NUMERICAL STUDY OF SOME SOLAR-WIND INTERACTION MODELS WITH SPACE OBJECTS

O. M. Belotserkovskii and V. Ya. Mitnitskii
Computing Center, Academy of Sciences
Moscow, USSR

ABSTRACT

Problems in space physics are discussed whose models, in simplified form, reduce to a supersonic flow scheme with a detached shock wave, namely:

Problem A. Solar-wind interaction with an intrinsic planetary magnetic field.

Problem B. Solar-wind interaction with the ionized component of the atmosphere of a comet.

Problem C. Solar-wind interaction with the ionosphere of a planet which does not possess its own magnetic field.

The numerical study of the above problems is performed with the use of magnetogasdynamic equations for an ideal single-fluid model. From the physical viewpoint, the problems are solved in terms of as simple phenomena as possible; the principal objective is to make recently-developed methods of numerical analysis of mixed flows applicable to space physics problems.

A common feature of all the problems in question is the assumption of the presence of a tangential discontinuity separating the solar plasma flow (the external flow) from some region (the magnetosphere, the ionosphere) surrounding a planet (the internal region). A detached shock wave is assumed to be present in the external flow.

PROBLEM A: SOLAR-WIND INTERACTION WITH AN INTRINSIC PLANETARY MAGNETIC FIELD

Supersonic and super-Alfvénic stationary flows around the magnetic cavity formed by a dipole oriented perpendicular to the oncoming flow velocity are investigated (figure 1). The magnetic field frozen-in to the solar plasma is assumed to be parallel to the undisturbed flow velocity. Magnetostatic equations are assumed to hold for the internal region.

A three-dimensional solution (r, θ, φ) is calculated. With respect to the angle, θ, a trigonometric approximation of functions is used to represent its values along several planes, φ_i. Then a boundary-value problem for two independent variables is numerically solved by means of the method of integral relations. The tangential discontinuity shape is found by minimizing the residual or differences in total pressure on both sides of the discontinuity

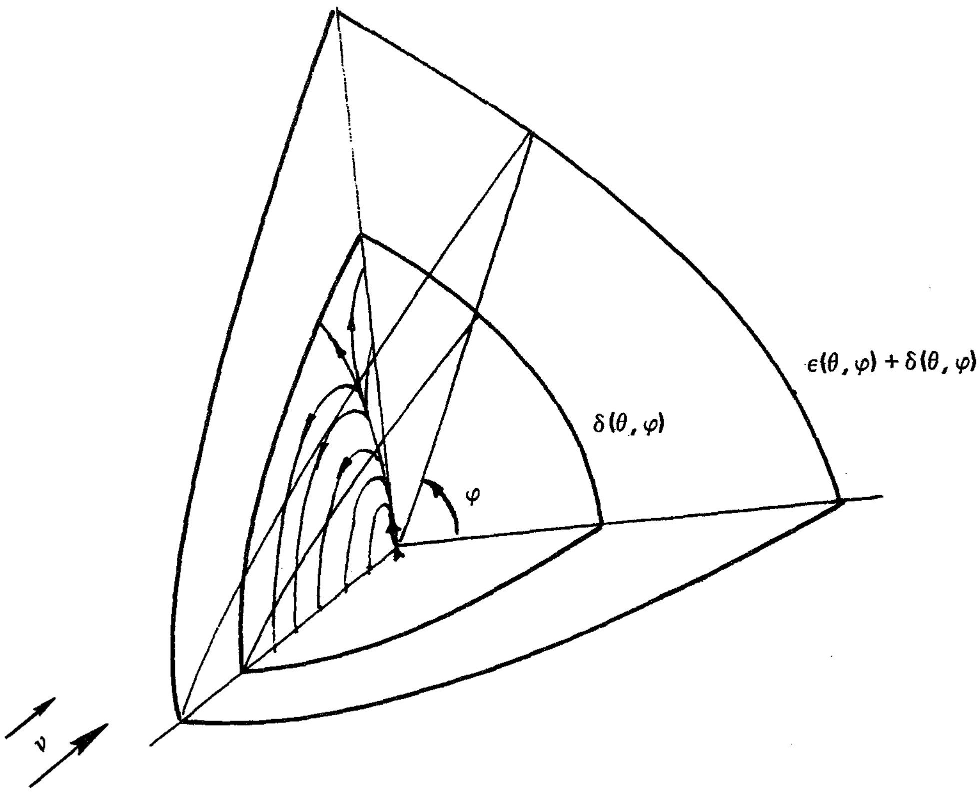

Figure 1. Supersonic and super-Alfvénic stationary flows around the magnetic cavity.

$$I = \sum_{i,j} (P - H^2/8\pi)_{ij}, \quad r_{ij} = \delta(\theta_i, \varphi_j).$$

The fact that the problem involves two mutually-perpendicular planes of symmetry permits a solution to be obtained in only one quadrant.

