NASA TECHNICAL MEMORANDUM

NASA TM X-73319

VECTOR WIND AND VECTOR WIND SHEAR MODELS 0 TO 27 KM ALTITUDE FOR CAPE KENNEDY, FLORIDA, AND VANDENBERG AFB, CALIFORNIA

By O. E. Smith
Space Sciences Laboratory

July 1976

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

(NASA-TM-X-73319) VECTOR WIND AND VECTOR WIND SHEAR MODELS 0 TO 27 km ALTITUDE FOR CAPE KENNEDY, FLORIDA, AND VANDENBERG AFB, CALIFORNIA (NASA) 102 p HC $5.50 CSCL 04B
N76-26764
Unclas
G3/47 44542

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA TM X-73319		

4. TITLE AND SUBTITLE
Vector Wind and Vector Wind Shear Models 0 to 27 km Altitude for Cape Kennedy, Florida, and Vandenberg AFB, California

5. REPORT DATE
July 1976

6. PERFORMING ORGANIZATION CODE

7. AUTHOR(S)
O. E. Smith

8. PERFORMING ORGANIZATION REPORT #

9. PERFORMING ORGANIZATION NAME AND ADDRESS
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

10. WORK UNIT NO.

11. CONTRACT OR GRANT NO.

12. SPONSORING AGENCY NAME AND ADDRESS
National Aeronautics and Space Administration
Washington, D.C. 20546

13. TYPE OF REPORT & PERIOD COVERED
Technical Memorandum

14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES
Prepared by Space Sciences Laboratory, Science and Engineering

ORIGINAL PAGE IS OF POOR QUALITY

16. ABSTRACT

This document provides the techniques to derive several statistical wind models. The techniques are from the properties of the multivariate normal probability function. Assuming that the winds can be considered as bivariate normally distributed, then (1) the wind components and conditional wind components are univariate normally distributed, (2) the wind speed is Rayleigh distributed, (3) the conditional distribution of wind speed given a wind direction is Rayleigh distributed, and (4) the frequency of wind direction can be derived. All of these distributions are derived from the 5-sample parameter of wind for the bivariate normal distribution. By further assuming that the winds at two altitudes are quadravariate normally distributed, then the vector wind shear is bivariate normally distributed and the modulus of the vector wind shear is Rayleigh distributed. The conditional probability of wind component shears given a wind component is normally distributed. Examples of these and other properties of the multivariate normal probability distribution function as applied to Cape Kennedy, Florida, and Vandenberg AFB, California, wind data samples are given. The consistent agreement between the derived probability estimates and empirical sample estimates supports the hypothesis that multivariate normality for these wind samples is a reasonable assumption.

A technique to develop a synthetic vector wind profile model of interest to aerospace vehicle applications is presented.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
	Unclassified — Unlimited Orvel E. Smith

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	103	NTIS

MSFC - Form 3292 (Rev December 1972)

For sale by National Technical Information Service, Springfield, Virginia 22151

ACKNOWLEDGMENTS

The author acknowledges the outstanding computer programming for the statistical parameters performed by Mr. G. W. Batts, Computer Sciences Corporation, and Mr. P. R. Harness of the Data Systems Laboratory, Marshall Space Flight Center. To Mr. J. C. Stephens, Systems Dynamics Laboratory, goes credit for programming the Synthetic Vector Wind Model.

The constructive comments and encouragement of Mr. W. W. Vaughan and Mr. S. C. Brown of the Aerospace Environment Division, Space Sciences Laboratory, are appreciated.

Credit is due to many colleagues who have made technical contributions through reviews and discussions of the investigations that have lead to this report.

