COLLEGE OF ENGINEERING

NSG-1150

TRE 76-1

UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE

DIFFUSIVITY IN TURBULENT FLUID
CONTAINING TWO DOMINANT SCALES,
AND COMPRESSIBLE SHEAR LAYER
ACCORDING TO A KINETIC THEORY.

by

Paul M. Chung

June, 1976

(NASA-CR-148146) DIFFUSIVITY IN TURBULENT FLUID CONTAINING TWO DOMINANT SCALES, AND COMPRESSIBLE SHEAR LAYER ACCORDING TO A KINETIC THEORY (Illinois Univ.) 36 p HC $4.00 CSCL 20D G3/34

N76-26420

Unclas 42159

DEPARTMENT OF ENERGY ENGINEERING

Diffusivity in Turbulent Fluid Containing Two Dominant Scales, and Compressible Shear Layer According to a Kinetic Theory

by

Paul M. Chung

Department of Energy Engineering

University of Illinois at Chicago Circle Campus

June, 1976

ACKNOWLEDGEMENT

This work is conducted under the NASA Grant NSG 1150. The encouragement and guidance of Dr. John Evans of the Langley Research Center are gratefully acknowledged.

I. INTRODUCTION

The physical and mathematical bases of the kinetic theory approach to the chemically reacting, incompressible, turbulent flows have been described in several of the previous papers.[1-5] Recently, it has been shown[6] that certain exact solutions of the governing kinetic equations can be obtained through construction of the appropriate Green's functions. However, the complexity of the kinetic equations is such that it is generally impractical to attempt to obtain the exact solutions except for the rather simple flow configurations such as the plane, shear mixing layer analyzed in Reference 6.

A study is being conducted to develop an approximate formalism which embodies the salient contributions of the kinetic theory but which employs a much simplified set of equations such that the chemical reactions taking place in the realistic flow fields can be analyzed.

This need to generate an approximate method of application of the kinetic theory to engineering problems becomes accentuated when one considers the compressible kinetic theory which is being constructed. Obviously, the compressibility increases the complexity of the governing equations, and, exact solution of the equations for the realistic flow configurations becomes extremely difficult.

The essence of development of the approximate method of solution is as follows.

We first develop a simplified moment relationship, from the two non-equilibrium-degree kinetic equation,[5] which will relate the effective length scale, Λ_e, to the mean turbulence energy, and perhaps a few other mean flow properties, and to the two given length scales, Λ_a and Λ_b. This length-scale equation is not needed for the simple flow fields such as the plane shear layer.

Next, we make use of the existing solution of the momentum equation for the particular flow problem. This is because the greatest difficulty in the application of the kinetic theory arises from the solution of the governing equation for the turbulent momentum field when the flow field is complicated. At the same time, it is known that the mean velocity, mean shear stress, etc., are usually quite well described by the existing theories such as the Prandtl's mixing-length theory. We assume an appropriate form of the distribution function, $f(t,\vec{x},\vec{u})d\vec{u}$, and from the existing solutions of the mean values, we determine the distribution function of the fluid elements.

Finally, the kinetic equations governing the chemical species and energy are solved for their distribution functions by the use of the $f(t,\vec{x},\vec{u})d\vec{u}$, since it is the mixing and chemical reactions which the existing theories are unable to describe correctly.[1-3] The effective length-scale equation is also employed in the solution of the kinetic equations when it is warranted.

The above process is simpler when the incompressible assumption is made, for then the distribution function of the fluid elements, $f(t,\vec{x},\vec{u})d\vec{u}$, can be determined independently of the solution of the kinetic equations for the energy and chemical species. In a compressible flow, determination of $fd\vec{u}$ from the existing solution of the momentum equations and solution of the kinetic equations become coupled. Since the existing solutions depend on the mean density variations only, an iterative scheme is expected to be satisfactory.

The present report describes the initial phase of development of the approximate formalism. It consists of two portions. The first contains the solution of the two nonequilibrium-degree kinetic equation[5] for the effective length scale and turbulence energy for a spatially homogeneous turbulence field with two characteristic length scales, wherein the source for one family of eddies exists. This solution is applied to the evaluation of the eddy diffusivity in the combustion chamber of an internal combustion engine. The result is compared with the other existing solution.[7] This is carried out to demonstrate the feasibility of obtaining an effective length-scale equation within the context of the kinetic theory.

The second portion of this report contains the formulation and partial solution of the compressible plane shear layer according to the approximate formalism discussed earlier.

II. HOMOGENEOUS FIELD WITH TURBULENCE SOURCE

Governing Equations

We consider an incompressible, homogeneous turbulence field consisting of two characteristic length scales. At time zero, the turbulence energies associated with both length scales are given. For time greater than zero, we consider that a source for the eddies associated with one of the two scales exists in the field. The two coupled Langevin equations for the two degrees of freedom, $\vec{U}$ and $\vec{V}$, given in References 5 and 6 are modified as,

$$\frac{dU_i'}{dt} = -\beta_a U_i' + 2SU_i' - \beta_c(U_i' - V_i') + A_i(t) + K_{a,i} \tag{1}$$

$$\frac{dV_i'}{dt} = -\beta_b V_i' - \beta_c(V_i' - U_i') + A_i(t) + K_{b,i} \tag{2}$$

where the $\vec{U}'$-degree of freedom is considered to have the source for $t > 0$. Also, as it is evident in Eq. (1), the source strength, $2SU_i'$, is considered to be proportional to U_i'. As we shall see subsequently, this particular form can imply, within the context of the present simplified analysis, the turbulence energy generation in proportion to the mean shear.

All symbols are defined in the Nomenclature. The physical meaning of the Langevin equations has been discussed in the previous papers,(5,6) and, therefore, it is not discussed here.

The Fokker-Planck equation governing the joint distribution function, $f_2(t,\vec{x},\vec{U}',\vec{V}')d\vec{U}'d\vec{V}'$, can be constructed from Eqs. (1) and (2) in the same manner as that shown in Reference 5. The moment equations governing the three components of the turbulence energy then can be derived from the Fokker-Planck equation in the same manner as that shown in Reference 6 as,

$$\frac{d\langle U_k U_k\rangle}{d\tau} = \langle W_k W_k\rangle^{1/2} \left\{ \left[\frac{8}{\langle W_k W_k\rangle^{1/2}} S(\tau) - 11 \right] \langle U_k U_k\rangle + (5 + M)\langle U_k V_k\rangle + M\langle V_k V_k\rangle \right\} \tag{3}$$

$$\frac{d\langle V_k V_k\rangle}{d\tau} = \langle W_k W_k\rangle^{1/2} \left[\langle U_k U_k\rangle + (5 + M)\langle U_k V_k\rangle - (4 + 7M)\langle U_k V_k\rangle \right] \tag{4}$$

$$\frac{d\langle U_k V_k\rangle}{d\tau} = \langle W_k W_k\rangle^{1/2} \left\{ 3\langle U_k U_k\rangle + (2 + M)\langle V_k V_k\rangle - \left[7 + 3M - \frac{4}{\langle W_k W_k\rangle^{1/2}} \hat{S}(\tau)\right] \langle U_k V_k\rangle \right\} \tag{5}$$

where

$$W_k W_k\rangle = \langle U_k U_k\rangle + 2\langle U_k V_k\rangle + \langle V_k V_k\rangle \tag{6}$$

In the above equations, all energy quantities are in dimensionless form as,

$$\begin{aligned} \langle U_k U_k\rangle &= \langle U_k' U_k'\rangle / \langle W_k' W_k'\rangle_o \\ \langle U_k V_k\rangle &= \langle U_k' U_k'\rangle / \langle W_k' W_k'\rangle_o \\ \langle V_k V_k\rangle &= \langle V_k' V_k'\rangle / \langle W_k' W_k'\rangle_o \end{aligned} \tag{7}$$

Also,

$$\begin{aligned} M &= \Lambda_a/\Lambda_b \\ \hat{S} &= S/\beta_{ao} \\ \beta_{ao} &= \langle W_k' W_k'\rangle_o^{1/2}/(2\Lambda_a) \end{aligned} \tag{8}$$

and

$$\tau = \frac{3\beta_{ao}}{2} t$$

The three coupled equations, Eqs. (3), (4) and (5), can be readily integrated as soon as a set of initial conditions and the source function, $\hat{S}(\tau)$, are specified. These values will be specified to simulate the physical problem found in the piston engines to be discussed in the subsequent subsection.

