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STOCHASTIC SIMULATION OF VERTICALLY NONHOMOGENEOUS GUSTS

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STOCHASTIC SIMULATION OF VERTICALLY NONHOMOGENEOUS GUSTS

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ABSTRACT

The small-scale horizontal gust structure of detailed wind profiles along the vertical in the first 20 km of the atmosphere is a vertically nonhomogeneous process. A linear stochastic model is developed for the process based on the process covariance function. This model is formulated through the use of a scaling hypothesis which transforms the nonhomogeneous gust process into a nondimensional gust process which is homogeneous in a nondimensional height coordinate. The velocity scaling parameter for the gust process is the gust standard deviation, and the length scale used to nondimensionalize the altitude is the vertical space lag associated with the first zero of the gust covariance function. State space theory is used to derive a digital filter from the model, which can be readily used to simulate gusts for space vehicle design applications.

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INTRODUCTION

The most striking characteristic of a vertical profile of the horizontal wind in the first 20 km of the atmosphere is its stochastic spatial variability. All vertical scales of motion from a few millimeters up to tens of kilometers are included in these profiles. To design space vehicles with sufficient structural and control system capability to withstand the stochastic "forcing" due to the detailed structure of these profiles (wind gusts), the design engineer requires a model of wind gusts which captures the stochastic character of the wind profile and yet, at the same time, permits ease of application. The scales of motion associated with vertical wavelength $\lambda < 2000$ m are responsible for the high-frequency excitations of the structural and control system modes of response of space vehicles [1]. The gusts associated with these small-scale wind profile features are the subject of this paper; however, before proceeding further, let us consider what is available to the engineer in order to focus on the contribution presented herein.

Various types of gust "inputs" are available to the aerospace engineering community for vehicle design. Included in these inputs are discrete gusts, spectra, and samples of wind profiles [2].

The discrete gust models generally consist of a gust shape (ramp buildup and decay, 1-cosine, etc.) with a gust amplitude

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determined in such a way as to be compatible with the accepted risk of compromising the vehicle design as a result of gust excitation, and a gust period which is varied so as to encompass the significant periods of vehicle response [2]. These models are relatively easy to apply to linear as well as to nonlinear vehicle equations of motion with constant as well as with time-dependent coefficients.

The spectral models consist of a one-dimensional wave number spectrum envelope which is derived from statistics of wind profile gust spectra [2, 3, 4]. To derive the gust spectra, the large-scale or synoptic-scale variations associated with wavelengths $\lambda > 2000$ m are removed from the wind profile, and the remaining profile is spectrally decomposed. This assumes statistical homogeneity of the gust process along the vertical—a point we shall return to later. To derive the gust spectrum envelope, a sample of gust profiles is Fourier decomposed, and marginal statistical distributions of the associated spectra are determined for a selected set of wave numbers. A risk of exceeding the gust spectrum at each wave number is then selected, and gust spectra consistent with this risk are determined from the marginal distributions for the selected wave numbers. The spectra associated with the assigned risk determine the gust spectrum design envelope [2]. The resulting spectrum envelope is used for what is called "frozen point" vehicle analyses, in which the vehicle equations

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of motion are linearized about a "nominal" flight trajectory. The resulting coefficients of the resulting linear differential equations are assigned values appropriate for a specific point in time on the nominal vehicle trajectory, and a response analysis is conducted locally about this selected time point [1]. The resulting linear, constant coefficient, differential equations are Fourier transformed, resulting in a system response function from which, in conjunction with the design gust spectrum, the vehicle response spectra are obtained. These results can then be used to make statistical design judgments. To do this, a Gaussian gust process is assumed. To apply the design spectrum to nonlinear vehicles with nonconstant coefficients, the spectrum can be factored and back-Fourier transformed to the vertical coordinate domain to yield a filter equation which has Gaussian white noise as a forcing function. The filter provides a series of random functions that have second moment statistics which resemble the design gust spectrum, and these random functions are used as a simulation of the design gust environment. These simulated gust space histories are then used to "drive" the nonlinear vehicle equations. A major difficulty here is that the resulting gust profiles do not exhibit certain vertical nonhomogeneous features. More will be said about this point later.

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A connection exists between gust spectra and discrete gusts in the sense that the discrete gust amplitude is derived from the gust spectrum.

The third available input consists of samples of measured wind profiles. The vehicle equations of motion are excited by the ensemble of measured wind profiles in the time domain, and the resulting vehicle responses are statistically analyzed. Usually, measured wind profile samples are used for final design verification. A sample of wind profiles for the first 18 km of the atmosphere with sufficient gust detail for space vehicle design studies is available for Cape Kennedy, Florida [2]. This sample consists of 150 detailed wind profiles for each month measured with the FPS-16 Radar/Jimsphere system [5, 6]. A program is now under way to collect sufficient data to prepare a similar sample for the Vandenberg Air Force Base to be used in design verification studies for the Space Shuttle. This sample will not be available until later 1977, so that an interim sample must be made available for preliminary studies. The gust model presented herein is now being used in the development of this interim sample with available rawinsonde data. To generate this detailed wind profile sample, the short wavelength ($\lambda \leq 2000$ m) information that is not contained in rawinsonde [7] data will be simulated with the stochastic gust model described herein. Approximately 12 years of rawinsonde wind data

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are available for Vandenberg Air Force Base, sufficient to generate the needed detailed wind profile sample.

A major criticism of the spectral models discussed previously is the assumption of statistical homogeneity along the vertical. It is a well-established fact that the wind structure along the vertical in the troposphere and lower stratosphere is not statistically homogeneous. Both experimental and theoretical evidence indicates this nonhomogeneous character. For example, on the theoretical side of the coin the basic gravity-wave theory of Hines [8] shows that the Fourier amplitudes of gravity-wave perturbations increase with altitude z as exp $(z/2H)$, where H is the atmospheric density scale height for an isothermal atmosphere. On the experimental side, extensive wind profile data are available which show nonhomogeneous behavior. For example, the eddy structure of the wind profile in the stratosphere and troposphere is markedly different in the sense that the amplitudes of the gusts are generally larger above the tropopause [9]. Nonhomogeneity of the wind profile gust statistics, and hence spectra, should be the normal state of affairs in view of the obvious vertical nonhomogeneous structure of atmospheric thermodynamic variables and associated synoptic (weather map) scale wind fields. Accordingly, the goal of the research reported herein is to provide a nonhomogeneous stochastic model of the horizontal gusts on the vertical profile of the wind which have wavelengths $\lambda < 2000$ m.