The author thanks the many people who edited, typed, and prepared this document for publication for doing an excellent job under pressure to meet an early publication date.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF ILLUSTRATIONS (Concluded)

LIST OF TABLES

TECHNICAL MEMORANDUM X-73319

VECTOR WIND AND VECTOR WIND SHEAR MODELS 0 TO 27 KM ALTITUDE FOR CAPE KENNEDY, FLORIDA, AND VANDENBERG AFB, CALIFORNIA

I. INTRODUCTION

A. Purpose

The investigations for this report were motivated by a desire to present a statistical description of upper air wind data samples in terms of probability models using rigorous mathematical probability properties of the multivariate normal probability distribution function. An attempt has been made to produce a comprehensive technical documentation of these investigations. The development of all subject material is not treated on an equal level because the primary objective is to illustrate known and potential applications of wind models derived from the probability theory. This report is not a treatise on the normal probability theory; however, it is an application of some well known and not so well known properties of normal probability distributions. Needs still exist for further analytical investigations to transfer normal probability theory to practical applications. These needs are recognized in simplifying the mathematical formulations, treatment of the sample data base, and coupling the statistical inferences to physical reality. For atmospheric winds the physical inferences from statistics include a large field of meteorology.

B. Coordinate System and Notations

Wind measurements are recorded in terms of magnitude and direction. The wind direction is measured in degrees clockwise from true north and is the direction from which the wind is blowing. The wind magnitude (the modulus of the vector) is the scalar quantity and is referred to as wind speed or scalar wind. A statistical description that accounts for the wind as a vector quantity is appropriate and requires a coordinate system.

For this report the standard meteorological coordinate system has been chosen for the wind statistics, all tables of statistical parameters, and related discussions because the coordinate system used in aerospace and related applied fields has not always been consistent.

Using Figure 1, the polar and Cartesian forms for the meteorological coordinate system are defined:

W = wind speed, scalar wind, or magnitude of the wind vector in m/s.

θ = wind direction. θ is measured in degrees clockwise from true north and is the direction from which the wind is blowing.

U = zonal wind component, positive west to east in m/s.

V = meridional wind component, positive south to north in m/s.

The components θ and W define the polar form, and the U–V components define the Cartesian forms:

$$U = -W \sin\theta \quad , \quad 0 \le \theta \le 360° \tag{1}$$

$$V = -W \cos\theta \quad . \tag{2}$$

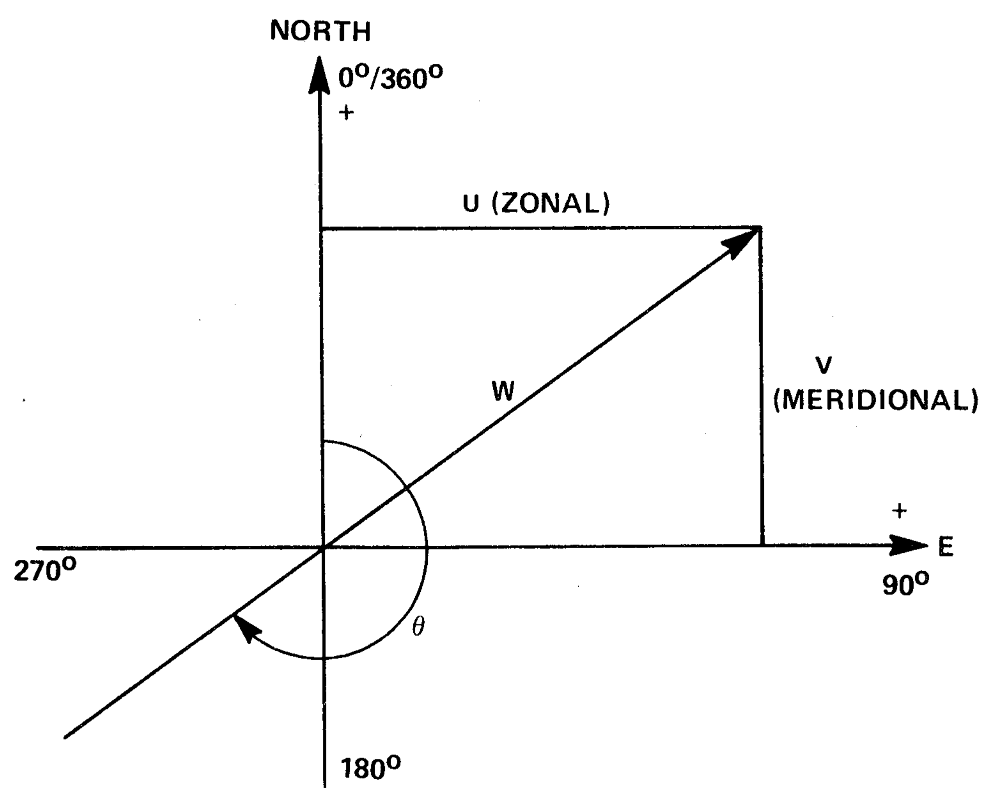


Figure 1. The meteorological coordinate system.

It is more convenient to work in the Cartesian coordinate form for the subsequent statistical analysis than in the polar form. The statistical tools are readily available for use in Cartesian form, whereas for the polar form the statistical treatment would be awkward and in some cases techniques would have to be developed.

C. Organization

The main body of this report is divided into sections. Section II contains information on probability distributions, Section III is a discussion of wind analysis, and vector wind profile models are presented in Section IV. These sections are followed by the conclusions and an appendix. Appendix A, entitled "A Program to Compute Conditional Bivariate Normal Parameters," was prepared by Michael C. Carter, University of Arkansas, under contract NAS8-31550 for the Aerospace Environment Division, Space Sciences Laboratory, Marshall Space Flight Center. Appendix B presents a description and an example of vector wind and vector wind shear statistical parameters available for Cape Kennedy, Florida, and Vandenberg AFB, California.

In Section II the normal probability distribution function and several properties or functions of bivariate normal variables are stated in general form. No attempt is made to show the derivations for the properties of these functions. The literature is cited for some of the lesser known distribution functions derivable from the properties of the multivariate normal probability distribution function. Since repeated applications of some of these functions are made, the relationships between functions of these variables are stated in this section as an aid to understanding the principles on which the wind analysis is based. Other extensions for engineering and scientific applications for derivable wind models should be investigated.

In Section III the wind analysis uses the principles given in Section II to illustrate that internally consistent probability distributions can be derived from sample estimates for the winds-aloft data. As an example, the distribution of wind speed can be derived from the five parameters of the bivariate normal distribution which are in close agreement with the empirical percentiles of wind speed. Another example is that the distribution of wind direction can also be derived from the five parameters of the bivariate normal distribution.

Section IV, "Vector Wind Profile Models," is the primary topic of interest in this report because of the potential applications to the flight performance analysis of aerospace vehicles. Several wind models are presented in this section. These models are called "synthetic vector wind profile models." The concept follows that of the synthetic scalar (wind speed) model derived from the conditional wind shears. A first extension of this concept was a synthetic component wind profile model that was investigated for the NASA Skylab program. We now have a further extension that produces a family of models for the vector wind profile. These models are presented only for technical information in this report.

II. PROBABILITY DISTRIBUTIONS

Since we have determined that the wind samples in the 1 to 27 km[1] altitude region over Cape Kennedy, Florida, and Vandenberg Air Force Base, California, can be treated as bivariate, normally distributed for practical applications, we have a powerful tool in the properties of the normal probability distribution function for modeling the wind for various aerospace design and operations planning problems. Because the statistical analysis of the wind-aloft sample presented in this report makes repeated use of some of the properties of the normal probability distribution, this section describes the pertinent properties. A generalized notation is used to present theoretical treatment and to avoid confusion between population parameters and the corresponding sample estimates of the parameters.

The probability distributions presented in analytical form are:

a. The normal (univariate) distribution

b. The bivariate normal distribution

c. The circular normal distribution

d. Functions of bivariate normal variables.

These distributions and some of their properties are introduced.

1. This assertion is supported by References 9 through 11.

A. Univariate Normal (Gaussian) Distribution

The normal, especially the univariate normal, probability distribution is the most widely known distribution. It is introduced here to establish the notations and because of its applications to wind component statistics.