A series of calculations is carried out achieving uniform accuracy at various stages and in various regions of the solution of the problem. In particular, we investigate the number of approximation planes, strips of the integral relations method, and the number of points required for the construction of the tangential discontinuity while attaining the necessary calculation accuracy which is required throughout the problem. As a result, it is found that for space objects whose shape only slightly deviates from axial symmetry, five approximation planes with respect to φ over a quadrant yield a calculation accuracy better than 1.5 percent. For achieving the same accuracy, it is sufficient to use the approximation of the integral relations method N = 2 (with $\theta < 60°$) and N = 3 (with $\theta < 120°$) and 10 X 5 points with respect to θ and φ for minimizing I.

Much attention is also attached to a technique of crossing singular points situated in the trans-sonic region of an external flow. This enables the calculations to be extended into a domain of hyperbolic equations (up to $\theta = 120°$). Calculations aimed at establishing the closure conditions of a boundary value problem in the magnetosphere tail are made as well. It appears that the closure of an internal elliptical problem takes place (within the accuracy of 1.5 percent in the forward part of the magnetosphere) when substituting the real tail part of the magnetopause by any ellipsoid whose semi-axes ratio is greater than 3.

Having made an analysis of the approximation errors and determined the dependence of the solution accuracy obtained upon the number of points in the numerical mesh chosen for the more complicated problem A (from the viewpoint of flow geometry), one need not be concerned for the accuracy of the results obtained in the solution of the simpler problems B and C.

The numerical solution is constructed in a dimensionless way depending upon the criteria of problem similarity: Mach number M, Alfvén Mach number M_A, adiabatic index γ, and the quantity

$$F = \frac{a^2}{R_E^6 P_\infty}$$

(Here, $a = 8.1 \times 10^{25}$ G cm^3, and R_E is the Earth's radius.) F characterizes the relationship of dipole strength to pressure in the undisturbed flow.

The calculations are made with the following parameters: M, M_A = 6, 8, 10, 12; γ = 1.2, 1.4, 1.67, 2.

Figure 2 shows the relation of ϵ_0 at the stagnation point to the values of M_∞ and γ. The quantity γ is seen to strongly influence ϵ_0 and, therefore, the whole flow pattern. This suggests the significance of determining some effective adiabatic index, γ_{eff}, for the solar plasma and, conversely, the probability of deriving γ_{eff} from the results of satellite experiments. A comparison of the solution obtained with satellite experimental data gives $\gamma_{eff} \cong 2$.

Figure 3 gives the dependence of the tangential discontinuity distance at the stagnation point, relative to the Earth's radius, r_0/R_E, upon certain parameters of the similarity problem. The relation r_0/R_E to Mach number, M_∞, in the upstream flow is presented for various values of γ and K = log (0.358 F). These curves indicate how the magnetopause location is connected to quasistationary changes in solar-wind conditions.

In figure 4, the families of shock waves and tangential discontinuities are given for various parameter values; figure 5 shows a typical calculation result with flow streamlines, constant density lines and sonic lines. (Figures 4 and 5 present in the upper half-plane, the flow in the plane $\varphi = \pi/2$; in the lower half-plane, the flow in the plane $\varphi = 0$.) The disagreement between the solution obtained and similar calculations by Spreiter et al. [1, 2] is less than 10 to 15 percent.

