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DEFINING CONSTANTS, EQUATIONS,
AND ABBREVIATED TABLES OF
THE 1975 U.S. STANDARD ATMOSPHERE

*R. A. Minzner, C. A. Reber, L. G. Jacchia,
F. T. Huang, A. E. Cole, A. J. Kantor,
T. J. Keneshea, S. P. Zimmerman,
and J. M. Forbes*

*Goddard Space Flight Center
Greenbelt, Md. 20771*



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16. Abstract <p>The <i>U.S. Standard Atmosphere, 1975</i> (COESA, 1975) is an idealized, steady-state representation of the earth's atmosphere from the surface of the earth to 1000-km altitude, as it is assumed to exist in a period of moderate solar activity. From 0 to 86 km, the atmospheric model is specified in terms of the hydrostatic equilibrium of a perfect gas, with that portion of the model from 0 to 51 geopotential kilometers (km') being identical with that of the <i>U.S. Standard Atmosphere, 1962</i> (COESA, 1962). Between 51 and 86 km, the defining temperature-height profile has been modified from that of the 1962 Standard to lower temperatures between 51 and 69.33 km, and to greater values between 69.33 and 86 km. Above 86 km, the model is defined in terms of quasi-dynamic considerations involving the vertical component of the flux of molecules of individual gas species. These conditions lead to the generation of independent number-density distributions of the major species, N₂, O₂, O, Ar, Ne, and H, consistent with observations. The detailed definitions of the model are presented along with graphs and abbreviated tables of the atmospheric properties of the 1975 Standard.</p>					
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FOREWORD

Tables of the U.S. Standard Atmosphere to heights in excess of 300 km have appeared in various editions over the past 18 years, the most recent being the *U.S. Standard Atmosphere, 1975* (COESA, 1975) to heights of 1000 km. These publications have resulted from the work stimulated and guided by the Committee for Extension to the Standard Atmosphere (COESA). This committee has a membership of about 30 organizations in government, industry, research institutions, and universities. The particular organizational representations in COESA resulted primarily from either the organization's scientific or administrative needs, or the organization's potential, for contributions to a standard atmosphere. In several cases, however, an organization's association with COESA stemmed largely from the interests and talents of individual scientists employed by that organization.

During the course of this work, COESA was led by three co-chairmen, each associated with one of the three principal sponsoring government agencies. These three agencies and the co-chairmen were: National Aeronautics and Space Administration (NASA), Maurice Dubin; National Oceanic and Atmospheric Administration (NOAA), Arnold R. Hull; and United States Air Force (USAF), K. S. W. Champion.

The requirements and viewpoints of the many organizations participating in COESA have shaped the format of the *U.S. Standard Atmosphere, 1975* (COESA, 1975). The scientific foundation for this Standard, however, is based upon the views of the international community of scientists including, to a large extent, the members of the Working Group of COESA. This Working Group, comprised of individual scientists representing nearly all of the 30 organizations in the parent committee, has based the U.S. Standard Atmosphere publication on current scientific and technical knowledge, rather than on policy concerns.

The Working Group established five task groups for the preparation of the 1975 Standard; some members contributed to more than one task group. Each of these task groups was charged with reviewing a particular category of available atmospheric data, and with preparing recommendations for possible revisions of the *U.S. Standard Atmosphere, 1962* (COESA, 1962). Three task groups (Task Groups I, II, and III) dealt with data involving the three overlapping height regions: 50 to 100 km, 80 to 200 km, and 140 to 1000 km, respectively. Task Group IV was charged with combining the recommendations of Task Groups I, II, and III into an internally-consistent model. Task Group V reviewed the present-day knowledge of minor constituents.

The membership of each of the five task groups is listed in the Foreword of the *U.S. Standard Atmosphere, 1975* (COESA, 1975). This book also includes an extensive section of detailed tables of the properties of the Standard, which applies to a mean, midlatitude atmosphere for heights from 0 to 1000 km. The rationale for the model and an extensive discussion of the minor atmospheric constituents are also included.

By contrast, *Defining Constants, Equations, and Abbreviated Tables of the 1975 U.S. Standard Atmosphere* emphasizes the definitions required to generate the tables of atmospheric properties of the Standard. This report also contains an abbreviated set of these tables, a set of graphs depicting the height profiles of these properties, and a discussion of two properties not considered in the 1975 Standard, that is, geopotential pressure scale height and geopotential density scale height.

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SYMBOLS

- a One of the constant coefficients used to specify the elliptical segment of the temperature-height profile, $T(Z)$
- a_i A set of species-dependent coefficients which, along with values of b_i , are used to define the set of height-dependent functions, D_i
- A Another constant coefficient used to specify the elliptical segment of $T(Z)$
- A_j A reaction-dependent rate coefficient used along with values B_j and C_j , to define k_j
- b A dimensionless subscript designating a set of integers $\{0, 1, 2, 3 \dots\}$ with 0 specifying sea-level conditions
- b_i A set of species-dependent exponents which, along with values of a_i , are used to define the set of height-dependent functions, D_i
- B_j A dimensionless, reaction-dependent exponent used in the expression for k_j
- C_j A reaction-dependent rate factor used as a part of an exponential expression in the definition of k_j
- C_s The height-dependent speed of sound
- D_i The set of height-dependent, species-dependent, molecular-diffusion coefficients for the atmospheric gas species O, O₂, Ar, He, and H
- $f(Z)$ The hydrostatic term in the height-dependent expression for n_i
- F_i The dimensionless, sea-level, fractional-volume concentration of the *ith* member of the set of atmospheric gas species
- F'_i The dimensionless, fractional-volume concentration of the *ith* member of the set of atmospheric gas species adjusted for 86-km height to account for the dissociation of O₂

g	The height-dependent acceleration of gravity ($g(Z)$) for 45°
g'_0	The adopted constant involved in the definition of the standard geopotential meter, and in the relationship between geopotential height and geometric height
H	The geopotential height used as the argument for all tables up to $84.8520 \text{ km}'$ (86 km)
H_p	The height-dependent, local pressure scale height of the mixture of gases comprising the atmosphere
H'_p	The height-dependent, local geopotential pressure scale height of the mixture of gases comprising the atmosphere, and dependent upon the single variable, T_M
H_ρ	The height-dependent, local density scale height of the mixture of gases comprising the atmosphere
H'_ρ	The height-dependent, local geopotential density scale height of the mixture of gases comprising the atmosphere
i	A dimensionless subscript designating the i th member of a set of gas species
j	A dimensionless subscript designating the j th member of a set of chemical reactions
k	The Boltzmann constant
k_j	The reaction rate of the j th chemical reaction
k_t	The height-dependent coefficient of thermal conductivity
K	The height-dependent, eddy-diffusion (or turbulent-diffusion) coefficient
L	The height-dependent, mean free path
$L_{K,b}$	A set of gradients of T with respect to Z
$L_{M,b}$	A set of gradients of T_M with respect to H
M	The height-dependent, mean molecular weight of the mixture of gases constituting the atmosphere

M_i	The set of molecular weights of the several atmospheric gas species
n_i	The set of height-dependent number densities of the several atmospheric gas species
N	The height-dependent, total number density of the mixture of neutral atmospheric gas particles
N_A	The Avogadro constant
P	The height-dependent, total atmospheric pressure
P_i	The partial pressure of the <i>ith</i> gas species
q_i	One set of six adopted sets of species-dependent constants, that is, sets q_i , Q_i , u_i , U_i , w_i , and W_i , all used in an empirical, species-dependent expression for the flux term $v_i/(D_i + K)$
Q_i	See q_i
r_0	The adopted, effective earth's radius used to compute $g(Z)$ for 45° North latitude, and used for relating H and Z at that latitude
R^*	The universal gas constant
S	The Sutherland constant, used in computing μ
t	The height-dependent Celsius temperature
T	The height-dependent, Kelvin kinetic temperature, defined as a function of Z for all heights above 86 km, and derived from T_M for heights below 86 km
T_c	The temperature coordinate of the center of the ellipse defining a portion of $T(Z)$
T_M	The height-dependent, molecular-scale temperature defined as a function of H for all heights from sea-level to 86 km
T_∞	The exospheric temperature
u_i	See q_i

U_i	See q_i
v_i	The flow velocity of the i th gas species
v_m	The height-dependent mole volume
V	The height-dependent mean particle speed
w_i	See q_i
W_i	See q_i
Z	Geometric height used as the argument of all tables at heights above 86 km
Z_c	The height coordinate of the center of the ellipse defining a portion of $T(Z)$
α_i	The set of species-dependent, thermal-diffusion factors
β	A constant used for computing μ
γ	A constant representing the ratio of specific heat at constant pressure to the specific heat at constant volume used to define C_s
Γ	The ratio g_0/g'_0
ϵ	A dimensionless factor relating F_i to F'_i
η	The height-dependent kinematic viscosity
λ	A coefficient used to specify the exponential expression defining a portion of $T(Z)$
μ	The height-dependent coefficient of dynamic viscosity
ν	The height-dependent mean collision frequency
ξ	A function of Z used in the exponential expression defining a portion of $T(Z)$
ρ	The height-dependent mass density of air
σ	The effective mean collision diameter used in defining L and ν

- τ A height-dependent coefficient representing the reduced height of the atomic hydrogen relative to a particular reference height, and used in the computation of $n(H)$ (number density of hydrogen)
- ϕ The vertical flux of atomic hydrogen
- Φ_G The potential energy per unit mass of gravitational attraction
- Φ_C The potential energy per unit mass associated with centrifugal force

DEFINING CONSTANTS, EQUATIONS, AND ABBREVIATED TABLES OF THE 1975 U.S. STANDARD ATMOSPHERE

R. A. Minzner and C. A. Reber
Goddard Space Flight Center

L. G. Jacchia
Smithsonian Astrophysical Observatory

F. T. Huang
Computer Science Corporation

A. E. Cole, A. J. Kantor, T. J. Keneshea,
S. P. Zimmerman, and J. M. Forbes
Air Force Cambridge Research Laboratories

INTRODUCTION

The *U.S. Standard Atmosphere, 1975* (COESA, 1975) is an idealized, steady-state representation of the earth's atmosphere from the surface of the earth to 1000-km altitude, as it is assumed to exist in a period of moderate solar activity. For heights from the surface to 51 geopotential kilometers (km'), this Standard is identical to the *U.S. Standard Atmosphere, 1962* (COESA, 1962), and is based on traditional definitions. These definitions, especially for heights below 20 km', were developed over a period of many years, and do not necessarily represent an average of the vast amount of atmospheric data available today from observations within that height region. For heights from 51 to 84.8520 km' (that is, 51.413 to 86 geometric kilometers), the 1975 Standard is consistent with averages of present-day atmospheric data, as represented by the traditional type of defining parameters. These include the linearly-segmented temperature-height profile, and the assumption of hydrostatic equilibrium, in which the air is treated as a homogeneous mixture of the several constituent gases.

At greater heights, however, where dissociation and diffusion processes produce significant departures from homogeneity, the definitions governing the 1975 Standard are more sophisticated than those used at lower altitudes. In this high-altitude regime, the hydrostatic equation, as applied to a mixed atmosphere, gives way to the more general equation for the vertical component of the flux for individual gas species (Colegrove et al., 1965; Keneshea and Zimmerman, 1970), which accounts for the relative change of composition with height. This flux equation simplifies to the hydrostatic equation for the special case when the atmospheric gases remain well mixed, as is the situation below 86 km.

The temperature-height profile between 86 and 1000 km is not expressed as a series of linear functions, as at lower altitudes. Rather, it is defined in terms of four successive functions chosen not only to provide a reasonable approximation to observations, but also to yield a continuous first derivative with respect to altitude over the entire height regime.

Observational data of various kinds provide the basis for independently determining various segments of this temperature-height profile. The observed temperatures at heights between 110 and 120 km were particularly important in imposing limits on the selection of the temperature-height function for that region. At the same time, the observed densities at 150 km and above strongly influenced the selection of both the temperature and the vertical extent of the low-temperature isothermal layer immediately above 86 km.

The mean temperatures derived from data sets associated with successive height regions were not necessarily continuous. In spite of this situation, it is necessary, for purposes of continuity and of mathematical reproducibility of the tables of this Standard, to express the temperature in a series of consecutive height functions from the surface to 1000 km. The expression for each successive function depends upon the end-point value of the preceding function, as well as upon certain terms and coefficients peculiar to the related height interval. This total temperature-height profile applied to the fundamental continuity models (that is, the hydrostatic equation and the equation of motion), along with all the ancillary required constants, coefficients, and functions, defines the *U.S. Standard Atmosphere, 1975* (COESA, 1975). The specification of this definition without any justification in terms of observed data is the purpose of this document.

The definition of this Standard is completely consistent with the tables of two international standard atmospheres, that of the International Civil Aviation Organization (ICAO, 1964) defined up to 32-km altitude, and that of the International Organization for Standardization (ISO, 1973) defined up to 50 km.

INTERNATIONAL SYSTEM OF UNITS

The *U.S. Standard Atmosphere, 1975* (COESA, 1975) is defined in terms of the International System (S.I.) of Units (Mechtley, 1973). A list of the symbols, names, and the related quantities of the applicable basic and derived S.I. units, as well as of the three non-standard metric units and one English unit employed in this Standard Atmosphere, is presented in table 1.

BASIC ASSUMPTIONS AND FORMULAS

Adopted Constants

For purposes of computation, it is necessary to establish numerical values for various constants appropriate to the earth's atmosphere. The adopted constants are grouped into three categories. Category I includes those constants which are common to many branches

Table 1
List of Units Applicable to the *U.S. Standard Atmosphere, 1975*
(COESA, 1975)

	Symbol	Name	Quantity
Basic S.I.	m	meter	Length
	kg	kilogram	Mass
	s	second	Time
	K	Kelvin	Thermodynamic temperature
	mol	mole	The amount of a substance
Derived S.I.	N	newton	Force (kg · m/s)
	Pa	pascal	Pressure (N/m ²)
	J	joule	Work, energy or quantity of heat (N · m)
	W	watt	Rate of energy (or heat) transfer (J/s)
Non-standard	mbar	millibar	Pressure 100 (N/m ²)
	torr at 0°C	torr	Pressure 133.322 (N/m ²)
	°C	Celsius degree	Temperature (Kelvin minus 273.15)
	m'	geopotential meter	Variable unit of length
English	ft	foot	Length (0.3048 m)*

*Exact definition

of the physical and chemical sciences, and are here considered to be fundamental constants. Some of these may be multi-valued as in the case of M_i representing the molecular weight of the i th gas species. Category I includes three single-valued and one multi-valued constant. Category II includes those constants which, in addition to the Category I constants and a suitable set of equations, are sufficient to define that portion of the 1975 Standard Atmosphere below 86 km. This category includes nine single-valued and three multi-valued constants. Category III includes all the remaining constants which, along with Category I and Category II constants and the related equations required for the low-altitude portion,

plus an expansion of that set of equations, are necessary to define that portion of the 1975 Standard Atmosphere above 86 km. This category includes 7 single-valued and 12 multi-valued constants.

The various single-valued and multi-valued constants are listed alphabetically by symbol within each of the three categories. The discussion of each of these constants includes the numerical value and dimensions, except that in the case of multi-valued constants, the values are given in one or another of five tables immediately following the listing of the three categories of constants.

Category I Constants

- k The Boltzmann constant, $k = 1.380\,622 \times 10^{-23} \text{ N} \cdot \text{m/K}$, is theoretically equal to the ratio R^*/N_A , and has a value consistent with the carbon-12 scale, as cited by Mechtly (1973).
- M_i The quantity M_i represents the set of the 10 values (kg/kmol) of molecular weight of particular atmospheric gases, listed in table 2. This set is based upon the carbon-12 isotope scale for which $C^{12} = 12$. This scale was adopted in 1961 at the Montreal meeting of the International Union of Pure and Applied Chemistry.
- N_A The Avogadro constant, $N_A = 6.022\,169 \times 10^{26} \text{ kmol}^{-1}$, is consistent with the carbon-12 scale, and is the value cited by Mechtly (1973).
- R^* The gas constant, $R^* = 8.314\,32 \times 10^3 \text{ N} \cdot \text{m}/(\text{kmol} \cdot \text{K})$, is consistent with the carbon-12 scale, but is not the value cited by Mechtly (1973). Rather, it is the value used in the *U.S. Standard Atmosphere, 1962* (COESA, 1962). This value is not exactly consistent with the above-listed values of k and N_A .

Category II Constants

- F The general member, F_i , of the set of values of fractional-volume concentrations of atmospheric gases (table 2) represents the assumed relative concentrations of the several gas species comprising dry air at sea level. The values of this set are identical to those given in the 1962 Standard. Except for minor modifications which are based upon CO_2 measurements (Keeling, 1960), these values are the same as those given by Glueckauf (1951) and are based upon the earlier work of Paneth (1939).
- g_0 The quantity, $g_0 = 9.80665 \text{ m/s}^2$, represents the adopted sea-level value of the acceleration of gravity for this Standard. This value is the one originally adopted by the International Committee on Weights and Measures in 1901 for 45° latitude. Even though it has since been shown to be too high by about five parts in 10,000 (List, 1968), this value has persisted in meteorology and in some standard atmospheres as the value associated with 45° latitude, even though it applies more precisely to a latitude of $45^\circ 32' 33''$.

- g'_0 The dimensional constant, $g'_0 = 9.80665 \text{ m}^2 / (\text{s}^2 \cdot \text{m}')$, selected to relate the standard geopotential meter (m') to geometric height is numerically equal to g_0 , but with appropriately different dimensions, that is, $\text{m}^2 / (\text{s}^2 \cdot \text{m}')$. This constant implicitly defines 1 m' as the vertical increment through which one must lift 1 kg to increase its potential energy by 9.80665 joules (J). The geometric length of this vertical increment varies inversely with the height-dependent value of g .
- H_b The general member, H_b , of the set of geopotential-height values listed in table 3 represents the base of each of eight successive atmospheric layers, defined to have particular constant values of $L_{M,b}$.
- $L_{M,b}$ The general member, $L_{M,b}$, of the set of seven values of gradient dT_M/dH of molecular scale temperature T_M (Minzner and Ripley, 1956) with respect to geopotential H listed in table 3 represents the fixed value appropriate throughout its related layer, H_b to H_{b+1} . The corresponding pairs of values of H_b and $L_{M,b}$ are based partly on tradition and partly on present-day observations. The first five of these pairs are identical to those of the first five layers of the 1962 Standard, while the remaining two pairs have been newly-selected to provide a reasonable fit to the presently-available atmospheric data. The first two values of the related sets have their origin in one of the earliest aeronautical standard atmospheres (Toussaint, 1919) and were approximated in the first U.S. Standard Atmosphere (Diehl, 1925).
- P_0 The standard sea-level atmospheric pressure, $P_0 = 1.013250 \times 10^5 \text{ Pa}$ (or N/m^2), was adopted in 1947 in Resolution 164 of the International Meteorological Organization. This corresponds to the pressure exerted by a column of mercury 0.760 m high, having a density of $1.35951 \times 10^4 \text{ kg}/\text{m}^3$, and subject to an acceleration due to gravity of $9.80665 \text{ m}/\text{s}^2$. This equivalency definition was adopted by the International Commission on Weights and Measures in 1948.
- r_0 The effective earth's radius for purposes of calculating geopotential altitude at any latitude is readily obtained from equations given by Harrison (1968). The value of $r_0 = 6.356766 \times 10^6 \text{ m}$ used in this Standard corresponds to the latitude for which $g = 9.80665 \text{ m}/\text{s}^2$.
- T_0 The standard sea-level temperature, $T_0 = 288.15 \text{ K}$, is based upon two international agreements. The first of these is Resolution 192 of the International Commission for Air Navigation which in 1924 adopted 15°C as the sea-level temperature of the International Standard Atmosphere. This value has been retained unchanged in all known standard atmospheres since that date. The second agreement is that of the 1954 Tenth General Conference on Weights and Measures which set the fixed point of the Kelvin temperature scale at the triple-point temperature 273.16 K , which is 0.01 K above the ice-point temperature at standard sea-level pressure.