From the solution of Eqs. (3), (4) and (5), the effective length scale, $\bar{\Lambda}_e(\tau)$, can be constructed as (see References 5 and 6):

$$\bar{\Lambda}_e = \frac{\Lambda_e}{\Lambda_a} = \frac{\langle W_k W_k \rangle}{[\langle U_k U_k \rangle + \langle U_k V_k \rangle] + M[\langle U_k V_k \rangle + \langle V_k V_k \rangle]} \qquad (9)$$

In the present problem, Λ_e represents the characteristic scale of the hypothetical family of the energy-containing eddies which would give the same time variation of the turbulence energy, $\langle W_k W_k \rangle$, of the field as that caused by interaction of the two families of the eddies, $\vec{U}$ and $\vec{V}$ modes. (See Reference 5.)

Eddy Diffusivity in Piston Engines

As it was stated in the Introduction, one of the objectives of the present report is to demonstrate a feasible means of deducing a length-scale equation usable within the context of the kinetic theory approach. We shall employ the present two-nonequilibrium-degree equations, Eqs. (3) - (5), to analyze a portion of the problem found in a piston engine first studied by Sirignano.[7]

Consider the sketch of a piston engine cylinder shown in Figure 1. In order to analyze the combustion taking place in the cylinder, the turbulent mixing of the fuel and air must be described. This mixing in the cylinder is controlled by at least two major families of the characteristic eddies with considerably different characteristic scales.

The fuel and air enter the cylinder turbulently through the valve. Therefore, their turbulence characteristics are of those dictated by the valve. Subsequently, however, larger scale eddies are generated by movement of the piston. In the analysis of Reference 7, which is based on the mixing-length theory, the eddy diffusivity, $D'(t)$, was approximated by the following equation.

$$D'(t) = \Lambda_a |u_p'| + \Lambda_b u_I' \exp[-u_I'(t - t_i)/\Lambda_b] \tag{10}$$

where u_p' and u_I' denote the instantaneous piston velocity and the initial intake velocity of the gases through the valve, respectively. Λ_a and Λ_b denote the scales of the eddies generated by the piston movement and the valve, respectively.

The rationale for Eq. (10) is that the characteristics of the smaller eddies initially generated through the valve decay exponentially whereas the piston velocity itself represents the mean turbulence velocity of the larger eddies generated by the piston at any given time.

In order to simulate this problem by the homogeneous solution of Eqs. (3), (4) and (5), we first consider that the $\vec{V}$-mode of the turbulence represents the smaller scale eddies generated by the valve, and that the $\vec{U}$-mode that of the larger scale eddies generated by the piston movement. Thus,

$$M = \Lambda_a/\Lambda_b > 1 \tag{11}$$

Then, we assume that the piston movement engenders the eddies by creating a shear layer along the wall. (See Figure 1.) If we let Λ_a denote the scale which is of the order of the shear layer, the mean velocity gradient will be of the order of $|u'_p|/\Lambda_a$. It is well known[8] that the generation rate of turbulence by shear is proportional to the shear stress times the mean velocity gradient. Within the context of the present simulation this rate would be of the order of $\langle U'_k U'_k \rangle \dfrac{|u'_p|}{\Lambda_a}$. For the present solution, we let

$$S = \frac{1}{2} \frac{|u'_p|}{\Lambda_a} \tag{12}$$

Then,

$$\hat{S} = S/\beta_{ao} = \frac{|u'_p|}{\langle W'_k W'_k \rangle_o^{1/2}} = |u_p| \tag{13}$$

and it can be readily shown from Eq. (3) that the rate of production of $\langle U'_k U'_k \rangle |u'_p|/\Lambda_a$.

Note that there exists a rather basic difference between the assumptions of Eq. (10) made in Reference 7 and those of the present equation. In Eq. (10), it is assumed that the piston velocity $|u'_p|$ itself represents the instantaneous mean turbulence velocity generated by the piston movement. In the present assumption of Eqs. (3) - (5), and Eq. (13), the piston velocity, $|u'_p|$, is considered to be the velocity which is responsible for the mean shear which gives a rise to the turbulence source. The instantaneous mean turbulence velocity generated by the piston movement is $<U_k'U_k'>^{1/2}$ which will be found from the solution of the governing equations, Eqs. (3) - (6).

From the solution of the present equations, variation of the effective length scale, Λ_e, is obtained from Eq. (9). Then, again within the context of the present simulation, we write,

$$D'(t) = \Lambda_e <W_k'W_k'>^{1/2} \tag{14}$$

The eddy diffusivities given by Eqs. (10) and (14) are nondimensionalized as,

<u>Reference 7</u>

$$D(\tau) = |u_p| + \frac{1}{M} <V_k V_k>_o^{1/2} \exp\left[- \frac{4}{3} M <V_k V_k>_o^{1/2} \tau \right] \tag{15}$$

<u>Present</u>

$$D(\tau) = \bar{\Lambda}_e <W_k W_k>^{1/2} \tag{16}$$

where

$$|u_p| = |u_p'| / <W_k'W_k'>_o^{1/2}, \quad D(\tau) = D'(t)/\Lambda_a <W_k'W_k'>_o^{1/2} \tag{17}$$

In writing Eq. (15), we took cognizance of the fact that the initial mean turbulence velocity of the intake gases, $<V_k'V_k'>_o^{1/2}$, is equal to the intake velocity u_I'. (See Equation 10.)

Solution of Eqs. (3), (4) and (5)

The initial conditions of the present equations for $<U_kU_k>$ and $<V_kV_k>$ are chosen such that some meaningful comparison can be made between the present solutions and Eq. (15) employed in Reference 7. It is pointed out at the outset, however, that a deatiled quantitative comparison between the two results is not possible. This is because there is no quantity in the expression for D'(t) of Reference 7 which corresponds to the present turbulence source strength, S. In Reference 7, as it has been already explained, a direct assumption on the $<U_k'U_k'>^{1/2}$ itself has been made as being equal to $|u_p'|$. In the present simulation, S was formulated by postulating a turbulence generation mechanism (Figure 1), and specification of its value beyond that of the order of magnitude is meaningless. Nevertheless, as we shall see, the comparison will elucidate certain basic qualitative aspects of the multi-scale mixing phenomenon.

For $\tau = 0$, we let,

$$<V_k'V_k'>_o^{1/2} = u_I'$$

$$<U_k'U_k'>_o^{1/2} = |u_p'| \tag{18}$$

and

$$<U_k'V_k'>_o^{1/2} = 0$$

in Eqs. (3) - (5).

Figures 2 through 5 show the typical results.

Discussion

Figure 2 shows the contribution of the smaller eddies generated by the intake valve, D_v, to the diffusivity for the constant piston velocity, $|u_p|$.

S-B denotes the results of Reference 7 given by Eq. (15). Since it has been assumed in Reference 7 that the contribution of the smaller eddies decays exponentially, the broken lines merely show these exponential decays for the two given scale ratios, Λ_b/Λ_a, of 1/3 and 1/10, respectively. The present solutions, on the other hand, show that the contribution of the smaller eddies does not completely decay but it persists. This is due to the fact that the smaller eddies continuously receive energy from the larger eddies which are being generated by the piston movement.

Figure 3 shows the diffusivity D generated by both the intake valve and piston. According to Eq. (10), D' monotonically decreases to the constant value, $\Lambda_a|u_p'|$, as D_v' dissipates exponentially. In the present solution, D initially increases with τ as the constant piston velocity begins to generate the eddies. However, D begins to decrease as D_v-dissipation comes to exceed the piston generation of the larger eddies. For τ of order one, the rate of generation of the larger-scale turbulence by u_p begins to be balanced by the turbulence dissipation of the smaller eddies, and D approaches an asymptotic value. As it has been alluded to, comparison, between Eq. (15) and the present solution, of the absolute levels of $D(\tau)$ is rather meaningless.

Figures 4 and 5 show $D_v(\tau)$ and $D(\tau)$ for the sinusoidal variation of u_p. Since the smaller eddies are assumed to be independent of u_p in Eq. (10), Figure 4 shows that D_v decays exponentially in the S-B solution as it did for the constant u_p (see Figure 2). In the present solution, D_v somewhat follows the sinusoidal piston movement but with a time lag after the initial decay period. The main point here is that the contribution of the smaller eddies to the diffusivity persists for all τ as the smaller eddies continuously receive energy from the larger eddies being generated by the piston movement.