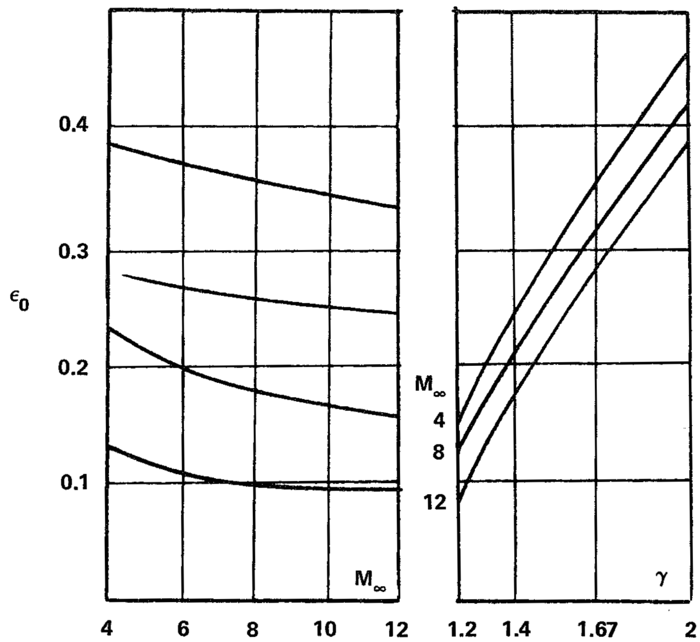

Figure 2. The relation of ϵ_0 at the stagnation point to the values of M_∞ and γ.

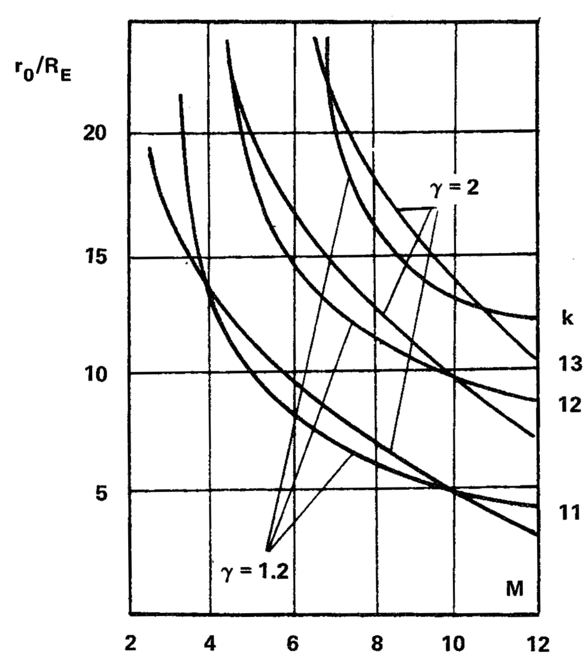

Figure 3. The dependence of the tangential discontinuity distance at the stagnation point.

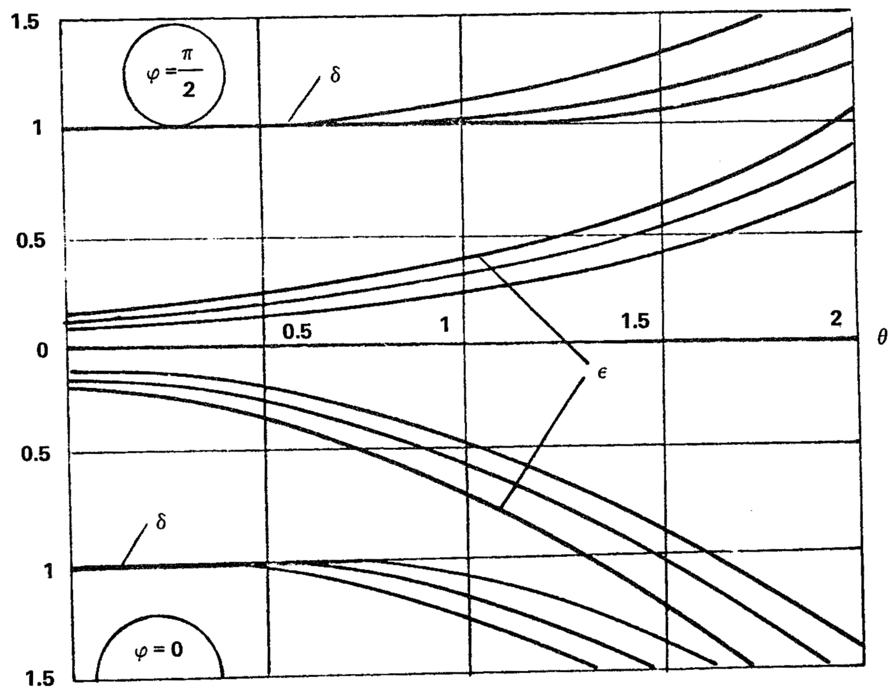

Figure 4. Families of shock waves and tangential discontinuities for various parameter values.