- S The Sutherland constant, $S = 110 \text{ K}$, (Hilsenrath et al., 1955) is a constant in the empirical expression for dynamic viscosity.
- β The quantity $\beta = 1.458 \times 10^6 \text{ kg}/(\text{s} \cdot \text{m} \cdot \text{K}^{1/2})$ (Hilsenrath et al., 1955) is a constant in the expression for dynamic viscosity.
- γ The ratio of specific heat of air at constant pressure to the specific heat of air at constant volume is a dimensionless quantity with an adopted value $\gamma = 1.4$. This is the value adopted by the Aerological Commission of the International Meteorological Organization in Toronto, 1948.
- σ The mean effective collision diameter, $\sigma = 3.65 \times 10^{-10} \text{ m}$, of gas molecules is a quantity which varies with gas species and temperature. The adopted value is assumed to apply in a dry, sea-level atmosphere. Above 85 km, the validity of the adopted value decreases with increasing altitude (Herschfelder et al., 1964; Chapman and Cowling, 1960) due to the change in atmospheric composition. For this reason, the number of significant figures in tabulations of quantities involving σ is reduced from that used for other tabulated quantities at heights above 86 km.

Category III Constants

- a_i The quantity a_i represents the general member of a set of five values ($\text{m}^{-1} \cdot \text{s}^{-1}$) of a species-dependent coefficient listed in table 4, and used in equation 8 for designating the height-dependent, molecular-diffusion coefficient D_i for the related gas species. (See b_i .)
- b_i The quantity b_i represents the general member of a set of five values (dimensionless) of a species-dependent exponent listed in table 4, and used, along with the corresponding value of a_i , in equation 8 for designating the height-dependent, molecular-diffusion coefficient for the related gas species. The particular values of a_i and b_i adopted for this Standard have been selected to yield a height variation of D_i consistent with observed number densities.
- K_7 The quantity $K_7 = 1.2 \times 10^2 \text{ m}^2/\text{s}$ is the adopted value of the eddy-diffusion coefficient K , at $Z_7 = 86 \text{ km}$, and in the height interval from 86 up to 91 km. Beginning at 91 km and extending up to 115 km, the value of K is defined by equation 7b. At 115 km the value of K equals K_{10} .
- K_{10} The quantity $K_{10} = 0.0 \text{ m}^2/\text{s}$ is the adopted value of the eddy-diffusion coefficient K at $Z_{10} = 120$ and throughout the height interval from 115 km through 1000 km.

- $L_{K,b}$ The two-valued set of gradients $L_{K,b} = dT/dZ$ listed in table 5 was specifically selected for this Standard to represent available observations. The first of these two values of $L_{K,b}$ is associated with the layer 86 to 91 km, and the second with the layer 110 to 120 km.
- $n(O)_7$ The quantity $n(O)_7 = 8.6 \times 10^{16} \text{ m}^{-3}$ is the number density of atomic oxygen assumed for this Standard to exist at $Z_7 = 86$ km. This value of atomic oxygen number density, along with other defined constants, leads to particular values of number density for N_2 , O_2 , Ar, and He at 86 km. (See Appendix A.)
- $n(H)_{11}$ The quantity $n(H)_{11} = 8.0 \times 10^{10} \text{ m}^{-3}$ is the assumed number density of atomic hydrogen at height $Z_{11} = 500$ km, and is used as the reference value in computing the height profile of atomic hydrogen between 150 and 1000 km.
- q_i The quantity q_i represents the first set of six sets of species-dependent coefficients listed in table 6 (that is, sets of q_i , Q_i , u_i , U_i , w_i , and W_i) the corresponding members of all six of which are simultaneously used in an empirical expression for the vertical transport term $v_i/(D_i + K)$ in the vertical flux equation for the particular gas species. The species-dependent values of all six sets have been selected for this Standard to adjust number-density profiles of the related gas species to particular boundary conditions at 150 and 450 km, as well as at 97 km in the case of atomic oxygen. These boundary conditions all represent observed or assumed average conditions. (See equation 37.)
- Q_i The quantity Q_i represents the second set of the six sets of constants described along with q_i above, and listed in table 6.
- T_9 The quantity $T_9 = 240$ K represents the kinetic temperature at $Z_9 = 110$ km. This temperature has been adopted along with the gradient $L_{K,9} = 12$ K/km to generate a linear segment of $T(Z)$ for this Standard between 110 and 120 km.
- T_∞ The quantity $T_\infty = 1000$ K represents the exospheric temperature, that is, the asymptote which the exponential function, representing $T(Z)$ above 120 km, closely approaches at heights above about 500 km, where the mean free path exceeds the scale height. The value of T_∞ adopted for this Standard is assumed to represent mean solar conditions.
- u_i The quantity u_i represents the third set of the six sets of constants described along with q_i above, and listed in table 6.
- U_i The quantity U_i represents the fourth set of the six sets of constants described along with q_i above, and listed in table 6.

- w_i The quantity w_i represents the fifth set of the six sets of constants described along with q_i above, and listed in table 6.
- W_i The quantity W_i represents the sixth set of the six sets of constants described along with q_i above, and listed in table 6.
- Z_b The quantity Z_b represents a set of six values of Z for b equal to 7 through 12. The values Z_7 , Z_8 , Z_9 , and Z_{10} correspond respectively, to the base of successive layers characterized by successive segments of the adopted temperature-height function for this Standard. The fifth value, Z_{11} , is the reference height for the atomic hydrogen calculation, while the sixth value, Z_{12} , represents the top of the region for which the tabular values of the Standard are given. These six values of Z_b , along with the designation of the type of temperature-height function associated with the first four of these values, plus the related value of $L_{K,b}$, for the two segments having a linear temperature-height function, are listed in table 5.
- α_i The quantity α_i represents a set of six adopted species-dependent, thermal-diffusion coefficients listed in table 4.
- ϕ The quantity $\phi = 7.2 \times 10^{11} \text{ m}^{-2} \cdot \text{s}^{-1}$ for the vertical flux is chosen as a compromise between the classical Jeans' escape flux for $T_\infty = 1000 \text{ K}$, with corrections to take into account deviations from a Maxwellian velocity distribution at the critical level (Brinkman, 1971) and the effects of charge exchange with H^+ and O^+ in the plasmasphere (Tinsley, 1973).

Table 2
Molecular Weights and Assumed Fractional-volume Composition of Sea-level Dry Air

Gas Species	Molecular Weight	Fractional Volume
	M_i (kg/kmol)	F_i (dimensionless)
N_2	28.013 4	0.780 84
O_2	31.998 8	0.209 476
Ar	39.948	0.009 34
CO_2	44.009 95	0.000 314
Ne	20.183	0.000 018 18
He	4.002 6	0.000 005 24
Kr	83.80	0.000 001 14
Xe	131.30	0.000 000 087
CH_4	16.043 03	0.000 002
H_2	2.015 94	0.000 000 5

Table 3
 The Adopted Reference Levels and Gradients of the Linearly-segmented
 Temperature-height Profile from the Surface to 86 Geometric
 Kilometers and the Related Derived Values of
 Molecular-scale Temperature

Subscript	Geopotential Height	Molecular-scale Temperature Gradient	Form of Function Relating T to H	Derived Values of Molecular-scale Temperature
b	H_b (km')	$L_{M,b}$ (K/km')		$T_{M,b}$ (K)
0	0	-6.5	Linear	288.150
1	11	0.0	Linear	216.650
2	20	+1.0	Linear	216.650
3	32	+2.8	Linear	228.650
4	47	0.0	Linear	270.650
5	51	-2.8	Linear	270.650
6	71	-2.0	Linear	214.650
7	84.8520			186.946

Note: These values of H_b and $L_{M,b}$ plus the defined sea-level value of T (equal to $T_{M,0}$) completely specify the geopotential-height profile of T_M from the surface to 86 geometric kilometers, as designated by the derived values of $T_{M,b}$ in this table.

Table 4
 A Set of Species-dependent, Thermal-diffusion Factors and Two Other Sets of
 Species-dependent Constants Required in Specifying the Height-dependent
 Function of the Molecular-diffusion Coefficient for the
 Several Species Listed

Gas	α_i (dimensionless)	$a_i = (m^1 \cdot s^{-1})$	b_i (dimensionless)
N ₂	0.00		0.750
O	0.00	6.986×10^{20}	0.750
O ₂	0.00	4.863×10^{20}	0.870
Ar	0.00	4.487×10^{20}	0.691
He	-0.40	1.700×10^{21}	0.500
H	-0.25	3.305×10^{21}	

Table 5
The Constants and Functions Adopted to Define the Four-layer Temperature-height Profile for Altitudes Between 86 and 1000 km, Plus the Derived Temperatures at the Boundary Heights of the Several Layers

Subscript	Geometric Height	Kinetic-temperature Gradient	Form of Function Relating T to Z	Derived Kinetic Temperature
b	Z _b (km)	L _{k,b} (K/km)		T (K)
7	86	0.0	Linear	186.87
8	91		Elliptical	186.87
9	110	12.0	Linear	
10	120		Exponential	360.00
11	500			999.24
12	1000			1000.00

Note: These adopted specifications, including the adopted kinetic temperature, 240 K at 110 km, and the kinetic temperature, 186.87 K derived from T_M at 86 km, plus the specific form of the exponential function, equation 31, and the requirement that dT/dZ be continuous from Z = 86 km to Z = 1000 km, define the height profile of T between these height limits. The specific form of the ellipse, equation 27, which satisfies the several adopted conditions is derived in Appendix B.

Table 6
Values of Six Sets of Species-dependent Coefficients Applicable to the Empirical Expression Representing the Flux Term v_i/(D_i + K) in the Equation for Number Density of the Four Species Listed

Gas	q _i (km ⁻³)	Q _i (km ⁻³)	u _i (km)	U _i (km)	w _i (km ⁻³)	W _i (km ⁻³)
O	-3.416248 × 10 ⁻³ *	-5.809644 × 10 ⁻⁴	97.0	56.90311	5.008765 × 10 ⁻⁴	2.706240 × 10 ⁻⁵
O ₂	0	1.366212 × 10 ⁻⁴		86.000		8.333333 × 10 ⁻⁵
Ar	0	9.434079 × 10 ⁻⁵		86.000		8.333333 × 10 ⁻⁵
He	0	-2.457369 × 10 ⁻⁴		86.000		6.666667 × 10 ⁻⁴

*This value of q_i applies only for 86 < Z < 97 km. For Z > 97 km, q_i = 0.0 km⁻³.

Equilibrium Assumptions

The air is assumed to be dry, and at heights sufficiently below 86 km, the atmosphere is assumed to be homogeneously mixed with a relative-volume composition leading to a constant mean molecular weight, M. In this height region of complete mixing, the air is treated as if it were a perfect gas, and the total pressure P, the temperature T, and the total density ρ at any point in the atmosphere are related by the equation of state, that is, the perfect gas law, one form of which is

$$P = \frac{\rho \cdot R^* \cdot T}{M} \quad (1)$$

where R^* is the universal gas constant. An alternate form of the equation of state, (equation 1) this one in terms of the total number density, N , and the Avogadro constant, N_A , is

$$P = \frac{N \cdot R^* \cdot T}{N_A} \quad (2)$$

This form represents the summation of P_i , the partial pressures of the individual gas species, where P_i is related to n_i , the number density of the i th gas species in the following expression:

$$P_i = n_i \cdot k \cdot T \quad (3)$$

where k is the Boltzmann constant, which is equivalent to R^*/N_A .

Within the height region of complete mixing, the atmosphere is assumed to be in hydrostatic equilibrium, and to be horizontally stratified so that dP , the differential of pressure, is related to dZ , the differential of geometric height, by the relationship

$$dP = -g \cdot \rho \cdot dZ \quad (4)$$

where g is the height-dependent acceleration of gravity. The elimination of ρ between equations 1 and 4 yields another well-known form of the hydrostatic equation, which serves as the basis for the low-altitude pressure calculation:

$$d \ln P = \frac{dP}{P} = \frac{-g \cdot M}{R^* \cdot T} \cdot dZ \quad (5)$$

Above 86 km the hydrostatic equilibrium of the atmosphere gradually breaks down as diffusion and vertical transport of individual gas species lead to the need for a dynamically-oriented model including diffusive separation. Under these conditions it is convenient to express the height variations in the atmospheric number density in terms of the vertical component of the flux of the molecules of individual gas species (Colegrove et al., 1965). In terms of the i th gas species, this expression is

$$\begin{aligned} n_i \cdot v_i + D_i \cdot \left(\frac{dn_i}{dZ} + \frac{n_i \cdot (1 + \alpha_i)}{T} \cdot \frac{dT}{dZ} + \frac{g \cdot n_i \cdot M_i}{R^* \cdot T} \right) \\ + K \cdot \left(\frac{dn_i}{dZ} + \frac{n_i}{T} \cdot \frac{dT}{dZ} + \frac{g \cdot n_i \cdot M}{R^* \cdot T} \right) = 0 \end{aligned} \quad (6)$$

where

- v_i = the vertical transport velocity of the i th species,
- D_i = the height-dependent, molecular-diffusion coefficient of the i th species diffusing through N_2 ,
- α_i = the thermal-diffusion factor (dimensionless) of the i th species,

M_i = the molecular weight of the i th species,

M = the molecular weight of the gas through which the i th species is diffusing, and

K = the height-dependent, eddy-diffusion coefficient.

The function K is defined differently in each of three height regions:

1. For $86 \leq Z < 95$ km,

$$K = K_7 = 1.2 \times 10^2 \text{ m}^2/\text{s} \quad (7a)$$

2. For $95 \leq Z < 115$ km,

$$K = K_7 \cdot \exp \left[1 - \frac{400}{400 - (Z - 95)^2} \right] \quad (7b)$$

3. For $115 \leq Z < 1000$,

$$K = K_{10} = 0.0 \text{ m}^2/\text{s} \quad (7c)$$

The function D_i is defined by

$$D_i = \frac{a_i}{\Sigma n_i} \cdot \left(\frac{T}{273.15} \right)^{b_i} \quad (8)$$

where a_i and b_i are the species-dependent constants defined in table 4, while T and Σn_i are both altitude-dependent quantities which are specified in detail below. The values of D_i , determined from these altitude-dependent quantities, and the defined constants a_i and b_i are plotted in figure 1 as a function of altitude for each of four species, O, O₂, Ar, and He.

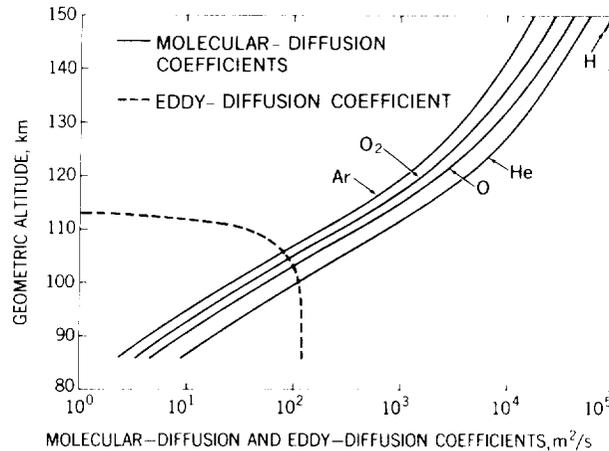


Figure 1. Molecular-diffusion and eddy-diffusion coefficients versus geometric altitude.

The value of D_i for atomic hydrogen, H, for heights just below 150 km is also shown in figure 1. This same figure contains a graph of K as a function of altitude. It is apparent that for heights sufficiently below 90 km, values of D_i are negligible compared with K , while above 115 km, the reverse is true. In addition, it is known that the flux velocity, v_i , for the various species becomes negligibly small at altitudes sufficiently below 90 km.

The information regarding the relative magnitudes of v_i , D_i , and K permits us to consider the application of equation 6 in each of several regimes. One of these regimes is for heights sufficiently below 90 km, such that v_i and D_i are both extremely small compared with K . Under these conditions, equation 6 reduces to the following form of the hydrostatic equation:

$$\frac{dn_i}{n_i} + \frac{dT}{T} = \frac{-g \cdot M}{R^* \cdot T} \cdot dZ \quad (9)$$

Since the left-hand side of this equation is seen through equation 3 to be equal to dP_i/P_i , equation 9 is seen to be the single-gas equivalent to equation 5. Consequently, while equation 6 was designed to describe the assumed equilibrium conditions of individual gases above 86 km, it is apparent that it also describes such conditions below that altitude. Here the partial pressure of each gas comprising the total pressure varies in accordance with the mean molecular weight of the mixture, as well as in accordance with the temperature and the acceleration of gravity. Nevertheless, equation 5, expressing total pressure, represents a convenient step in the development of equations for computing total pressure versus geometric height, when suitable functions are introduced to account for the altitude variation in T , M , and g .

It has been customary in standard-atmosphere calculations to effectively eliminate the variable portion of the acceleration of gravity from equation 5 by the transformation of the independent variable Z to geopotential altitude H . This simplifies both the integration of equation 5 and the resulting expression for computing pressure. The relationship between geometric and geopotential altitude depends upon the concept of gravity.

Gravity and Geopotential Altitude

Viewed in the ordinary manner from a frame of reference fixed in the earth, the atmosphere is subject to the force of gravity. The force of gravity is the resultant (vector sum) of two forces: (1) the gravitational attraction in accordance with Newton's universal law of gravitation, and (2) the centrifugal force, which results from the choice of an earthbound, rotating frame of reference.

The gravity field, being a conservative field, can be derived conveniently from the gravity potential energy per unit mass, that is, from the geopotential Φ . This is given by

$$\Phi = \Phi_G + \Phi_C \quad (10)$$

where Φ_G is the potential energy per unit mass of gravitational attraction, and Φ_C is the potential energy per unit mass associated with the centrifugal force. The gravity per unit mass is

$$\mathbf{g} = \nabla\Phi \quad (11)$$

where $\nabla\Phi$ is the gradient (ascendant) of the geopotential. The acceleration due to gravity is denoted by g and is defined as the magnitude of \mathbf{g} , that is,

$$g = |\mathbf{g}| = |\nabla\Phi| \quad (12)$$

When moving along an external normal from any point on the surface Φ_1 to a point on the surface Φ_2 infinitely close to the first surface, so that $\Phi_2 = \Phi_1 + d\Phi$, the incremental work performed by shifting a unit mass from the first surface to the second will be

$$d\Phi = g \cdot dZ \quad (13)$$

hence

$$\Phi = \int_0^Z g \cdot dZ. \quad (14)$$

The unit of measurement of geopotential (Appendix C) is the standard geopotential meter which represents the work done by lifting a unit mass one geometric meter, through a region in which the acceleration of gravity is uniformly 9.80665 m/s^2 .

The geopotential of any point with respect to mean sea level (assumed zero potential), expressed in geopotential meters, is called geopotential altitude. Therefore, geopotential altitude, H , is given by

$$H = \frac{\Phi}{g'_0} = \frac{1}{g'_0} \cdot \int_0^Z g \cdot dZ \quad (15)$$

and is expressed in geopotential meters (m') when the unit geopotential, g'_0 , is set equal to $9.80665 \text{ m}^2/(\text{s}^2 \cdot m')$.