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As noted previously, the motivation for the research reported herein and the selection of the scales of motion to be analyzed is the need for a nonhomogeneous gust model within the aerospace engineering community for space vehicle design applications. The main approach is to model the correlation function. This will, by necessity, involve a transformation of the vertically nonhomogeneous random zonal and meridional gust processes u(z) and v(z) into the homogeneous (we hope) processes $u(z)/\sigma_u(z) = \xi[z/L(z)]$ and $v(z)/\sigma_v(z) = \zeta[z/L(z)]$, where $\sigma_u(z)$ and $\sigma_v(z)$ are the zonal and meridional gust standard deviations at height z. The quantity L(z) is a length scale which is defined such that upon stretching transformation of the gust altitude coordinate, the random processes $\xi(t)$ and $\zeta(t)$ are statistically homogeneous in the nondimensional coordinate $t = z/L(z)$. A spectrum function shall be derived from the correlation functions of $\xi(t)$ and $\zeta(t)$ in terms of a nondimensional wave number. Factorization of this spectrum function will then lead to a set of first-order state differential equations for $\xi(t)$ and $\zeta(t)$ driven by white noise random processes. These state equations facilitate the simulation of the homogeneous processes $\xi(t)$ and $\zeta(t)$ which, in turn, can be transformed to simulations of the nonhomogeneous random processes u(z) and v(z). The simulations of u(z) and v(z) can then be used for space vehicle design studies.

EMPIRICAL GUST STATISTICS

To develop a stochastic model of horizontal wind gusts suitable for simulation of wind profiles along the vertical for gust wavelengths $\lambda < 2000$ m, a sample of detailed wind profile observations measured at 25-m intervals with the FPS-16/Jimsphere wind measurement system [6] was selected for analysis. Each wind profile observation consisted of a zonal and a meridional wind profile in the 1 to 18 km altitude band. The total sample consisted of approximately 150 wind profile observations each for the months of January, February, and March. The zonal and meridional wind profiles for each observation were digitally filtered with Martin-Grahm filters specially developed by DeMandel and Krivo [10] for detailed wind profile analysis with a cutoff at $\lambda \simeq 2000$ m characterized by a very sharp "roll-off," so that essentially all information associated with $\lambda \leq 2000$ m was retained with a minimum of extraneous information associated with $\lambda > 2000$ m retained in the gust profiles. The "DC components" of the gust profiles were removed by averaging the ensemble of zonal and meridional filtered profiles separately at each altitude and subtracting the resulting ensemble average zonal and meridional profiles from each of the respective profiles in the ensemble. The DC components revealed no detectable nonzero trend along the vertical and were statistically equal to zero. We denote the short wavelength ($\lambda \leq 2000$ m) zonal and

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meridional gusts at height z with the DC components removed as u(z) and v(z), respectively.

The empirical two-point covariance functions

$$R_u(z_r, z) = \left\langle \frac{u(z_r)}{\sigma_u(z_r)} \frac{u(z)}{\sigma_u(z)} \right\rangle \qquad (1)$$

and

$$R_v(z_r, z) = \left\langle \frac{v(z_r)}{\sigma_v(z_r)} \frac{v(z)}{\sigma_v(z)} \right\rangle \qquad (2)$$

were determined for z_r incremented at 1000-m intervals and z incremented at 25-m intervals relative to altitude z_r for $z < z_r$. The angular brackets in Eqs. (1) and (2) denote ensemble averages, and

$$\sigma_u^2(z) = \langle u^2(z) \rangle \ , \ \sigma_v^2 = \langle v^2(z) \rangle \ . \qquad (3)$$

Figures 1 and 2 contain the experimental estimates of R_u and R_v versus nondimensional space lag $(z_r - z)/L(z_r)$. The length scale $L(z_r)$ is the value of $z_r - z$ associated with the first zero of R. The experimental values of σ^2 and L are given in Figures 3 and 4. The data in Figures 1 through 4 appear to show that the statistics of u and v are equal, so that in the analysis that follows we shall pool these statistics. This is physically reasonable. The following formulae summarize the σ and L data:

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$$\sigma(z) = 1.3077 \text{ m sec}^{-1} \;, \quad z < 9160 \text{ m} \tag{4}$$

$$\sigma(z) = 0.346 \exp(1.45 \times 10^{-4} z) \;, \quad z \geq 9160 \text{ m}$$

$$L(z) = 310 + 0.0129z, \quad z < 9160 \text{ m} \tag{5}$$

$$L(z) = 428 \text{ m} \;, \quad z \geq 9160 \text{ m}$$

where all units are in the MKS system.

AUTOCOVARIANCE AND SPECTRAL FUNCTIONS

It appears that the scaling of u(z) and v(z) indicated in Eqs. (1) and (2) and the nondimensionalization of $(z_r - z)$ by division with $L(z_r)$ result in a reasonable collapse of the covariance data and, thus, provide a basis for the development of a statistical "law" or model of detailed wind profile gusts. Let us express the nondimensional lag as

$$\frac{z_r}{L(z_r)} - \frac{z}{L(z_r)} = \frac{z_r}{L(z_r)} - \frac{z}{L(z)(1+\epsilon)} \tag{6}$$

where

$$\epsilon = \frac{L(z_r) - L(z)}{L(z)} \quad . \tag{7}$$

The quantity $\epsilon \simeq 0.10$, so that

$$\frac{z_r}{L(z_r)} - \frac{z}{L(z_r)} \simeq \frac{z_r}{L(z_r)} - \frac{z}{L(z)} \quad . \tag{8}$$

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Thus, the data in Figures 1 and 2 appear to be indicating that a length scale $L_o(z)$ exists such that the random processes $\xi(t) = u(z)/\sigma(z)$ and $\zeta(t) = v(z)/\sigma(z)$ are homogeneous processes relative to the coordinate $t = z/L_o(z)$, where $L(z)$ is an estimate of $L_o(z)$. This conjecture is the basis for the development that follows.

