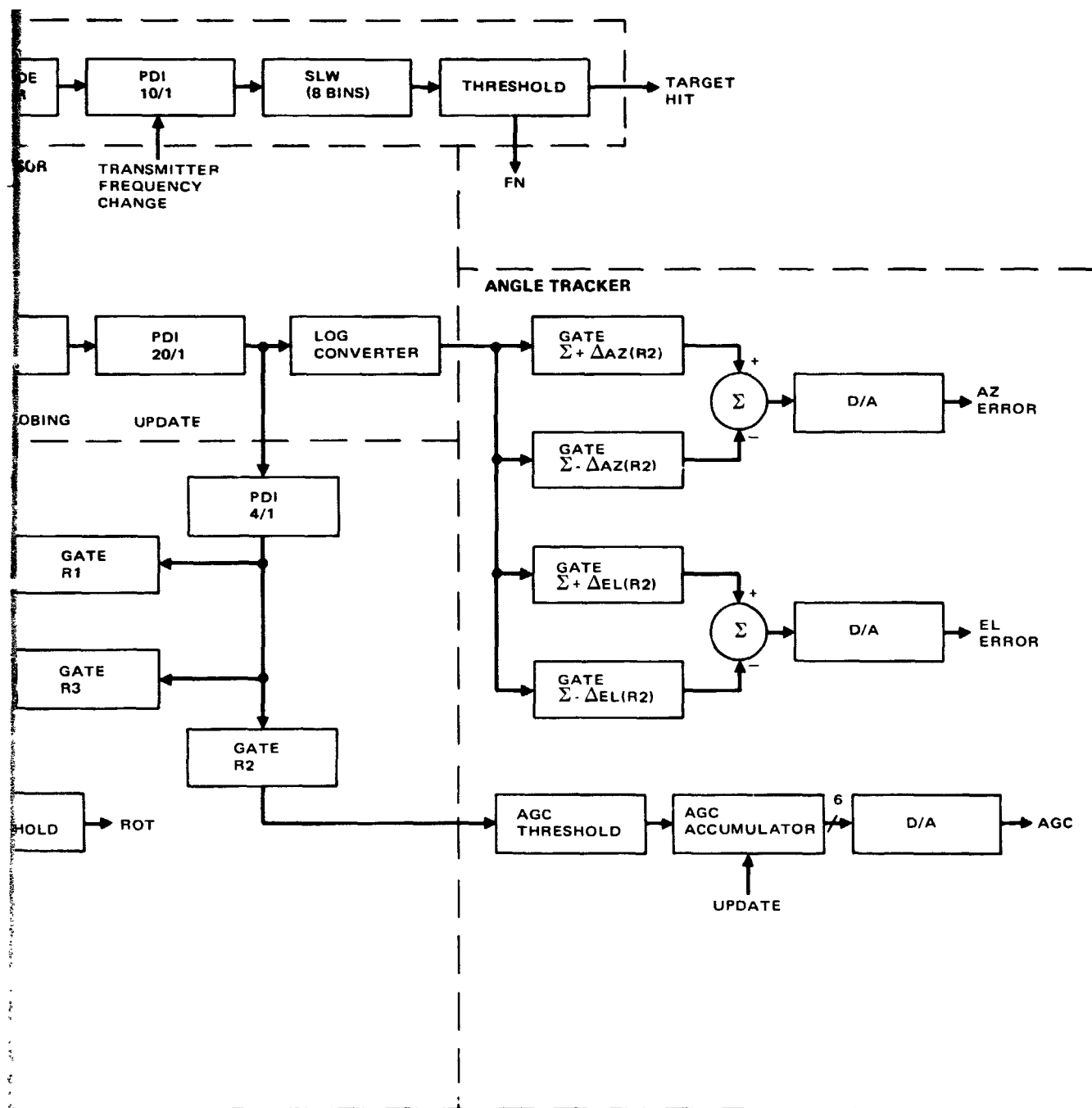


FIGURE 3-21. RADAR SIGNAL PROCESSOR BLOCK DIAGRAM

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mode, the AGC is determined from the largest signal in the three range gates.

3.3.3.6 Timing and Control

The timing and control (T&C) function provides the synchronous signals for the signal processor and the other functions of the radar system. The T&C operates from a master clock and derives the range sampling gates and the transmitter gate. Also derived from the clock are the transmitter frequency change command and the antenna lobing signals. The timing and control also contains a range register which measures the target range in terms of master clock cycles. This register is continually updated by the range tracker.

3.3.3.7 Power Supply

The power supply operates from the 28 volt prime power supply. A block diagram of the power supply function is shown in Figure 3-22. Prime power is regulated to remove variations due to the battery charge level. This regulated voltage is then converted to the power forms required by the signal processing functions. Regulation of these forms is obtained by feedback to the prime regulator. Signal processor power requirements are minimized by utilizing low power "C" MOS integrated circuits and switching power to the high speed A/D converter. Power is applied to the converter during the sample interval. The estimated power required for the signal processor is 21 watts.

3.3.3.8 Physical Design

The digital and analog circuit modules consist of two 11.4 x 14 cm (4.5 x 5.5 in.) printed wiring boards (PWBs) bonded to a 11.4 x 15.2 x 0.08 cm (4.5 x 6.0 x 0.3 in.) thick heat sink. Each digital module has a capacity of 1 integrated circuits divided equally on the two PWBs. The modules are mounted on a 1.3 cm (0.5 in.) center line. Each module has a connector with approximately 175 input/output pins. The power supply module has a thickness of 6.4 cm (2.5 in.). A sketch of the radar signal processor unit is shown in Figure 3-23.

These modules are contained in the standard 16.8 x 17.8 cm (6.6 x 7 in.) enclosure. The total length required is 27.94 (11 in.). The 16.8 cm (6.6 in.) width includes the two 2.0 cm (0.8 in.) wide mounting flanges at the base of the unit. The total volume is 8320 cm³ (508 in³).

The modules are cooled by conducting the heat through the heat sink to the chassis walls and then through the base flange from which it is conducted into the cold wall. Most of the modules dissipate less than 0.5 watt, however, one logic module dissipates 2.5 watts, one has 4.5 watts, and the power supply dissipates approximately 10 watts. The two logic modules may

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require the use of copper rather than aluminum in the heat sinks. The power supply will be bolted to the base near the mounting flanges to assure a low thermal resistance to the cold wall.

Unit weight is calculated below and is exclusive of chassis connectors.

	<u>Kilograms</u>	<u>Pounds</u>
13 modules (at 0.19 kg (0.42 lb))	2.5	5.5
1 power supply module	0.91	7.0
1 harness assembly	0.23	0.5
Chassis	<u>1.59</u>	<u>3.5</u>
	5.23	11.5

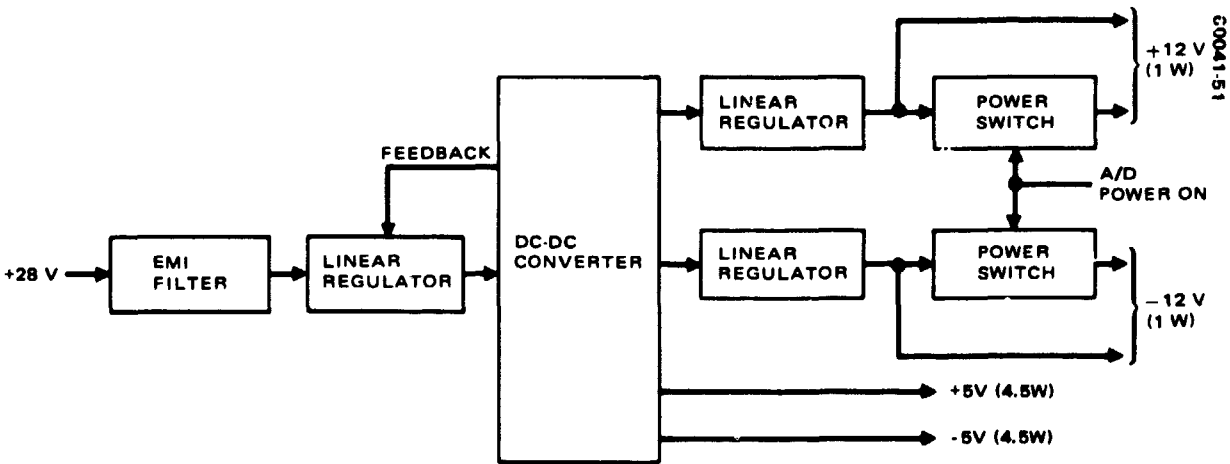


FIGURE 3-22. RADAR SIGNAL PROCESSOR POWER SUPPLY

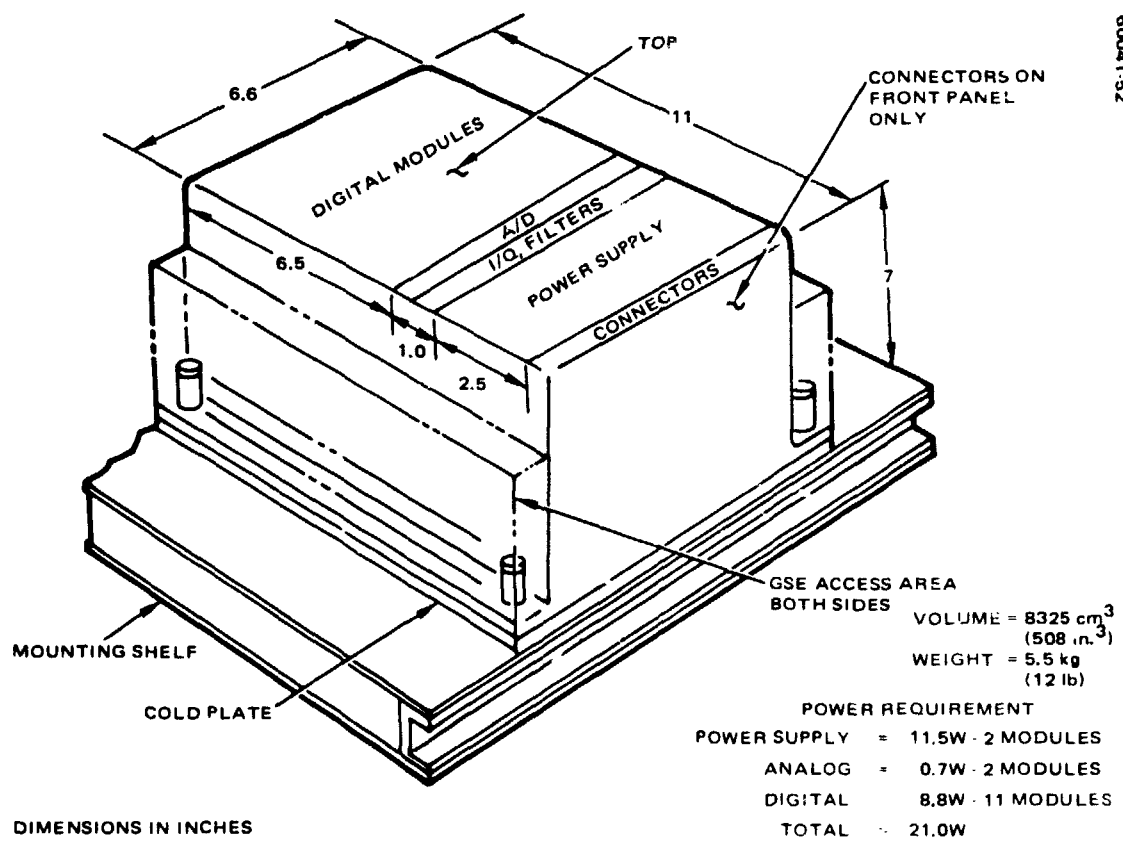


FIGURE 3-23. RADAR SIGNAL PROCESSOR UNIT

3.4 ANTENNA SUBSYSTEM

The antenna electrical requirements for the space shuttle TDRSS communication link are summarized in Table 3-8. Space available for antenna stowage limits the size to approximately 63.5 cm (25 in.) in diameter and a depth of 23 cm (9 in.). Gain requirement given in Table 3-8 is the gain at the input to the radiating structure (i. e. , horn feed in the case of a reflector antenna) and does not include the loss in connecting lines from antenna to transmitter or receiver, or loss in other necessary circuitry such as switches, circulators, etc.

Because of the requirement for polarization diversity, only two types of antennas were considered in the study: front fed paraboloidal reflector antenna and cassegrain reflector antenna.

3.4.1 Front Fed Reflector Antenna

The front fed reflector antenna consists of a paraboloidal reflector with a five element feed placed at its focus. The physical characteristics of this antenna are shown in Figure 3-24 and the electrical characteristics are given in Table 3-9. The reflector diameter and focal distance (F/D) ratio are constrained by the gain and depth requirements. The F/D ratio and reflector diameter were adjusted to yield maximum gain while maintaining an overall antenna depth of 23 cm (9 in.).

The feed horn for the reflector consists of a square horn and four polyrod radiators as shown in Figure 3-25. The radiators are linearly polarized and used only in the radar mode where they provide the azimuth and elevation difference patterns. The horn is used in both the radar and communication link modes, hence, must be capable of switching from linear to circular polarization to function properly. The aperture dimensions of the horn were chosen to provide a reflector illumination taper of approximately 10 dB.

The horn is fed by a short section of square waveguide with two inputs; one for each of the two dominant orthogonal modes in the square guide. The input guide is followed by a short section of guide containing irises whose purpose is to introduce a 90° phase shift between two orthogonal modes and

TABLE 3-8. ANTENNA ELECTRICAL REQUIREMENTS

Frequency	13.75 to 13.8 GHz: receive 14.98 to 15.18 GHz transmit
Polarization	Circular for communication; linear for radar
Gain at input to antenna	35.2 at 13.775 GHz 36.0 at 15.08 GHz

TABLE 3-9. FRONT FED ANTENNA
GAIN/LOSS BUDGET, dB

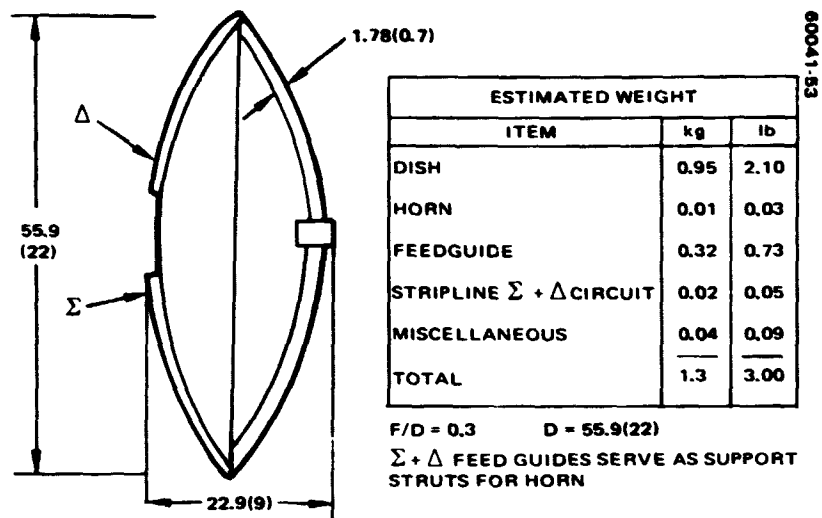
f = 13.775 GHz	
Area gain ($\pi D/\lambda$) ²	38.13
Illumination eff (taper loss)	-0.71
Spillover loss	-0.75
Blockage loss	-0.05
Strut blockage	-0.15
Cross polarization	-0.10
Reflector, $1/2R$	-0.06
Feed loss	-0.40
VSWR loss	-0.20
Net gain	35.71
f = 15.08 GHz	
Area gain ($\pi D/\lambda$) ²	38.91
Illumination eff (taper loss)	-0.89
Spillover loss	-0.68
Blockage loss	-0.06
Strut blockage	-0.15
Cross polarization	-0.10
Reflector, $1/2R$	-0.06
Feed loss	-0.40
VSWR loss	-0.20
Net gain	36.37

thus, obtain circular polarization. In the horn shown in Figure 3-25, linear polarization is obtained when both inputs in the square guide are simultaneously connected to the transmitter. Either sense of circular polarization is obtained when either of the inputs is energized singly.

The polyrod radiators are endfire type antennas made of quartz or a ceramic foam material with a dielectric constant in the range of 2.5 to 4.0. The dielectric rods are approximately 1-1/2 wavelengths in length and are tapered from approximately $\lambda/2$ in diameter of the base to 0.3λ at the tip. The radiation patterns of the polyrod radiators have a 10 dB width, approximately equal to the angle subtended by the dish.

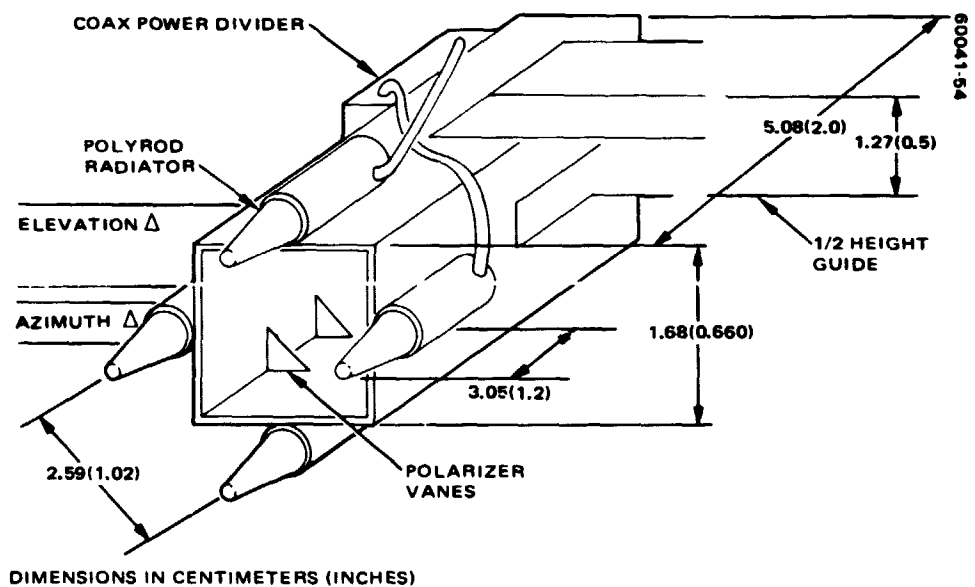
The secondary radiation patterns of the reflector are given in tabular form in Table 3-10.

The relatively low tracking slope exhibited by the front fed reflector is due to the small F/D ratio required to meet the depth constraint. The tracking slope can be improved to equal that of the cassegrain antenna by simply increasing the reflector focal distance by approximately 1 inch.



DIMENSIONS IN CENTIMETERS (INCHES)

FIGURE 3-24. FRONT FED REFLECTOR ANTENNA DISH



DIMENSIONS IN CENTIMETERS (INCHES)

FIGURE 3-25. FEED HORN FOR FRONT FED DISH

TABLE 3-10. FRONT FED REFLECTOR

Σ			Δ	
Theta, deg	H-Plane, dB	E-Plane, dB	Theta, deg	Decibels
Radiation Patterns, f = 13.775 GHz				
0.01	-0.01	-0.01	0.01	-50.56
0.11	-0.03	-0.02	0.11	-29.74
0.21	-0.09	-0.07	0.21	-24.13
0.31	-0.18	-0.15	0.31	-20.75
0.41	-0.32	-0.27	0.41	-18.34
0.51	-0.49	-0.41	0.51	-16.46
0.61	-0.70	-0.58	0.61	-14.93
0.71	-0.95	-0.79	0.71	-13.64
0.81	-1.24	-1.03	0.81	-12.53
0.91	-1.57	-1.30	0.91	-11.56
1.01	-1.95	-1.61	1.01	-10.70
1.11	-2.37	-1.95	1.11	-9.93
1.21	-2.84	-2.33	1.21	-9.24
1.31	-3.35	-2.74	1.31	-8.61
1.41	-3.92	-3.19	1.41	-8.04
1.51	-4.54	-3.68	1.51	-7.53
1.61	-5.22	-4.21	1.61	-7.06
1.71	-5.96	-4.78	1.71	-6.63
1.81	-6.77	-5.40	1.81	-6.24
1.91	-7.65	-6.06	1.91	-5.89
2.01	-8.62	-6.77	2.01	-5.57
2.11	-9.68	-7.53	2.11	-5.28
2.21	-10.85	-8.34	2.21	-5.03
2.31	-12.14	-9.22	2.31	-4.80
2.41	-13.58	-10.15	2.41	-4.60
2.51	-15.21	-11.15	2.51	-4.43
2.61	-17.07	-12.22	2.61	-4.29
2.71	-19.24	-13.38	2.71	-4.17
2.81	-21.86	-14.62	2.81	-4.08
2.91	-25.21	-15.96	2.91	-4.01
3.01	-29.95	-17.42	3.01	-3.97
3.11	-38.83	-19.00	3.11	-3.96
3.21	-44.16	-20.73	3.21	-3.97
3.31	-33.50	-22.65	3.31	-4.01
3.41	-29.52	-24.79	3.41	-4.07
3.51	-27.30	-27.21	3.51	-4.15
3.61	-25.94	-30.00	3.61	-4.27
3.71	-25.12	-33.35	3.71	-4.41
3.81	-24.68	-37.56	3.81	-4.57
3.91	-24.55	-43.47	3.91	-4.76
4.01	-24.68	-55.09	4.01	-4.98

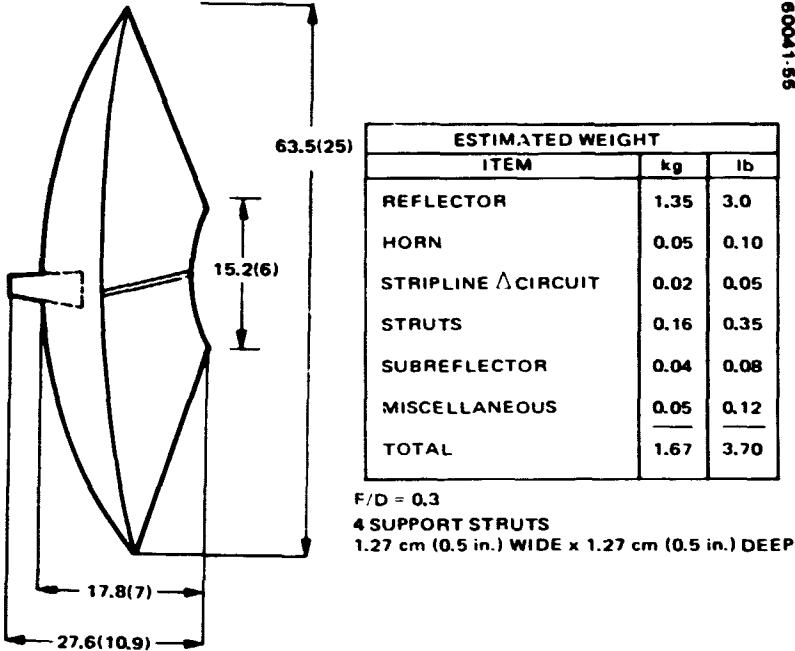
Table 3-10 (continued)

Σ			Δ	
Theta, deg	H-Plane, dB	E-Plane, dB	Theta, deg	Decibels
Radiation Patterns, f = 15.08 GHz				
0.01	-0.01	-0.01	0.01	-55.10
0.11	-0.03	-0.03	0.11	-34.26
0.21	-0.10	-0.08	0.21	-28.61
0.31	-0.22	-0.17	0.31	-25.18
0.41	-0.38	-0.30	0.41	-22.68
0.51	-0.58	-0.46	0.51	-20.69
0.61	-0.83	-0.66	0.61	-19.04
0.71	-1.13	-0.89	0.71	-17.60
0.81	-1.47	-1.16	0.81	-16.33
0.91	-1.87	-1.47	0.91	-15.19
1.01	-2.32	-1.82	1.01	-14.14
1.11	-2.82	-2.20	1.11	-13.18
1.21	-3.38	-2.63	1.21	-12.29
1.31	-4.01	-3.10	1.31	-11.46
1.41	-4.69	-3.60	1.41	-10.69
1.51	-5.45	-4.16	1.51	-9.97
1.61	-6.28	-4.76	1.61	-9.30
1.71	-7.19	-5.40	1.71	-8.67
1.81	-8.20	-6.09	1.81	-8.09
1.91	-9.30	-6.84	1.91	-7.54
2.01	-10.53	-7.64	2.01	-7.04
2.11	-11.89	-8.50	2.11	-6.57
2.21	-13.42	-9.41	2.21	-6.15
2.31	-15.15	-10.39	2.31	-5.76
2.41	-17.15	-11.44	2.41	-5.41
2.51	-19.50	-12.56	2.51	-5.09
2.61	-22.38	-13.76	2.61	-4.82
2.71	-26.14	-15.04	2.71	-4.57
2.81	-31.77	-16.42	2.81	-4.37
2.91	-45.21	-17.89	2.91	-4.20
3.01	-38.71	-19.48	3.01	-4.07
3.11	-31.85	-21.18	3.11	-3.97
3.21	-28.75	-23.02	3.21	-3.91
3.31	-27.02	-24.99	3.31	-3.88
3.41	-26.03	-27.12	3.41	-3.89
3.51	-25.54	-29.41	3.51	-3.94
3.61	-25.44	-31.83	3.61	-4.02
3.71	-25.67	-34.37	3.71	-4.14
3.81	-26.21	-36.91	3.81	-4.30
3.91	-27.08	-39.26	3.91	-4.49
4.01	-28.32	-41.12	4.01	-4.72

Table 3-10 (continued)

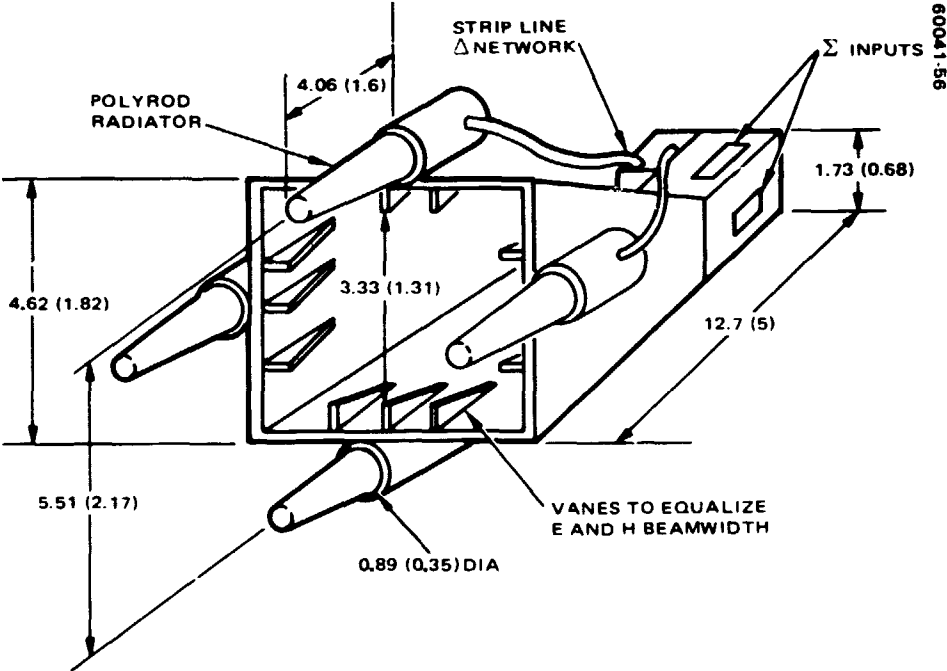
Theta, deg	Theta/BW	Track Slope	Theta, deg	Theta/BW	Track Slope
Tracking Slope, k_m					
f = 13.775 GHz			f = 15.08 GHz		
0.05	0.02	0.77	0.05	0.02	0.42
0.15	0.06	0.77	0.15	0.06	0.42
0.25	0.10	0.78	0.25	0.10	0.43
0.35	0.13	0.78	0.35	0.15	0.45
0.45	0.17	0.79	0.45	0.19	0.47
0.55	0.21	0.80	0.55	0.23	0.50
0.65	0.25	0.82	0.65	0.27	0.54
0.75	0.29	0.83	0.75	0.31	0.57
0.85	0.33	0.85	0.85	0.35	0.62
0.95	0.37	0.87	0.95	0.40	0.68
1.05	0.40	0.90	1.05	0.44	0.74
1.15	0.44	0.92	1.15	0.48	0.81
1.25	0.48	0.96	1.25	0.52	0.89
1.35	0.52	0.99	1.35	0.56	0.98
1.45	0.56	1.01	1.45	0.60	1.08
1.55	0.60	1.06	1.55	0.65	1.19
1.65	0.63	1.11	1.65	0.69	1.34
1.75	0.67	1.15	1.75	0.73	1.48
1.85	0.71	1.19	1.85	0.77	1.69
1.95	0.75	1.26	1.95	0.81	1.86
2.05	0.79	1.33	2.05	0.85	2.15
2.15	0.83	1.34	2.15	0.90	2.39
2.25	0.87	1.46	2.25	0.94	2.81
2.35	0.90	1.53	2.35	0.98	3.26
2.45	0.94	1.58	2.45	1.02	3.98

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DIMENSIONS IN CENTIMETERS (INCHES)

FIGURE 3-26. CASSEGRAIN ANTENNA



DIMENSIONS IN CENTIMETERS (INCHES)

FIGURE 3-27. FEED HORN FOR CASSEGRAIN ANTENNA

However, the overall antenna depth is increased by the same amount and no longer fits in the available space.

3.4.2 Cassegrain Antenna

The cassegrain antenna consists of a main reflector 63.5 cm (25 in.) in diameter, a 15.2 cm (6 in.) hyperboloidal subreflector, and five element feed assembly placed at one of the foci of the subreflector. The physical characteristics of the cassegrain antenna are shown in Figure 3-26 and the electrical characteristics are summarized in Table 3-11.

The feed system similar to that of the front fed reflector is shown in Figure 3-27. The feed is composed of a horn used to obtain the sum pattern and four polyrod antennas that provide the azimuth and elevation difference patterns. The polyrod radiators are similar to those used for the front fed

TABLE 3-11. CASSEGRAIN ANTENNA
GAIN/LOSS BUDGET, dB

f = 13.775 GHz		
Area gain ($\pi D/\lambda$) ²		39.24
Illumination eff (taper loss)		-0.48
Spillover loss		-1.17
Diffraction spillover		-0.66
Subreflector blockage		-0.84
Strut blockage		-0.23
Cross polarization		-0.06
Reflector, I ² R loss		-0.06
Feed loss		-0.20
VSWR loss (1.5:1)		-0.20
Net gain		35.34
f = 15.08 GHz		
Area gain ($\pi D/\lambda$) ²		40.02
Illumination eff (taper loss)		-0.69
Spillover loss		-0.87
Diffraction spillover		-0.66
Subreflector blockage		-0.92
Strut blockage		-0.23
Cross polarization		-0.06
Reflector, I ² R loss		-0.06
Feed loss		-0.20
VSWR loss (1.5:1)		-0.20
Net gain		36.13

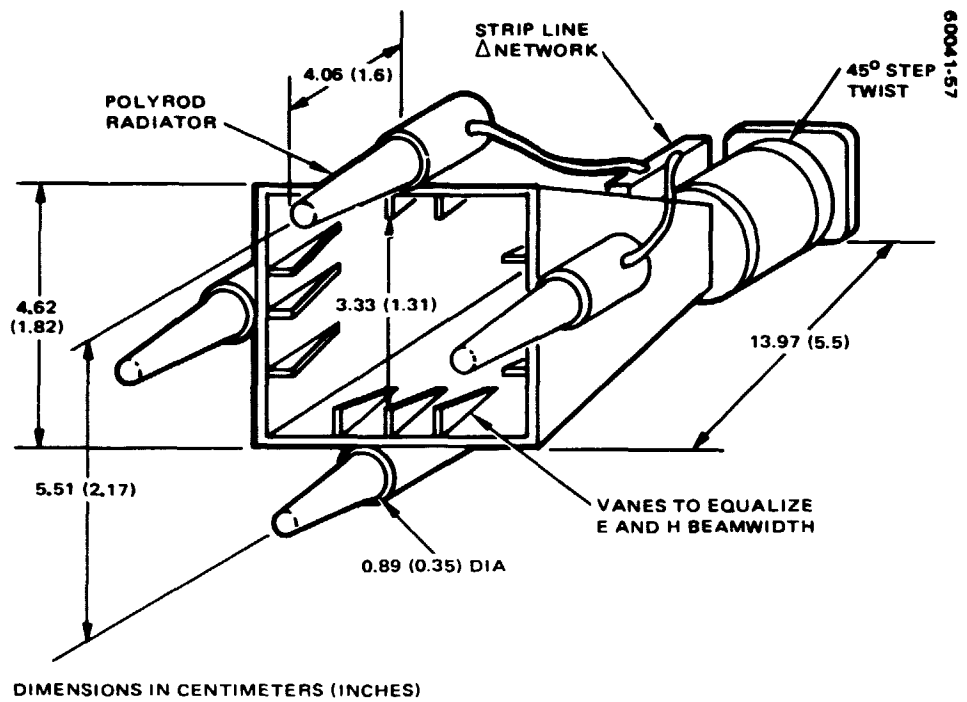


FIGURE 3-28. ALTERNATE FEED HORN FOR CASSEGRAIN ANTENNA

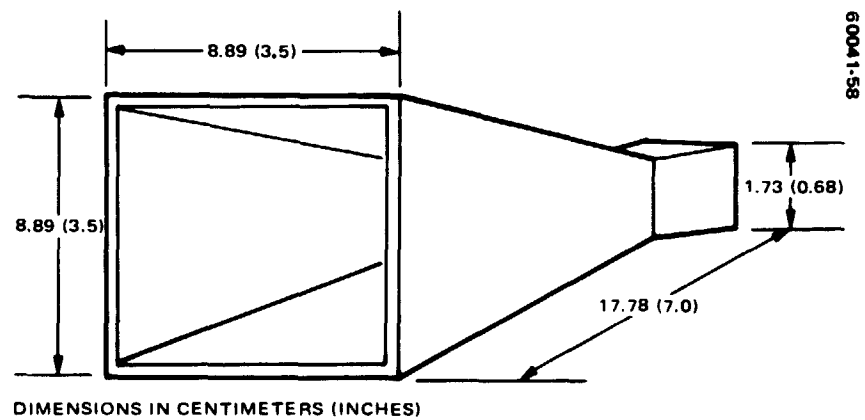


FIGURE 3-29. ACQUISITION HORN

reflector but are approximately one-half wavelength longer to provide a 10 dB beamwidth equal to the angle subtended by the subreflector.

The square sum horn is also similar to that used with the paraboloidal reflector but has a larger aperture since a smaller 10 dB beamwidth is required. The E- and H-plane beamwidths of the feed are made equal by means of the vanes extending from the aperture toward the horn throat. The vanes are spaced so as to prevent propagation of an E-field parallel to the vanes. Thus, in effect, narrowing the E-plane dimension of the aperture provides equal E- and H-plane beamwidths.

The dual polarization capability is provided in the same manner as the front fed reflector. The horn has two orthogonal input ports and linear polarization is radiated when both ports are energized. Either sense of circular polarization may be obtained by exciting the ports singly. An alternate method of obtaining the dual polarization capability is illustrated in Figure 3-28. The two orthogonal input ports are replaced by a single port, followed by a step twist section that can be twisted 45° by a stepper motor. In the 0° position circular polarization is radiated; linear polarization is radiated in the 45° position.

The secondary radiation patterns of the cassegrain antenna are given in tabular form in Table 3-12. The sidelobes are considerably higher than those observed for the front fed reflector due to relatively larger aperture blockage by the subreflector. The gain of the cassegrain is also lower than for the paraboloid, but the tracking slope is much higher. The poorer pattern and gain performance of the cassegrain in comparison to the front fed reflector is due to the relatively small size of the cassegrain antenna. In general, cassegrain antennas, whose diameter is less than approximately 100 wavelengths, have less gain than a single reflector antenna of comparable size. This is primarily caused by the subreflector which has considerable aperture blockage and, at the same time, is small enough so that spillover losses due to subreflector diffraction effects are high.

3.4.3 Acquisition Antenna

A simple horn antenna is used for the initial acquisition of the satellite. The dimensions of the horn are given in Figure 3-29. The acquisition horn is similar in design to the feed horn for the cassegrain antenna except that vanes are not used to equalize the E- and H-beamwidths. The short section of the square input guide contains an iris type polarizer to convert linear polarization into circular polarization. The electrical characteristics of the horn are summarized in Table 3-13.

The horn is mounted at the edge of the cassegrain main reflector. Isolation between the acquisition horn and cassegrain antenna is determined primarily by the relative gains of the feed and acquisition horns in the direction defined by the ray connecting the two horns. The gain of acquisition antenna is down 20 to 30 dB in this direction, while the radiation from the feed horn is down approximately 10 dB. Hence, the isolation of the acquisition horn is 30 to 40 dB relative to the feed horn.

TABLE 3-12. CASSEGRAIN ANTENNA

Σ			Δ	
Theta, deg	H-Plane, dB	E-Plane, dB	Theta, deg	Decibels
Radiation Patterns, f = 13.775 GHz				
0.01	-0.01	-0.01	0.01	-45.12
0.11	-0.04	-0.04	0.11	-24.31
0.21	-0.12	-0.12	0.21	-18.72
0.31	-0.25	-0.25	0.31	-15.38
0.41	-0.43	-0.44	0.41	-13.02
0.51	-0.66	-0.68	0.51	-11.20
0.61	-0.95	-0.97	0.61	-9.75
0.71	-1.30	-1.32	0.71	-8.55
0.81	-1.70	-1.74	0.81	-7.54
0.91	-2.16	-2.21	0.91	-6.68
1.01	-2.69	-2.75	1.01	-5.95
1.11	-3.29	-3.37	1.11	-5.32
1.21	-3.96	-4.06	1.21	-4.79
1.31	-4.71	-4.83	1.31	-4.33
1.41	-5.56	-5.70	1.41	-3.95
1.51	-6.50	-6.68	1.51	-3.62
1.61	-7.56	-7.78	1.61	-3.36
1.71	-8.76	-9.03	1.71	-3.16
1.81	-10.12	-10.46	1.81	-3.01
1.91	-11.69	-12.11	1.91	-2.90
2.01	-13.53	-14.07	2.01	-2.85
2.11	-15.73	-16.47	2.11	-2.84
2.21	-18.50	-19.55	2.21	-2.88
2.31	-22.25	-23.94	2.31	-2.97
2.41	-28.23	-32.03	2.41	-3.10
2.51	-48.45	-40.07	2.51	-3.27
2.61	-31.17	-27.71	2.61	-3.49
2.71	-25.13	-23.27	2.71	-3.76
2.81	-22.00	-20.70	2.81	-4.07
2.91	-20.02	-19.00	2.91	-4.43
3.01	-18.67	-17.84	3.01	-4.85
3.11	-17.74	-17.03	3.11	-5.31
3.21	-17.11	-16.49	3.21	-5.83
3.31	-16.71	-16.16	3.31	-6.41
3.41	-16.50	-16.01	3.41	-7.06
3.51	-16.45	-16.01	3.51	-7.77
3.61	-16.54	-16.14	3.61	-8.56
3.71	-16.75	-16.40	3.71	-9.43
3.81	-17.09	-16.79	3.81	-10.40
3.91	-17.55	-17.28	3.91	-11.48
4.01	-18.11	-17.89	4.01	-12.69

Table 3-12 (continued)

Σ			Δ	
Theta, deg	H-Plane, dB	E-Plane, dB	Theta, deg	Decibels
Radiation Patterns, f = 15.08 GHz				
0.01	-0.01	-0.01	0.01	-46.30
0.11	-0.04	-0.04	0.11	-25.48
0.21	-0.13	-0.14	0.21	-19.88
0.31	-0.28	-0.29	0.31	-16.53
0.41	-0.50	-0.51	0.41	-14.14
0.51	-0.77	-0.78	0.51	-12.30
0.61	-1.10	-1.13	0.61	-10.81
0.71	-1.51	-1.54	0.71	-9.57
0.81	-1.98	-2.02	0.81	-8.52
0.91	-2.52	-2.58	0.91	-7.62
1.01	-3.14	-3.21	1.01	-6.84
1.11	-3.84	-3.94	1.11	-6.16
1.21	-4.64	-4.76	1.21	-5.56
1.31	-5.54	-5.69	1.31	-5.05
1.41	-6.55	-6.74	1.41	-4.60
1.51	-7.70	-7.93	1.51	-4.22
1.61	-9.01	-9.29	1.61	-3.90
1.71	-10.51	-10.87	1.71	-3.63
1.81	-12.26	-12.73	1.81	-3.42
1.91	-14.34	-14.97	1.91	-3.25
2.01	-16.91	-17.80	2.01	-3.14
2.11	-20.28	-21.65	2.11	-3.08
2.21	-25.26	-27.89	2.21	-3.07
2.31	-35.82	-54.50	2.31	-3.10
2.41	-34.82	-29.81	2.41	-3.19
2.51	-26.32	-24.12	2.51	-3.33
2.61	-22.59	-21.13	2.61	-3.52
2.71	-20.36	-19.25	2.71	-3.76
2.81	-18.90	-18.00	2.81	-4.06
2.91	-17.92	-17.15	2.91	-4.41
3.01	-17.27	-16.61	3.01	-4.83
3.11	-16.89	-16.30	3.11	-5.32
3.21	-16.71	-16.18	3.21	-5.87
3.31	-16.70	-16.23	3.31	-6.50
3.41	-16.85	-16.43	3.41	-7.21
3.51	-17.14	-16.76	3.51	-8.01
3.61	-17.55	-17.22	3.61	-8.92
3.71	-18.09	-17.81	3.71	-9.95
3.81	-18.74	-18.51	3.81	-11.11
3.91	-19.15	-19.32	3.91	-12.45
4.01	-20.38	-20.25	4.01	-13.99

Table 3-12 (continued)

Theta, deg	Theta/BW	Track Slope	Theta, deg	Theta/BW	Track Slope
Tracking Slope, km					
f = 13.775 GHz			f = 15.08 GHz		
0.05	0.02	1.16	0.05	0.02	1.01
0.15	0.07	1.16	0.15	0.07	1.02
0.25	0.12	1.16	0.25	0.12	1.02
0.35	0.17	1.16	0.35	0.17	1.03
0.45	0.21	1.16	0.45	0.21	1.04
0.55	0.26	1.15	0.55	0.26	1.06
0.65	0.31	1.15	0.65	0.31	1.07
0.75	0.36	1.15	0.75	0.36	1.09
0.85	0.40	1.14	0.85	0.40	1.11
0.95	0.45	1.12	0.95	0.45	1.13
1.05	0.50	1.12	1.05	0.50	1.16
1.15	0.55	1.08	1.15	0.55	1.20
1.25	0.60	1.08	1.25	0.60	1.20
1.35	0.64	1.03	1.35	0.64	1.25
1.45	0.69	1.03	1.45	0.69	1.25
1.55	0.74	0.94	1.55	0.74	1.26
1.65	0.79	0.85	1.65	0.79	1.30
1.75	0.83	0.75	1.75	0.83	1.25
1.85	0.88	0.66	1.85	0.88	1.29
1.95	0.93	0.37	1.95	0.93	1.11
2.05	0.98	0.09	2.05	0.98	0.86
2.15	1.02	-0.50	2.15	1.02	0.23
2.25	1.07	-1.63	2.25	1.07	-1.72
2.35	1.12	-4.06	2.35	1.12	-8.84
2.45	1.17	-23.53	2.45	1.17	-7.86

TABLE 3-13. ELECTRICAL PROPERTIES OF
ACQUISITION HORN

Peak gain	20.5 dB
3 dB beamwidths	
E-plane	16°
H-plane	20°
Polarization	Circular
Isolation from cassegrain feed	30 to 40 dB

3.5 GIMBAL AND DRIVE SUBSYSTEM

3.5.1 Requirements

The gimbal and drive scheme envisioned for this application is based on satisfying a broad range of mission requirements with a simple and straightforward two axis gimbal using proven elements. The requirements and baseline considerations are summarized below and in Table 3-14.

- 1) **Payload mounting:** The payload is a monolithic type package consisting of a cassegrain antenna and associated electronics. The payload is installed so that its combined center of gravity lies at the intercept of the two axes. Payload balance is required for 1) inertial control capability, and 2) to minimize the influence of external disturbances on antenna pointing.

TABLE 3-14. GIMBAL MECHANISM FOR SHUTTLE ORBITER
KU-B AND RADAR/COMMUNICATION

Requirements	Primary Axis	Secondary Axis
Payload package		
Movable weight, kg (lb)	26.3 (58)	13.1 (29)
Movable inertia, N/m ² (slug-ft ²)	0.0077 (0.5)	0.0019 (0.15)
CG location	At axis intercept	
Range (from latch), deg	±180	-35 down to +85 up
Maximum rates (IIIB raster), rad/sec ²	1.03	11.6
Cable bending torque, kg-m (ft-lb)	0.097 (0.7)	0.069 (0.5)
Bearing friction torque, kg-m (ft-lb)	0.028 (0.2)	0.028 (0.2)
Total motor torque,* kg-m (ft-lb)	0.521 (3.77)	0.579 (4.19)
Tracking accuracy, deg		
Radar	±0.57	
Communications	±0.2	
Gyro rate sensing range (track), deg/sec	±10	
Gyro rate null (track), deg/sec	±0.01	
Initial pointing accuracy, deg	±0.1	
Gimbal orthogonality, deg (mr)	0.028 (0.5)	
Cable interconnections to payload	4 coax 10 shielded leads 5 temperature sensor leads 7 heater leads	
Launch lock/relatch	Must relatch automatically if spacecraft power is lost	
Operational temperature range, °C (°F)	-50 to +130 (-58 to +266)	

*Including 3.5X margin for bearing and cable torques.