With geopotential altitude defined as in equation 15, the differential of equation 15 may be expressed as

$$g'_0 \cdot dH = g \cdot dZ. \quad (16)$$

This expression, introduced into equation 5, will reduce the number of variables prior to its integration, thereby leading to an expression for computing pressure as a function of geopotential height.

The inverse-square law of gravitation provides an expression for g as a function of altitude with sufficient accuracy for most model-atmosphere computations:

$$g = g_0 \cdot \left(\frac{r_0}{r_0 + Z} \right)^2 \quad (17)$$

where r_0 is the effective radius of the earth at a specific latitude as given by Lambert's equations (List, 1968). Such a value of r_0 takes into account the centrifugal acceleration at the particular latitude. For this Standard, the value of r_0 is taken as 6356.766 km, and is consistent with the adopted value of $g_0 = 9.80665 \text{ m/s}^2$ for the sea-level value of the acceleration of gravity. The variation of g as a function of geometric altitude is depicted in figure 2.

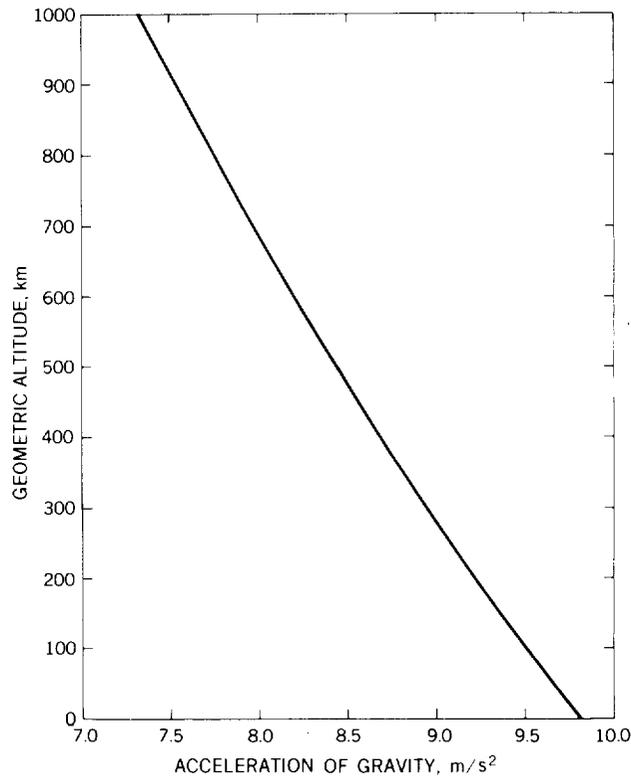


Figure 2. Acceleration of gravity versus geometric altitude.

Integration of equation 15, after substitution of equation 17 for g , yields

$$H = \frac{g_0}{g'_0} \cdot \left(\frac{r_0 \cdot Z}{r_0 + Z} \right) = \Gamma \cdot \left(\frac{r_0 \cdot Z}{r_0 + Z} \right) \quad (18)$$

or

$$Z = \frac{r_0 \cdot H}{\Gamma \cdot r_0 - H} \quad (19)$$

where $\Gamma = g_0/g'_0 = 1 \text{ m'/m}$.

Differences between geopotential altitudes obtained from equation 18 for various values of Z , and those computed from the more complex relationship used in developing the *U.S. Standard Atmosphere, 1962*, (COESA, 1962) are small. For example, values of H computed from equation 18 are approximately 0.2, 0.4, and 33.3 m greater at 90, 120, and 700 km, respectively, than those obtained from the relationship used in the 1962 Standard. In the 1975 Standard, geopotential altitude is used explicitly only at heights below 86 geometric kilometers.

The transformation from Z to H (equation 16) in the development of the pressure-height relationship, for heights between the surface of the earth and 86-km altitude, makes it necessary to define the altitude variation of T and M in terms of H . It is convenient, therefore, to determine the sea-level value of M , as well as the extent of any height dependence of this quantity, between the surface of the earth and 86-km altitude. Then, for this low-altitude regime, the two variables, T and M , are combined with the constant M_0 into a single variable T_M , which is then defined as a function of H .

Mean Molecular Weight

The mean molecular weight, M , of a mixture of gases is by definition

$$M = \frac{\sum (n_i \cdot M_i)}{\sum n_i} \quad (20)$$

where n_i and M_i are the number density and defined molecular weight, respectively, of the i th gas species. In that part of the atmosphere, between the surface of the earth and about 80-km altitude, mixing is dominant, and the effect of diffusion and photochemical processes upon M is negligible. In this region, the fractional composition of each species is assumed to remain constant at the defined value, F_i , and M remains constant at its sea-level value, M_0 . For these conditions, n_i is equal to the product of F_i times the total number density, N , so that equation 20 may be rewritten as

$$M = M_0 = \frac{\sum [F_i \cdot N(Z) \cdot M_i]}{\sum [F_i \cdot N(Z)]} = \frac{\sum (F_i \cdot M_i)}{\sum F_i} \quad (21)$$

The right-hand element of this equation results from the process of factoring $N(Z)$ out of each term of both the numerator and the denominator of the preceding fraction, so that, in spite of the altitude dependence of N , M is seen analytically to equal M_0 over the entire altitude region of complete mixing. When the defined values of F_i and M_i (from table 2)

are introduced into equation 21, M_0 is found to be 28.9644 kg/kmol. At 86 km (84.852 km'), however, the defined value of atomic-oxygen number density ($8.6 \times 10^{16} \text{ m}^{-3}$) is seen (in Appendix A) to lead to a value of $M = 28.9522 \text{ kg/kmol}$, about 0.04 percent less than M_0 . To produce a smooth transition from this value of M to M_0 , the altitude profile of M has been arbitrarily defined at intervals of 0.5 km', for altitudes between 79.006 and 84.852 km', in terms of the ratio M/M_0 as given in table 7. These ratio values have been interpolated from those initially selected for intervals of 0.5 geometric kilometers between 80 and 86 km to satisfy the boundary conditions of $M = M_0 = 28.9644$ at 80 km, and $M = 28.9522$ at 86 km, and to satisfy a condition of smoothly-decreasing first differences in M within the height interval, 80 to 86 km.

Table 7
Molecular-weight Ratio versus Geopotential and Geometric Altitudes in Meters

H	Z	M/M_0	Z	H	M/M_0
79000	79994.1	1.000000	80000	79005.7	1.000000
79500	80506.9	0.999996	80500	79493.3	0.999996
80000	81019.6	0.999988	81000	79980.8	0.999989
80500	81532.5	0.999969	81500	80468.2	0.999971
81000	82045.4	0.999938	82000	80955.7	0.999941
81500	82558.6	0.999904	82500	81443.0	0.999909
82000	83071.5	0.999864	83000	81930.2	0.999870
82500	83584.8	0.000822	83500	82417.3	0.999829
83000	84098.0	0.999778	84000	82904.4	0.999786
83500	84611.4	0.999731	84500	83391.4	0.999741
84000	85124.8	0.999681	85000	83878.4	0.999694
84500	85638.4	0.999679	85500	84365.2	0.999641
			86000	84852.0	0.999578

These arbitrarily-assigned values of M/M_0 may be used for correcting a number of parameters of this Standard, if the tabulations are to correctly fit the model in the fifth, and perhaps in the fourth, significant figures within this height region. This after-the-fact correction is required because these values of M/M_0 were not included in the program used for computing the tables of this Standard below 86 km, and hence, the tabulations of some of the properties may show a discontinuity of up to 0.04 percent between 85.5 and 86 km. This situation exists particularly for four properties in addition to molecular weight: kinetic temperature, total number density, mean-free path, and collision frequency. For these five parameters, the discrepancy in the detailed tables between 80 and 86 km can be readily remedied by a simple multiplication or division: Tabulated values of M , T , and L must be multiplied by the corresponding values of M/M_0 from table 7; tabulated values of N and ν must be divided by the corresponding values of M/M_0 .

Three other properties—dynamic viscosity, kinematic viscosity, and thermal conductivity (which are tabulated only for heights below 86 km)—have similar discrepancies for heights immediately below 86 km. These values are not so simply corrected, however, because of the empirical nature of their respective defining functions. Rather, these quantities must be recalculated in terms of a suitably-corrected set of values of T , if the precisely correct values are desired for geometric altitudes between 80- and 86-km altitude.

Molecular-scale Temperature versus Geopotential Altitude (0 to 84.3520 km')

The molecular-scale temperature, T_M (Minzner et al., 1958), at a point is defined as the product of the kinetic temperature, T , at that point times the ratio, M_0/M , where M is the mean molecular weight of air at that point and M_0 is the sea-level value of M discussed above (see Appendix C). Analytically,

$$T_M = T \cdot \frac{M_0}{M} \quad (22)$$

(When T is expressed in the Kelvin scale, T_M is also expressed in that scale.)

The principle virtue of the parameter T_M is that it combines the variable portion of M with the variable T into a single new variable, in a manner somewhat similar to the combining of the variable portion of g with Z to form the new variable H . When both of these transformations are introduced into equation 5, and when T_M is expressed as a linear function of H , the resulting differential equation has an exact integral. Under these conditions, the computation of P versus H becomes a simple process not requiring numerical integration. Traditionally, standard atmospheres have defined temperature as a linear function of height to eliminate the need for numerical integration in the computation of pressure versus height. This Standard follows the tradition to heights up to 86 km, and the function T_M versus H is expressed as a series of seven successive linear equations. The general form of these linear equations is

$$T_M = T_{M,b} + L_{M,b} \cdot (H - H_b) \quad (23)$$

with the value of subscript b ranging from 0 to 6 in accordance with each of seven successive layers. The value of $T_{M,b}$ for the first layer ($b = 0$) is 288.15 K, identical to the sea-level value of T , since at this level $M = M_0$. With this value of $T_{M,b}$ defined, and the set of six values of H_b and the six corresponding values of $L_{M,b}$ defined in table 3, the function T_M of H is completely defined from the surface to 84.852 km' (86 km). A graph of this function compared to the similar function of the 1962 Standard is shown in figure 3. From the surface of the earth to the 51-km' altitude, this profile is identical to that of the 1962 Standard. The profile from 51 to 84.852 km' was selected in accordance with present-day data, and abbreviated tables of thermodynamic properties of the atmosphere based upon this temperature-height profile were published by Kantor and Cole (1973).

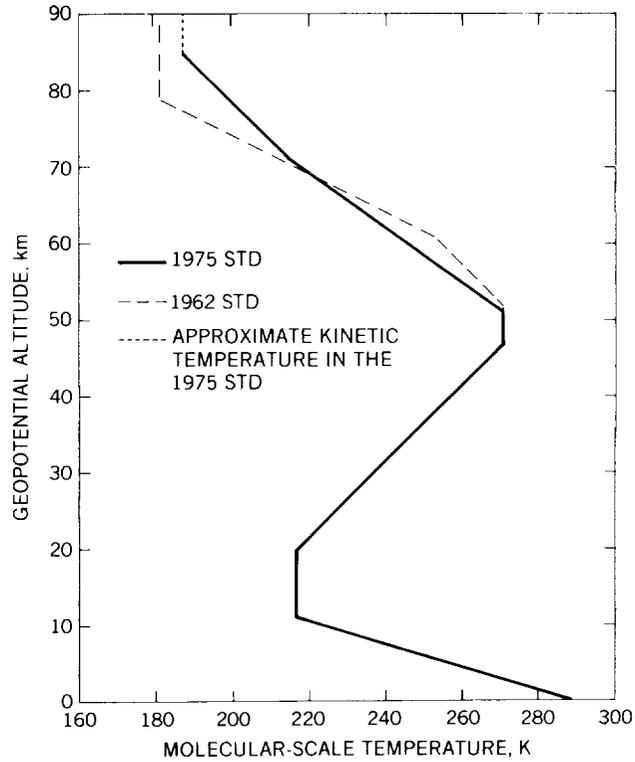


Figure 3. Molecular-scale temperature versus geopotential altitude.

Kinetic Temperature versus Geometric Altitude (0 to 1000 km)

Between the surface of the earth and 86-km altitude, kinetic temperature is based upon the defined values of T_M . In the lowest 80 km of this region, where M is constant at M_0 , T is equal to T_M in accordance with equation 22. Between 80 and 86 km, however, the ratio M/M_0 is assumed to decrease from 1 to 0.9995788, as indicated in table 7, such that the values of T correspondingly decrease from those of T_M . Thus, at $Z_7 = 86$ km, a form of equation 22 shows that T_7 has a value 186.8673 K, that is, 0.0787 K smaller than that of T_M at that height.

At heights above 86 km, values of T_M are no longer defined, and geopotential is no longer the primary argument. Instead, the temperature-altitude profile is defined in terms of four successive functions, each of which is specified in such a way that the first derivative of T with respect to Z is continuous over the entire altitude region, 86 to 1000 km. These four functions begin successively at the first four base heights, Z_b , listed in table 5, and are designed to represent the following conditions:

1. An isothermal layer from 86 to 91 km;
2. A layer in which $T(Z)$ has the form of an ellipse from 91 to 110 km;

3. A constant, positive-gradient layer from 110 to 120 km; and
4. A layer in which T increases exponentially toward an asymptote, as Z increases from 120 to 1000 km.

86 to 91 km

For the layer from $Z_7 = 86$ km to $Z_8 = 91$ km, the temperature-altitude function is defined to be isothermally linear with respect to geometric altitude, so that the gradient of T with respect to Z, is zero (see table 5). Thus, the standard form of the linear function, which is

$$T = T_b + L_{K,b} \cdot (Z - Z_b) \quad (24)$$

degenerates to

$$T = T_7 = 186.8673 \text{ K} \quad (25)$$

and by definition

$$\frac{dT}{dZ} = 0.0 \text{ K/km} \quad (26)$$

The value of T_7 is derived from one version of equation 22 in which T_M is replaced by $T_{M,7} = 186.946$ from equation 23 or from table 3, and M/M_0 is replaced by the value 0.9995788 from table 7. Thus, $T_7 = 186.8673$ K. Since the kinetic temperature, T, is defined to be constant for the entire layer, Z_7 to Z_8 , the temperature at Z_8 is $T_8 = T_7 = 186.8673$ K, and the gradient, dT/dZ , at Z_8 is $L_{K,8} = 0.0$ K/km, the same as for $L_{K,7}$.

91 to 110 km

For the layer $Z_8 = 91$ km to $Z_9 = 110$ km, the temperature-altitude function is defined to be a segment of an ellipse expressed by

$$T = T_C + A \cdot \left[1 - \left(\frac{Z - Z_8}{a} \right)^2 \right]^{1/2} \quad (27)$$

where

$$T_C = 263.1905 \text{ K, derived in Appendix B,}$$

$$A = -76.3232 \text{ K, derived in Appendix B,}$$

$$a = -19.9429 \text{ km, derived in Appendix B,}$$

and Z is limited to values from 91 to 110 km.

Equation 27 is derived in Appendix B from the basic equation for an ellipse, to meet the values of T_8 and $L_{K,8}$ derived above, as well as the defined values $T_9 = 240$ K and $L_{K,9} = 12$ K/km, for $Z_9 = 110$ km. With these restraints, the values of T_c , a , and A are found to be those cited above.

The expression for dT/dZ related to equation 27 is

$$\frac{dT}{dZ} = \frac{-A}{a} \cdot \left(\frac{Z - Z_8}{a} \right) \cdot \left[1 - \left(\frac{Z - Z_8}{a} \right)^2 \right]^{-1/2} \quad (28)$$

110 to 120 km

For the layer $Z_9 = 110$ km to $Z_{10} = 120$ km, $T(Z)$ has the form of equation 24, where subscript b is 9, such that T_b and $L_{K,b}$ are, respectively, the defined quantities T_9 and $L_{K,9}$ (see Category III Constants and table 5 respectively), while Z is limited to the range 110 to 120 km. Thus,

$$T = T_9 + L_{K,9}(Z - Z_9) \quad (29)$$

and

$$\frac{dT}{dZ} = L_{K,9} = 12.0 \text{ K/km} \quad (30)$$

Since dT/dZ is constant over the entire layer, $L_{K,10}$, the value of dT/dZ at Z_{10} , is identical to $L_{K,9}$ (that is, 12 K/km) while the value of T_{10} at Z_{10} is found from equation 29 to be 360 K.

120 to 1000 km

For the layer $Z_{10} = 120$ to $Z_{12} = 1000$ km $T(Z)$ is defined to have the exponential form (Walker, 1965)

$$T = T_\infty - (T_\infty - T_{10}) \cdot \exp(-\lambda \cdot \xi) \quad (31)$$

such that

$$\frac{dT}{dZ} = \lambda \cdot (T_\infty - T_{10}) \cdot \left(\frac{r_0 + Z_{10}}{r_0 + Z} \right)^2 \cdot \exp(-\lambda \cdot \xi) \quad (32)$$

because

$$\lambda = L_{K,9}/(T_\infty - T_{10}) = 0.01875,$$

and

$$\xi = \xi(Z) = (Z - Z_{10}) \cdot (r_0 + Z_{10})/(r_0 + Z)$$

In the above expressions, T_{∞} equals the defined value 1000 K. A graph of T versus Z from 0- to 1000-km altitude is given in figure 4. The upper portion of this profile was selected to be consistent with satellite drag data (Jacchia, 1971), while the mid-portion, particularly between 86 and 200 km, and to some extent in the region 200 to 450 km, was selected* to be consistent with observed temperatures and satellite observations of composition data (Hedin et al., 1972).

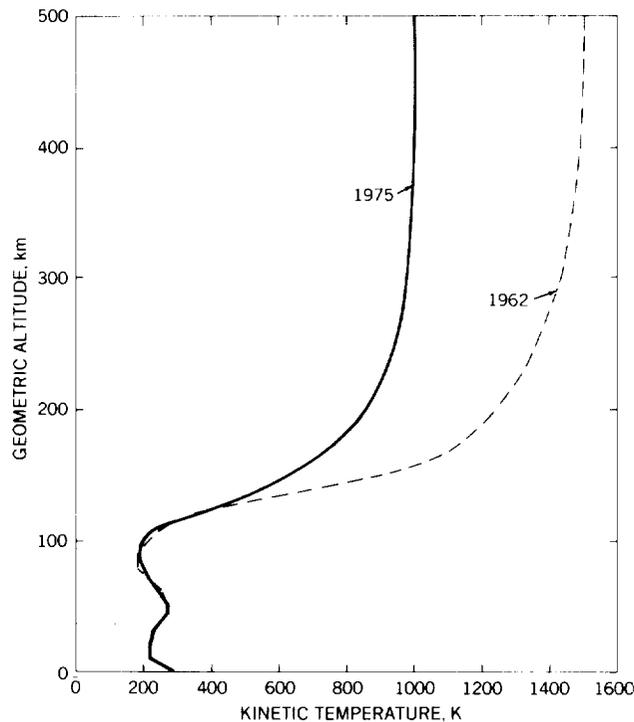


Figure 4. Kinetic temperature versus geometric altitude.

COMPUTATIONAL EQUATIONS

The tables of this Standard have been computed in two height regions, 0 to 86 km (84.852 km'), and 86 to 1000 km, because the computations for each region are based on compatible but different sets of initial conditions. These two different sets of initial conditions lead to two different computational procedures. Consequently, the following discussion of computational equations, which is presented according to a series of atmospheric parameters, does not necessarily flow in the order in which the calculation is actually performed for each altitude region. The equations used for computing the various

*Minzner, R. A., C. A. Reber, K. S. W. Champion, F. T. Huang, O. K. Moe, A. O. Nier, G. R. Swenson, S. P. Zimmerman, "The 1975 Standard Atmosphere Above 86-km Altitude: Recommendations of Task Group II to COESA," 1974, to be published as NASA SP.

properties of the atmosphere for altitudes below 86 km are, with certain noted exceptions, equivalent to those used in the 1962 Standard, and the various equations involving T_M stem from expressions used in the ARDC Model Atmosphere, 1956 (Minzner, 1956).