The empirical autocovariances can be approximated in functional form by

$$R(t) = \langle \xi(t)\, \xi(t+\tau) \rangle = \langle \zeta(t)\, \zeta(t+\tau) \rangle \tag{9a}$$

$$= \exp(-D|\tau|) \left\{ \cos(B|\tau|) - \frac{D}{B} \sin(B|\tau|) \right\} \tag{9b}$$

where B and D are nondimensional constants equal to 1.122 and 0.539, respectively, and

$$\tau = \frac{z_r}{L(z_r)} - \frac{z}{L(z)} \quad . \tag{10}$$

A comparison of the empirical and functional forms of the autocovariance given in Figure 5 shows good agreement between the two.

Fourier transformation of the autocovariance yields a spectrum $\phi(\omega)$; and since $R(\tau)$ is an even function, the spectrum can be written as

$$\phi(\omega) = 2 \int_0^{\infty} R(\tau) \cos \omega\tau \, d\tau \tag{11a}$$

$$\phi(\omega) = \frac{4D\omega^2}{[D^2 + (B-\omega)^2][D^2 + (B-\omega)^2]} \tag{11b}$$

where ω is a nondimensional wave number.

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SYSTEM TRANSFER FUNCTION

In general, the output spectrum of a linear system can be written as

$$\phi(\omega) = H(\omega)\, H^*(\omega)\, \psi(\omega) \tag{12}$$

where $\psi(\omega)$ is the input process spectrum, $H(\omega)$ is a system transfer function, and superscript star denotes complex conjugation. If the input process is white Gaussian noise with unit spectral density, so that $\psi(\omega) = 1$, then $\phi(\omega) = H(\omega)\, H^*(\omega)$ and the spectrum given by Eq. (11b) can be factored to yield

$$H(s) = \frac{2D^{\frac{1}{2}} s}{(s + D - iB)\,(s + D + iB)} = \frac{Y(s)}{W(s)} \tag{13}$$

to within an arbitrary phase angle which we shall take as zero, $s = i\omega$ and $Y(s)$ and $W(s)$ denote the Laplace transforms of $\xi(t)$ or $\zeta(t)$ and the white noise input process, respectively.

STATE SPACE SYSTEM

A system of first-order differential equations can be written in terms of state variables $x_j(t)$ which will yield the system transfer function given by Eq. (13), namely

$$\dot{x}_i = a_{ij}\, x_j + d_i\, w(t) \tag{14}$$

where $w(t)$ is a white noise process with unit spectral density, a_{ij} and

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d_i are functions of D and B [see Eq. (13)], and the Einstein summation convention on repeated indices is understood [11]. In our particular application the state variables $x_i(t)$ are linearly related to $\xi(t)$ and $\zeta(t)$ through the expression

$$y(t) = e_i\, x_i(t) \tag{15}$$

where y(t) is either $\xi(t)$ or $\zeta(t)$. After a number of mathematical steps, it is easily verified that the coefficients a_{ij}, d_i, and e_i are given by

$$[a_{ij}] = \begin{bmatrix} 0 & 1 \\ -(D^2+B^2) & -2D \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1.5494 & -1.0780 \end{bmatrix} \tag{16}$$

$$[d_i] = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{17}$$

$$[e_i] = \begin{bmatrix} 0 \\ 2D^{\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4683 \end{bmatrix} \quad . \tag{18}$$

The state space system given by Eq. (14) will yield the system transfer function, Eq. (15). The flowgraph for the state space representation of the gusts is given in Figure 6. This system is stable and will serve as a basis for developing digital filters for the simulation of zonal and meridional gust profiles.

DISCRETE STATE SPACE SYSTEM

For use on digital computers, the state equations must be converted to a discrete time system. One procedure for achieving this goal is to pass the signal through a zero order holding device which samples at unit intervals of time and holds the signal value constant between samples [12].

The solution to Eq. (14) is given by

$$x_i(t) = \psi_{ij}(t - t_o)\, x_j(t_o) + \int_{t_o}^{t} \psi_{ij}(t - t')\, d_j\, w(t')\, dt' \tag{19}$$

where $x_j(t_o)$ is an initial condition. The quantity $\psi_{ij}(t)$ is called the fundamental matrix [13]. Now since w(t) is considered constant over the interval T, Eq. (19) can be evaluated at $t = (K+i)T$, where K is an integer, so that

$$x_i(K+1) = \psi_{ij}(T)\, x_j(K) + \Lambda_i(T)\, w(K) \tag{20}$$

where

$$\Lambda_i(T) = \int_0^T \psi_{ij}(t')\, d_j\, dt' \quad . \tag{21}$$

According to the theory of discrete random processes, it is readily shown that the matrix $\psi_{ij}(t)$ is given by

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$$\psi_{ij}(t) = \begin{bmatrix} e^{-Dt}(\cos Bt + \frac{D}{B} \sin Bt) & \frac{1}{B} e^{-Dt} \sin Bt \\ -\frac{D^2+B^2}{B} e^{-Dt} \sin Bt & e^{-Dt}(\cos Bt - \frac{D}{B} \sin Bt) \end{bmatrix} \tag{22}$$

The asymptotic behavior of Eq. (22) as $t \to 0$ is given by

$$\psi_{ij}(t) = \begin{bmatrix} 1 & t \\ D(D^2+B^2)t & 1-2Dt \end{bmatrix} \tag{23}$$

and from Eq. (21) we obtain

$$\Lambda_i(T) = \int_0^T \begin{bmatrix} t' \\ 1-2Dt' \end{bmatrix} dt' = \begin{bmatrix} 0 \\ T \end{bmatrix} \tag{24}$$

to within first order in T.

