COSMIC RAY MODULATION IN THREE DIMENSIONS

N76-24131

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Abstract. A brief critique of spherically symmetric conventional modulation theory is supplied. Estimates are made of the cosmic ray intensity at high solar latitudes. Direct evidence for significant off-ecliptic cosmic ray gradients is reviewed in support of the requirement for an off-ecliptic spacecraft mission. The possibility of measuring the galactic spectrum is discussed.

1. Spherically symmetric modulation theory and its problems.

Cosmic ray modulation arises because motion of the energetic particles along the spiral interplanetary magnetic field lines is controlled by scattering due to magnetic irregularities which are being convected outwards by the solar wind. Inward diffusion is balanced both by outward convection and energy loss of the particles as they suffer adiabatic deceleration in the expanding wind. A Fokker-Planck equation expressing these effects in a spherically symmetric, steady-state situation is (Parker 1965, Gleeson and Axford 1967)

$$\frac{1}{r^2}\frac{\delta}{\delta r}\left(r^2VU - r^2K_r\frac{\delta U}{\delta r}\right) = \frac{2}{3}\frac{V}{r}\frac{\delta}{\delta T}(\alpha TU) \qquad (1)$$

Here U is the differential particle number density at position r and kinetic energy $T, \alpha = \frac{T + 2T_o}{T + T_o}$ with T_o as rest mass energy, V is the solar wind velocity and K_r, the effective radial diffusion coefficient, is given by $K_r = K_{\parallel}\cos^2\psi + K_{\perp}\sin^2\psi$ for $K_{\parallel}$ and $K_{\perp}$ respectively the parallel and perpendicular diffusion coefficients with $\psi = \cos^{-1}(\underline{B}.\underline{r} / |\underline{B}|\,|\underline{r}|)$.

Particle streaming $\underline{S}$ is due to diffusion in the wind frame plus an additional term involving a Lorentz transformation to the rest frame via the Compton-Getting factor C such that

$$\underline{S} = C\underline{V}U - \tilde{K}.\mathrm{grad}\ U \tag{2}$$

with $C = 1 - \frac{1}{3U} \frac{\delta}{\delta T} (\alpha TU)$ and $\tilde{K}$ as the tensor diffusion coefficient.

At magnetic rigidities exceeding about 1 GV, radial streaming is usually negligible and under these circumstances the Fokker-Planck (1) can be integrated to give a relation between the Galactic density $U(r_g, T_g)$ and the observed density $U(r,T)$,

$$U(r, T) \doteq U(r_g, T_g) \exp \left(- \int_r^{r_g} \frac{CV}{K_r} dr\right) \tag{3}$$

for spherical symmetry with r_g determined by the effective outer bound to the interplanetary scattering process.

It is important for Astrophysics to know the energy spectra of various nuclear species of cosmic rays in the galaxy and a common method for achieving this "demodulation" is as follows:

(a) Estimate the Galactic electron spectrum from radio synchrotron data.

(b) Compare the near-earth electron spectrun with the Galactic spectrum to find the magnitude and rigidity dependence of $\int_r^{r_g} (CV/K_r).dr$.

(c) Use the results of (b) to correct the near-earth proton and heavy nuclei spectra for modulation.

Various objections can be raised to this scheme. First, the average Galactic electron spectrum derived from radio data may not represent the local spectrum and in any case one must be sure of the model enabling the effects of local, cold, absorbing interstellar clouds (Goldstein et al. 1970a) to be taken into account. Second, there is no adequate theoretical explanation as yet for the magnitude, rigidity dependence and radial dependence of K_r, as witnessed by investigations of solar proton diffusion (e.g. Webb et al. 1973) and the measurement of cosmic ray radial gradients (<10%/AU) which are much less than those expected on the basis of theoretical K_r values (e.g. Webber and Lezniak, 1973). It is important to know the r dependence of the integrand in (3) since adiabatic deceleration is inversely proportional to r. This importance becomes apparent in our third point, based on the work of Goldstein et al. (1970b) and Gleeson and

