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AN ALTERNATIVE OPTION TO THE DUAL-PROBE OUT-OF-ECLIPTIC MISSION VIA JUPITER SWINGBY

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We have recently conducted a preliminary study on the possibility of combining the out-of-ecliptic (OOE) mission with a solar-probe mission. In particular, we have been looking at the possibility of having a high-inclination OOE probe complemented by a second probe going from Jupiter to the sun along a rectilinear path (at least for the segment from 0.3 a.u. inward to the sun).

The scientific interest in approaching close to the sun is obvious since it enhances observation of particles, fields, and gravitational harmonics. Our particular choice of path results from the associated simplicity of the spacecraft configuration needed to provide, for example, good thermal control, a drag-free system, and good communications with the earth.

A preliminary error analysis conducted by J. D. Anderson of the Jet Propulsion Laboratory leads to very interesting conclusions for an elliptical orbit with a perihelion distance of $16R_{\odot}$. Assuming that the nongravitational forces are compensated by a drag-free system (with three degrees of freedom) and that the spacecraft is tracked down to perihelion, the quadrupole moment of the sun can be determined with an accuracy of 3 parts in 10^7. Since the estimated value of J_2 ranges from 3×10^{-5} (applying Dicke's theory) to 1×10^{-7} (assuming rigid rotation of the interior with the observed surface), the interest in determining this moment with an accuracy of at least 1 part in 10^7 is clear. We remember that J_2 gives a fundamental constraint to the moment of inertia (or the ratio $C/MR_{\odot}^2$) and, therefore, on the internal density distribution of the sun.

As mentioned above, the result obtained by Anderson implies a three-axis drag-free system with an accuracy of 10^{-8} cm/sec^2. A drag-free system having this accuracy has recently been flown in the TRIAD satellite. If, however, we choose a solar-impact trajectory, then by using a spinning spacecraft, a one-axis drag-free system can be implemented that requires much less complexity. In fact, a spacecraft spinning about an axis aligned with the rectilinear path would allow 1) gyrostabilization (from 0.3 a.u. to the sun), 2) an easier design for thermal shielding, and 3) a one-degree-of-freedom drag-free system. In particular, a sphere with an electrostatic suspension is free to move along the spin axis with no exchange of forces along the path. The displacement of the sphere along the spin axis will be sensed, causing the thruster (oriented along the spin axis) to compensate the nongravitational forces along the path to the desired accuracy. The spacecraft will be forced to follow the proof mass and, therefore, to follow a purely gravitational path. Transverse forces should be 4 orders of magnitude smaller and need not be compensated. The drag-free system can be calibrated when the probe is far from the sun (5 a.u.) in order to find the equilibrium position along the spin axis of the proof mass in the gravity field of the spacecraft.

During the 3.5 days that the spacecraft will spend in going from 0.25 a.u. to 0.01 a.u. and closer, the earth–sun-probe geometry will permit the earth to be in the beam of the 0.2-m-diameter antenna pointing parallel to the spacecraft spin axis. The Jupiter-swingby technique has enough flexibility to enable the mission to be timed so that the earth–spacecraft line remains within a few degrees of the direction of the spacecraft track. A 20-cm dish mounted on the spacecraft operating in the X band has a beamwidth of 8° and a gain of 27 db. By using a 64-m dish on the earth, this will allow a transmission data rate from 50 to 100 bits/sec, even with the noise of the sun in the background. Doppler tracking using two frequencies in the X band (8 and 12 GHz) should yield a relative-velocity measurement accuracy of the order of 10^{-2} cm/sec (1 σ) with 60-sec integration time.

From Jupiter inward to 0.5 a.u., the spacecraft will operate in the Pioneer mode, performing selected experiments related to the solar stereoscopic and OOE missions. The two-frequency radio-science experiment will allow the integrated electron content to be determined at each instant, and perhaps a weighted component of the integrated magnetic field (from Faraday rotation).

From 0.5 a.u. inward, however, the mission will become more sun-oriented. The spacecraft spin axis will be directed sunward at this time, and the drag-free servo system will be activated. Some relevant probe parameters inside 0.5 a.u. are given in Table 1.

Table 1. Probe parameters.

Solar Distance (a.u.)	($R_\odot$)	Time to $4R_\odot$ (days)	Velocity (km/sec)	Equilibrium Temperature (°K)
0.5	100	10.5	54	215
0.4	80	7.4	60	235
0.3	60	4.8	70	275
0.2	40	2.6	85	335
0.1	20	0.9	121	470
0.05	10	0.25	171	680
0.02	4	0	270	1060

The temperatures shown in the table are surface temperatures related to a properly designed reflective heat shield that covers the "front" side of the spacecraft; the internal temperatures of the spacecraft will not necessarily be so high. Furthermore, the last few solar radii are traversed in less than an hour, during which time, thermal equilibrium will not be established. It is entirely possible that the system will survive to 2 $R_\odot$! If it does, a straightforward calculation shows that tracking to a doppler accuracy of 10^{-2} cm/sec over the half-hour interval required for the spacecraft to fall from 3 to $2R_\odot$ would permit J_2 to be determined to an accuracy of 10^{-8}. Since a realistic estimate of the magnitude of J_2 is about 10^{-7}, as is shown in the Appendix, an extremely valuable result would be guaranteed.

Obviously, many details of engineering design and scientific applicability remain to be worked out for this mission. But the preliminary effort so far expended appears more than sufficient to warrant this further pursuit.