Thus, the quantities $\psi_{ij}(T)$ and $\Lambda_i(T)$ are known functions of T and the system parameters D and B, and Eq. (20) thus permits the digital simulation of $x_i(K)$; to obtain the digital gust processes we merely use Eq. (15), so that

$$y(K) = e_i x_i(K) \quad . \tag{25}$$

DISCUSSION OF RESULTS

To apply the above model one first generates the discrete Gaussian noise process w(K). Substitution of the noise process into the recursive filter given by Eq. (20) and application of Eq. (24) yields the nondimensional gust processes $\xi(t) = u(z)/\sigma(z)$ and $\zeta(t) = v(z)/\sigma(z)$. To recover the gust processes $u(z)$ and $v(z)$ we merely stretch the t-coordinate with $z = tL(z)$ and multiply $\xi(t)$ and $\zeta(t)$ by $\sigma(z)$. Note that $\xi(t)$ and $\zeta(t)$ are homogeneous processes and the above transformation yields a non-homogeneous gust process with the proper length (L) and velocity (σ) scales.

The input noise process is Gaussian with zero mean and standard deviation equal to σ_w. What is the correct value of σ_w? The answer to this question is directly related to the selection of the value of T. We demand that the noise spectrum $\psi(\omega)$ in Eq. (12) possess unit value over the nondimensional wave number band of concern. Since the noise process consists of a stair function with the time increment of each stairstep being T, we may write [14]

$$\psi(\omega) = \left[\frac{\sin(\omega T/2)}{\omega T/2}\right]^2 \tag{26}$$

where $T \ll 1$. Integrations of Eq. (26) over the domain $-\infty < \omega < \infty$ yields

$$\sigma_w = \left[\frac{1}{T}\int_{-\infty}^{\infty}\left(\frac{\sin\eta}{\eta}\right)^2 d\eta\right]^{\frac{1}{2}} \propto T^{-\frac{1}{2}} \quad . \tag{27}$$

Thus, the selection of T automatically determines the appropriate value of σ_w. A more detailed discussion of this point is given in Reference 15. The wind data used to develop the stochastic model

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herein was digitized at 25-m intervals along the vertical, and the data processing procedures used to derive the wind profiles from the FPS-16 tracking data were designed so that nearly all (~90 percent) of the spectral energy of the gust process existed at wavelength λ = 100 m with zero spectral energy at λ = 50 m [16]. This means the Nyquist wavelength is 50 m, which corresponds to a vertical digitization interval $\Delta z \simeq 25$ m in z-space. Since the typical value of L = 400 m (see Figure 5), then the digitational interval of the input noise process in t-space should be no larger than $T \simeq \Delta z/L \simeq 0.06$. Figure 5 gives the results of a 1000-point simulation with T = 0.06 and is in good agreement with the desired autocorrelation.

CONCLUDING COMMENTS

The statistical model reported in this paper was developed by assuming the gust process could be rendered nondimensional by scaling it with the gust standard deviation $\sigma(z)$ and nondimensionalizing the altitude with a length scale L(z). However, classical theory of continuous media and dimensional analysis concepts demands that additional parameters exist by which to scale $\sigma(z)$ and L(z). This is a crucial point because to apply the model to locations other than the Kennedy Space Center, the parameters $\sigma(z)$ and L(z) must be determined. Nondimensional functions for $\sigma(z)$ and L(z) would permit this objective determination through the estimation of the additional scaling parameters. At the present time these nondimensional functions for $\sigma(z)$ and $L(z)$ are not available; however, analyses of detailed wind profile data from the Vandenberg Air Force Base appear to show that the model presented herein can be applied to that site. Nevertheless, the technical issue of determining the nondimensional functions for $\sigma(z)$ and $L(z)$ remains. This problem is a complicated one, and it is not within the scope of this paper. However, a number of comments can be made about the problem. The model presented herein was based solely on Kennedy Space Center data, so that the profiles of $\sigma(z)$ and $L(z)$ depend on the climatological distribution of the dynamic meteorological situations over the Kennedy Space Center because all of the data were combined to obtain $\sigma(z)$ and $L(z)$. Thus, any set of nondimensional functions for $\sigma(z)$ and $L(z)$ should reflect the same "mix" of meteorological conditions. A possible solution to this problem is to partition the data according to meteorological conditions (convective atmospheres, gravity waves, etc.) and to develop models for each condition. These subclass models would most likely be applicable to more sites than the one presented herein. Application of the subclass models to other locations would be accomplished by applying the climatology of the conditionalizing meteorological conditions that were used to delineate the subclass models. This would be an extremely tedious procedure,

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and sufficient data do not exist at this time to accomplish the task; and even if they did exist, the intended engineering applications may not warrant the detail implied above.

We might speculate on the functional forms of the nondimensional functions for $\sigma(z)$ and $L(z)$. We require meteorological parameters which characterize thermal stratification and wind profile shear. We also have available the acceleration of gravity g and the Coriolis parameter $f = 2\Omega \sin \phi$, Ω being the rotation rate of the earth and ϕ the latitude of the location. The stratification parameter could be the density scale height

$$\bar{H} = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial z}\right)^{-1} \tag{28}$$

where ρ is atmospheric density. A wind profile shear parameter could be

$$S = \left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right] \tag{29}$$

where $\bar{u}$ and $\bar{v}$ are zonal and meridional components of the mean wind profile. This does not exhaust the possible parameters one might use. A dimensional analysis yields

$$\frac{\sigma}{L(g/H)^{\frac{1}{2}}} = F_1\left(\frac{z}{H}, \frac{g}{HS}, \frac{g}{Hf^2}\right) \tag{30}$$

$$\frac{L}{H} = F_2\left(\frac{z}{H}, \frac{g}{HS}, \frac{g}{Hf^2}\right) \tag{31}$$

where F_1 and F_2 are "universal" functions. The quantity g/HS is related to the Richardson number which, in turn, is a measure of the dynamic stability of the atmosphere. The quantity g/Hf^2 is a ratio of time scales of motions governed by the stratification ($(g/H)^{\frac{1}{2}} \sim 10^{-2}$ rad sec^{-1}) and Coriolis forces ($f \sim 10^{-4}$ rad sec^{-1} in midlatitudes). To develop the argument further we shall assume that g/HS and g/Hf^2 are relatively unimportant and substitute typical values of H for the Kennedy Space Center [17] into Eqs. (4) and (5) to obtain