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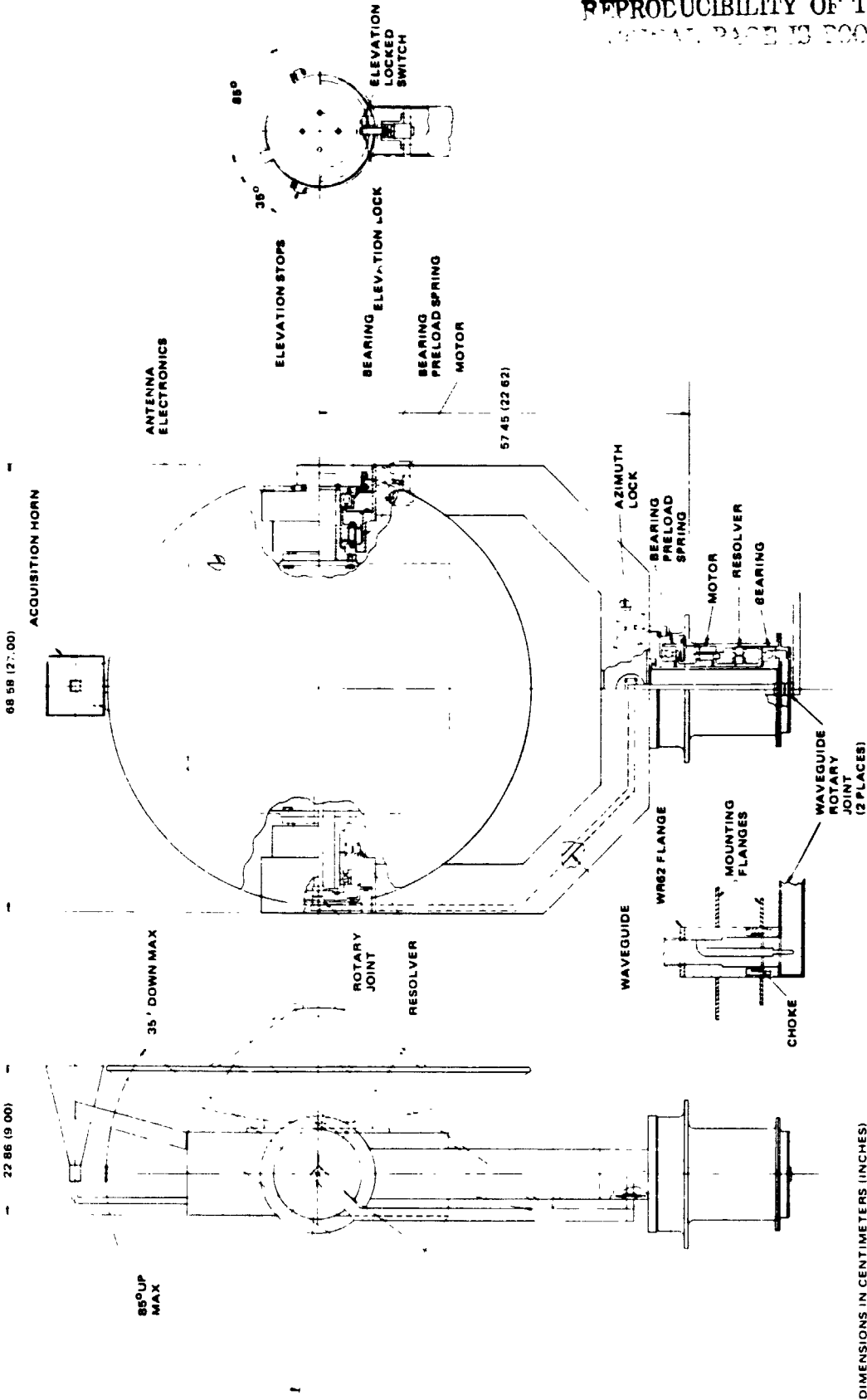


FIGURE 3-30. TWO-AXIS GIMBAL GENERAL ARRANGEMENT

DIMENSIONS IN CENTIMETERS (INCHES)

- 2) Overall package: The gimbal mechanism and its payload are mounted onto the main communication/radar equipment compartment. The entire arrangement is mounted on a two position deployment device (shuttle provided) and must stow within the confines between the shuttle payload dynamic envelope and the bay door dynamic envelope.
- 3) Restraint: Each axis of the gimbal mechanism requires anti-rotation restraint to maintain position within the stowage envelope during launch and ascent, and must be restowable in this position after use for reentry and landing. Positive relatch to the stowed position must take place automatically if vehicle power is lost.
- 4) Range and rates: Gimbal range requirements are a function of the orientation of the primary axis as controlled by the shuttle provided package deployment link. The gimbal yoke connecting the two axes is tailored accordingly to accommodate the maximum mission range envisioned. Worst case gimbal rate requirements are those associated with the raster scan pattern associated with radar Mission IIIB.

3.5.2 General Description

3.5.2.1 Package

Figure 3-30 shows the general arrangement of the two axis gimbal. As mentioned previously, the size of the yoke is dictated by the requirement to look 35° below the gimbal horizon without interfering with the dish or blocking the beam.

A central pedestal type arrangement was considered to achieve a more compact and lighter weight gimbal structure; however, this packaging concept would require splitting the electronic package and is not amenable to the cassegrain type feed, nor to the stowage volume in providing the -35° range capability.

Electrical interconnections across the two axes to serve the gimballed antenna electronics package consists of a single channel of WR 62 waveguide, four flexible coaxial cables, 10 shielded leads, and approximately 12 leads for serving temperature sensors and heaters. In addition to these payload required connections, the several wires associated with controlling the secondary gimbal axis must cross the primary axis. Also, the cables to the inertial rate sensors located within the payload package are a part of the interconnection requirements.

These cable bundles (not shown in the figure) would be routed in service loops to assure noninterference with gimbal action while minimizing any bias torques associated with cable flexing. Since continuous rotation is not a

requirement for either axis, the use of slip rings is not practical (or economical) due to RFI requirements, impedance matching, brush friction losses, and package size and weight. Flexing of cables in well designed service loops is based on proven design practices used on many other Hughes programs.

The single channel WR 62 waveguide is connected across each axis through a coaxial probe type RF rotary joint. This is a noncontacting design that uses the gimbal bearings for support, and is selected to provide uniform RF performance independent of look angle.

3.5.2.2 Motors and Control

Each axis is powered by a full rotation brush type dc torque motor sized to provide the required dynamic torques of the Mission IIIB raster scan, and to provide adequate torque margin for the expected cable and bearing torques. Use of a brush type motor is compatible with this application. It is also more simple in construction than brushless types and uses simpler control electronics. Operational brush wear life has been demonstrated on other applications as being well in excess of 1×10^7 revolutions in an outer space environment and reliability is well established. Motor design data are listed below.

Peak torque	0.62 kg-m (4.5 ft-lb)
Electrical time constant	1.6 ms
Stall power at peak torque	216 watts
Brush friction torque	0.014 kg-m (0.1 ft-lb)
Ripple torque (average to peak)	5 percent
No load speed	36 rad/sec
Motor weight	1.36 kg (3.0 lb)
Torque sensitivity	0.059 kg-m/amp (0.43 ft-lb/amp)

Initial pointing control, position telemetry, and other functions are provided by including a simple dual speed resolver for each axis. A 1 and 16 speed resolver is picked for this application due to its extreme reliability, packaging compatibility, and ease of processing to secure the one part in 2^{13} resolution required. The resolver is packaged as part of the gimbal bearing system. It has no contacting parts and is a direct reading device requiring no search for a reference signal from which to initialize a register.

Several alternates to this resolver concept exist. An optical incremental encoder would perform the same function with a direct digital system, but would be more susceptible to temperature and other problems. In a final system study, a tradeoff would be conducted to select the best device to be used. For the purpose of this preliminary study, however, the resolver concept is presented as a workable system based on a proven capability. Resolver design data are shown below.

Input voltage	4 volts rms
Frequency	4.8 kHz
Transformation ratio	0.91
Phase shift	2.4°
Angular accuracy error (proportional voltage)	1.0 arcmin maximum

Actual gimbal control error signals would be provided by the two axis rate integrating gyros contained within the payload package. Selected gyros would be the fully qualified Northrop G-6 type packaged units, previously flown on several space vehicle applications. These gyros are small, lightweight, fit easily within the electronics package, and provide the proper error signal levels.

3.5.3 Bearings and Lubrication

Two precision angular contact bearings are used in each axis to support the gimbal. Motors, resolvers, and waveguide rotary joints are suspended within each bearing pair. The angular contact bearings pairs are mounted in a back-to-back configuration to provide the greatest effective separation and, therefore the most favorable resistance moment arm.

The bearings are installed with a light interference fit onto both the rotating and stationary members. One bearing in each pair has its outer race mounted onto a thin diaphragm steel spring designed to produce a low spring rate in the thrust direction, while providing a very high radial stiffness. While duplex pairs would have a higher radial load capacity, compliance analyses show that very small radial temperature gradients cause unacceptable increases in friction. This sensitivity could be relieved by slip fits at the bearing ring interface, but equal radial load sharing would then not be assured. Single bearings provide a more predictable, and therefore more reliable suspension than duplex pairs.

With the diaphragm spring loaded system shown, a temperature difference of 5°F across the shaft to the housing causes only a 0.45 kg (1 lb) change in preload, compared to approximately 36 kg (80 lb) that could occur in a system with all races fixed. Because it does not depend on critical fits, this

system of axial compensation is very reliable compared to the conventional sliding race. The axial compensation method has been qualified for high level vibration using 90 and 150 mm bore bearings for despin assemblies. A limit stop is provided to control motion during the launch and vibration loads. Because the axial compensation system eliminates radial free play at the bearing interfaces, structural stiffness is assured with a consequent improvement in pointing stability. Moreover, the absence of unpredictable free play permits more accurate definition of gimbal dynamics characteristics.

In each bearing pair, the spring mounted bearing that provides axial relief is located on the side opposite the rotary joint gap. Placing the hard mounted bearing adjacent to the joint gap avoids problems of gap change due to bearing movement.

To avoid introducing problems due to thermal gradients within an axis, beryllium is selected as the shaft and housing material because of its thermal coefficient match between the 440 CVEM bearing material and the steel preload diaphragm spring.

On the secondary axis where the two bearings are separated by the electronics package, bearing axial stiffness is dependent upon the structural tie of the box rather than the stiffness of the yoke. Since both the yoke and box are fabricated from aluminum, additional spring travel is required in this axis to maintain bearing preload uniformity over the temperature range.

The bearings in this system are sized to withstand the payload weight during launch vibration. The allocated space for the gimbal package is such that there is no adjacent structure for providing a hard locking point that would bypass launch or landing loads from the bearings. The locks, therefore, are only antirotational devices that do not provide lateral restraint. The heaviest loads on a bearing will be seen by the upper bearing of the primary axis due to the offset moment of the payload package. Exact dynamic loading to which the bearings will be subjected is not fully known until dynamic studies are complete. Therefore, for the purpose of preliminary bearing sizing a value of 50 g static equivalent load was selected. Load ratings of 350,000 psi mean Hertzian stress were used. Selected bearings are as follows:

Upper azimuth bearing 100 mm bore,	150 mm O.D.
Lower azimuth bearing 85 mm bore,	130 mm O.D.
Elevation bearings (both) 65 mm bore,	100 mm O.D.
Bearing preload	22.7 kg (50 lb)/pair

Bearing lubrication for this application would be selected not only from the standpoint of successful operation in a space mission over the temperature range, but also from the standpoint of ground environment to which

the gimbals would be subjected. An oil or grease system could be compounded to provide uniform torque characteristics over the -44°C (-58°F) to 130°C (-266°F) temperature range. However, this type system attracts particulate matter in a ground environment (particularly the dust generated by the shuttle landing).

Dry films provide very uniform operation over the temperature range without this strong attraction to airborne particles. Certain types of dry films (as are certain types of oils) are, however, susceptible to moisture contamination that might be present during prelaunch operations that could affect bearing operation.

Hughes has developed a sputter plating technique for applying dry film lubricants to bearings that shows the best promise of providing a durable film with the least amount of compromise to overall performance. Sputtering with molybdenum disulfide might cause moisture contamination concerns, however other materials such as gold over lead could produce the desired results. Dry film lubricants would be the baseline, and selection of the exact material would be the result of an evaluation and test program before final selection of these materials are made.

The sputtering application process provides a very uniform thin film of lubricant without any binder that tightly adheres to the balls and races and does not upset the internal geometry of the bearing. The resultant bearing is extremely smooth running with excellent wear life capabilities, and is compatible with a space environment in close proximity to optical surfaces. The ball separator is fabricated from a sputtered bronze material that provides a minute transfer of lubrication for replenishment as the bearing operates and provides freedom from sloughing off of particles that would cause debris buildup problems.

3.5.4 Antirotation Latches

Redundant solenoid latches, one set for each axis, are envisioned to provide the unlock/relock capability. Since the load is balanced about each axis of operation, loads on the locks are quite minimal. The two solenoids for one axis would be arranged with a bell crank between, where one is push action and the other pull action. An auxiliary spring on the bell crank would augment the dropping in force.

The requirement that the axes relatch automatically should vehicle power be lost presents an interesting problem. First, something must trigger the relatch sequence; then the axes must be driven to the "home" position; and finally, an auxiliary cockpit indication of relatch would be highly desirable. Since the latch position for the secondary axis is somewhat midrange (this would probably hold true for the primary axis also), some direction sensing is required to find the "home" position.

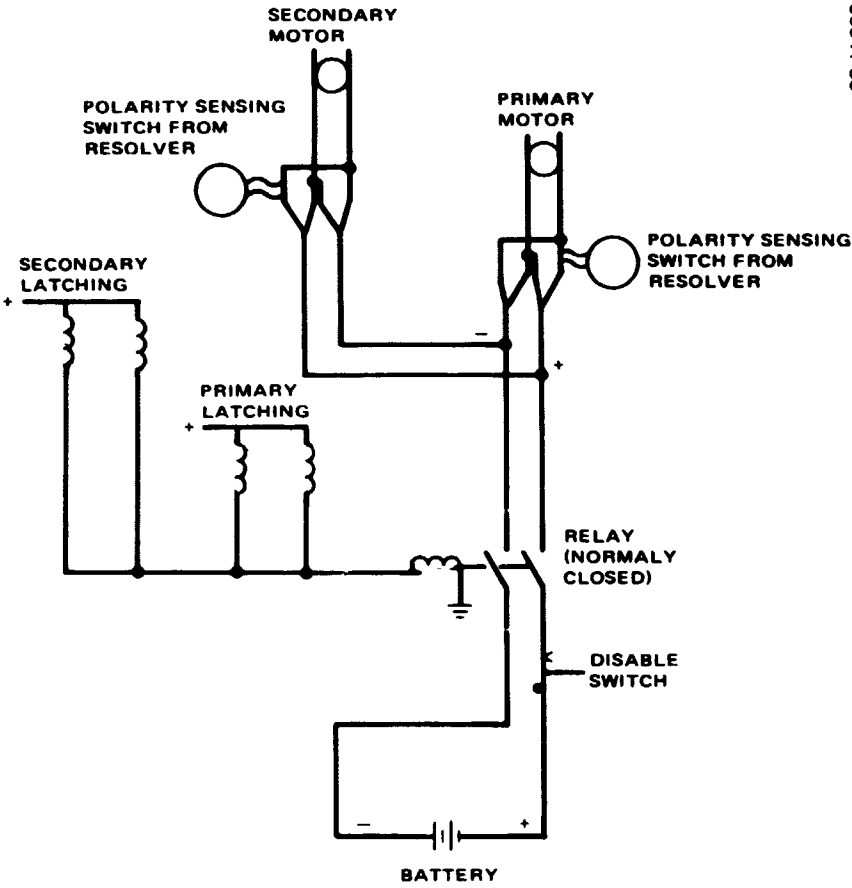


FIGURE 3-31. AUXILIARY BATTERY FUNCTION

Mechanical or pneumatic devices could be made to work but the linkages and levers would add weight and other complications to the mechanism. Springs would not be desirable for "homing" since they would add significant bias torques to the operational capabilities of the unit. For example, a spring of sufficient torque to drive the axis to the latch position would require about 0.28 kg-m (2 ft-lb) steady bias to the axis drivers. This not only adds to the weight and power dissipation of the motor, but also has the effect of cancelling out the inertial control capabilities of the pointing logic.

The technique envisioned to provide this function would be to furnish an auxiliary battery to drive the gimbal motors to the "home" position as shown in Figure 3-31. The solenoids of the latches would remain energized during operation and, unless the emergency provisions were overridden during a normal relatch loss of solenoid, power would initiate the sequence. The latches would drop against the plate, and remain spring loaded against the plate until the drop-in position is reached. The auxiliary battery system would be held open by another relay that would drop in when power is lost. One other feature that must be included is a polarity switching device to tell the battery which way to send the motors to find "home". This would be a continually operating device during operation that would sense resolver position and keep the battery polarity always pointing to "home".

A refinement of this system would be to provide an auxiliary motor or an auxiliary set of motor windings for driving power should motor problems cause power loss.

Each latching device contains a latch status switch to indicate relatch. With auxiliary power as part of the automatic system, the cockpit indication would be available also, on this emergency basis as well as a normal basis.

3.6 GIMBAL SERVO SUBSYSTEM

This section covers the analysis and preliminary design of the gimbal servo systems.

3.6.1 Servo Performance Requirements

The servo performance requirements are based on the general requirements given in subsection 3.5. The servo must acquire and track both the TDRSS for communications and low orbit targets for radar. In the preliminary design of the servo, it is assumed that the only difference between radar and communication tracking is the sensor scale factor, noise level, and target dynamics.

The more important servo performance requirements are summarized in Table 3-15 for the radar and communication modes. The radar requirements given represent a worst case composite of the Missions I/II and IIIB requirements. The given scan range represents the LOS scan requirements about the orbiter (-Z axis). The actual gimbal range requirements are slightly greater as discussed in subsection 3.4.2. The 64 scan lines provide 1.41° resolution, which for a 2.0° beamwidth represents a 30 percent overlap. This is adequate for the low inertial target rates.

The 3.5 deg/sec base motion includes a 2.5 deg/sec orbiter rate (earth pointing mode) plus up to 1 deg/sec limit cycle rate from the reaction jet control system. The sensor field of view (or beamwidth) of 2.0° and the associated dwell time is based on the preliminary design work discussed earlier. The target rate given for the radar acquisition is based on an estimate for the IIIB mission. The inertial target rate for the communications acquisition is derived from the worst case kinematic motion between the orbiter and a TDRSS satellite.

For the track mode the target motion and the base stability are assumed to be the same as for acquisition. The tracking accuracy requirement and power spectral density (PSD) are based on the analysis of subsections 4.1 (radar) and 4.2 (communication).

The slew rate and gimbal pointing accuracy requirements are based on considerations of operator convenience and accuracy of a single speed resolver. The basic manual operational mode involves the operator slewing at a constant rate while observing a readout of gimbal position as discussed in more detail later.

The disturbance torque inputs come from two sources: orbiter translational motion during jet operation which causes a reaction torque due to gimbal imbalance, and restraint torques due to the electrical leads. The latter is a source of large torques because of the coax leads that cross the gimbal interface. Fortunately, the gimbal servo design is not particularly sensitive to the disturbance torques.

TABLE 3-15. ANTENNA SERVO PERFORMANCE REQUIREMENTS

Modes and Disturbances	Radar*	Communication	Comments
ACQUISITION MODE			
Type scan	Raster	Raster**	
Range, deg	±45	±4	Each axis
Lines	64	6	
Line resolution, deg	1.41	1.5	
Scan rate, deg/sec	133	10	Elevation axis
Gimbal acceleration, deg/sec ²	667	100	Elevation axis
Frame time, sec	60	6	
Base stability, deg/sec	3.5	3.5	Each axis
Sensor FOV, deg	2.0	2.7	Cassegrain antenna
Dwell time, ms	21	2 x 10 ³	
Target rate, deg/sec	0.06	0.02	LOS rate
TRACK MODE			
Target rate, deg/sec	0.06	0.02	Estimate
Base stability, deg/sec	3.5	3.5	
Tracking accuracy, deg	±0.57	±0.28	
Power spectral density, deg ² /Hz	0.106	5 x 10 ⁻⁵	
Angle rate accuracy, deg/sec	0.008	—	
Sensor FOV, deg	±1.2	±1.35	
Sensor linear FOV, deg	±1	±1	
MANUAL MODE			
Slew rate, deg/sec	±10	±10	Each axis
Gimbal pointing accuracy, deg	±0.1	±0.1	
DISTURBANCES			
Orbiter motion, kg-m (ft-lb)	0.0028 (0.02)	0.0028 (0.02)	
Lead torque, kg-m (ft-lb)	0.07 (0.5)	0.07 (0.5)	Elevation axis
Total, kg-m (ft-lb)	0.073 (0.52)	0.073 (0.52)	

*Worst case composite from Missions I, II, and IIIB.

**Same as radar miniscan.

3.6.2 Gimbal Angle Coverage

The basic requirement for gimbal angle coverage is 4π sr. This is not practical for a two gimbal system however because of the gimbal lock problem. Since some of the coverage is blocked by the gimbal and orbiter structure, in any case rather than go to a three gimbal system, it is possible to orient the two gimbal system such that the region lost due to gimbal lock is superimposed on that already lost due to the orbiter geometry. This

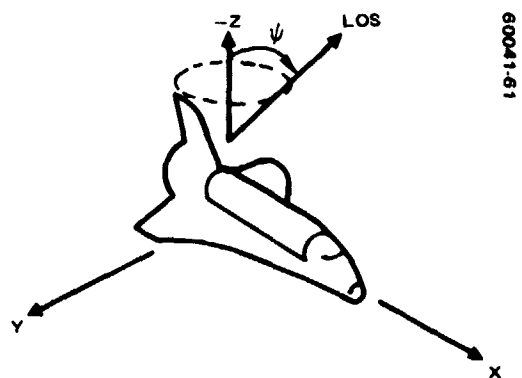


FIGURE 3-32. ORBITER AXES DEFINITION

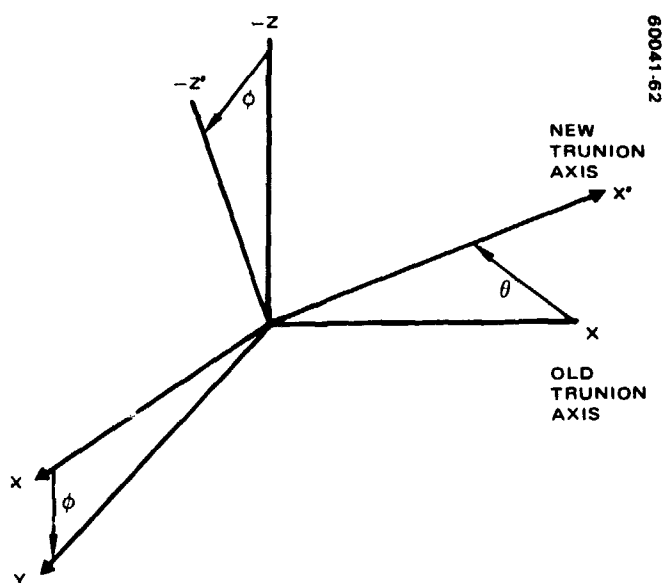


FIGURE 3-33. DEPLOYMENT ROOM AXES DEFINITION

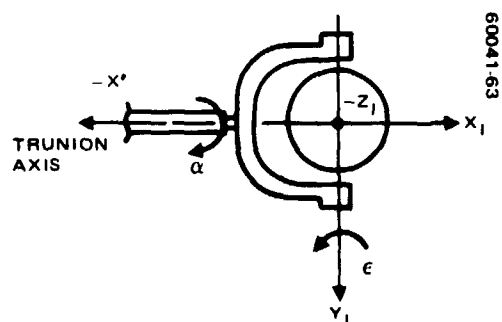


FIGURE 3-34. GIMBAL AXES DEFINITION

problem is analyzed in depth by Strelow and Werbow.* The deployment selected here is based on the results of that analysis.

As mentioned above, the basic problem is to superimpose the azimuth trunion structure blockage onto the blockage caused by the orbiter structure. At the same time it is desirable to keep the gimbal lock region of the gimbal system well away from the orbiter "vertical" axis because of the high scan rates required for the radar.

The basic approach is illustrated in Figures 3-32 through 3-34. The orbiter axes are defined in Figure 3-32. The -Z-axis is directed along the vertical. Generally, it is the area above the orbiter (defined by a half-cone angle Ψ) that is of most interest. This requirement could be best satisfied (were it the only requirement) by orienting the trunion axis of the gimbals along the -X-axis. But as discussed, it is desirable to orient the gimbal trunion axis away from the X-axis in order to optimize the antenna coverage. The new orientation of the gimbal reference is obtained by orienting the deployment boom at angles ϕ and θ relative to the original XYZ, etc., as illustrated in Figure 3-33. Gimbal rotations are now defined relative to the XYZ as illustrated in Figure 3-34.

An important consideration for this arrangement, of course, is how a LOS direction relative to the -Z-axis (as defined by Ψ) is related to the gimbal angles, and the extent of coverage provided in the XYZ axes. It can be shown that the angle Ψ is related to the boom deployment angles (ϕ , θ) and gimbal angles (α , ϵ) as follows

$$\Psi = \cos^{-1} \left[-\cos \phi \sin \theta \sin \epsilon - \sin \phi \sin \alpha \cos \epsilon + \cos \phi \cos \theta \cos \alpha \cos \epsilon \right]$$

The relationships among these parameters are further illustrated in Figure 3-35 for the case of $\phi = 60^\circ$ and $\theta = 75^\circ$. The entire spherical coverage of the LOS is depicted in terms of the gimbal angles required for the LOS. Complete coverage requires $\pm 180^\circ$ for α and $\pm 90^\circ$ for ϵ . But because of physical constraints on gimbal size, discussed in subsection 3.3, only 35° is provided in one direction of inner gimbal travel. This is illustrated in Figure 3-35 by the crosshatching.

The choice of θ and ϕ is based on the results of the Strelow study which shows that for optimum antenna coverage, the angles should be as chosen. But for this choice, one edge of the 120° cone coincides with the gimbal lock position. Therefore, it is necessary to modify the ϕ and θ coordinates slightly. By increasing ϕ to 64° , the required elevation angle is reduced in magnitude from 90 to 85° . The resultant change in antenna coverage is negligible.

*Strelow, R. E. and Werbow, M., Antenna Deployment/Antenna Coverage Study for Rendezvous Radar System and KuBand Data System,

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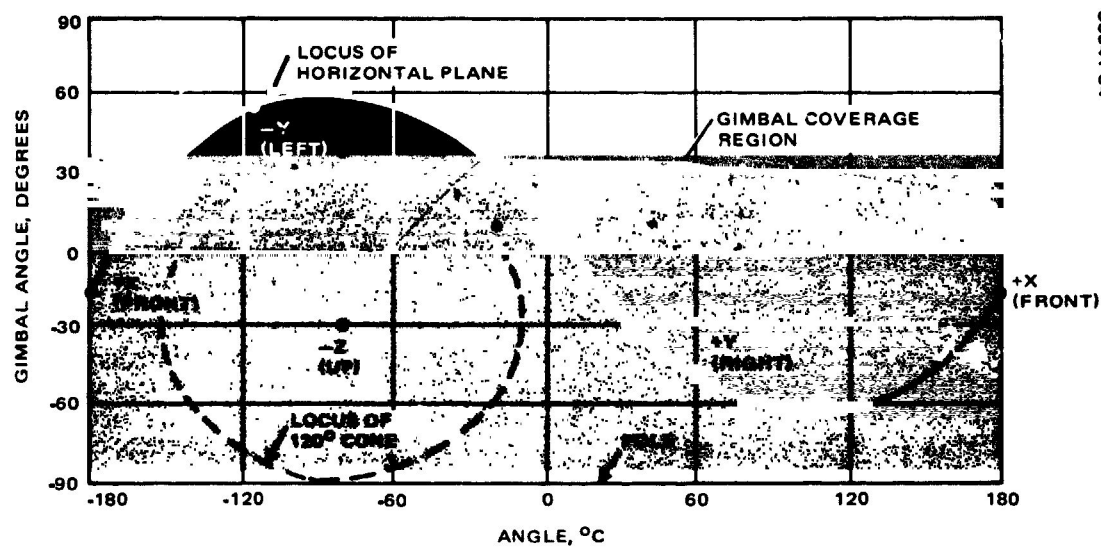


FIGURE 3-35. GIMBAL ANGLE COVERAGE RELATIVE TO VEHICLE AXES

The locus of the horizontal plane is also sketched in Figure 3-35. The -Y-axis is not covered by the gimbals (this is for a single antenna on the right side of the orbiter), but the orbiter fuselage blocks that LOS direction. Therefore, the arrangement shown is probably the optimum in terms of field of view coverage traded against gimbal weight.

3.6.3 Servo Preliminary Design

The design configuration for the gimbal control servo is governed by the radar requirements to scan rapidly during acquisition, to stabilize a tracking line of sight inertially, and to provide a high resolution LOS rate readout during radar track. To meet these requirements, a rate integrating gyro (RIG) is used to inertially stabilize the antenna against base motion disturbances and to provide the required rate resolution. Thus, the antenna is basically rate stabilized. To provide gimbal pointing or closed loop target track, a loop is closed around the RIG through a position sensor, thereby operating the rate stabilized antenna in the "erect" mode.

A functional block diagram of the system is shown in Figure 3-36. The antenna is driven by a dc torquer in each axis. Gimbal position readout is provided by a resolver for each axis, and a RIG is physically mounted to each axis with the appropriate alignment. The hardware characteristics of these are discussed in subsection 3.3.

Several operational modes are provided for in the switching logic shown in the diagram. To understand how the logic works, consider first that the antenna system is in the manual mode. That is, the switch marked MANUAL/AUTO is in the manual position. For this case, the drive signal to the gimbal torquer for each axis comes directly from the slew command switch. For a fixed slew rate, the slew command will be a dc voltage of given polarity, which after conversion to an equivalent current command is applied to the RIG torquer coil. This torques the RIG gimbal about its output axis, producing an output voltage that is applied to the antenna gimbal thereby causing the RIG case to be rotated about its input axis and generate a gyroscopic torque to counter the torque produced by the command current. Thus, the RIG operated in this way constitutes a rate stabilization system without the requirement for an electronic loop. The loop is effectively closed through the gyrodynamics, which represents a simpler implementation required with a rate gyro.

When no slew command is present, the RIG stabilized antenna is inertially held to within the rate drift capabilities of the gyro (less than 0.01 deg/sec for the selected gyro). Thus, the simple servo configuration consisting of the torque motor driven by an RIG output constitutes both a rate control system and an inertial position control system. To control the antenna relative to the orbiter or a moving target, however, requires the closure of an outer electronic loop.

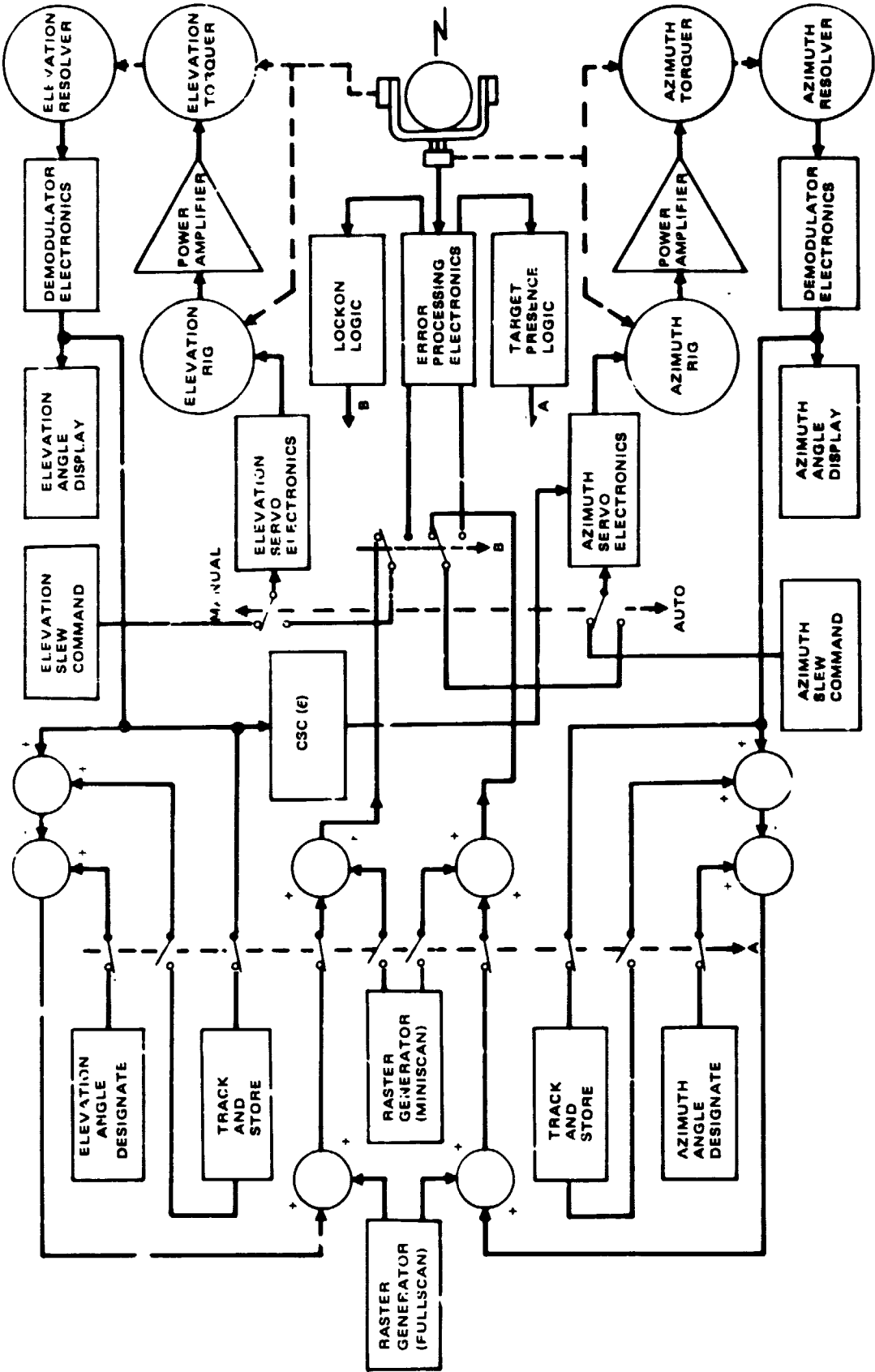


FIGURE 3-36. SERVO SYSTEM FUNCTIONAL BLOCK DIAGRAM

Two outer loops are provided: a tracking loop (which accepts inputs from either the radar tracking sensor or the communications tracking error detector) and a gimbal control loop using the gimbal mounted resolver. The resolver loop is used to automatically control gimbal pointing relative to the orbiter. This is required during acquisition to accept target designation angles and for the elevation axis during track to compensate for the cosecant effect in the gimbal kinematics. Otherwise, it is assumed that gimbal pointing will be performed manually where the operator reads the gimbal angle displays and controls position through the slew controls. It would be fairly simple, however, to add a pointing control where the gimbal angles are set through a resolver control unit, then nulled within the accuracy of the gimbal mounted resolvers.

The acquisition procedure for the radar is described as follows. Assume the switch marked AUTO/MAN is in the automatic position. A raster scan is automatically generated and applied to the rate loop through the normally closed (NC) contacts of switches A and B. Superimposed on the raster scan is a position error signal from the outer resolver loop, using commands from the GPC and angle designation electronics. The position commands are applied every T_s seconds, the sample rate, which would be chosen on the basis of operational considerations (such as, how often new information is available and the level of confidence in it). At the time the outer loop is closed, the gimbals will be quickly slewed to the designated position while the raster continues. For small movements the response time of the resolver loop is a fraction of a second. The outer loop is then opened and the raster continues to cycle until the target presence is received (discrete A). At the receipt of this signal, indicating the target has swept across the detector, switch A is energized. This closes the outer loop again, but the commanded position is the gimbal position at the time the discrete A is received. This position is continuously tracked and stored. The application of the stored position brings the gimbals back to approximately the location where the target was "sighted". Depending on a number of factors, such as orbiter motion and gimbal velocity at the time of target presence, this step may bring the target onto the detector for capture. If not, the miniscan is initiated which covers a smaller field (10° by 10°) at a much slower rate. In the final design it probably will not be necessary to include both the miniscan and the track and store capability. The miniscan is included because of the requirement to quickly reacquire in case of loss of track. An alternative approach is to use the miniscan, but instead of tracking the target position in orbiter coordinates, the outer loop is simply opened, which will quickly bring the gimbals to rest in inertial coordinates. The miniscan must be large enough in this case to accommodate the coast range of the rate system. A final choice must be made on the basis of more detail design study and simulation.

However the LOS is locked onto the target, this event produces discrete B which closes the tracking loop through the servo gain shaping electronics. It is assumed that scale differences between the radar error detector and the communication error detector are resolved in the error processing electronics so that the same dynamic shaping would apply to both cases.

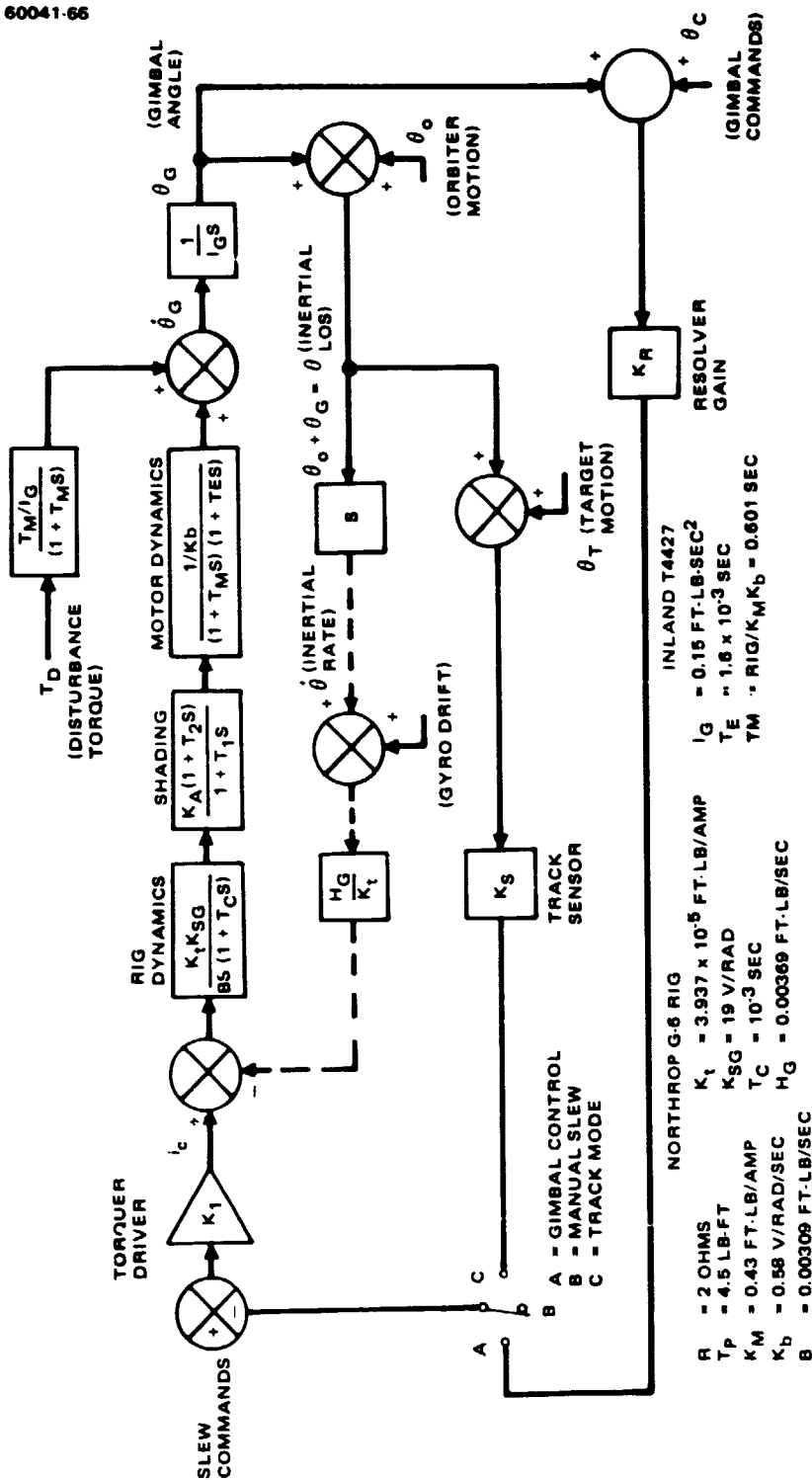


FIGURE 3-37. SINGLE-AXIS SERVO DIAGRAM

It should be noted that loss of target will cause discrete B to be lost right away but discrete A has a built-in delay on loss so that the miniscan can search for the target before the decision is made to resume full raster scanning.

3.6.4 Servo Performance Analysis

A single axis servo block diagram of the gimbal control system is shown in Figure 3-37. For preliminary design only a single axis was studied. This is valid because there is no rigid body dynamic coupling between the axes. There are two loops: an inner rate loop and an outer position loop. The position loop operates as either a track loop or as a gimbal control loop. The inner loop is shown dotted since the loop is closed through the gyro dynamics of the RIG rather than through an electronic loop, as would be the case for a rate gyro.

The rate loop blocks are arranged to illustrate the fact that rate commands are generated by current commands into the gyro torquer. These torque commands are countered by gyro torque through the antenna gimbal dynamics. The RIG dynamics are represented by its characteristic time constant T_G , gyro damping B , and scale factors K_t and K_{SG} for the torquer and output signal generator, respectively. The gyro is followed by shaping networks and a power amplifier to drive the gimbal torquers.

The torquer is represented by its mechanical time constant, electrical time constant, and back EMF constant. The gimbal dynamics for preliminary design purposes are taken as the rigid body I_G .

The total angle θ of the LOS in inertial space is the sum of the orbiter angle θ_o and gimbal angle θ_G . (θ_G represents either the azimuth α or elevation ϵ). The gyro senses the rate of change of the inertial angle. The track sensor senses a difference between the inertial angle and the target angle θ_{T1} whereas, the resolver reads the difference between commanded gimbal angle and the actual (provided the automatic gimbal pointing mode is included). The slew rate commands are introduced directly into the RIG torquer through a proportional current source.

Disturbances to the loop include external torques caused by electrical leads crossing the gimbals, gyro drift, and orbiter motion. As shown in the block diagram arrangement, the torque disturbances must be transmitted through the motor dynamics.

The preliminary design for the loop is defined in Table 3-16. The mechanical time constant for the torquer includes the gimbal inertia added to the motor inertia.

The design philosophy has been to make the bandwidth of the inner loop (15 Hz) large compared to that of the outer tracking loop, but not large

TABLE 3-16. SERVO PRELIMINARY DESIGN PARAMETERS

Parameter	Symbol	Value
GIMBAL TORQUER		
Winding resistance, ohms	R	2
Peak torque, kg-m (ft-lb)	T _P	0.62 (4.5)
Back EMF constant, V/rad/sec	K _b	0.58
Torque constant, kg-m/amp (ft-lb/amp)	K _M	0.059 (0.43)
Mechanical time constant, sec	T _M	0.6
Electrical time constant, sec	T _E	1.6×10^{-3}
Gimbal inertia, kg-m-sec ² (ft-lb-sec ²)	I _G	0.021 (0.15)
RIG		
Time constant, sec	T _C	10^3
Momentum, kg-m-sec (ft-lb-sec)	H _G	5.1×10^{-4} (3.7×10^{-3})
Damping, kg-m-sec (ft-lb-sec)	B	5.1×10^{-4} (3.7×10^{-3})
Torque constant, deg/sec/mA	K _T	0.6
Scale factor, V/rad	K _{SG}	19
INNER LOOP DESIGN		
Power amplifier gain, V/V	K _A	163
Lag time constant, sec	T ₁	0.015
Lead time constant, sec	T ₂	0.0015
Velocity constant, sec	K _V	5330
Rate loop bandwidth, Hz	—	15
OUTER LOOP DESIGN		
Resolver scale factor, V/rad	K _R	60
Sensor scale factor, V/rad	K _S	590
Gyro driver gain, amps/V	K _I	163
Resolver loop bandwidth, Hz	—	10
Track loop bandwidth, Hz	—	1

enough to cause problems with structural resonance. The bandwidth of the resolver loop is made as large as practical (10 Hz) in order to minimize the effect of orbiter rates (up to 3.5 deg/sec) on inertial pointing of the gimbal LOS. At the same time, the tracking bandwidth is kept small in order to minimize the effect of sensor noise in the loop.

The steady state tracking and pointing errors caused by the major error sources are summarized in Table 3-17. For the gimbal pointing mode, the resolver readout is used to null gimbal position commands. It is assumed that the basic accuracy and jitter of the resolver and processing electronics is equivalent to 0.1° noise (3σ). The noise equivalent angles given for the tracking mode are based on a 1 Hz noise bandwidth for both track loops. The

TABLE 3-17. SUMMARY OF STEADY-STATE ERRORS

Error Source	Operational Mode			
	Gimbal Pointing	Radar Track	Communication Track	Manual Hold (Inertial)
Sensor noise (1 Hz bandwidth), deg rms	—	0.33	0.008	—
Disturbance torque(0.1 kg-m (0.7 ft-lb))	0	0	0	0.03
Base motion (3.5 deg/sec)	0.06	0	0	0.001
Sensor accuracy	0.1	—	—	—
Radar target motion (0.23 deg/sec)	—	0.04	—	—
TDRSS target motion (0.017 deg/sec)	—	—	0.0026	—
Gyro drift (0.01 deg/sec)	0.0002	0.002	0.002	0.01*
RSS total (3 σ)	0.12	0.332	0.0154	0.032

*1 second interval.

power spectral density for the two cases is based on the analyses presented in subsection 4.1 (communication) and 4.2 (radar) and are given for the worst case tracking geometry.

The last column of Table 3-17 (Manual Hold, Inertial) corresponds to the LOS jitter while in the inertial hold position (manual/auto switch in manual but with no slew voltage applied). The sensor noise does not apply as such since it is, in fact, the gyro drift rate carried in another row below.