APPENDIX

J_2 OF THE SUN

The value of J_2 can be inferred for a rotating axially symmetric body by means of the first-order formula (Jeffries, 1970)

$$\frac{3}{2} J_2 = f - \frac{1}{2} m \quad , \qquad (1)$$

where f is the flattening and $m = \omega^2 R_e / g_e$ is the ratio of centrifugal force at the equator to gravity at the equator. The assumptions made are that the gravitational potential is given by its first two terms only,

$$V_g = \frac{\mu}{r} \left[1 - J_2 \left(\frac{R_E}{r} \right)^2 P_2 (\sin \phi) \right] \quad , \qquad (2)$$

and that the surface is rotating uniformly so that the centrifugal force can be derived from the potential

$$V_c = \frac{1}{2} \omega^2 r^2 \cos^2 \phi \qquad (3)$$

Then the actual surface will be a level surface of the potential $V = V_g + V_c$.

If the surface is not rotating uniformly, we can modify equation (1) by assuming that, at any latitude, the surface will be perpendicular to the resultant of the gravity force given by the gradient of equation (2) and the centrifugal force equal to $\omega^2(\phi) r \cos\phi$ and directed away from the axis of rotation. Assuming that the shape of the surface is given by $r = R_E (1-Y)$, we find, to first order,

$$\frac{dY}{d\phi} = (3J_2 + \frac{R_E^3\,\omega^2(\phi)}{\mu})\sin\phi\cos\phi \quad , \tag{4}$$

We assume that the angular velocity of the sun's surface can be approximated by $\omega(\phi) = \omega_0 - \omega_2 \sin^2\phi$ and obtain

$$\frac{3}{2}\ J_2 = f - \frac{1}{2}\ m_0 \left[1 - \frac{\omega_2}{\omega_0} + \frac{1}{3}\left(\frac{\omega_2}{\omega_0}\right)^2\right] \quad , \tag{5}$$

where $m_0 = \omega_0^{\,2} R_e / g_e$. For the sun, we have $m = 2.14 \times 10^{-5}$, $\omega_0 =$ 14.4 °/day, and $\omega_2 \cong 4.5$°/day, so the second term in equation (5) lies between 7.7×10^{-6} and 10.7×10^{-6}, depending on whether the differential rotation is included or not. The best determination of the flattening of the sun (Hill, 1974) is $f = (9.6 \pm 6.5) \times 10^{-6}$. It is clear that J_2 cannot be derived with any accuracy from equation (5), since it is the difference between two not very well-known quantities of nearly equal magnitude.

If the sun is rotating uniformly and if the density distribution is known, J_2 can be directly calculated. Following an analysis by Sterne (1939a), we define the "apsidal motion coefficient,"

$$k = \frac{3 - \eta_s}{4 + 2\eta_s} \quad ,$$

where η_s is the value at the surface of the variable η, which is zero at $r = 0$ and which satisfies Radeau's equation:

$$r\frac{d\eta}{dr} + \eta^2 - \eta - 6 + \frac{6\rho}{\rho_m}(\eta + 1) = 0 \quad . \tag{6}$$

In equation (6), ρ is the density at r and ρ_m is the mean density interior to r. Then,

$$J_2 = \frac{2}{3}\ km \quad , \tag{7}$$

where m has been previously defined. The coefficient k depends solely on the distribution of mass within the star, ranging from zero for a completely concentrated star to 3/4 for a homogeneous star. Values of k have been calculated (Motz, 1952) for solar models by Schwarzschild (1946) and by Epstein (1951). Motz obtained $k = 0.00585$ and 0.00599, which leads to $J_2 = 8.3 \times 10^{-8}$ and 8.5×10^{-8}, respectively. Calculating J_2 for three later solar models, we found $J_2 = 1.56 \times 10^{-7}$ for a zero-age sun (Schwarzschild, 1958)and $J_2 = 1.41 \times 10^{-7}$ and 1.20×10^{-7} for two models of the present sun (Weymann, 1957, and Sears, 1964). Although we do not at present have detailed calculations of later solar models, we note that a recent one (Hoyle, 1975),proposed to explain the low neutrino emission from the sun, has the unusually low central density of 75 g/cm^3. The ratio of central to mean density is then 53.2, which is quite close to 54.2, the ratio of central to mean density of the "standard model," a polytrope of index 3. Russell (1928) found $k = 0.0144$ for a polytrope of index 3, so we consider 2×10^{-7} to be a reasonable upper limit to the value of J_2 for a uniformly rotating sun.

It is of interest to consider a lower bound to k and,hence, to J_2. The most concentrated star with a given central density is the generalized Roche model, which consists of a homogeneous core, with a density equal to the central density containing all the star's mass, and an envelope with infinitesimal density. Radeau's equation can then be solved analytically (Sterne, 1939b) to obtain

$$J_2 = \frac{1}{2} m \left(\frac{\rho_c}{\bar{\rho}} \right)^{-5/3} \tag{8}$$

With current estimates of the central density of the sun ranging from about 75 to about 150 g/cm^3, we find the lower limit of J_2 to be between 1.4×10^{-8} and 4.5×10^{-9}.

References

Epstein, I. 1951, Ap.J., 114, 4380.

Hill, H.A., et al. 1974, Phys. Rev. Letters, 33, 25, 1497.

Hoyle, F., 1975, Ap.J. (Letters), 197, L127.

Jeffries, H., 1970, The Earth, Fifth ed., Cambridge, p. 171.

Motz, L., 1952, Ap.J., 115, 562.

Russell, H. N., 1928, M.N., 88, 641.

Schwarzschild, M., 1946, Ap.J., 104, 203.

Schwarzschild, M., 1958, Structure and Evolution of the Stars, Princeton.

Sears, R.L., 1964, Ap.J., 140, 477.

Sterne, T.E., 1939a, M.N., 99, 451.

Sterne, T.E., 1939b, M.N., 99, 670.

Weymann, R., 1957, Ap.J., 126, 208.