Pressure

Three different equations are used to compute pressure, P , in various height regimes of this Standard. One of these equations applies to heights above 86 km, while the other two apply to the height regime from the surface of the earth up to 86-km altitude, within which regime the argument of the computation is geopotential. Consequently, expressions for computing pressure as a function of geopotential altitude stem from the integration of equation 5 after replacing $g \cdot dZ$ by its equivalent $g'_0 dH$ from equation 16, and after replacing the ratio M/T by its equivalent M_0/T_0 in accordance with equation 22. Two forms result from this integration—one is for the case when $L_{M,b}$ for a particular layer is equal to zero, and the other when the value of $L_{M,b}$ is not zero. The latter of these two expressions is

$$P = P_b \cdot \left[\frac{T_{M,b}}{T_{M,b} + L_{M,b} \cdot (H - H_b)} \right]^{\frac{g'_0 \cdot M_0}{R^* \cdot L_{M,b}}} \quad (33a)$$

and the former is

$$P = P_b \cdot \exp \left[\frac{-g'_0 \cdot M_0 (H - H_b)}{R^* \cdot T_{M,b}} \right] \quad (33b)$$

Equation 33a is used for layers associated with values of subscript b equal to 0, 2, 3, 5 and 6; equation 33b is used for layers associated with values of subscript b equal to 1 and 4.

In these equations, g'_0 , M_0 , and R^* are each defined single-valued constants, while $L_{M,b}$ and H_b are each defined multi-valued constants in accordance with the value of b as indicated in table 3. In each equation, H may have values ranging from H_b to H_{b+1} . The quantity $T_{M,b}$ is a multi-valued constant listed in table 3 with values derived from equation 23 in accordance with the several values of b and the corresponding defined values of $L_{M,b}$ and H_b . The reference-level value of P_b for $b = 0$ is the defined sea-level value, $P_0 = 101.3250$ kPa (equivalently 101325 N/m² or 1013.25 mbar). The values of P_b for $b = 1$ through $b = 6$ are obtained from the application of the appropriate member of the pair of equations 33a and 33b for the case when $H = H_{b+1}$.

These two equations applied successively yield the pressure for any desired geopotential altitude from sea level to H_7 , where H_7 is the geopotential altitude corresponding to the geometric altitude $Z_7 = 86$ km. Pressures for H from 0 to -5 km' may also be computed from equation 33a when $b = 0$.

For Z equal to 86 km and above the value of pressure is computed as a function of geometric altitude, Z , and involves the altitude profile of kinetic temperature, T , rather than that of T_M , in an expression in which the total pressure, P , is equal to the sum of the partial pressures for the individual species as expressed by equation 3. Thus, for $Z = 86$ to 1000 km,

$$P = \sum P_i = \sum n_i \cdot k \cdot T = \frac{\sum n_i \cdot R^* \cdot T}{N_A} \quad (33c)$$

In this expression,

k = the Boltzmann constant, defined in Category I,

T = $T(Z)$ defined in equations 25, 27, 29, and 31 for successive layers,

$\sum n_i$ = the sum of the number densities of the individual gas species comprising the atmosphere at altitude Z above 86 km, as described below.

Neither n_i , the number densities of individual species, nor $\sum n_i$, the sum of the individual number densities, is known directly. Consequently, pressures above 86 km cannot be computed without first determining n_i for each of the significant gas species.

Number Density of Individual Species

In the height region of complete mixing (0 to 80 km), n_i , the number density of any particular major gas species varies with altitude in accordance with the altitude variation of the total number density, N . For this height region, the value of the i th species is, therefore, given by

$$n_i = F_i \cdot N \quad (34)$$

where F_i is the constant fractional volume coefficient given for each species in table 2. Since the values of N listed in the detailed tables of this Standard are not completely consistent with the basic definition of the model between 80 and 85 km, as previously discussed, values of n_i calculated from tabulated values of N for this limited height regime, must be corrected by dividing by the appropriate values of M/M_0 from table 7. At altitudes above 86 km, equation 34 no longer applies, since the model assumes the existence of various processes which lead to particular differing height variations in the number-density values of several individual species, N_2 , O, O_2 , Ar, He, and H, each governed by equation 6. Ideally, the set of equation 6, each member of which is associated with a particular species, should be solved simultaneously, since the number densities of all the species are coupled through the expressions for molecular diffusion which are included in equation 6. Such a solution would require an inordinate amount of computation, however, and a simpler approach was desired. This was achieved with negligible loss of validity by some simplifying approximations, and by calculating the number densities of individual species one at a time in the order $n(N_2)$, $n(O)$, $n(O_2)$, $n(Ar)$, $n(He)$, and $n(H)$. For all species except hydrogen (which is discussed later), we divide equation 6 by n_i and integrate directly, to obtain the following set of simultaneous equations, one for each gas species:

$$n_i = n_{i,7} \cdot \frac{T_7}{T} \cdot \exp \left\{ - \int_{Z_7}^Z \left[f(Z) + \left(\frac{v_i}{D_i + K} \right) \right] dZ \right\} \quad (35)$$

In this set of equations,

$n_{i,7}$ = the set of species-dependent, number-density values for $Z = Z_7 = 86$ km, one member for each of the five designated species, as derived in Appendix A and listed here,

N	$1.129794 \times 10^{20} \text{ m}^{-3}$
O	$8.6 \times 10^{16} \text{ m}^{-3}$
O ₂	$3.030898 \times 10^{19} \text{ m}^{-3}$
Ar	$1.351400 \times 10^{18} \text{ m}^{-3}$
He	$7.5817 \times 10^{10} \text{ m}^{-3}$

T_7 = 186.8673 K, the value of T at Z_7 , as specified in equation 25,

T = T(Z) defined by equations 25, 27, 29, and 31 for the appropriate altitude regions,

f(Z) = the function written as equation 36 below,

$v_i/(D_i + K)$ = the set of empirical functions written as equation 37 below

For f(Z) we have

$$f(Z) = \frac{g}{R^* \cdot T} \cdot \left(\frac{D_i}{D_i + K} \right) \cdot \left[M_i + \frac{M \cdot K}{D_i} + \frac{\alpha_i \cdot R^*}{g} \cdot \frac{dT}{dZ} \right] \quad (36)$$

where

D_i = $D_i(Z)$ as defined by equation 8 for the *i*th species,

K = K(Z) as defined by equations 7a, 7b, and 7c,

M_i = the molecular weight of the *i*th species as defined in table 2,

α_i = the thermal diffusion coefficient for the *i*th species as defined in table 4,

dT/dZ = one of equations 26, 28, 30, or 32, as appropriate to the altitude region,

M = M(Z), with special considerations mentioned below.

For $[v_i/(D_i + K)]$ we have the following set of empirical expressions.

$$\begin{aligned} \frac{v_i}{D_i + K} = & Q_i \cdot (Z - U_i)^2 \cdot \exp [-W_i \cdot (Z - U_i)^3] \\ & + q_i \cdot (u_i - Z)^2 \cdot \exp [-w_i \cdot (u_i - Z)^3] \end{aligned} \quad (37)$$

The set of expressions represented by equation 37, while representing a function of both D_i and K , does not directly involve calculated values of either of these coefficients. Rather, it involves a series of six other coefficients which, for each of four species, have been empirically selected to adjust the number-density profile of the related species to particular values in agreement with observations. The defined values of the six sets of species-dependent coefficients— q_i , Q_i , u_i , U_i , w_i , and W_i used in equation 37—are listed in table 6. The values of q_i and U_i were selected so that for O_2 , Ar, and He, the quantity $v_i/(D_i + K)$ becomes zero at exactly 86 km. For atomic oxygen, however, all six of these coefficients contribute to maximizing this quantity for $Z = 86$ km.

Molecular Nitrogen

Molecular nitrogen (N_2) is the first species for which n is calculated. On the average, the distribution of N_2 is close to that for static equilibrium, and hence, for this species, we may neglect the transport velocity, thereby eliminating the term $[v_i/(D_i + K)]$ from that version of equation 35 applying to N_2 . This species is dominant above and below the turbopause, and its molecular weight is close to the mean molecular weight in the lower thermosphere, where mixing still dominates the distribution process. The effect of mixing up to 100-km altitude is approximated therefore by two additional adjustments to equation 35 as applied to N_2 . Both adjustments are implicit in $f(Z)$; these are neglecting K and replacing M_i by the mean molecular weight M which, for the altitude region 86 to 100 km, is approximated by M_0 . With these three adjustments, that version of equation 35 applying to N_2 reduces to

$$n(N_2) = n(N_2)_7 \cdot \frac{T_7}{T} \cdot \exp \left\{ - \int_{Z_7}^Z \frac{M \cdot g}{R^* \cdot T} \cdot dZ \right\} \quad (38)$$

where

$$M = M_0 \text{ for } Z \leq 100 \text{ km, and}$$

$$M = M(N_2) \text{ for } Z > 100 \text{ km.}$$

Figure 5 shows a graph of $n(N_2)$ versus Z .

Species O, O₂, Ar, and He

As noted above, after the calculation of $n(N_2)$ has been performed, the values of n_i for the next four species are calculated from equation 35 in the order O, O_2 , Ar, and He. In the case of O and O_2 , the problem of mutual diffusion is simplified by considering N_2 as the stationary background gas (as described in the previous section). For Ar and He, which are minor constituents in the lower thermosphere, it is more realistic to use the sum of the number densities of N_2 , O, and O_2 as the background gas in evaluating the molecular-diffusion coefficient, D_i , and the mean-molecular weight, M , except below 100 km where

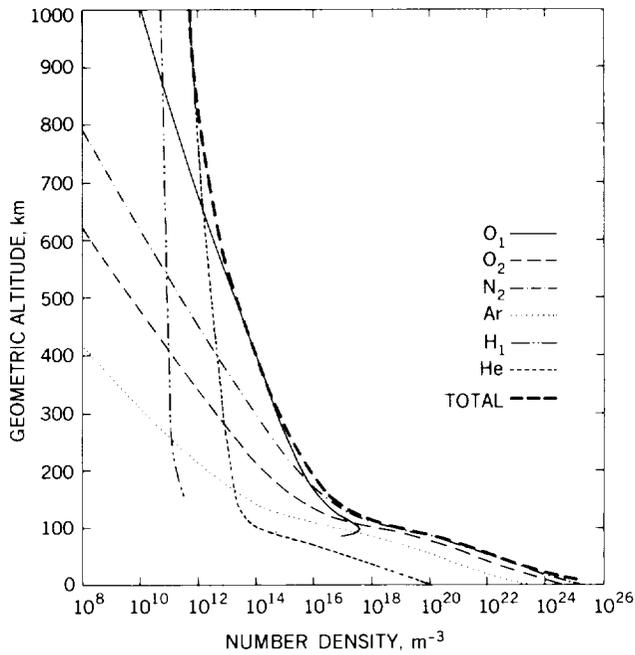


Figure 5. Number density of individual species and total number density versus geometric altitude.

M is taken to be the sea-level value, M_0 . This latter choice is to maintain consistency with the method for calculating $n(N_2)$.

In equation 37, defining $[v_i/(D_i + K)]$, the coefficients q_i , Q_i , u_i , U_i , w_i , and W_i , which (except for q_i) are constant for a particular species, are each adjusted such that appropriate densities are obtained at 450 km for O and He, and at 150 km for O, O_2 , He, and Ar. The constant q_i , and hence the second term of equation 37, is zero for all species except atomic oxygen, and is also zero for atomic oxygen above 97 km; the extra term for atomic oxygen is needed below 97 km to generate a maximum in the density-height profile at the selected height of 97 km. This maximum results from the increased loss of atomic oxygen by recombination at lower altitudes. The flux terms for O and O_2 are based on, and lead (qualitatively) to the same results as those derived from the much more detailed calculations by Colegrove et al. (1965) and Keneshea and Zimmerman (1970) and discussed in Appendix D.

A further computational simplification is realized above 115 km where the eddy-diffusion coefficient becomes zero. For these altitudes, the set of expressions represented by equation 35 becomes uncoupled, and each member reduces to a form where the integration is performed only on the sum of three terms:

1. The barometric term for the particular species (that is, the right-hand side of equation 5),

2. The thermal-diffusion term $(\alpha_i/T) \cdot (dT/dZ)$,
3. A simplified velocity term.

In the case of O, O₂, and Ar, the thermal-diffusion term is zero. Also, as may be shown, the velocity term, $[v_i/(D_i + K)]$, becomes small above 120 km and, with the exception of atomic hydrogen, each species considered is nearly in diffusive equilibrium at these heights. For the present model, however, this situation becomes exactly true only at altitudes above 150 km.

The altitude profile of number density for each of the species O, O₂, Ar, and He is given in figure 5, along with that for N₂.

Atomic Hydrogen

For various reasons, the height distribution of the number density of atomic hydrogen, $n(H)$, is defined only for heights from 150 to 1000 km. Below 150 km, the concentration of H is negligible compared with the concentrations of O, O₂, Ar, and He. The defining expression for $n(H)$, like the expression for $n(N_2)$, $n(O)$, and so on, is derived from equation 6. The solution for $n(H)$, however, is expressed in terms of the vertical flux, $n(H) \cdot v(H)$ represented by ϕ , rather than in terms of $v(H)$, because it is the flux which is considered known for H. In this model, only that contribution to ϕ due to planetary escape from the exosphere is considered.

Since K is zero for the altitude region of interest, the particular version of equation 6 applied to H is correspondingly simplified, and one possible solution to the resulting expression is

$$n(H) = \left[n(H)_{11} - \int_{Z_{11}}^Z \frac{\phi}{D(H)} \cdot \left(\frac{T}{T_{11}} \right)^{1+a(H)} \cdot (\exp \tau) \cdot dZ \right] \cdot \left(\frac{T_{11}}{T} \right)^{1+a(H)} \cdot (\exp -\tau) \quad (39)$$

where

- $n(H)_{11} = 8.0 \times 10^{10} \text{ m}^{-3}$, the number density of H at $Z_{11} = 500 \text{ km}$, as defined in Category III Constants,
- $D(H)$ = the molecular diffusion coefficient for hydrogen given by equation 8 in which the values of a_i and b_i are as defined in table 4,
- $\phi = 7.2 \times 10^{11} \text{ m}^{-2} \cdot \text{s}^{-1}$, the vertical flux of H, as defined in Category III Constants,
- $T = T(Z)$ as specified by equation 31,
- $T_{11} = 999.2356 \text{ K}$, the temperature derived from equation 31 for $Z = Z_{11}$,

$\alpha(H)$ = the thermal diffusion coefficient for H, -0.25 (dimensionless), as defined in table 4,

τ = $\tau(Z)$ defined in equation 40

$$\tau = \int_{Z_{11}}^Z \frac{g \cdot M(H)}{R^* \cdot T} \cdot dZ \quad (40)$$

Because $D(H)$ becomes very large compared with ϕ for heights above 500 km, the value of the integral term in equation 39 can be neglected at these heights, and atomic hydrogen is then essentially in diffusive equilibrium. Figure 5 depicts the graph of $n(H)$ as a function of Z , along with those of other species.

Mean Molecular Weight (Above 86 km)

Equations 35 through 39 permit the calculation of the number densities of the species N_2 , O , O_2 , Ar , He , and H for heights above 150 km, and of the first five of these species for heights between 86 and 150 km, where $n(H)$ is insignificant compared with $n(N_2)$. These number densities permit the calculation of several atmospheric parameters in the height region 86 to 1000 km. The first is mean molecular weight using equation 20. These values of M , along with those implicit in table 7 for Z from 80 to 86 km, plus the invariant value, M_0 , for heights from 0 to 80 km, are shown in figure 6.

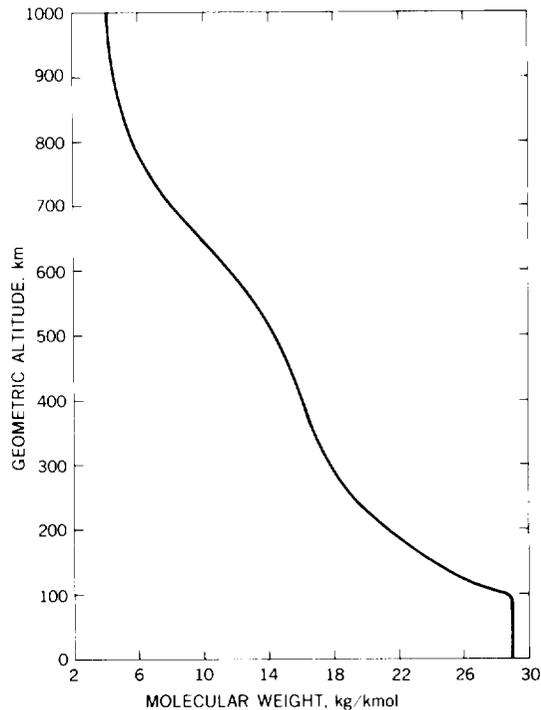


Figure 6. Mean molecular weight versus geometric altitude.

Pressure and Mass Density (Above 86 km)

The number densities of the several species also permit us now to compute total pressure and mass density for heights from 86 to 1000 km, using equation 33c, and one version of equation 1. Figure 7 depicts these values as well as those for heights below 86 km computed from equations 33a and 33b. Finally, these individual number densities permit the calculation of total number density, $N = \sum n_i$, at heights from 86 to 1000 km.

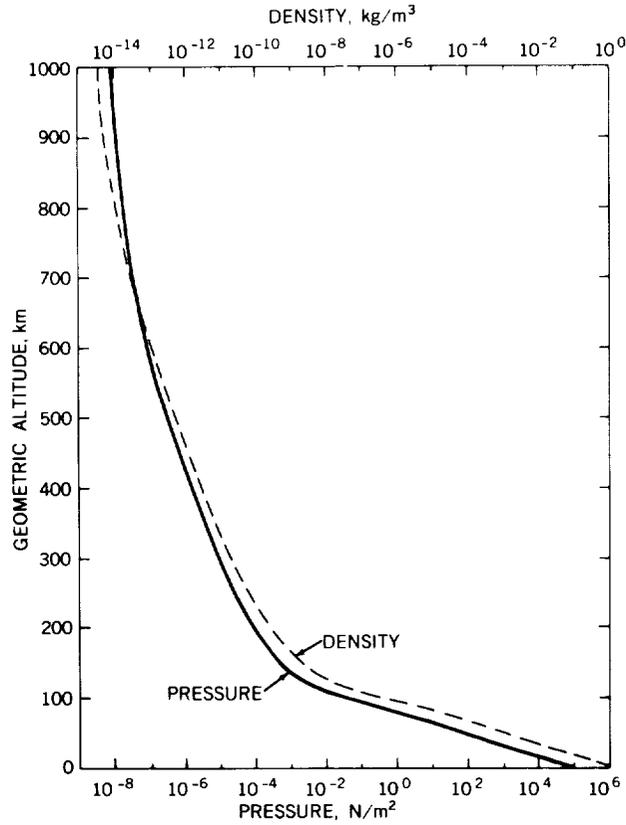


Figure 7. Total pressure and mass density versus geometric altitude.

