ON THE QUESTION OF THE ENERGY OF THE PRECESSIONAL DYNAMO

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The problem of the effectiveness of precession in the generation of the geomagnetic field has been revived and discussed in the recent literature [1 through 6]. Estimates of the capability of the Poincare force to produce motions in the core which satisfy the requirements for generation of a magnetic field [1 through 4, 7], as well as estimates of the energy which precession is able to transfer to the dynamo process, serve as the criteria on which the conclusions are based.

In connection with the first criterion, very strong doubts were expressed because of the diurnal change of the sign of the Poincare force [4, 8]. A possibility of surmounting that doubt was suggested [8] by the necessity to consider nonlinear effects, plausible in the case of core-mantle coupling. The mechanism by which precession transfers the rotational, kinetic energy of Earth into the energy of its magnetic field is far from being clear. But it is generally accepted that the mechanism is associated with the hydromagnetic stresses which originate between the mantle and the core. These precess at somewhat different rates and angles due to differences in the dynamic compression of the mantle and the core. Nevertheless, different processes of core-mantle coupling, proposed by various authors, have led to significantly different estimates of the energetics of a precession dynamo.

According to the estimates of Malkus [1] and Stacey [2], the coupling mechanism is able to transfer into the core an energy of approximately 2-3 $\times$ 10^{17} ergs/s. According to estimates of Rochester et al. [4], the amount of energy that can be transferred does not exceed 10^{15} ergs/s, if the flow in the core remains stable. According to the estimates of Loper [7], an even lower value of energy transfer is realized, 3.5 $\times$ 10^{14} ergs/s, and this is dissipated in the boundary layer between the mantle and the core and cannot provide any energy for the dynamo process. (Interested readers should refer to the original articles [4, 7] which discuss in sufficient detail the bases for these appreciable differences and estimates.)

In the publications of Dolginov [5, 6], it was shown that the present magnetic states of the Earth, Jupiter, Mars, Moon, Venus, and Mercury can be described by a formula developed under the assumption that the magnetic states of these planets can be compared in terms of a similarity model. The bases for the scaling, which occurs due to the effectiveness of the dynamo mechanism and maintained by precession, is that the numerical value of the dipole field of a planet can be calculated by comparison with the field of Earth by the formula

$$H_{oi} = K\,H_{OE}\frac{R_i^3\,\Omega_i\,\omega_i \sin\alpha_i\,\zeta_i\,\eta_E}{R_{CE}^3\,\epsilon^3\,\Omega_E\,\omega_E \sin\alpha_E\,\zeta_E\,\eta_i} \tag{1}$$

where subscript i means the planet and E, the Earth while

$\vec{\Omega}$ = the rate of precession,

$\vec{\omega}$ = the angular velocity of rotation,

α = the angle between $\vec{\Omega}$ and $\vec{\omega}$,

R_{CE} = the radius of the liquid core of Earth,

ζ = density,

η = magnetic viscosity,

H_o = strength of the magnetic field at the equator,

ϵ = the ratio between the radius of the planet and the radius of the liquid core.

The small scatter of values of the coefficient K [5, 6] indicates how remarkably well the magnetic fields of the planets obey such a relationship.

It is natural to discuss the physical significance of the proportionality coefficient K. Assume that the equation for field intensity is given by

$$H = K\frac{R^3\,\Omega\,\omega \sin\alpha\,\zeta}{\eta} \tag{2}$$

where H is the strength of the planetary field. Let us consider the dimensions of the coefficient K using CGS units. This gives

$$K = \frac{cm^{-1/2}\,gm^{1/2}\,s^{-1}\,cm^2\,s^{-1}}{cm^3\,s^{-2}\,gm\,cm^{-3}} = cm^{3/2}\,gm^{-1/2} = \frac{1 = 1}{|D|\ \ [\epsilon H]} \tag{3}$$

where we see that K has the dimensions inverse to electric induction. In that case, the following equation can be written

$$HE = \frac{R_c^3\,\omega\,\Omega \sin\alpha\,\zeta}{\epsilon\eta} \tag{4}$$

where ϵ is the dielectric permeability. The left-hand side is the Poynting vector while the right-hand side, where all the quantities are known, has the dimensions of ergs/cm^2/s. Thus, equation 4 represents the density of the flux of electromagnetic energy from the core to the mantle per unit time.

Let us estimate the energy flux for the Earth, which is associated with precession, assuming typical values for the parameters as follows:

$$R_C^3 = 3.8 \times 10^{25}\ \text{cm}^3 \qquad \omega = 7.29 \times 10^{-5}\ \text{rad/s}$$

$$\Omega = 8 \times 10^{-12}\ \text{rad/s} \qquad \epsilon = 1\ \text{s}^2/\text{cm}^2$$

$$\eta = 1.2 \times 10^{4}\ \text{cm}^2/\text{s} \qquad \zeta = 11.5\ \text{gm/cm}^3$$

$$4\pi R_C^2 = 1.5 \times 10^{18}\ \text{cm}^2$$

This yields

$$HE = 9.5 \times 10^{24}\ \text{ergs/s}.$$

In fact, as MacDonald has shown, the dynamic compression of the core is approximately 1/400 as compared to 1/300 for the mantle. Therefore, the core should precess with a rate which is 3/4 of that for the mantle and only 1/4 of the Poincare force takes part in the generation of the field. Therefore, the energy associated with precession cannot be larger than 2.5×10^{24} ergs/s.

Let us compare this value with estimates of the energy of the geomagnetic field in different models and with estimates of the energy of the field calculated from the data of spherical harmonic analysis. According to Verosub and Cox [9], the energy of the dipole magnetic field, calculated from the international reference model of 1965, at the boundary of the core is 5.4×10^{25} ergs. The non-dipole portion is 1.9×10^{25} ergs. The rate of ohmic dissipation of energy in the core Q, in the model of Bullard and Gellman [10] is 9×10^{17} ergs/s, while in the model of Braginskii [8] it is 10^{19} ergs/s. Pekeris et al. [11] give a value 10^{16} - 10^{17} ergs/s.

Thus, the derived value of energy, which is associated with precession, is in reasonable agreement with the observed external energy of the geomagnetic field and with the rate of dissipation of energy in the core according to various estimates. Having in mind the largest value of Q (10^{19} ergs/s), it can be noted that the efficiency factor for the precession mechanism is extremely low. Perhaps a somewhat lower value of electrical conductivity in the core can be assumed on the basis of this work and the unknown details of the mechanism.

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