A normally distributed random variable, X, has the probability density function (p.d.f.),

$$f(X) = \frac{1}{\sqrt{2\pi}\,\sigma_X} e^{-\frac{(X-\xi)^2}{2\sigma_X^2}} \qquad -\infty \leqq X \leqq \infty \quad . \tag{3}$$

The constants ξ and σ_X^2 are the mean and the variance or parameters of the distribution. The square root of the variance is called the standard deviation; i.e., $\sqrt{\sigma_X^2} = \sigma_X$.

The normal probability distribution function (PDF) is

$$F(X) = \int_{-\infty}^{X} f(X)\, dX \quad . \tag{4}$$

This function is sometimes called the cumulative distribution function. Because it cannot be integrated in closed form, it is widely tabulated for zero means and unit variances as follows. Set

$$t = \frac{X - \xi}{\sigma_X} \quad ;$$

the p.d.f. is then

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \tag{5}$$

for unit variance and there is no loss of generality. The PDF is

$$F(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt \quad . \tag{6}$$

To emphasize the connotation of probability, $F(t)$ is shown in Table 1 as $P\{X\}$ versus selected values of t. The t-values in Table 1 are used as multiplier factors for the standard deviation to express the probability that a normally distributed variable, X, is less than or equal to a given value.

$$P\{X \leqq \xi + t\sigma_X\} = \text{probability, } \underline{p} \quad . \tag{7}$$

Specifically, when $t = 1.6449$, the probability that X, the random normal variable, is less than or equal to the mean, ξ, plus 1.6449 standard deviations is 0.95. The 95th percentile of X is that value of X which is less than or equal to the mean plus 1.6449 standard deviations.

Also given in Table 1 are the numerical values to express the probability that X falls in the interval X_1 and X_2; i.e.,

$$P\{X_1 = \xi - t\sigma \leqq X \leqq X_2 = \xi + t\sigma\} \quad .$$

For $t = 1.9602$, the probability that X lies in the interval X_1 and X_2 is 0.95. Since the total area under the p.d.f, equation (3), is unity, 95 percent of the probability area is between X_1 and X_2 for this example. The values X_1 and X_2 would be called the inter-95th percentile range. In common terms, 95 percent of the X's fall within X_1 and X_2.

TABLE 1. VALUES OF t FOR STANDARDIZED NORMAL (UNIVARIATE) DISTRIBUTION FOR PERCENTILES AND INTERPERCENTILE RANGES

t	P(X)	X	$P\{X_1 \leq X \leq X_2\}$ (%)
-3.0000	0.00135	ξ - 3.0000 σ	99.73 (0.27)
-2.5758	0.00500	ξ - 2.5758 σ	99 (1.00)
-2.3263	0.01000	ξ - 2.3263 σ	98 (2.00)
-2.2365	0.01266	ξ - 2.2365 σ	97.468 (2.532)
-2.0000	0.02275	ξ - 2.0000 σ	95.45 (4.55)
-1.9602	0.02500	ξ - 1.9602 σ	95 (5)
-1.6449	0.05000	ξ - 1.6449 σ	90 (10)
-1.2816	0.10000	ξ - 1.2816 σ	80 (20)
-1.0000	0.15866	ξ - 1.0000 σ	68.268 (31.732)
-0.8416	0.20000	ξ - 0.8416 σ	60 (40)
-0.6745	0.25000	ξ - 0.6745 σ	50 (50)
-0.2533	0.40000	ξ - 0.2533 σ	20 (80)
0.0000	0.50000	ξ	
0.2533	0.60000	ξ + 0.2533 σ	
0.6745	0.75000	ξ + 0.6745 σ	
0.8416	0.80000	ξ + 0.8614 σ	
1.0000	0.84134	ξ + 1.0000 σ	
1.2816	0.90000	ξ +1.2816 σ	
1.6449	0.95000	ξ + 1.6449 σ	
1.9602	0.97502	ξ +1.9602 σ	
2.0000	0.97725	ξ + 2.0000 σ	
2.2365	0.98734	ξ +2.2365 σ	
2.3263	0.99000	ξ + 2.3263 σ	
2.5758	0.99500	ξ + 2.5758 σ	
3.0000	0.99865	ξ 3.0000 σ	