Urch (1971) who, for certain choices of r dependence and reasonable magnitudes of K_r, find that adiabatic deceleration may completely exclude all $\lesssim$100 MeV/nucleon particles from the inner solar system. Thus no certain knowledge of the lowest energy Galactic primaries is possible. Fourth, the cause of time variations in the modulation is unknown, with not enough solar cycle variation in the solar wind parameters being observed near earth to account for the known changes in the cosmic ray flux (e.g. Hedgecock et al. 1972). For example, the power spectrum of magnetic irregularities at 10^{-4} Hz should change by a factor >2 in order to account for the observed modulation of 1 GV particles between 1965 and 1969 according to resonant, wave/particle interaction theory while in practice the change is <10% (Hedgecock 1975). Off-ecliptic control of modulation via the effects far beyond 1 AU of solar streams emerging from the zones of maximum solar activity may be the only way to explain the cosmic ray 11-year cycle. Hence careful study by off-ecliptic spacecraft is required.

2. The off-ecliptic route to the boundary.

It has been thought that a direct determination of the Galactic cosmic ray charge and energy spectrum can be achieved employing ecliptic plane spacecraft trajectories to the outer planets. In this way the problems of section 1 are all by-passed. However, the boundary to modulation could be as far as 100 AU and the low, measured cosmic ray density gradients seen by Pioneer 10 render this approach uncertain. Alternatively, we note that cosmic rays have an easier inward motion over the solar poles where the geometric path length along the interplanetary field lines is much shorter than in the tightly wound spiral regime of the equatorial plane (Lietti and Quenby 1968). An appropriate Fokker-Planck equation for the steady state which relinquishes the requirement of spherical symmetry and takes into account the spiral geometry is then

$$\frac{1}{A}\frac{\delta}{\delta s}\left(AK_{\parallel}\frac{\delta U}{\delta s}\right) = \frac{1}{r^2}\frac{\delta}{\delta r}(r^2 CVU) \qquad (4)$$

if $K_{\perp} = 0$ for path length s along a magnetic flux tube of area A. Allowing $K_{\parallel} = K_e r \sin^q\theta$ where θ is solar latitude yields

$$U/U_g = (r/r_g)^{CVr/K_e} \quad \exp - \left\{ \frac{CVr_g^{\,2}\sin^{2-q}\theta}{2K_e r} \right\} \tag{5}$$

and if $q = o$ and $U/U_g = 1/e$ at 1 GV, we obtain the following table for the percentage residual modulation at 1AU:

	$\theta = 90^0$	$\theta = 30^0$	$\theta = 0^0$
U/U_g (at 1 GV)	63%	22%	9%

Thus on a simple model for scattering, a spacecraft passing over the poles at 1 AU may see ∿90% of the unmodulated intensity and therefore get a better measurement of galactic conditions than is available at Jovian distances.

3. Direct evidence for off-ecliptic gradients

Observations confined to the ecliptic plane can only reveal the existence of off-ecliptic effects by noting modifications to cosmic ray streaming from the expectations of the spherically symmetric model case. Equation (2) in a more general form is

$$\underline{S} = CU\underline{V} - K_{\parallel}\left(\frac{\delta U}{\delta r}\right)_{\parallel} - \frac{v^2}{3\omega}\left(\frac{\delta U}{\delta r}\right) \times \underline{B} \tag{6}$$

where v is energetic particle velocity and ω is cyclotron frequency. It has been assumed that direct slippage of particles across field lines makes only a small streaming contribution ($K_\perp/K_\parallel \stackrel{\sim}{\sim} \frac{1}{\omega^2\tau^2} \ll 1$, τ being time to travel one parallel mean free path). Furthermore, we assume the short-circuiting by scattering of that perpendicular gradient which would cancel out the anisotropy due to $(\underline{E} \times \underline{B})_\perp$ drift in the non-scattering limit when Liouville's theorem applies to the particle intensity. At high rigidities, some few to a hundred GV, the radial streaming is negligible over long periods with the third term on the right of (b) cancelling out on average. Then the first two terms combine to give the streaming from the east or 1800 hr LT anisotropy. When, however, the anisotropy is studied in practice as a function of sign of the interplanetary field sector structure, two effects of the third term become apparent. A north-south anisotropy arises due to $\left(\frac{\delta U}{\delta r}\right)_{radial} \times \underline{B}$, or the effect of the radial gradient and an ecliptic plane anisotropy arises due to $\left(\frac{\delta U}{\delta r}\right)_z \times \underline{B}$, or the effect of off-ecliptic gradient. Hashim and Bercovitch (1972) find $G_z = 5.5\ R^{-0.6}$%/AU directed north → south in 1967/68 for the latter effect, possibly physically resulting from the excess northern hemisphere solar activity suggested at that time by coronal green line 5303A emission.