Figure 5 shows the variations of $D(\tau)$ for the sinusoidal variation of u_p. In the S-B solution of Eq. (15), D_v almost completely dissipates for $\tau \geq \pi/4$, and D becomes sinusoidal as is u_p. The present solution shows that there exists a substantial time lag between the diffusivity and the piston velocity since the $u_p(\tau)$ determines the instantaneous turbulence source, $S(\tau)$, but not the instantaneous turbulence energy.

III. COMPRESSIBLE PLANE SHEAR LAYER

As the initial development of the formalism discussed in Section I, which will utilize the existing solution of the mean momentum equation in the solution of the kinetic equations, the compressible plane shear layer is being analyzed. This section describes the analysis. The final numerical results are not as yet available. However, all basic features of the solution expected as well as the method and assumptions made are discussed herein.

Existing Solution of the Momentum Equation

There are several solutions of the mean flow properties available. For convenience of the present analysis, we shall employ the compressible solution obtainable via the Howarth transformation from the incompressible solution given in Reference 9.

We first define the density-averaged ordinate,

$$\xi = \frac{\sigma}{x} \int_0^y \frac{\langle\rho\rangle}{\rho_\infty} \, dy \tag{19}$$

Then, we employ the Ting-Libby compressibility modification (Reference 10) of the incompressible eddy diffusivity of momentum as,

$$\varepsilon = 0.014\, b u_{\infty} \left(\frac{\rho_{\infty}}{\langle \rho \rangle} \right)^{2} x \tag{20}$$

With Eqs. (19) and (20), and with appropriate similarity assumptions, the governing mean momentum equation of the compressible plane shear layer based on the mixing-length theory becomes identical to that for the incompressible flow whose solution is given in Reference 9. The momentum equation transformed on the similarity plane is then,

$$\frac{d^3 F}{d\xi^3} + 2\sigma F \frac{d^2 F}{d\xi^2} = 0 \tag{21}$$

with the boundary conditions,

$$\begin{aligned} \frac{dF}{d\xi}(\infty) &= 1 + \frac{u_{\infty} - u_{-\infty}}{u_{\infty} + u_{-\infty}} \\ \frac{dF}{d\xi}(-\infty) &= 1 - \frac{u_{\infty} - u_{-\infty}}{u_{\infty} + u_{-\infty}} \end{aligned} \tag{22}$$

From the solution of Eq. (21), the mean velocity and shear stress profiles can be established as,

$$\langle u \rangle = \frac{u_{\infty} + u_{-\infty}}{2} \left(1 + \frac{u_{\infty} - u_{-\infty}}{u_{\infty} + u_{-\infty}} \operatorname{erf} \xi \right) \tag{23}$$

$$\langle v \rangle = -\frac{1}{2} \frac{\rho_{\infty}}{\langle \rho \rangle} \left(\frac{u_{\infty} + u_{-\infty}}{u_{\infty}} \right) \left[F(\xi) - \xi \frac{dF(\xi)}{d\xi} \right] \tag{24}$$

$$\langle UV\rangle = -(0.014)\, b\sigma^2 \left(\frac{u_\infty + u_{-\infty}}{u_\infty}\right) \frac{\rho_\infty}{\langle\rho\rangle} \quad \frac{d^2F(\xi)}{d\xi^2} \tag{25}$$

Reference 9 suggests that the values of the constants σ and b be 13.5 and 0.098, respectively. The functions, F and $dF/d\xi$, are obtained from Reference 9.

Kinetic Equation for Compressible Flow

There are substantial experimental indications (see References 11, 12 and 13), which seem to show that combustion does not alter the fundamental structure of the turbulence field. In the absence of definite contrary experimental data, we shall assume that the high-turbulence-Reynolds-number energy cascading structure of the turbulence field upon which the incompressible kinetic theory has been built remains valid. Then, the kinetic equation can be constructed to include the density variation in basically the same manner as the incompressible kinetic equation of Reference 1. This compressible equation is,

$$- (\langle u_k\rangle + U_k)\left(\frac{\partial\rho zf}{\partial x_k} - \frac{\partial\langle u_m\rangle}{\partial x_k}\frac{\partial\rho zf}{\partial U_m}\right) - \beta\langle\rho\rangle\left[2\frac{\partial}{\partial U_k}(fzU_k)\right.$$

$$\left. + \frac{\langle U_mU_m\rangle}{3}\frac{\partial^2 fz}{\partial U_k\partial U_k}\right] - \frac{\partial}{\partial U_k}\left(fz\frac{\partial\langle p\rangle}{\partial x_k}\right) - \frac{\partial}{\partial U_k}(\Gamma_k fz\rho)$$

$$- f\rho\omega = 0 \tag{26}$$

The structure of the turbulence field enters into the kinetic equation through the eddy-interaction terms contained in the square brackets, and it is seen that these terms are identical to those derived for the incompressible kinetic equation since it has been assumed that the fundamental structure of the turbulence field is unaltered by the compressibility.

Γ_k, in Eq. (26), represents the direct turbulence generation by the compressibility. This term should engender, among other terms, the correlation $< P \frac{\partial U_k}{\partial x_k} >$ in the second order moment of Eq. (26). It is our present feeling that this term is not important unless there exist shocks in the flow field (Reference 14). Therefore, for the present initial analysis of flows without shocks, we shall let

$$\Gamma_k = 0$$

Γ_k will be analyzed in detail in a future paper.

Once the solution of Eq. (26) is obtained for a given flow problem, all one-point moments of engineering interest can be readily constructed as, for instance,

$$\begin{aligned} <u_i> &= \int f u_i d\vec{U} \\ <\rho U_i U_j> &= \int \rho(\vec{x},\vec{U}) U_i U_j \; f(\vec{x},\vec{U}) d\vec{U} \end{aligned} \tag{27}$$

Before Eq. (26) can be solved, a specific relationship must be found between the density, temperature, and the pressure.

There are several plausible relationships that can be employed to relate the fluctuating and mean components of ρ, t, and P. For the present study, we shall try the relationship suggested in Reference 15 for ideal gases, which is,

$$\frac{t - \langle t \rangle}{\langle t \rangle} = - \frac{\rho - \langle \rho \rangle}{\langle \rho \rangle} \tag{28}$$

where $t - \langle t \rangle$ and $\rho - \langle \rho \rangle$ denote the fluctuating components of the temperature and density, respectively. Equation (28) can be rewritten as,

$$\frac{\rho}{\langle \rho \rangle} = 2 - \frac{t}{\langle t \rangle} \tag{29}$$

Solution of Compressible Plane Shear Layer

According to the approximate concept set forth in Section I, we shall make use of the existing solutions given by Eqs. (23), (24), and (25). The particular problem to be analyzed is the plane shear layer shown in Figure 6. In addition to the mean velocity difference, u_∞ and $u_{-\infty}$, it is considered that the two streams are at different mean temperatures of t_∞ and $t_{-\infty}$, respectively. We seek the solution of the kinetic equation, Eq. (26), for the temperature and density profiles, as well as the other turbulence quantities, with the aid of Eqs. (23) - (25).

As an initial try, we assume that the probability distribution function of the fluid elements, f, and those of the temperature and density, tf and ρf, can be approximated by bimodal functions. Thus, we write,

$$f = f_1 + f_2 \tag{30}$$

where

$$f_1 = \frac{1}{\left(\frac{2}{3}\pi E_1\right)^{3/2}} \exp\left[-\frac{(u-u_{o1})^2 + V^2 + W^2}{2E_1/3}\right] \qquad \text{for } V \geq 0 \qquad (31)$$

$$f_2 = \frac{1}{\left(\frac{2}{3}\pi E_2\right)^{3/2}} \exp\left[-\frac{(u-u_{o2})^2 + V^2 + W^2}{2E_2/3}\right] \qquad \text{for } V \leq 0 \qquad (32)$$

with $f_1 = 0$ for $V \leq 0$ and $f_2 = 0$ for $V \geq 0$, as it has been done earlier. (See References 1 - 5.)