External disturbance torques (e.g., lead torque) cause no steady state errors when the outer loop is closed (gimbal pointing and track modes). In the manual slew mode, the torque produced offset error is determined by the inner loop gain. A constant base rate (orbiter rotational rate) causes the same type of standoff error as does the external torque for the manual hold mode. Base motion causes an error in the gimbal pointing mode due to the gyro response to inertial motion.

The target rate of 0.23 deg/sec during radar track is estimated on the basis of rendezvous geometry for Mission IIIB. The value of 0.017 deg/sec for communication track is the worst case inertial LOS rate for two satellites separated by synchronous attitude.

The effect of gyro drift is to produce an error for all operational modes. For the inertial hold mode, of course, a constant drift rate will produce an increasing error with time since the inner loop sees the drift as a rate command.

From Table 3-17 it is apparent that the steady state errors are caused principally by the sensor noise. For the radar track mode the resulting error is acceptable. This value is for a distance of 12 miles. For a 10 mile distance the error drops to 0.26° (3σ). The communications tracking error is quite low and is obviously not a problem.

The angular rate resolution of the servo is a function only of the rate integration gyro chosen. Discussions with vendors indicate that rate integrating gyros with sufficient resolution are available.

3.7 DEPLOYED ASSEMBLY PACKAGING

3.7.1 General Description

The deployed Ku-band radar/communications subsystem electronics is contained in two main packages, the antenna electronics unit and transmitter/receiver electronics unit. The deployed units are shown in Figures 3-38 and 3-39 with various antenna configurations that are under consideration. Packaging studies were concerned primarily with the spatial arrangement of components within the antenna electronics unit.

3.7.2 Antenna Electronics Unit

The initial component arrangements included installation of the transmitter TWTA and elements of the receiver group inside the antenna box as shown in Figure 3-40. This arrangement was studied for the communications system only. The configuration did not include radar components and used a helix TWTA. Preliminary thermal considerations indicated an undesirable thermal interchange between the antenna package, back into the shuttle orbiter. Also, an adequate heat sinking area could not be provided for the TWTA. The physical characteristics of this package are presented in Table 3-18.

TABLE 3-18. SHUTTLE KU-BAND ANTENNA ELECTRONICS
PHYSICAL CHARACTERISTICS

Description	Quantity	Mass, kg (lb)	Power, W (at 28 V)
RECEIVER GROUP			
Diplexer	1	0.18 (0.4)	—
15 dB WG coupler	1	0.06 (0.13)	—
Parametric amplifier	1	0.45 (1.0)	7.0
Error modulator	1	0.68 (1.5)	—
Mixer chain	1	0.86 (1.9)	4.1
IF amplifier	2	0.23 (0.5)	3.4
TRANSMITTER GROUP			
TWTA	1	5.90 (13.00)	90.0
Frequency upconverter	1	0.65 (1.44)	0.6
Frequency multiplexer chain	1	0.79 (1.75)	4.5
MISCELLANEOUS			
Housing	1	2.49 (5.5)	
Cables	Lot	1.59 (3.5)	
Waveguide	Lot	0.32 (0.7)	
Brackets, etc.	—	0.23 (0.5)	
Total		14.43 (32.0)	109.6 (dissipated)

By removing the primary thermal source from the antenna package, the thermal dissipation was greatly reduced, as well as the package size and weight. The component arrangements as shown in Figure 3-41 consist mainly of Ku-band waveguide switches and RF components. However, Ku-band waveguide rotary joints will have to be employed in order to interface the TWTA/modulator located in the transmitter/receiver unit and the antenna package. The waveguide run will be routed between the two deployed boxes via the gimbal structure as shown in subsection 3.5. It was found to be feasible to package the receiver front end in the antenna package to eliminate one set of rotary joints.

The antenna package will also provide the mounting interface for the antenna and the outer gimbal mechanism. The dc and RF cable bundles will also have to be routed through the gimbal structure down into the transmitter/receiver box. It is estimated that the interunit wiring will consist of one low frequency signal cable bundle and four flexible 50 ohm coax cables.

3.7.3 Weight Estimate (Table 3-19)

The following preliminary weight estimate is provided for the deployed Ku-band radar/communication subsystem. It should be noted that the deployed boom is not a part of this weight study. Also, the material weights of the boxes and other structural elements of this subsystem are calculated based on aluminum. Subsequent weight reduction studies, stress, and dynamic analysis will reduce these figures.

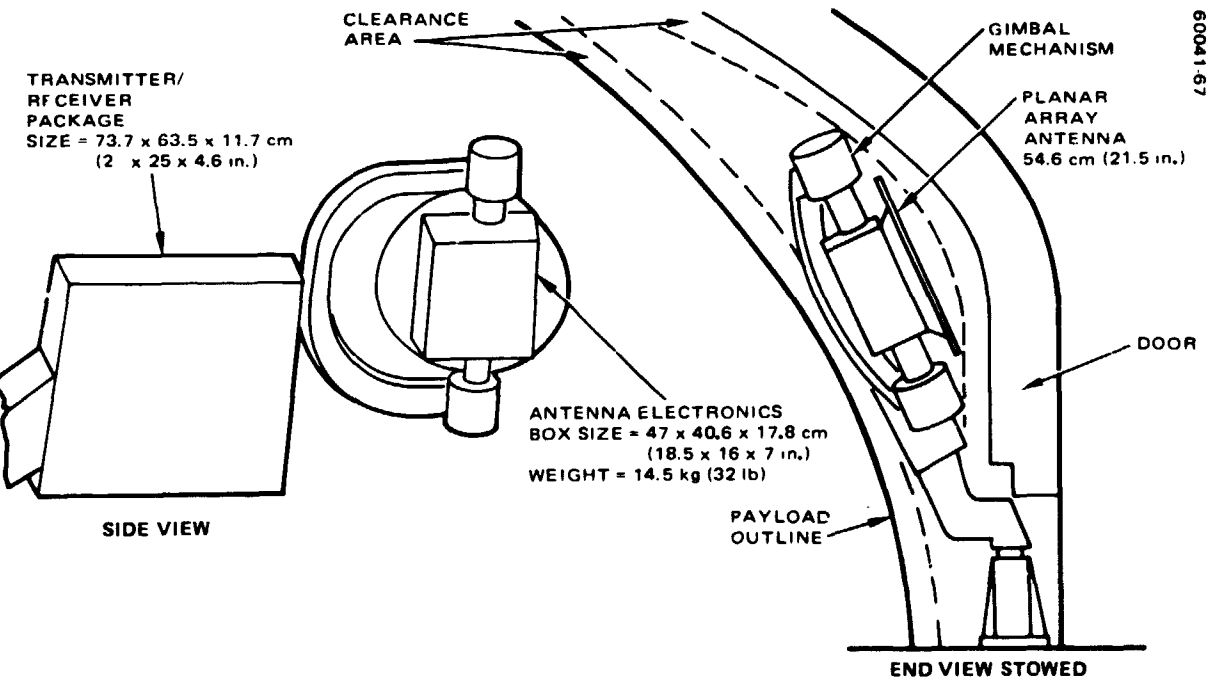
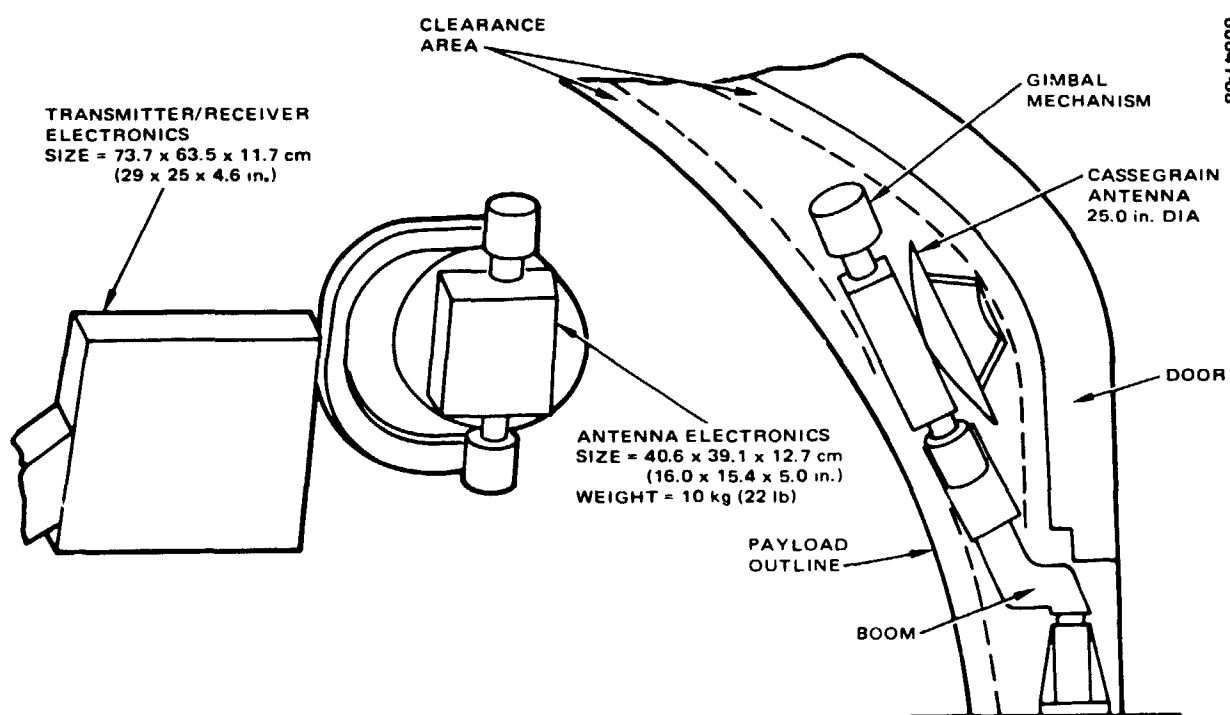


FIGURE 3-38. ANTENNA ELECTRONIC UNITS WITH PLANAR ARRAY ANTENNA – COMMUNICATIONS ONLY

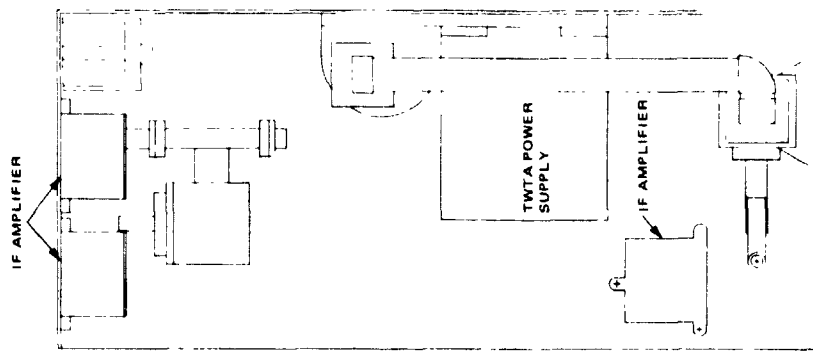
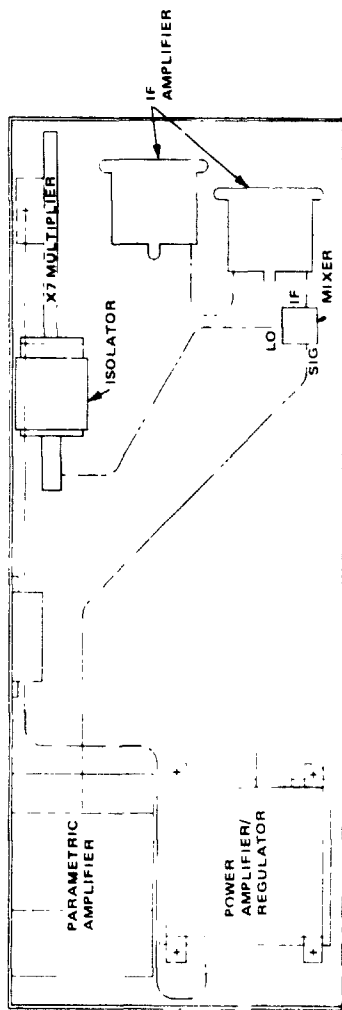


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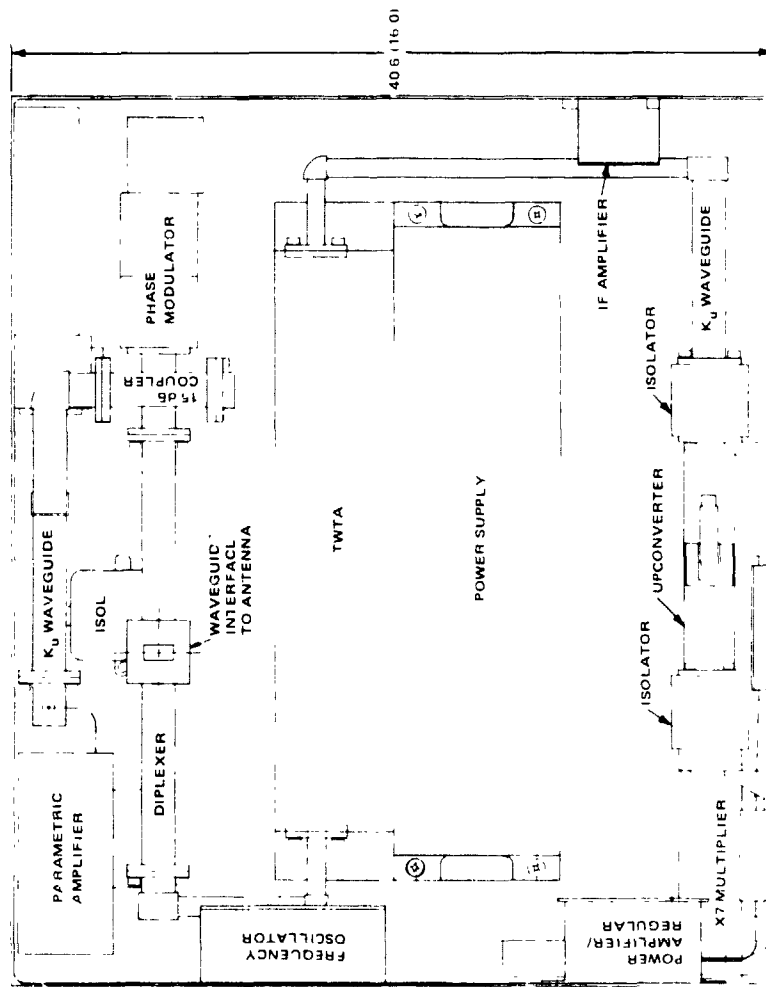
FIGURE 3-39. RADAR/COMMUNICATIONS WITH CASSEGRAIN ANTENNA

TABLE 3-19. PRELIMINARY WEIGHTS OF DEPLOYED PACKAGE

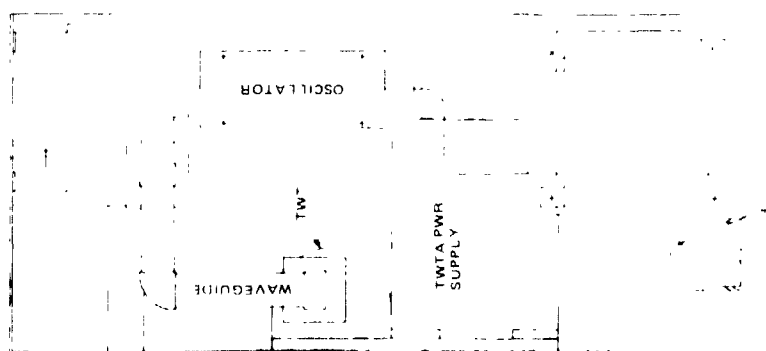
Description	Quantity	Total Weight, kg (lb)
TWTA/modulator	1	17.24 (38.0)
Parametric amplifier	2	0.91 (2.0)
Upconverter	1	0.68 (1.5)
Multiplier chain	1	0.77 (1.7)
Radar exciter	1	2.23 (4.7)
Radar receiver	1	0.68 (1.5)
Communications receiver	1	0.68 (1.5)
Rotary joints	2	1.13 (2.5)
Parabolic antenna	1	1.36 (3.0)
Horn antenna	1	0.68 (1.5)
Polarizer	1	0.23 (0.5)
BPF	2	0.36 (0.8)
Circulator switch	3	0.68 (1.5)
Directional coupler	2	0.18 (0.4)
Transmitter/receiver switch	1	0.14 (0.3)
Power monitor	1	0.18 (0.4)
Chassis and structure	2	9.07 (20.0)
Gimbal and structure	1	18.14 (40.0)
Thermal doubler and control circuits	1	6.12 (13.5)
DC cabling	Lot	3.18 (7.0)
Coax cables	Lot	1.13 (2.5)
Waveguide	Lot	0.45 (1.0)
Brackets, miscellaneous hardware	Lot	0.91 (2.0)
		Total weight 67.1 kg (147.8)



END VIEW

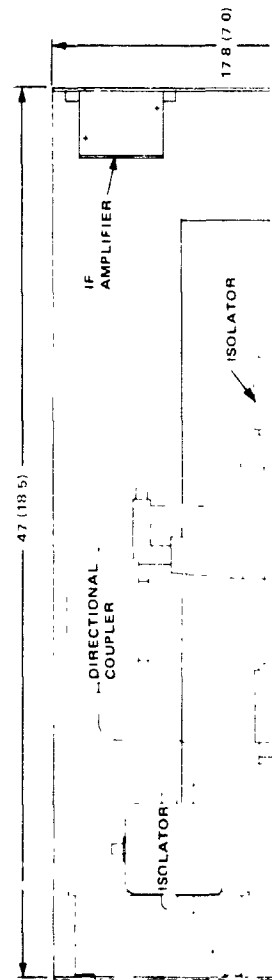


TOP VIEW



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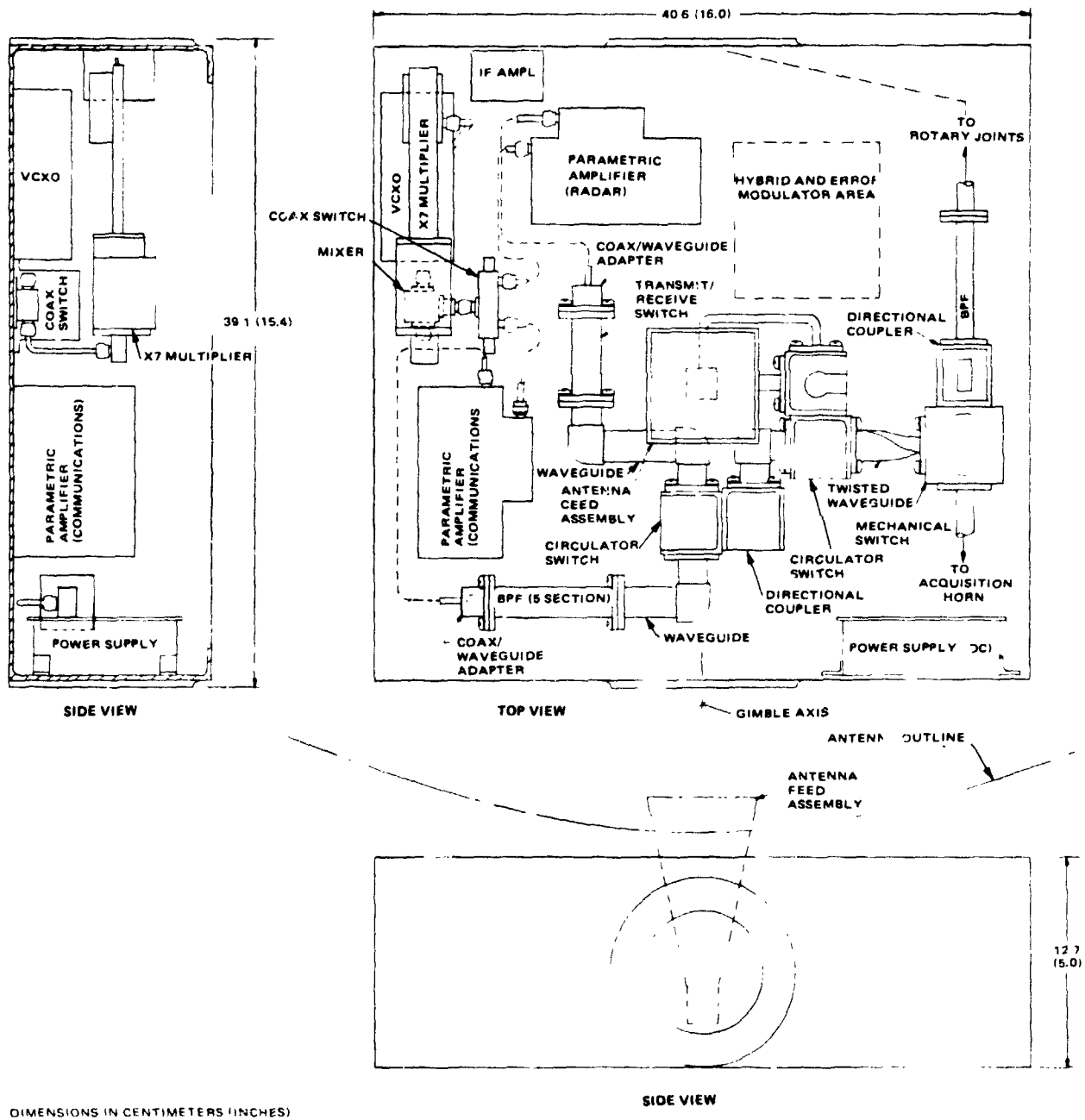


FIGURE 3-41. ANTENNA ELECTRONICS UNIT - INTEGRATED SYSTEM

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3.8 THERMAL SUBSYSTEM

3.8.1 Thermal Control

The Ku-band integrated rendezvous radar and communication subsystem thermal control is a significant problem only for the deployed assembly which must operate independently of the space shuttle active heat rejection system. The remaining packages, which are contained in the avionics bay, have access to the shuttle thermal control system and are not considered here.

3.8.2 Environment (Table 3-20)

The Ku-band radar/communication deployed assembly will be subjected to a varying environment. In its stowed position the assembly may be exposed to the deep space environment when shuttle equipment bay doors are in the open position. When shuttle equipment bay doors are closed, the equipment is protected by the relatively benign environment existing within the shuttle equipment bay. Subsequent to deployment, the subsystem will be exposed to the deep space environment, space shuttle external surfaces, and a view of the shuttle's coolant radiators.

Environmental temperature extremes and shuttle properties of interest are summarized below. These data were used as boundary conditions in sizing radiator surfaces for the communications package.

The values for earth emission and albedo loads used in this study are 70 Btu/hr·ft² and 0.36, respectively.

3.8.3 Design Requirements

Radar/communication subsystem design temperature requirements are summarized in Table 3-21. These limits have been chosen to provide high reliability, consistent with the design life of the communications equipment. Design and qualification limits have been derived from other Hughes communication satellite programs where identical or similar hardware is utilized. The Ku-band TWT has a very broad operational temperature range for an external collector design. However, these limits have been narrowed to be compatible with TWTs flown on commercial satellites whereby

TABLE 3-20. SHUTTLE THERMAL PARAMETERS

Parameters	Temperatures °C (°F)		Solar Absorptance	Emittance
	Maximum	Minimum		
Radiators	121 (250)	156 (-250)	0.15	0.7
Cabin exterior	66 (150)	129 (-200)	0.40	0.85

0-2

TABLE 3-21. RADAR/COMMUNICATION SUBSYSTEM TEMPERATURE REQUIREMENTS, °C (°F)

Component	Design		Qualification	
	Maximum	Minimum	Maximum	Minimum
COMMUNICATION				
TWT*				
Collector	235 (455)	-135 (-211)	250 (482)	-150 (-238)
Body	50 (122)	10 (50)	65 (149)	-5 (23)
Power supply	40 (104)	0 (32)	55 (131)	-15 (+5)
Electronics	40 (104)	0 (32)	55 (131)	-15 (+5)
Motors	130 (266)	-50 (-58)	145 (293)	-65 (-85)
RADAR				
Electronics	40 (104)	0	55 (131)	-15 (+5)
Antenna	130 (266)	-150 (-238)	145 (293)	-165 (-265)
Feed	TBD	TBD	TBD	TBD
Motors	130 (266)	-50 (-58)	145 (293)	-65 (-85)
Resolvers	130 (266)	-50 (-58)	145 (293)	-65 (-85)

*Revision may be required based on collector/body conductive coupling and final configuration.

the collectors are hard mounted to an equipment platform. Drive motor temperature limits (-50° to 130° C) are consistent with drive motors used to orient antenna systems on Hughes commercial satellites.

Radar/communication subsystem power requirements are summarized in Table 3-22 for the proposed operational modes. For the purpose of this study, the thermal dissipation is assumed to be the same, 145 watts, for all modes.

TABLE 3-22. RADAR/COMMUNICATION SYSTEM POWER SUMMARY, W

Component	Operational Modes*		
	Communication	Radar Search	Radar Track
TWT	80	28	7
Power supply	35	35	35
Modulator**	(20)	20	20
Communication electronics	30	30	30
Total	145	123	92

*During nonoperational periods, 145 W are dissipated through the thermostatically controlled heating elements distributed over the radiator surface.

**Dissipation not included in thermal balance. During radar operation total dissipation is reduced due to lower average power and use of modulation.

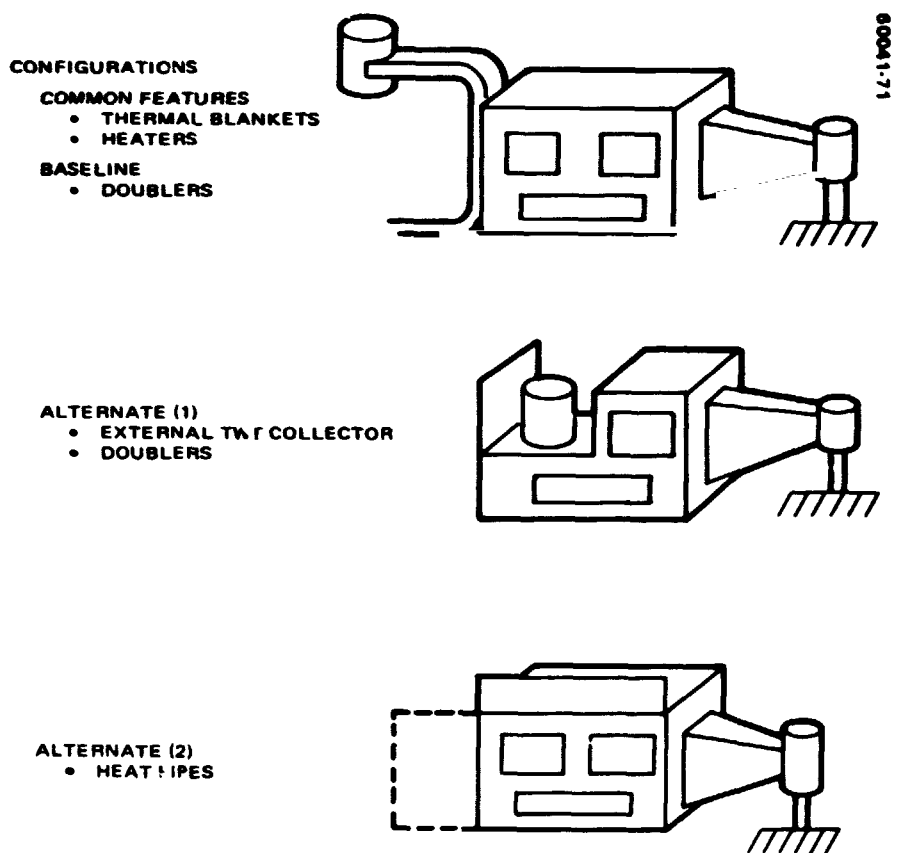


FIGURE 3-42. THERMAL DESIGN CONFIGURATIONS

3.8.4 Thermal Design Concept (Figure 3-42)

The radar/communication subsystem will utilize passive thermal control techniques to provide adequate temperature control. A radiator surface characterized by a low solar absorptance and high infrared emittance surface finish will be used to reject internal dissipation to the space environment during operational periods. The design is augmented with thermostatically controlled heaters to provide acceptable temperatures during nonoperational periods. Thermal doublers are used to distribute thermal energy from regions having high power densities and reject it to the external environment via radiation. Thermal blankets are employed to radiatively decouple the electronic compartments from the widely varying external environment, and to minimize the required heater power during nonoperational periods. The radiator surfaces employ aluminized teflon ($\alpha_s = 0.15$, $\epsilon \approx 0.70$).

3.8.5 Thermal Performance

Radar/communication subsystem steady state thermal performance is summarized in Table 3-23. These data present the maximum and minimum operational temperatures. The maximum temperatures occur when the compartment radiator is facing the earth; hence, the earth emission and albedo thermal loads are maximum and the radiator surface has a minimum view of deep space. It is readily apparent that this condition sizes the thermal doublers. Minimum temperatures occur when the compartment radiator views deep space and receives no solar irradiation, earth emission, or albedo thermal loads. Calculations indicate that the power supply and electronics exhibit maximum and minimum temperatures of 40° and 0°C, (104° and 32°F), respectively. The TWT collector exhibits a maximum temperature of 55°C (131°F) for the worst case condition.

During periods when the payload is inhibited, heaters are employed to maintain acceptable temperature levels. Thermal control heaters are thermostatically activated. Thermostats are used to enable a heater driver

TABLE 3-23. RADAR/COMMUNICATION SYSTEM THERMAL PERFORMANCE SUMMARY, ~°C (°F)

Component	Operational		Heater Mode
	Maximum	Minimum	
TWT			
Collector	55 (131)	16 (61)	5 (41)
Body	41 (106)	5 (41)	5 (41)
POWER SUPPLY	35 (95)	0 (32)	5 (41)
COMMUNICATION ELECTRONICS			
Drive motors	40 (104)	5 (41)	5 (41)

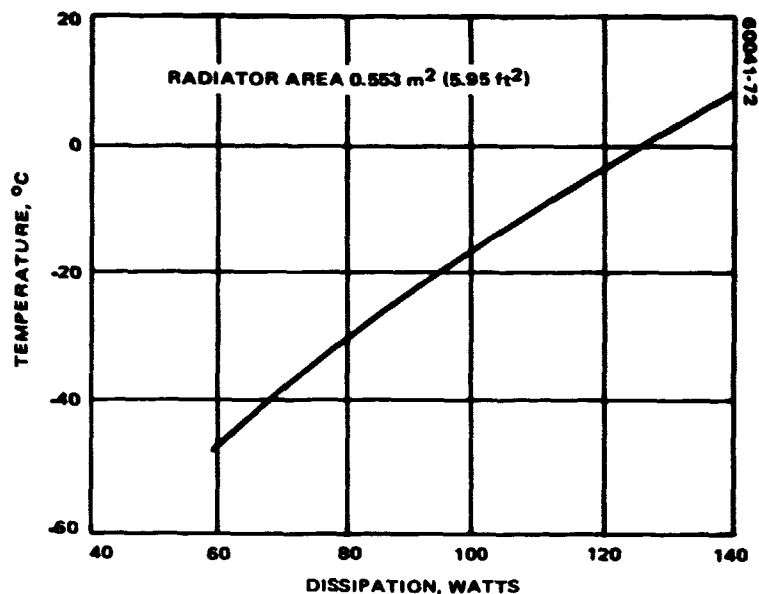


FIGURE 3-43. COMMUNICATION COMPARTMENT BULK TEMPERATURE VERSUS HEATER DISSIPATION

that supplies current to multiple daleohm heating elements distributed across the radiator surface. The heater distribution is selected to provide an average radiator surface temperature of 5°C (41°F). The electronics compartment bulk temperature is presented in Figure 3-43 as a function of heater dissipation. These data indicate that 67 watts are required to maintain compartment temperatures above -40°C (-40°F) (survival limit).

Radar/communication subsystem thermal control weight requirements are summarized in Table 3-24. These data indicate that approximately 7 kg (15.4 lb) are required to provide acceptable temperature control for each unit using conventional aluminum doublers. The use of beryllium doublers will result in a weight savings of 1.4 kg (3 lb). Preliminary estimates indicate that radiator weight can be reduced from 4.5 kg (9.9 lb) (aluminum) to approximately 1.1 kg (2.5 lb) for a radiator employing heat pipes embedded in a honeycomb substrate. Preliminary analyses indicate that the acceleration loads resulting from the motion (roll, pitch, yaw) of the shuttle are insignificant and will not have adverse effects on heat pipe operations. Therefore, the heat pipe radiator design will continue to be carried as the alternate until the full matrix of acceleration loads resulting from all shuttle motions can be evaluated to provide assurance that the communication subsystem thermal control system will be adequate for all mission phases.

**TABLE 3-24. RADAR/COMMUNICATION SYSTEM THERMAL
CONTROL WEIGHT SUMMARY, kg (lb)**

Component	No. of Units	Aluminum	Beryllium
THERMAL DOUBLERS			
TWT	1	2.81 (6.2)	1.95 (4.3)
Power supply	1	0.68 (1.5)	0.50 (1.1)
Communication electronics	1	1.00 (2.2)	0.68 (1.5)
THERMAL BLANKETS			
Communications package	1	0.68 (1.5)	0.68 (1.5)
Radar	1	0.23 (0.5)	0.23 (0.5)
THERMAL CONTROL HEATERS			
Heaters (daleohms)	40	0.82 (1.8)	0.82 (1.8)
Heater driver	2	0.68 (1.5)	0.68 (1.5)
Thermostats	4	0.09 (0.2)	0.09 (0.2)
Total		6.99 (15.4)	5.63 (12.4)

3.9 SYSTEM WEIGHT AND POWER SUMMARY

The estimated weight and power for the integrated system by assembly is given in Table 3-25. Weight of the deployed assembly is based on the detailed estimate of subsection 3.7. Power dissipation for this unit is detailed in subsection 3.8. The weight of the deployed assembly does not include the deployment boom. The radar signal processor is included in the A-1 and A-2 assemblies and not in the signal processor. Power estimates do not include peak dissipation of the antenna drive motor during the radar high speed search since the duration is only 1 minute.

TABLE 3-25. ESTIMATED WEIGHT AND POWER FOR INTEGRATED SYSTEM

Assembly	Power, W	Weight, kg (lb)
DEPLOYED ASSEMBLY	200	67.1 (148)
AVIONICS BAY UNITS		
Radar/communications A-1	2	10.9 (24)
Radar/communications A-2	2	9.1 (20)
Signal processor	18	6.8 (15)
Communications B-1	15	9.1 (20)
Communications B-2	15	6.8 (15)
Total	288	109.8 (242)

4. SYSTEM PERFORMANCE ANALYSIS

The detailed communication and radar analyses performed during the study are presented in this section.

4.1 COMMUNICATION PERFORMANCE ANALYSES AND TRADEOFFS

The communication analyses were primarily concerned with the problem of acquiring the Tracking and Data Relay System Satellite (TDRSS). Two cases were considered. The first was to despread the spread spectrum communication signal before angle acquisition. The second case was to accomplish angle acquisition and to despread the spread spectrum signal independently.

The remaining analysis tasks included techniques for maintaining despread delay lock loop tracking after acquisition, phase locked loop acquisition time, phase locked loop signal-to-noise ratio and its effect on bit error rate, and angle tracking thermal noise performance.

The derivation of angle tracking thermal noise is presented in Appendix A. Appendix D is an analysis of interference due to transmitter distortion.

4.1.1 Communications Performance Summary

4.1.1.1 Forward Link Circuit Margins

Circuit margins for the forward link have been computed using several different assumed conditions regarding both data type and shuttle communication equipment characteristics. For the shuttle communication package, two antenna and two receiver implementations have been considered. These implementations and their characteristics are tabulated in Table 4-1.

Both biphasic and unbalanced quadriphase have been considered for the forward link data. Biphasic data are assumed to be sent at a 216 kbps data rate, with a maximum error rate objective of 10^{-4} .

TABLE 4-1. ANTENNA AND RECEIVER SUMMARY

Antenna type	On-axis gain, dB
Cassegrain	35.3
Front fed	35.7
Receiver	System noise temperature, °K
Mixer front end	960 (29.8 dB - °K)
Paramp	480 (26.8 dB - °K)

In the unbalanced quadriphase case, 72 kbps is sent uncoded on one channel, again with a $P_e = 10^{-4}$ objective, while the other channel carries 1 Mbps data, convolutionally encoded as above ($V = 3$, $K = 7$), with a 10^{-6} error probability objective.

Circuit margins for these various conditions are computed in Table 4-2. A "baseline" calculation is carried out using the cassegrain antenna/mixer front end combination. Circuit margins for the other combinations are arrived at by appropriately incrementing those of the baseline. Note that the increment between antenna types is 0.4 dB and between receivers, 3 dB (see Table 4-1).

4.1.1.2 Forward Link Acquisition Performance

Subsections 4.1.2 through 4.1.8 present a number of analytical studies regarding forward link signal acquisition and tracking. The results of these studies indicate forward link acquisition performance is as follows:

Pseudo-noise (PN) despread acquisition time (see subsection 4.1.4.5)	<1 second
Phase lock acquisition time (see subsection 4.1.8)	<0.2 second
Target acquisition (0.99 probability) (see subsection 4.1.7)	
Interlocked despread/spatial scan	5 to 10 seconds
Independent spatial scan	2 to 3 seconds

TABLE 4-2. FORWARD LINK CIRCUIT MARGIN SUMMARY

Parameter	Unbalanced Quadriphase				Biphase	
	1 Mbps Channel		72 kbps Channel		216 kbps Channel	
	Cassegrain Antenna	Front Fed Antenna	Cassegrain Antenna	Front Fed Antenna	Cassegrain Antenna	Front Fed Antenna
TDRSS net EIRP, dBW	49.0					
Polarization loss, dB	0.1					
Space loss, 14 GHz, 42,253 km (22,800 n.mi.) dB	-207.8					
Modulation loss, dB	<u>-1.0</u>		<u>-7.0</u>		<u>0.0</u>	
Received signal, relative to isotropic, dBW	-159.9		-165.9		-158.9	
Shuttle antenna gain (cassegrain), dB	35.3	35.7	35.3	35.7	35.3	35.7
Received power, dBW	<u>-124.6</u>		<u>-130.6</u>		<u>-123.6</u>	
System noise temperature (mixer), dB-°K	29.8		29.8		29.8	
Boltzmann's constant dBW/°K-Hz	-228.6		-228.6		-228.6	
Bit rate bandwidth, dB-Hz	60.0		48.6		53.3	
Total noise, dBW	<u>-138.8</u>		<u>-140.2</u>		<u>-150.2</u>	
E _b /N ₀ , dB	14.2		19.6		22.4	
Spread spectrum degradaton, dB	-1.0					
TDRSS SNR degradation, dB	-1.0					
Bit sync degradation, dB	-2.0					
Effective E _b /N ₀ , dB	10.2		15.6		17.9	
Required E _b /N ₀ dB	<u>5.0</u>		<u>8.4</u>		<u>8.4</u>	
Margin, dB mixer front end	5.2	5.6	7.2	7.6	9.5	9.9
Margin, dB paramp	8.2	8.6	10.2	10.6	12.5	13.4

The preceding analyses were carried out on the basis of the following receiving system G/T values for the cassegrain antenna and the front fed antenna:

<u>Receiver Front End</u>	<u>Cassegrain G/T</u>	<u>Front Fed G/T</u>
Mixer	5.1	5.5
Paramp	8.1	8.5

The above values are in agreement with those given in subsection 3.1.

4.1.2 Background Assumptions

An analytical evaluation of the acquisition and tracking functions of the shuttle Ku-band integrated communications/radar subsystem is given in this subsection. Acquisition involves four main functions: 1) spatial search for signal angle-of-arrival, 2) despreading, 3) establishing phase lock, and 4) establishing bit synch. These functions are to be completed within 60 seconds of initiation, given the actual presence of a signal having the proper characteristics.

Concurrent with performing the acquisition sequence, the communication subsystem also will begin generating antenna steering signals and initiate switchover of antenna positioning subsystem to autotrack mode of operation.

Table 4-3 lists characteristics of the received signal which are pertinent to acquisition and upon which the present design analysis is based.

TABLE 4-3. FORWARD LINK SIGNAL CHARACTERISTICS

Frequency	13.8 GHz
EIRP	49.0 dBW
Maximum doppler	
Compensated	±7.5 kHz
Uncompensated	±350 kHz
Data (biphase-L modulation, all cases)	
Alternative A. Unbalanced quadriphase	
In-phase channel rate	≤3(10 ⁶) symbols/sec
Quadrature channel rate	72(10 ³) symbols/sec
Power ratio, in-phase/quadrature	4:1
Alternative B. Biphase	216(10 ³) symbols/sec
PN spread sequence rate	14.28(10 ⁶) chips/sec
PN spread sequence length	2047 bits

Table 4-4 similarly gives assumed characteristics of the integrated communications/radar subsystem. Finally, Table 4-5 using information from the preceding two tables, develops quantitative estimates of normalized on-axis received signal strength P_0/N_0 , a basic quantity in the analysis to follow.

4.1.3 Alternative Methods for Spatial Search and Despreading

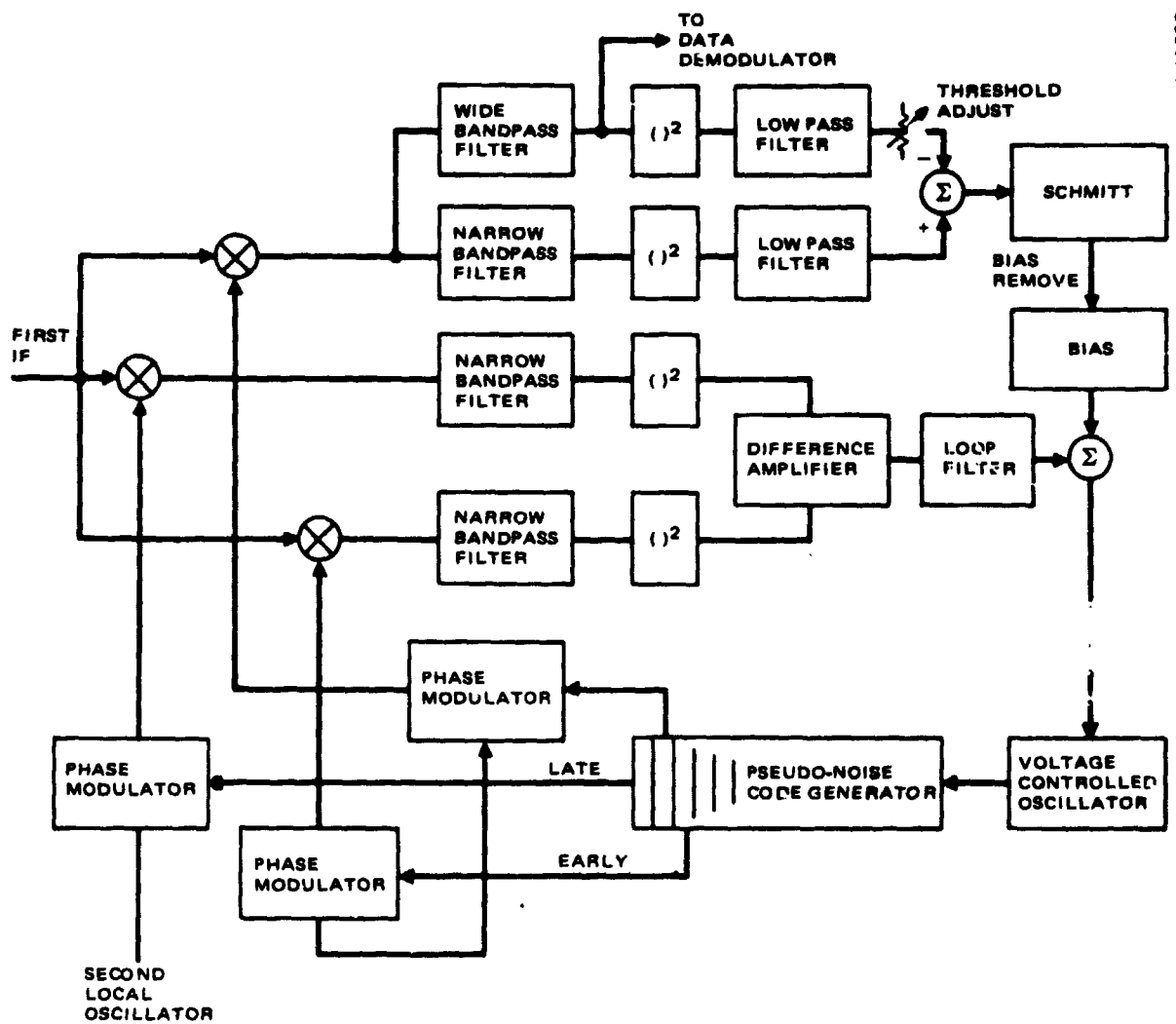
Two different approaches have been considered with regard to implementing the functions of spatial search and PN despreading. In one approach, the two functions proceed independently. The communication subsystem searches in space for the spectrally spread communication signal, while the despreader simultaneously searches for a correlation match. In the second approach, the despread and spatial search functions are interlocked. Spatial acquisition is based on sensing the despread communication signal and thus is contingent on the despreader already having achieved synchronization.

TABLE 4-4. SHUTTLE RECEIVING SYSTEM CHARACTERISTICS

G/T	
Mixer	5.1 dB
Paramp	8.1 dB
Polarization/VSWR loss	0.5 dB (included in G/T)
Antenna 3 dB beamwidth	2.1°
Frequency stability	±5 parts in 10 ⁶

TABLE 4-5. COMPUTATION OF NORMALIZED RECEIVED SIGNAL LEVEL P_0/N_0

TDRSS EIRP	49.0 dBW
Path loss	-207.8 dB (22,786 n.mi.)
Received signal, isotropic	-158.3 dBW
Shuttle G/T	
Mixer	5.1 dB-°K
Paramp	8.1 dB-°K
Boltzmann's constant	-228.6 dB/°K
P_0/N_0 on-axis	
Mixer	74.9 dB
Paramp	77.9 dB



60041-1

FIGURE 4-1. PSEUDO-NOISE DESPREADER

The interlocked or despread-first case puts a constraint on the maximum rate that the antenna may be allowed to scan. Antenna motion must be slow enough to allow the PN despreader to search through all possible time-match positions during the time a potential signal source remains in the antenna beam. In the other case (spatial acquisition before despread), the antenna scan rate is limited, in principle at least, only by the integration time required by the radiometer-type receiver. This would permit antenna slew rates one or two orders of magnitude faster than in the despread-first approach, though as a practical matter mechanical considerations likely would prevent much of this apparent speed advantage from being realized.