$$\frac{\sigma}{L(g/H)^{\frac{1}{2}}} = \begin{cases} \dfrac{0.13}{1+0.381\, z/H}, & \dfrac{z}{H} \leq 1 \\ 0.0228 \exp(1.131\, z/H), & \dfrac{z}{H} > 1 \end{cases} \tag{32}$$

$$\frac{L}{H} = \begin{cases} 0.0338 + 0.0129\, z/H, & \dfrac{z}{H} \leq 1 \\ 0.0549, & \dfrac{z}{H} > 1 \end{cases} \tag{33}$$

where all numerical coefficients are nondimensional quantities. These expressions should not be taken seriously but rather are intended to give one an idea of what might be expected if an attempt were made to experimentally determine F_1 and F_2. Note that we have used L to estimate a length scale to obtain a scaling velocity $[L(g/H)^{\frac{1}{2}}]$, because L is a characteristic length of the eddies, while $(g/H)^{-\frac{1}{2}}$ is a characteristic time scale. It should be noted that $\sigma/L(g/H)^{\frac{1}{2}}$ is nearly

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equal to a constant in the region $z/H < 1$, which can be explained by convective mixing of the troposphere over the Kennedy Space Center. On the other hand, for $z/H > 1$ the quantity $\sigma/L(g/H)^{\frac{1}{2}}$ has exponential behavior, and we attribute this result to the fact that in the region $z/H > 1$ the atmosphere is stably stratified and the small scale dynamics essentially consist of gravity waves.

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FIGURE CAPTIONS

Figure 1. Covariance function for the zonal component of the gust velocity vector.

Figure 2. Covariance function for the meridional component of the gust velocity vector.

Figure 3. Zonal and meridional gust variances as a function of altitude z.

Figure 4. Zonal and meridional gust length scale as a function of altitude z. The gust length scale is the vertical space lag associated with the first zero of the covariance function.

Figure 5. The autocorrelation function $R(\tau)$. The solid curve corresponds to the empirical autocorrelation derived from averaging the data in Figures 1 and 2. The dashed curve corresponds to the mathematical function given by Eq. (9b) with B = 1.122 and D = 0.539. The data points correspond to the correlation function of the nondimensional gust process $\zeta(t)$ or $\xi(t)$ derived from a 1000-point simulation using the recursive filter given by Eq. (20) with T = 0.06.

Figure 6. State space representation of the nondimensional gust process $y(t)$.

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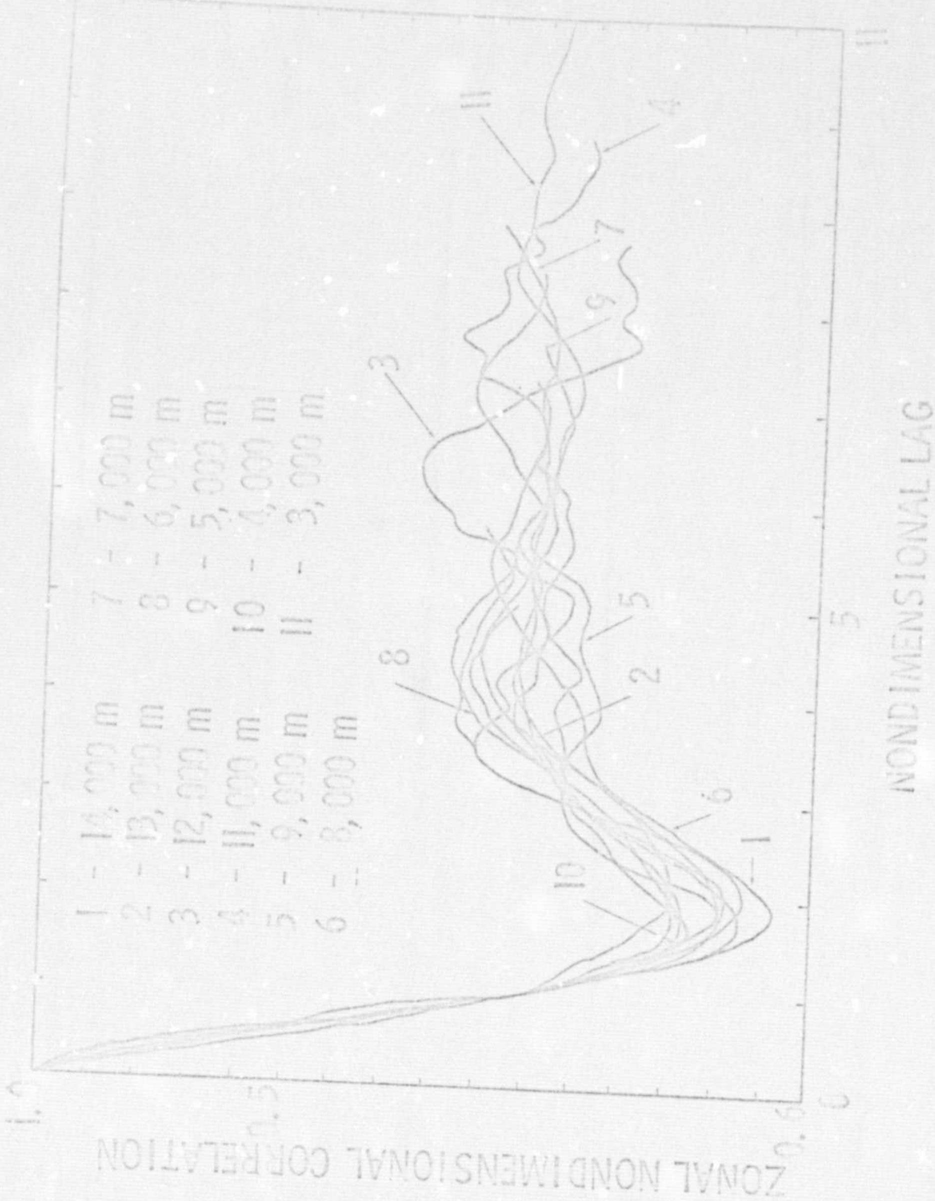