Using the assumed bimodal form of the distribution function of Eq. (30), we derive,

$$\langle u \rangle = \frac{1}{2}(u_{o1} + u_{o2}) \qquad (33)$$

$$\langle v \rangle = \frac{1}{\sqrt{6\pi}}(\sqrt{E_1} - \sqrt{E_2}) \qquad (34)$$

$$\langle UV \rangle = \frac{1}{2\sqrt{6\pi}}(u_{o1} - u_{o2})(\sqrt{E_1} + \sqrt{E_2}) \qquad (35)$$

$$\langle U_k U_k \rangle = \frac{1}{4}(u_{o1}^2 + u_{o2}^2) + \left(\frac{1}{2} - \frac{1}{6\pi}\right)(E_1 + E_2) - \frac{u_{o1}u_{o2}}{2} + \frac{1}{3\pi}\sqrt{E_1 E_2} \qquad (36)$$

The first step of the present approximate solution is to determine the functions $u_{o1}(\xi)$, $u_{o2}(\xi)$, $E_1(\xi)$, and $E_2(\xi)$ by matching Eqs. (33) - (36) with the corresponding moments obtained from the existing solution of the momentum equation. Equations (23), (24) and (25) give $\langle u \rangle$, $\langle v \rangle$, and $\langle UV \rangle$, as functions of ξ. However, the first order mixing-length theory, from which these functions have been obtained, does not give the turbulence energy, $\langle U_k U_k \rangle$. We shall assume that

$$\langle U_k U_k \rangle = -5 \langle UV \rangle \tag{37}$$

Equations (23), (24), (25) and (37), show that these four mean quantities are given provided that the density ratio $\rho_\infty/\langle\rho\rangle$ is known. This is to be expected since in a compressible flow the momentum and energy equations are coupled.

The present proposed method of solution is as follows. For a given problem with $u_{-\infty}/u_\infty$ and $t_{-\infty}/t_\infty(=\rho_\infty/\rho_{-\infty})$, the density ratio, $\langle\rho\rangle/\rho_\infty$, is assumed. Equations (23) - (25) and (37) then give the four moments, $\langle u\rangle$, $\langle v\rangle$, $\langle UV\rangle$, and $\langle U_k U_k\rangle$ as functions of ξ. With these moments given, Eqs. (33) - (36) constitute a set of four nonlinear algebraic equations for the four unknown functions, $u_{o1}(\xi)$, $u_{o2}(\xi)$, $E_1(\xi)$, and $E_2(\xi)$. Solution of these equations for the four functions results in the distribution function f through Eq. (30). With the f known, the kinetic equation, Eq. (26), is solved for the distribution function of the density, and, therefore, of the temperature through Eq. (29). The mean density profile is then evaluated as,

$$\frac{\langle\rho\rangle}{\rho_\infty} = \frac{1}{\rho_\infty} \int \rho(\vec{\xi},\vec{U})\; f(\vec{\xi},\vec{U})d\vec{U} \tag{38}$$

With the use of the $\langle\rho\rangle/\rho_\infty$ evaluated by the above equation, the moments, $\langle u\rangle$, $\langle v\rangle$, $\langle UV\rangle$, and $\langle U_k U_k\rangle$ are recalculated from Eqs. (23) - (25) and (37). Solution of Eq. (26) is then repeated. This iteration continues until a satisfactory convergence on the density is attained.

We shall now discuss the solution of the kinetic equation, Eq. (26).

Commensurate with the bimodal assumption of f (see Eq. 30), Eq. (26) is solved also by an approximate bimodal method. As it has been done earlier (References 1 - 5), we let,

$$t(\xi,\vec{U}) = t_1(\xi,V) + t_2(\xi,V) \tag{39}$$

$$\rho(\xi,\vec{U}) = \rho_1(\xi,V) + \rho_2(\xi,V) \tag{40}$$

where t_1 and ρ_1 are zero for $V \leq 0$, and t_2 and ρ_2 are zero for $V \geq 0$.

Now, we generate two moment equations from Eq. (26) in the following manner. We first let z = t and integrate Eq. (26) termwise with respect to the velocity space. This produces the thermal energy conservation equation. Next, we multiply Eq. (26) through by V, with z = t, and integrate the resulting equation termwise with respect to the velocity space. This results in the turbulent thermal-energy transport flux equation. In both of the above equations, we set $\Gamma_k = \omega = 0$. The resulting moment equations are as follows where the appropriate similarity transformation commensurate with the momentum equation, Eq. (21), has been performed.

$$-\xi \frac{d}{d\xi}(\langle\rho\theta\rangle\langle u\rangle) + \sigma \frac{\langle\rho\rangle}{\rho_\infty} \frac{d}{d\xi}(\langle\rho\theta\rangle\langle v\rangle) = 0 \tag{41}$$

$$\begin{aligned} -\xi \frac{d}{d\xi}\langle\rho\theta UV\rangle &+ \sigma \frac{\langle\rho\rangle}{\rho_\infty} \frac{d}{d\xi}\langle\rho\theta V^2\rangle \\ &+ \sigma \frac{\langle\rho\rangle}{\rho_\infty}\langle\rho\theta\rangle\langle v\rangle \frac{d\langle v\rangle}{d\xi} - \langle\rho\theta\rangle\langle u\rangle\,\xi\,\frac{d\langle v\rangle}{d\xi} \\ &+ 3\hat{\beta}\frac{\langle\rho\rangle}{\rho_\infty}\langle\theta V\rangle = 0 \end{aligned} \tag{42}$$

where

$$\hat{\beta} = \frac{\langle U_k U_k \rangle^{1/2}}{2u_\infty} \tag{43}$$

$$\langle\rho\theta\rangle\langle u\rangle = 2 \frac{\langle\rho\rangle}{\rho_\infty} \frac{\theta_1\theta_2}{\theta_1 + \theta_2} \langle u\rangle \tag{44}$$

$$\langle\rho\theta\rangle\langle v\rangle = \frac{2}{\sqrt{6\pi}} \frac{\langle\rho\rangle}{\rho_\infty} \frac{\theta_1\theta_2}{\theta_1 + \theta_2} (\sqrt{E_1} - \sqrt{E_2}) \tag{45}$$

$$\langle\rho\theta UV\rangle = \frac{1}{\sqrt{6\pi}} \frac{\langle\rho\rangle}{\rho_\infty} (u_{o1} - u_{o2}) (\sqrt{E_1} + \sqrt{E_2}) \frac{\theta_1\theta_2}{\theta_1 + \theta_2} \tag{46}$$

$$\langle\rho\theta V^2\rangle = \frac{1}{3} \frac{\langle\rho\rangle}{\rho_\infty} \left[(E_1 + E_2) + \frac{1}{\pi} (\sqrt{E_1} - \sqrt{E_2})^2 \right] \frac{\theta_1\theta_2}{\theta_1 + \theta_2} \tag{47}$$

$$\langle\theta V\rangle = \frac{1}{2\sqrt{6\pi}} (\theta_1 - \theta_2) (\sqrt{E_1} + \sqrt{E_2}) \tag{48}$$

$$\theta_1 = t_1/t_\infty \quad , \qquad \theta_2 = t_2/t_\infty \tag{49}$$

In Eqs. (41) through (48), the functions, $\langle u\rangle$, u_{o1}, u_{o2}, $\langle v\rangle$, $\sqrt{E_1}$ and $\sqrt{E_2}$, represent the dimensionless quantities which have been normalized by u_∞. Also, ρ represents a dimensionless quantity which has been normalized by ρ_∞. The same symbols as for the previous dimensional quantities have been employed for convenience.

Solutions

A typical solution of Eqs. (33) - (36) for the functions, u_{o1}, u_{o2}, $\sqrt{E_1}$, and $\sqrt{E_2}$ comprising the distribution function of the fluid elements f is shown in Figure 7, for a given density ratio $\langle\rho\rangle/\rho_\infty$.

As it was mentioned earlier, solution of the kinetic equations, Eqs. (41) and (42), has not yet been completed. Different numerical techniques to accelerate the convergence, with respect to $\langle\rho\rangle/\rho_\infty$, are being tried. Detailed numerical analysis of these equations will constitute a future paper.

IV. CONCLUDING REMARKS

As an initial development of a formalism which would substantially simplify application of the kinetic theory concept to the turbulent chemically reacting flow problems of engineering interest, the following two problems have been analyzed.