The interlocked or despread-first approach, though possibly more time consuming, still performs well within required time objectives and leads, furthermore, to a simpler implementation. Therefore, this system is taken as the baseline in the present analysis. The major portion of analytical effort concerns this case.

4.1.4 Despread-First Implementation

The circuitry for implementing the despreading function is illustrated in Figure 4-1. Despreading is done in conjunction with downconversion to the second IF. The second LO is biphase modulated with a locally generated version of the PN spreading sequence. Means are provided to establish and maintain proper phasing between the two PN sequences, local and incoming.

A delay lock loop of the envelope correlating type employing an early/late gate arrangement is used to maintain synchronism once established. To establish synchronism initially, a search procedure is performed. Search is carried out by clocking the local PN sequence generator at a slightly higher rate than the chip rate of the incoming sequence, so the codes appear to slip past each other in time. At some point in time, the codes coincide. A large buildup of signal energy occurs in a correlation detector, signaling the end of the search phase.

4.1.4.1 Adaptive Threshold Correlation Detector

Figure 4-2 details the portion of Figure 4-1 that senses when incoming and locally generated PN sequences are synchronized. The circuitry consists of two parallel channels, one of relatively large bandwidth and the other narrow. Each has a noncoherent detector, modeled as square law, followed by a low pass filter.

The wideband or reference channel functions to provide an adaptive reference voltage level for detecting signal buildup in the narrow channel. The reference adjusts to long term changes that might occur in gain or noise characteristics of the receiver prior to the second mixer, and prevents false alarms due to such factors as a strong signal or noise source (e.g., the sun) entering the receive antenna beam. In particular, the adaptive reference

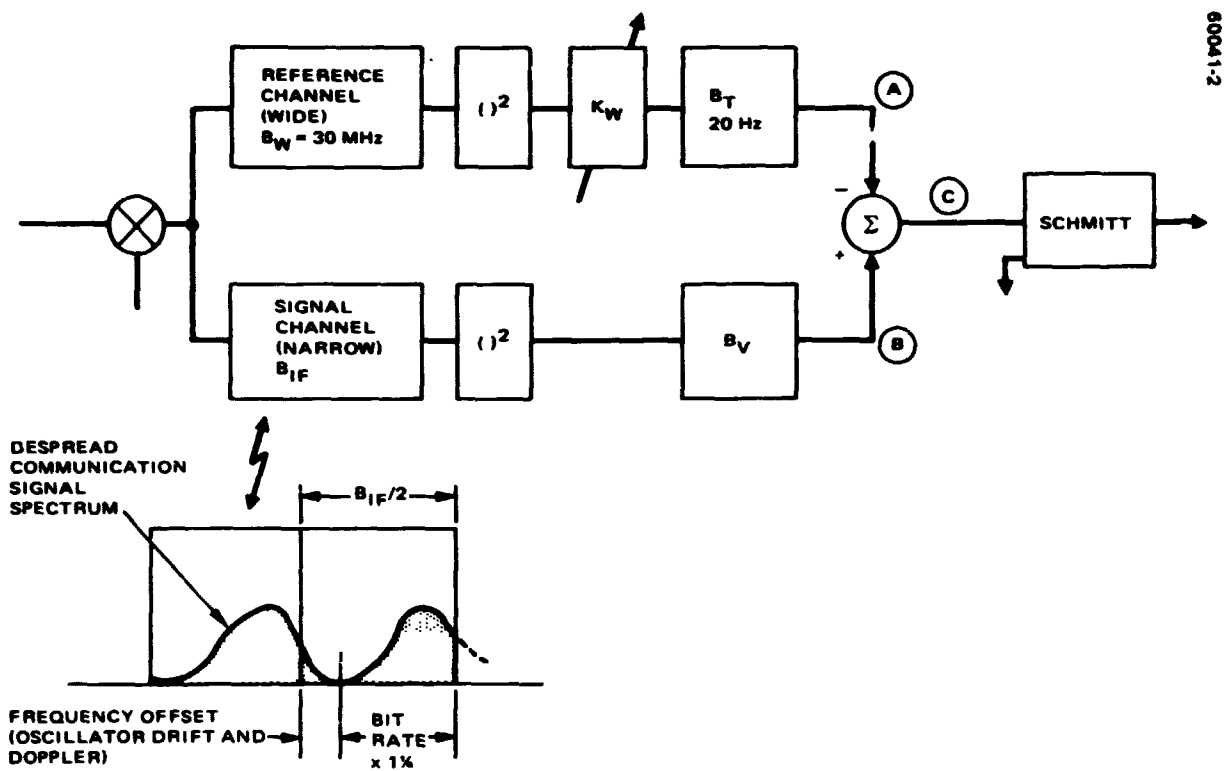


FIGURE 4-2. ADAPTIVE THRESHOLD PSEUDO-NOISE SYNCH SENSOR

guards against a premature acquisition signal being produced when the TDRSS signal first enters the antenna beam prior to actual PN synchronization.

The reference channel bandwidth B_W is selected to pass roughly equal amounts of signal-plus-noise whether PN synchronization has occurred or not. The postdetection filter in this channel, B_T , is narrow enough to reduce fluctuations due to noise to a negligible level, yet wide enough to follow the change in average level that occurs when a signal source (e.g., the TDRSS) enters the antenna beam.

The bandwidth B_{IF} of the narrowband or "signal" channel is selected on the basis of the despread communication signal spectrum width and frequency uncertainties. The criterion used is that, under worst case frequency offset conditions, no more than 1 dB of (despread) signal power should fall outside the B_{IF} passband. The inset in Figure 4-6 shows that this implies a half-bandwidth equal to maximum frequency offset plus about 1.25 times bit rate.

Frequency offset results from oscillator drift and doppler shift. The bit rate in question, in the case of unbalanced quadriphase, is that of the low data rate channel. Table 4-6 computes half-bandwidth as defined in Figure 4-2 for various conditions, and Table 4-7 lists B_{IF} .

With no despread correlation, only a small amount of the broad spectrum reaching B_{IF} will pass through. When correlation is achieved, the communication signal spectrum "collapses" to fall within B_{IF} , resulting in a sharp rise in B_{IF} output level.

TABLE 4-6. HALF-BANDWIDTH, kHz

Item	72 kbps (Compensated)	216 kbps (Not Compensated)	72 kbps (Compensated)	216 kbps (Not Compensated)
Oscillator drift (5 ppm)	70	70	70	70
Doppler	8	8	350	350
Data rate times 1.25	<u>90</u>	<u>270</u>	<u>90</u>	<u>270</u>
Half-bandwidth	168	348	510	690

TABLE 4-7. BANDWIDTH OF SIGNAL
BRANCH B_{IF} , kHz

Signal Type	Doppler Compensated	No Doppler Compensation
Quadriphase	340	1020
Biphase	700	1380

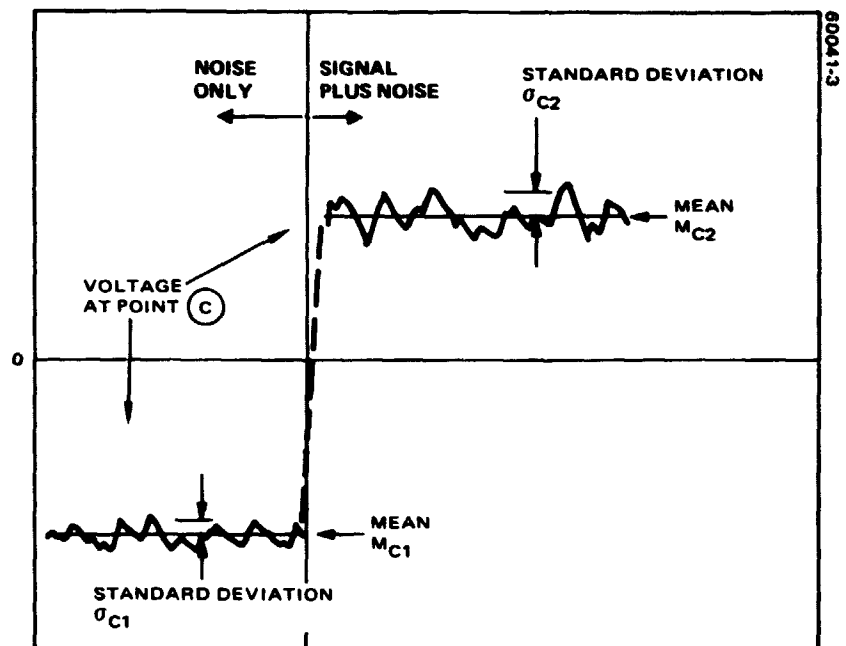


FIGURE 4-3. MECHANICS OF SYNCH DETECTION

4.1.4.2 Analytical Model of Despread Acquisition

Figure 4-3 illustrates the functional relationships involved in the sensing of PN correlation. Action of the square-law detector in the wide channel of Figure 4-2 produces a voltage at point A, having mean M_A and standard deviation σ_A . Similarly, at point B mean and variance are M_B and σ_B . Differencing produces a decision variable, point C, with mean $M_C = M_B - M_A$ and standard deviation

$$\sigma_C = \left(\sigma_A^2 + \sigma_B^2 \right)^{1/2}.$$

The gain factor K_W in the wide channel is so adjusted that when there is no received signal the decision variable has a negative mean. To avoid false alarms this mean must be several times larger in magnitude than the standard deviation. The no-signal situation is denoted by adding a second subscript of 1 to M_C and σ_C . Thus the no signal design criterion can be stated

$$-M_{C1} \geq k_1 \sigma_{C1} \quad (1)$$

where k_1 is a constant picked to give adequately low false alarm probability, P_{FA} . The latter is computed from

$$P_{FA} = \frac{1}{2} \operatorname{erfc} \left(k_1 / \sqrt{2} \right) \quad (2)$$

where

$$\operatorname{erfc}(x) \triangleq 1 - \frac{2}{\pi^{1/2}} \int_0^x e^{-u^2} du \approx e^{-x^2} / \pi^{1/2} x \quad (3)$$

When a signal is present and PN correlation is achieved, the decision voltage (see Figure 4-3) rises from a negative to a positive value. The passing of this voltage through zero provides the indication that PN search has ended. The correlated signal case is denoted by the subscript 2 appended to M_C and σ_C . In the small likelihood of missing the event, mean again must

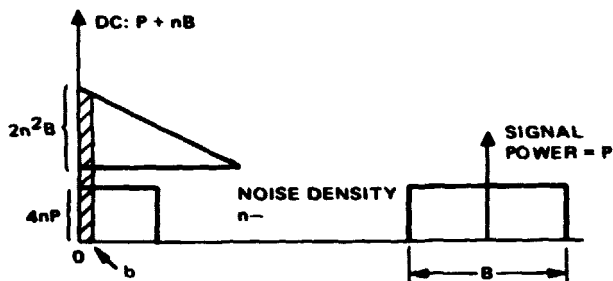
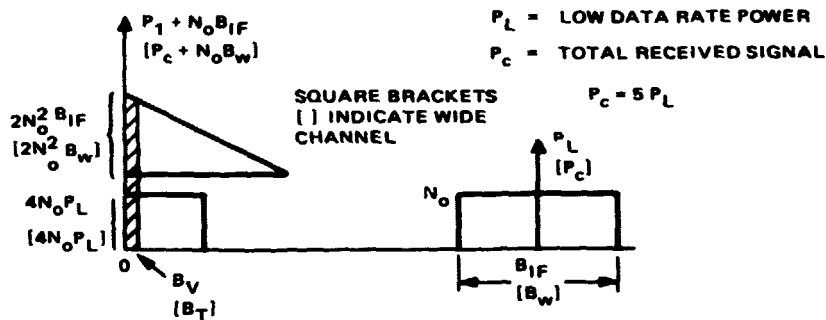


FIGURE 4.4. SQUARE LAW DETECTOR FUNDAMENTALS



PARAMETER	GENERAL CASE FIGURE 4.1-4	PRESENT CASE	
		NARROW	WIDE
INPUT NOISE DENSITY	n	N_o	N_o
INPUT BANDWIDTH	B	B_{IF}	B_W
SIGNAL POWER	P	P_L	P_c
OUTPUT BANDWIDTH	b	B_V	B_T
OUTPUT MEAN	$P + nB$	$P_L + N_o B_{IF}$	$P_c + N_o B_W$
OUTPUT VARIANCE	$(4nP + 2n^2B)b$	$(4N_o P_L + 2N_o^2 B_{IF})B_V$	$(4N_o P_c + 2N_o^2 B_W)B_T$

FIGURE 4.5. DETECTOR INPUT/OUTPUT RELATIONS IN PSEUDO-NOISE SYNCH DETECTOR

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exceed standard deviation by an adequate factor, leading to the second design criterion:

$$M_{C2} \geq k_2 \sigma_{C2} \quad (4)$$

Design consists of choosing a k_1 and then adjusting the various circuit parameters that govern the quantities M_{C2} and σ_{C2} so as to satisfy Equation 1. Having done this, the second criterion (Equation 4) tells how much received signal power is required to meet missed correlation objectives.

To carry out the above it is necessary to express M_C and σ_C in terms of circuit parameters. This is done with the aid of square law detector theory, the fundamentals of which are reviewed in Figure 4-4. It is seen in Figure 4-4 that the output of such a detector has a mean (d-c) equal numerically to the combined input signal and noise power (ignoring trivial proportionality constants). In addition, there is a fluctuating component consisting of two parts, one having a rectangular and the other a triangular power spectrum.

Figure 4-4 depicts the case where the detector input consists of gaussian noise accompanied by an unmodulated sinewave. The model in the figure will be assumed to apply to the circuit under consideration, even though the input signal in the present case is phase modulated.

With the above assumption, adapting Figure 4-4 to the present problem gives the results shown in Figure 4-5 from which

$$M_{C1} = N_o(B_{IF} - K_W B_W) \quad (5)$$

$$\sigma_{C1} = N_o(2B_V B_{IF})^{1/2} \quad (6)$$

$$M_{C2} = P_L + N_o B_{IF} - K_W(P_c + N_o B_W) \quad (7)$$

$$\sigma_{C2} = \left(2N_o^2 B_V B_{IF} + 4N_o P_L B_V \right)^{1/2} \quad (8)$$

In Equations 7 and 8 the term P_c represents the power in the total despread communication signal, whereas P_L is that portion going through the narrow channel filter B_{IF} only. In biphase, P_L and P_c are the same, while in unbalanced quadriphase, P_L is only the low data rate portion of P_c (i. e., $P_L = P_c/5$).

Standard deviations (Equations 6 and 8) are the same as those of the narrow channel only (i. e., σ_A). K_W and B_T in the wide channel are such as to make the wide channel deviation σ_B negligible.

Applying Equations 5 and 6 to Equation 1 leads to an expression for the wide channel gain term

$$K_W = \frac{B_{IF} + k_1 (2B_V B_{IF})^{1/2}}{B_W} \quad (9)$$

Given values for the other terms in Equation 9, the gain setting K_W can be calculated. This then is used in Equation 7 in conjunction with Equations 8 and 4 to compute received signal power requirements.

4.1.4.3 Pointing Loss Margin and Effective Beamwidth

The foregoing procedure yields a received signal power requirement expressed as a ratio P_C/N_O . This is the amount of (normalized) power needed to effect PN acquisition with a given reliability. The value of P_C/N_O thus determined, compared with the on-axis expected value P_O/N_O from Table 4-5, then gives what is termed pointing loss margin.

Pointing loss margin provides an indication of how much antenna pointing error can be tolerated (i. e., how near beam center the TDRSS must be) for reliable PN acquisition. The tolerable amount of point error is referred to as effective beamwidth, the total solid angle effectively being observed at a given moment. Clearly, the greater the effective beamwidth (pointing loss margin), the smaller the amount of antenna scanning needed to search out a given spatial region.

4.1.4.4 Postdetection Bandwidth in Narrow Channel

The term B_V in Equation 9 is the bandwidth of the narrow channel postdetection filter (Figure 4-2). The function of this filter is to reduce random fluctuation of the decision variable. This is achieved by making B_V small. B_V , however, also sets a limit on how fast the PN search can proceed. The search rate must be slow enough to allow the B_V output to build up during the interval when a correlation match exists. For quick PN search B_V must be large.

In the interlocked or despread-first method of signal acquisition the conflicting requirements on B_V are of particular significance. It is desirable to perform spatial search as rapidly as possible. Any potential "target," however, must be allowed to dwell within the antenna effective beamwidth long enough for a complete PN sequence search.

In order not to inhibit antenna scan rate excessively then B_V should be large. Large B_V , on the other hand, increases the decision variable variance, in turn increasing the received signal level requirement (for given reliability), and thus gives reduced pointing loss margin and smaller effective beamwidth.

The relation between B_V and the time requirement to scan through the PN spreading sequence is developed as follows. It will be assumed that the scan rate (i. e. , differential clock rate between received and locally generated code) must be such that the codes will not slip more than the one-half chip in time $1/B_V$. Expressed in symbols

$$R_{CL} - R_{CR} \leq B_V/2 \quad (10)$$

where

R_{CL} = local clock rate, by hypothesis always larger than R_{CR}

R_{CR} = received clock rate

Because of doppler and shuttle receiver oscillator drift, some uncertainty will exist in the actual slip rate achieved between local and received PN sequences.

The nominal local clock rate must be set so as to keep slip rate below its maximum allowable value during periods when actual clock rate drifts to the high end of its frequency tolerance range and when doppler is at the maximum opening rate. Thus from Equation 10:

$$R_{CLN}(1 + k_o) = B_V/2 + R_{CR} = B_V/2 + R_{CX}(1 - k_D) \quad (11)$$

where R_{CLN} stands for nominal local clock rate, R_{CX} is transmitted clock rate, and k_o and k_D , respectively, are oscillator and doppler frequency offset factors

Then when the local clock drift and doppler are at their other extremes, actual slip rate will fall to a minimum

$$R_{MIN} = R_{CLN}(1 - k_o) - R_{CX}(1 + k_D) \quad (12)$$

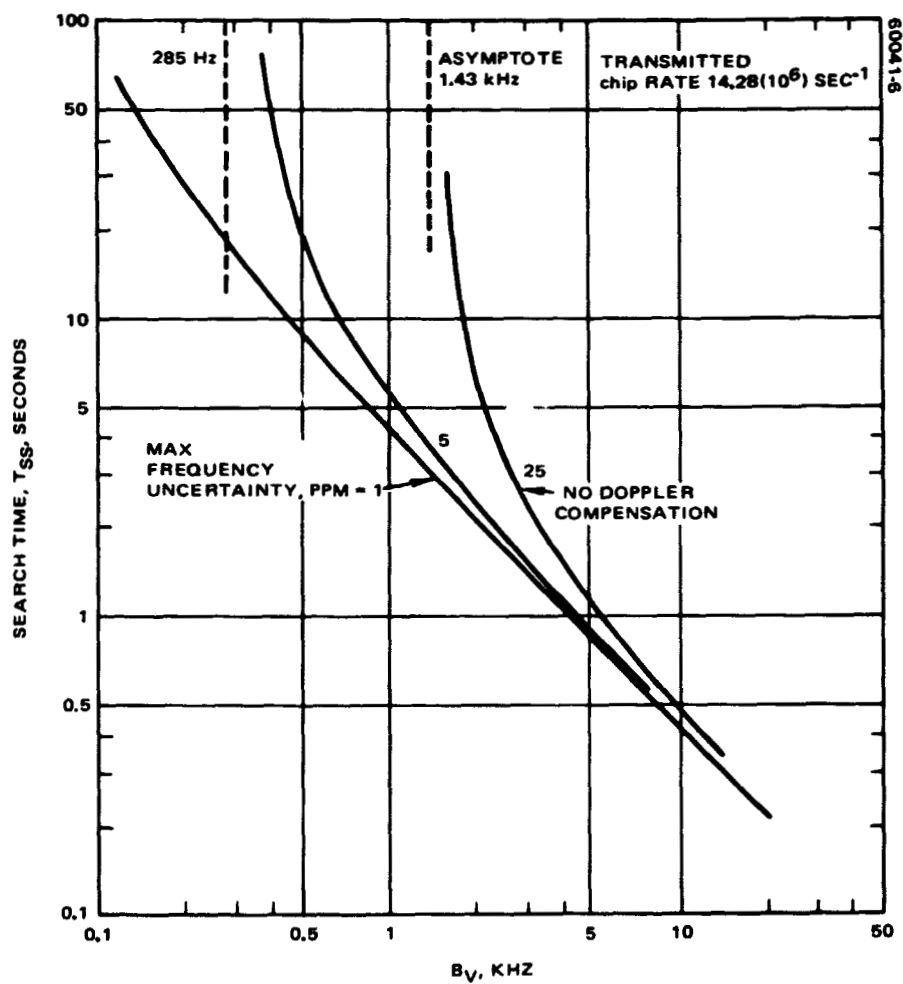


FIGURE 4-6. TIME REQUIRED TO SEARCH SEQUENCE OF LENGTH 2047

At this minimum scan rate it will take a maximum of L/R_{MIN} seconds to scan the entire L-chip sequence. Solving for this maximum sequence scan time T_{SS} using Equations 11 and 12:

$$T_{\text{SS}} = \frac{2L}{B_V - 4R_{\text{CX}}(k_o + k_D)} \tag{13}$$

T_{SS} is the time that TDRSS must be allowed to dwell within the antenna effective beamwidth to provide adequate PN search time under worst case oscillator drift and doppler conditions.

A plot of Equation 13 appears in Figure 4-6 for three values of $k_o + k_D$. The $k_o + k_D = 25$ PPM curve represents an uncorrected-doppler situation, whereas the 5 PPM curve reflects the case where uncompensated doppler is negligible and oscillator drift predominates.

4.1.4.5 Pointing Loss Margin Calculation for Despread-First Implementation

A value of about 1 second will be taken as a reasonable antenna beam dwell time requirement for the despread-first type of operation. Then from Figure 4-6, the bandwidth B_V may be set at 5 kHz.

For purposes of calculating margin, the constants k_1 and k_2 in Equations 1 and 4 will be set respectively at $5\sqrt{2}$ and $5/\sqrt{2}$. False alarm and missed correlation then will be roughly "7-sigma" and "3.5-sigma" events. The reason for the unbalance is that in a typical search situation there will be a large number of false alarm opportunities per missed correlation opportunity. For example, with $B_V = 5$ kHz about 10^4 independent trials will occur per second, or some 10^6 during an extended search of 100 seconds. Thus for small overall false alarm probability, the probability per trial must be very small. The selected k_1 and k_2 values yield $P_{\text{FA}} \approx 10^{-12}$ and a missed correlation probability of about $2(10^{-4})$.

The parameter values used in computing pointing loss margin are summarized in Table 4-8.

TABLE 4-8. PARAMETERS FOR SIGNAL ACQUISITION
USING DESPREAD-FIRST IMPLEMENTATION

Reference (wide) channel width, B_W	30 MHz
Reference channel postdetection bandwidth, B_T	20 Hz
Signal channel postdetection bandwidth, B_V	5 kHz
k_1	$5\sqrt{2}$
k_2	$5/\sqrt{2}$

TABLE 4-9. DESPREAD-FIRST POINTING LOSS MARGIN

Signal Type and Conditions	BIF, kHz (Table 4-7)	KW (Equation 9)	P _c /N ₀ , dB	Bandwidth Loss Allowed, dB	Correlation Loss Allowed, dB	Total, dB-Hz	P ₀ /N ₀ , dB-Hz (Table 4-5)		Margin, dB	
							Mixer	Paramp	Mixer	Paramp
Unbalanced quadrature Doppler compensated	340	0.025	67.5	1	2.5	71.0	74.9	77.9	3.9	6.9
Doppler compen- sated with center 10 MHz notched out of BW	340	0.038	66.9	1	2.5	70.4	74.9	77.9	4.5	7.5
Biphase Doppler not compensated	1380	0.074	62.4	1	2.5	65.9	74.9	77.9	9.0	12.0
Doppler compensated	700	0.43	61.1	1	2.5	64.6	74.9	77.9	10.3	13.3

Equations 4, 7, and 8 can be manipulated to

$$\frac{P_L}{N_o} = \frac{(2B_{V})^{1/2}}{1 - \frac{K_W P_c}{P_L}} \left\{ k_1 (B_{IF})^{1/2} + k_2 \left[B_{IF} + 2 \left(\frac{P_L}{N_o} \right) \right] \right\}^{1/2} \quad (14)$$

Equation 14 can be made into a quadratic and solved for P_L/N_o using the quadratic formula. The expressions are unwieldy though simple in concept. Pointing loss margin computations then proceed as in Table 4-9, where such margins are shown for several conditions.

4.1.4.6 Likelihood of DLL Unlock

PN search is produced by biasing the frequency of the voltage controlled oscillator (VCO) that clocks the PN generator. When signal is present and correlation occurs, the VCO bias is removed. Control of VCO frequency passes to the feedback control loop (i. e., the DLL itself). Under these conditions of signal present and delay lock loop (DLL) tracking, there always will be some chance that the VCO search bias could be restored inadvertently due to noise. The conditions for this can be seen from Figure 4-3. What is required is for the wavy line in the "signal plus noise" region to dip below the zero level.

It can be appreciated that, given reasonable M_{C2} and σ_{C2} values, such a dip could be quite rare and, furthermore, of brief duration. The question might be raised, however, as to what effect such an event would have on the DLL were it to occur.

The answer is that there would be very little if any noticeable effect. The DLL, in fact, could operate properly with the bias continually applied. The bias would represent a small loop stress that would have to be overcome by a small time mismatch. This, in turn, would translate to a steady state loss in effective received signal strength, which is the reason for removing the bias during normal tracking operation. However, reapplying bias for brief, rare periods would have little average effect. In particular, the DLL would not "unlock" on these occasions unless predisposed by the simultaneous presence of an unusually high noise-induced loop phase error.

Though not likely to cause difficulty in normal operation, it still might be well to avoid brief random reapplications of VCO bias due to the above cause. A simple way to effect this would be to provide for altering the bandwidth of B_V simultaneously with removing VCO bias.

For example, consider an implementation wherein B_V is halved when the search bias is removed. The complementary error function (erfc) argument (Equation 3) may be shown to be inversely proportional to $B_V^{1/2}$. The type of event being considered, namely accidentally reapplying VCO bias, is the same kind of occurrence as missing correlation in the first place, which was found earlier (see subsection 4.1.4.5) to have a probability of about $2(10^{-4})$ under a certain set of conditions. Increasing the erfc argument by $\sqrt{2}$ (by cutting B_V in half) can be shown to reduce the above probability to 10^{-7} .

An estimate of mean time between occurrences can be obtained by observing that the new B_V value, 2.5 kHz, implies about $5(10^3)$ independent trials per second. Thus, some 2000 seconds or 33 minutes on the average would pass between events. This compares with about 0.5 second for $B_V = 5$ kHz. Clearly, further reduction in B_V would extend the mean occurrence time beyond any practical requirements.

4.1.4.7 Delay Lock Loop Bandwidth

The DLL should be wide enough to allow quick pull-in, yet narrow enough to keep noise-caused jitter acceptably small. In the present case, these two constraints happen to be fairly loose, allowing some latitude in the actual choice of loop bandwidth.

As a starting point, a desirable objective for pull-in time might be to have pull-in essentially completed in the time the relative code slip equals one chip. Earlier, the quantity B_V^{-1} (reciprocal of the correlation detector low pass filter (LPF) bandwidth) was set equal to one-half this same interval. Thus the pull-in time objective can be stated in terms of B_V

$$T_{PI} = 2/B_V$$

Phase plane trajectory studies have shown pull-in time T_{PI} to equal approximately twice the inverse DLL bandwidth B_{DL}

$$T_{PI} \approx 2/B_{DL}$$

It is seen then that the stated objective for pull-in time is met by making

$$B_{DL} = B_V$$

It remains to verify whether such a bandwidth under steady state tracking conditions will keep noise jitter effects small.

RMS jitter for a DLL of the type under consideration here is given by*

$$\sigma_T = \frac{B_{DL}}{2(P_c/N_o)} + \left[\frac{B_{IF} B_{DL}}{(P_c/N_o)^2} \right]^{1/2} \tag{15}$$

and effective signal loss caused by this jitter is approximately**

$$L_{DB} = -10 \log (1 - 1.6 \sigma_T) \tag{16}$$

Table 4-10 below shows the results of applying Equations 15 and 16 to the options considered in subsection 4.1.4.5 (see Table 4-9). Since steady state on-axis tracking is being considered, the P_c/N_o term in Equation 15 can be taken, for biphase communication, to be the same as the P_c/N_o of Table 4-5. In the quadriphase case P_c/N_o is this figure reduced by 7 dB, the power-split factor.

TABLE 4-10. DLL TRACKING ERROR (σ_T) AND SIGNAL LOSS (L) DUE TO NOISE, $B_{DL} = 5 \text{ kHz}$

Signal Type and Conditions	B _F , kHz	P _c /N _o , dB		σ _T Chips		L _{DB} , dB	
		Mixer	Paramp	Mixer	Paramp	Mixer	Paramp
Quadriphase with doppler compensation	340	67.9	70.9	0.02	0.014	0.14	0.1
Biphase							
Doppler not compensated	1380	74.9	77.9	0.0087	0.006	0.06	0.04
Doppler compensated	700	74.9	74.9	0.0085	0.006	0.06	0.04

* Gill, W. J., A Comparison of Delay-Lock Tracking-Loop Implementations, IEEE Trans. AES, July 1966, pp. 415-423.

** De Couvreur, G. A., Effect of Random Synchronization Errors in PN and PSK Systems, IEEE Trans. AES, January 1970, pp. 98-100.

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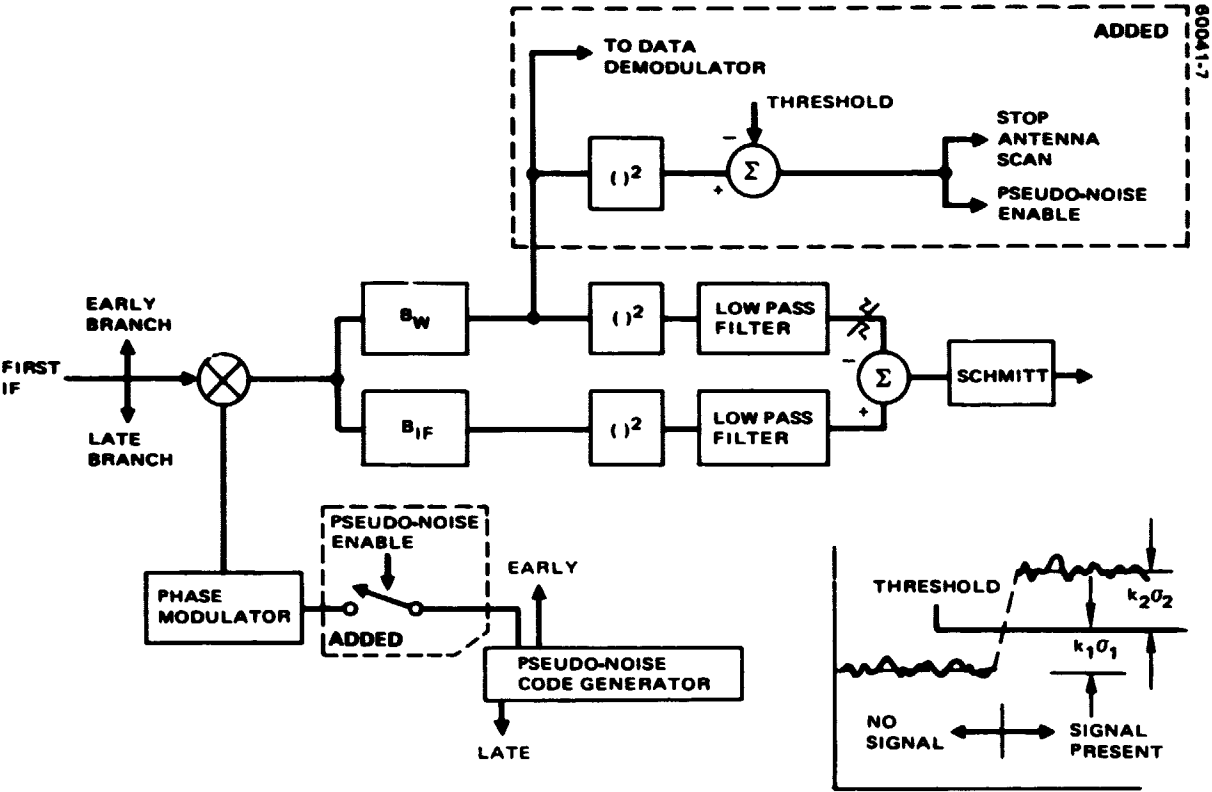


FIGURE 4-7. ADDITIONS FOR INDEPENDENT SPATIAL SEARCH

4.1.5 Independent Spatial and Despread Search

Spatial search and PN despread acquisition can be made independent by providing separate threshold detection circuitry for each function.

Figure 4-7 shows modifications to the previously considered circuitry (see Figure 4-1) that provide for separate spatial acquisition. Modification consists essentially of an additional noncoherent detector (square law) with associated threshold circuitry. Switching is provided to disable PN modulation of the main LO until spatial acquisition is achieved. This is done to avoid additional spreading of the incoming spread signal, which occurs when the local and incoming PN sequences are not synchronized.

Referring to the inset in Figure 4-7, required signal power P_c to satisfy the detection criterion is seen to be

$$P_c = k_1 \sigma_1^2 + k_2 \sigma_2^2$$

where k_1 and k_2 , as in subsection 4.1.4.2, establish false alarm and missed detection probabilities.

Further

$$\sigma_1^2 = 2N_o^2 B_D B_W$$

and

$$\sigma_2^2 = 2N_o^2 B_D B_W + 4N_o B_D P_c$$

from square law detector theory. Here B_D is bandwidth of the LPF in the detector output.

P_c , normalized to noise density N_o , can be solved for from the above relation

$$\frac{P_c}{N_o} = A + \sqrt{A^2 - B} \quad (17)$$

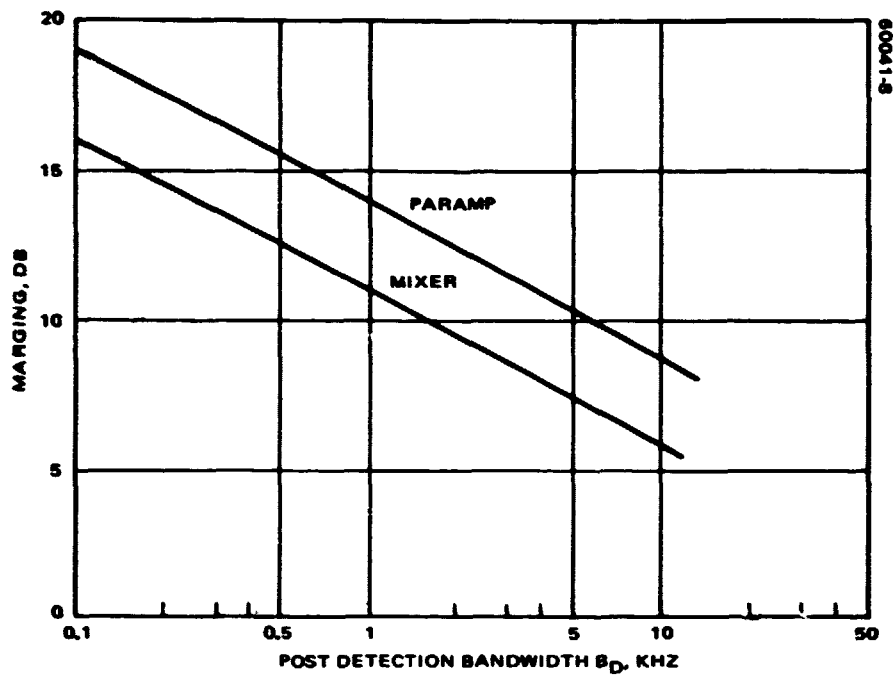


FIGURE 4-8. POINTING LOSS MARGIN VERSUS POSTDETECTION BANDWIDTH FOR INDEPENDENT SPATIAL SEARCH CASE

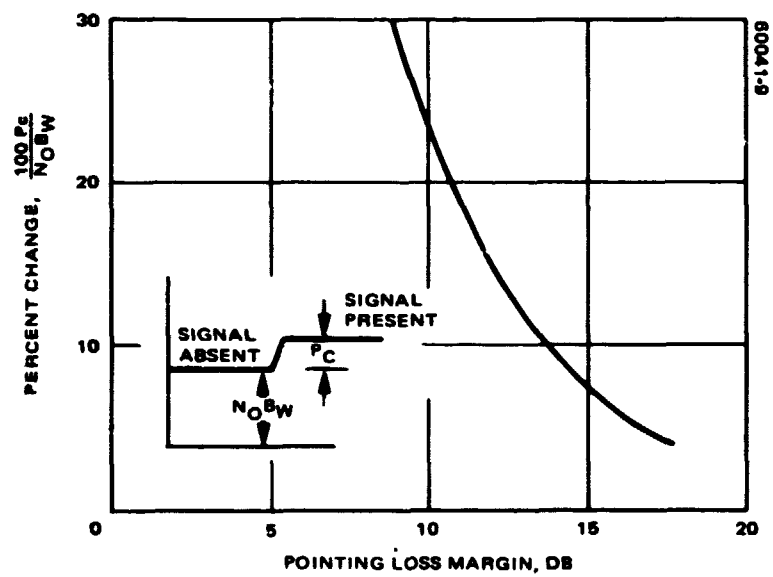


FIGURE 4-9. LEVEL CHANGE WITH MINIMUM SIGNAL

where

$$A = k_1(2B_D B_W)^{1/2} + 2k_2^2 B_D$$

$$B = (k_1^2 - k_2^2) 2 B_D B_W$$

Given k_1 , k_2 , and bandwidth B_W , P_c/N_0 can be evaluated as a function of lowpass filter bandwidth B_D . In principle, the power needed for detection can be made arbitrarily small by sufficiently narrowing B_D . However, the fractional level change that must be sensed, between signal absent and signal present, becomes proportionately smaller as the signal to be detected becomes weaker. The problem of accurately setting and maintaining the threshold becomes increasingly difficult. Also, as a practical matter, there is a limit on how small a signal it would be desirable to be able to detect. Too much sensitivity raises the problem of sidelobe reception.

Figure 4-8 is a plot of pointing loss margin as a function of B_D . This is determined by subtracting P_c/N_0 , computed using Equation 17, from on-axis level P_0/N_0 (see Table 4-5). The threshold stability problem is illustrated in Figure 4-9. Here the percentage level change that must be sensed is plotted as a function of pointing loss margin. Note that the threshold itself must be held accurately at a certain point within this range. Thus the tolerance on threshold setting would be perhaps an order of magnitude smaller than the indicated level change.

4.1.6 Spatial Acquisition and Tracking

4.1.6.1 Estimated Search Time

A rough estimate of the time required to effect spatial acquisition can be obtained by considering the geometrical relationships depicted in Figure 4-10. A series of concentric circles represents the probability volume within which the target lies. Each circle is labeled with a number representing the probability that the target lies inside that circle (assuming a two-dimensional gaussian probability density function). Superimposed is a small circle representing the shuttle antenna field of view. This small circle is to be scanned over the probability volume until it has swept out all the area contained within the 0.99 contour.

The amount of scanning required is defined by the ratio of the two areas: the 0.99 circle to the antenna FOV circle. The size of the antenna field of view (FOV) is determined by the pointing loss margin, M_{PL} . As indicated in Figure 4-10, the FOV radius θ is

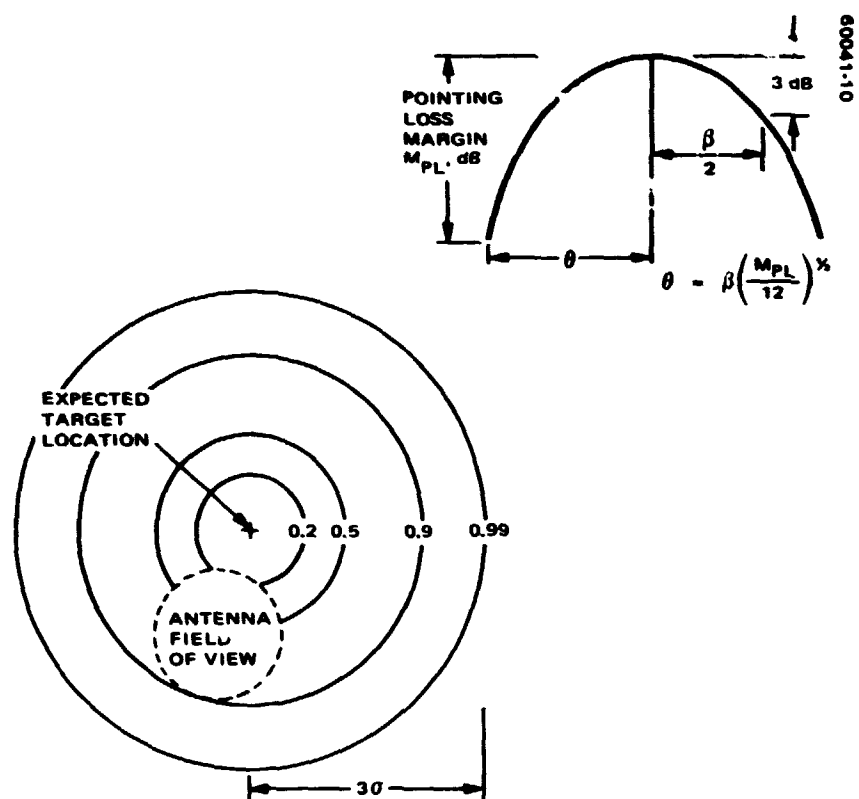


FIGURE 4-10. SPATIAL SEARCH PARAMETERS

$$\theta = \beta \left(\frac{M_{PL}}{12} \right)^{1/2}$$

where β is antenna 3 dB beamwidth and M_{PL} is in dB.

The 0.99 probability circle radius is known to be 3.03σ , σ being the gaussian standard deviation, assumed equal in both dimensions. The area ratio (AR) in question is the square of the radius ratio:

$$AR = \frac{110}{M_{PL}} \left(\frac{\sigma}{\beta} \right)^2 \quad (18)$$

4.1.6.2 Despread-First Case

In the despread-first implementation, it was found that about 1 second of dwell time should be devoted to each new area observed to allow PN synchronization to occur. If the uncertainty region could be scanned with no overlap, that is, with no redundant scanning of regions once examined, then Equation 18 would be a direct estimate of the time in seconds needed.

In reality, it is impossible to ensure dwelling at least 1 second on every point without observing some points for longer periods. This has the effect of lengthening total search time. A reasonable estimate for the degree of lengthening is a factor of between 1.5 and 2. Applying a factor of 2 to Equation 18 gives for search time T_{SCH}

$$T_{SCH} = \frac{220}{M_{PL}} \left(\frac{\sigma}{\beta} \right)^2 \text{ seconds} \quad (19)$$

A plot of Equation 19 as a function of margin M_{PL} appears in Figure 4-11, with the ratio of rms designation error to beamwidth, σ/β , as a parameter. For reference, it might be noted that preliminary designation error estimates place 3σ at about 2.5° . Using the current cassegrain design figure of $\beta = 2.1^\circ$ would put the present system on the $\sigma/\beta = 0.4$ line in Figure 4-11. Callouts on the figure indicate the search time requirements for the various options of Table 4-9.

4.1.6.3 Independent Spatial Search Case

In the alternative implementation where spatial search proceeds independently of PN sync search, the antenna rate of motion is not

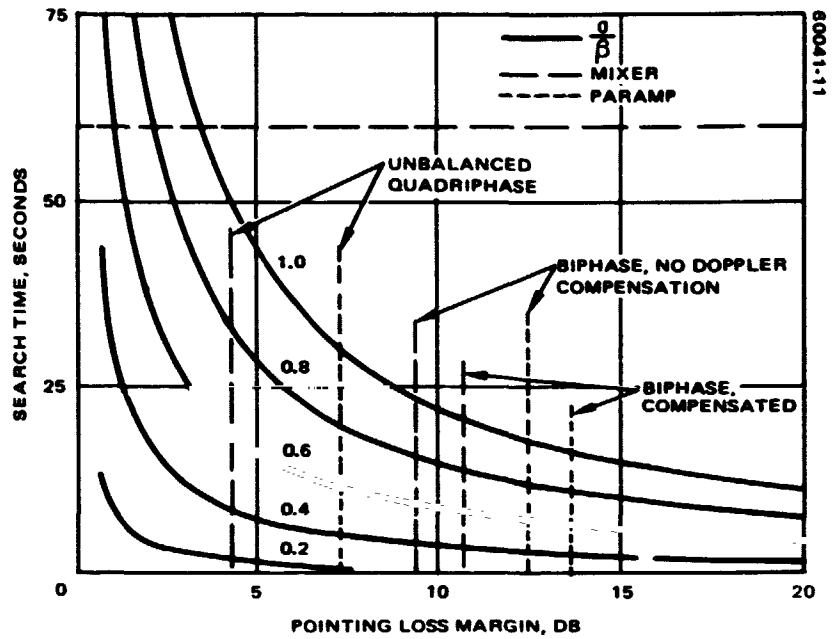


FIGURE 4-11. TDRSS ANGLE ACQUISITION WITH DESPREAD ACQUISITION FIRST

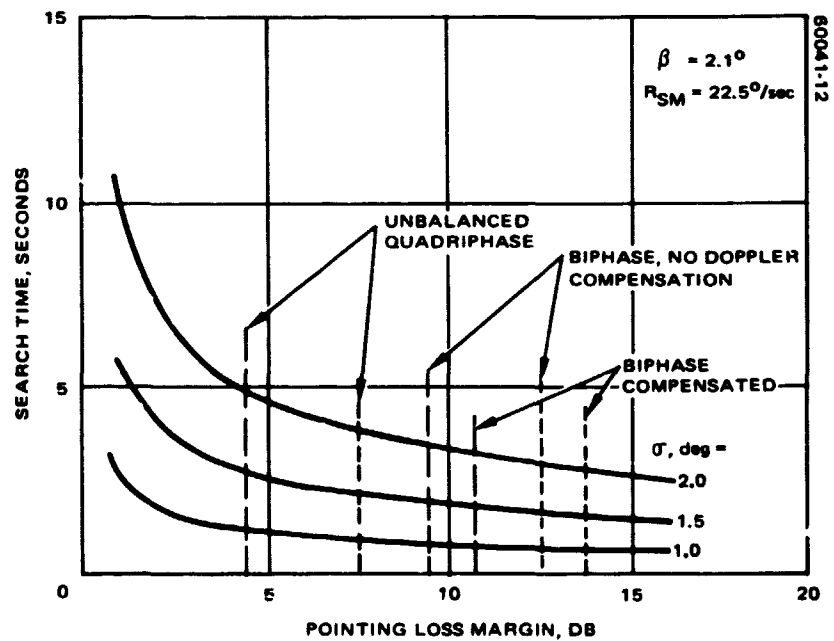


FIGURE 4-12. INDEPENDENT SPATIAL SEARCH

constrained by PN search considerations. The antenna can be slewed at the maximum rate permitted by mechanical limitations. Total search time is estimated by multiplying Equation 18 (which in a sense is the number of "cells" to be observed) times the time needed to move from one cell to the next. The latter is the distance 20 divided by antenna slew rate R_{SM} . Search time, again allowing for 2:1 lengthening due to overlap, thus is

$$T_{SCH} = \frac{220}{M_{PL}} \left(\frac{\sigma}{\beta} \right)^2 \cdot \frac{2\beta(M_{PL}/12)^{1/2}}{R_{SM}} = \frac{120\sigma^2}{R_{SM}\beta M_{PL}^{1/2}} \text{ seconds} \quad (20)$$

Equation 20 is plotted in Figure 4-12 as a function of M_{PL} .