Total Number Density

From equations 2, 22, and 33c, it is apparent that total number density, N , the number of neutral gas particles per unit volume of the atmosphere, may be expressed in any one of the following three equivalent forms:

$$N = \frac{M_0 \cdot N_A \cdot P}{T_M \cdot R^* \cdot M} = \frac{N_A \cdot P}{R^* \cdot T} = \sum n_i \quad (41)$$

The three forms are selected to satisfy three types of calculations:

1. Those depending upon values of T_M ,
2. Those depending upon values of T , and
3. Those depending upon values of Σn_i .

This format will be followed in specifying the computational equations, insofar as possible, for the balance of the quantities discussed in this document. A graph of the altitude variation of total number density is presented in figure 5, along with the number densities of individual species. That portion of equation 41 involving T_M is of particular interest in calculating N for heights from 0 to 86 km.

Mass Density

From equations 1, 22, and 33c, one may write the following three forms of computational equations for mass density, ρ :

$$\rho = \frac{P \cdot M_0}{R^* \cdot T_M} = \frac{P \cdot M}{R^* \cdot T} = \frac{\Sigma (n_i \cdot M_i)}{N_A} \quad (42)$$

The altitude-dependent variations of this quantity are shown in figure 7 along with those of pressure.

Mole Volume

Mole volume, v_m , of air is defined as the volume of one mole of air, where one mole of air is the amount consisting of a number of neutral particles equal to N_A . In S.I. units, the quantity v_m should specify the number of cubic meters containing 1 kmol of air. Since M has the dimensions of kg/kmol, and ρ has the dimensions of kg/m³, the ratio M/ρ , with the units m³/kmol, provides the definition of mole volume. In terms of equations 1, 22, and 33c, this ratio may be equated to the following series of expressions:

$$v_m = \frac{R^* \cdot M \cdot T_M}{M_0 \cdot P} = \frac{R^* \cdot T}{P} = \frac{N_A}{\Sigma n_i} \quad (43)$$

This quantity, while not tabulated in this Standard, is shown graphically in figure 8.

Scale Height

Pressure Scale Height

The quantity $R^* \cdot T/(g \cdot M)$, which has dimensions of length, is commonly associated with the concept of scale height and is the defining form of pressure scale height, H_p , used in this model. Thus, equations 20 and 22 may be written:

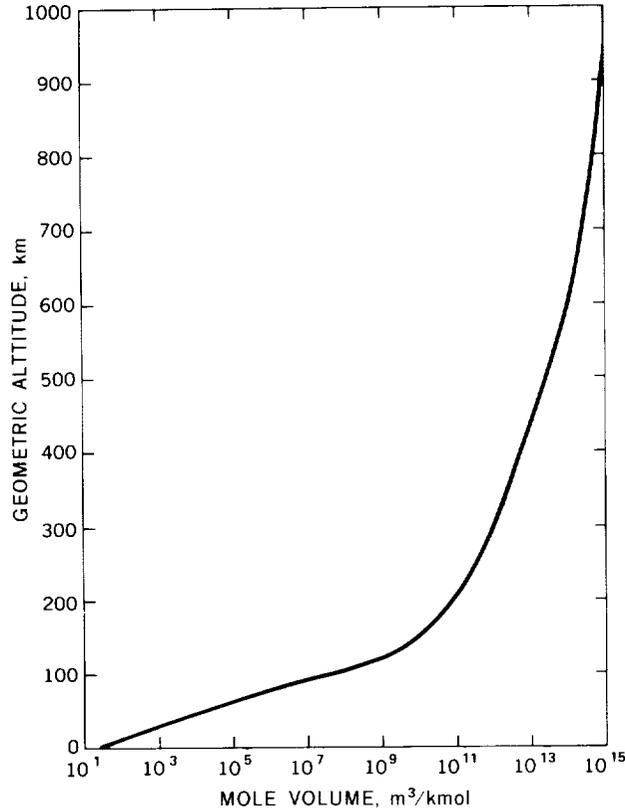


Figure 8. Mole volume versus geometric altitude.

$$H_P = \frac{R^* \cdot T_M}{g \cdot M_0} = \frac{R^* \cdot T}{g \cdot M} = \frac{R^* \cdot T \cdot \sum n_i}{g \cdot \sum (n_i \cdot M_i)} \quad (44)$$

The reciprocal of this quantity, which appears on the right-hand side of equation 5, is seen to equal the slope of the function $\ln P$ versus Z at height Z in the regions where hydrostatic equilibrium or diffusive equilibrium holds. In the present model, this condition is true for heights below 80 km (complete mixing), and is essentially true above approximately 120 km where diffusive equilibrium is nearly satisfied, and where each individual species is governed by equation 4.

In the 80- to 120-km region, where the transition from a completely mixed atmosphere to one in diffusive equilibrium takes place, the situation is complicated by the competition between three processes—molecular diffusion, eddy diffusion, and dissociation of molecular oxygen. These processes result in a vertical transport, such that equations 4 and 5 are no longer exactly true in this 40-km layer. Since molecular nitrogen is the dominant species in this altitude range, however, and since this species has a zero transport velocity in this model, the pressure scale height is still a good indicator of the rate of change of pressure in this height region.

It should be noted that equations 4 and 5 also become invalid at very high altitudes (the exospheric region) due to the infrequent collisions between neutral particles. Thus, in this region, the significance of H_p as a measure of $d \ln P / dZ$ again loses validity.

In equation 44, both g and T_M or all three of g , T , and M are functions of Z such that H_p is the local value of geometric pressure scale height. This quantity, which is the particular scale height tabulated in this Standard, and which is plotted in figure 9, is frequently but incorrectly associated with the altitude increment over which the pressure decreases by exactly a factor of $1/e$. The conditions necessary for the pressure to decrease by exactly that factor over an altitude increment of a single pressure scale height would be for the variables T , g , and M all to remain constant over that altitude interval. Since g may never be constant over any altitude interval, this particular concept of pressure decrease can rarely, if ever, apply exactly to H_p .

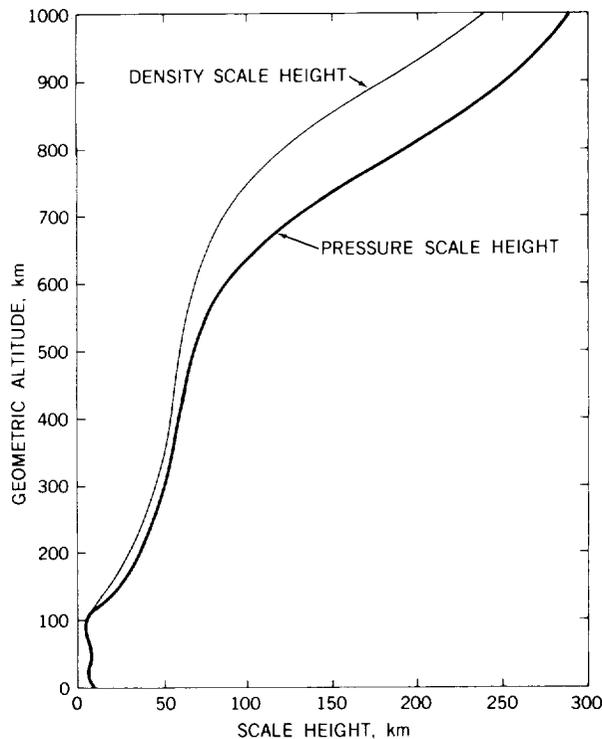


Figure 9. Pressure scale height and density scale height versus geometric altitude.

With the replacement of $g \cdot dZ$ in equation 5 by $g'_0 \cdot dH$ in accordance with equation 16, and with the replacement of T/M in equation 5 by T_M/M_0 in accordance with equation 22, as in the development of equations 33a and 33b, an expression for the slope of the function $\ln P$ versus geopotential height is developed. The negative reciprocal of the expression

for this slope is called geopotential pressure scale height, H'_p , and it has the same dimensions as geopotential height. The quantity H'_p is implicit in the computational pressure equations 33a and 33b. This concept of scale height may be expressed as follows:

$$H'_p = \left(\frac{-d \ln P}{dH} \right)^{-1} = \frac{R^* \cdot T_M}{g'_0 \cdot M_0} = \frac{R^* \cdot T}{g'_0 \cdot M} \quad (45)$$

Within a layer of constant T_M , or of constant ratio T/M , the pressure does, in fact, decrease by exactly 1/e of its initial value when the geopotential altitude increases by exactly one geopotential scale height. This is precisely the situation expressed by equation 33b which, in terms of H'_p , may be rewritten as

$$P = P_b \cdot \exp \left(\frac{H - H_b}{-H'_p} \right) \quad (46)$$

Equation 45 is rigorously valid only below 80 km, while equation 46 applies only within isothermal layers below 80 km.

Density Scale Height

Because of the relationship between H_p and the slope of $\ln P$ versus Z it is convenient to apply the name geometric density scale height, H_ρ , to the negative reciprocal of the slope of $\ln \rho$ versus Z . Using the equation of state (equation 1) to relate $d \ln \rho$ to $d \ln P$, one may define

$$H_\rho = \frac{H_p}{1 + H_p \cdot \left(\frac{d \ln T_M}{dZ} \right)} = \frac{H_p}{1 + H_p \cdot \left(\frac{d \ln T}{dZ} - \frac{d \ln M}{dZ} \right)} \quad (47)$$

The relationships implied between H_ρ and $d \ln \rho / dZ$ are subject to the same limitations as those between H_p and $d \ln P / dZ$ expressed above, that is, H_ρ is only an approximation to $(d \ln \rho / dZ)^{-1}$ between 80 and 120 km and in the exosphere, where the approximation becomes increasingly invalid with increasing altitude.

Within these limitations, it is apparent that in layers where T_M does not change with changing altitude, that is, where $(d \ln T_M / dZ) = 0$, H_ρ is equal to H_p . Within such layers, the slope of $\ln P$ versus Z at any particular altitude Z is identical to the slope of $\ln P$ versus Z , but neither P nor ρ decreases by a factor of 1/e when altitude increases by an amount $H_p = H_\rho$.

While density scale height is not tabulated in this Standard, values of this quantity are shown graphically with those of H_p in figure 9.

If, however, a geopotential density scale height, H'_ρ , is defined as the negative reciprocal of the slope of $\ln\rho$ with respect to geopotential, one may write

$$H'_\rho = \left(\frac{-d\ln\rho}{dH} \right)^{-1} = \frac{H'_p}{1 + H'_p \cdot \left(\frac{d\ln T_M}{dH} \right)} = \frac{H'_p}{1 + H'_p \cdot \left(\frac{d\ln T}{dH} - \frac{d\ln M}{dH} \right)} \quad (48)$$

This quantity is equal to H'_p in a layer within which T_M is constant, and for this condition, H'_ρ specifies the altitude change for which ρ changes by a factor of $1/e$. The quantity H'_ρ permits the writing of a density-altitude equation analogous to the pressure-altitude equation, equation 33b, but in terms of H'_ρ :

$$\rho = \rho_b \cdot \exp\left(\frac{H - H_b}{-H'_\rho} \right) \quad (49)$$

As in the case of equation 45, equation 49 is rigorously valid only below 80 km, while equation 49 applies only within isothermal layers below 80 km.

Mean Air-particle Speed

The mean air-particle speed, V , is the arithmetic average of the speeds of all air particles in the volume element being considered. All particles are considered to be neutral. For a valid average to occur, there must be a sufficient number of particles involved to represent mean conditions. Pressure and temperature gradients within the volume must also be negligible. The analytical expression for V is closely related to that for the speed of sound, and is proportional to the ratio T/M . Thus, in terms of equations 20 and 22,

$$V = \left[\frac{8 \cdot R^* \cdot T_M}{\pi \cdot M_0} \right]^{1/2} = \left[\frac{8 \cdot R^* \cdot T}{\pi \cdot M} \right]^{1/2} = \left[\frac{8 \cdot R^* \cdot T \cdot \Sigma n_i}{\pi \cdot \Sigma (n_i \cdot M_i)} \right]^{1/2} \quad (50)$$

The variation of particle speed with geometric altitude is shown in figure 10.

Mean Free Path

The mean free path, L , is the mean value of the distances traveled by each of the neutral particles, in a selected volume, between successive collisions with other particles in that volume. As in the case of V , a meaningful average requires that the selected volume be big enough to contain a large number of particles. The computational form for L is

$$L = \frac{2^{1/2} \cdot R^* \cdot M \cdot T_M}{2\pi \cdot N_A \cdot \sigma^2 \cdot M_0 \cdot P} = \frac{2^{1/2} \cdot R^* \cdot T}{2\pi \cdot N_A \cdot \sigma^2 \cdot P} = \frac{2^{1/2}}{2\pi \cdot \sigma^2 \cdot \Sigma n_i} \quad (51)$$

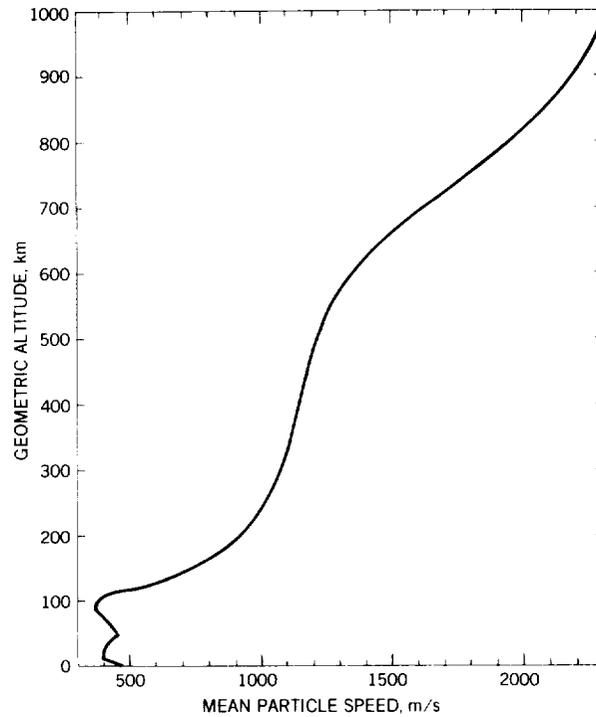


Figure 10. Mean air-particle speed versus geometric altitude.

where σ is the effective collision diameter of the mean air molecules. The adopted value of σ , that is, 3.65×10^{-10} m is suitable for that part of the atmosphere below about 86 km, which is dominated by N_2 and O_2 . Above this height, the value of σ , which depends upon composition in a complicated manner, begins to change significantly so that tabulations with four significant figures are no longer valid. At great altitudes, this expression for L is valid only under assumptions that hold M , T_M , P , and σ constant throughout the volume used. Figure 11 depicts the mean free path in terms of altitude.

Mean Collision Frequency

The mean collision frequency, ν , is the average speed of the air particles within a selected volume divided by the mean free path, L , of the particles within that volume. That is,

$$\nu = \frac{V}{L} \quad (52)$$

and in computation form:

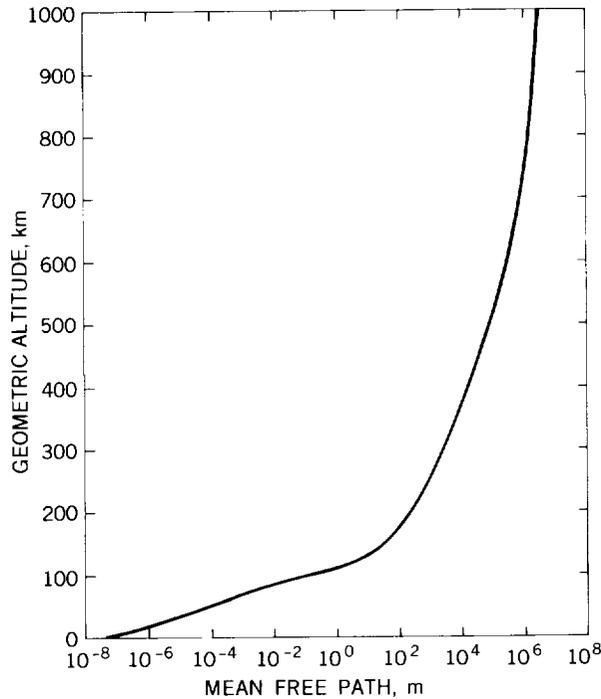


Figure 11. Mean free path versus geometric altitude.

$$\begin{aligned}
 \nu &= 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot M_0 \cdot P^2}{R^* \cdot M^2 \cdot T_M} \right]^{1/2} = 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot P^2}{R^* \cdot M \cdot T} \right]^{1/2} \\
 &= 4N_A \cdot \sigma^2 \cdot \left[\frac{\pi \cdot P^2 \cdot \Sigma n_i}{R^* \cdot T \cdot \Sigma (n_i \cdot M_i)} \right]^{1/2}
 \end{aligned} \tag{53}$$

Note that σ is again involved in this quantity, and hence ν has limitations similar to those of mean free path. The foregoing expressions are taken to apply to neutral particles only, since no considerations involving charged particles are introduced for purposes of developing the tables and graphs of this Standard.

Figure 12 graphically displays the variation of collision frequency with altitude. See *Mean Air-particle Speed* of this document for a discussion of the assumptions under which equation 53 is valid at great altitudes.

Speed of Sound

The expression adopted for the speed of sound, C_s , is

$$C_s = \left(\frac{\gamma \cdot R^* \cdot T_M}{M_0} \right)^{1/2} \tag{54}$$

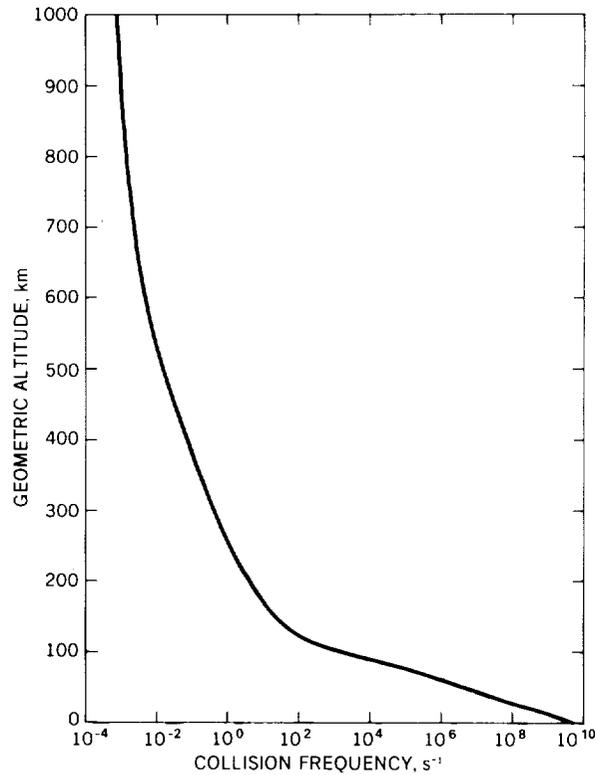


Figure 12. Mean collision frequency versus geometric altitude.

where γ is the ratio of specific heat of air at constant pressure to that at constant volume, and is taken to be 1.4 exact (dimensionless), as defined in Category II Constants. Equation 54 for speed of sound applies only when the sound wave is a small perturbation on the ambient condition. Calculated values of C_s have been found to vary slightly from experimentally determined values.

The limitations of the concept of speed of sound due to extreme attenuation are also of concern. The attenuation, which exists at sea level for high frequencies, applies to successively-lower frequencies as atmospheric pressure decreases, or as the mean free path increases. For this reason, the concept of speed of sound (except for frequencies approaching zero) progressively loses its range of applicability at high altitudes. Hence, the listing of the values for speed of sound are not given for heights above 86 km. Figure 13 shows the variation with altitude of the computed speed of sound.