For the purpose of eventually deducing a length-scale equation commensurate with the kinetic theory, the two-nonequilibrium degree kinetic equation developed earlier has been solved for a homogeneous turbulence field with two characteristic families of the energy-containing eddies wherein a turbulence source exists for one of the two families. Solutions have been obtained which would simulate the turbulence which exists in the combustion chamber of an internal combustion engine. There, the family of the smaller eddies generated through the intake valve decays while the larger-scale eddies are continuously generated by the subsequent piston movement.

A comparison was made between the eddy diffusivities deduced from the present solution and those obtained elsewhere by the mixing-length theory. Among other things, the present solution shows that the contribution of the smaller eddies to the diffusivity does not quickly dissipate as was predicted by the existing mixing-length theory. Through the interaction of the eddies, it was found that the smaller eddies continuously receive energy from the larger eddies in such a way that the effect of the smaller eddies on the diffusivity persists.

The governing equations deduced relate the effective length scale, Λ_e, to the turbulence energy, turbulence-energy source strength, and the two given length scales of the two families of the eddies. It seems very feasible that such equation could be solved in conjunction with the simplified formalism being developed.

The fundamental concepts of the simplified formalism being proposed have been discussed in the text. These concepts are being applied to the solution of a compressible plane shear layer. A simple, bimodal approximation has been employed to deduce the distribution function from the existing, mixing-length solution of the momentum equation. A governing set of the moment equations for the temperature and density fields has been constructed according to the kinetic theory by the use of the distribution function. This set of the equations is coupled with the existing solution of the momentum equation through the density.

Numerical solution has not yet been completed. Different approximations will be tested in the future for the deduction of the distribution function of the fluid-elements from the existing solution of the momentum equation. This seems to be one of the key points which may dictate the outcome of the proposed formalism.

REFERENCES

1. Chung, P.M., "A Simplified Statistical Model of Turbulent, Chemically Reacting Shear Flows," AIAA Journal, Vol. 7, Oct. 1969, pp. 1982-1991.

2. Chung, P.M., "Chemically Reaction in a Turbulent-Flow Field with Uniform Velocity Gradient," Physics of Fluids, Vol. 13, May 1970, pp. 1153-1165.

3. Chung, P.M., "Diffusion Flame in Homologous Turbulent Shear Flows," Physics of Fluids, Vol. 15, Oct. 1972, pp. 1735-1746.

4. Chung, P.M., "Turbulence Description of Couette Flow," Physics of Fluids, Vol. 16, July 1973, pp. 980-987.

5. Bywater, R.J., and Chung, P.M., "Turbulent Flow Fields with Two Dynamically Significant Scales," AIAA Paper No. 73-646 (1973).

6. Hong, Z.C., "Turbulent Chemically Reacting Flows According to a Kinetic Theory," Ph.D. Thesis. Dept. of Energy Engineering, University of Illinois at Chicago, Aug. 1975.

7. Sirignano, W.A., "One-Dimensional Analysis of Combustion in a Spark-Ignition Engine," Combustion Science and Technology, Vol. 7, May 1973, pp. 99-108.

8. Hinze, J.O., Turbulence, McGraw-Hill, N.Y. 1959.

9. Schlichting, H., Boundary Layer Theory,McGraw-Hill, New York, 1955. Chapt. 23.

10. Ting, L. and Libby, P.A., "Remarks on the Eddy Vicosity in Compressible Mixing Flows." Jour. Aerospace Science, Vol. 27, 1960, pp. 797-798.

11. Guenther, R. and Simon, H., "Turbulence Intensity, Spectral Density Functions, and Eulerian Scales of Emission in Turbulent Diffusion Flames." 12th International Symposium on Combustion, Pittsburgh, Pa., 1969.

12. Beer, J., "Combustion Aerodynamics" Chapt. 3., Combustion Technology, Edited by Palmer, H. and Beer, J., Academic Press, New York, 1974.

13. Baker, R., Bourke, P. and Whitelaw, J. "Measurements of Instantaneous Velocity in Laminar and Turbulent Diffusion Flames Using an Optical Anemometer." Journal of the Inst. of Fuel, Dec. 1973, pp. 388-395.

14. Oh, Y.H. and Bushnell, D.M., "Influence of External Disturbances and Compressibility on Free Turbulent Mixing," Aerodynamic Analyses Requiring Advanced Computers, Part 1. Conference at Laugley Research Center, March 4-6, 1975. NASA.

15. Cebeci,T. and Smith, A.M.O., Analysis of Turbulent Boundary Layers, Chapt. 3 Academic Press, New York, 1974.

NOMENCLATURE

$A_i(t)$	Random acceleration of fluid element by the small equilibrium eddies.
b	constant for eddy diffusivity
$D(\tau)$	dimensionless diffusivity defined by Eq. (17).
$D'(t)$	eddy diffusivity.
$D_v(\tau)$	contribution to $D(\tau)$ by the valve-generated smaller eddies.
E_1, E_2,	functions defining f_1 and f_2 respectively.
$F(\xi)$	dimensionless stream function for plane shear layer.
f	distribution function of fluid elements.
f_1, f_2	functions defined by Eq. (30).
$f_2(t, \vec{x}, \vec{U}', \vec{V}')$	one-point joint distribution function for $\vec{U}'$and $\vec{V}'$ modes.
$K_{a,i}$, $K_{b,i}$	forces due to molecular viscosity.
M	Λ_a/Λ_b
P	flucluating portion of pressure.
p	instantaneous pressure
S	turbulence source strength due to piston movement.
$\hat{S}$	S/β_{ao}
t	time, or temperature (Sec. III)
t_I	time at which value opens
U	x-component of the velocity relative to the mean velocity in Sec. II'.
$\vec{U}'$, U'_i	$\vec{U}'$-mode velocity. Contribution of the family of larger eddies to $\vec{W}'$.
$\vec{U}$,U_i	relative velocity in sec. III.
$\langle U_kU_k\rangle$, $\langle U_kV_k\rangle$, $\langle V_kV_r\rangle$, $\langle W_kW_k\rangle$	dimensionless energies defined by Eqs. (7).
$\langle U_kU_k\rangle$	Turbulence energy in sec.III.

U	x-component of the absolute velocity.
$\vec{u}$, u_i	instantaneous absolute velocity in sec. III.
u'_I	mean turbulence velocity of the value-generated eddies.
u'_p	piston velocity
u_p	$u'_p/\langle W'_k W'_k\rangle_o^{1/2}$
u_{o1},u_{o2}	functions defined by Eqs. (31) and (32).
V	y-component of the velocity relative to the mean velocity.
$\vec{V}$', V'_i	$\vec{V}$'-mode velocity. Contribution of the family of smaller eddies to $\vec{W}$'.
$\vec{U}$,U_i	relative velocity in sec. III.
v	y-component of the absolute velocity.
W	z-component of the velocity relative to the mean velocity in sec. III.
$\vec{W}'$, W'_i	velocity relative to the mean velocity. $W'_i = U'_i + V'_i$
$\vec{x}$, x_i	postion vector.
x, y, z	cartesion components of $\vec{x}$
z	scalar quantity.
β	$\langle U_k U_k\rangle^{1/2}/2\Lambda$ in sec. III.
β_a, β_b	$\langle W'_k W'_k\rangle^{1/2}/2\Lambda_a$ and $\langle W'_k W'_k\rangle^{1/2}/2\Lambda_b$ respectively.
$\hat{\beta}$	dimensionless quantity defined by Eq. (43)
Γ_k	direct effect of compressibility on turbulence
ε	eddy diffusivity
θ	t/t_∞
Λ	characteristic length scale of plane shear layer ($=x/c$).
Λ_a, Λ_b	characteristic scales of $\vec{U}$' and $\vec{V}$' modes respectively.
Λ_e	effective scale
$\bar{\Lambda}_e$	Λ_e/Λ_a
ξ	similarity variable defined by Eq. (19)

ρ	density
σ	constant
τ	$3\beta_{ao}t/2$
ω	source for z due to chemical reaction
$\langle\ \rangle$	ensemble average

Subscripts

o	initial
∞, $-\infty$	averaged quantities at $y = \infty$ and $y = -\infty$ respectively.