The present antenna design is capable of a maximum slew rate of some 90 deg/sec. In searching out a limited spatial region, however, the maximum rate likely would be attained only infrequently due to frequent starting, stopping, and reversing of the direction of travel. Therefore, in plotting Equation 20, slew rate R_{SM} was "derated" to one-fourth its maximum value.

4.1.7 Antenna Pointing Error Due to Thermal Noise

Thermal noise will cause an autotracking antenna to "wander" about the true radio line of sight. The mean-square magnitude of this wander may be estimated by means of*

$$\sigma_A^2 = \frac{4k\beta^2 B_s}{k_m^2 S_I(G/T)} \alpha_R \frac{1 + \frac{kB_{RF}}{2S_I(G/T)}}{1 - \frac{2kB_s}{S_I(G/T)}} (\text{degrees})^2 \quad (21)$$

where

k = Boltzmann's constant

B_s = servo bandwidth, Hz

β = antenna beamwidth, degrees

* See Appendix A for derivation.

S_I = received signal power relative to isotropic, watts

k_m = normalized difference pattern gain slope

B_{RF} = RF bandwidth, Hz

The term α_R (attenuation ratio) is the ratio between the difference and sum channel circuit losses, including coupler loss, from the monopulse feed to the tracking coupler output. This is approximately equal to the coupler loss in the difference channel. A reasonable value for the latter is 15 dB, from which $\alpha_R = 32$.

Two antenna designs currently under consideration exhibit the following characteristics:

Type	β , deg	k_m
Front fed	2.6	0.77
Cassegrain	2.1	1.02

Other quantities in the present system are

B_{RF}	30 MHz
S_I (see Table 4-5)	-158.3 dBW
G/T (see Table 4-4)	
Mixer	5.1 dB
Paramp	8.1 dB

Using the foregoing in Equation 21 gives the results listed in Table 4-11. The first two columns are mean square pointing error per Hertz of B_s bandwidth, while the latter two give rms error assuming $B_s = 1$ Hz.

TABLE 4-11. ANTENNA POINTING ERROR DUE TO THERMAL NOISE (TWO-AXES)

Antenna	σ_A^2/B_s (degrees) ²		RMS Error (degrees) with 1 Hz B_s	
	Mixer	Paramp	Mixer	Paramp
Front fed	60(10 ⁻⁶)	25(10 ⁻⁶)	0.0078	0.005
Cassegrain	16(10 ⁻⁶)	7(10 ⁻⁶)	0.004	0.0026

4.1.8 Phase Lock Loop

4.1.8.1 Bandwidth

PLL bandwidth selection is constrained by two considerations. Large bandwidth allows rapid lockup, but excessive width degrades communication (BER) performance by allowing too much noise to enter the loop and cause random phase error.

The probability density function of loop phase error ϕ , for the second order PLL under practical SNR conditions, is given quite closely by*

$$p(\phi) = \frac{\exp(\alpha \cos \phi)}{2\pi I_0(\alpha)} ; |\phi| \leq \pi \quad (22)$$

where α is the reciprocal of loop mean-square phase error and is equal to $1/\sigma_\phi^2$.

Furthermore, the probability of symbol decision error, given a particular value of ϕ , is

$$P_E(\phi) \triangleq \text{prob (error}/\phi) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2R} \cos \phi}^{\infty} \exp(-X^2/2) dX \quad (23)$$

In Equation 23, R is the normalized signal energy per symbol

$$R = p \dot{P}_c / N_o R_s \quad (24)$$

where P_c is total signal power, N_o is noise density, and R_s is symbol rate. The factor p in Equation 24 is of significance only in the case of quadriphase signaling. It represents the proportion of total power P_c allocated to the signal channel in question, I or Q. In biphase, p is unity.

In the type of demodulator under consideration ("modulation wipe-off"), the term α in Equation 22 is**

*Lindsey, W.C., "PSK Signal Detection with Noisy Reference Signals," IEEE Trans. AES, July 1966, pp. 393-401.

** B_L is one-sided loop noise bandwidth: $B_L \triangleq \int_0^\infty |H(f)|^2 df$

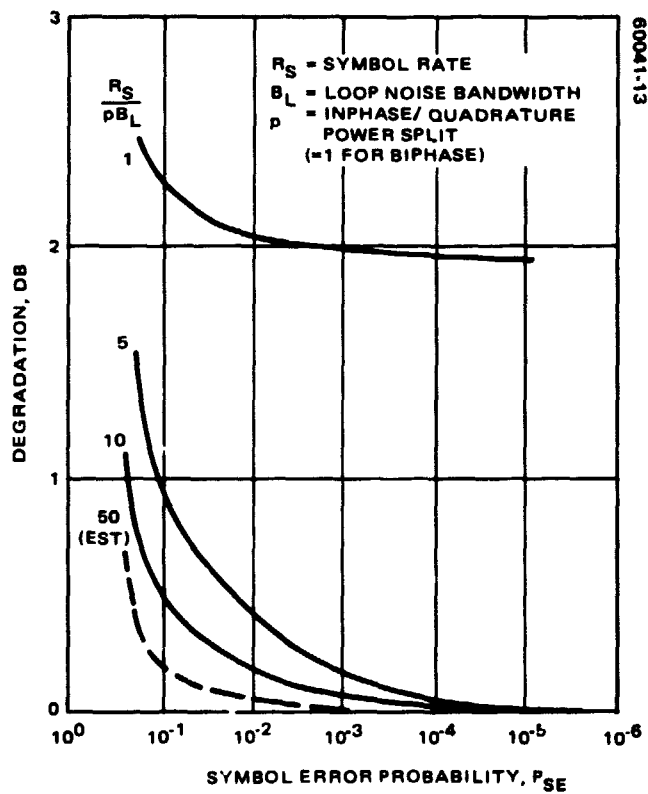


FIGURE 4-13. DEGRADATION DUE TO LOOP PHASE NOISE

$$\alpha = \frac{1}{\sigma_{\phi}^2} = P_c / N_o B_L$$

This can be rewritten in terms of the quantity R defined in Equation 24 and another symbol δ defined as

$$\delta = R_s / p B_L \quad (25)$$

The new expression for loop SNR thus is

$$\alpha = \delta R$$

The purpose of the notation change is simply to be compatible with that used in the cited Lindsey paper.

Overall symbol error probability P_{SE} is found by multiplying Equation 22 by the conditional error probability of Equation 23 and integrating overall ϕ . In terms of the defined notation:

$$P_{SE} = \frac{1}{\pi} \int_0^{\pi} \frac{\exp(\delta R \cos \phi)}{I_o(\delta R)} d\phi \int_{\sqrt{2R} \cos}^{\infty} \frac{\exp(-X^2/2)}{\sqrt{2\pi}} dX \quad (26)$$

By evaluating Equation 26 Lindsey generated a family of curves giving symbol error probability as a function of R, with δ as a parameter. From Lindsey's curves, a determination can be made of the amount of degradation caused by loop noise for various ratios of symbol rate to bandwidth. This is plotted in Figure 4-13.

As can be seen in Figure 4-13 for a given ratio δ , degradation increases with increasing error rate. The degradation represents the added amount of signal power (decibels) that would be needed to achieve a given error probability relative to that needed with an ideal noise-free loop.

The results shown in Figure 4-13 have primary significance with regard to convolutionally encoded data. This is because such data can tolerate quite high symbol error rates, typically in the range of 0.1 or higher. In order not to detract from the potential usefulness of the coding, the PLL itself should be capable of operating at the low signal levels thus implied, without contributing excessive degradation of its own. To satisfy this requirement it is estimated in Figure 4-13 that a δ of around 50 or more should be sought. That is, loop bandwidth should be limited to a few percent of the symbol rate.

4.1.8.2 Quadriphase Case

In the quadriphase data format of Table 4-3, the in-phase channel carries $3(10^6)$ symbols/sec of convolutionally encoded data. The power split factor p is 0.8 for this channel. If a δ of 125 is assumed, then from Equation 25, $B_L = 3(10^6)/(125)(0.8) = 30$ kHz.

The suitability of this bandwidth from the standpoint of the quadrature channel is checked by again referring to Equation 25, this time using the above computed B_L value, together with quadrature channel power split and data rate which are $p = 0.2$ and $R_s = 72$ kbps, respectively. Solving for δ gives $\delta = 12$. Referring to Figure 4-13, a loop with such a value of δ would cause negligible degradation as long as ideal BER is less than about 10^{-3} , i. e., the normal operating region for uncoded data.

The conclusion is that a 30 kHz loop bandwidth should be satisfactory from a BER degradation point of view. Even less degradation could be ensured by making B_L smaller. A reason to keep it large, however, has to do with pull-in time. The time required to pull in is given by Gardner*

$$T_p \approx \frac{4.2(\Delta f)^2}{B_L^3} \text{ sec} \tag{27}$$

Using $B_L = 30$ kHz Equation 27 yields the pull-in times in Table 4-12. These figures indicate that pull-in time would be negligible with or without doppler compensation, and no frequency sweep would be needed.

TABLE 4-12. PULL-IN TIME, SECOND-ORDER LOOP
 $B_L = 30$ kHz, $\zeta = \sqrt{2}/2$

Condition	Doppler, kHz	Oscillator drift, kHz	Total, Δf , kHz	T_p , sec
No doppler compensation	350	70	420	0.02
Doppler compensated	8	70	78	0.001

* Gardner, F. M., Phaselock Techniques, Eq. (4-32), Wiley, 1966.

4. 1. 8. 3 Biphase Case

With biphase, symbol rate is $216(10^3)$ per second. To avoid appreciable BER degradation, a reasonable upper limit on B_L would be about 5 kHz. From Equation 25 this gives a δ of about 40 ($p = 1$). Pull-in time from Equation 25 would be about 0.2 second with doppler prediction.

In the absence of doppler prediction a frequency search would have to be implemented, as the required pull-in range relative to loop bandwidth is too large for automatic pull-in to be practical.

Maximum permissible sweep rate for certain acquisition is about $0.28 B_L^{1/2}$ *, which for a 5 kHz loop translates to some 7 MHz/sec. Allowing considerable derating to accommodate noise effects, parameter tolerances, etc., the ± 420 kHz maximum frequency uncertainty range could be readily swept in 0.2 second.

4. 1. 8. 4 Summary

Phase lock loop parameters and performance are summarized in Table 4-13.

TABLE 4-13. PHASE LOCK PARAMETER SUMMARY

Signal Type and Condition	Frequency Uncertainty, kHz	Loop Bandwidth, kHz	Pull-in Time, sec
Unbalanced quadriphase			
Doppler compensated	70	30	0.01
No doppler compensation	420	30	0.02
Biphase			
Doppler compensated	70	5	0.2
No doppler compensation	420	5	0.2*

*With frequency sweep.

*Reference Gardner, Figure 4-10.

4.2 RADAR PERFORMANCE ANALYSES AND TRADEOFFS

This section presents the radar analyses and tradeoffs that have been performed to provide an integrated Ku-band radar/communications subsystem design. Estimates of performance and tradeoff/design considerations are discussed leading to the selection of radar subsystem parameters and design. The discussions are divided into two major subsections which correspond to the modes of radar operation. They are:

4.2.1 Search and Detection

4.2.2 Tracking Analysis

The design presented will meet all orbiter radar requirements. However, further tradeoffs, especially in the cost versus performance area (which are beyond the scope of this study), must still be performed to fully optimize the radar design.

4.2.1 Search and Detection

The search and detection requirements for the rendezvous radar are given in Table 4-14.

TABLE 4-14. SEARCH AND DETECTION REQUIREMENTS

	Mission I/II, Passive	Mission IIIB, Passive/ Cooperative*	Mission I/II, Cooperative
Maximum detection range	22.23 km (12 n.mi.)	14.82 km (8 n.mi.)	555.93 km (300 n.mi.)
Required detection			
Probability	0.99	0.99	0.99
False alarm rate	1/10 min	1/10 min	1/10 min
Acquisition time	1 min	1 min	Not critical (5 minutes)
Search volume uncertainty (half-cone angle)	20°	45°	5°
Target radar cross section	1 m ²	1 m ²	NA
Target RCS characteristic	(Swerling I)	(Swerling I)	NA
Target stabilization	±10° each axis 5 deg/min	±10° each min 5 deg/min	±10° each min 5 deg/min

*Cooperative = beacon mode of operation.

4.2.1.1 Probability of Detection

The specified target characteristic is that of a Swerling I type (slow scintillation, exponential radar cross section distribution). The required signal-to-noise ratios for detection will be very high (e. g. , greater than 30 dB for a 0.99 detection probability, P_D) unless frequency diversity is employed to decorrelate the target radar cross section. Figure 4-14 shows the integrated signal-to-noise energy ratio (E/N_0) for a given probability of detection and number of radar transmitter frequencies and is, in actuality, a plot of a Swerling II case for N sample of postdetection integration. Figure 4-14 is based on the assumption that the radar cross sections are uncorrelated for each of the N sample (transmitter frequencies) and that the radar cross section follows an ideal exponential distribution. From Figure 4-14 there is a minimum E/N_0 requirement of 18.6 dB for $P_D = 0.99$ and 10 radar frequencies. This is a theoretical improvement of 13.2 dB over a single frequency system. The actual improvement may be less for a real target. It was found* that the maximum actual improvement for frequency diversity may be 10 dB for real targets which indicates that fewer than 10 frequencies are required (perhaps as few as 5). There is a cost tradeoff here, however, as tracking 10 frequencies appear to be more desirable.

To ensure independent samples, the change in transmit frequency (ΔF) should be

$$\Delta F = \frac{C}{2L}$$

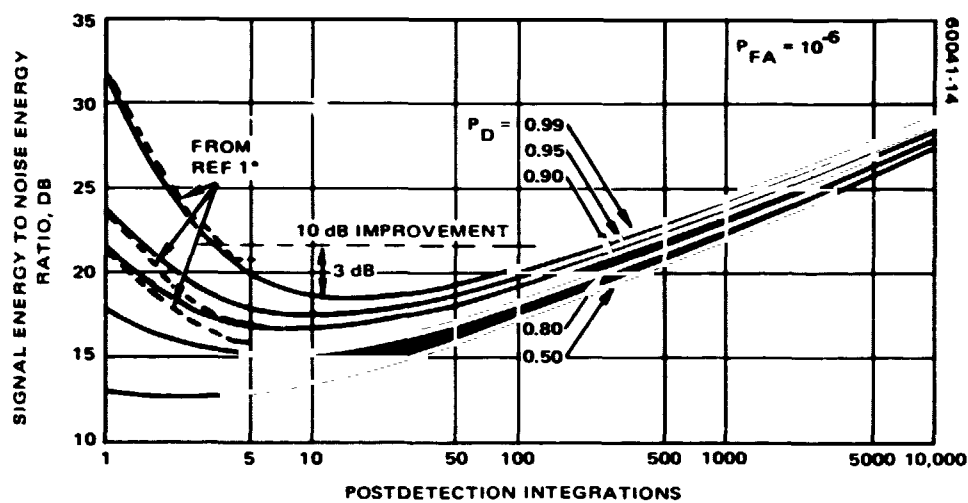
where

C = velocity of light

L = target length

For example, a 6 meter target requires a frequency change of 25 MHz to decorrelate the return. Selecting a total of 10 frequencies over a 225 MHz total bandwidth will provide some decorrelation for targets as small as 1 meter in length.

*Tice, D. P., "The Effect of Frequency Agility on the Radar Detectability of Spacecraft," Westinghouse Electric Corporation, 26 September 1973.



*TICE, D.P., "THE EFFECT OF FREQUENCY AGILITY ON THE RADAR DETECTABILITY OF SPACECRAFT," WESTINGHOUSE ELECTRIC CORPORATION, 26 SEPTEMBER 1973.

FIGURE 4-14. SIGNAL ENERGY VERSUS POSTDETECTION INTEGRATION

4.2.1.2 Design Considerations and Parameter Selection

The major items that must be considered in the radar design include the following:

Transmitter	Peak power Average power Bandwidth Coherency Frequency agility
Receiver	Noise figure Bandwidth
Waveform	Type (PRF, pulse width coding, coherency)
Signal processor	Type (coherent, noncoherent analog), method of tracking
Losses	RF Processing Other (beam shape, field degradation, etc.)
Target characteristics	Radar cross section Cross section Statistics

By examination of the radar range equation

$$E/N_o = \frac{P_{AV} G^2 \lambda^2 \sigma L T_D}{(4\pi)^3 k T_{eff} R^4}$$

where

E/N_o = signal energy to noise energy ratio

P_{AV} = average transmitter power

G = antenna gain

λ = transmitted radar wavelength

σ = target radar cross section

L = system losses, including RF, processing, beam shape, system degeneration, target fluctuation losses

T_D = dwell time on target

k = Boltzman's constant

T_{eff} = effective noise temperature

R = range to target

Insight can be gained in the selection of the various radar parameters. The quantities which may readily be varied are P_{AV} , G , and T_D . Both system losses and effective noise temperature (T_{eff}) are minimized, while the range is fixed. E/N_0 is determined by the detection probability required and by the target radar cross section statistics. The major parameter that can be varied is P_{AV} which is minimized by choosing the processing concept. The following paragraphs present tradeoffs leading to the processing concepts and parameter selection.

4.2.1.3 Waveform and Processing Selection

In selecting the waveform and method of processing, consideration must be given to reflected ground clutter to ensure that it does not compete with the target signal or cause false alarms. The effects of clutter can be eliminated by either one of the following techniques:

- 1) Change transmitter rf frequencies at short intervals so that ground clutter will be out of band as compared to target signals.
- 2) Design antenna far sidelobe levels so that received clutter is negligible.

Since the round trip time to the earth's surface is approximately 1.2 ms (altitude = 185 km (100 n. mi.)), the first technique would require changing transmitter frequencies every 1.2 ms. The second technique can be easily accomplished if the antenna sidelobes (at 90° and beyond) are 45 dB or more below the peak antenna gain. The second method was selected since it allows complete freedom in regards to frequency switching. Antennas with far sidelobes of these levels are readily achievable.

The worst case interference would result if the earth's surface were a perfect reflection and the received power is given by the beacon equation

$$P_R = \frac{P_T G_{SL}^2 \lambda^2 L}{(4\pi)^2 (2h)^2}$$

and the signal-to-noise ratio is given in Table 4-15.

For example, the clutter-to-noise ratio is -26.5 dB. This is comparable to about a 0 dB target signal-to-noise ratio for a target range of 22 km (12 n. mi.). Degradation of the antenna sidelobes to -40 dB (with respect to peak) would result in marginal performance since this interfering signal would only be about 6 dB below the target signal. It should be noted that this calculation assumes a very ideal case and, normally, the reflected radar energy will be scattered diffusely rather than all specularly. Therefore, even for an antenna with a far sidelobe down only -40 dB, the clutter-to-noise would still be negligible. This is demonstrated for diffuse clutter as shown in Figure 4-15 and Table 4-16. It can be shown that the clutter power in a range gate is given by

$$\frac{P_c}{N} = \frac{P_T G_{SL}^2 \lambda^2 L}{(4\pi)^3 k T_E B_N} \eta \frac{2\pi h}{3R^3} \left[1 - \frac{1}{(1 + C\tau/2R)^3} \right]$$

where $R \geq h$ and this yields for $R = h$.

TABLE 4-15. SPECULAR CLUTTER-TO-NOISE RATIO, SC/N

$\frac{SC}{N} = \frac{P_{RC}}{N} = \frac{P_T G_{SL}^2 \lambda^2 L}{(4\pi)^2 (2h)^2 k T_E B_N}$			
Plus (+) Decibels		Minus (-) Decibels	
$P_T = 50 \text{ W}$	17.0	$G_{SL}^2 = 27.6^*$	($G_{SL} = -13.8$)
$k =$	<u>228.6</u>	$\lambda^2 = 34.0$	($\lambda = 2 \text{ cm}$)
	245.6	$L = 6.0$	
		$(4\pi)^2 = 22.0$	
		$T_E = 28.1$	($T_E = 650^{\circ}\text{K}$)
		$B_N = 43.0$	($B_N = 1.2/\tau$)
		$(2h)^2 = 111.4$	($\tau = 60 \mu\text{s}$)
		<u>272.1</u>	($h = 185 \text{ km (100 n.mi.)}$)
	-272.1 245.6 SC/N = -26.5		

*Sidelobe gain, $G_{SL} = 36.2 \cdot 50 = -13.8$.

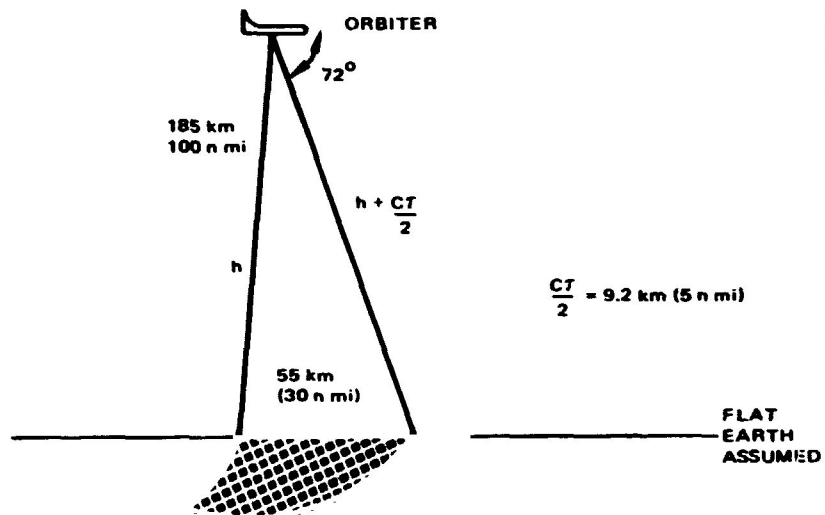


FIGURE 4-15. DIFFUSE CLUTTER GEOMETRY AND MODEL

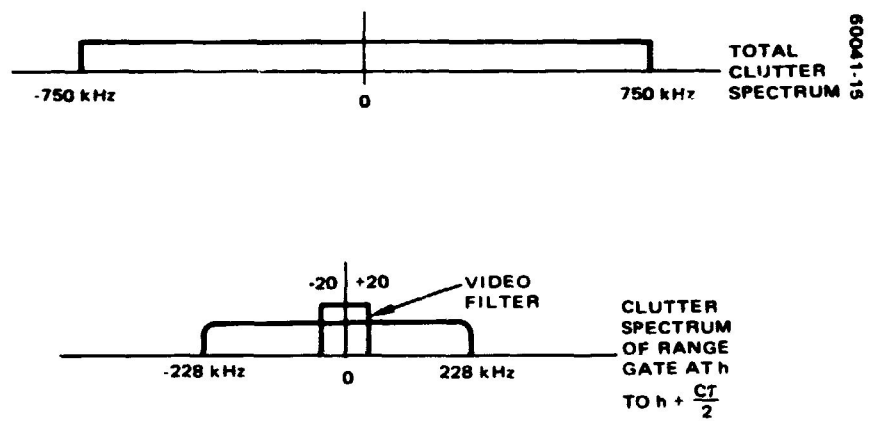


FIGURE 4-16. CLUTTER SPECTRUM

TABLE 4-16. DIFFUSE CLUTTER-TO-NOISE RATIO

Plus (+) Decibels		Minus (-) Decibels	
$P_T =$	17	G_{S1}^2	27.6
$k =$	228.6	λ^2	34.0
$\eta =$	10	L	6.0
$2\pi =$	8	$(4\pi)^3$	33.0
		T_E	28.1
	263.6	B_N	43.0
		3	4.8
		h^2	105.3
		$1.1/(1 + C_T/2R)^3$	8.7
			290.5

This model assumes a clutter backscatter coefficient of +10. This is valid for vertical incidence but the backscatter coefficient quickly drops off as the grazing angle decreases (see Nathanson* for example, pg 237) and as a result, the above calculation is conservative.

The total doppler spread of the clutter return will range from about 750,000 Hz opening to 750,000 Hz closing (orbit velocity 7.6 km/sec (25,000 fps)). For the above example, the doppler in the first range gate (h to $h + C\tau/2$) will range between ± 228 kHz as shown in Figure 4-16. The pulse matching filter (matched to $C\tau/2$ ($= 60 \mu\text{sec}$)) will further reduce the clutter noise by 10 dB because of the mismatch between the two spectra. Because of the wide clutter spectrum, clutter will be exactly like white noise. At worst, clutter will raise the noise level in the receiver.

From the above analysis, the antenna for sidelobes (beyond 90°) must be at least 40 dB, and more desirably 50 dB below, the peak antenna gain.

4.2.1.4 Processing Selection

The radar signal processing can employ either noncoherent or coherent processing of the radar signals. The major tradeoff is between complexity and processing efficiency. Noncoherent processing requires only a magnitude detector, video integrator, and thresholding device. Coherent processing requires the formation of doppler filters. There is approximately a 6 dB difference between coherent and noncoherent processing for this application. Doppler filters are easily mechanized by digital processing and since speeds are very slow, low power CMOS logic can be used for this application. Doppler processing is also required in order to

*Nathanson, F. E., Radar Design Principles, McGraw-Hill, 1969.

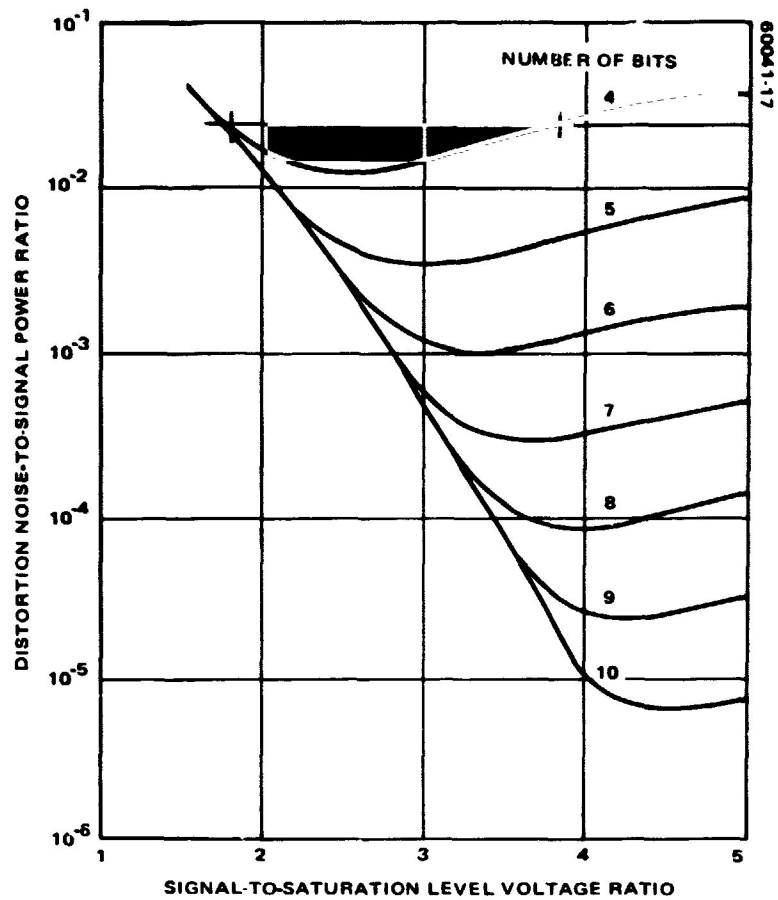


FIGURE 4-17. ANALOG TO DIGITAL CONVERSION SATURATION AND QUANTIZATION NOISE LEVELS

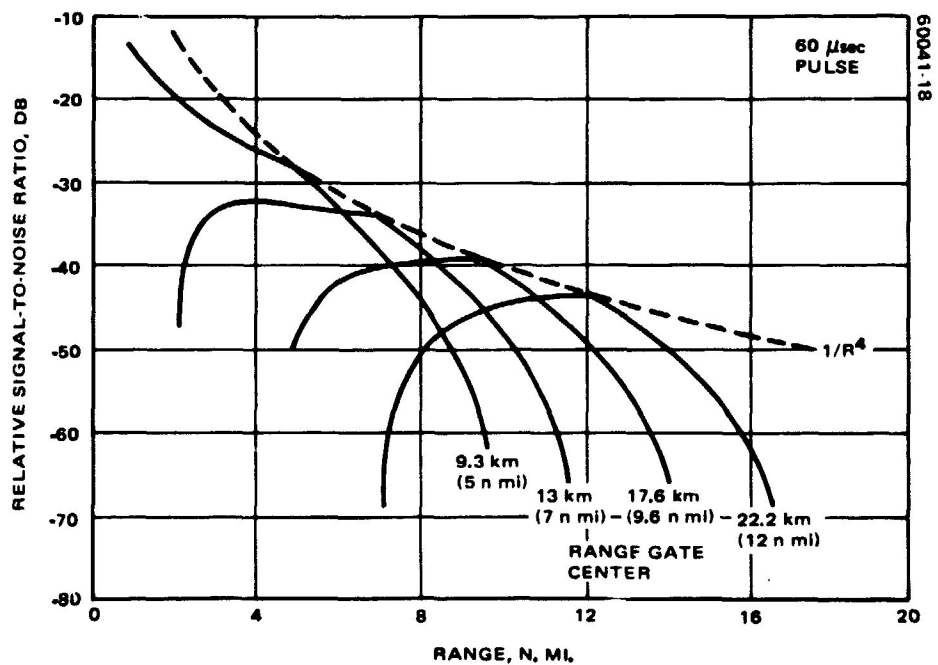


FIGURE 4-18. RANGE GATE STRADDLING LOSS

meet the closing rate measurement requirement. Since the transmitter is coherent, a coherent pulse train for the radar and processing can easily be obtained.

The signal processor employs digital processing for both the search and track functions and is described in subsection 3.3. The analog-to-digital (A/D) converter requirements differ for search and track. For search, a 1 bit quantizer could be used which would provide a signal processing loss of about 1.5 to 2.0 dB. By using more bits in the A/D, this processing loss can be eliminated. Four bits would be more than adequate for search. Figure 4-17 shows the level of quantization and saturation noise for gaussian input signals (Rayleigh envelope). The figure is plotted as a function of the ratio of saturation to rms signal level.* For 4 bits, this noise level is down 16 to 19 dB below the signal level, which is acceptable for both search and track processing.

The number of range gates to be processed is determined by examining Figure 4-18 which shows the amplitude of the signal relative to the range gate center position. By placing three range gates at 9.3, 17.6, and 22 km (5, 9.5, 12 n.mi.), contiguous range coverage is obtained from 22.2 to 2 km (12 to 1 n.mi.). The 13 km (7 n.mi.) range gate would be used only in Mission IIIB and for reacquisition.

4.2.1.5 Antenna Scan Tradeoffs

The major tradeoff between a linear raster scan and a circular spiral scan is between the complexity of generating drive signals and the torque required for each application. A linear scan shaped to the uncertainty volume is compared to a spiral scan with constant angular rate. Figures 4-19 and 4-20 show the dwell time, angular scan rate, and angular accelerations required for scan turnaround as a function of turnaround time. There is a minimum acceleration required for each mission. A turnaround time of 0.3 second is selected since it minimizes the acceleration required and leads to an acceptable dwell time.

The spiral scan with constant angular rate results in a dwell time that varies inversely with the sine of the angle off the scan center. Figure 4-21 shows the deployed assembly coordinate system and definition of the spiral scan.

It can be shown that

$$\xi = \sin^{-1} (-\sin\theta \sin\rho)$$

$$\psi = \sin^{-1} \left(\frac{\cos\theta \sin\rho}{\cos\xi} \right)$$

* Gray, G. A. and Zeoli, G. W., "Quantization and Saturation Noise Due to Analog-to-Digital Conversion," IEEE Transactions on Aerospace and Electronic Systems, January 1971.

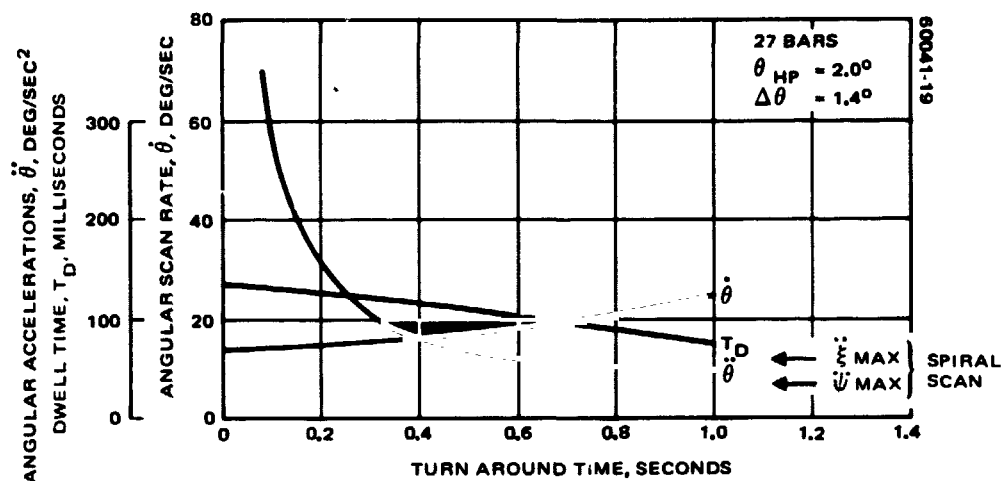


FIGURE 4-19. RASTER SCAN 20° HALF CONE ANGLE SECTOR

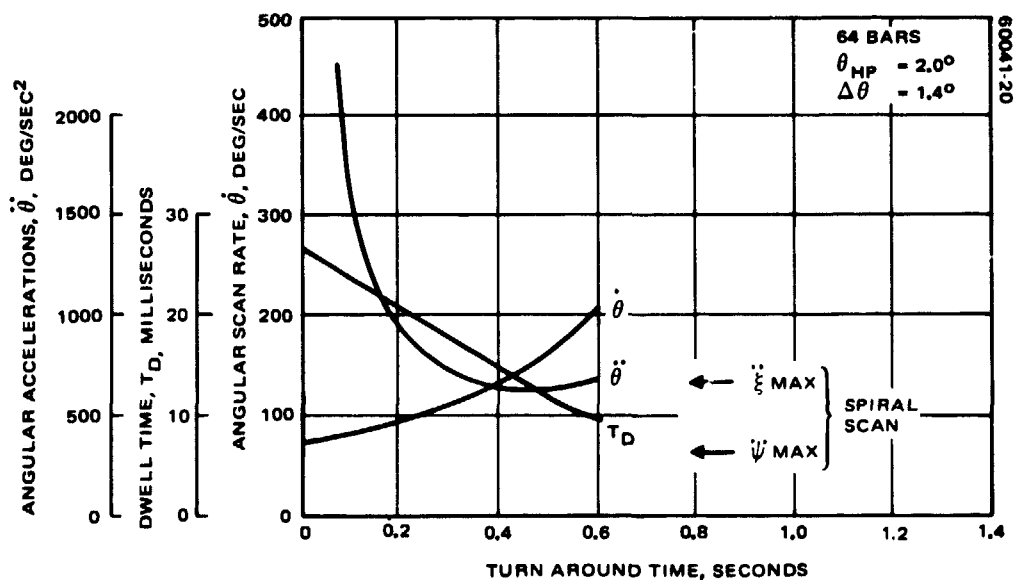


FIGURE 4-20. RASTER SCAN 45° HALF CONE ANGLE SECTOR

and

$$\begin{aligned}\dot{\xi} &= -\dot{\theta} \sin \psi \\ \dot{\psi} &= \dot{\theta} \cos \psi \tan \xi\end{aligned}$$

where

$$\dot{\theta} = \text{constant}$$

and

$$\begin{aligned}\ddot{\xi} &= -\dot{\theta}^2 \cos \psi \tan \xi \\ \ddot{\psi} &= \dot{\theta}^2 \cdot \frac{\sin 2\psi}{2}\end{aligned}$$

from which

$$\begin{aligned}\dot{\xi} \text{ max} &= \dot{\theta} \sin \rho_0 \\ \dot{\psi} \text{ max} &= \dot{\theta} \tan \rho_0\end{aligned}$$

and

$$\begin{aligned}\ddot{\xi} \text{ max} &= \dot{\theta}^2 \tan \rho_0 \\ \ddot{\psi} \text{ max} &= \dot{\theta}^2 \sin^2 2 \rho_0 / 2\end{aligned}$$

where ρ_0 is the maximum search angle (20° for Mission I/II, 45° for Mission IIIB). The above accelerations couple sinusoidally from one axis to the other. For a 60 second frame time, the following maximum accelerations occur (in deg/sec^2).

	$\ddot{\xi}$	$\ddot{\psi}$
Mission I/II	45	40
Mission IIIB	643	321

These values are also shown on Figures 4-19 and 4-20.

Since there is no great difference in the acceleration, the raster scan is more attractive because the high accelerations are required only on the inner gimbal (ξ) while the outer gimbal accelerations are very small ($<10 \text{ deg/sec}^2$) for all angles. Also, since the dwell time varies inversely as the sine of the angle ρ , the dwell time at the edge of the spiral scan will be less than that of the raster scan, 69 versus 122 ms for Mission I/II and 14 versus 18 ms for Mission IIIB. Since the spiral scan inefficiently allows

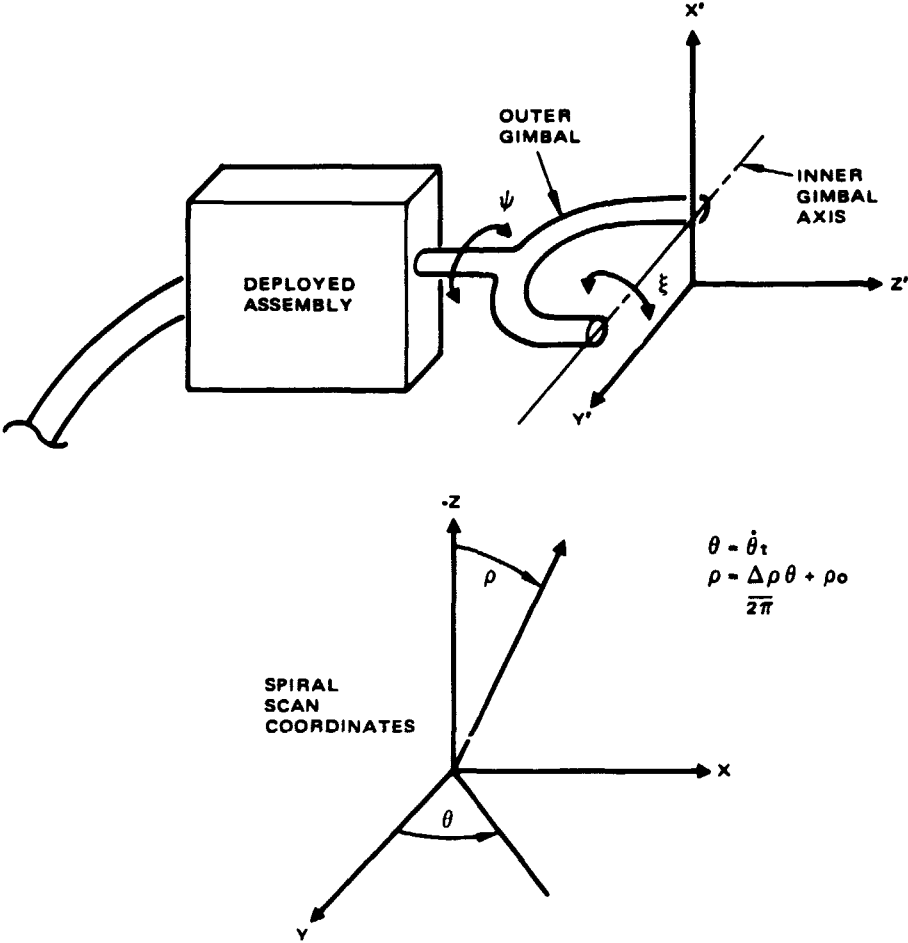


FIGURE 4-21. DEPLOYED ASSEMBLY COORDINATE SYSTEM

the dwell time to vary, and since the acceleration requirements are nearly the same for both types of scan, the raster scan shaped to the target uncertainty volume is chosen for the rendezvous radar.

4.2.1.6 Search Performance

The predicted search performance is tabulated in Tables 4-17 and 4-18. For the chosen parameters and mechanization, these tables show that there is a theoretical margin of 9 to 10 dB. For actual targets, this margin can drop by 3 to 6 dB because of the target radar cross section characteristics and statistics. Maximum improvement of about 10 dB* and improvement of 4 to 9 dB** depends on the target. Therefore, the present radar margin for search is acceptable. Further cost type tradeoffs may dictate that other parameters be changed somewhat to reduce this margin.

Table 4-19 shows the single pulse R_0 range for the 60 μ sec pulse. This radar will always operate with a signal-to-noise ratio of 0 dB or greater for both search and track.

TABLE 4-17. MISSION I/II SEARCH PERFORMANCE
12 n.mi., 20° Half-Cone Angle

$\frac{E}{N_o} = \frac{P_{AV} G^2 \lambda^2 \sigma L T_D}{(4\pi)^3 k T_E R^4}$			
P_{AV}	14.5 W	11.6	
G	36.1 dB	72.2	
λ	2 cm		34
σ	1 m ²	0.0	
T_D	122 ms*		9.1
$(4\pi)^3$	—		33.0
R	22.2 km (12 n.mi.)		173.8
k	1.38×10^{-23}	228.6	
T_E	650°K		28.1
L^{**}	6.5 dB		6.5
		312.40	284.50
		284.50	
		27.9 dB = E/N_o	
		18.6 = required E/N_o	
		9.3 dB max margin	

*0.3 sec turnaround.

** L_{RF} = 2.8 dB, L_{BS} = 1.7 dB, L_{PROC} = 2.0 dB.

*Reference Tice.

**Kennedy, R. W., "The Spatial and Spectral Characteristics of the Radar Cross Section of Satellite-Type Targets," AFAL-TR-66-17, March 1966.

TABLE 4-18. MISSION IIIB SEARCH PERFORMANCE
7 n.mi., 20° Half-Cone Angle

PAV	14.5 W	11.6	
G	36.1 dB	72.2	
λ	2 cm		34.0
σ	1 m ²	0.0	
T _D	18 ms*		17.4
(4 π) ³	—		33.0
R	13 km (7 n.mi.)		164.5
k	1.38 x 10 ⁻²³	228.6	
T _E	650° K		28.1
L	6.5 dB		6.5
		312.40	283.5
		283.5	
		+ 28.9 dB = E/N ₀	
		18.6 Require E/N ₀	
		10.3 max margin	

*0.3 sec turnaround.

TABLE 4-19. SINGLE PULSE, R₀
SIGNAL-TO-NOISE RATIO
 $\tau = 60 \mu\text{sec}$

$\frac{S}{N} = \frac{P_p G^2 \lambda^2 \sigma L \tau}{(4\pi)^3 k T_E R^4}$			
P _p	50	17.0	
G	36.1 dB	72.2	
λ	2 cm		34
σ	1 m ²	0.0	
τ	60 μsec		42.2
(4 π) ³			33.0
k	1.38 x 10 ⁻²³	228.6	
T _E	650°		28.1
L	6.5		6.5
		317.8	143.8
		143.8	
		174.0	
$R_0^4(m) = \text{antilog}_{10} 17.4$			
$R_0 = 22.387 \text{ km (12.09 n.mi.)}$			

4.2.2 Tracking Analysis

4.2.2.1 Range Tracker

The range tracker is a conventional early/late gate tracker that employs an $\alpha - \beta$ tracker mechanization to close the range tracking loop. The digital signals are magnitude detected and postdetection integrated (summed) over both pulses and rf frequencies. A log difference discriminant is used to generate the tracking error signal. The functional form of the range tracker is shown in Figure 4-22 and Table 4-20 lists the tracking requirements.