Dynamic Viscosity

The coefficient of dynamic viscosity, μ , is defined as a coefficient of internal friction developed where gas regions move adjacent to each other at different velocities. The

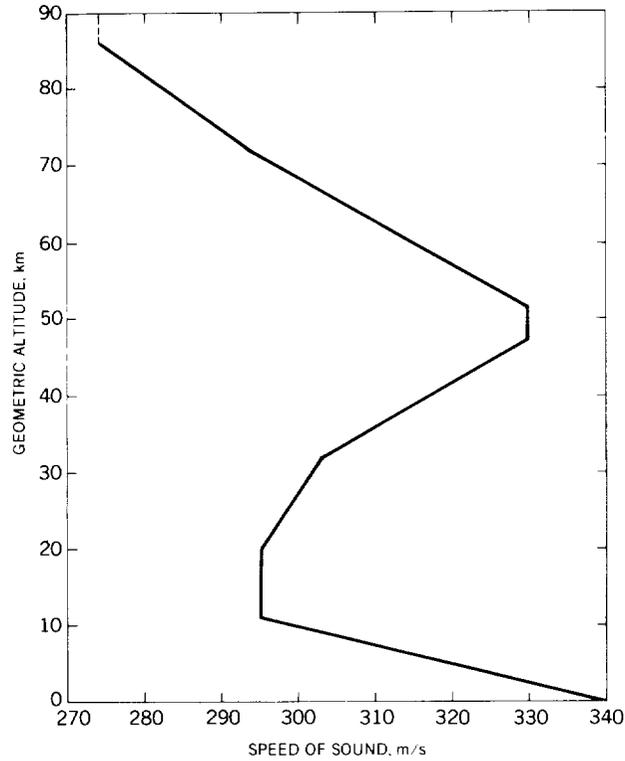


Figure 13. Speed of sound versus geometric altitude.

following expression, basically from kinetic theory but with constants derived from experiment, is used for computation of this quantity:

$$\mu = \frac{\beta \cdot T^{3/2}}{T + S} \quad (55)$$

In this equation, β is an exact constant equal to $1.458 \times 10^{-6} \text{ kg}/(\text{s} \cdot \text{m} \cdot \text{K}^{1/2})$, and S is Sutherland's constant, equal to 110.4 K (exact), both defined in Category II Constants. Because of the empirical nature of this equation, no attempt has been made to transform it into one involving T_M .

Equation 55 fails for conditions of very high and very low temperatures, and under conditions occurring at great altitudes. Consequently, tabular entries for coefficient of dynamic viscosity are not given for heights above 86 km. For these reasons, caution is also necessary in making measurements involving probes and other objects which are small with respect to the mean free path of molecules, particularly in the region of 32 to 86 km.

The variation of dynamic viscosity with altitude is shown in figure 14.

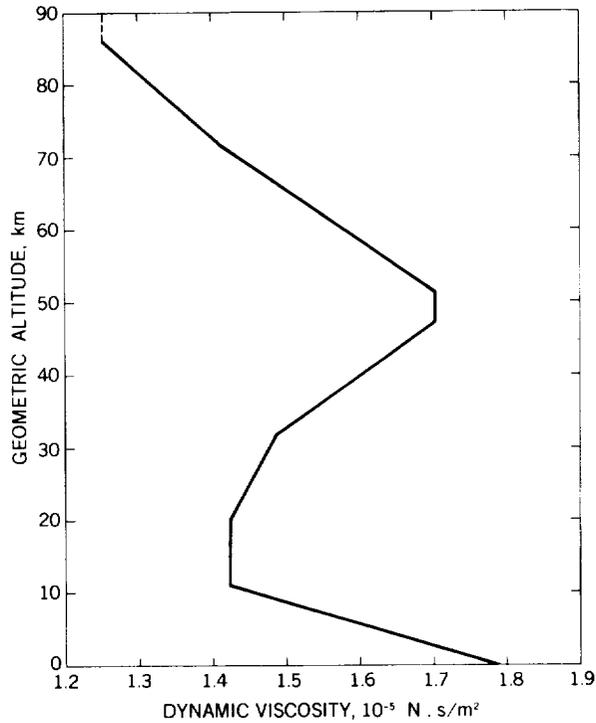


Figure 14. Dynamic viscosity versus geometric altitude.

Kinematic Viscosity

Kinematic viscosity, η , is defined as the ratio of the dynamic viscosity of a gas to the density of that gas, that is,

$$\eta = \frac{\mu}{\rho} \quad (56)$$

Limitations of this equation are comparable to those associated with dynamic viscosity, and consequently, tabular entries of kinematic viscosity are also not given for heights above 86 km. Figure 15 is a graphical representation of the variation of kinematic viscosity with altitude.

Coefficient of Thermal Conductivity

The empirical expression adopted for purposes of developing tabular values of the coefficient of thermal conductivity, k_t , for heights up to the 86-km level is as follows:

$$k_t = \frac{2.64638 \times 10^{-3} \cdot T^{3/2}}{T + 245.4 \times 10^{-(12/T)}} \quad (57)$$

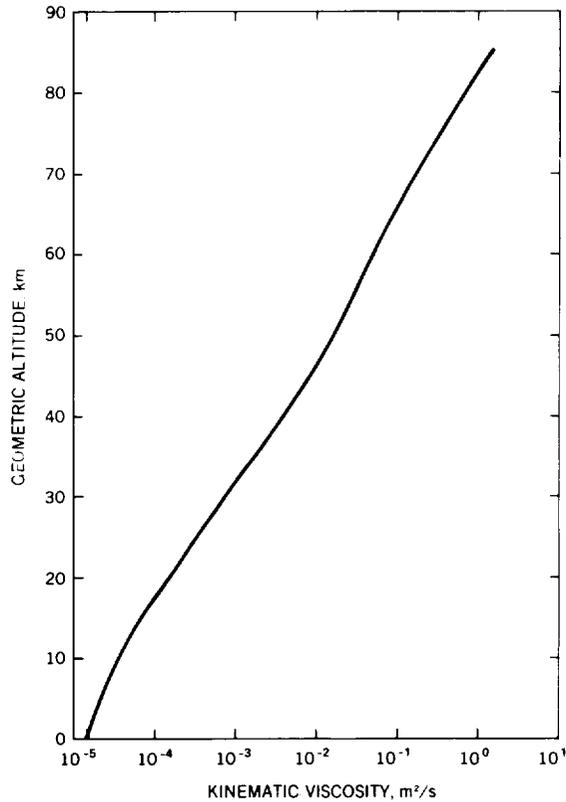


Figure 15. Kinematic viscosity versus geometric altitude.

This expression differs from that used in the *U.S. Standard Atmosphere, 1962* (COESA, 1962), in that the numerical constant has been adjusted to accommodate a conversion of the related energy unit from the temperature-dependent kilogram calorie to the invariant joule. Thus, the values of k_t in units of $J/(s \cdot m \cdot K)$ or $W/(m \cdot K)$ are greater than the values of k_t in units of $kcal/(s \cdot m \cdot K)$ by a factor of exactly 4.1858×10^3 , when the kilocalorie is assumed to be the one for $15^\circ C$. Kinetic-theory determinations of thermal conductivity of some monatomic gases agree well with observation. For these gases, thermal conductivity is directly proportional to the dynamic viscosity. Modification of the simple theory has accounted in part for differences introduced by polyatomic molecules and by mixtures of gases. Tabular entry of values for coefficient of thermal conductivity is terminated at 86 km. The variation with height of this quantity is shown in figure 16.

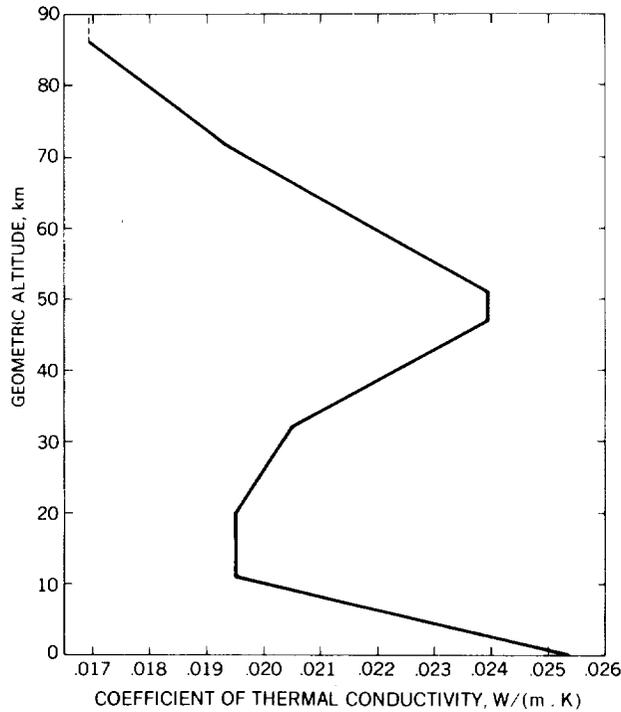


Figure 16. Thermal conductivity versus geometric altitude.

SUMMARY OF TABULAR VALUES OF ATMOSPHERIC PROPERTIES

Sea-level Values

The sea-level values of 15 of the atmospheric properties discussed in this Standard are as follows:

Symbol	Sea-level Value
$C_{s,0}$	3.40294×10^2 m/s
g_0	9.80665 m/s ²
$H_{p,0}$	8.4345×10^3 m
$k_{t,0}$	2.5326×10^{-3} J/(s · m · K) or W/(m · K)
L_0	6.6328×10^{-8} m
$v_{m,0}$	2.3643×10^1 m ³ /kmol
M_0	2.89644×10^1 kg/kmol
N_0	2.5470×10^{25} m ⁻³
P_0	1.01325×10^5 N/m ²
T_0	2.8815×10^2 K
V_0	4.5894×10^2 m/s
η_0	1.4607×10^{-5} m ² /s
μ_0	1.7894×10^{-5} kg/(m · s)
ν_0	6.9193×10^9 s ⁻¹
ρ_0	1.2250 kg/m ³

The sea-level values for g , P , and T are defined quantities; the remainder are quantities calculated from the preceding equations.

Conversion of Metric to English Units

For those who have need to work in the English System of Units, the conversion factors listed in table 8 are applicable to the atmospheric parameters tabulated or shown graphically in this Standard. For other transformations, see Mechtly (1973).

Table 8
Metric to English Conversion Factors for Properties of the
U.S. Standard Atmosphere, 1975 (COESA, 1975)

Symbol	To Convert from Metric Units	To English Units	Divide by
C_s	m/s	ft/s	$3.048 \times 10^{-1} *$
g	m/s^2	ft/s^2	$3.048 \times 10^{-1} *$
H_p	m	ft	$3.048 \times 10^{-1} *$
k_t	$W/(m \cdot K)$	$BTU/(ft \cdot s \cdot ^\circ R)$	$6.226\ 477\ 504 \times 10^{-3}$
L	m	ft	$3.048 \times 10^{-1} *$
v_m	$m^3/kmol$	$ft^3/lbmol$	$6.242\ 796\ 057 \times 10^{-2}$
M	$kg/kmol$	$lb/lbmol$	1.000*
N	m^{-3}	ft^{-3}	$3.531\ 466\ 672 \times 10^2$
P	mbar	in Hg (32°F)	$3.386\ 389 \times 10^1$
T or T_M	K	$^\circ R$	$5/9*$
V	m/s	ft/s	$3.048 \times 10^{-1} *$
η	m^2/s	ft^2/s	$9.290\ 304 \times 10^{-2} *$
μ	$N \cdot s/m^2$	$lb/(ft \cdot s)$	1.488 163 944
ν	s^{-1}	s^{-1}	1.000
ρ	kg/m^3	lb/ft^3	$1.601\ 846\ 3 \times 10^1$

*Exact definition.

Tables of Atmospheric Properties

Detailed tables of the height-dependent values of the various atmospheric properties defined by this standard have been prepared elsewhere (COESA, 1975). Abbreviated versions of these tables are presented at the end of this section in tables 9 through 15. The first four of these tables, 9 through 12, list atmospheric properties as a function of the boundary heights of the seven layers adopted to define the temperature-height profile between the surface of the earth and 86-km altitude. These boundary heights are specified in integer multiples of one geopotential kilometer, although the equivalent values of geometric height are also given.

Tables 13, 14, and 15 list the atmospheric properties of this Standard for 14 selected altitudes in the 86- to 1000-km interval. These altitudes include the boundary heights of the four layers adopted to define the temperature-height profile in this height region. Because of the large interval, 120 to 1000 km, comprising the highest layer of this model, the values of the various atmospheric properties in tables 13, 14, and 15 have been given for nine additional heights within this layer.

Only three tables (13, 14, and 15) are used to list the properties of this Standard for heights from 86 to 1000 km, as compared with four tables for the height region 0 to 86 km. This is due to the fact that, in this Standard, none of the four quantities—speed of sound, dynamic viscosity, kinematic viscosity, and thermal conductivity—is defined for heights above 86 km. Consequently, table 12, which lists the values of these properties below 86 km, has no counterpart for heights above 86 km. Table 11, listing number densities of five atmospheric gas species for heights from 0 to 86 km, has no counterpart in the detailed tables of the COESA document (COESA, 1975).

ACKNOWLEDGMENTS

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National Aeronautics and Space Administration
Greenbelt, Maryland January 2, 1976

Table 9
Temperature, Pressure, and Density versus Geopotential Altitude, 0 to 86 km

Altitude		Temperature			Pressure			Density	
H	Z	T	t	T _M	P	P	P/P ₀	ρ	ρ/ρ ₀
km'	km	K	°C	K	mbar	torr	—	kg/m ³	—
0.0000	0.0000	288.150	15.000	288.150	1.013250 +3	7.60000 +2	1.00000 +0	1.224999 +0	1.0000 +0
11.0000	11.0190	216.650	-56.500	216.650	2.263206 +2	1.69754 +2	2.23361 -1	3.639178 -1	2.9708 -1
20.0000	20.0631	216.650	-56.500	216.650	5.474889 +1	4.10650 +1	5.40330 -2	8.803480 -2	7.1865 -2
32.0000	32.1619	228.650	-44.500	228.650	8.680187 +0	6.51068 +0	8.56667 -3	1.322500 -2	1.0796 -2
47.0000	47.3500	270.650	- 2.500	270.650	1.109063 +0	8.31866 -1	1.09456 -3	1.427532 -3	1.1653 -3
51.0000	51.4124	270.650	- 2.500	270.650	6.693887 -1	5.02083 -1	6.60635 -4	8.616049 -4	7.0335 -4
71.0000	71.8019	214.650	-58.500	214.650	3.956420 -2	2.96756 -2	3.90468 -5	6.421099 -5	5.2417 -5
84.8520	86.0000	186.867	-86.283	186.946	3.733836 -3	2.80061 -3	3.68501 -6	6.957879 -6	5.6799 -6

Table 10
Seven Secondary Atmospheric Properties versus Geopotential Altitude, 0 to 86 km

Altitude		Acceleration Due to Gravity	Pressure Scale Height	Number Density	Mean Particle Speed	Mean Collision Frequency	Mean Free Path	Mean Molecular Weight
H km'	Z km							
0.0000	0.0000	9.80665	8.4345	2.5471 +25	458.94	6.9193 +9	6.6328 -8	28.9644
11.0000	11.0190	9.77274	6.3636	7.5669 +24	397.95	1.7824 +9	2.2327 -7	28.9644
20.0000	20.0631	9.74504	6.3817	1.8305 +24	397.95	4.3117 +8	9.2295 -7	28.9644
32.0000	32.1619	9.70817	6.7608	2.7499 +23	408.82	6.6542 +7	6.1438 -6	28.9644
47.0000	47.3500	9.66217	8.0407	2.9683 +22	444.79	7.8146 +6	5.6918 -5	28.9644
51.0000	51.4124	9.64992	8.0509	1.7915 +22	444.79	4.7166 +6	9.4303 -5	28.9644
71.0000	71.8019	9.58881	6.4258	1.3351 +21	396.11	3.1303 +5	1.2654 -3	28.9644
84.8520	86.0000	9.54659	5.6212	1.4473 +20	396.67	3.1667 +4	1.1674 -2	28.9522

Table 11
Number Densities of Five Species versus Geopotential Altitude, 0 to 86 km

Altitude		Number Densities				
H km'	Z km	n(N ₂) m ⁻³	n(O) m ⁻³	n(O ₂) m ⁻³	n(A) m ⁻³	n(He) m ⁻³
0.0000	0.0000	1.9888 +25	—	5.3353 +24	2.3789 +23	1.3346 +20
11.0000	11.0190	5.9082 +24	—	1.5850 +24	7.0671 +22	3.9648 +19
20.0000	20.0631	1.4292 +24	—	3.8342 +23	1.7096 +22	9.5912 +18
32.0000	32.1619	2.1471 +23	—	5.7599 +22	2.5682 +21	1.4408 +18
47.0000	47.3500	2.3176 +22	—	6.2174 +21	2.7722 +20	1.5553 +17
51.0000	51.4124	1.3988 +22	—	3.7526 +21	1.6732 +20	9.3870 +16
71.0000	71.8019	1.0425 +21	—	2.7966 +20	1.2469 +19	6.9957 +15
84.8520	86.0000	1.1298 +20	8.6 +16	3.0309 +19	1.35140 +18	7.5817 +14

Table 12
Sound Speed, Dynamic Viscosity, Kinematic Viscosity, and Thermal Conductivity
versus Geopotential Altitude, 0 to 86 km

Altitude		Sound Speed C_s m/s	Dynamic Viscosity		Kinematic Viscosity		Thermal Conductivity	
H km'	Z km		μ kg/(m · s)	μ/μ_0	η m ² /s	η/η_0	k W/(m · K)	k/k ₀
0.0000	0.0000	340.30	1.7894 - 5	1.0000 + 0	1.4607 - 5	1.0000 + 0	2.5326 - 2	1.0000 + 0
11.0000	11.0190	295.07	1.4216 - 5	7.9447 - 1	3.9064 - 5	2.6743 + 0	1.9505 - 2	7.7015 - 1
20.0000	20.0631	295.07	1.4216 - 5	7.9447 - 1	1.6148 - 4	1.1055 + 1	1.9505 - 2	7.7015 - 1
32.0000	32.1619	303.13	1.4868 - 5	8.3090 - 1	1.1242 - 3	7.6964 + 1	2.0510 - 2	8.0983 - 1
47.0000	47.3500	329.80	1.7037 - 5	9.5211 - 1	1.1935 - 2	8.1703 + 2	2.3938 - 2	9.4521 - 1
51.0000	51.4124	329.80	1.7037 - 5	9.5211 - 1	1.9773 - 2	1.3537 + 3	2.3938 - 2	9.4521 - 1
71.0000	71.8019	293.71	1.4106 - 5	7.8832 - 1	2.1968 - 1	1.5039 + 4	1.9336 - 2	7.6349 - 1
84.8520	86.0000	274.04	1.2529 - 5	7.0018 - 1	1.8007 + 0	1.2327 + 5	1.6962 - 2	6.6976 - 1

Table 13
Temperature, Pressure, and Density versus Geometric Altitude, 86 to 1000 km