The previous discussion refers to the first derivative of density, but studies of the second harmonic of the cosmic ray intensity can reveal the presence of a rising or falling, symmetric, off-ecliptic gradient via a dependence on the second derivative. Lietti and Quenby (1968) essentially use a version of (5) for the rising gradient case to predict a second harmonic with direction of maximum perpendicular to $\underline{B}$ and amplitude $a_2 = \frac{1}{2}\frac{\rho^2}{r^2}\frac{\delta^2 U}{\delta\theta^2} \sim 0.005P$ % at rigidity P, cyclotron radius ρ. This expression is in reasonable accord with observations although Nagashima et al. (1972) claim that a cylindrical pitch angle particle distribution about $\underline{B}$ with a different physical cause better fits cosmic ray anisotropy data.

With finite $K_{\perp}$, the symmetrical gradient will either feed particles into the equatorial plane or draw them off to higher latitudes, thus setting up radial streaming. A correction to the Fokker-Planck (1) is employed by adding

$$\mathrm{div}\left[-\frac{K_{\parallel}}{\omega^2\tau^2}\left(\frac{\delta U}{\delta r}\right)\right]$$

to the right hand side. Dyer et al. (1974) in particular evoke this streaming for a falling gradient to explain a sunward flow at $\gtrsim$ 1 GV seen by a satellite detector which is too large to be explained by any energy loss effects in a spherically symmetric model. These authors require maximum modulation over the sunspot zones with meridianal flow patterns set up to draw particles down from $\theta = 0^0$ and up from $\theta = 90^0$. Cecchini et al. (1975) have developed a computational model to confirm the above model. Chief features are:

$$K = K_o\beta P\ f(\theta)\begin{cases} \exp\ {}^{r}/_{r_o} / \exp(1) & r < 1\ \mathrm{AU} \\ {}^{r}/_{r_o} & r > 1\ \mathrm{AU} \end{cases}$$

$$K = \varepsilon K_o\beta P_o\ f(\theta)\begin{cases} ({}^{r}/r_o)^3 & r < 1\ \mathrm{AU} \\ ({}^{r}/r_o) & r > 1\ \mathrm{AU} \end{cases}$$

$$f(\theta) = 1 + \cos\theta\ (-3 + 5\cos^2\theta)\ ;$$

$$K_o = 2.2\ 10^{19}\ \mathrm{cm}^2/_{\mathrm{sec\ Mev}}\ \text{at 1AU}\ ;$$

$\varepsilon = 0.5$, 1AU; V_w = const.; P_o = 100 MV; $U_g \,\alpha (T + T0)^{-2.75}$

Thus $K_\perp/K_\parallel = 0.05$ at 1 GV. The results of employing an alternating gradient technique to solve the corrected Fokker-Planck, with finite $K_\perp$, is to predict an inward streaming $\sim$0.3% in amplitude between 2 and 10^{-1} GeV, a radial gradient $\sim$10%/AU at 1.1 GeV between 1 and 10 AU and a ratio $U(\theta)/U(\theta = \frac{\pi}{2})$ at 1 AU varying from 2.5 at $\theta = 0$ to 0.7 at $\theta = 60^0$ for 1.1 GeV protons. Hence it is possible to explain the radial streaming with reasonable gradients and $K_\perp/K_\parallel$ ratios.

4. Conclusions.

We have shown that study of cosmic ray modulation by integrating the transport equation outwards in the ecliptic plane, assuming spherical symmetry, encounters various problems. The transport processes and boundary conditions are insufficiently well understood, modulation may be controlled by off-ecliptic gradients and assymetries can have noticeable effects on solar equatorial plane observations. Three-dimensional study of the solar cavity cosmic ray distribution is required to:

(a) Measure off-ecliptic gradients and streaming.

(b) Enhance understanding of the solar control of intensity time variations.

(c) Gain better knowledge of boundary conditions, especially the possibility of measuring a near-Galactic energy spectrum over the solar poles.

Objectives (a) and (c) are satisfied by a Jovian swing-by mission but (b) requires a direct injection at 1 AU spacecraft for detailed time variation studies on solar wind and solar parameters.