TABLE 4-20. RADAR TRACKING REQUIREMENTS

	Mission I/II, Passive	Mission IIIB, Passive/Cooperative	Mission I/II, Cooperative
Tracking range			
Ma:	22.2 km (12 n.mi.)	14.8 km (8 n.mi.)	555 km (300 n.mi.)
Min	30 m (100 ft)	30 m (100 ft)	30 m (100 ft)
Tracking accuracy			
Range	Greater of 0.01 R or 30 m (100 ft) [R < 18.5 km (10 n.mi.)]	Greater of 0.01 R or 30 m (100 ft) [R < 9.3 km (5 n.mi.)] 119 m (390 ft) (9.3 km (5 n.mi.) < R < 55.5 km (30 n.mi.)]	Greater of 0.01 R or 30 m (100 ft) [R < 9.3 km (5 n.mi.)] 119 m (390 ft) (9.3 km (5 n.mi.) < R < 55.5 km (30 n.mi.)]
Range rate	± 0.305 m/sec (± 1 fps)	± 0.305 m/sec (± 1 fps)	± 0.305 m/sec (± 1 fps) R < 56 km (30 n.mi.)
Angle	± 10 mr random 60 mr bias	± 10 mr random 60 mr bias	± 10 mr random 60 mr bias R < 56 km (30 n.mi.)
Angular rate	0.14 mr/sec	0.14 mr/sec	0.14 mr/sec R < 56 km (30 n.mi.)
Angular rate			
Acquisition	4 mr/sec	4 mr/sec	4 mr/sec
Track	4 deg/sec	4 deg/sec	5 deg/sec
Target stabilization	$\pm 10^0$ each axis	$\pm 10^0$ each axis	$\pm 10^0$ each axis
Limit cycle	5 deg/min	5 deg/min	5 deg/min

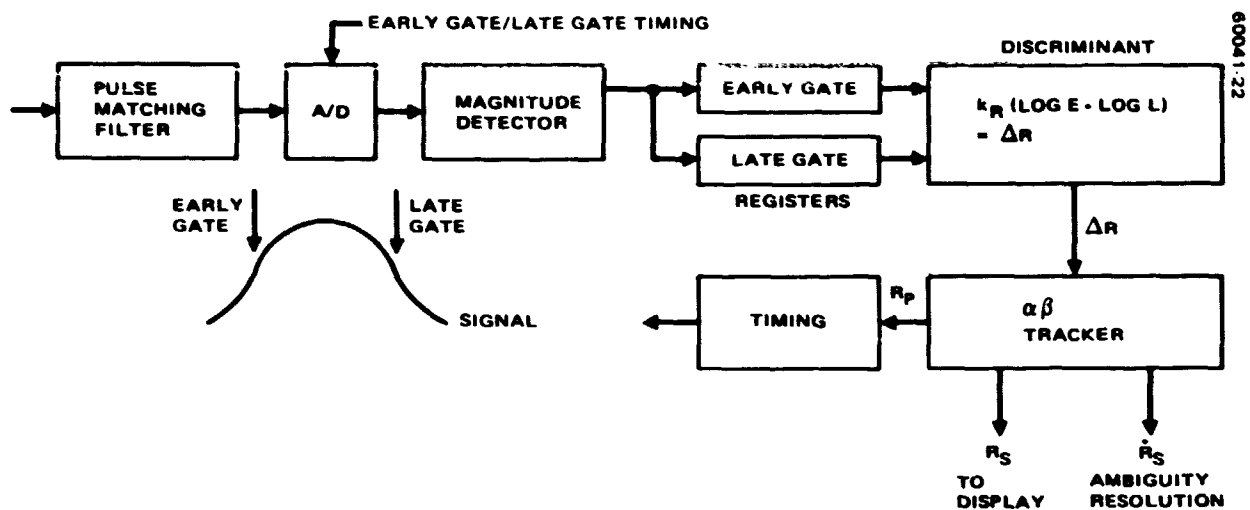


FIGURE 4-22. RANGE TRACKER FUNCTIONAL BLOCK DIAGRAM

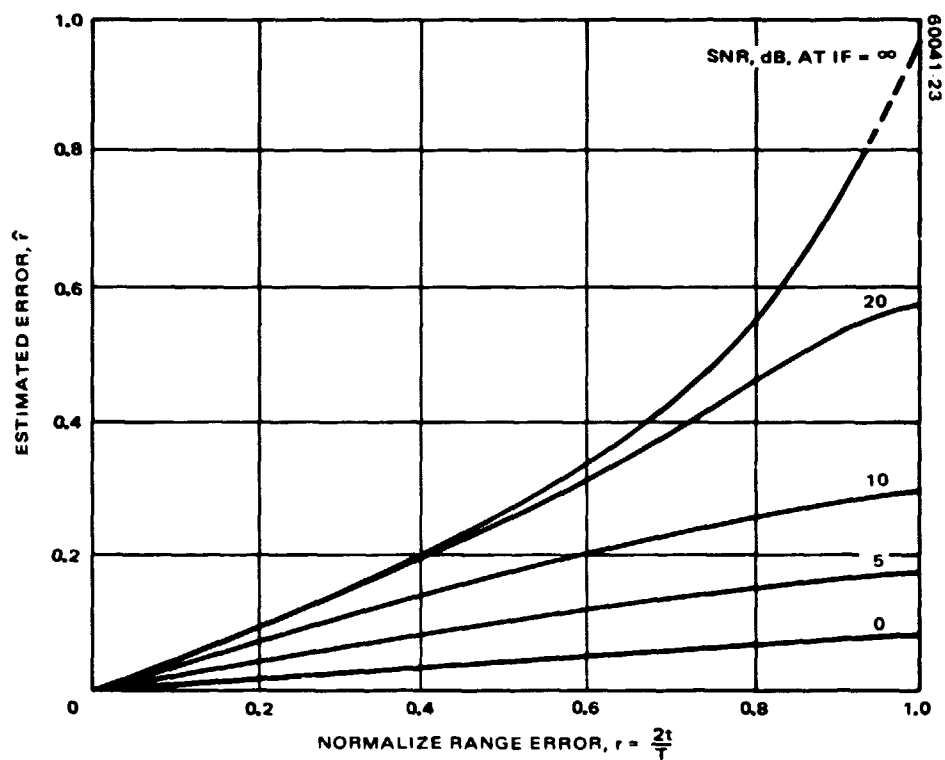


FIGURE 4-23. RANGE DISCRIMINANT CHARACTERISTIC

The range discriminant is formed by taking the logarithm of the sum of pulses for the early and late gate

$$\left(\log \left(\sum_{f=1}^{10} \sum_{n=1}^{64} P_{Enf} \right) - \log \left(\sum_{f=1}^{10} \sum_{n=1}^{64} P_{Lnf} \right) \right)$$

Intuitively, this is better than summing the logarithms over the 10 RF frequencies, for example

$$\sum_{f=1}^{10} \left\{ \left(\log \left(\sum_{n=1}^{64} P_{Enf} \right) - \log \left(\sum_{n=1}^{64} P_{Lnf} \right) \right) \right\}$$

because in the case of a signal fade, the second algorithm could lead to a large error. On the other hand, the first algorithm weights the largest amplitude signal the most, and results in more nearly centroid tracking. *

The characteristic of the first algorithm for a point target is shown in Figure 4-23 which gives the discriminant characteristics and standard deviation of the measurement as a function of the signal-to-noise ratios at the pulse matching filter.

The range tracker performance is a function of the gain of the tracker, the signal-to-noise ratio, pulse length, and target extent. The gain determines both the transient response behavior and the noise smoothing. The signal-to-noise ratio affects both the discriminator slope characteristic and noise out of the discriminant. Both the pulse length and target extent determine the accuracy to which the target can be tracked.

An α - β tracker is chosen for the range track because of its simplicity of mechanization and its adequacy in meeting the tracking requirements.

The single pulse signal-to-noise ratio is shown in Figure 4-24 for the three pulse lengths which are used during track. As indicated earlier, the

* Several studies have shown that a strong signal is more likely to be from the centroid of the target complex, while weak signals are caused by destructive interference, resulting in an apparent target center off of the true centroid. This is true for both range and angle tracking. For example see:

Dunn, J. H. and Howard, D. D., "Radar Target Amplitude, Angle, and Doppler Scintillation from Analysis of the Echo Signal Propagating in Space," IEEE Transactions on Microwave Theory and Techniques, Volume MTT-16, No. 9, September 1968.

Graf, E. R. and Nagle, H. T., "Frequency Diversity and Signal Processing to Reduce Glint Pointing Error," AD 747-348, Engineering Experimental Station, Auburn University, Alabama, 30 June 1972.

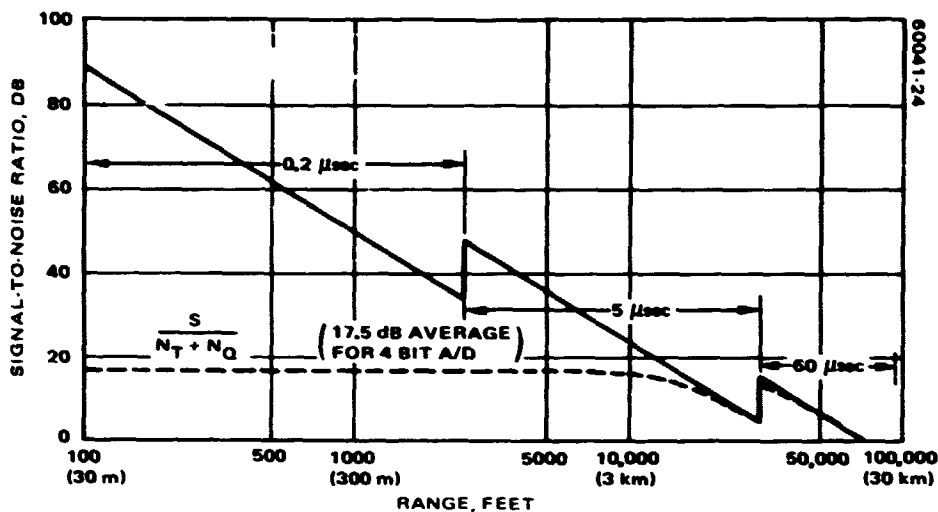


FIGURE 4-24. SINGLE PULSE SIGNAL-TO-NOISE RATIO

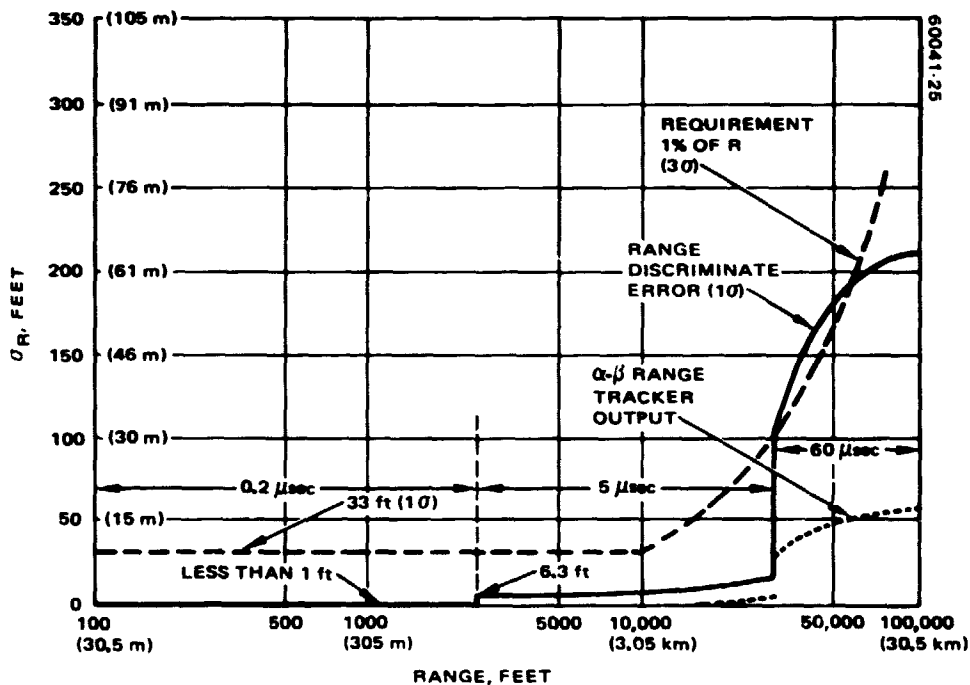


FIGURE 4-25. RANGE TRACKING PERFORMANCE POINT TARGET

pulse length is changed at the range corresponding to the pulse length's unambiguous range (i. e., at 9.15 km (30K feet) and at 762 meters (2500 feet)). Figure 4-24 also shows the effective signal-to-noise ratio at the output of the A/D converter. The signal input to the A/D converter will vary over a 6 dB region because of AGC action; the output signal-to-saturation plus quantization noise will vary from 16 to 19 dB below the signal level depending on the input signal level.* An average value of 17.5 dB will be used in the analysis.

The error out of the discriminant as a function of range is shown in Figure 4-25 along with the range requirement (1 percent of range or 100 feet (30 meters), 3σ). Except for long ranges, the range error out of the discriminant is below the required value. Closing the range tracker loop through the α - β tracker will reduce the range error to below the requirement.

The range discriminant will produce an output every 133 ms (640 pulses). The output is used to update the α - β tracker. The gain α of the α - β tracker can be between 0 and 1 ($\beta = \alpha^2/2 - \alpha$) and is selected to provide a compromise between noise smoothing and transient response. An α of 1 results in a "first order" hold mechanization and provides no noise smoothing. Figure 4-26 shows the noise reduction ratio (square root of variance reduction ratio and transient error response as a function of α). The following equations are used to evaluate the α - β tracker performance.

$$K_{\text{rate}} = \sqrt{\frac{2\beta^2}{T^2\alpha(4-2\alpha-\beta)}}$$

$$K_{\text{position}} = \sqrt{\frac{2\alpha^2 + \beta(2-3\alpha)}{\alpha(4-2\alpha-\beta)}}$$

$$\text{Transient position error} = \frac{\alpha-1}{\beta} T^2$$

$$\text{Transient rate error } \beta = \frac{\alpha-2}{2\beta} T$$

where

$$\beta = \frac{\alpha^2}{2-\alpha}$$

The transient error is the steady state lag error caused by a unit acceleration input.

* See Figure 4-17 and reference Gray and Zeoli.

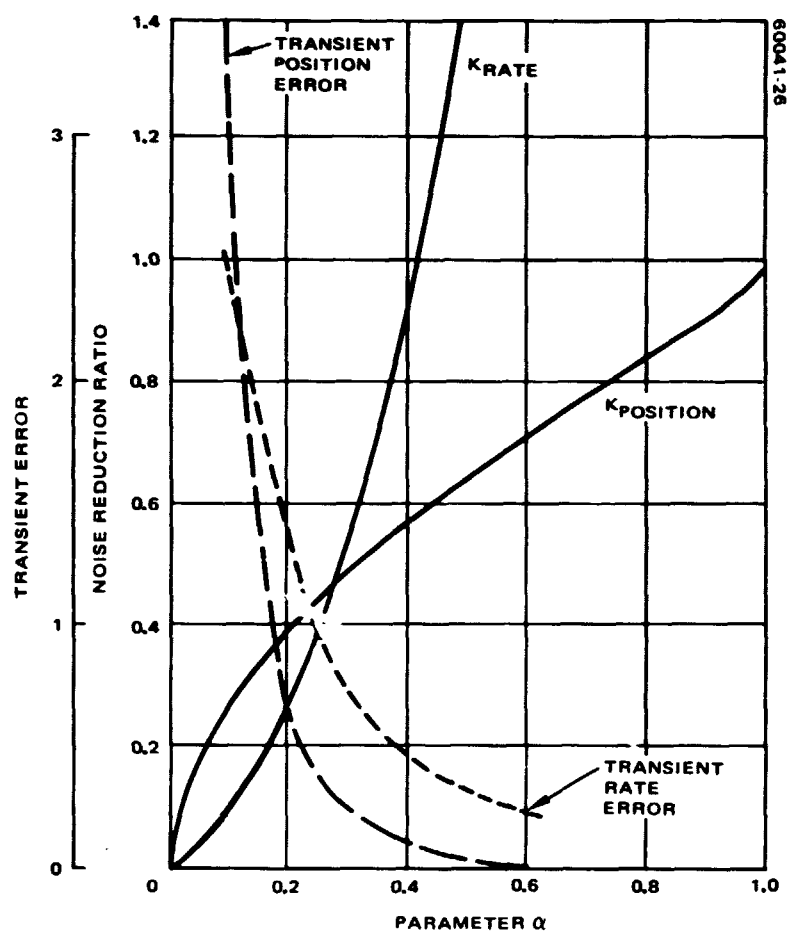


FIGURE 4-26. α - β TRACKER CHARACTERISTICS

A value of $\alpha = 0.1$ is chosen for the range tracker gain. This gain will result in a transient position error of 0.5 meter (1.5 feet) at the end of a 20 second deceleration period. The output range position noise will be reduced to 27 percent of the input noise and the range rate noise is 0.075 per second of the input range noise. The range tracker noise values are shown in Figure 4-25 as a function of range. For all ranges of interest, the range noise error is less than 15 meters (50 feet) and the range rate error is less than 5 m/sec (15 fps).

Because of the accuracy that the range discriminant produces and because of the mild acceleration capability required, the basic $\alpha - \beta$ tracker capability far exceeds the requirements. Other errors, such as instrumentation errors, timing jitters, and target extent, will dominate the tracker response.

The tracking slope of the range discriminant decreases as the signal-to-noise decreases, as shown in Figure 4-23. At acquisition, the signal-to-noise will be near 0 dB. The range tracker will be more sluggish since the gain is reduced by a factor of 5 at the low signal-to-noise ratios. This will increase the dynamic lag during decelerations. However, if uncorrected, the maximum error would be 30 meters (100 feet) for a 20 second deceleration. This error would be well within the limits of the 60 μ sec tracking pulse, 10 km (30K feet) and a break lock will not occur.

From Figure 4-25, it can be seen that the tracking error for a point target will be small and the major contributions to range tracking error will be other factors, such as target extent. From Appendix B, the range discriminant error caused by target extent can be expressed as

$$\sigma_R^2 \cong \frac{X_r^2}{2(N_f - 1)}$$

where

X_r = radius of gyration in range direction

N_f = number of rf transmitter frequencies

Since the target extent error is correlated from one discriminant output to the next, the range tracker will have very little effect on reducing this error. The discriminant will produce an output that is close to the centroid of the target extent. For 10 RF frequencies and a 20 meter (60 foot) length target where the scatterers are uniformly distributed along the target, the extended target tracking error is approximately

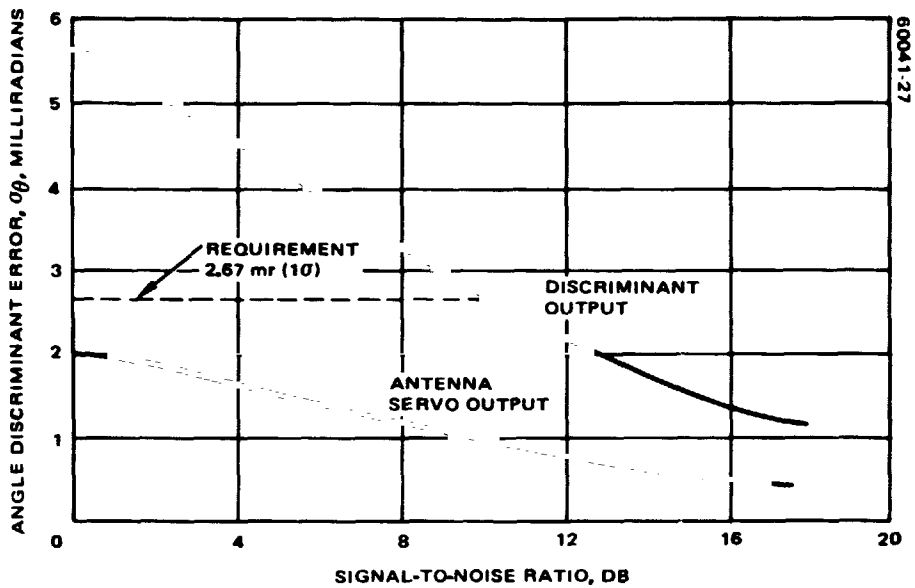


FIGURE 4-27. ANGLE TRACKING ERROR 1σ POINT TARGET

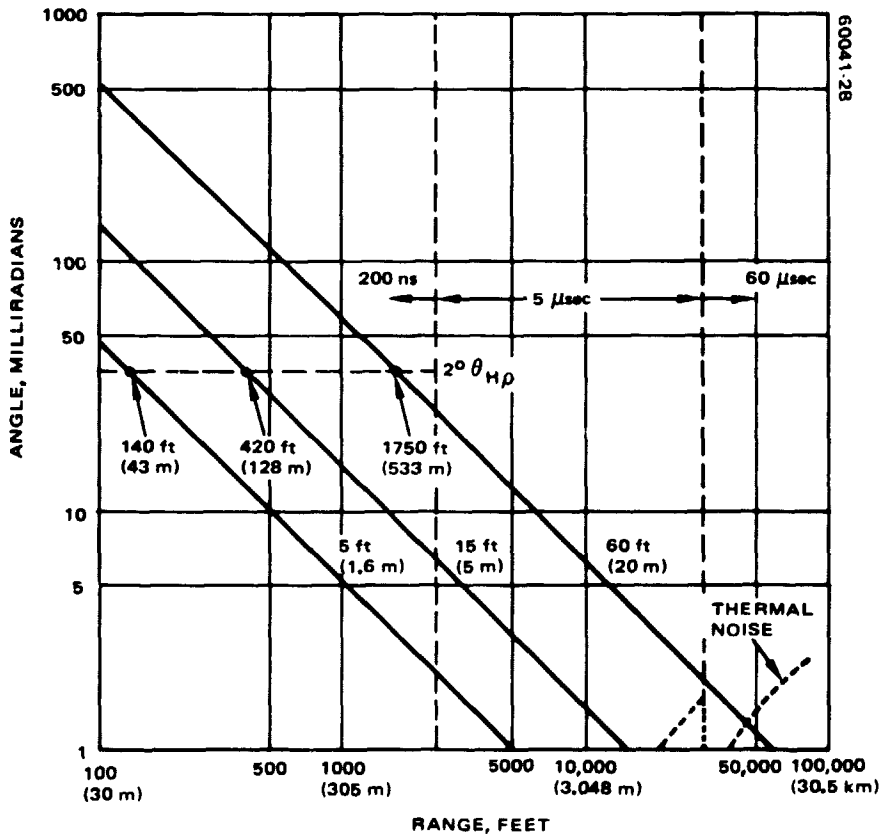


FIGURE 4-28. TARGET ANGULAR EXTENT

$$\sigma_R = \frac{L}{\sqrt{24} \sqrt{(N - 1)}} = 1.3 \text{ meters (4 feet)}$$

which is well within the range tracking requirements.

4.2.2.2 Angle Tracking

The analysis and performance predictions of the angle tracker for the rendezvous radar follows the same methods as used for the range tracker. The angle tracking mechanization uses sequential lobing by adding the difference signal to the sum signal and the logarithmic discriminant to form the angular error signals. Each discriminant is formed by a total of 160 pulses (16 pulses on each of 10 RF frequencies). The pulses are PDied (summed) before the discriminant is formed. The angle discriminant error (for a point target) is given by

$$\sigma_\theta \equiv k_\theta (2(1 - R^2)/N - 1)^{1/2}$$

where the terms are defined in Appendix B and shown in Figure 4-27 as a function of signal-to-noise ratio. The action of the angle tracking is to reduce this error by N_θ where N_θ is the square root of the discriminant sample rate to the antenna servo bandwidth ($N_\theta \approx 2.74$). The predicted noise output from the antenna servo is also shown in Figure 4-27. This error is negligible except at the long ranges.

Target angular extent is dominant at the shorter ranges. Figure 4-28 plots the target angular extent for the 5 by 20 meter cylinder target (15 by 60 feet) and the 1.6 meter (5 feet) sphere target. The use of frequency agility will reduce the effective angular extent of the target. The output of the discriminant is of the form

$$\sigma_\theta^2 = \frac{\theta_r^2}{2(N - 1)}$$

where θ_r = radius of gyration of the target. With 10 RF frequencies, the target extent noise will be reduced to about one-fifteenth of its radius of gyration, assuming uniformly distributed targets.

When the target extent is on the order of the beamwidth, the target radius of gyration must be further weighted by the antenna beamwidth. When the range to the target is such that the antenna beamwidth is smaller than the angular extent of the target, the angle tracking servo will cause the antenna to wander over the target and track the strongest return in the beamwidth.

4. 2. 2. 3 Closing Rate Measurement

The target closing rate is determined by measuring the doppler shift by means of doppler filtering. The doppler is determined by locating which of the 64 doppler filters have the two largest amplitude outputs. A logarithmic discriminant is formed similar to the range and angle discriminant by using the outputs of the two adjacent filters. The characteristic of the discriminant is shown in Figure 4-29. Since the signal-to-noise in the doppler filter is extremely high (>18 dB), the estimate will be extremely accurate. Because of the nonlinear behavior of the discriminant when the target is at the peak response ($0.5 f/B$), a bias error of up to $^{\circ}$ Hz can exist (~ 0.1 m/sec (0.3 fps)). The random error is evaluated by the methods outlined in Appendices B and C. The random error is less than 1 Hz (1°) for any doppler position shown in Figure 4-26.

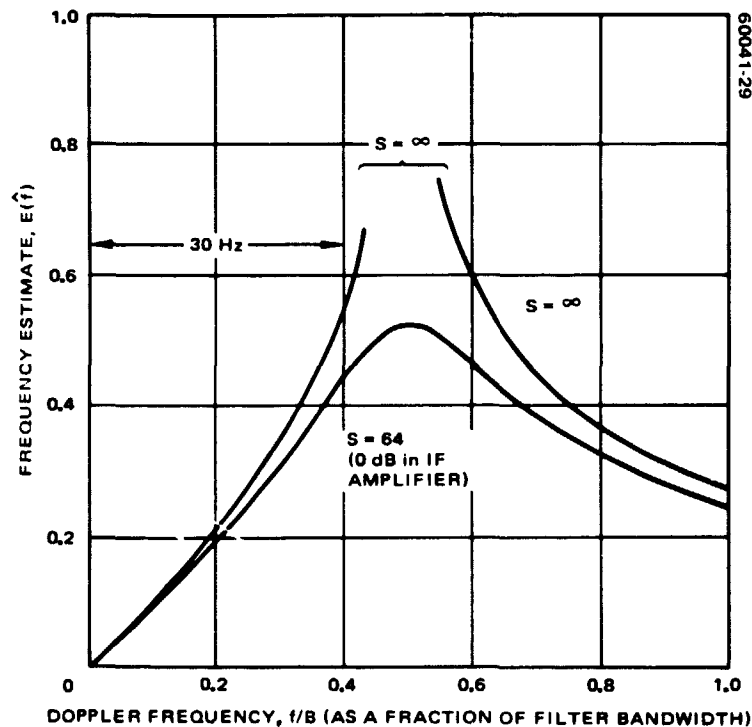


FIGURE 4-29. EXPECTED VALUE OF FREQUENCY ESTIMATE VERSUS DOPPLER FREQUENCY

APPENDIX A. THERMAL NOISE ERRORS IN SHUTTLE AUTOTRACK SYSTEM

This memo deals with the use of time-shared monopulse to autotrack a communication signal. The communications signal in question employs unbalanced quadriphase for transmitting digital data. It is desired to compare the use of AM versus PM for handling the monopulse pointing-error information. The AM approach appears to afford the better thermal noise performance.

I. INTRODUCTION

Figure A-1 shows two ways of implementing time-shared monopulse. The general principles are assumed well known and will not be elaborated on. The difference pattern signal in one axis (say E1) is modulated by a switching function $s(t)$ and then summed with the sum pattern signal. The other axis (Az, not shown) is treated similarly by another switching function orthogonal in time.

Two different systems evolve from the above procedure, depending on whether the difference signal is summed in-phase or in quadrature with the sum signal. In-phase summing results in AM, in which case the modulation (switched difference signal) can be recovered by non-coherent detection. Quadrature summation produces phase modulation, and a coherent phase detector is needed to get the switched difference signal back. In either case the switched difference signal, once recovered is multiplied by the switching signal, to recover polarity.

The analysis to follow compares thermal noise performance of the above AM and PM monopulse approaches, for the case where the communication signal uses unbalanced quadriphase.

II. UNBALANCED QUADRIPHASE COMMUNICATION SIGNAL

The communication signal is formed by the quadrature summation of two independently biphas modulated signals

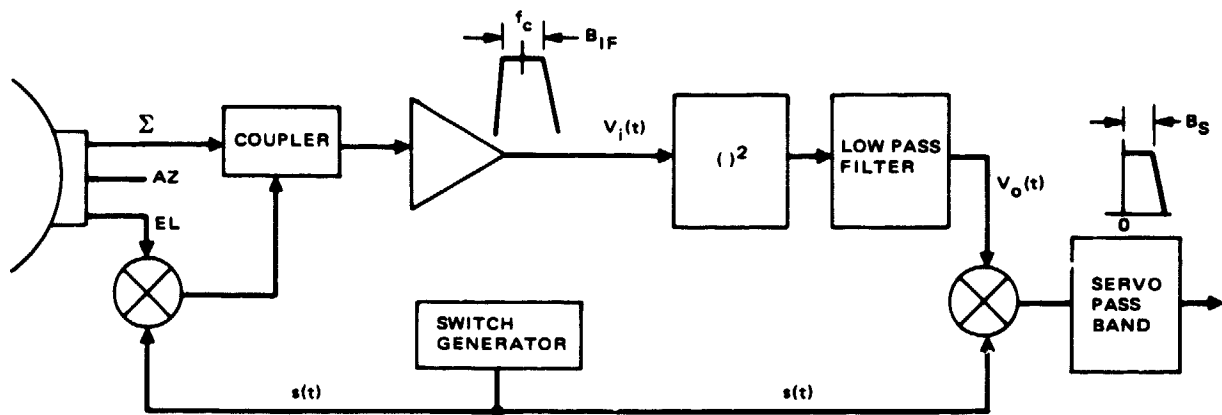
$$A \sin [\omega_c t + (\pi/2) \mu_a(t)] = A \mu_a(t) \cos \omega_c t$$

and

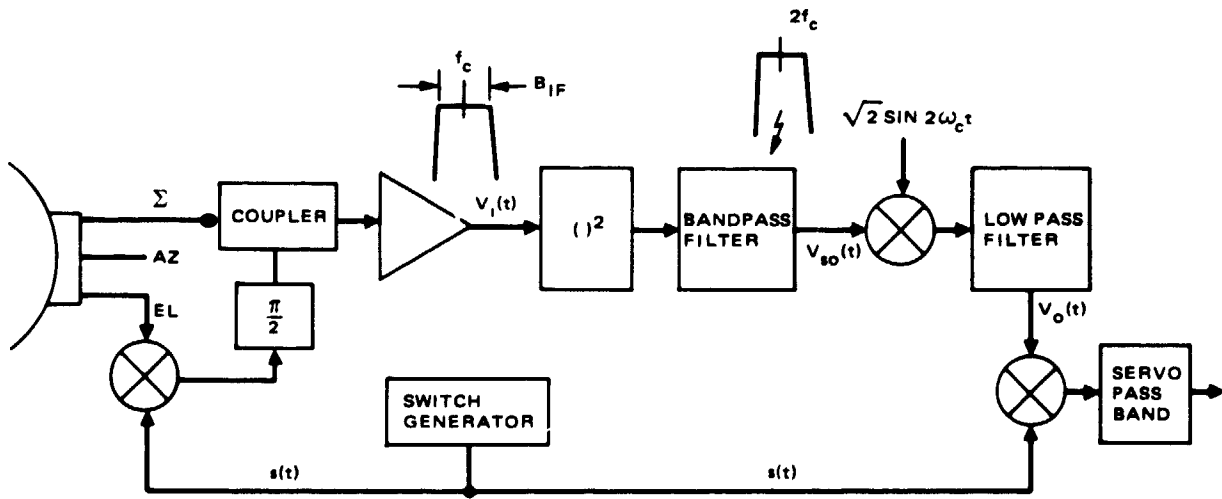
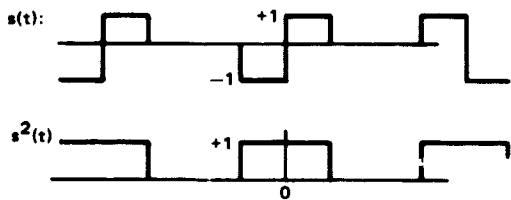
$$B \cos [\omega_c t + (\pi/2) \mu_b(t)] = B \mu_b(t) \sin \omega_c t$$

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a) AM APPROACH



b) PM APPROACH

FIGURE A-1. TIME SHARED MONOPULSE IMPLEMENTATIONS

where $\mu_a(t)$ and $\mu_b(t)$ are independent binary data streams.

Summation yields

$$C(t) = \sqrt{2P_c} \cos [\omega_c t + D(t)]$$

where

$$P_c = \text{signal power}$$

$$\sqrt{2P_c} = (A^2 + B^2)^{\frac{1}{2}}$$

$$D(t) = \tan^{-1} [B\mu_b(t)/A\mu_a(t)]$$

The following facts about phase angle $D(t)$ will be used later:

$$\sin D(t) = \frac{B}{\sqrt{A^2 + B^2}} \mu_b(t)$$

$$\cos D(t) = \frac{A}{\sqrt{A^2 + B^2}} \quad (\text{constant})$$

$$\sin 2D(t) = \frac{2AB}{A^2 + B^2} \mu_a(t)\mu_b(t)$$

$$\cos 2D(t) = \frac{A^2 - B^2}{A^2 + B^2}$$

III. NOISE

The signal to be autotracked is accompanied by noise $N(t)$, expressible in terms of uncorrelated in-phase and quadrature components:

$$N(t) = N_i(t) \cos \omega_c t + N_q(t) \sin \omega_c t$$

with the usual properties

$$\overline{N^2(t)} = \overline{N_i^2(t)} = \overline{N_q^2(t)} = N_o B_{IF}$$

where N_0 is input one-sided noise density and B_{IF} is i-f bandwidth.

The autocorrelation functions and two-sided power spectra of $N_1(t)$ and $N_q(t)$ are identical, and will be denoted respectively by $R_{NB}(\tau)$ and $W_{NB}(f)$. $W_{NB}(f)$ is assumed rectangular of height N_0 , and extending from $-B_{IF}/2$ to $+B_{IF}/2$.

IV. SWITCHING FUNCTION

Several properties of the switching function $s(t)$ are of importance to the ensuing analysis:

1. Being periodic $s(t)$ has a power spectrum $W(f)$ composed of delta functions (line spectrum). The same goes for $W_{s2}(f)$, the power spectrum of $s^2(t)$. It can be seen almost from inspection that $W_s(f)$ has no term at zero (dc) and that $W_{s2}(0)$ is a delta function with weight 1/4. Only this one term falls within the servo passband. All other lines of both $W_s(f)$ and $W_{s2}(f)$ fall well outside.
2. $s^3(t) = s(t)$
3. $\overline{s(t)s^2(t + \tau)} = 0$ for $\tau \neq 0$
4. $\overline{s(t)s^2(t + \tau)} = -\overline{s(t)s^2(t - \tau)}$

2,3 and 4 are obvious from geometrical considerations.

5. All the main components of the spectra $W_s(f)$ and $W_{s2}(f)$ are clustered in a very narrow region compared to the i-f bandwidth B_{IF} . The total integrated power in each of these spectra is 1/2, due to the 50% duty factor of $s(t)$.

V. AM CASE

The noncoherent demodulator in the AM system is modeled as a square law detector. The detector input is

$$V_i(t) = \sqrt{2P_c} \cos [\omega_c t + D(t)] [1 + ms(t)] + N(t)$$

Here m (mod index) is the voltage ratio between difference and sum channel r-f signals at the point where they are summed (coupler). This fact will be used later (Attachment) to get performance expressed in terms of receiving system parameters (antenna gain etc.)

After squaring $V_i(t)$, only the low-frequency portions are retained, and these are then multiplied by the switching function $s(t)$:

$$\begin{aligned}
V_o(t) = & P_c [1 + 2ms(t) + m^2 s^2(t)] s(t) \\
& + \sqrt{2P_c} [1 + ms(t)] [N_i(t) \cos D(t) + N_q(t) \sin D(t)] s(t) \\
& + \frac{1}{2} [N_i^2(t) + N_q^2(t)] s(t)
\end{aligned} \tag{10}$$

In (10), the top line (on right hand side) is output caused by antenna pointing error ("signal" component) and the two lower lines are noise. What is wanted is to know how much of the power represented by these terms finds its way into the autotrack servo passband. This is determined from an analysis of the power spectra, which in turn are computed from autocorrelation functions.

1. First part of $V_o(t)$

For convenience the three portions of $V_o(t)$ in (10) will be treated separately, and will be distinguished by an additional subscript 1,2 or 3. Thus the top line will be called $V_{o1}(t)$, and its autocorrelation function:

$$R_{o1}(\tau) = \overline{V_{o1}(t)V_{o1}(t+\tau)}$$

Performing the indicated operations and making use of the properties of $s(t)$ given in Section IV leads to

$$R_{o1}(\tau) = P_c^2 [(1 + m^2)^2 R_s(\tau) + 4m^2 R_{s2}(\tau)]$$

The power spectrum, the Fourier transform, is

$$W_{o1}(f) = P_c^2 [(1 + m^2)^2 W_s(f) + 4m^2 W_{s2}(f)] \tag{11}$$

As noted in Section IV the only portion of the spectra in (11) that falls within the servo passband is the d-c component of $W_{s2}(f)$, namely a delta function of weight 1/4. Thus the "signal" component of the output is

$$P_S = P_c^2 m^2 \tag{12}$$

2. Second part

Proceeding with the second line in (10) the autocorrelation function is found to be

$$R_{o2}(\tau) = [R_s(\tau) + m^2 R_{s2}(\tau)] R_{NB}(\tau) [A^2 + B^2 R_a(\tau)]$$

where $R_a(\tau)$ is the autocorrelation function of binary data stream $\mu_a(t)$.

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Fourier transforming:

$$W_o(f) = [W_s(f) + m^2 W_{s2}(f)] * W_{NB}(f) * [A^2 \delta(f) + B^2 W_a(f)]$$

where the * represents convolution, and $W_a(f) \Leftrightarrow R_a(\tau)$.

Given the narrowness of $W(t)$ and $W_{s2}(f)$ relative to the noise spectrum $W_{NB}(f)$ (Section IV), the indicated convolution yields negligible spreading. The net effect of the convolution is just to multiply $W_{NB}(f)$ by $\frac{1}{2}$:

$$W_{o2}(f) = \frac{(1+m^2)}{2} [A^2 W_{NB}(f) + B^2 W_{NB}(f) * W_a(f)] \quad (13)$$

If nearly all components of the data signal spectrum $W_a(f)$ fall within the range of $W_{NB}(f)$, that is, within the i-f bandwidth, then the convolution at the far right in (13) will yield a density, near $f = 0$, substantially the same as that of $W_{NB}(f)$, namely N_o . Thus in the region near d-c

$$W_{o2}(o) = \frac{(1+m^2)}{2} N_o (A^2 + B^2) = P_c (1+m^2) N_o$$

and the noise entering the servo passband (B_s Hz one-sided) is

$$P_{NI} = 2W_{o2}(o)B_s = 2P_c (1+m^2) N_o B_s \quad (14)$$

3. Third part of $V_o(t)$

Turning to the final line in (10) one finds

$$R_{o3}(\tau) = \overline{V_{o3}(t) V_{o3}(t+\tau)} = \frac{1}{2} [\overline{2N_i^2(t)N_i^2(t+\tau)} + \overline{2N_i^2(t)N_q^2(t)}] R_s(\tau) \quad (15)$$

Inside the brackets in (15), the two mean-square terms on the right each equal $N B_{IF}$. The other bracketed term is reduced using the rule for the expectation of the product of gaussian variables:

$$\langle X_1 X_2 X_3 X_4 \rangle = \langle X_1 X_2 \rangle \langle X_3 X_4 \rangle + \langle X_1 X_3 \rangle \langle X_2 X_4 \rangle + \langle X_1 X_4 \rangle \langle X_2 X_3 \rangle$$

leading to

$$R_{o3}(\tau) = [R_{NB}^2(\tau) + (N_o B_{IF})^2] R_s(\tau)$$

and the spectrum

$$W_{o3}(\tau) = W_{NB}(f) * W_{NB}(f) * W_s(f) + (N_o B_{IF})^2 \delta(f) * W_s(f) \quad (16)$$

The term on the right with the delta function produces nothing within the servo bandwidth. In the other term, spreading due to $W_s(f)$ again is negligible and the only resultant effect is multiplication by $\frac{1}{2}$.^s The convolution of $W_{NB}(f)$ with itself yields a triangular shape having height $N_o^2 B_{IF}$. Thus, in a narrow band (the servo bandwidth $2 B_s$) around zero, noise power is

$$P_{N2} = 2W_{o3}(0) B_s = N_o^2 B_{IF} B_s \quad (17)$$

To summarize then, the AM implementation provides servo passband SNR given by the ratio of (12) to the sum of (14) plus (17):

$$SNR_{AM} = \frac{P_c^2 m^2}{2P_c (1 + m^2) N_o B_s + N_o^2 B_{IF} B_s} \quad (18)$$

Note that there is nothing in (18) related to the data stream being transmitted; tracking is independent of the presence or absence of data.

VI PM Case

In the PM implementation the input to the squarer is

$$V_i(t) = \sqrt{2P_c} \left\{ \cos[\omega_c t + D(t)] + ms(t) \sin[\omega_c t + D(t)] \right\} + N(t) \quad (19)$$

The squarer now acts as a frequency doubler rather than as a noncoherent detector. Thus when (19) is squared the second harmonic components are retained:

$$\begin{aligned} V_{so}(t) = P_c \left\{ [1 - m^2 s^2(t)] \cos 2[\omega_c t + D(t)] + \right. & \quad (t) \sin 2[\omega_c t + D(t)] \left. \right\} \\ + \sqrt{2P_c} \left\{ [N_i(t) + ms(t)N_q(t)] \cos[2\omega_c t + D(t)] + [N_q(t) + \right. & \\ ms(t)N_i(t)] \sin[2\omega_c t + D(t)] \left. \right\} + \frac{N_i^2(t) - N_q^2(t)}{2} \cos 2\omega_c t + & \\ N_i(t)N_q(t) \sin 2\omega_c t & \end{aligned}$$

$V_{so}(t)$ next is multiplied by $\sqrt{2} \sin 2\omega_c t$ in the coherent demodulator. The low-frequency products from this latter operation are retained. The low-frequency products then are multiplied by the switching signal $s(t)$. The result is

$$V_o(t) = \sqrt{2} P_c \left[(m^2 - 1) \frac{AB}{A^2 + B^2} \mu_a(t) \mu_b(t) s(t) + m \frac{A^2 - B^2}{A^2 + B^2} s^2(t) \right] \\ + \sqrt{P_c} [N_q(t)s(t) + ms^2(t)N_i(t)] + (1/\sqrt{2}) N_i(t)N_q(t)s(t) \quad (20)$$

1. First part of (20)

Proceeding as in the AM case the spectrum of $V_{o1}(t)$ is found

$$W_{o1}(f) = 2P_c^2 \left\{ \left[(m^2 - 1) \frac{AB}{A^2 + B^2} \right]^2 W_a(f) * W_b(f) * W_s(f) + \left[\frac{m(A^2 - B^2)}{A^2 + B^2} \right]^2 W_{s2}(f) \right\} \quad (21)$$

The first part of (21) (to the right of the equal sign) is communication signal energy "leaking" into the autotrack system. This could be made negligible by proper choice of data signal format, for example by using split-phase (Manchester) coding on the higher data rate signal. Such a choice keeps spectral content low in the low-frequency region.

It will be assumed that such is the case and that the term in question is negligible in (21). Then as seen before the only component falling into the servo passband is the $\frac{1}{2}$ weight "d-c" delta function of $W_{s2}(f)$. This gives the "signal" component:

$$P_s = \frac{P_c^2 m^2}{2} \left(\frac{A^2 - B^2}{A + B^2} \right)^2 \quad (22)$$

2. Second part of (20)

Spectrum is found to be

$$W_{o2}(f) = P_c [W_{NB}(f) * W_s(f) + m^2 W_{NB}(f) * W_{s2}(f)]$$

Again the main effect of convolution is just to multiply $W_{NB}(0)$, noise density near zero, ($=N_0$), by $\frac{1}{2}$. Noise in the servo passband thus is

$$P_{N1} = 2W_{o2}(0)B_s = P_c (1 + m^2) N_0 B_s \quad (23)$$

3. Third part of (20)

$$W_{o3}(f) = \frac{1}{2} W_{NB}(f) * W_{NB}(f) * W_s(f)$$

$$W_{o3}(o) = \frac{1}{2} N_o^2 B_s$$

$$P_{N2} = 2W_{o3}(o)B_s = \frac{1}{2} N_o^2 B_{IF} B_s \quad (21)$$

Servo SNR is (22) divided by (23) plus (24):

$$SNR_{PM} = \frac{P_c^2 m^2}{2P_c(1+m^2)N_o B_s + N_o^2 B_{IF} B_s} \left(\frac{A^2 - B^2}{A^2 + B^2} \right)^2 \quad (25)$$

VII Conclusions

Except for the parenthesized term on the far right in (25), the PM case result is identical to that for AM, Eq (18). The parenthesized term represents a loss in SNR due to the presence of the unbalanced quadriphase communication signal. There could also be further SNR reduction caused by data crosstalk effects noted in connection with (21), depending on actual data signal format.

Two extreme cases are of interest in (25). If the quadriphase is balanced rather than unbalanced, then $A = B$ and SNR goes to zero, that is, there is no "signal" (see Eq. 22). At the other extreme $B \rightarrow 0$, meaning the comm signal is biphasic instead of quadriphase. The parenthesized factor in (25) goes to unity. The data crosstalk term in (21) also disappears, as $W_b(f)$ also is zero. Thus with a biphasic data signal, the AM and PM autotrack implementation perform identically, and performance in both cases is unaffected by whether data is being transmitted or not.

ATTACHMENT

Thermal-Induced Pointing Error in Terms of Specifiable Receiver Parameters

Let G and T respectively, represent antenna sum channel power gain (on axis), and system effective noise temperature. Both are referred to the same point, the antenna output terminals. Then noise density N_0 used in the body of this memo can be rewritten as kT where k is Boltzmann's constant. Similarly, P_c can be expressed as $S_I G$, S_I standing for received signal power relative to isotropic.