Altitude		Temperature			Pressure			Density	
Z km	H km'	T K	T °C	T _M K	P mbar	P torr	P/P ₀	ρ kg/m ³	ρ/ρ_0
86.0	84.852	186.87	-86.28	186.95	3.7338 - 3	2.8006 - 3	3.6850 - 6	6.958 - 6	5.680 - 6
91.0	89.716	186.87	-86.28	187.36	1.5380 - 3	1.1536 - 3	1.5179 - 6	2.860 - 6	2.335 - 6
110.0	108.129	240.00	-33.15	254.93	7.1040 - 5	5.3284 - 5	7.0111 - 8	9.708 - 8	7.925 - 8
120.0	117.777	360.00	86.85	397.91	2.5382 - 5	1.9038 - 5	2.5050 - 8	2.222 - 8	1.814 - 8
150.0	146.542	634.39	361.24	762.34	4.5421 - 6	3.4069 - 6	4.4827 - 9	2.076 - 9	1.694 - 9
200.0	193.899	854.56	581.41	1161.84	8.4732 - 7	6.3554 - 7	8.3624 - 10	2.541 - 10	2.074 - 10
300.0	286.480	976.01	702.86	1594.82	8.7699 - 8	6.5780 - 8	8.6552 - 11	1.916 - 11	1.564 - 11
400.0	376.320	995.83	722.68	1804.53	1.4517 - 8	1.0888 - 8	1.4327 - 11	2.802 - 12	2.288 - 12
500.0	463.539	999.24	726.09	2019.70	3.0234 - 9	2.2677 - 9	2.9834 - 12	5.215 - 13	4.257 - 13
600.0	548.252	999.85	726.70	2517.13	8.2126 - 10	6.1599 - 10	8.1052 - 13	1.137 - 13	9.278 - 14
700.0	630.563	999.97	726.82	3621.33	3.1907 - 10	2.3932 - 10	3.1489 - 13	3.069 - 14	2.506 - 14
800.0	710.574	999.99	726.84	5225.13	1.7036 - 10	1.2778 - 10	1.6813 - 13	1.136 - 14	9.272 - 15
900.0	788.380	1000.00	726.85	6577.15	1.0873 - 10	8.1555 - 11	1.0731 - 13	5.759 - 15	4.701 - 15
1000.0	864.071	1000.00	726.85	7351.17	7.5137 - 11	5.6357 - 11	7.4154 - 14	3.561 - 15	2.907 - 16

Table 14
Seven Secondary Atmospheric Properties versus Geometric Altitude, 86 to 1000 km

Altitude		Acceleration Due to Gravity	Pressure Scale Height	Number Density	Mean Particle Speed	Mean Collision Frequency	Mean Free Path	Mean Molecular Weight
Z km	H km'	g m/s ²	H _p km	N m ⁻³	V m/s	ν s ⁻¹	L m	M kg/kmol
86.0	84.852	9.5466	5.621 +0	1.447 +20	369.7	3.17 +4	1.17 -2	28.95
91.0	89.716	9.5318	5.642 +0	5.962 +19	370.1	1.31 +4	2.83 -2	28.89
110.0	108.129	9.4759	7.723 +0	2.144 +18	431.7	5.48 +2	7.88 -1	27.27
120.0	117.777	9.4466	1.209 +1	5.107 +17	539.3	1.63 +2	3.31 +0	26.20
150.0	146.542	9.3597	2.338 +1	5.186 +16	746.5	2.3 +1	3.3 +1	24.10
200.0	193.899	9.2175	3.618 +1	7.182 +15	921.6	3.9 +0	2.4 +2	21.30
300.0	286.480	8.9427	5.119 +1	6.508 +14	1079.7	4.2 -1	2.6 +3	17.73
400.0	376.320	8.6799	5.968 +1	1.056 +14	1148.5	7.2 -2	1.6 +4	15.93
500.0	463.539	8.4286	6.879 +1	2.192 +13	1215.1	1.6 -2	7.7 +4	14.33
600.0	548.252	8.1880	8.824 +1	5.949 +12	1356.4	4.8 -3	2.8 +5	11.51
700.0	630.563	7.9576	1.306 +2	2.311 +12	1627.0	2.2 -3	7.3 +5	8.00
800.0	710.574	7.7368	1.939 +2	1.234 +12	1954.3	1.4 -3	1.4 +6	5.54
900.0	788.380	7.5250	2.509 +2	7.876 +11	2192.7	1.0 -3	2.1 +6	4.40
1000.0	864.071	7.3218	2.882 +2	5.442 +11	2318.1	7.5 -4	3.1 +6	3.94

Table 15
Number Densities of Six Atmospheric Species versus Geometric Altitude, 86 to 1000 km

Altitude		Number Density					
Z km	H km'	n(N ₂) m ⁻³	n(O) m ⁻³	n(O ₂) m ⁻³	n(Ar) m ⁻³	n(He) m ⁻³	n(H) m ⁻³
86.0	84.852	1.130 +20	8.600 +16	3.031 +19	1.351 +18	7.582 +14	
91.0	89.716	4.643 +19	2.953 +17	1.234 +19	5.478 +17	3.419 +14	
110.0	108.129	1.641 +18	2.302 +17	2.621 +17	1.046 +16	5.821 +13	
120.0	117.777	3.726 +17	9.274 +16	4.395 +16	1.366 +15	3.888 +13	
150.0	146.542	3.124 +16	1.780 +16	2.750 +15	5.000 +13	2.106 +13	3.767 +11
200.0	193.899	2.925 +15	4.050 +15	1.918 +14	1.938 +12	1.310 +13	1.630 +11
300.0	286.480	9.593 +13	5.433 +14	3.942 +12	1.568 +10	7.566 +12	1.049 +11
400.0	376.320	4.669 +12	9.584 +13	1.252 +11	2.124 + 8	4.868 +12	8.961 +10
500.0	463.539	2.592 +11	1.836 +13	4.607 + 9	3.446 + 6	3.215 +12	8.000 +10
600.0	548.252	1.575 +10	3.707 +12	1.880 + 8	6.351 + 4	2.154 +12	7.231 +10
700.0	630.563	1.038 + 9	7.840 +11	8.410 + 6	1.313 + 3	1.461 +12	6.556 +10
800.0	710.574	7.377 + 7	1.732 +11	4.105 + 5	3.027 + 1	1.001 +12	5.961 +10
900.0	788.380	5.641 + 6	3.989 +10	2.177 + 4	7.742 - 1	6.933 +11	5.434 +10
1000.0	864.071	4.626 + 5	9.562 + 9	1.251 + 3	2.188 - 2	4.850 +11	4.067 +10

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APPENDIX A

BOUNDARY-VALUE NUMBER DENSITIES OF ATMOSPHERIC CONSTITUENTS

The boundary-value neutral number densities of the several constituents defined to comprise the U.S. Standard Atmosphere at 86 km and above were determined using a deductive process based upon several assumptions. It was decided to include as constituents of this model atmosphere only those species which are known to contribute significantly to the total number density in any portion of the atmosphere between 86 and 1000 km, because of either their mixing distribution below the turbopause or their diffusive distribution above this height. Those gases which appear never to contribute more than about 0.5 percent of the total composition at any height within this region, or which for various reasons do not exhibit predictable behavior, were purposely omitted. Using these guidelines, the following gases were included: molecular nitrogen, N_2 ; molecular oxygen, O_2 ; argon, Ar; helium, He; and atomic oxygen, O. Atomic hydrogen, H, was included at heights above 149 km, but was not included in boundary-value considerations at 86 km. The remaining neutral gases which were used in establishing the sea-level value of the mean molecular weight, but which are not used at other heights in this model, are listed with the major gases and their respective contribution to the sea-level mean molecular weight in table 16.

Table 16
Sea-level Atmospheric Composition

Species	Fractional Volume F_i	Molecular Weight of Species M_i	$F_i M_i$
N_2	0.78084	28.0134	21.87398326
O_2	0.209476	31.9988	6.70298063
Ar	0.00934	39.948	0.37311432
CO_2	0.000314	44.00995	0.01381912
Ne	0.00001818	20.183	0.00036693
He	0.00000524	4.0026	0.00002097
Kr	0.00000114	83.80	0.00009553
Xe	0.000000087	131.30	0.00001142
CH_4	0.000002	16.04303	0.00003208
H_2	0.0000005	2.01594	0.00000101
	<hr/> $\Sigma F_i = 0.99999714$		<hr/> $\Sigma(F_i \cdot M_i) = 28.964425$

The first three of the gases used in this model comprise more than 0.9996 of the air in any unit volume at sea level, as is evident from summing the fractional composition F_i over these three species in table 16. Since the fractional volumes of these major species do not change significantly below the mesopause, which in this model is located at 86-km altitude, the sea-level fractional composition can be assumed to be approximately correct at 86 km. It is believed, however, that photochemical processes lead to small quantities of atomic oxygen in this height region, and a fractional amount of about 0.00059 by volume, or exactly 8.6×10^{16} atoms per m^3 , was agreed upon by Task Group II as an acceptable concentration of O for 86 km.

The introduction of the fixed amount of atomic oxygen at this height, and the simultaneous elimination of some minor species made it necessary to adjust the fractional concentrations of each of the four remaining species from their known sea-level values, F_i , by a common unknown factor, ϵ , to the 86-km fractional composition values, F'_i , such that

$$F'_i = \epsilon \cdot F_i \quad (58)$$

The 86-km fractional composition of atomic oxygen is equal to the ratio of $n(O)$ to N , where $n(O)$ is equal to $8.6 \times 10^{16} m^{-3}$, the adopted atomic-oxygen number density for that height, and N is the unknown total number density at that height. The sum of the 86-km fractional composition of the remaining four species is

$$\sum_4 F'_i = \epsilon \cdot \sum_4 F_i \quad (59)$$

The sum of the total of the 86-km fractional compositions, (that is, of the five species adopted to comprise the model at this height) must equal unity in accordance with the expression:

$$\epsilon \cdot \sum_4 F_i + \frac{n(O)}{N} = 1 \quad (60)$$

The total number density, N , is expressible in terms of the mean molecular weight, M , Avogadro's constant, N_A , and the mass density, ρ , the value of which is known at 86 km from other considerations. This relationship is

$$N = \frac{N_A \cdot \rho}{M} \quad (61)$$

The mean molecular weight at 86 km is the sum of the products $F'_i \cdot M_i$ over the five gases comprising the model at this height. For the atomic oxygen this product is

$$F'(O) \cdot M(O) = \frac{n(O) \cdot M(O)}{N} \quad (62)$$

while the sum of the products of the remaining four gases is expressible as

$$\sum_4 F'_i M_i = \epsilon \cdot \sum_4 F_i M_i \quad (63)$$

such that the mean molecular weight, M , at 86 km is expressed as

$$M = \epsilon \cdot \sum_4 F_i M_i + \frac{n(O) \cdot M(O)}{N} \quad (64)$$

Eliminating M between equations 61 and 64, and solving for ϵ yields

$$\epsilon = \frac{N_A \cdot \rho - n(O) \cdot M(O)}{N \cdot \sum_4 F_i M_i} \quad (65)$$

The elimination of ϵ between equations 60 and 65 leads to the following expression for total number density:

$$N = \frac{\left(\sum_4 F_i \right) \cdot [N_A \cdot \rho - n(O) \cdot M(O)]}{\sum_4 F_i \cdot M_i} + n(O) \quad (66)$$

From table 16 the value of $\sum_4 F_i$, the sum of F_i for the four species N_2 , O_2 , Ar, and He is seen to be 0.99966124, while the value of $\sum_4 F_i M_i$ for the same four species is seen to be 28.95009918. The value of $M(O)$ is taken to be one half of the value of $M(O_2)$, also given in table 16. The value of $8.6 \times 10^{16} \text{ m}^{-3}$ was adopted for $n(O)$, as previously stated, and N_A has the standard value $6.022169 \times 10^{26} \text{ kmol}^{-1}$. The value of ρ at 86 km is found to be $6.957880 \times 10^{-6} \text{ kg/m}^3$. These values, introduced into equation 66, yield a number density of $1.447265 \times 10^{20} \text{ m}^{-3}$ at 86 km. This value introduced into equation 58 leads to $\epsilon = 0.99974445$, while equation 64 then yields $M = 28.952208$ for the molecular weight at 86 km.

For the 86-km height, the values of F'_i , the fractional composition of each of the five species comprising the model at that height, are given in table 17 as the product $\epsilon \cdot F'_i$, along with the corresponding products $F'_i \cdot M_i$, and the corresponding values of $N \cdot F'_i$, the number densities of the five gas species comprising the model atmosphere at 86-km height.

The value of $\sum_5 F'_i$, the sum of the five values of F'_i listed in table 17 is seen to be 0.999999999, essentially the unity value which it should have. The sum of the five values of $F'_i \cdot M_i$ and of the five values of n_i , that is, $\sum_5 F'_i \cdot M_i$ and $\sum_5 n_i$, both of which are also given in table 17, shows essentially exact agreement with the value of their respective equivalents, M and N , computed independently. Thus, the validity of the computation is established.

Table 17
Number Densities and Molecular Weight at 86 km

	$F'_i = \epsilon F_i$	$F'_i \cdot m_i$ (kg/kmol)	$n_i = F'_i \cdot N$ (m^{-3})
N ₂	0.7806404557	21.86839334	$1.129793736 \times 10^{20}$
O ₂	0.2094224682	6.701267675	$0.3030898426 \times 10^{20}$
Ar	0.00933761315	0.3730189704	$0.0135140022 \times 10^{20}$
He	0.00000523866	0.0000209683	$0.0000075817 \times 10^{20}$
O	0.00059422421	0.0095072308	0.00086×10^{20}
	$\Sigma_5 F'_i = 0.99999999992$	$\Sigma_5 F'_i \cdot M_i = 28.9522082$	$\Sigma_5 n_i = 1.447265163 \times 10^{20}$

APPENDIX B

A SEGMENT OF AN ELLIPSE TO EXPRESS TEMPERATURE VERSUS HEIGHT

It is desired to determine the expression for a temperature function for a limited height region, $Z_8 = 91$ to $Z_9 = 110$ km, in the plane defined by Z and T , such that the slope of the function at each of the end points exactly matches a prescribed value. At $Z = Z_8$, where $T = T_8 = 186.8673$ K, the derivative of the function with respect to Z must be zero, to match the slope of the temperature-height profile in the isothermal layer between 86 and 91 km. At $Z = Z_9$, where $T = T_9 = 240$ K, the derivative of T with respect to Z must be 12 K/km to match the slope of a layer of constant temperature-height gradient between 110 and 120 km. A suitably adjusted ellipse will satisfy these conditions.

The general equation of an ellipse in terms of Z and T with center at $Z = 0$ and $T = 0$ is

$$\frac{Z^2}{a^2} + \frac{T^2}{A^2} = 1 \quad (67)$$

With the center shifted to $Z = Z_c$ and $T = T_c$ the expression becomes

$$\frac{(Z - Z_c)^2}{a^2} + \frac{(T - T_c)^2}{A^2} = 1 \quad (68)$$

The derivative of equation 68 with respect to Z is

$$\frac{2(Z - Z_c)}{a^2} + \frac{2(T - T_c)}{A^2} \cdot \frac{dT}{dZ} = 0 \quad (69)$$

To meet the condition for $dT/dZ = 0$ at $Z = Z_8$, equation 69 is evaluated for those conditions, and it is found that $Z_c = Z_8$, such that equation 68 may be rewritten as

$$\frac{(Z - Z_8)^2}{a^2} + \frac{(T - T_c)^2}{A^2} = 1 \quad (70)$$

Evaluating equation 70 for $Z = Z_8$ and $T = T_8$ leads to

$$A = T_8 - T_c \quad (71)$$

Substituting Z_c for its equal Z_8 in equation 69, and evaluating that expression for $Z = Z_9$, where $T = T_9$ and where (dT/dZ) has the particular value $L_{K,9} = 12$, and finally solving the resulting expression for $1/a^2$ yields

$$\frac{1}{a^2} = \frac{-(T_9 - T_c) \cdot L_{K,9}}{A^2 \cdot (Z_9 - Z_8)} \quad (72)$$

Evaluating equation 70 at $Z = Z_9$, where $T = T_9$, and solving for $1/a^2$ yields

$$\frac{1}{a^2} = \frac{A^2 - (T_9 - T_c)^2}{A^2 \cdot (Z_9 - Z_8)} \quad (73)$$

Eliminating $1/a^2$ between equations 72 and 73, and solving for T_c leads to

$$T_c = \frac{L_{K,9} \cdot (Z_9 - Z_8) \cdot T_9 + T_8^2 - T_9^2}{L_{K,9} \cdot (Z_9 - Z_8) + 2T_8 - 2T_9} \quad (74)$$

The elimination of A between equations 71 and 73 yields

$$a = \frac{(Z_9 - Z_8) \cdot (T_8 - T_c)}{[(T_8 - T_c)^2 - (T_9 - T_c)^2]^{1/2}} \quad (75)$$

Finally, solving equation 70 for T yields the functional expression

$$T(Z) = T_c + A \cdot \left[1 - \left(\frac{Z - Z_8}{a} \right)^2 \right]^{1/2} \quad (76)$$

The evaluation of equations 71, 74 and 75, in accordance with $Z_8 = 91$ km, $T_8 = 186.8673$ K, $Z_9 = 110$ km, $T_9 = 240$ K, and $L_{K,9} = 12$ K/km, yields the following values for the three constants in equation 76:

$$T_c = 263.1905 \text{ K}$$

$$A = -76.3232 \text{ K}$$

$$a = -19.9429 \text{ km}$$

Since it was shown that $Z_c = Z_8$, the ellipse which meets the required derivative and temperature conditions, has its center at $Z = 91$ km and $T = 263.1905$ K, and equation 76 represents the function which meets the required conditions.

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APPENDIX C

GEOPOTENTIAL AND MOLECULAR-SCALE TEMPERATURE

The concept of geopotential as a measure of height was introduced by Bjercknes et al. (1910) who, according to Harrison (1954), made use of the term *dynamic height* in referring to the geopotential of a point because in meteorology, geopotential is preferable to geometric height as a representation of the vertical coordinate of the point. Bjercknes proposed the term *geodynamic meter* (gdm), or *dynamic meter* for short, as the name of the unit of dynamic height. With a slight revision in the definition of this concept, the Aerological Commission of the International Meteorological Organization in 1947 adopted the *geopotential meter* (gpm) defined such that $1 \text{ gpm} = 0.98 \text{ gdm} = 9.8 \text{ m}^2/\text{s}^2$. This definition related the geopotential meter directly to the sea-level value of the acceleration of gravity.

In the U.S. Standard Atmosphere of 1955, published in NACA Report 1235 (1955), the *standard geopotential meter* (sgpm) was defined in terms of the so-called standard sea-level value of the acceleration of gravity, 9.80665 m/s^2 , such that $1 \text{ sgpm} = 9.80665 \text{ m}^2/\text{s}^2$. The use of the standard geopotential meter (now designated as m') was continued in the Air Research and Development Command (ARDC) Model Atmosphere, 1956 (Minzner and Ripley, 1956); in the U.S. Extension to the ICAO Standard Atmosphere (Minzner et al., 1958); in the ARDC Model Atmosphere, 1959 (Minzner et al., 1959), in the *U.S. Standard Atmosphere, 1962* (COESA, 1962); as well as in the 1975 revision of that Standard Atmosphere (COESA, 1975). Implicitly, geopotential has been used as a measure of height in all standard and model atmospheres, in which the tabulated values of atmospheric properties have been calculated on the basis of a value of the acceleration of gravity which is invariant with height.