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COSMIC RAY ACCESS AT POLAR HELIOGRAPHIC LATITUDES

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Abstract

Based on a modified WKB analysis of the interplanetary irregularity spectra, a discussion of the radial dependence of the radial cosmic-ray diffusion coefficient at polar heliographic latitudes is presented. At 1 AU radial distance the parameters are taken to equal those observed in the ecliptic. In the sense of a present best estimate it is argued that relativistic nuclei should have significantly easier access to 1 AU at the pole than in the ecliptic. The reverse may very well be true for the direct access of very low rigidity particles.

1. Introduction

The access of galactic cosmic rays to the inner solar system is regulated by the cumulative effect of the irregular fields which scatter the particles on their way in. To determine the degree of this access ... heliographic latitudes that are significantly different from the solar equatorial plane, one has to rely on theoretical estimates. These conventionally assume a diffusive propagation scheme. Of primary importance is then the spatial diffusion coefficient or, more generally, the diffusion tensor, and in particular its spatial variation in the solar system.

In the earlier work of Võlk et al. (1974) the irregular fields were assumed to be Alfvên waves of solar origin, convected outwards by the solar wind. In a (on the average) stationary interplanetary medium with axial symmetry around the solar rotation axis, this led to a radial direction for almost all wave normals beyond about 1 AU. Assuming the effective average magnetic field to be in the ideal spiral field direction and the wave amplitudes to vary according to the (WKB) approximation of geometrical optics, a radial dependence of the coefficient for diffusion along the average field was calculated. As a sensitive function of the angle between wave normals and average field, the value of this diffusion coefficient, or equivalently, of the mean free path, varies from a minimum at 0 degrees to infinity at 90 degrees. Since in the solar equatorial plane the angle between the radial and the spiral field direction is large already at 1 AU, and increases with radial distance, deviations of the wave normals from the radial direction have been discussed by Richter (1974) in the context of the solar wind stream structure; see also Hollweg (1975).

We do not intend to evaluate here the effects of deviations from the simple picture of Völk et al. (1974) at low latitudes and refer to a future publication (Morfill et al., in preparation). We shall rather concentrate on the situation at polar heliographic latitudes. There the assumption of all wave vectors $\underline{k}$ being radial and parallel to the average field $\underline{B}$ leads to the minimum value of the radial diffusion coefficient κ as far as its dependence on the angle between $\underline{k}$ and $\underline{B}$ is concerned. Thus, within the WKB approximation for the radial development of the scattering centers, this provides a lower limit for κ. Such a lower limit is of interest, if it can be argued, or at least be speculated in a reasonable way that even in this case there is essentially unimpeded access to about 1 AU for a significantly larger part of the galactic energy spectrum than in the solar equatorial plane.

We shall give a short discussion here, motivated by three considerations. The first one is that in two earlier publications this author was associated with (Völk et al., 1974; Völk, 1975), a rather different result was believed to hold true. Another reason is given by the present discussion of an ex-ecliptical probe to explore the sun and the interplanetary medium. The final reason is that via easy access at the poles much larger regions of interplanetary space might possibly be populated by particles of galactic origin.

In the next section we shall present the general behavior of κ in a modified WKB analysis, using simple approximations to two rather different measured interplanetary power spectra. In the last section the relation to the expected actual situation at the heliographic pole is discussed.

2. WKB Analysis

Consider the simplest, axisymmetric, interplanetary medium, where all quantities depend on heliocentric distance r and where only the ideal spiral field $\underline{B}$ with components

$$(1)\quad B_r = B_{r_0}\left(r_0/r\right)^2 ; \quad B_\Theta = 0 ; \quad B_\phi = B_{r_0}\cdot\frac{r_0}{r}\cdot\frac{r_0\cdot\Omega_s\cdot\sin\theta}{V}$$

and the power spectrum P (f,r,θ) may in addition depend on heliographic latitude θ. Here r_o is a radial reference level, B_{ro} is independent of θ; ϕ denotes heliographic longitude, $\Omega_s \approx 2.65 \times 10^{-6}$ cps is the angular frequency of the sun's rotation, V is the solar wind speed (assumed to be radial and constant), and f is the wave frequency seen by an observer at rest relative to the sun's center. Then it is simple to show (Völk et al., 1974) that for $\theta \longrightarrow \frac{\pi}{2}$ the radial diffusion coefficient is given by

$$(2)\quad K = \beta\, R^2\, \frac{2c}{(V+|\underline{v}_A|)}\, \frac{|\delta B_k(r_1,\theta)|^2}{|\delta B_k(r,\theta)|^2}$$

$$\cdot\int_0^1 d\mu\cdot\mu\cdot(1-\mu^2)\left\{P\left(f=\frac{1}{2\pi}\left|\frac{B(V+|\underline{v}_A|)}{R\cdot\mu}\right|, \theta=\frac{\pi}{2}, r_1\right)\right\}^{-1}$$