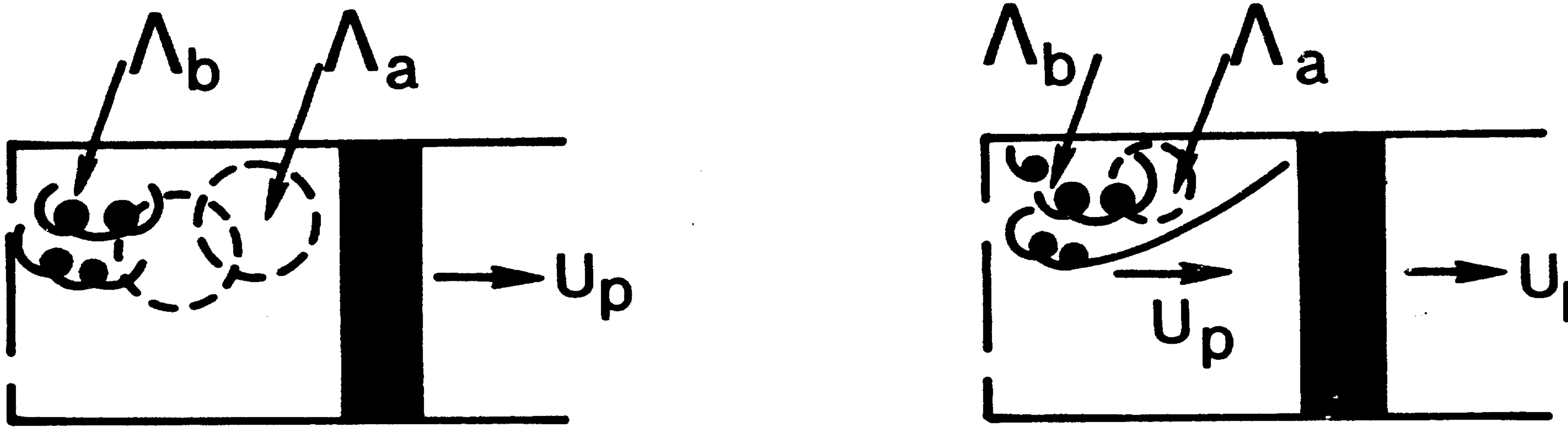

Fig 1. Sketch showing generation of two families of the eddies.

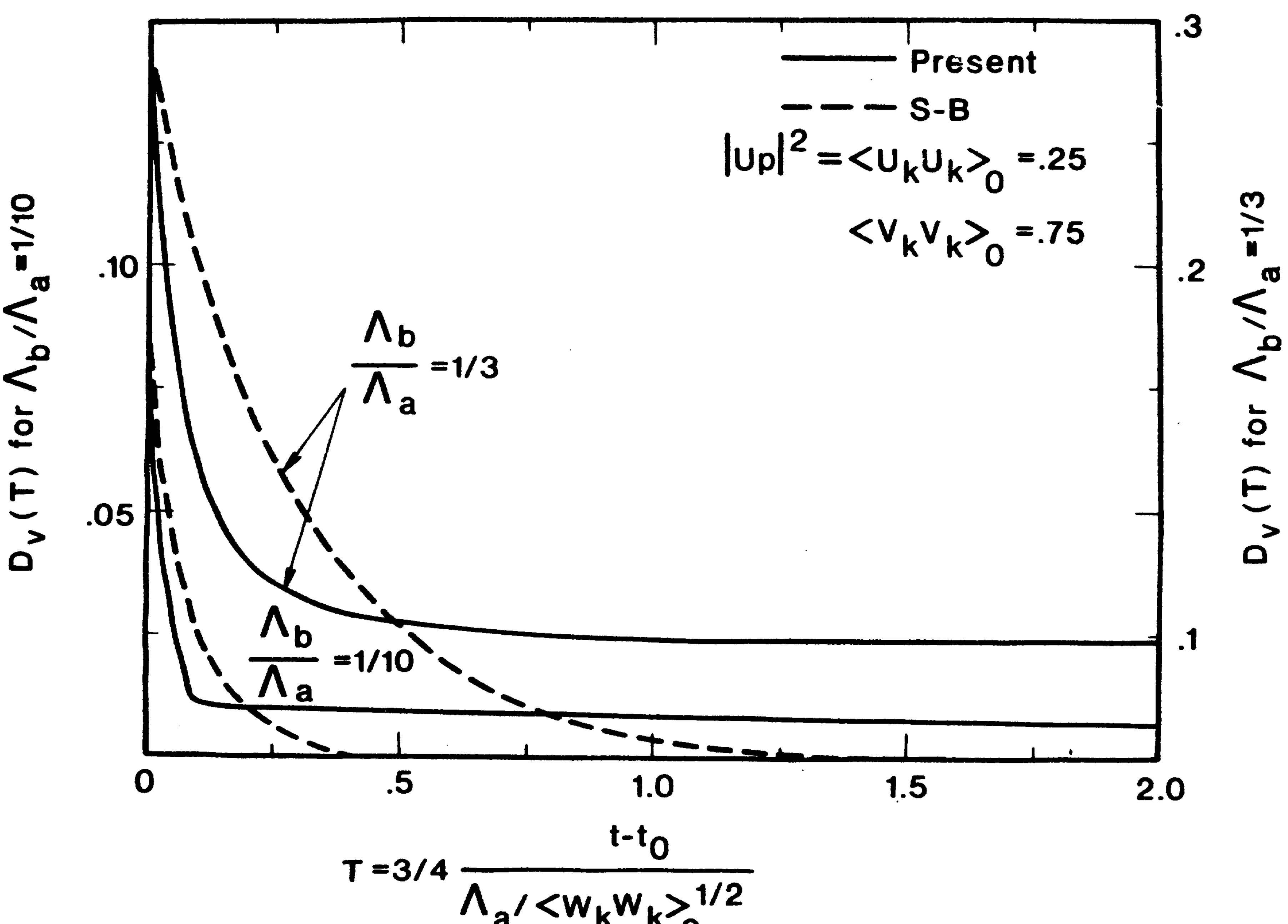

Fig 2. Contribution of smaller eddies to the diffusivity for constant piston velocity.