Figure 1

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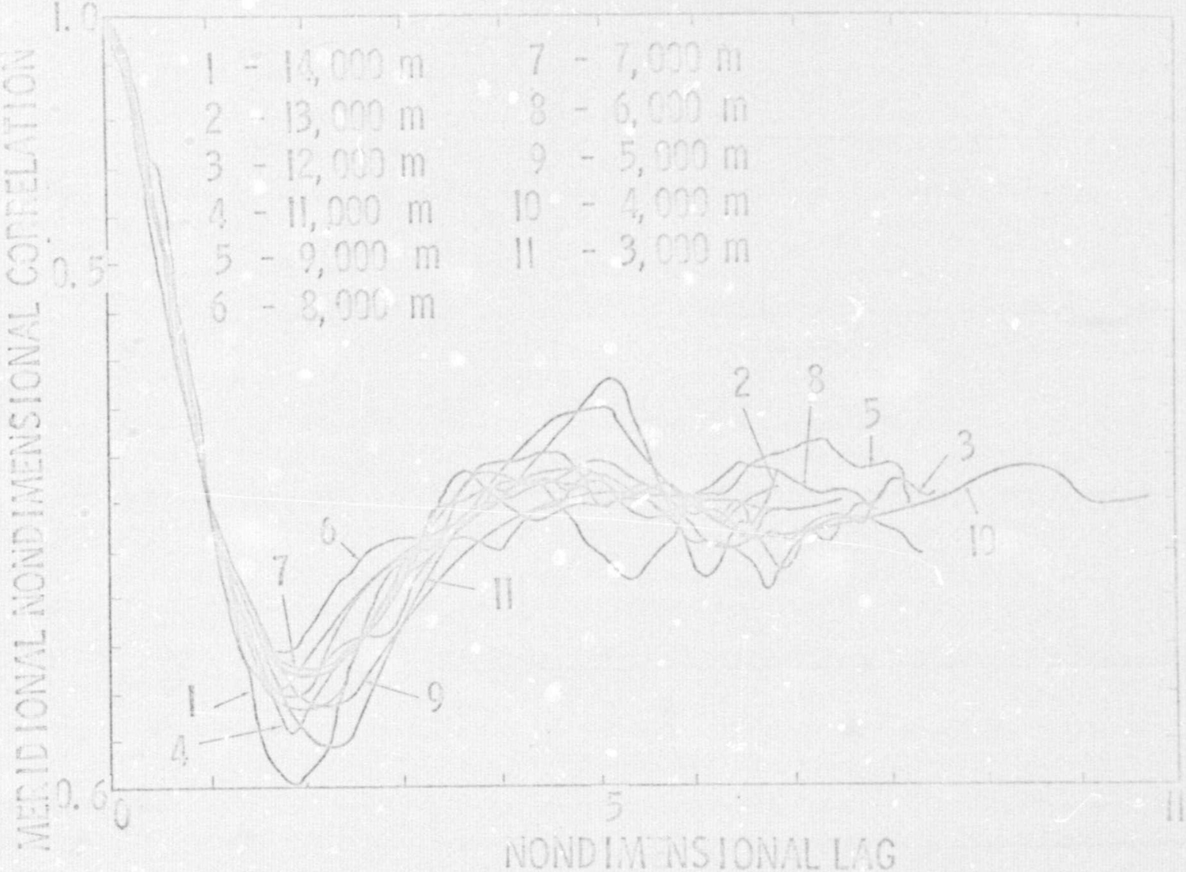

Figure 2

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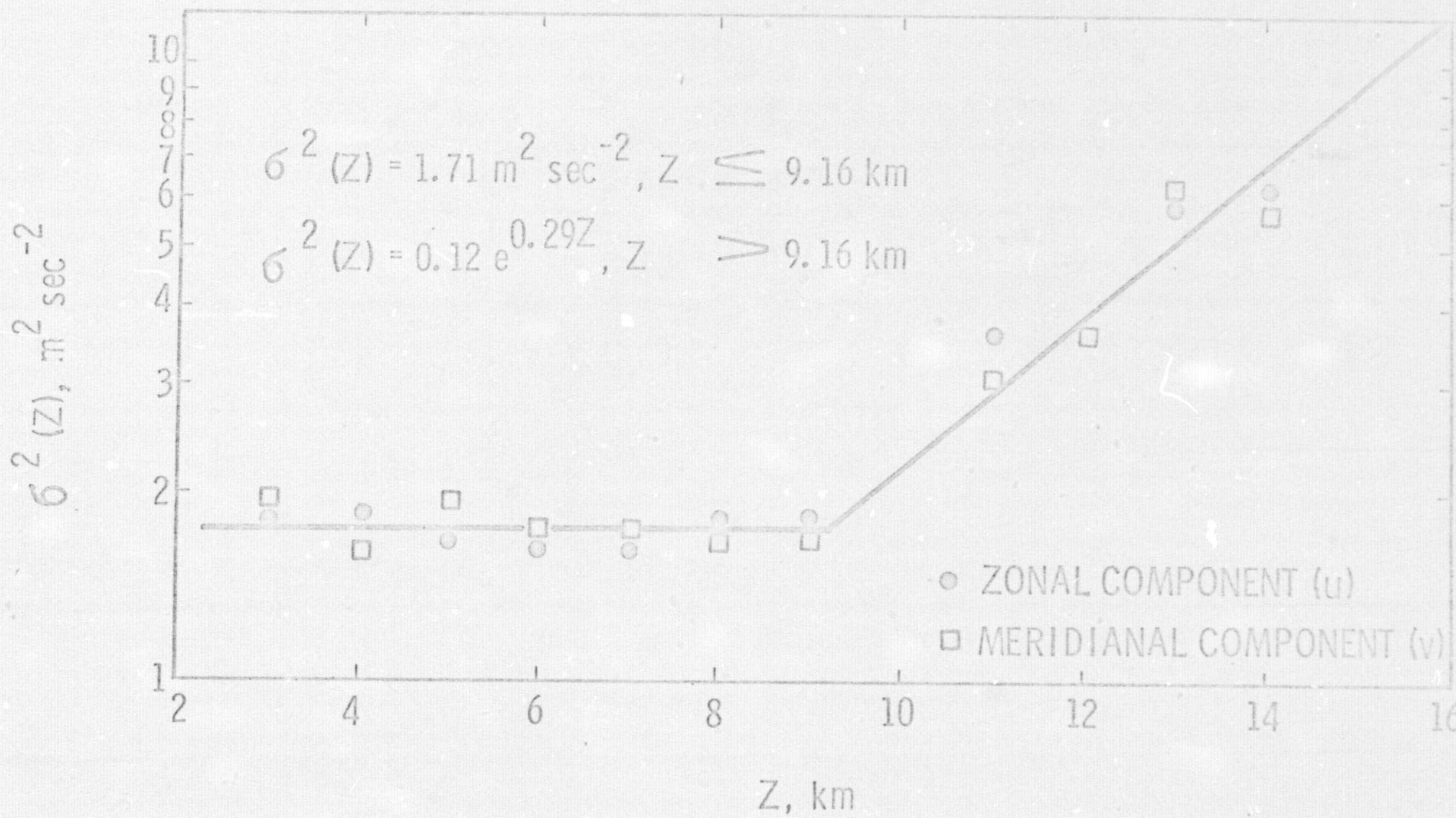