In equation (2) we have $\beta = \frac{w}{c}$, the ratio of particle velocity w to the velocity of light c; R denotes particle rigidity; $\underline{v}_A = \underline{B}\cdot(4\pi\rho)^{-1}$ is the (vectorial) Alfvén - speed, where ρ is the average solar wind mass density; $\mu = w_\parallel / w$ is the cosine of the particle's pitch angle, where $w_\parallel$ is the velocity component parallel to $\underline{B}$; $r_1 = 1$ AU. The amplification factor for the Alfvén wave amplitudes δB_k

$$(3)\quad \frac{|\delta B_k(r)|^2}{|\delta B_k(r_1)|^2} = \left(\frac{V + V_{Ar}(r_1)}{V + V_{Ar}(r)}\right)^2 \cdot \frac{V_{Ar}(r)}{V_{Ar}(r_1)} \left(\frac{r_1}{r}\right)^2 \approx \left(\frac{r_1}{r}\right)^3$$

is independent of wavevector $\underline{k}$ and Θ. If both r_{11} and r are large compared to the Alfvénic critical radius ($\lesssim 20\ R_\odot$ in the equatorial plane), then the approximation on the far r.h.s. of equation (3) holds in this lowest order WKB approximation. However, following equations (1) and (3) we have (at $\Theta = 90°$): $\langle \delta \underline{B}^2(r, \Theta) \rangle \;/\; \underline{B}^2(r,\Theta) \sim r$, where $\langle \delta B^2 \rangle$ is the total power in the fluctuations. Thus, the possibility arises that $\langle \delta B^2 \rangle / \underline{B}^2 > 1$ beyond some radial distance in which case we expect wave propagation to be rather drastically altered by nonlinear effects that ultimately should lead to dissipation of wave energy. To take this possible effect into account, we modify equation (3) so that for all r

$$(3a)\quad \langle \delta B^2(r) \rangle \lesssim \underline{B}^2(r)$$

A similar device has been used by Hollweg (1973) and appears as a simple if crude way to take the inadequacy of the linearised WKB approximation into account.

It is clear physically that equations (1), (2), (3) and (3a) also hold if, at given r_o, B_{ro}, V and ρ are different from their values at $\Theta \approx 0$, as long as their dependence on Θ is weak enough, such that for example $\frac{1}{V}\frac{dV}{d\Theta} \ll 1$. For the rest of this section, however, we will consider $B_r\ (r_1)$, V, $\rho\ (r_1)$ and $P(f, \Theta, r_1)$ as given by their values in the equatorial plane.

Observed power spectra at $\theta \approx 0$ and $r \approx r_1$ (e.g. Jokipii and Coleman, 1968; Bercovitch, 1971), generally approximate rather well a power law for $P(f, r_1)$ at high frequencies, while flattening at low frequencies. Qualitatively, an analytical form

$$P(f, r_1) = C_f \cdot \frac{f_0^{\,q}}{1+(f/f_0)^{-q}} \tag{4}$$

with parameters C_f, f_0, and $q < 0$, is not an unreasonable representation. With equation (4) the general character of κ (r) can be inferred. For small enough rigidity R, so that $|\underline{B}| \cdot V/R > 2\pi f_0$ we have $\kappa \sim r^{3+2q}$ whereas $\kappa \sim r^3$ for $|\underline{B}| \cdot V/R \ll 2\pi f_0$. The exponent of r is additively increased by unity whenever the supplementary restriction (3a) comes into force. For our present discussion we choose $q = -3/2$. Thus, $\kappa \approx$ const (or $\sim r$) for small $R \cdot r^2$, whereas $\kappa \sim r^3$ (or $\sim r^4$) for large $R \cdot r^2$. At sufficiently large r, κ will be $\sim r^3$ (or $\sim r^4$) for any R. At fixed values of q and R, the transition of κ to the r^3 (or r^4) dependence increases with decreasing f, cf. equation (2). As an aside we should mention that for this value of q and for the present case of $\underline{k} \parallel \underline{B}$ the quasilinear expression for κ used in writing in equation (2) is numerically quite satisfactory and the modifications in the region around $\mu = 0$ (e.g. Jones et al., 1973) therefore not essential.