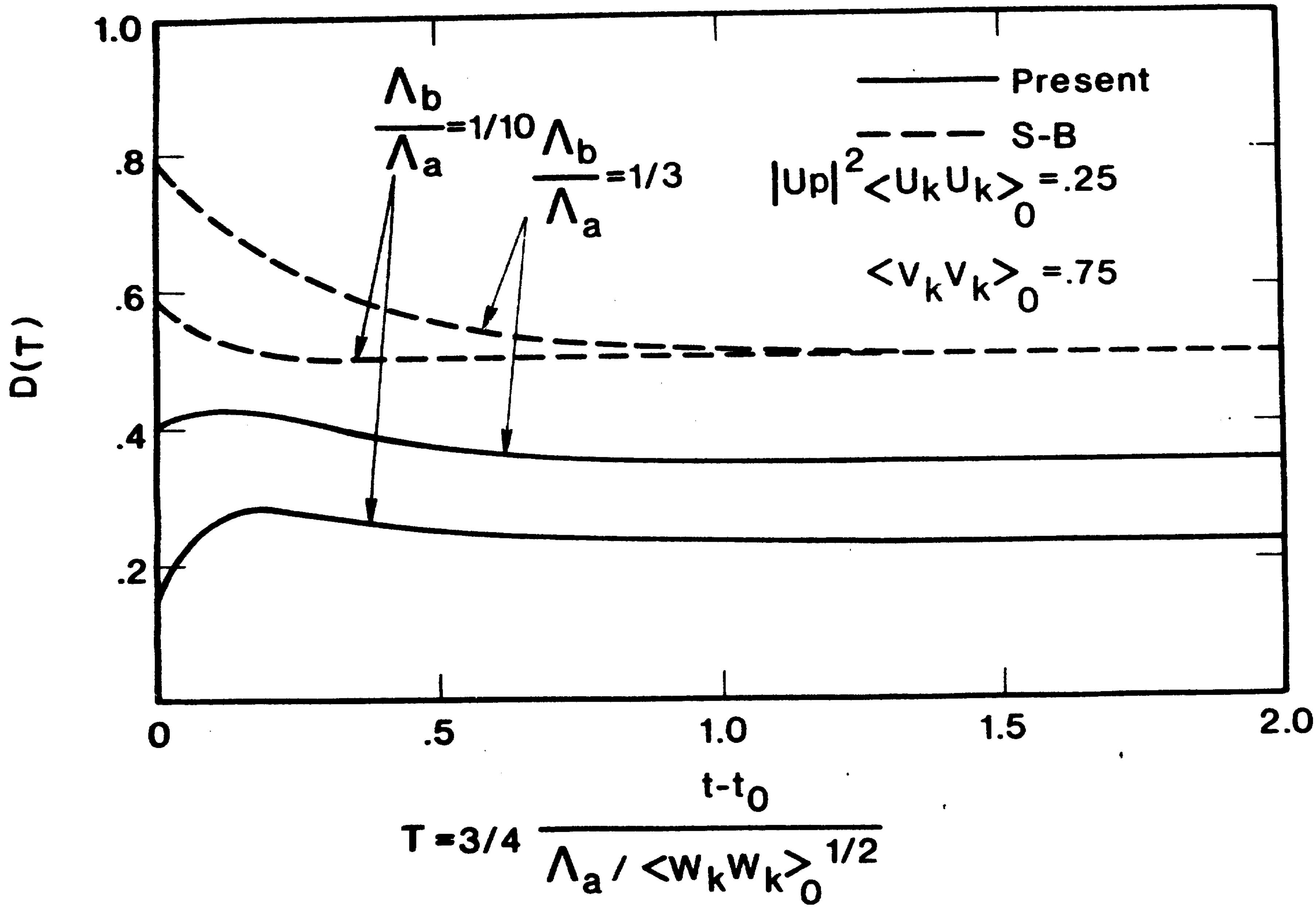

Fig 3. Diffusivity for constant piston velocity.

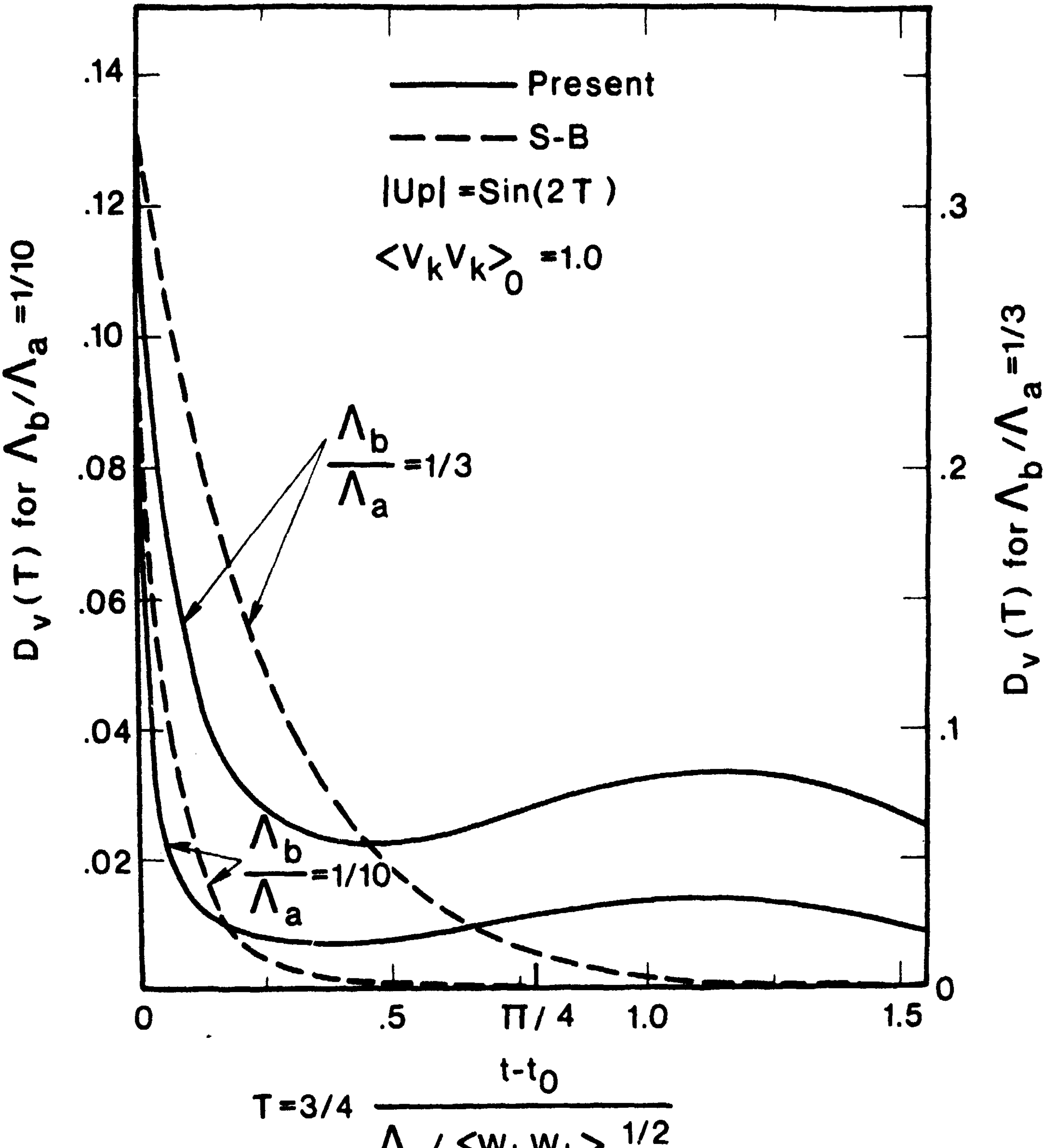

Fig 4. Contribution of smaller eddies to the diffusivity for sinusoidal piston velocity.