The numerical value of the height of a particular point above sea level, in regions where the sea-level value of the acceleration of gravity is equal to or less than 9.80665 m/s^2 , is smaller when expressed in geopotential than when expressed in geometric meters. One standard geopotential meter is exactly equal to one geometric meter only under very specialized and perhaps hypothetical conditions. These conditions involve the existence of a region with a gravitational-field value of exactly 9.80665 m/s^2 (generally near the earth's surface), and with a zero vertical gradient of that field over a height interval of at least one geometric meter. The concept of $1 \text{ m}'$ is more precisely defined, however, in terms of a specific change in potential energy of a unit mass, such that $1 \text{ m}'$ is exactly the height increment through which one must lift one kilogram (mass) in order to increase its potential energy by 9.80665 J . The geometric equivalent of this height increment is essentially one geometric meter (m) at sea level, but increases to more than 1 m with increasing height, in a manner inversely proportional to the corresponding decrement of the acceleration of gravity.

The concept of geopotential has some very meaningful uses in meteorology and aeronomy. One of its principal advantages is the reduction from 4 to 3 of the number of variables in the hydrostatic equation when geopotential is introduced. This feature is particularly desirable in expressions involving the integrated forms of the hydrostatic equation. Thus, the use of this concept, either implicitly or explicitly in the definition of standard atmospheres produced prior to the development of high-speed digital computers, simplified the defining equations and reduced the work involved in manual computation of the related tables. The use of geopotential has been maintained in all subsequent standard atmospheres in order to avoid any revision of the lower portion of the tables which have represented the established standard for the past 50 years.

The introduction of the concept of molecular-scale temperature, T_M , came at a time when standard-atmosphere tables were being extended to heights where the composition, and hence the mean molecular weight, were unknown. The concept (without a particular name) was first applied to atmospheric models by the Rocket Panel (1952) in a paper which referred to a "derived 'Temperature' based upon an assumed constant mean molecular weight μ of 28.966 g/mole for the atmosphere." The use of this special kind of temperature not only avoided the problem of determining or guessing at a value of mean molecular weight at high altitudes, but also eased the problem of hand or desk-computer calculations by leading to simpler equations than would have resulted if specific height functions had been introduced for both kinetic temperature and mean molecular weight.

The name *molecular-scale temperature* for this derived temperature first appeared in the ARDC Model Atmosphere, 1959. In this model, molecular-scale temperature, T_M , at any height Z was related to the kinetic temperature, T , at height Z by the relationship $T_M(Z) = T(Z) [M_0/M(Z)]$ where M_0 was the currently accepted sea-level value of the mean molecular weight of air, 28.9644 kg/kmol, and M was the implicit value of the mean molecular weight of air at height Z . This concept was carried into the U.S. Extension to the ICAO Standard Atmosphere, 1958; the ARDC Model Atmosphere, 1959; the *U.S. Standard Atmosphere, 1962* (COESA, 1962), and has been carried into the 1975 revision to that Standard Atmosphere (COESA, 1975).

The temperature-height profile of each of the first three of these four previously published models was defined in terms of T_M (in units of Kelvin, K) as a function of geopotential height H (in units of m') over the entire height range of these models. Since many aeronomers studying the upper atmosphere are unfamiliar with these quantities and prefer the quantities kinetic temperature, T , and geometric heights, Z , it was decided that, in the *U.S. Standard Atmosphere, 1962* (COESA, 1962) the model should be divided at 90 geometric kilometers, with temperatures in that part of the model from 0 to 90 km defined in terms of T_M and H , and in that part above 90 km defined in terms of T and Z . This procedure has been continued in the 1975 U.S. Standard Atmosphere, but in this revision, the division occurs at 86 geometric kilometers where the geopotential is 84.8520 km' , and $M = 28.9522$ kg/kmol.

Before the name molecular-scale temperature was used in model atmospheres, Brombacher (1953), in an attempt to simplify the calculation of standard atmospheres, combined the height-dependent acceleration of gravity, the height-dependent molecular weight, and the height-dependent kinetic temperature into a single variable, which he called scale-height temperature. The use of this variable, however meritorious, was never adopted in U.S. Standard Atmospheres.

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APPENDIX D
THE CALCULATION OF A DYNAMIC MODEL FOR THE
1975 U.S. STANDARD ATMOSPHERE

INTRODUCTION

The objective of this appendix is to describe the procedure for the calculation of a dynamic model of the earth's atmosphere between 50 and 150 km, comprised of an internally consistent set of diurnally averaged gas concentrations versus altitude, one for each of the four major atmospheric gas species in this height region, that is, nitrogen, molecular and atomic oxygen, and argon, where each of these concentration profiles meets the following two conditions:

1. The concentration values versus height are the result of a time-dependent, photochemical-transport calculation which incorporates measured chemical-reaction-rate constants, solar-radiation fluxes, and turbulent diffusion coefficients into coupled sets of equations of motion and continuity.
2. The calculated number densities of each of the four species at 150 km fall within particular limits recommended by the COESA Working Group.

This sophisticated and detailed calculation serves to establish the physical basis for the generation of dynamic models of the earth's atmosphere, and yields height profiles of number-density flux values which are approximated by artificially adjusted functions for the calculation of the 1975 U.S. Standard Atmosphere.

BASIC CONSIDERATIONS

The species considered are O, O₂, O₃, O¹D, O₂(¹Δ_g), OH, H, HO₂, H₂O, H₂O₂, H₂, Ar, and He. The number densities, from 50 to 150 km, are obtained through a semi-implicit, finite-difference solution of a system of mass- and momentum-conservation equations (Shimazaki, 1967; Keneshea and Zimmerman, 1970). In these calculations, thermal-diffusion factors for the species, H, H₂ and He have the values respectively of -0.39, -0.31, and -0.36 (Zimmerman and Keneshea, 1975). The numerical approach is essentially that introduced by Shimazaki (1967), but modified at the boundaries and in the volume integrations, following George et al. (1972). Table 18 lists the chemical reactions and the associated rate constants actually used in the generation of the resulting concentration profiles. It should be noted, however, that refined rate-constant measurements, made since these calculations were completed, indicate the need for a revision of some of the listed values. The current calculations have not been updated with these new rate-constant values, however, since the changes have only a negligible influence on the concentrations of O, O₂, Ar,

Table 18
 The Chemical Reactions and Associated Reaction Rates k_j
 Expressed in the Form of the Value of Rate Coefficients
 A_j , B_j , and C_j where $k_j = A_j \cdot (T/300)^{B_j} \times \exp(C_j/T)$

	REACTION				A_j	B_j	C_j
1	O	+ O	+ M	\rightarrow O ₂ + M	3.00E-33	-2.9	
2	O	+ O ₂	+ M	\rightarrow O ₃ + M	5.50E-34	-2.6	
3	O	+ O ₃		\rightarrow O ₂ + O ₂	1.20E-11		-2.00E+03
4	H	+ O ₃		\rightarrow O ₂ + OH	2.60E-11		
5	OH	+ O		\rightarrow H + O ₂	5.00E-11		
6	OH	+ O ₃		\rightarrow HO ₂ + O ₂	4.00E-14		
7	H	+ O ₂	+ M	\rightarrow HO ₂ + M	7.40E-33		6.10E+02
8	HO ₂	+ O		\rightarrow OH + O ₂	1.00E-11		
9	HO ₂	+ O ₃		\rightarrow OH + O ₂ + O ₂	1.00E-17		
10	OH	+ OH		\rightarrow H ₂ O + O	2.00E-12		
11	OH	+ HO ₂		\rightarrow H ₂ O + O ₂	2.00E-10		
12	H	+ HO ₂		\rightarrow H ₂ + O ₂	3.00E-12		
13	H	+ HO ₂		\rightarrow OH + OH	1.00E-11		
14	O	+ H ₂		\rightarrow OH + H	7.00E-11		-5.10E+03
15	HO ₂	+ HO ₂		\rightarrow H ₂ O ₂ + O ₂	3.00E-12		
16	OH	+ H ₂ O ₂		\rightarrow H ₂ O + HO ₂	1.70E-11		-9.00E+02
17	O	+ H ₂ O ₂		\rightarrow OH + HO ₂	4.00E-15		
18	H	+ H ₂ O ₂		\rightarrow H ₂ + HO ₂	3.90E-11		-4.60E+03
19	O ¹ D	+ O ₃		\rightarrow O ₂ + O ₂	3.00E-10		
20	O ¹ D	+ O ₂		\rightarrow O + O ₂	6.00E-11		
21	O ¹ D	+ N ₂		\rightarrow O + N ₂	9.00E-11		
22	O ¹ D	+ H ₂		\rightarrow OH + H	1.00E-11		
23	O ¹ D	+ H ₂ O		\rightarrow OH + OH	1.00E-11		
24	O ₂ ¹ Δg	+ O ₃		\rightarrow O ₂ + O ₃	3.00E-15		
25	O ₂ ¹ Δg	+ M		\rightarrow O ₂ + M	4.40E-19		
26	O ₂ ¹ Δg	+ H		\rightarrow OH + O	1.10E-14		
27	O ₂ ¹ Δg			\rightarrow O ₂	2.58E-04		
28	O ₂	+ hν		\rightarrow O + O			
29	O ₂	+ hν		\rightarrow O ¹ D + O			
30	O ₃	+ hν		\rightarrow O ₂ + O			
31	O ₃	+ hν		\rightarrow O ¹ D + O ₂ ¹ Δg			
32	H ₂ O	+ hν		\rightarrow OH + H			
33	H ₂ O ₂	+ hν		\rightarrow OH + OH			

Note: The units of the two-body reaction rates are cm³ s⁻¹, while those for the three-body reaction rates are cm⁶ s⁻¹.

and He in the altitude region above 80 km. The intensity of the solar-radiation flux used in these calculations is 0.65 of that shown in figure 17, which depicts the Ackerman (1971) values of solar-radiation flux versus wavelength. The absorption cross sections were taken from various sources. For O_2 and O_3 , these cross sections were taken from the compilation of Ackerman (1971) with the exception of those for the Schumann-Runge bands of O_2 , for which region the values measured by Hudson and Mahle (1972) were used. The adopted absorption cross sections for water vapor and hydrogen peroxide are those reported by Watanabe and Zelikoff (1953), and by Volman (1963), respectively. The temperature-height profile up to 150 km and the values of mean molecular weight up to the turbopause are those recommended by the Working Group of COESA. Using these data, the initial species distributions were calculated assuming complete mixing up to the turbopause, and diffusive equilibrium above it.

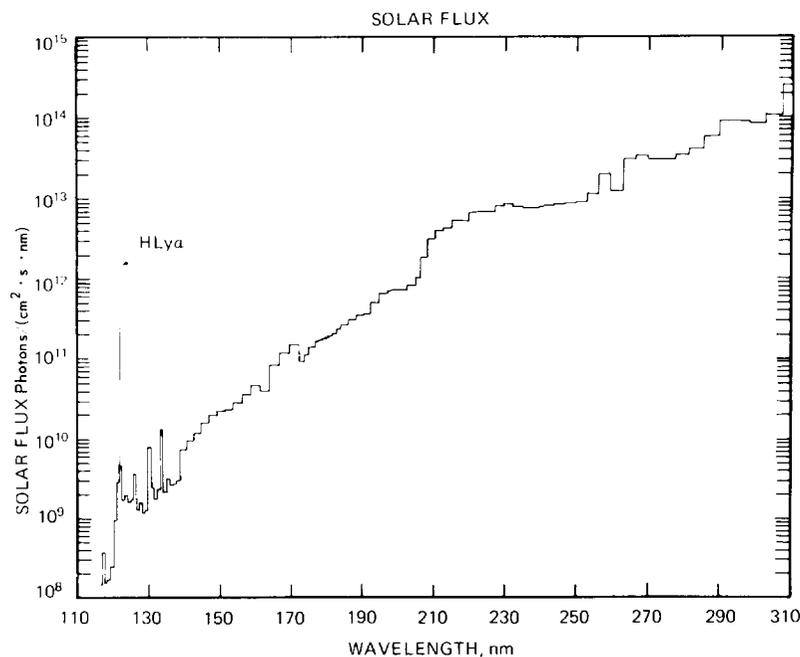


Figure 17. Solar radiation flux versus wavelength in the region from 115 to 310 nm as reported by Ackerman.

The total number density was obtained by integrating the hydrostatic equation, where the sea-level values of mass density and of mean molecular weight were taken from the *U.S. Standard Atmosphere, 1962* (COESA, 1962).

Beginning with these static profiles, the steady-state solution of all species was determined. The time-dependent calculations were then allowed to proceed for 15 solution days using a semi-implicit, finite-difference technique, a variable-time step up to 30 minutes, and a fixed-height step of 100 m. This stringent height step was shown to be necessary to restrict the errors generated by species gradients when height steps larger than 100 m were used.

The height-dependent, turbulent diffusion coefficients used are shown in figure 18, and are based upon observations of turbulence in chemical trails (Philbrick et al., 1973). These values are derived (Zimmerman and Trowbridge, 1973) from the fluctuation dynamics observed in rocket-borne chemical releases, and are valid from about 88 to 112 km. Because of the lack of chemical-tracer wind and turbulence measurements in the altitude region between 50 and 88 km, an exponential fit has been assumed between the reported value of $1 \times 10^5 \text{ cm}^2 \text{ s}^{-1}$ (Beaudoin et al., 1967) at 50 km, and the values at 88 km.

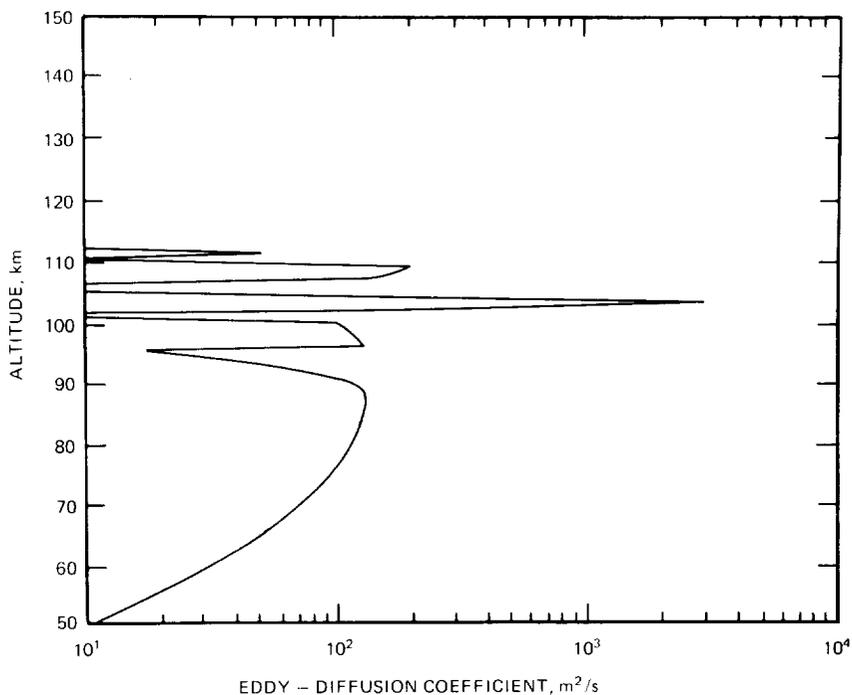


Figure 18. Eddy-diffusion coefficient versus altitude.

RESULTS

The time-dependent calculations were continued for the above mentioned period of time, after which the species concentrations reproduced themselves to within 1 percent over a diurnal cycle, a condition which is called arriving at diurnal reproducibility. The diurnal averages of the concentration of O, O₂, and Ar are then calculated and extrapolated to 250 km by assuming diffusive equilibrium without thermal diffusion above the 150-km boundary. Figure 19, depicting the height profiles of the N₂ concentration and temperature, shows the initial conditions used in these one-dimensional calculations. Figure 20 shows the resulting diurnally averaged height profiles of O, O₂, and Ar, each of which is in good agreement with the 150-km values recommended by the COESA Working Group, and shown as error bars.

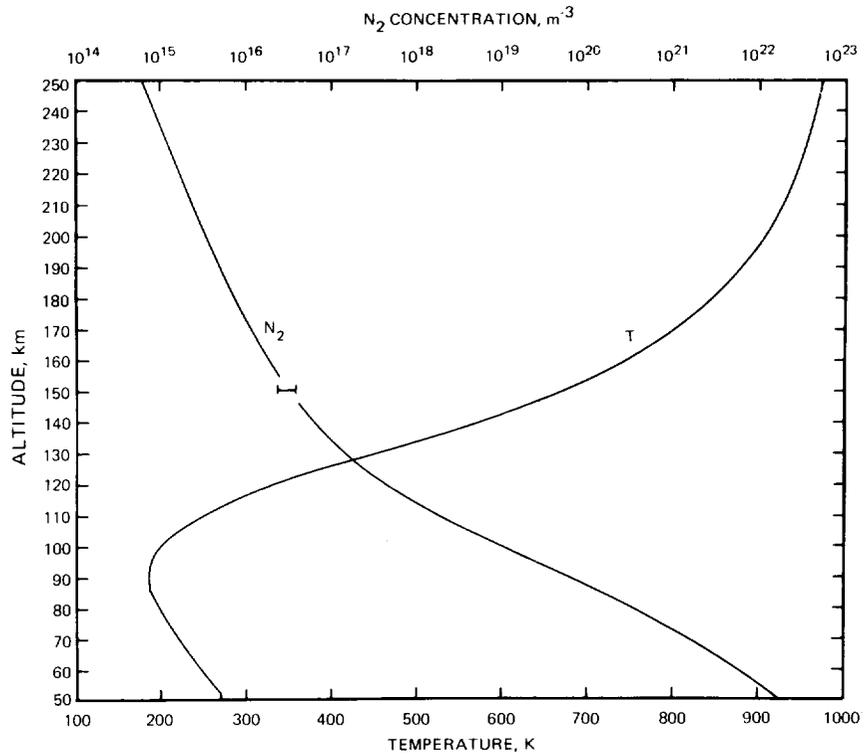


Figure 19. Altitude profile of kinetic temperature and molecular nitrogen concentration with the range of 150-km values recommended by the COESA Working Group indicated by an error bar.

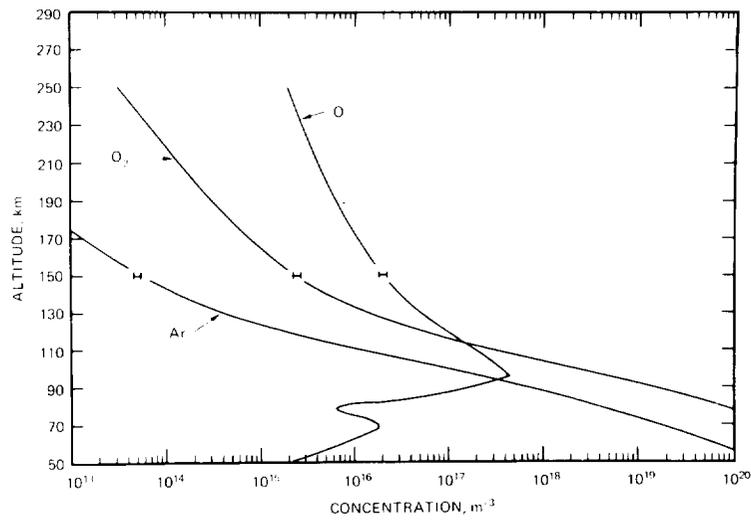


Figure 20. Altitude profiles of diurnally-averaged concentrations of O, O₂, and Ar, with the range of 150-km values recommended by the COESA Working Group indicated by corresponding error bars.

Thus, it has been demonstrated that an internally self consistent model of the density structure of the upper mesosphere and lower thermosphere may be calculated from measured values of solar radiation flux, chemical-reaction-rate constants, and derivatives of measured vertical-turbulent-transport parameters deduced from chemical-trail studies.

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