Numerical results are shown in Figures 1 and 2. Figure 1 is calculated using the spectrum of Jokipii and Coleman (1968) where, in reference to equation (4), $C_f \approx 16 \times 10^{-3} \cdot \gamma^2 (\mathrm{Hz})^{-1-q}$; $f_0 \approx 7 \times 10^{-5} \cdot \mathrm{Hz}$; $q \approx -3/2$, and Alfvén wave propagation was started at $r_0 = 20\ R_\odot$ (solar radii). Figure 2 uses the spectrum published by Bercovitch (1971) which we roughly approximate by $C_f \approx 6 \times 10^{-3} \cdot \gamma^2 (\mathrm{Hz})^{-1-q}$; $f_0 \approx 7 \times 10^{-6} \cdot \mathrm{Hz}$; $q \approx -3/2$. Although the results are not very much different for both cases near $r = 1$ AU, the very different f_o-values lead to strong differences in the onset distance of the r^3 (or rather r^4) law. We should mention here that we used here the component of the magnetic spectral tensor perpendicular to the ecliptic to represent the total power per frequency interval. For true axisymmetry of the spectrum all κ values in Figures 1 and2 should be multiplied by a factor 1/2. This is an unavoidable uncertainty

The interesting aspect of these results is that they imply little modulation for relativistic nuclei, where adiabatic deceleration is small, considering the value of f_o in Figure 2 as a rather extreme lower limit to the actual situation. For 1 GeV protons, for example, a 10-20 percent modulation is estimated in the diffusion convection approximation.

3. Discussion

The above results were obtained by fitting, at 1. AU, the average solar wind parameters as well as power spectra by the corresponding quantities observed at 1 AU in the ecliptic; the spatial dependence of the spectra assumed a modified WKB approximation for the irregularities. In reality, the polar region of the corona may well be a large, stationary coronal hole, resulting in a somewhat (perhaps fifty percent) larger flow speed and, possibly, somewhat more power in the frequency region $f \gtrsim f_o$. All this would lead to a moderately increased modulation.

On the other hand, it may very well be that the power in frequencies $\lesssim f_o$ is much smaller at $\theta = \pi/2$ than near $\theta = 0$. We have taken these fluctuations also to be Alfvénic which leads to the amplitude variation given in equation (3). In reality, the part in the spectrum with $f \lesssim f_o$ may well be due to the solar wind stream structure (Goldstein and Siscoe, 1972). If the latter is assumed to be absent at $\theta = 90^o$, also the power at $f \lesssim f_o$ would be absent with a corresponding decrease in modulation. In this light, also the possibility of increased scattering at larger distances due to local production of waves - a situation that is quite likely at $\theta \approx 0$ - appears rather weak. Irregularities produced by enhanced (compared to the ecliptic) turbulence due to radially increasing departures from thermal equilibrium at the poles should be assumed to have small scales, irrelevant for cosmic rays even in the case of the Firehose instability. It should be added that in contrast to a popular feeling this result for κ and the consequent argument for modulation has little to do with the shorter geometrical path along $\underline{B}$ of a galactic particle to, say, 1 AU, but rather to the strong decrease with r of the magnetic field at the poles.

Thus, barring unknown new effects, the present best estimate is that cosmic ray access at the poles should be significantly better than at $\theta \approx 0$ for relativistic nuclei. For very low-rigidity particles on the other hand, the sharp increase of κ with r occurs only at such a large radial distance that their direct access may be at least as strongly prohibited as in the equatorial plane. However, for this last kind of statement, the present estimate is not well suited.

Acknowledgment:

The author would like to thank Dr. G. Morfill for discussions and the numerical calculation of the results.