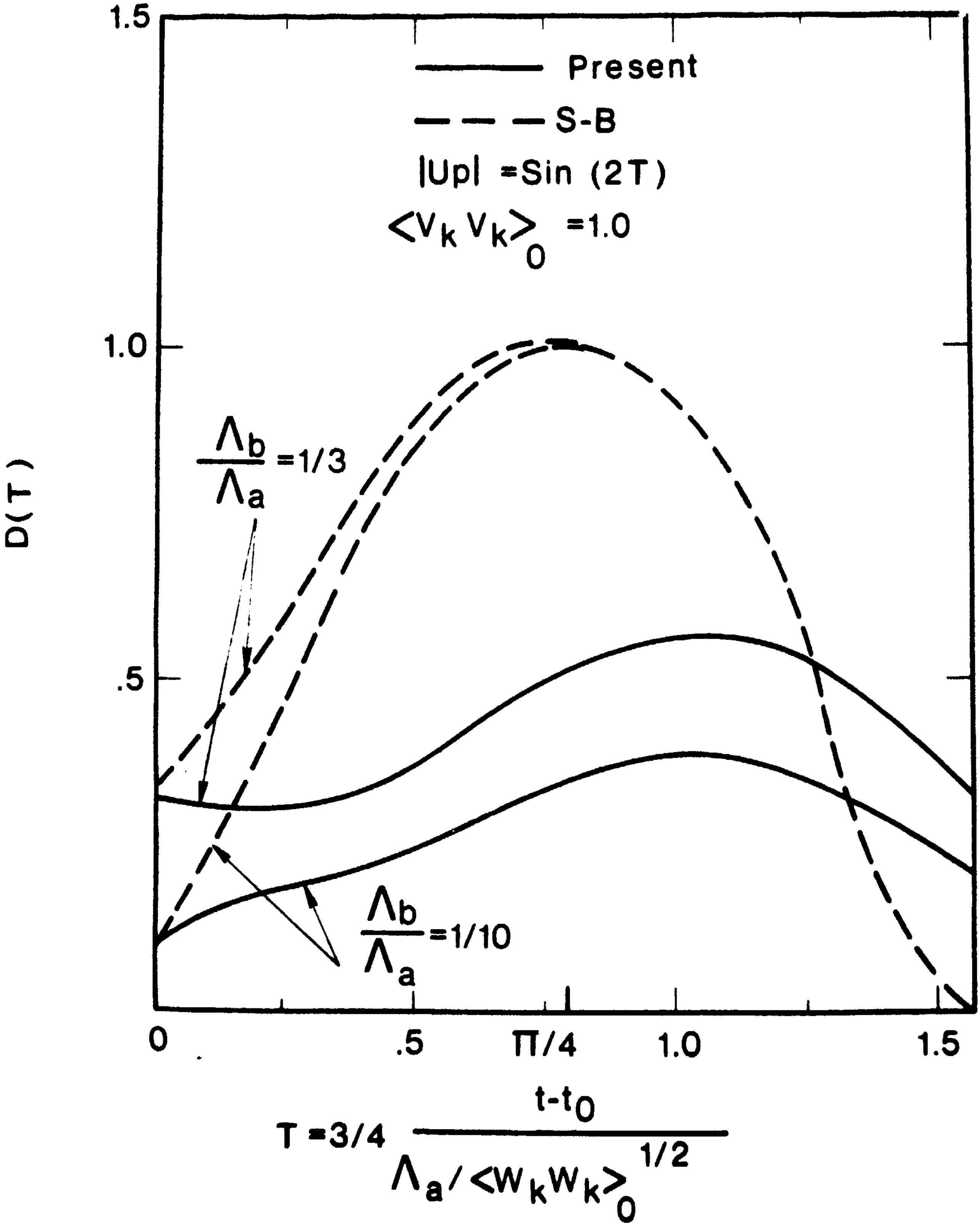

Fig 5. Diffusivity for sinusoidal piston velocity.

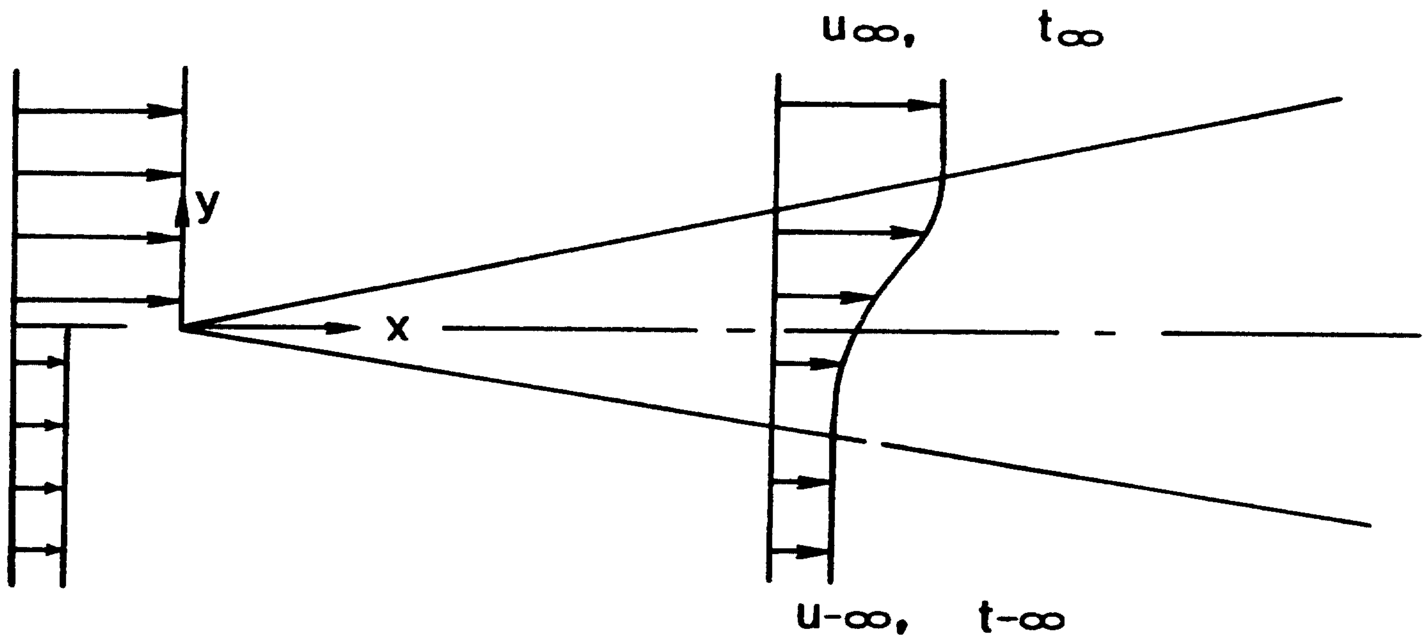

Fig 6. Turbulent plane shear layer.

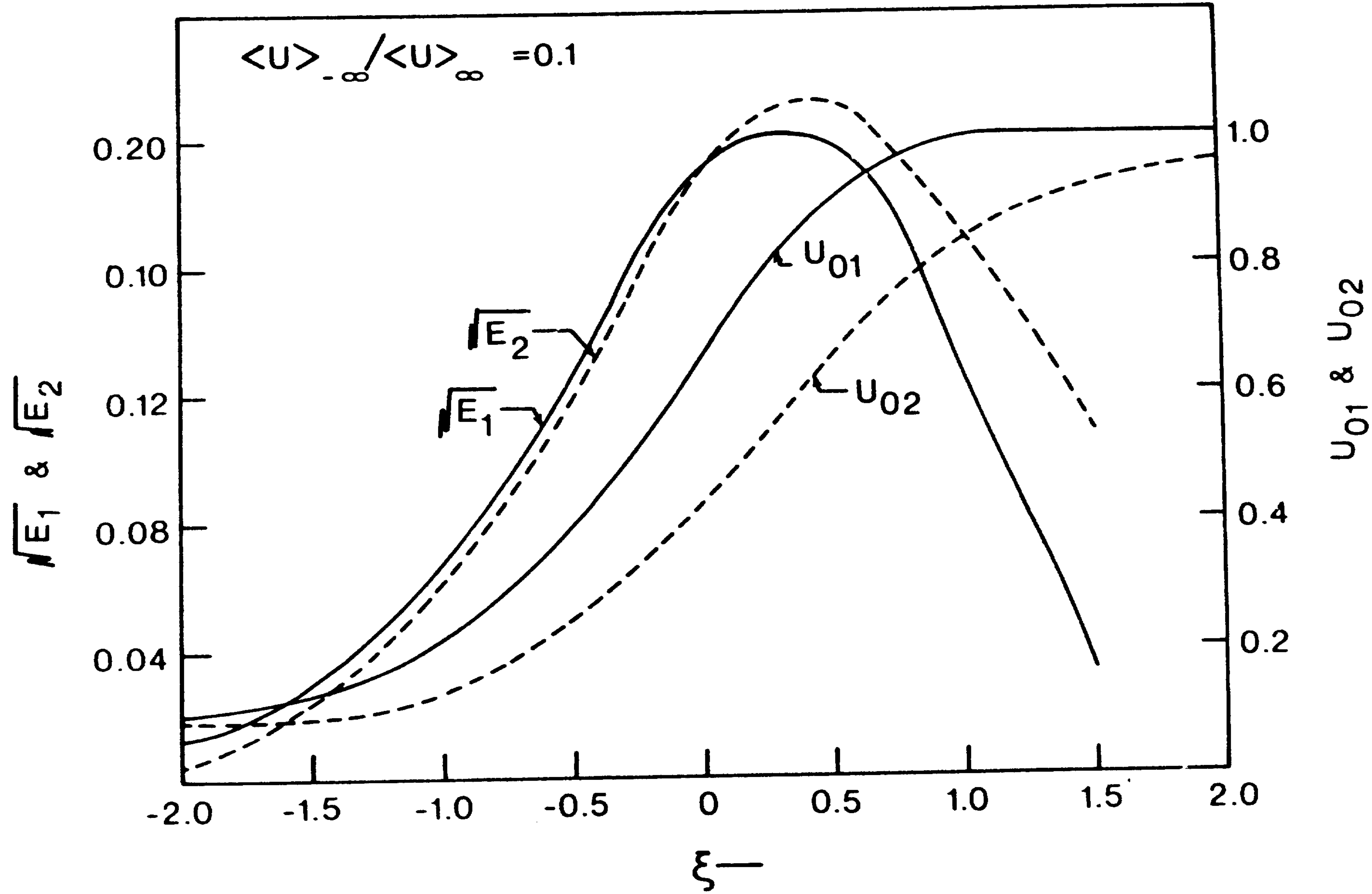

Fig 7. Functions comprising distribution functions for $\langle \rho \rangle_{\infty} / \langle \rho \rangle_{\infty} = 10$.