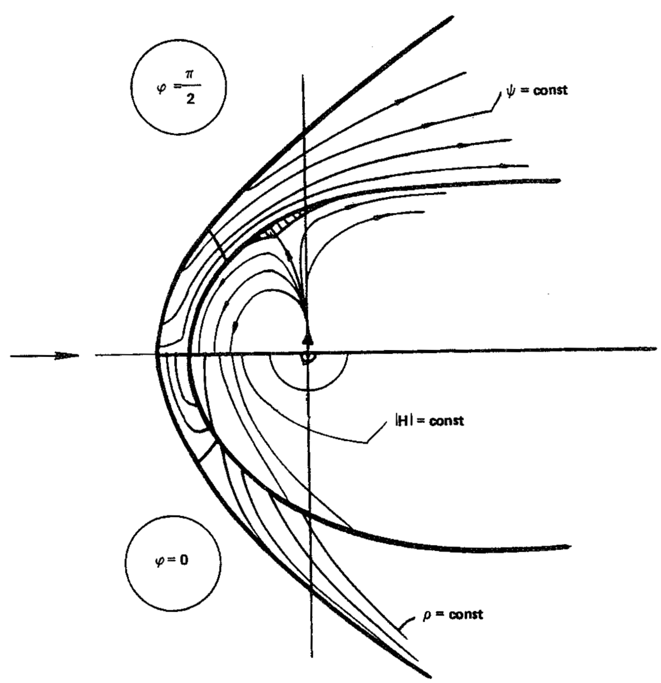

Figure 5. A typical calculation result.

In addition to the numerical investigations, an exact solution of a two-dimensional problem concerned with a captured plasma region in the vicinity of the magnetopause neutral point (a so-called cusp) was obtained. The problem is reduced to finding a conformal mapping $z(\xi)$ between the upper half-plane of the complex plane $\xi = \Delta + i\eta$ with a magnetic dipole at the point $i\sqrt{3}$ and an infinitely conducting boundary on the axis and the upper part of a plane $z = x + iy$ contiguous to an infinitely conducting fluid and to a dipole on the imaginary axis.

The conformal mapping obtained is

$$z = \int_0^{\xi} e^{W(\omega)}\, d\omega + iY_0$$

where

$$W(\xi) = \frac{\sqrt{\xi^2 - 1}}{i\pi} \int_{-1}^{+1} \frac{u(t)dt}{\sqrt{t^2 - 1}\,(t - \xi)}$$

$$u(t) = \ln \frac{\beta\,|t|}{(t^2 + 3)^2} \qquad \beta = \frac{4\sqrt{3}\,M_d}{\sqrt{8\pi}\,P_0}$$

M_d is dipole intensity, P_0 is pressure in a decelerated plasma.

The magnetosphere boundary line in the neighborhood of the neutral point is

$$X_b = \beta \int_0^{\Delta} \frac{|t|}{(t^2 + 3)^2} \cos(\text{Im } W)\, dt$$

$$Y_b = Y_0 + \int_0^{\Delta} \frac{|t|}{(t^2 + 3)^2} \sin(\text{Im } W)\, dt$$

Figures 6 and 7 represent the function Im $W(\xi)$ = Arg $(dz/d\xi)$ and boundary lines obtained with various β.

An investigation has also been carried out in the vicinity of the Sun-Earth line with arbitrary orientation of velocity and field vectors in the undisturbed flow. For this purpose, the functions were derived in the form of their expansions in a series with respect to a small departure from the axis. The results indicate that the presence of a field component perpendicular to the velocity in the undisturbed flow can give rise to a sharp drop in density in the vicinity of the stagnation point of the magnetosphere.

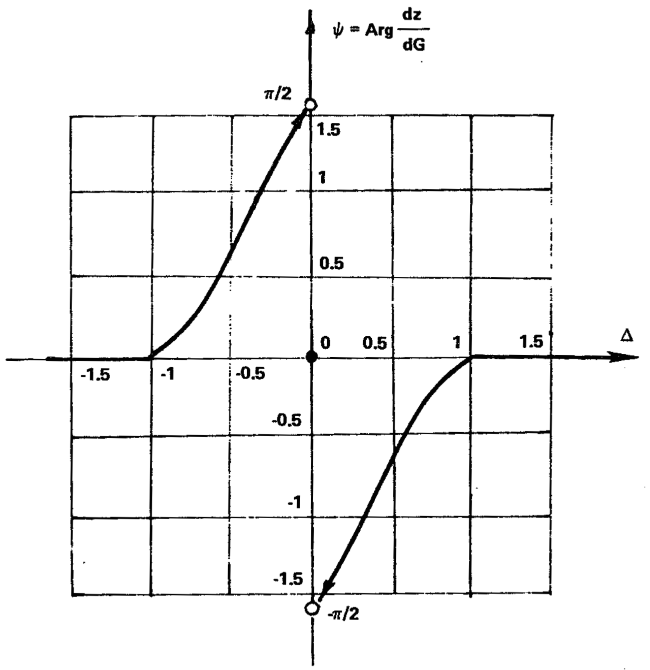

Figure 6. Representation of the function Im W(ξ) = Arg (dz/dξ).