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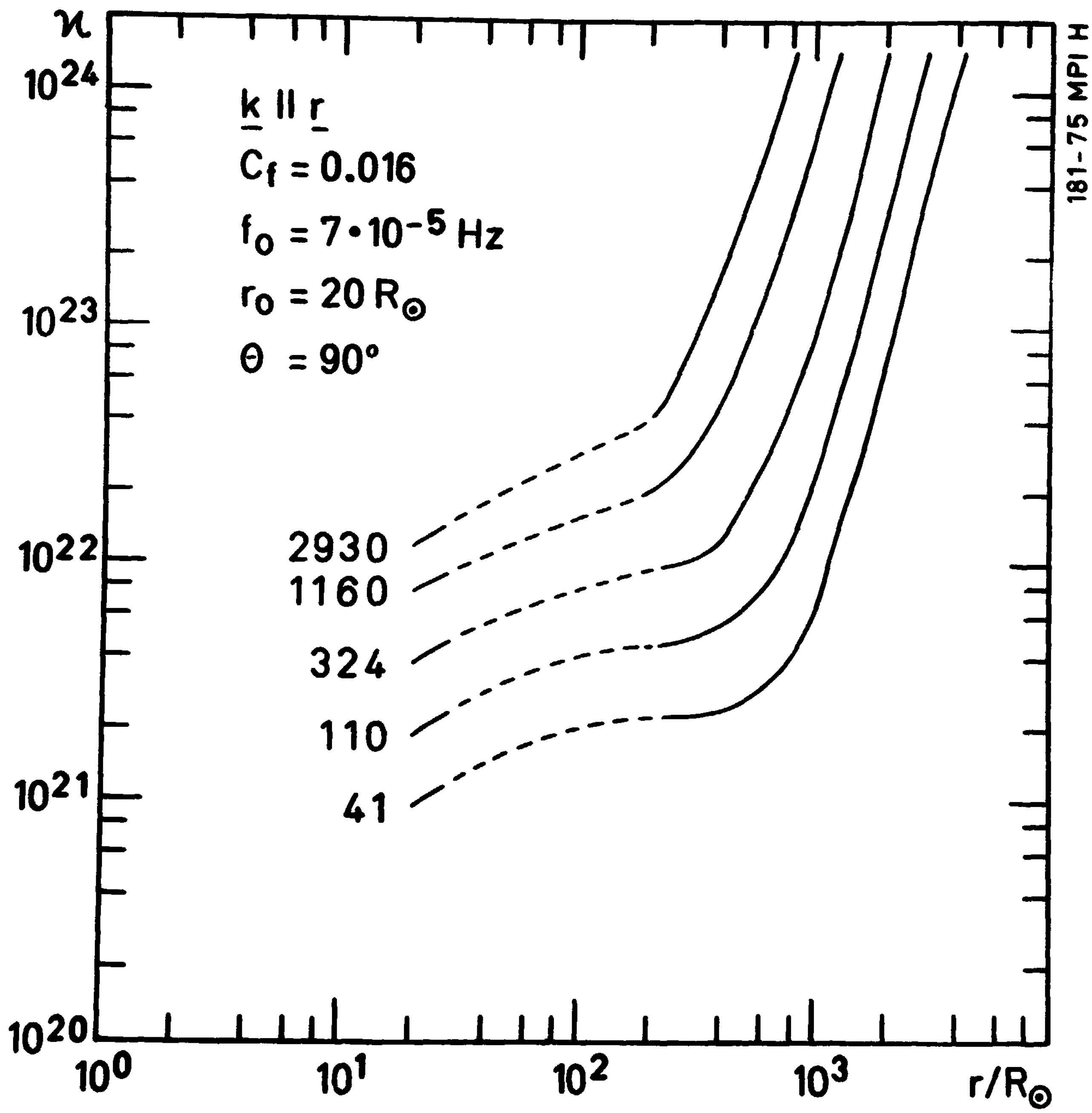
Figure Captions

Figure 1: The radial diffusion coefficient κ at heliographic latitude $\theta = 90^{o}$ as a function of radial distance r in solar radii $R_{\odot}$ for various proton energies. The power spectrum $P(f)=C_f \cdot f^q$ with the values $C_f = 16 \times 10^{-3} \gamma^2 (Hz)^{-1-q}$, $f_o = 7 \times 10^{-5}$ Hz (Jokipii and Coleman, 1968). Wave normals $\underline{k}$ are assumed to be radial. The calculation was started at $r_o = 20\ R_{\odot}$.

Figure 2: The same as Figure 1 with values of C_f and f_o adapted to the spectra of Bercovitch (1971).