Next L_Σ and L_Δ can represent all dB losses in sum and difference signal paths, respectively, between the above mentioned reference point and the output of the coupler where the signals are combined. For convenience define numerical attenuation factors

$$\alpha_s = \text{antilog}_{10} (-L_\Sigma / 10) \quad (A1A)$$

$$\alpha_D = \text{antilog}_{10} (-L_\Delta / 10) \quad (A1B)$$

Mod index m was stated to be the ratio of difference to sum channel volts. Define normalized gain slope K_G as ratio of difference pattern slope (volts/degree) to sum pattern on-axis voltage output:

$$K_G = \frac{\Delta'(\theta)}{\Sigma(\theta)} \quad (A2)$$

Then m can be expressed

$$m = \frac{\Delta'(\theta)\epsilon}{\Sigma(\theta)} \left(\frac{\alpha_D}{\alpha_s} \right)^{\frac{1}{2}} = \left(\frac{\alpha_D}{\alpha_s} \right)^{\frac{1}{2}} K_G \epsilon$$

where ϵ is pointing error (degrees).

Pointing error due to thermal noise is computed by letting the mean-square value of (12) equal the sum of (14) and (17). Making the substitutions indicated above, a little algebra leads to mean-square pointing error in (degrees)²:

$$\overline{\epsilon^2} = \frac{2kB_s}{K_G^2 S_I (G/T)} \cdot \frac{\alpha_s}{\alpha_D} \cdot \frac{\left[1 + \frac{kB_{IF}}{2S_I (G/T)} \right]}{\left[1 - \frac{2kB_s}{S_I (G/T)} \right]} \quad (A3)$$

(A3) gives pointing error in one axis only. In a two axis system total MS error would be computed by solving (A3) separately for each axis and adding the two results. In general, normal gain slope K_G defined in (A2) will differ slightly between axes. Otherwise everything is substantially the same.

Sometimes instead of the normalized gain slope K_G a different pattern figure of merit, k_m , is available, related to K_G by

$$k_m = K_G \beta$$

where β is 3-dB beamwidth.

Substituting k_m for K_G in (A3) gives MS pointing error in (beamwidths)² instead of degrees. It can of course be converted back to (degrees)² by multiplying by β^2 .

(A3) is a useful form in that it gives pointing error in terms of fundamental system parameters. In the absence of measured data, a value of k_m between 0.5 and 1 can be used for preliminary estimates. Likewise, reasonable predictions can be made by substituting the coupling loss value (stated numerically) for the quantity α_S/α_D . Thus, for instance for a 15-dB coupler the quantity to use would be $\text{antilog}_{10}(15/10) = 32$.

Finally, (A3) applies to the AM system with square law detection.

APPENDIX B. ESTIMATION OF TARGET RANGE, RANGE RATE, AND DIRECTION

This appendix describes the estimators used to measure the range, range rate, and direction of the target and the errors caused by receiver noise and target scintillation.

It is assumed here that the amplitudes of the target echoes are random quantities having Rayleigh distributions (i. e., Swerling 1 or 2 type targets). Such distributions are typical of many complex radar targets. The noise is assumed to have this same distribution. As a result, the in phase and quadrature components into which the received signals can be resolved are zero-mean gaussian random processes.

As a result of the above assumptions regarding the signal statistics, the statistical character of a pair of signals is fully described by giving the elements of their covariance matrix (i. e., their expected powers and correlation coefficient).

The estimators employed are functions of the sums of samples of the received signals. Such sums of random variables tend to have distributions which approach gaussian distributions as the number of elements in the sum increases even when the random variables being summed have distributions which depart markedly from gaussian distributions. This makes the estimators relatively insensitive to the signal statistics, so that the statistical assumptions are not as restrictive as they would be for some other form of estimator. The use of estimators which are functions of sums of independent samples of the signals suppresses the large errors that some estimators exhibit during fades of the received signals. As a result the estimators employed here have lower variances than estimators that do not use this or some equivalent technique.

The target range, range rate, and direction are measured by estimators which are mathematical duals of each other. In each of the three cases, the information is contained in a pair of voltages which are nominally in phase and whose difference is approximately proportional to the target parameter being measured. In that case, it then follows that the logarithm of the ratio of the expected powers of the two signals is also approximately proportional to the target parameter. An unbiased estimate of this signal parameter (i. e., the logarithm of the ratio of the expected powers) can be constructed by averaging samples of the mean square values of these two

voltages, and then taking the logarithm of the ratio of the two averages. The resulting estimator then has the form

$$\hat{x} = k_x \hat{z} = k_x \log (\underline{U}^t \underline{U} / \underline{V}^t \underline{V})$$

where $\underline{U}$ and $\underline{V}$ are column vectors whose components, U_i and V_i , are samples of the two voltages. $\underline{U}^t$ and $\underline{V}^t$ are the Hermitian conjugates of $\underline{U}$ and $\underline{V}$ respectively. $\hat{x}$ represents one of the estimators. The components, U_i and V_i are complex numbers whose real and imaginary parts are equal to the components of the voltage in phase with and in quadrature with a reference voltage used to heterodyne the received signals to base band prior to the analog-to-digital converter. From the foregoing definitions, it is seen that $\underline{U}^t \underline{U}$ is simply a compact notation for the sum of a sequence of outputs from a square law envelope detector. k_x is a constant of proportionality.

A very similar estimate can be constructed using averages of the amplitudes of the two voltages. Such an estimator has the form

$$\hat{x}_1 = 2 k_x \log \left(\left(\sum_i |U_i| \right) / \left(\sum_i |V_i| \right) \right)$$

Both estimators perform quite similarly since $\underline{U}^t \underline{U}$ and $k_u (\sum_i |U_i|)^2$ are, with an appropriate choice of the constant k_u , both unbiased estimates of the mean square value of the population of samples U_i . However, the standard deviation of $k_u (\sum_i |U_i|)^2$ is about 5 percent larger than that of $\underline{U}^t \underline{U}$. The estimator $\hat{x}$ is much easier to analyze than the estimator $\hat{x}_1$. The analyses presented here are based on the use of $\underline{U}^t \underline{U}$ and $\underline{V}^t \underline{V}$ as estimates of the mean square values of the amplitudes of the two voltages. The difference in the variances of the two estimators is too small to control which form is implemented.

The signals U_i and V_i can be readily recombined in hybrids to yield sum and difference signals or to yield two signals such that the information is contained in a phase difference. These new signals can be used in a variety of estimators. The particular estimator used here has a number of marked advantages. The estimators lend themselves to economical implementations that are insensitive to phase errors. The estimators can be applied to sequential lobing or to monopulse systems of angle estimation. The variances of the estimators are comparable to the best of the available alternates. The estimators used here are somewhat more nonlinear for large errors than some of the alternate available estimators; however, this is relatively unimportant when the design is such that large errors are avoided by tracking the target or other means.

The properties of the estimator $\hat{z}$ are derived in Appendix C. It is shown that the expected value of $\hat{z}$ is a function of the elements of the covariance matrix of U_i and V_i . (These elements are defined later on in this derivation.) Specifically,

$$E(\hat{z}) = \log(P_u/P_v)$$

where P_u and P_v are the expected powers of the signals U_i and V_i . The variance of $\hat{z}$ is shown to be a function of N , the number of independent sample pairs, U_i and V_i , and R , the normalized correlation coefficient of U_i and V_i . In turn, R is a function of the elements of the covariance matrix of U_i and V_i .

A graph of the variance is given in Figure B-1. The variance can also be calculated from

$$\sigma_z^2 = 2(1 - R^2)/(N - b)$$

where b is a number between 0.39 and 1.0. The correct value of b is approximated by the empirical formula

$$b = (0.5 - 0.1/N)(1 + R^2)$$

This value of b yields values of σ_z^2 accurate to 0.001 for N as small as $N = 4$. The error due to the approximate value of b decreases rapidly with increasing values of N . In the applications considered here, the above formula for the variance has negligible error.

The derivation of the probability density of $\hat{z}$, in Appendix C, assumes that the samples U_i and V_i are statistically independent of the samples U_j and V_j when $i \neq j$. When this is not the case, the formulas for the variance of $\hat{z}$ require modification. In particular, in a number of the applications encountered here, the number of independent target echoes is determined by the number frequencies used to illuminate the target; however, the number of independent samples of the noise is determined by the number of sample pairs used in the estimator.

If N_s and N_n are used to denote the number of independent samples of the target echo and the noise respectively, and if $\sigma_z^2(S, N)$ is used to denote the variance for a signal-to-noise power ratio of S and for N independent samples of the noise and echo, then a suitable approximation for the variance is

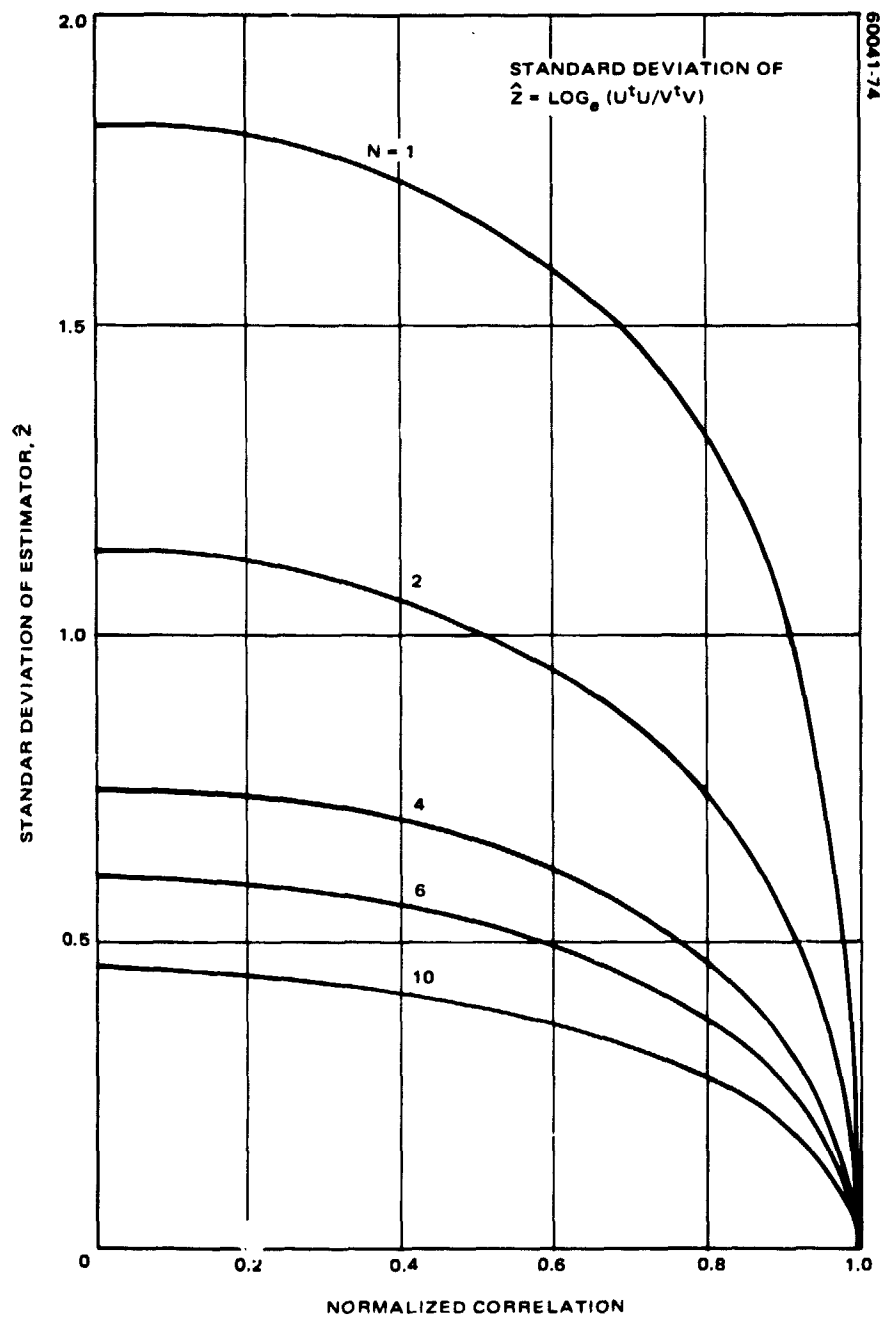


FIGURE B-1. STANDARD DEVIATION OF ESTIMATOR VERSUS
 NORMALIZED CORRELATION

$$\sigma_z^2(S, N_s, N_n) = \sigma_z^2(\omega, N_s) + \frac{\sigma_z^2(S, N_s) - \sigma_z^2(\omega, N_s)}{(N_n - b)/(N_s - b)}$$

In the above equation, $\sigma_z^2(\omega, N_s)$ is the variance due to target scintillation. The second term is the part of the variance due to the noise and the noise-signal beats. It is approximately proportional to the reciprocal of $N_n - b$. The formula is a very close approximation for very high and very low signal-to-noise power ratios and for all signal-to-noise power ratios when $N_n = N_s$.

The elements of the covariance matrix of the signal sample pairs, U_i and V_i , are defined by

$$P_u = E(U_i^* U_i)$$

$$P_v = E(V_i^* V_i)$$

$$P_{uv} = E(U_i^* V_i) = P_{vu}^*$$

$$R^2 = (P_{uv}^* P_{vu}) / (P_u P_v)$$

$E(\quad)$ denotes the expected value of its argument. The superscript $*$ is used to denote the complex conjugate of a quantity. Although P_{uv} is in general complex, it is assumed here that the designer has chosen to make P_{uv} real. Thus $P_{uv} = P_{vu}$. Although P_u , as defined above, is equal to the mean square value of the complex amplitude U_i , it will be referred to as the expected power of the voltage from which the samples U_i are drawn.

Since the noise is assumed to be independent of the target echo, the covariance matrix of the signal (noise plus echo) is the sum of the covariance matrices of the noise and the target echo taken alone. The noise in the two channels is assumed to be independent and to have equal powers. Thus the elements of the covariance matrix of the noise alone are

$$P_u = P_v = P_n$$

$$P_{uv} = P_{vu} = R = 0.0$$

where P_n is the noise power.

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When the target is a point target, the target echo is a product of a possibly random voltage and a deterministic function of the target parameter x (and other elements of the radar range equation).

$$U_i = f_u(x) s_i$$

$$V_i = f_v(x) s_i$$

where s_i is a possibly random voltage. The elements of the covariance matrix are then

$$P_u = P_s (f_u(x))^2$$

$$P_v = P_s (f_v(x))^2$$

$$P_{uv} = P_s f_u(x) f_v(x)$$

where P_s is the expected value of $s_i^* s_i$. When the target consists of scattering areas distributed over a range of values of the target parameter x , it is usually appropriate to add the powers of the echoes returned from the different parts of the target. (This procedure is appropriate when the returns from the different parts of the target are expected to be uncorrelated when the correlation is averaged over the set of samples in the estimate.) In that case the evaluation of the elements of the covariance matrix takes the form of an integral over the extent of the target.

$$P_u = \int (f_u(x))^2 P(x) dx$$

$$P_v = \int (f_v(x))^2 P(x) dx$$

$$P_{uv} = \int (f_u(x) f_v(x)) P(x) dx$$

The quantity $P(x)$ is the power returned per unit of the parameter x .

When $f_u(x)$ and $f_v(x)$ are linear functions of the target parameter x , the elements of the covariance matrix of the target echo can be expressed as functions of the location of the target centroid, x_c , and the radius of gyration, x_r , of the distribution of scattering areas about the centroid. As an example, let

$$U_i = s_i f_u(x) = s_i(1 + dx)$$

$$V_i = s_i f_v(x) = s_i(1 - dx)$$

for a scatterer within the region occupied by the target. Then, integration over the extent of the target yields the following results for the elements of the scattering matrix of noise plus echo:

$$P_u = P_s \left(1 + 2 d x_c + d^2 x_c^2 + d^2 x_r^2 \right) + P_n$$

$$P_v = P_s \left(1 - 2 d x_c + d^2 x_c^2 + d^2 x_r^2 \right) + P_n$$

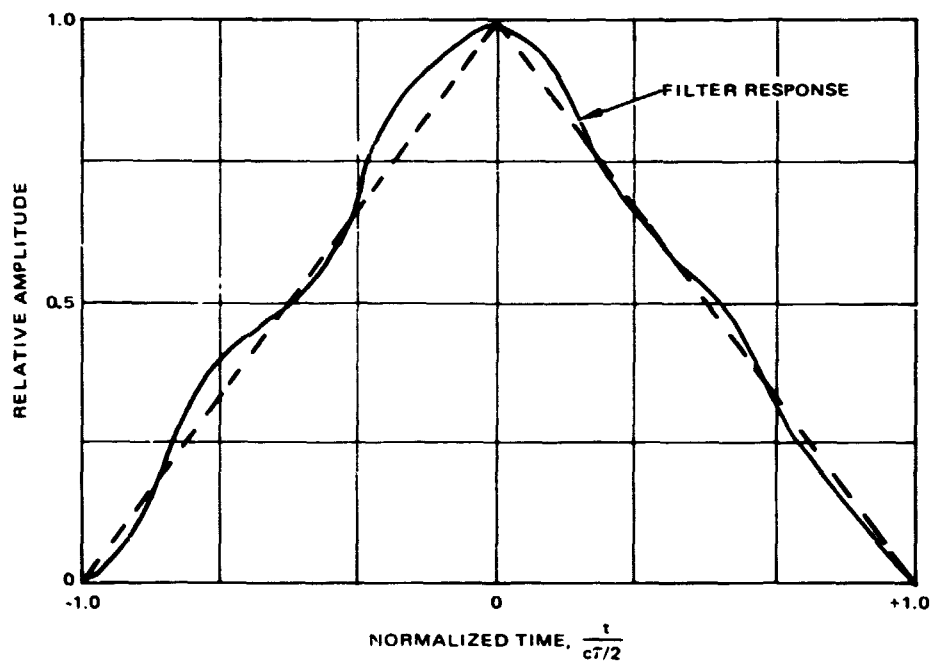
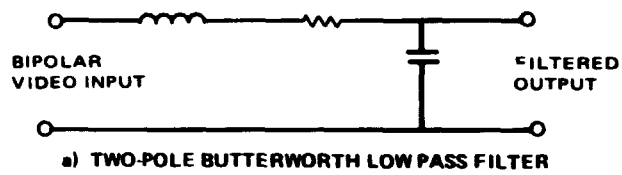
$$P_{uv} = P_s \left(1 - d^2 x_c^2 - d^2 x_r^2 \right)$$

If $k_x = 1/(4d)$, the expected value of the estimator $\hat{x}$ is approximately equal to x_c for small values of x_c , x_r , and P_n/P_s . At very high signal to noise power ratios, the estimation accuracy is limited by the radius of gyration x_r . In that case, when x_c and x_r are small, the variance is found to be approximately equal to

$$\sigma_x^2 = x_r^2 / (2(N - b))$$

ACCURACY OF RANGE ESTIMATOR

In the case of the estimator of the target range relative to the tracking gate, the signals U_i and V_i are early and late samples of the received signals. The two samples are separated in time by a time equal to the transmitted pulse length.



b) PULSE RESPONSE OF LOW-PASS FILTER FOLLOWED BY SUMMING TWO SAMPLES SEPARATED BY ONE-HALF PULSE WIDTH

FIGURE B-2. RESPONSE OF VIDEO FILTERING

The receiver is assumed to be matched to a rectangular pulse. This leads to a triangular response to the transmitted pulse signals. Further, the autocorrelation function of the noise is also triangular and falls to negligible values for times greater than one pulse width apart. Hence, the noise voltages contributing to the early and late gate samples are assumed to be uncorrelated. (This choice is conservative since the presence of some correlation makes the actual errors smaller than the error computed with the assumption that the noise is uncorrelated.) The assumed receiver characteristic is quite close to the performance attainable with relatively simple predetection filtering circuits. As an example, the response shown in Figure B-2 can be obtained using a two-pole low-pass Butterworth filter (also shown in the figure) followed by 2 to 1 digital presampling of two samples separated by one-half a pulse width. The transfer function of the analog filter is

$$F(s) = a^2 / (s^2 + 2as + a^2)$$

$$F(s) = 1 / (LCs^2 + RCs + 1)$$

$$a = 4\pi/T$$

where the pulse width, T , is twice the predetector sampling period, T_s . With this choice of parameters, the aliasing loss (the increase in noise due to aliasing) is about 0.5 dB. Alternate circuit designs of the receiver filters yield the same errors in range if the signal-to-noise ratio is unaffected and the slope of the sampled pulse waveform remains the same at the sampling points.

As a result of the assumptions made, the received voltages are a linear function of the tracking error $t = 2R/c$ when the sum of the error and the target range extent is less than half the transmitted pulse width, T . Hence, for a point target,

$$U_i = (v_i/2) (1 + 2t/T) + N_{ui}$$

$$V_i = (v_i/2) (1 - 2t/T) + N_{vi}$$

where v_i is the peak of the signal echo received in response to the i th illuminating pulse. N_{ui} and N_{vi} are the two samples of the noise voltage. Comparing the above with the results for the estimator $\hat{x}$ it is seen that x is the dual of t , d is the dual of $2/T$, and P_s is the dual of $E(v_i^2/4)$, and P_n is the dual of the expected powers in N_{ui} and N_{vi} . With these identifications, the elements of the covariance matrix become

$$P_u = P_s \left(1 + 4t/T + 4(t_c^2 + t_r^2)/T^2 \right) + P_n$$

$$P_v = P_s \left(1 - 4t/T + 4(t_c^2 + t_r^2)/T^2 \right) + P_n$$

$$P_{uv} = P_s \left(1 - 4(t_c^2 + t_r^2)/T^2 \right)$$

The signal-to-noise power ratio at the peak of the pulse is

$$S = 4 P_s / P_n$$

The dual of k_x is $k_t = T/8$ if the range error is measured in units of time. The estimate can be changed to yield an estimate in units of length by replacing k_t with $k_r = cT/16$. c is the speed of propagation of the radar signal.

ACCURACY OF ESTIMATE OF TARGET DIRECTION

The information on target direction relative to the antenna boresight plane in one of the angular dimensions is contained in the signals received at the sum and one of the difference ports of the antenna. The received voltages are added or subtracted to obtain two sequentially lobed voltages. Samples of the complex amplitudes of these voltages will be denoted by U_i and V_i :

$$U_i = S_i + aD_i + N_i$$

$$V_i = S_{i+1} - aD_{i+1} + N_{i+1}$$

In the above equations S_i and D_i denote the sum and difference signal voltages (i. e., complex amplitudes) at time t_i , N_i denotes the receiver noise voltage, and a is a design parameter ($a = 0.1778$). It is here assumed that the design of the antenna and associated microwave circuits is such that S_i and D_i are in phase when the target has negligible angular extent (i. e., is a point target).

The voltages D_i and S_i depend on the target direction, the antenna pattern voltage gains $G_D(\theta)$ and $G_S(\theta)$, the instantaneous target cross section A_i , and the other parameters in the radar equation. If the product of these other parameters is denoted by the parameter B ,

$$D_i = B \sqrt{A_i} G_D(\theta) G_S(\theta)$$

$$S_i = B \sqrt{A_i} G_S^2(\theta)$$

The above equations assume that the sum antenna pattern is used to illuminate the target. The expected value of A_i is the radar cross section of the target. Near the antenna boresight axis, $G_S(\theta)$ can be considered as constant while $G_D(\theta)$ varies linearly with θ , the angle to the target relative to the direction of the boresight axis. This linear approximation is used in this appendix.

For small values of θ , the antenna design is such that

$$G_D(\theta) = 0.85 G_S(0) \theta$$

when θ is measured in units equal to the antenna half-power beamwidth. Then, for a point target

$$D_i = (0.85)\theta S_i$$

and U_i and V_i become

$$U_i = S_i(1 + d \theta) + N_i$$

$$V_i = S_{i+1}(1 - d \theta) + N_{i+1}$$

$$d = (0.85)a = (0.85)(0.1778) = 0.151$$

For larger angles off the boresight axis, d is not a constant and $d\theta$ should be replaced by $aG_D(\theta)/G_S(\theta)$. Also $G_S(\theta)$ should be used to compute the sum channel signal-to-noise power ratio.

The expected value of $\hat{\theta}$ is given by

$$E(\hat{\theta}) = k_6 \log_c(P_u/P_v)$$

A comparison of the equations defining U_i and V_i in the cases of the estimators $\hat{x}$ and $\hat{\theta}$ reveals that they are duals. That is, x is the dual of θ . Thus the covariance matrix elements are

$$\begin{aligned} P_u &= P_s (1 + 2d \theta_c + d^2 (\theta_c^2 + \theta_r^2)) + P_n \\ P_v &= P_s (1 - 2d \theta_c + d^2 (\theta_c^2 + \theta_r^2)) + P_n \\ P_{uv} &= P_s (1 - d^2 (\theta_c^2 + \theta_r^2)) \end{aligned}$$

At long ranges the target angular extent is negligible. Then, for a target on the boresight axis, R is given approximately by

$$R^2 = S^2 / (S^2 + 2S + 1)$$

where S is the signal-to-noise power ratio in the sum channel. The standard deviation of $\hat{\theta}$ is given by

$$\sigma_{\theta} = \left(2(1 - R^2) / (N_n - b) \right)^{1/2}$$

VELOCITY MEASUREMENT ACCURACY

The velocity measurement accuracy is determined by the accuracy with which the doppler shift of the echo from the target is determined. The possibly ambiguous doppler measurement is made by observing which filters contain the target return and then using the logarithm of the ratio of the signal powers in two adjacent filters to estimate the doppler shift more accurately. Any ambiguities are resolved by utilizing the coarse estimate of target radial velocity that is available in the velocity memory of the range tracker.

The doppler filters are finite impulse response filters formed by adding M weighted samples of the signal spaced T seconds apart. The filter centers are at $m/(MT)$, $m = 0, 1, 2, \dots, M - 1$. The weights all have the same magnitudes. Thus the transfer functions of the filters are

$$F_m(s) = \sum_{n=1}^M \exp(nsT - j2\pi n m/M)/M$$

$$F_m(s) = \exp(jaM) \sin(aM)/(M \sin(a))$$

$$a = \pi(fT - m/M)$$

The doppler filter 4 dB bandwidth is approximately equal to $1/(MT)$. The doppler frequency can be expressed as a fraction of the doppler bandwidth by making the substitution $b = f/(MT) = f/B$. Then, $a = \pi(b - m)/M$ and

$$F_m(s) = \exp(j\pi(b - m)) \sin(\pi(b - m))/(M \sin(\pi(b - m)/M))$$

Let U_i and V_i be the i^{th} sample of N samples of the outputs of two adjacent doppler filters. Then an estimator of the frequency of a signal straddling the two filters is given by

$$\hat{f} = k_f \hat{z} = k_f \log_e (\underline{U}^t \underline{U} / \underline{V}^t \underline{V})$$

The properties of the estimator, $\hat{z}$, are given in Appendix C. It is shown that the estimator is an unbiased estimate of the logarithm of the power out of the two filters. Hence the expected value of the estimator $\hat{f}$ is given by

$$E(\hat{f}) = k_f \log_e \left[\frac{1 + S \{ \sin(h_1)/(M \sin(h_1/M)) \}^2}{1 + S \{ \sin(h_2)/(M \sin(h_2/M)) \}^2} \right]$$

$$h_1 = \pi(y + 0.5)$$

$$h_2 = \pi(y - 0.5)$$

$$k_f = -0.125/(MT)$$

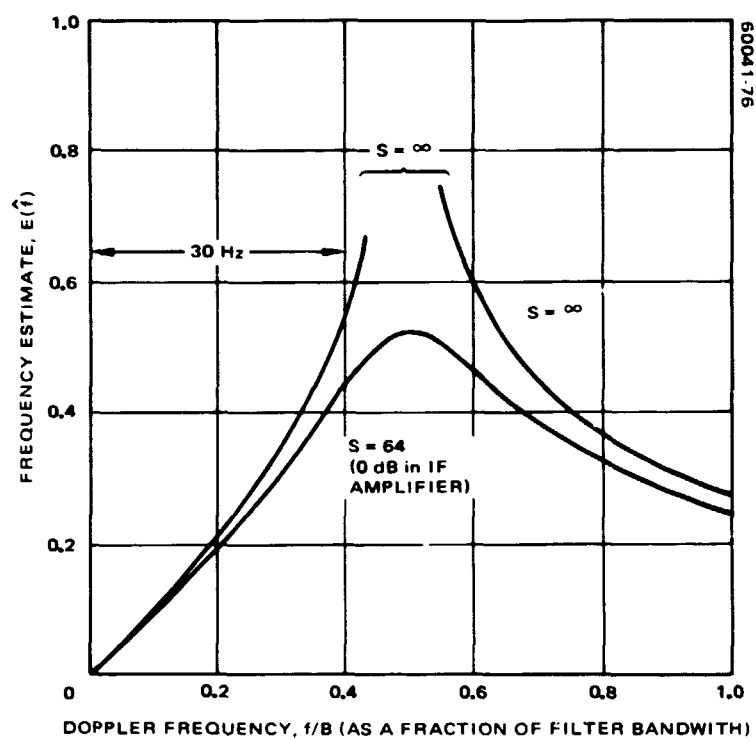


FIGURE B-3. EXPECTED VALUE OF FREQUENCY ESTIMATE
VERSUS DOPPLER FREQUENCY

In the above equation, y is the frequency of the signal expressed as a fraction of the 4 dB doppler filter bandwidth $1/(MT)$ and measured relative to the midpoint between two filters. S is the signal-to-noise power ratio for a signal centered in a doppler filter. The expected value of the estimator, $\hat{f}$, is plotted in Figure B-3 versus y in the same units as y . It is seen that the estimator is approximately linear near the midpoint of the filters provided the signal-to-noise power ratio in the doppler filter is high. However, the departure from linearity becomes quite large when the signal is far from the midpoint between the two filters. It is seen that estimator becomes ambiguous near the center of a doppler filter, the point farthest from the midpoint between the two filters. However, there should be no difficulty in determining the frequency within less than 30 Hz. The estimator exhibits odd symmetry about the midpoint between the two filters. The linearity can be increased by weighting the sample amplitudes to yield somewhat wider filters. Alternatively, it would be easy to construct filters of twice the width using uniform weighting and half the number of samples while still maintaining the same spacing between the filters.

The variance of the estimator, $\hat{z}$, is derived in Appendix C. It is shown to be a function of N , the number of independent values of the signal pairs (U_i and V_i) included in the estimator, and R , the normalized correlation coefficient of the signals, U_i and V_i . For $N \geq 10$, the variance is given by

$$\sigma_z^2 = 2(1 - R^2)/(N - 1)$$

Therefore, the variance of $\hat{f}$ is given by

$$\sigma_f^2 = E(\hat{f}^2) = (1/8) (1 - R^2)/((N - 1) (MT)^2)$$

The normalized correlation coefficient is defined by

$$R^2 = \frac{E(U_i^* V_i) E(U_i V_i^*)}{E(U_i^* U_i) E(V_i^* V_i)}$$

The powers of the two signals, normalized to a noise level of unity, have already been used to compute the expected value of $\hat{f}$. The powers are

$$P_u = E(U_i^* U_i) = 1 + S(\sin(h_1)/(M \sin(h_1/M)))^2$$

$$P_v = E(V_i^* V_i) = 1 + S(\sin(h_2)/(M \sin(h_2/M)))^2$$

$$h_1 = \pi(y + 0.5)$$

$$h_2 = \pi(y - 0.5)$$

For the filter spacing and shape chosen, the correlation of the noise voltages in the two filters is zero despite the fact that the filters overlap. The square of the magnitude of the correlation coefficient of the two signals is found to be

$$E(U_i^* V_i) E(U_i V_i^*) = (P_u - 1) (P_v - 1)$$

Therefore,

$$1 - R^2 = (1/P_u) + 1/P_v - 1/(P_u P_v)$$

The formulas given above apply to a point target whose doppler spectrum has negligible width. When the spectral lines have a finite width, P_u , P_v , the correlation between the signals must be computed as an integral over the spectrum width. This is warranted only for very high signal-to-noise power ratios where the line width of the spectrum is significant. In that case it is found that the variance of the frequency estimator caused by the finite spectral width of the signal is given by

$$\sigma_f^2 = f_r^2 / (2(N - 1))$$

where f_r is the radius of gyration of the spectral lines. The derivation of this result can be obtained by following the similar derivations for the errors in range and angle caused by target scintillation. The error due to the target scintillation can be added to any residual errors caused by noise.

When numerical values are introduced into the equations giving the standard deviation of the estimator, $\hat{f}$, it is found that the errors due to noise are less than about 1 Hz even for IF signal-to-noise power ratios as low as 0 dB. This is due to the large improvement in the signal-to-noise power ratio which results from passing the signals through doppler filters, as shown in Table B-1.

TABLE B-1. RANDOM FREQUENCY ERROR DUE
TO NOISE

(Signal-to-noise power ratio = 0 dB
in IF amplifier)

<u>Doppler Frequency Filter Bandwidth</u>	<u>Standard Deviation, Hz</u>
0.0	0.275
0.1	0.294
0.2	0.357
0.3	0.496
0.4	0.773
0.5	1.021

SCINTILLATION

Scintillation is characteristic of complex targets containing a number of scatterers. When scintillation is present, estimates of target parameters such as target range, direction, and echo doppler shift can fluctuate violently during fades of the sum signal even though the sum signal stays well above the noise level. This is easily understood when the estimate is the ratio of a sample of a difference signal to a sample of a sum signal. However, this scintillation occurs regardless of whether the estimate uses a sum signal. An example of an estimator that does not use a sum signal is given by a phase monopulse angle estimator. With this type of estimator, large errors are associated with phase angles near 180° since a phase measurement of $+180^\circ$ cannot be generally distinguished from one of -180° . This ambiguous situation, though, is the very one that causes the sum signal to fade.

The accuracy can be improved somewhat by averaging a number of statistically independent estimates of the target parameter; however, the accuracy remains relatively low if the average does not give larger weights to data associated with larger sum signals.

The errors can be minimized by combining frequency agility with the use of weighted averages. Frequency agility increases the rate of target scintillation so that more independent data samples can be obtained in the available averaging time.

The weighting method can take a number of forms. Some investigators have reported good results if about half the data — that corresponding to the data having less than the average sum signal strength — is simply discarded. However, statistical estimation theory provides theoretical bounds to the attainable accuracy and leads to algorithms which use more of the data allowing a close approach to the theoretical accuracy.

When such theory is applied to a set of N independent samples of sum and difference signals, the angle estimator takes the form

$$\hat{\theta} = k_{\theta} \frac{\sum_{n=1}^N S_n^* D_n}{\sum_{n=1}^N S_n^* S_n}$$

where the S_n and D_n are samples of the complex amplitudes of the sum and difference voltages. Clearly, a deep fade of a particular sample, S_j , tends to make the contribution of $S_j^* D_j$ and $S_j^* S_j$ to the numerator and denominator small, minimizing the overall error.

The ratio of sums is the maximum likelihood estimate of the ratio P_{SD}/P_S where P_{SD} is the correlation coefficient of the sum and difference signals and P_S is the expected value of the sum signal power. The ratio P_{SD}/P_S is a nearly linear function of target direction for high values of the signal-to-noise power ratio. The variance of $\hat{\theta}$ is given by

$$\sigma_{\theta}^2 = k_{\theta}^2 \frac{C^2}{2(N-1)}$$

where C^2 is a function of the elements of the covariance matrix of the signals. The Cramer-Rao bound, the minimum variance theoretically achievable, is

$$k_{\theta}^2 C^2 / (2N)$$

Good accuracy is not limited to systems using sum and difference signals or even to the algorithms used in the above example. However, alternative formulations use weighted averages and frequency agility. An example of an alternative is

$$\hat{\theta}_L = k_L \hat{Z} = k_L \log_c \left[\frac{\sum U_n^* U_n}{\sum V_n^* V_n} \right]$$

$$U_n = S_n + D_n$$

$$V_n = S_n - D_n$$

$\hat{Z}$ is an unbiased maximum likelihood estimate of $\log_e (P_u/P_v)$ where P_u and P_v are the expected powers of U and V . The variance of $\hat{\theta}_r$ is slightly less than $\hat{\theta}$. For targets of small angular extent, the expected value of $\hat{Z}$ is approximately proportional to the target direction relative to the boresight axis.

APPENDIX C. PROPERTIES OF MAXIMUM LIKELIHOOD

$$\text{ESTIMATOR } \hat{z} = \text{Log}_e (\underline{U}^t \underline{U} / \underline{V}^t \underline{V})$$

$\underline{U}$ and $\underline{V}$ are vectors whose N complex components $U_i, V_i, i = 1, 2, 3, \dots, N$, have amplitudes and arguments equal to the amplitudes and phases of a sequence of statistically independent samples of voltages $u(t)$ and $v(t)$. The real and imaginary parts of these components are Gaussian random variables with zero means. The only non-zero elements of the covariance matrix of these components are

$$\begin{aligned} P_U &= E(U_i^* U_i) \\ P_V &= E(V_i^* V_i) \\ P_{UV} &= E(U_i^* V_i) = P_{VU}^* \\ P_{VU} &= E(V_i^* U_i) \end{aligned} \tag{C-1}$$

Further P_U, P_V, P_{UV} and P_{VU} are assumed to be independent of i . The following additional parameters are useful in simplifying the derivations which follow

$$\begin{aligned} F &= A + jB = P_{UV} / P_V \\ C^2 &= (P_U / P_V) - F^* F \\ D^2 &= A^2 + B^2 = F^* F \\ \rho^2 &= P_{UV}^2 / (P_V P_U) \end{aligned} \tag{C-2}$$

It is also useful to define a vector, $\underline{T}$ given by

$$\underline{U} = F^* \underline{V} + C \underline{T} \tag{C-3}$$

3
This definition makes the component, T_1 , of $\underline{T}$ statistically independent of V_1 but with the same mean square value as V_1 . This is readily shown using C-1, C-2 and C-3 above to compute the expected values of $T_1^* T_1$ and $T_1^* V_1$.

As a first step in finding the properties of $\hat{z}$, the probability density of $z_a = (\underline{U}^t \underline{U}) / (\underline{V}^t \underline{V})$ will be found. This task is simplified by introducing the substitution

$$\underline{U} = \underline{F}^* \underline{V} + \underline{C} \underline{T} \quad (C-4)$$

Then

$$z_a = \underline{F}^* \underline{F} + [\underline{C}^t (\underline{V}^t \underline{T}) + \underline{F}^* \underline{C} \underline{T}^t \underline{V} + \underline{C}^2 \underline{T}^t \underline{T}] / (\underline{V}^t \underline{V}) \quad (C-5)$$

It is clear from the definition of z_a that it is independent of the phase of the reference oscillator. Further since U_1 and V_1 are independent of U_j and V_j when $i \neq j$ the choice of reference oscillator phase may be made separately for each sample pair. It is convenient to choose the reference phases so that all the values of V_1 are real. The coordinates in which the vectors are defined are now rotated so that the N^{th} coordinate axis is parallel to the vector $\underline{V}$. This causes all the components of $\underline{V}$ except V_N to vanish and $V_N^2 = \underline{V}^t \underline{V}$. With this choice of coordinates and phases z_a becomes

$$z_a = \underline{F}^* \underline{F} + \underline{C}^2 \underline{T}^t \underline{T} / (\underline{V}^t \underline{V}) + \underline{C} (\underline{F}^t \underline{T}_N + \underline{F}^* \underline{T}_N^*) / (\underline{V}^t \underline{V})^{1/2}$$

Let

$$\underline{F} = \underline{A} + j \underline{B}$$

$$\underline{T}_N = \underline{x} + j \underline{y}$$

Then, z_a may be arranged to read

$$z_a = C^2(\underline{T}_1^t \underline{T}_1 - \underline{T}_N^* \underline{T}_N) / (\underline{V}_1^t \underline{V}_1) + (A + Cx / (\underline{V}_1^t \underline{V}_1)^{1/2})^2 + (B - Cy / (\underline{V}_1^t \underline{V}_1)^{1/2})^2$$

The value of z_a is a voltage ratio and hence independent of the receiver gains provided all channels have the same gain. It is convenient to choose the gains so that the variance of real and imaginary parts of V_1 (and thus also of T_1) is unity. Then the probability distributions of x , y , R ($R^2 = (\underline{T}_1^t \underline{T}_1 - \underline{T}_N^* \underline{T}_N)$), and Q , ($Q^2 = \underline{V}_1^t \underline{V}_1$) are

$$P_R(R) dR = R^{2N-3} \exp(-R^2/2) dR / (2^{N-2} (N-2)!)$$

$$P_Q(Q) dQ = Q^{2N-1} \exp(-Q^2/2) dQ / (2^{N-1} (N-1)!)$$

$$P_{xy}(x, y) dx dy = \exp(-(x^2 + y^2)/2) dx dy / (2\pi)$$

$$0 < R, Q < \infty$$

$$-\infty < x, y < \infty$$

The joint density of R , Q , x and y is

$$P_1(x, y, R, Q) = \frac{R^{2N-3} Q^{2N-1} \exp(-(x^2 + y^2 + Q^2 + R^2)/2)}{2\pi 2^{2N-3} (N-1)! (N-2)!}$$

The probability density of $W^4 = z_a$ is desired

$$W^4 = C^2 R^2 / Q^2 + (A + Cx/Q)^2 + (B - Cy/Q)^2$$

The desired result may be obtained by finding a transformation to new variables V, W, ϕ, θ and then integrating with respect to V, ϕ and θ . A suitable transformation is

$$Cx = VW \cos \phi \cos \theta - AV/W$$

$$Cy = VW \sin \phi \cos \theta + BV/W$$

$$Cz = VW \sin \theta$$

$$Q = V/W$$

The Jacobian of the transformation is

$$|J| = 2W(V/C)^3 \cos \theta$$

Therefore, the new joint density is

$$P_2(\phi, \theta, V, W) = \frac{V^{4N-1} \cos \theta (\sin \theta)^{2N-3}}{C^{2N} W^{2N-3} \pi (N-1)! (N-2)!} \exp[-V^2/(2C^2) \{W^2 + (A^2 + B^2 + C^2)/W^2 - 2 \cos \theta (A \cos \phi - B \sin \phi)\}]$$

$$0 < V, W < \infty$$

$$0 < \theta < \pi/2$$

$$0 < \phi < 2\pi$$

The above expression can be somewhat simplified by the substitutions

$$\alpha = \phi + \arctan(-B/(-A))$$

$$D = (A^2 + B^2)^{1/2}$$

$$w = W^2$$

$$v = V^2/(2C)$$

Then

$$P_3(\alpha, \theta, w, v) = \frac{2v^{2N-1} \cos \theta (\sin \theta)^{2N-3}}{w \pi (N-1)! (N-2)!} \exp[-(v/C) \{w + (D^2 + C^2)/w + 2D \cos \theta \cos \alpha\}]$$

But

$$\int_0^{2\pi} \exp(-a \cos(x)) dx = 2\pi I_0(a)$$

Therefore integrating with respect to α , the result is

$$P_4(\theta, w, v) = \frac{4v^{2N-1} \cos \theta (\sin \theta)^{2N-3}}{w (N-1)! (N-2)!} \exp[-(v/C) \{w + (D^2 + C^2)\}] I_0(2v(D/C) \cos \theta)$$

With $a = 2v D/C$, the desired integration with respect to θ is

$$I_0 = \int_0^{\pi/2} \cos \theta (\sin \theta)^{2N-3} I_0(a \cos \theta) d\theta$$

Let

$$x = a \cos \theta$$

$$dx = -a \sin \theta d\theta$$

$$(\sin \theta)^{2N-4} = (1 - \cos^2 \theta)^{N-2}$$

$$= \sum_{i=0}^{N-2} \frac{(N-2)! (-x^2/a^2)^i}{i! (N-2-i)!}$$

The integral is then

$$\sum_{i=0}^{N-2} \left[\frac{(-1)^i (N-2)!}{a^{2i+2} i! (N-2-i)!} \int_0^a x^{2i+1} I_0(x) dx \right]$$

The integral can be evaluated by repeated integration by parts using the relation

$$d(z^{m+1} I_{m+1}(z)) = z^{m+1} I_m(z) dz$$

In the first step

$$\begin{aligned} dv &= x I_0(x) dx & u &= x^{2i} \\ v &= x I_1(x) & du &= 2i x^{2i-1} \end{aligned}$$

Thus, denoting the integral over x by I_x ,

$$I_x = a^{2i+1} I_1(a) - \int_0^a x^{2i} I_1(x) dx$$

Continuing the result is

$$I_x = \sum_{k=0}^i \left[\frac{(i)!}{(i-k)!} (-2)^k a^{2i+1-k} I_{k+1}(a) \right]$$

Thus I_θ is

$$I_\theta = \sum_{i=0}^{N-2} \sum_{k=0}^i \left[\frac{(-1)^{i+k} (N-2)! (2/a)^k I_{k+1}(a)}{a(1-k)! (N-2-i)!} \right]$$

Inverting the order of summation yields

$$I_{\theta} = \sum_{k=0}^{N-2} \sum_{i=k}^{N-2} \frac{(-1)^{i+k} (N-2)! (2/a)^k I_{k+1}(a)}{a(1-k)! (N-2-1)!}$$

Now let $i = k+m$.

$$I_{\theta} = \sum_{k=0}^{N-2} \sum_{m=0}^{N-2-k} \frac{(-1)^m (N-2)! (2/a)^k I_{1+k}(a)}{a m! (N-2-k-m)!}$$

But

$$\sum_{m=0}^{N-2-k} \left[\frac{(N-2-k)! (-1)^m}{(N-2-k-m)! m!} \right] = (1-1)^{N-2-k}$$

Hence all the terms vanish for $k \neq N-2$. The only term left is that for $k = N-2$ and $m = 0$. Thus

$$I_{\theta} = (N-2)! (2/a)^{N-2} I_{N-1}(a)/a$$

The density thus becomes

$$P_5(w, v) = \frac{4v^{2N-1} 2^{N-2}}{w(N-1)! a^{N-1}} I_{N-1}(a) \exp \left[-\frac{v}{C} \left\{ W + \frac{D^2 + C^2}{W} \right\} \right]$$

Now make the variable change $v = aC/(2D)$. The result is

$$P_{wa}(w, a) = \left(\frac{C^2}{2D^2} \right)^N \frac{a^N I_{N-1}(a)}{w(N-1)!} \exp \left[-a \frac{W^2 + D^2 + C^2}{2Dw} \right]$$

The integration with respect to a has the form

$$I_a = \int_0^\infty a^N I_{N-1}(a) \exp(-ab) da$$

$$b = w^2 + D^2 + C^2/(2Dw)$$

The Laplace transform of $a^N I_{N-1}(a)$ exists since b is greater than 1. It is

$$I_a = \Gamma(N + \frac{1}{2}) 2^N b(b^2 - 1)^{-N-1/2} / \Gamma(\frac{1}{2})$$

Therefore

$$P_w(w) dw = \left(\frac{C^2}{D^2}\right)^N \frac{\Gamma(N + \frac{1}{2}) b dw}{\Gamma(\frac{1}{2}) (N-1)! w(b^2 - 1)^{N+1/2}}$$

But

$$b = (w^2 + D^2 + C^2)/(2Dw)$$

Therefore

$$P_w(w) dw = \frac{C^{2N} 2^{2N} w^{2N-1} \Gamma(N + \frac{1}{2})}{\Gamma(\frac{1}{2}) (N-1)!} \frac{(w^2 + C^2 + D^2) dw}{((w^2 + C^2 + D^2)^2 - 4w^2 D^2)^{N+1/2}}$$

The distribution of $U = w^2 = W^4$ is thus

$$P_U(U) dU = \frac{C^{2N} 2^{2N-1} \Gamma(N + \frac{1}{2})}{\Gamma(\frac{1}{2}) (N-1)!} \frac{U^{N-1} (U + C^2 + D^2) dU}{[(U + C^2 - D^2)^2 + 4C^2 D^2]^{N+1/2}}$$

It is convenient to let

$$x = \frac{P_V}{P_U} U$$

Then using the definitions of C, D and ρ ,

$$P_x(x) dx = \frac{(2N-1)!}{[(N-1)!]^2} (1 - \rho^2)^N \frac{x^{N-1}(x+1) dx}{[(x+1)^2 - 4\rho^2 x]^{N+1/2}}$$

The expected values of $\hat{z} = \log_e U$, $(\hat{z})^2$ and the variance of $\hat{z}$, σ_z^2 are given by

$$E(\hat{z}) = E(\log_e U) = \log_e \frac{P_U}{P_V} + E(\log_e x)$$

$$E(\hat{z}^2) = (\log_e \frac{P_U}{P_V})^2 + 2 \log_e \frac{P_U}{P_V} E(\log_e x) + E((\log_e x)^2)$$

$$\sigma_z^2 = E(\hat{z}^2) - E^2(\hat{z})$$

It is easily shown that $E(\log_e x)$ is zero so that $\hat{z}$ is an unbiased estimator of $\log_e P_U/P_V$. Further this estimate maximizes the likelihood function of $\hat{z}$.

$$E(\hat{z}) = \log_e \frac{P_U}{P_V}$$

$$\sigma_z^2 = E((\log_e x)^2)$$

To show the first result let $x = e^y$ and then multiply numerator and denominator by $\exp(-y(N+1/2))$. The result is

$$\begin{aligned} E((\log_e x)^a) &= \int_0^\infty (\log_e x)^a P(x) dx \\ &= \int_{-\infty}^\infty A y^a \frac{(e^{y/2} + e^{-y/2}) dy}{[e^y + e^{-y} + 2 - 4\rho^2]^{N+1/2}} \end{aligned}$$

$$A = (1 - \rho^2)^N (2N-1)! / ((N-1)!)^2$$

The integral must exist for positive $N \geq 1$ and $1 - \rho^2 > 0$ for then the integral is bounded by a constant times $y^2 e^{-|yN|}$. If $a = 1$ the integrand is an odd function of y and thus the integral must vanish.

$$E(\log_e x) = 0$$

When $a = 2$ the integral is even and may be evaluated numerically.

$$E((\log_e x)^2) = 2 \int_0^{\infty} Ay^2 \frac{(1 + e^{-y}) e^{-Ny} dy}{[e^{-2y} + (2 - 4\rho^2) e^{-y} + 1]^{N+1/2}}$$

The results of such an evaluation are given in Figure B-1, for N equal to 1, 2, 4, 6, and 10. When N is equal to or greater than 10, the variance of the estimator z is given approximately by

$$\sigma_z^2 = 2 (1 - R^2) / (N - 1)$$

APPENDIX D. CROSSTALK ON TDRS FORWARD DATA LINK CAUSED BY TRANSMISSION DEVIATIONS AND IMPLEMENTATION OFFSETS

Introduction

A contemplated TDRS forward data link implementation would send two simultaneous independent binary data streams, by bi-phase modulating two carriers at the same frequency but orthogonal in phase. Bit rates of the two data streams would differ widely from each other and there also would be a substantial difference between power levels of the two carriers. This memo investigates crosstalk between the two data streams due to various imperfections, and provides means for estimating the degree to which crosstalk might degrade BER performance.