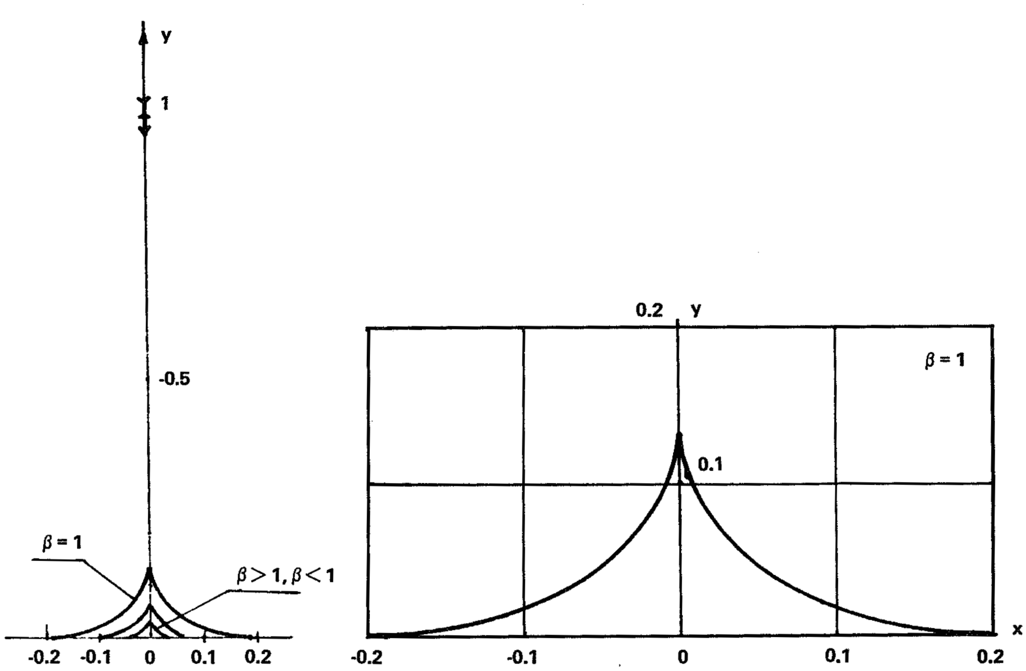

Figure 7. Boundary lines obtained with various β.

PROBLEM B: SOLAR-WIND INTERACTION WITH THE IONIZED COMPONENT OF THE ATMOSPHERE OF A COMET

Biermann et al. [3] suggested that the solar-wind interaction with the ionized component of a comet forms a shock wave.

The simplest model of the solar-wind interaction with a comet is a subsonic source in a supersonic gas flow. The source gas and the solar plasma are separated by a tangential discontinuity where the pressure balance is maintained. The discontinuity surface at infinity is shown to asymptotically tend to become cylindrical. As compared to the previous problem, there is a substantial simplification, that is, the possibility of treating an axisymmetrical problem. Except for slight variations, the solution of the problem is similar to the above case. However, there are two modifications which arise in defining the tangential discontinuity line. The first modification (the same as in problem A) is the fitting of the external and the internal solutions in accordance with a minimum residual balance of total pressures. The second one consists in a simultaneous integration of the external and internal problems and in the construction of a shock wave and a tangential discontinuity during crossing from the stagnation point to the periphery. The accuracy of the results, in both modifications, is practically identical but with much less use of computer time in the second case.

Figure 8 shows the distances of the shock wave and the tangential discontinuity at the stagnation point as well as the radius of the cylindrical part of the tangential discontinuity R_∞ depending upon oncoming flow parameters. Figure 9 represents a characteristic source flow pattern.

PROBLEM C: SOLAR-WIND INTERACTION WITH THE IONOSPHERE OF A PLANET WHICH DOES NOT HAVE ITS OWN MAGNETIC FIELD

At present, there is no universally-accepted point of view concerning the mechanism of solar-wind interaction with such planets although the existence of a bow shock is recognized by almost all investigators. The simplest model (due to Spreiter et al., 1967) is the direct contact of the solar plasma with the ionosphere of a planet and the maintenance of a pressure balance at a tangential discontinuity. On the other hand, there can occur a magnetic barrier due to currents produced either by solar-wind motions or by some dynamo in the planet's ionosphere. Finally, a barrier of this kind can have a substantially nonstationary character.