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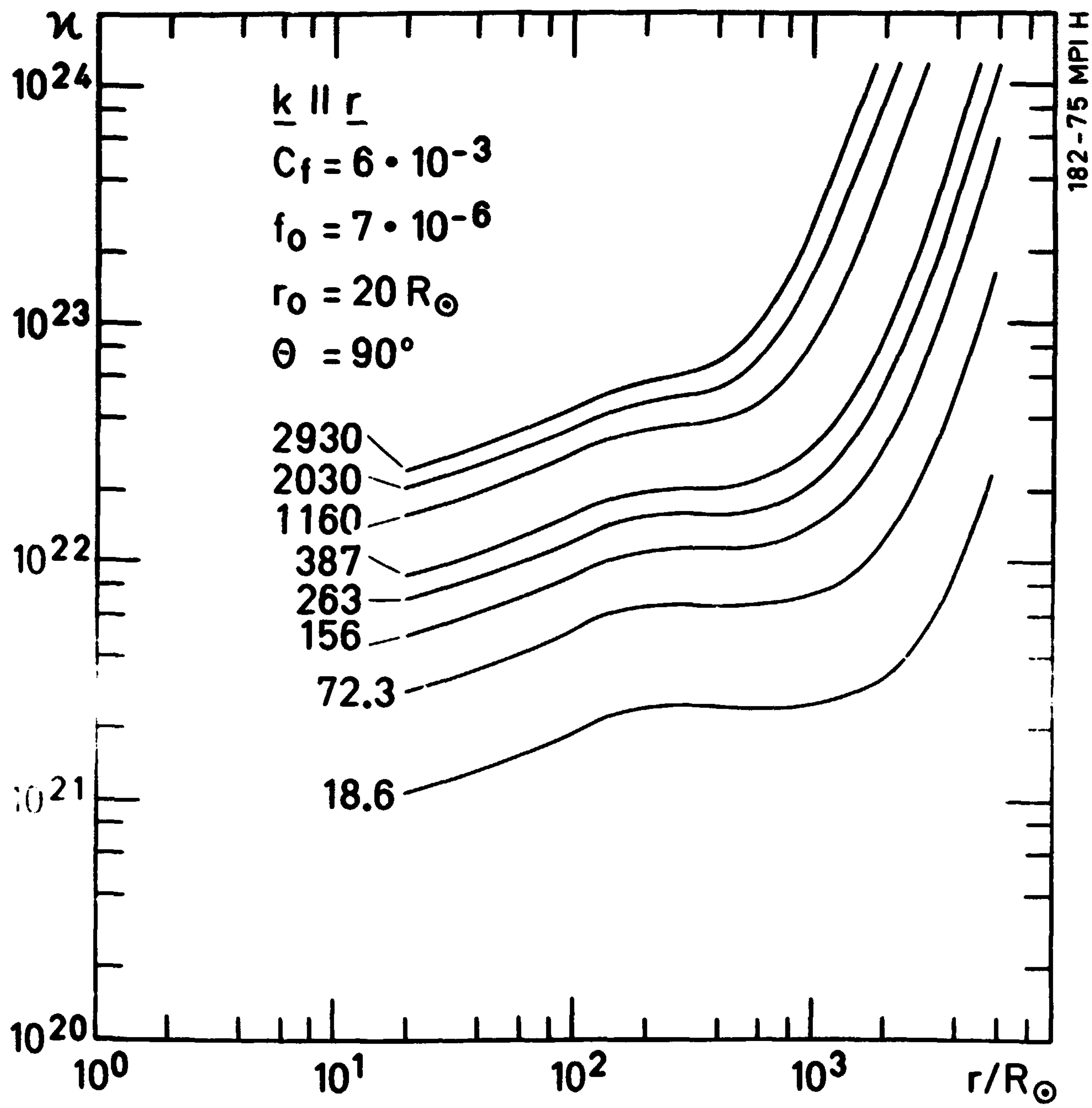

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10²⁴
10²³
10²²
10²¹
10²⁰
k ll r
Cf = 6 • 10⁻³
f0 = 7 • 10⁻⁶
r0 = 20 R⊙
Θ = 90°
2930
2030
1160
387
263
156
72.3
18.6
10⁰
10¹
10²
10³
r/R⊙
182-75 MPI H