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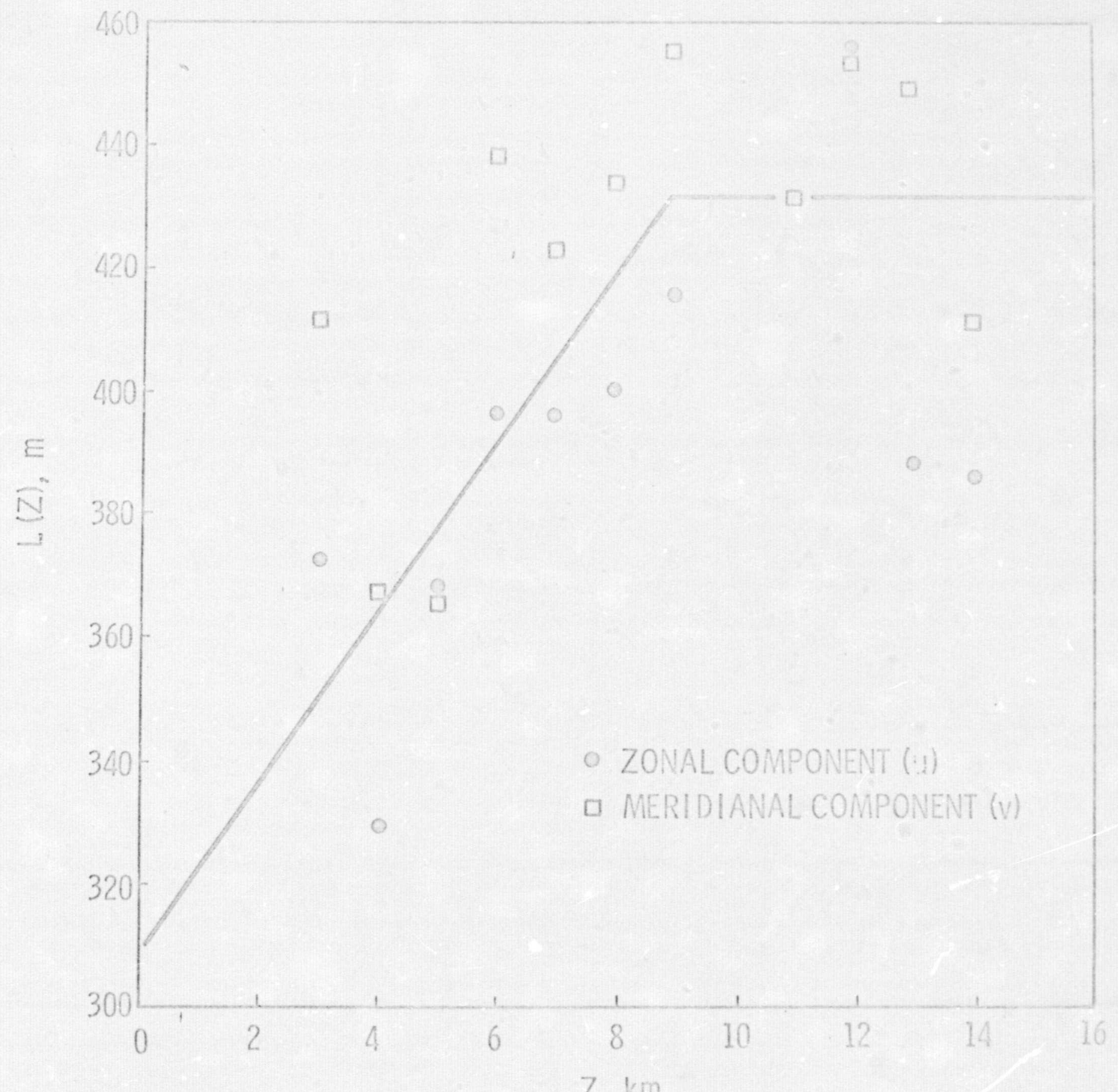