The supersonic stationary flow has been calculated in an atmosphere bound by the gravitational force around a spherical object [4]. The technique for solving the problem is like those described above except for the fact that terms responsible for the gravity are added to an equation of motion. In this case the equation of motion, as compared to an ordinary magnetohydrodynamic equation, will have the form:

$$(\vec{V},\nabla)\,\vec{V} - \frac{1}{4\pi\rho}(\vec{H},\nabla)\,\vec{H} = -\frac{1}{\rho}\,\mathrm{grad}\left(P + \frac{H^2}{8\pi}\right) + \frac{GM_p\vec{r}}{r^3}$$

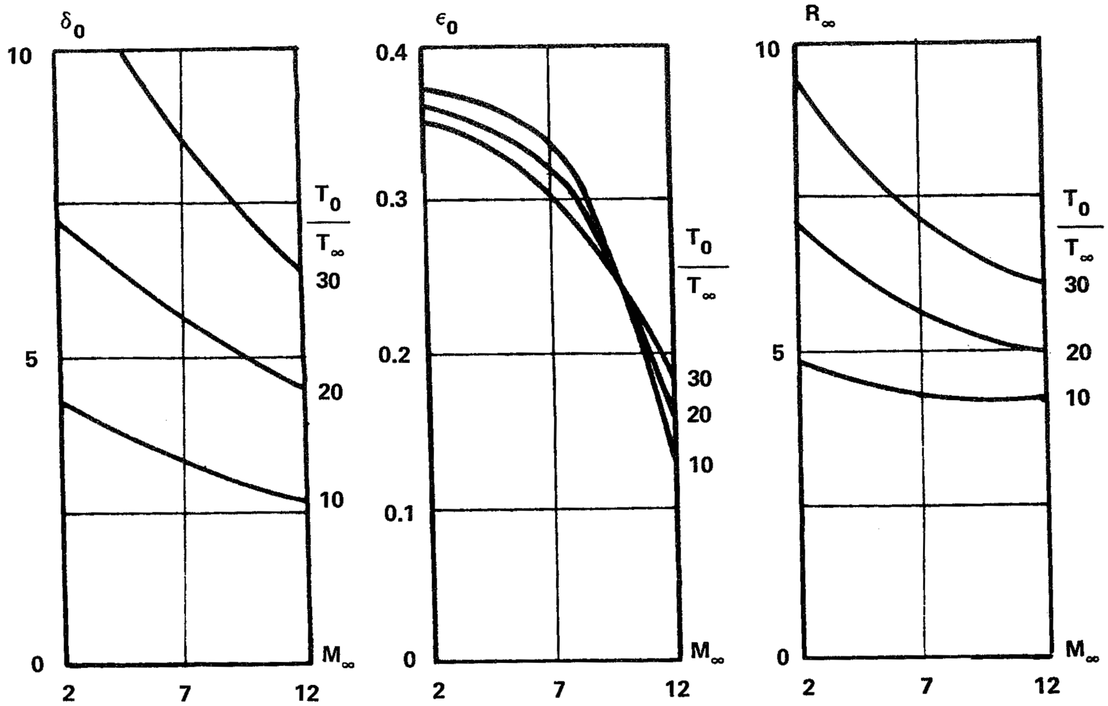

Figure 8. The distances of the shock wave and the tangential discontinuity at the stagnation point.