Figure D-1 shows idealization of the two received data streams, i.e., low and high data rate, in the absence of thermal noise. Each bit stream has superimposed on it a small distorted replica of the cross-channel bit stream. These cross-channel streams will interfere with bit decisions that are to be made on the in-channel signal and thus constitute performance degradation.

From an analytical viewpoint, the important characteristic of Figure D-1 is the large ratio between bit rates in the two streams, (up to 40:1 with presently-conceived signal design). This large discrepancy is crucial to special analytical approaches devised for use in the present study to compute crosstalk effects. A case in point is high data rate crosstalk appearing in the low-rate channel. It is clear that each low-rate bit decision will include crosstalk effects from a large number of high-rate bits of random polarity. Because of the large number of individual contributors involved, the net effect of crosstalk can be assumed of least reasonably to approximate that of gaussian noise. This means that only the mean (d-c) and variance of the crosstalk need be computed in order to know its effect on low data rate BER.

The high data rate BER is somewhat different. Here crosstalk is from the low-rate channel. The crosstalk appears mainly as a low level square wave of long period, riding on a small d-c. Each polarity persists during many high data rate bit decisions. The effect of crosstalk thus is primarily to bias high-rate decision by a "d-c" offset that can assume, with equal likelihood, one of two different values. This simple arrangement becomes upset every so often when the low rate bit stream switches from one polarity to the other, causing a transient to occur during the bit decision interval of a high-rate bit. The relative infrequency of these events makes it possible either to ignore them completely or to compute their effect using simplifying assumptions, without greatly affecting accuracy of the overall results.

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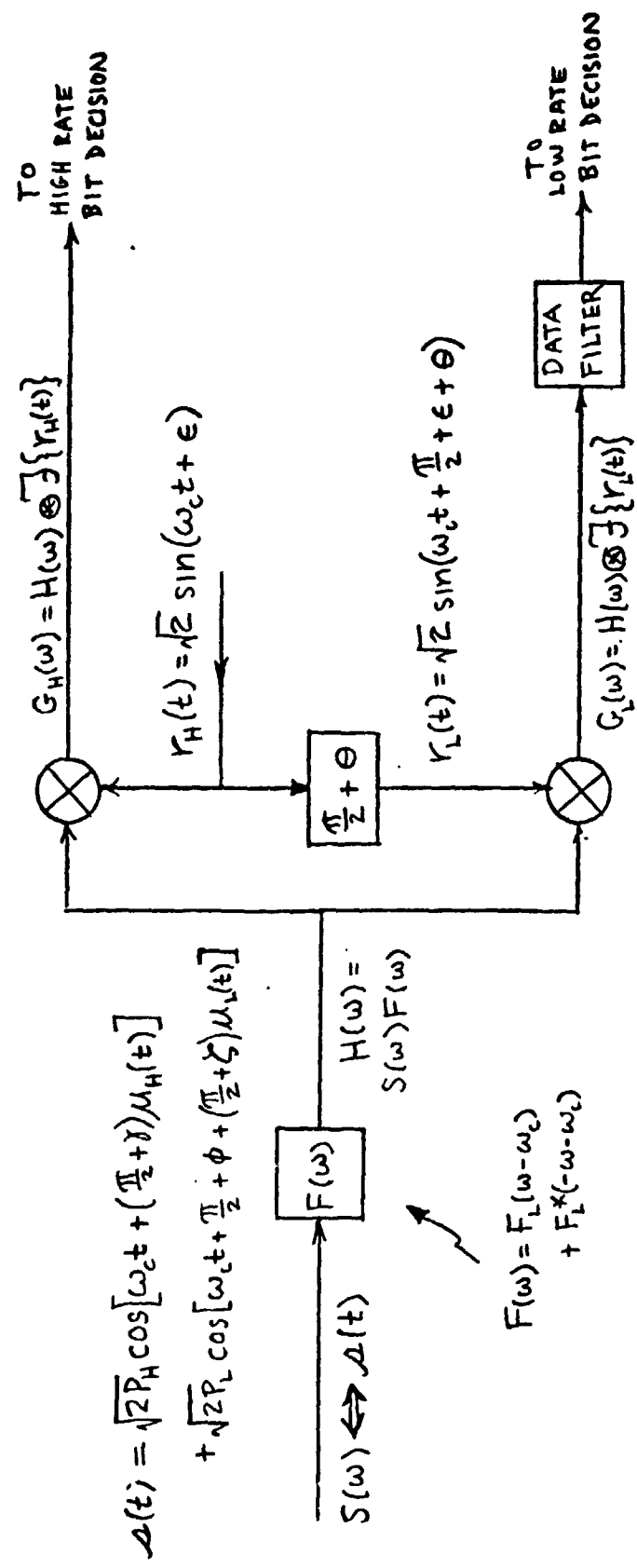


Figure D-1. Unbalanced Quadriphase Transmission Model

Analysis

I. Analytical Model

The system to be analyzed is modeled in Figure D-2. A bandpass filter $F(\omega)$ represents spectral shaping that may occur in the transmission path. It can be represented in terms of a lowpass equivalent $F_L(\omega)$ for analytical convenience. One of the present objectives is to see what effect various assumed shapes for $F_L(\omega)$ will have on crosstalk. Any departures of $F_L(\omega)$ from constant amplitude-vs-frequency or linear phase-vs-frequency are referred to as transmission deviations.

The Figure D-2 model includes a number of implementation imperfections or offsets. Thus, the two data carriers, ideally orthogonal, actually are $\pi/2 + \theta$ apart, where $\theta \neq 0$ represents an implementation error. At the receive end, the two references also miss orthogonality by θ radians, and furthermore, are rotated ϕ radians from where they ought to be. Finally, the carrier mod indices are off from the ideal of $\pm \pi/2$ radians by the amounts γ and ζ radians respectively.

The input signal $s(t)$ can be written

$$s(t) = \sqrt{2P_H} \operatorname{Re}\{m_H(t) e^{j\omega_c t}\} + \sqrt{2P_L} \operatorname{Re}\{m_L(t) e^{j\omega_c t}\}$$

where $m_H(t)$ and $m_L(t)$ are complex envelopes:

$$m_H(t) = e^{j(\frac{\pi}{2} + \gamma)} u_H(t)$$

$$m_L(t) = e^{j\left[\left(\frac{\pi}{2} + \zeta\right)u_L(t) + \frac{\pi}{2} + \theta\right]}$$

$u(t)$ is a binary data stream and the H and L subscripts denote high and low-rate channels respectively.

Then the spectrum of $s(t)$ can be expressed* as

$$S(\omega) = \sqrt{\frac{P_H}{2}} \left[M_H(\omega - \omega_c) + M_H^*(-\omega - \omega_c) \right] + \sqrt{\frac{P_L}{2}} e^{j(\frac{\pi}{2} + \theta)} \left[M_L(\omega - \omega_c) + M_L^*(-\omega - \omega_c) \right] \quad (1)$$

where $M_H(\omega)$ and $M_L(\omega)$ are the Fourier transforms of $m_H(t)$ and $m_L(t)$:

* Stein & Jones, "Modern Communication Principles," Chapter 3

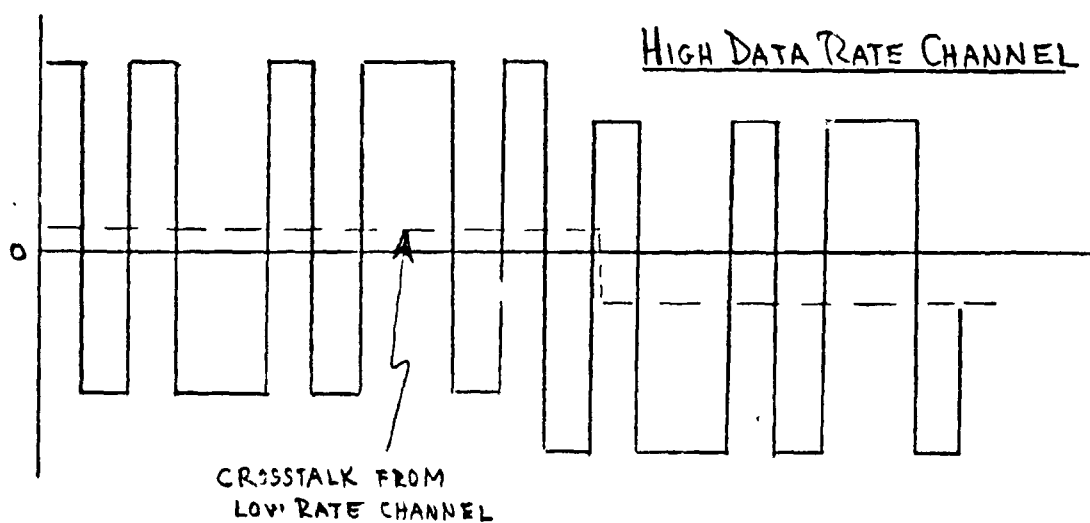
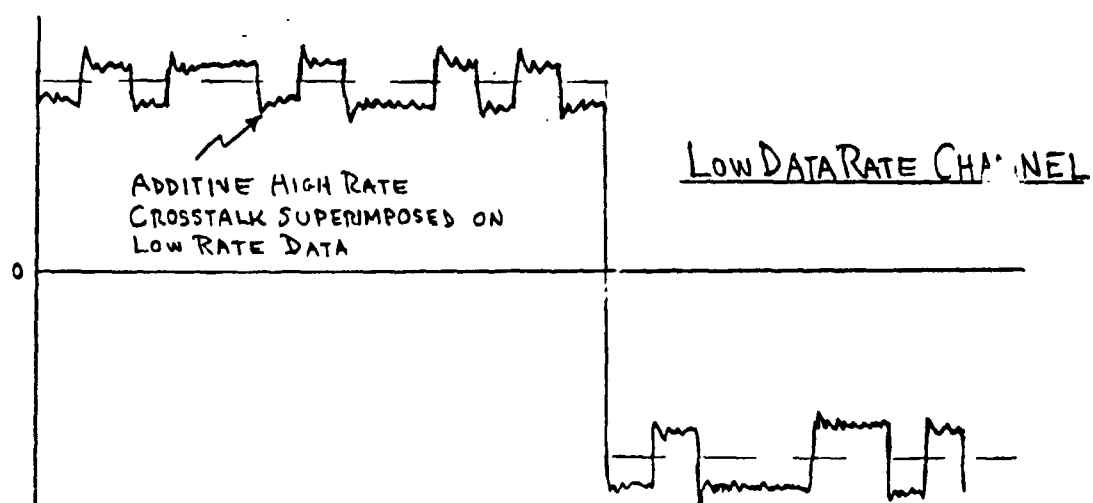


Figure D-2. Data and Crosstalk Waveforms

$$M_H(\omega) = \int_{-\infty}^{\infty} m_H(t) e^{-j\omega t} dt = -\sin \gamma \delta(\omega) + j \cos \gamma U_H(\omega)$$

$$M_L(\omega) = [-j \sin \zeta \delta(\omega) - \cos \zeta U_L(\omega)] e^{j\theta}$$

In the above, $U_i(\omega) \Leftrightarrow u_i(t)$ for $i = H, L$. Also, since $U_i(t)$ is real

$$U_i^*(-\omega) = U_i(\omega).$$

Starting with expression (1) for $S(\omega)$ and carrying through the operations indicated in Figure D-2 leads to expressions for the spectra of the signals at the high-rate and low-rate channel demodulator outputs:

$$G_H(\omega) = P_H^{\frac{1}{2}} \left\{ -\sin \gamma \sin \delta(\omega) - \cos \gamma U_H(\omega) \left[\frac{e^{j\epsilon} F_L^*(-\omega) + e^{-j\epsilon} F_L(\omega)}{2} \right] \right. \\ \left. + P_L^{\frac{1}{2}} \left\{ \sin \zeta \cos(\epsilon - \theta) \delta(\omega) + \cos \zeta U_L(\omega) \left[\frac{e^{j(\epsilon - \theta)} F_L^*(-\omega) - e^{-j(\epsilon - \theta)} F_L(\omega)}{2j} \right] \right\} \right\} \quad (2)$$

$$G_L(\omega) = P_H^{\frac{1}{2}} \left\{ -\sin \gamma \cos(\epsilon - \theta) \delta(\omega) + \cos \gamma U_H(\omega) \left[\frac{e^{j(\epsilon + \theta)} F_L^*(-\omega) - e^{-j(\epsilon + \theta)} F_L(\omega)}{2j} \right] \right\} \\ - P_L^{\frac{1}{2}} \left\{ \sin \zeta \sin(\epsilon + \theta - \theta) \delta(\omega) + \cos \zeta U_L(\omega) \left[\frac{e^{j(\epsilon + \theta - \theta)} F_L^*(-\omega) + e^{-j(\epsilon + \theta - \theta)} F_L(\omega)}{2} \right] \right\} \quad (3)$$

II. Crosstalk in Low-Data Rate Channel

In Eq. (3) the first half of the right hand side represents crosstalk from the high-rate channel. Though complicated by the presence of numerous interacting parameters, the expression consists simply of a d-c offset (the delta function) plus a low-level replica of the high data rate bit stream, represented by the spectrum $U_H(\omega)$. The spectrum is distorted by multiplication with the bracketed term.

A. Crosstalk due to Transmission Deviations

A feel for the implications of (3) is best obtained by considering various terms separately. Initially all the implementation offsets θ , ϵ , γ , etc. will be assumed zero. The crosstalk remaining then will be due only to transmission deviations. The crosstalk power spectrum will be denoted $X_L(\omega)$. $X_L(\omega)$ is the squared magnitude of the Eq (3) crosstalk expression. With the foregoing assumption of "perfect" implementation it is seen that

$$X_L(\omega) = P_H \left| U_H(\omega) \right|^2 F_{DEV} \tag{4}$$

where

$$F_{DEV} = \left[F_L^*(-\omega) - F_L(\omega) \right] \left[F_L(-\omega) - F_L^*(\omega) \right] / 4$$

Crosstalk power due to any (lowpass equivalent) transmission characteristic $F_L(\omega)$ can be computed using (4). $F_L(\omega)$ could be a measured channel frequency response. Alternatively, a "classical" filter shape, Tschebyshev, Bessel, etc., could be tried. Finally, a third possibility, the one to be pursued here, is to let $F_L(\omega)$ take the form of various idealized amplitude and phase distortions by which transmission deviations frequently are specified, i.e., parabolic and cubic phase, phase and amplitude ripple, and amplitude tilt.

These are listed in Table D-1 below along with the resulting F_{DEF} for each. F_{DEV} is given first in terms of the general proportionality constants (a's and k's) and in the last column these constants have been re-stated in terms more commonly used in specifications.

TABLE D-1. SPECTRAL SHAPING FACTOR, F_{DEV} DUE TO VARIOUS DEVIATIONS

Deviation Type	$F_L(\omega)$	F_{DEV}	F_{DEV} in Terms of Spec Values
Parabolic phase	$\exp \left(-jk_2\omega^2 \right)$	$\sin^2 k_2\omega^2$	$\sin^2 \left(\frac{r_2^2 \omega^2}{4\pi^2 R_H^2} \right)$
Cubic Phase	$\exp \left(-jk_3\omega^3 \right)$	0	0
Phase ripple	$\exp \left[ja_p \cos(k_p\omega + \theta_p) \right]$	$a_p^2 \sin^2 \theta_p \sin^2 k_p\omega$	$a_p^2 \sin^2 \left(\frac{N_{PR}\omega}{2R_H} \right)$ *
Amplitude ripple	$1+a_A \cos(k_A\omega + \theta_A)$	$a_A^2 \sin^2 \theta_A \sin^2 k_A\omega$	$\frac{R_{DB}}{300} \sin^2 \left(\frac{N_{AR}\omega}{2R_H} \right)$ *
Amplitude tilt	$1+k_T\omega$	$k_T^2 \omega^2$	$\frac{T_{DB}}{12100} \left(\frac{\omega}{R_H} \right)^2$

* Assuming $\theta_p = \theta_A = \pi/2$ (worst case)

TABLE D-1. (Continued)

R_H	"High" bit rate, b/s
r_2	Parabolic phase deviation $\pm R_H$ from band center, rad.
a_p	Phase ripple amplitude, rad p-p. (Assumed $\leq 1/2$)
N_{PR}	No. of cycles of phase ripple across $\pm R_H$ passband.
R_{DB}	Amplitude ripple in dB, p-p
N_{AR}	No. of cycles of amplitude ripple across $\pm R_H$ passband.
T_{DB}	DB amplitude tilt across $\pm R_H$ passband

The crosstalk power spectrum X_L given by (4) will be shaped by a lowpass data filter before reaching the point where it actually affects bit decisions. The effective crosstalk power N_{XL} will be

$$N_{XL} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_L(\omega) \left| F_{DL}(\omega) \right|^2 d\omega \quad (5)$$

where F_{DL} is frequency response of the data filter (in the low data rate channel).

It is of interest to estimate the ratio of effective crosstalk power N_{XL} to signal power P_L in typical situations, as this provides one measure of the degrading effect of crosstalk. From (4) and (5) the ratio is

$$\frac{N_{XL}}{P_L} = \frac{R_{PD}}{2\pi} \int_{-\infty}^{\infty} \left| U_H(\omega) \right|^2 \left| F_{DL}(\omega) \right|^2 F_{DEV} d\omega \quad (6)$$

where R_{PD} ("power division ratio") is ratio between high-rate signal power P_H and low-rate signal power P_L .

Exact values for crosstalk power obviously depend on many factors including data filter shape plus many parameters of the various F_{DEV} expressions. Useful insight however can be obtained by means of quite crude approximations that simplify computation immensely. One such approximation is to substitute for F_{DL} an ideal lowpass filter cutting off at frequency R_L (the low-rate channel bit rate).

For $U_H(\omega)$ two possibilities will be considered. One assumes the high-rate bit stream to be a random series of 1's and 0's of equal probability, leading to an isosceles-triangle autocorrelation function, and power spectrum:

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$$W_{RB}(\omega) = \left| U_{H_{RB}}(\omega) \right|^2 = \frac{1}{R_H} \frac{\sin^2\left(\frac{\omega}{2R_H}\right)}{\left(\frac{\omega}{2R_H}\right)^2} \quad (7)$$

where the subscript RB indicates "random binary."

Because of the assumed high ratio between R_H and R_L (high and low bit rates) only the central portion of (7) will be involved in the integration (6). Thus (7) can be approximated simply by

$$W_{RB}(\omega) \approx 1/R_H \quad (8)$$

The other bit-stream possibility will be split-phase or Manchester encoding whereby

$$W_M(\omega) = \frac{1}{R_H} \frac{\sin^4\left(\frac{\omega}{4R_H}\right)}{\left(\frac{\omega}{4R_H}\right)^2} \quad (9)$$

Again only the central portion will be involved, leading to the approximation

$$W_M(\omega) \approx \frac{\omega^2}{16R_H^3} \quad (10)$$

Crosstalk estimates for various deviations are obtained below.

Parabolic Phase

Again invoking the $R_L \ll R_H$ assumption, the sine term for reasonable r_2 (say $r_2 \leq 1$) can be replaced by its argument. Thus the approximation of (6):

$$\frac{N_{XL}}{P_L} = 2R_{PD} \int_0^{R_L} W_k(f) \frac{r_2^2 f^4}{R_H^4} df \quad k = RB, M$$

from which

$$\text{Random Binary:} \quad \frac{N_{XL}}{P_L} = \frac{2}{5} R_{PD} r_2^2 \left(\frac{R_L}{R_H} \right)^5 \quad (11)$$

$$\text{Manchester case:} \quad \frac{N_{XL}}{P_L} = \frac{\pi^2}{14} R_{PD} r_2^2 \left(\frac{R_L}{R_H} \right)^7 \quad (12)$$

Given no more than one radian of parabolic phase r_2 , a power division ratio not exceeding 10 dB, and data rate ratio R_H/R_L at least 10:1, (11) says crosstalk will be in the neighborhood of 40 dB down, and in (12) 60 dB down. Given gaussian amplitude statistics this would make crosstalk "noise" quite negligible.

Amplitude Tilt

$$\frac{N_{XL}}{P_L} = 2 R_{PD} \int_0^{R_L} W_k(f) \frac{4\pi^2 f^2 T_{DB}^2}{12100 R_H^2} df$$

$$\text{Random Binary: } \frac{N_{XL}}{P_L} = \frac{1}{460} R_{PD} T_{DB}^2 \left(\frac{R_L}{R_H} \right)^3 \quad (13)$$

$$\text{Manchester: } \frac{N_{XL}}{P_L} = \frac{1}{310} R_{PD} T_{DB}^2 \left(\frac{R_L}{R_H} \right)^5 \quad (14)$$

With the previous constraints on R_H/R_L and power division ratio R_{PD} , crosstalk due to a dB or two or amplitude tilt again will be about 40 dB down in the first case and 60 dB down in the other.

Amplitude Ripple

From Table D-1:

$$F_{DEV} = \frac{R_{DB}^2}{300} \sin^2 \left(\frac{N_{AR} \omega}{2R_H} \right)$$

which assumes worst-case phasing of the ripple relative to band center. Rough estimates of crosstalk can be obtained for two extreme cases. One extreme is for ripple period to be great relative to low rate data filter bandwidth R_L . Then the sine term again can be replaced by its argument, and

$$\frac{N_{XL}}{P_L} = 2R_{PD} \int_0^{R_L} W_k(f) \frac{R_{DB}^2 N_{AR}^2 \omega^2}{300 R_H^2} df$$

$$\text{Random Binary: } \frac{N_{XL}}{P_L} = \frac{1}{50} R_{PD} R_{DB}^2 N_{AR}^2 \left(\frac{R_L}{R_H} \right)^3 \quad (15)$$

$$\text{Manchester: } \frac{N_{XL}}{P_L} = \frac{1}{300} R_{PD} R_{DB}^2 N_{AR}^2 \left(\frac{R_L}{R_H} \right)^5 \quad (16)$$

(15) and (16) hold only for small numbers of ripples, the number depending on the R_H/R_L ratio. For $R_H/R_L = 10$, $N_{AR} \approx 1$ or 2, maximum. Then for a dB or two of ripple, crosstalk would be 20-30 dB down in the first case and 50-60 dB down in the other.

At the other extreme the number of ripple cycles, N_{AR} , could be large, such that one or more complete cycles lie within the $0-R_L$ Hz passband of the data filter. In this case the $\sin^2$ term can be crudely approximated by the number $1/2$:

$$\frac{N_{XL}}{P_L} = \frac{2R_{PD} R_{DB}^2}{300} \cdot \frac{1}{2} \int_0^{R_L} w_k(f) df$$

$$\text{Random Binary: } \frac{N_{XL}}{P_L} = \frac{1}{300} R_{PD} R_{DB}^2 \left(\frac{R_L}{R_H} \right) \quad (17)$$

$$\text{Manchester: } \frac{N_{XL}}{P_L} = \frac{1}{360} R_{PD} R_{DB}^2 \left(\frac{R_L}{R_H} \right)^3 \quad (18)$$

In (17) 2 dB of ripple could produce crosstalk noise less than 20 dB down, a level capable of noticeable performance impact. This result is only a rough estimate. It does indicate the desirability of a more detailed analysis using actual system parameters should a system of this type be contemplated.

With Manchester coding, (18), estimated crosstalk is down some 40 dB (assuming 2 dB p-p ripple), as compared to the 60 dB found when N_{AR} was small. For intermediate values of N_{AR} , intermediate amounts of crosstalk i.e., 40-60 dB down, would be expected, indicating negligible performance impact. These are maximum values since worst-case phasing was assumed, as noted at the outset.

Phase Ripple

The shaping factor F_{DEV} for phase ripple has the same form as for amplitude (Table D-1). Therefore, the same crosstalk expressions result, except only that a_p^2 is substituted for $R_{DB}^2/300$. Worst case results (N_{PR} large):

$$\text{Random Binary: } \frac{N_{XL}}{P_L} = a_p^2 R_{PD} \left(\frac{R_L}{R_H} \right) \quad (19)$$

$$\text{Manchester: } \frac{N_{XL}}{P_L} = \frac{a_p^2}{1.2} R_{PD} \left(\frac{R_L}{R_H} \right)^3 \quad (20)$$

(19) shows crosstalk noise could be a problem. For instance, if the power division ratio R_{PD} were to be selected so as just to compensate for bit rate difference, crosstalk noise would be down only by the factor a_p^2 . A typical value for a_p might be 1/4 radian (phase counterpart of a 2-dB amplitude ripple p-p). In such a case N_{XL} would only be 12 dB below P_L , which could have serious performance implications. Again all this really indicates is that a more careful analysis of this case, e.g., starting over from (6) would be warranted in the event of an actual system design.

In (20), the cubic relationship suppresses crosstalk an additional 20 dB or more ($R_H/R_L \geq 10:1$) relative to that in (19), providing reasonable assurance that crosstalk would not be a problem here.

B. Crosstalk Due to Implementation Offsets

The foregoing assumed no implementation offsets. The effects of such offsets now will be considered, with the complementing assumption of ideal transmission. The latter implies $F_L(\omega) = A e^{-jK\omega}$, i.e., flat amplitude and constant delay. The trivial constants A and K , further will be set equal to 1 and 0 respectively for convenience. Then the crosstalk portion of (3) becomes

$$G_{XL}(\omega) = P_H^{\frac{1}{2}} \left[-\sin \gamma \cos (\epsilon + \theta) \delta(\omega) + \cos \gamma \sin (\epsilon + \theta) U_H(\omega) \right]$$

or as a function of time

$$g_{XL}(t) = A_H + B_H u_H(t)$$

where

$$A_H = -P_H^{\frac{1}{2}} \sin \gamma \cos (\epsilon + \theta) \quad (21)$$

$$B_H = P_H^{\frac{1}{2}} \cos \gamma \sin (\epsilon + \theta) \quad (22)$$

Crosstalk thus consists of a d-c voltage given by (21) plus a low-level replica of the high-rate data stream.

Similarly, the desired signal in the low data rate channel is, from (3):

where

$$g_{SL} = A_L u_L(t) \quad (23)$$

$$A_L = -P_L^{\frac{1}{2}} \cos \zeta \cos (\epsilon + \theta - \phi) \quad (24)$$

In (23) a $\sin \zeta \sin (\epsilon + \theta)$ term is omitted on grounds of being the product of two small numbers, for parameter range of interest, and therefore, negligible.

At this point it is convenient to switch data filter models, using an integrate-and-dump instead of the ideal LPF previously considered. Also, attention will be confined for the moment to the case where $u_L(t)$ is the random binary type of signal. The I-D filter just before dumping at the end of one $1/R_L$ integration period has a voltage of

$$V_{RB} = \int_{R_L^{-1}}^{R_L^{-1}} [g_{SL}(t) + g_{XL}(t)] dt = \frac{A_H \pm A_L}{R_L} + B_H \int_{R_L^{-1}}^{R_L^{-1}} u_H(t) dt \quad (25)$$

The high-rate bit stream $u_H(t)$ is assumed to have a mean of zero (equal 1's and 0's on the average). Therefore, the mean of V_{RB} will be just the first part of (25) on the right:

$$\bar{V}_{RB} = \frac{A_H \pm A_L}{R_L} \quad (26)$$

The $\pm$ sign depends on whether the low rate bit $u_L(t)$ was a 1 or 0 during the particular $1/R_L$ interval integrated.

Bit decisions are based on whether V_{RB} is positive or negative. Errors result from the fact that V_{RB} fluctuates around its average value (26). Part of the fluctuation is due to the last term in (25) and part due to thermal noise. It is known that the latter, given an input power density of N_0 , causes a variance

$$\sigma_N^2 = N_0 / 2T \quad (27)$$

The variance due to crosstalk furthermore is shown in the Attachment to equal (random binary case)

$$\sigma_{XRB}^2 = B_H^2 / R_H R_L \quad (28)$$

It is at this point that the key assumption is invoked, namely that V_{RB} , Eq (25) (whose variance is given by (28)), is normally distributed. Given this, bit error probability is known to be

$$P_{E_{RB}} = \frac{1}{2} \operatorname{erfc} \left[\frac{|\bar{V}_{RB}|}{\left[2 (\sigma_N^2 + \sigma_{XRB}^2) \right]^{\frac{1}{2}}} \right] \quad (29)$$

where

$$\operatorname{erfc}(X) = 1 - \frac{2}{\sqrt{\pi}} \int_0^X e^{-u^2} du \quad (30)$$

(29) applies to random binary data. With Manchester-coding, the I-D filter is preceded by a "decoding" operation that brings both halves of each split-phase signalling element back to the same polarity. In the process, the cross-talk -induced d-c offset A_H given by (21) is eliminated. Thus the decision variable average, Manchester case, is

$$\bar{V}_M = \pm A_L / R_L \quad (31)$$

and the variance (Attachment) is approximately

$$\sigma_{XM}^2 = B_H^2 / 6R_H^2 \quad (32)$$

Error probability with Manchester coding is found by substituting (31) and (32) for the corresponding RB-subscripted terms in (29)

Because of the number of parameters involved, a comprehensive examination of degradation will not be attempted. Useful insight can be obtained however, by making certain approximations. By restricting the errors γ , ϵ , θ , etc. to small values, say 0.1 or less, the cosine terms in (21)-(24) can be set to unity and the sine terms replaced by their arguments without excessive error. In this case (29) can be manipulated to

Random Binary:

$$P_E = \frac{1}{2} \operatorname{erfc} \left[\left(\frac{E_b (1 \pm 2\gamma R_{PD})^{\frac{1}{2}}}{N_o + 2E_b \frac{R_{PD}}{R_{BR}} (\theta + \epsilon)^2} \right)^{\frac{1}{2}} \right] \quad (33)$$

Manchester:

$$P_E = \frac{1}{2} \operatorname{erfc} \left[\left(\frac{\bar{E}_b}{N_o + E_b \frac{R_{PD}}{3R_{BR}^2} (\theta + \epsilon)^2} \right)^{\frac{1}{2}} \right]$$

The above are to be compared with the familiar

$$P_E = \frac{1}{2} \operatorname{erfc} \left[\left(\frac{E_b}{N_o} \right)^{\frac{1}{2}} \right] \quad (34)$$

that is known to apply to ideal PSK.

In the above $E_b = P_L/R_L$ = energy/bit, R_{BR} is "bit rate ratio" R_H/R_L , and R_{PD} is power division ratio, defined earlier.

(33) shows that with RB, any mod index error γ that might exist will be augmented by the power division ratio, and that special attention therefore, would be merited in keeping γ small in an unbalanced-power situation. With Manchester coding the rectification operation or "Manchester decoding" that takes place prior to bit decisions, removes sensitivity to γ .

Crosstalk caused by θ and ϵ errors produces a noise density term that adds to thermal noise density. Manchester coding again is less sensitive to these errors, as the effect is reduced by the square instead of only the first power of bit rate ratio R_{BR} .

A convenient measure of degradation is the amount of additional signal energy required to overcome imperfect implementation effects, i.e., to achieve a given bit error rate in the imperfect versus an ideal system. This is readily determined in the Manchester case; it is necessary only to equalize arguments in (33) and (34). Carrying out the algebra leads to

$$\text{DB degradation} \approx 1.5 \frac{R_{PD}}{R_{BR}} (\theta + \epsilon)^2 \frac{E_b}{N_o} \quad (35)$$

In (35) E_b/N_o is the ratio ("energy contrast ratio") needed in an ideal system to achieve some given BER, and (35) gives the number of dB that must be added to achieve the same BER with imperfections present.

As an example consider a BER of 10^{-5} . Then E_b/N_o is known to be about 9.1 (9.6 dB). Assume that $R_{PD} = R_{BR} = 10$, and that $\theta = \epsilon = 0.1$. Then (35) shows about 0.06 dB added power would be needed to overcome the effects of the phase errors, i.e., 9.66 instead of 9.6 dB for the 10^{-5} BER.

Degradation is much more difficult to compute in the RB case. P_E must be calculated using both + and - signs in (33) and the average taken. In addition, of course, there is one more parameter γ to consider.

Typical results appear in Figure D-3. The curves show added signal power needed to maintain a BER of 10^{-5} in the presence of the imperfections indicated. Again, R_{PD} and R_{BR} both are taken to be 10. The relative sensitivity to mod index error γ is apparent, as about 4 dB degradation results from an γ of only 10 percent, while a similar magnitude for either θ or ϵ degrades performance less than 1 dB.

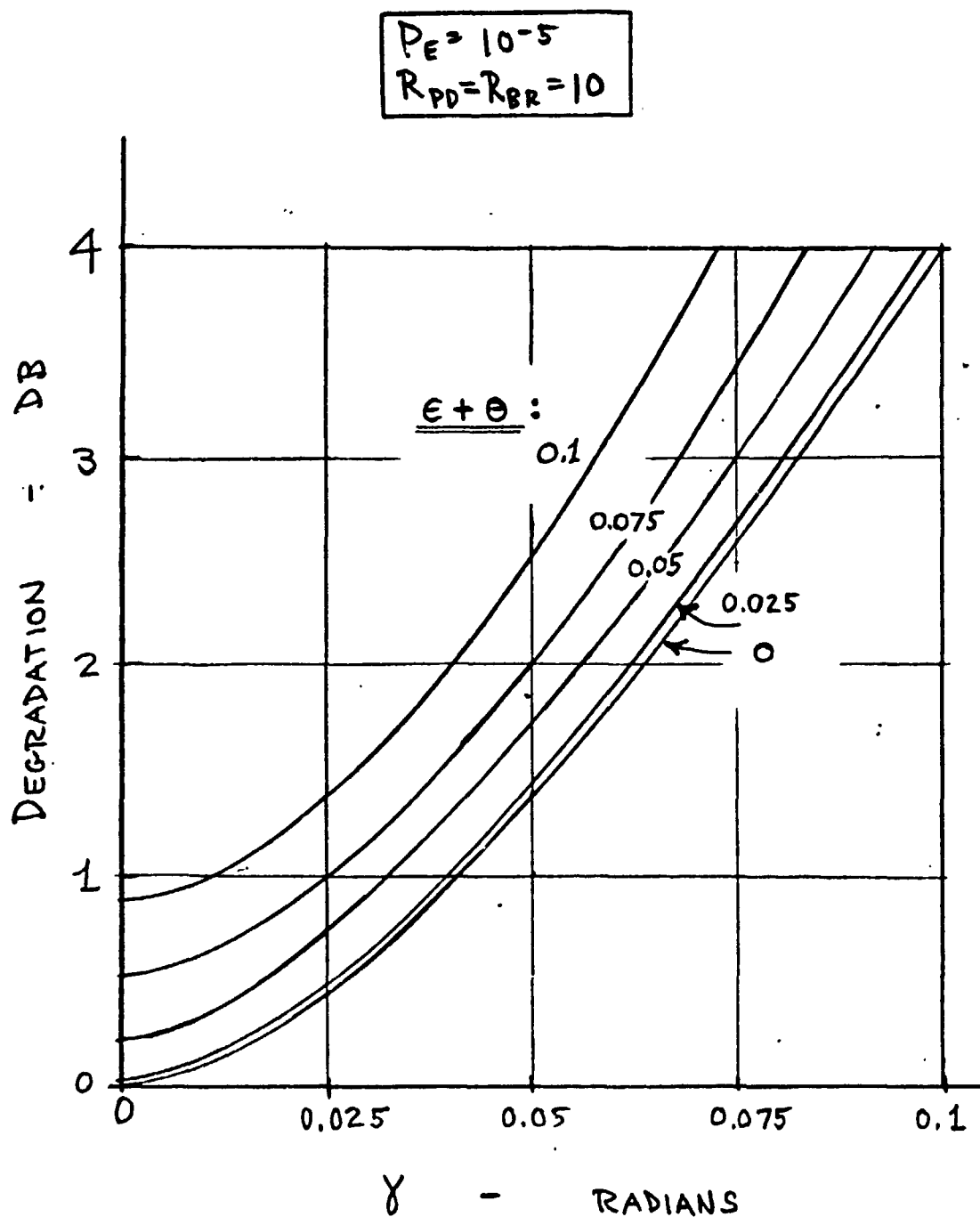


Figure D-3. Crosstalk Degradation due to Implementation Errors
Random Binary Data

III. Crosstalk in High Data Rate Channel

Attention has been confined thus far to crosstalk appearing in the low data rate channel. The low data rate bit stream also can produce crosstalk in the high rate channel, but this latter crosstalk appears to offer less potential for harm. As pointed out in connection with Figure D-1, in the high data rate channel the crosstalk waveform would appear slowly-varying relative to the desired-signal bit period. Bit decisions for the most part would be influenced as though by a constant d-c bias. Thus, the extra noise terms in the denominators of (33) are not present in the high-rate channel case.

Without the extra noise term in (33) Manchester coding is seen to reduce to the "ideal" form, (34), indicating that to first order, crosstalk effect in the high rate channel is zero.

In the random-binary case, following a derivation similar to that leading to (33) gives, for crosstalk in the high-rate channel:

$$P_E = \frac{1}{2} \operatorname{erfc} \left[\left(\frac{E_b \left\{ 1 \pm 2R_{PD}^{-\frac{1}{2}} \left[c \pm (c - \phi) \right] \right\}}{N_o} \right)^{\frac{1}{2}} \right] \quad (36)$$

As would be expected, the power division ratio R_{PD} is seen in this case, to help decrease phase offset effects. Also, the extra "noise density" term that appeared in the denominator of (33) is absent from (36). Given comparable parameter tolerances then (36) represents substantially less degradation than (33).

(36) ignores possible effects of transmission deviations. Certain kinds of deviations, e.g. parabolic phase and amplitude tilt, couple "spikes" of energy into the cross channel during signal transitions from one polarity to the other. No completely satisfactory model has been developed for estimating the effects of these transients, but preliminary analysis and certain plausibility arguments lead to the conclusion that their effects on BER would not be great.

Plausibility arguments include 1) the small amplitude of the low rate signal relative to the high, and 2) relative infrequency of the transient events because of bit rate difference.

IV. Conclusions

Crosstalk will be felt more in the low data rate channel than in the high. If crosstalk from the high rate data stream into the low is kept within acceptable limits, then crosstalk in the other direction should be negligible.

Equations (11)-(20) give approximate expressions for crosstalk level due to transmission deviations, notation being defined in Table D-1. Effects of implementation errors are shown in (33). Biasing power division ratio toward

the low data rate channel, that is allocating more power than just in proportion to bit rate, is effective against all sources of crosstalk.

Random binary data: High rate mod index error is the most serious source of potential degradation, as its effect is magnified by the power division ratio.

Transmission deviations if kept within normal tolerances (which in fact would be dictated by considerations other than crosstalk) appear unlikely to produce noticeable crosstalk. Possible exceptions are certain extreme cases of amplitudes and phase ripple. More refined analysis and/or simulation is indicated to resolve this point.

Manchester coded data: Crosstalk degradation generally is considerably less with Manchester than with random binary data. Manchester is relatively insensitive to implementation errors that cause d-c bias, the biggest contributor to degradation in the random binary case. In addition, the spectral shape of Manchester (low density around spectrum center) makes it less sensitive to transmission deviations. Power division bias (see above) is more effective with Manchester as a means of reducing crosstalk effects.

With normal system tolerances on transmission deviations and implementation errors, crosstalk degradation (Manchester case) should be well under 0.1 dB.

ATTACHMENT. VARIANCE OF V_i ; $i = RB, M$

$$V_i = \bar{V}_i + B_H \int_0^a u_i(t) dt \quad a = R_L^{-1}$$

$$\text{Var} \{V_i\} \triangleq \sigma_i^2 \triangleq \langle (V_i - \bar{V}_i)^2 \rangle = B_H^2 \left\langle \left[\int_0^a u_i(t) dt \right]^2 \right\rangle$$

$$\sigma_i^2 = B_H^2 \left\langle \int_0^a \int_0^a u_i(t) u_i(u) dt du \right\rangle = B_H^2 \int_0^a \int_0^a R_i(t-u) dt du$$

$$= B_H^2 \int_0^a \int_0^a \int_{-\infty}^{\infty} W_i(f) e^{j\omega(t-u)} df dt du$$

$$= B_H^2 \int_{-\infty}^{\infty} W_i(f) \left[\int_0^a e^{j\omega t} dt \right] \left[\int_0^a e^{-j\omega u} du \right] df$$

$$\sigma_i^2 = \frac{2B_H^2}{R_L^2} \int_0^{\infty} W_i(f) \frac{\sin^2\left(\frac{\pi f}{R_L}\right)}{\left(\frac{\pi f}{R_L}\right)^2} df$$

Random Binary:

$$W_{RB}(f) = \frac{1}{R_H} \frac{\sin^2 \frac{\pi f}{R_H}}{\left(\frac{\pi f}{R_H}\right)^2}$$

$$\sigma_{RB}^2 = \frac{B_H^2}{3R_H} \left(\frac{3R_H - R_L}{R_H R_L} \right) \approx \frac{B_H^2}{R_H R_L}$$

Manchester:

$$W_m(f) = \frac{1}{R_H} \frac{\sin^4\left(\frac{\pi f}{2R_H}\right)}{\left(\frac{\pi f}{2R_H}\right)^2}$$

$$\sigma_M^2 \approx \frac{B_H^2}{6R_H^2}$$