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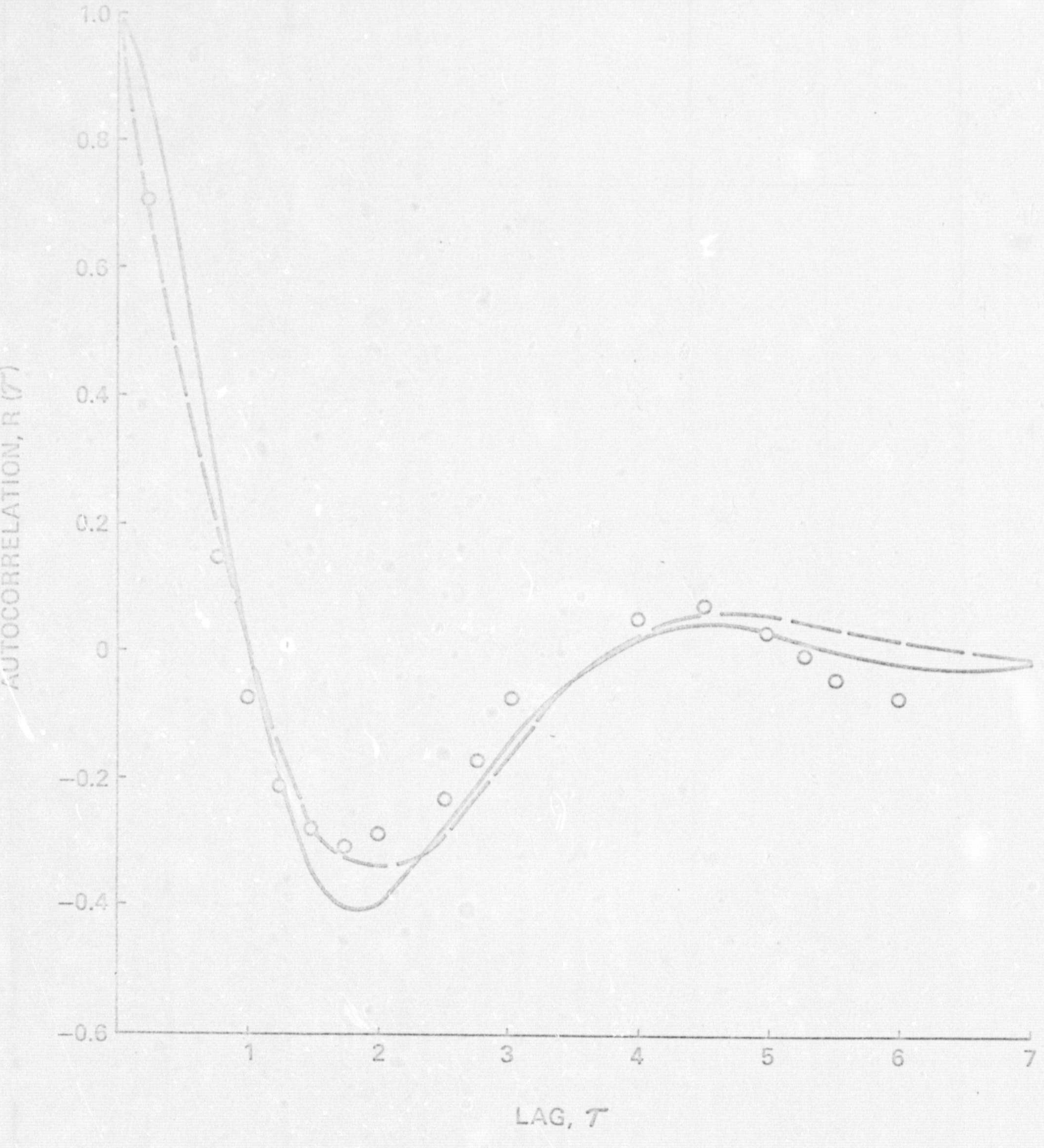

Figure 5

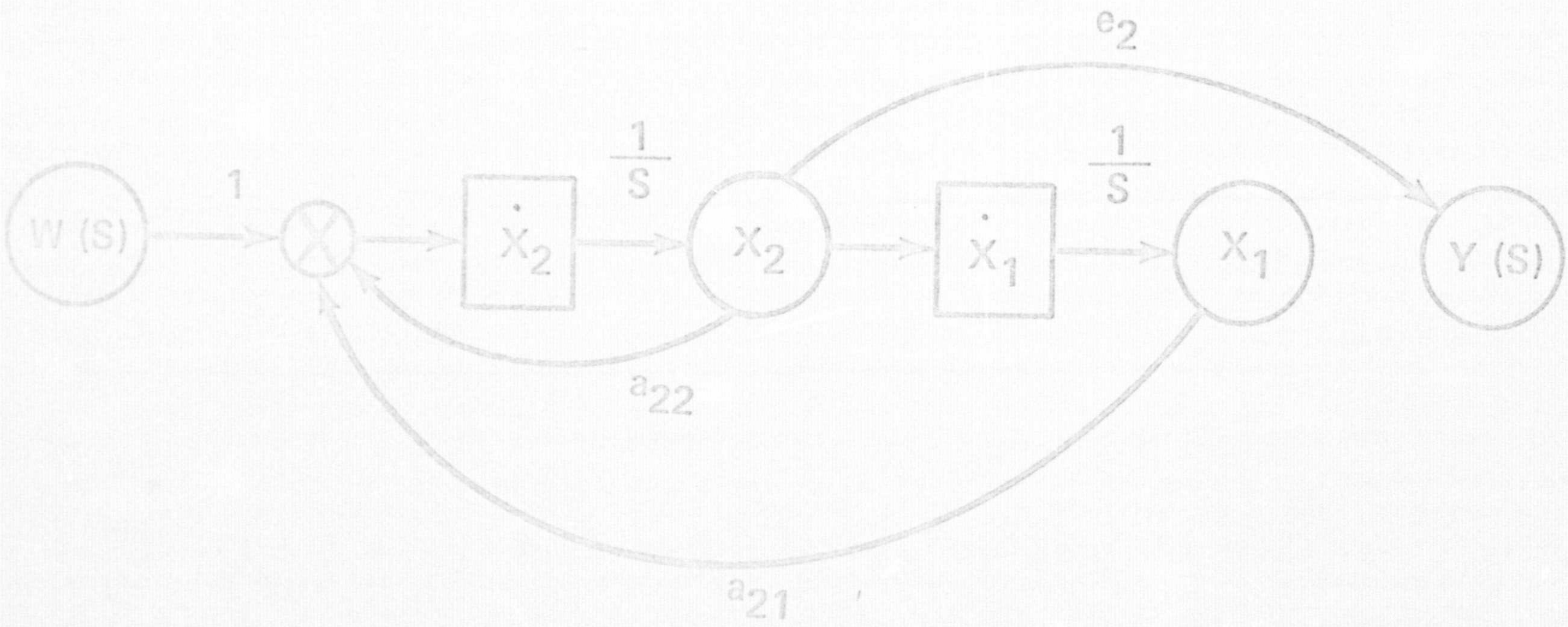

Figure 6