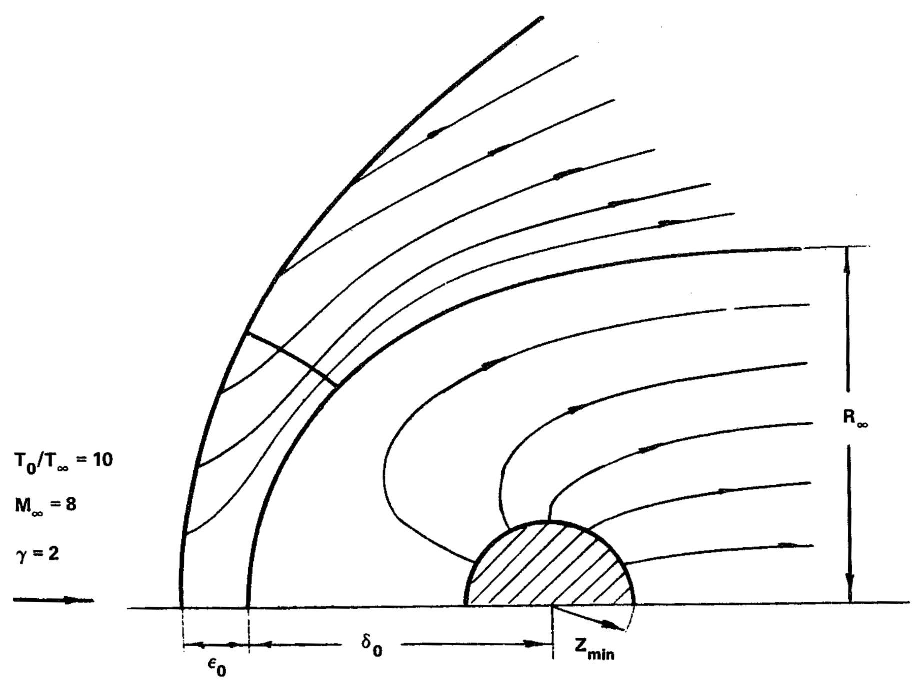

Figure 9. A characteristic source flow pattern.

Here G is the universal gravitational constant, M_p is the planet mass. Neglecting electromagnetic phenomena, the internal problem solution is given by a barometric height formula:

$$P = P_* \exp\left[-\int_{r*}^{r} K(r) \frac{dr}{r^2} \right], \qquad K = \frac{GM_p \mu}{RT}$$

where

P_* = the pressure at some altitude r_*,

μ = the molecular weight of atmospheric gas,

T = its temperature.

In figure 10 is given the dependence of the dimensionless distance from the coordinate origin to the stagnation point of the tangential discontinuity, r_0/r_*, upon M_∞ in the oncoming flow for three values of K/r_* and two values of $\ln(P_*/P_1)$. P_1 is the pressure immediately after the gas passes through the shock wave. The families of shock waves and tangential discontinuities in figure 11 are presented as dependent upon the parameter K/r_0, whose change has a more pronounced effect on the discontinuity line.

An attempt is also made to estimate the influence on the interaction of magnetic fields generated by the solar wind itself (for a stationary case). For this purpose the following model has been considered. A spherical layer of finite conductivity, whose external radius is approximately equal to the distance from the planet center to the stagnation point of the tangential discontinuity, obtained from the previous solution, is placed in the flow of an infinitely-conducting plasma whose frozen-in magnetic field in the undisturbed flow is perpendicular to the velocity at infinity. As the solution to the external problem, an electric field distribution is obtained

$$\vec{E} = -\frac{1}{c}[\vec{V} \times \vec{H}]$$

along the spherical layer surface. Then the internal problem is solved, that is, currents are found flowing in the spherical layer driven by this electric field and the magnetic field that results from them. The solution results in the analysis of the induced magnetic field contribution $\Delta\vec{H}$, at the external boundary of the spherical layer, to the total pressure balance at this boundary. The calculations are done under the assumption that the spherical layer has a scalar conductivity.

As a result, it is found that in a plane intersecting the planetary center and perpendicular to the magnetic field in the undisturbed flow, the induced magnetic field contribution is insignificant, that is, the ionopause shape in this plane does not vary in taking account of an electromagnetic interaction.

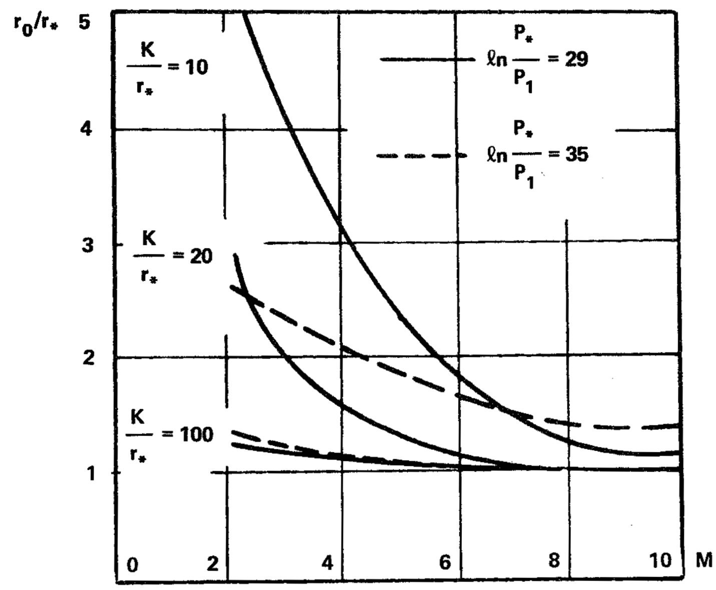

Figure 10. The dependence of the dimensionless distance from the coordinate origin to the stagnation point of the tangential discontinuity.

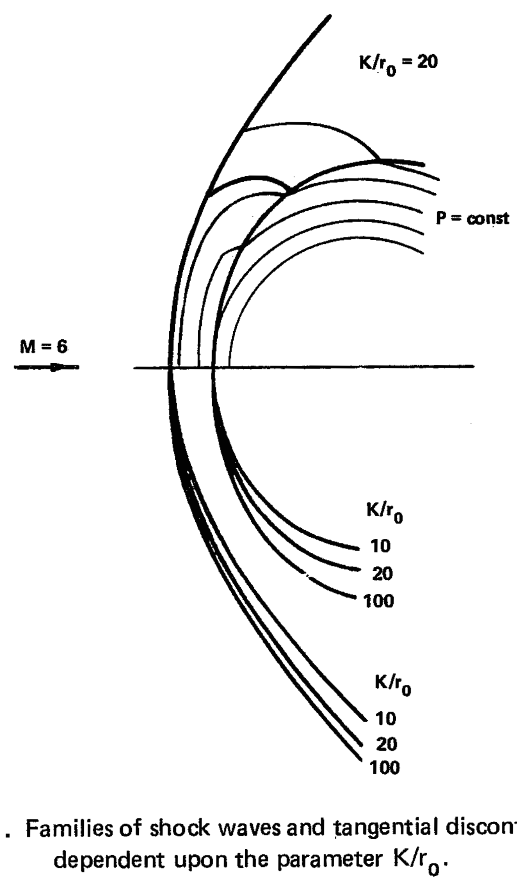

Figure 11. Families of shock waves and tangential discontinuities dependent upon the parameter K/r_0.

In the plane containing the center of the planet and parallel to the magnetic field vector in the undisturbed flow, there is obtained a distribution $\Delta H^2/8\pi$ depending upon the parameter $H_\infty V_\infty \sigma/c^2 r_0$, where σ is the spherical layer conductivity and r_0 is its thickness. The maximum addition to the pressure balance at the boundary due to induction is at the stagnation point. The estimates show that for a substantial change in the ionopause shape, by means of the mechanism involved, rather large intensities of the magnetic field frozen-in to the solar plasma ($H_\infty \sim 100\ \gamma$) and large values of ionospheric conductivities (10^{-2} - 10^{-1} mhos/m) are required.

BRIEF CONCLUSIONS

1. The complete three-dimensional solution of a stationary, self-consistent problem is obtained in the forward part of the interaction region for an infinitely-conducting plasma flow around a magnetic dipole.

2. It is shown that with a gasdynamic approach to an interaction of this kind, the choice of an effective index for the plasma adiabatic index is of crucial importance for the flow pattern.

3. The closure condition is obtained for an internal elliptic problem (in the search for a solution concerned with the forward region).

4. An accurate solution is obtained for a two-dimensional problem at a neutral point.

5. The flow in the vicinity of the Sun-Earth line is examined for the case of the field and the velocity being nonaligned in the undisturbed flow; it is found that the presence of a perpendicular field component causes a drop in the density at the magnetopause stagnation point.

6. The solution of a self-consistent problem is obtained for supersonic source flow, the simplest model of flow around a comet.

7. The solution to the problem of a gasdynamic flow around a nonmagnetic planet, but possessing a gravitationally-bound atmosphere, is obtained; the induction effect of the secondary field, produced by the solar wind for a stationary case, is evaluated and turns out to be insignificant with characteristic values of space plasma parameters.

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QUESTIONS

Belotserkovskii/Spreiter: I want to make a comment and not a question. First, I congratulate you and your young colleagues on the progress in these difficult problems. I believe we are now in a new era in which the magnetohydrodynamic model has a firmly established position in solar-wind planetary interactions and that further consequences of the theory should be explored more completely and in greater detail as part of the continuing investigations. Secondly, I wish to inject a note of caution regarding your statements on the extensions to include viscosity effects in the tail. Viscosity cannot be considered as a scalar quantity; it is a tensor quantity and extremely anisotropic as shown, for example, by the work of Braginskii. Proper attention must be given to this property if a realistic representation is to be obtained.