where $X_1 = \xi - t\sigma$
and $X_2 = \xi + t\sigma$

B. Bivariate Normal Distribution

Extensive use will be made of the bivariate normal distribution and some of its properties in the presentation of wind statistics. The properties will be stated without proofs because they are well known or well treated in textbooks dealing with this subject.

The bivariate normal probability density function (B.p.d.f) is

$$f(X,Y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[\frac{-1}{2(1-\rho^2)}\left\{\frac{(X-\bar{X})^2}{\sigma_x^2} - \frac{2\rho(X-\bar{X})(Y-\bar{Y})}{\sigma_x\sigma_y} + \frac{(Y-\bar{Y})^2}{\sigma_y^2}\right\}\right] \quad -\infty \leqq X \leqq \infty \quad \text{and} \quad -\infty \leqq Y \leqq \infty \quad . \tag{8}$$

For typographical simplicity let the means for population parameters be represented by $\bar{X}$ and $\bar{Y}$. Equation (8) expresses the joint probability relationship between two random variables X and Y that are bivariate normally distributed. This function is completely described by the five parameters: the means $\bar{X}$ and $\bar{Y}$, the standard deviations σ_x and σ_y, and the linear correlation coefficient ρ. The linear correlation coefficient ρ characterizes the dependence between the random variables X and Y. Note that the marginal distributions of equation (8) are normal. The converse is not necessarily true. When the two variables X and Y are correlated, it is a necessary, but not sufficient, condition for the marginal distributions f(X) and f(Y) to be normal and the joint distribution to be bivariate normal [1]. Work by H. Crutcher and L. Falls [2] gives a statistical test for multivariate normality for sample variables.

As is the case for the univariate normal probability density function, the bivariate normal probability density function is of little interest for practical problems, although both have important roles in theoretical analysis. The probability distribution function for the bivariate normal distribution does not take on a simple form as is the case for the univariate normal distribution. By setting the terms of the exponent of equation (8) equal to a constant, λ^2, we have

$$\frac{(X - \bar{X})^2}{\sigma_x^2} - \frac{2\rho(X - \bar{X})(Y - \bar{Y})}{\sigma_x \sigma_y} + \frac{(Y - \bar{Y})^2}{\sigma_y^2} = \lambda^2 \quad . \tag{9}$$

Equation (9) is recognized as a family of ellipses depending on the value of λ^2. The density function has constant value on these ellipses; therefore, the ellipses of equation (9) are referred to as ellipses of equal probability. For most applications the interest is in determining the probability that a point (X, Y) falls inside a given ellipse. Hence, the value for λ^2 must be obtained to define a given ellipse. This is done by setting

$$P(\lambda) = \iint_{R(\lambda)} f(X, Y)\, dX\, dY \quad , \tag{10}$$

where $R(\lambda)$ defines the region bounded by the ellipse of equation (9). The integration of equation (10) is obtained by changing the variables X and Y to polar coordinates. The result is

$$P(\lambda) = 1 - e^{\frac{-\lambda^2}{2(1-\rho^2)}} \quad . \tag{11}$$

Solving for λ^2 and replacing $P(\lambda)$ by p, we have

$$\lambda^2 = -2(1 - \rho^2) \ln (1 - p) \quad . \tag{12}$$

Selected values for

$$\lambda_e = \sqrt{2}\sqrt{-\ln (1 - p)} \tag{13}$$

are given in Table 2. A complete derivation of equation (11) is given by Gnedenko [3].

TABLE 2. VALUES OF λ FOR BIVARIATE NORMAL DISTRIBUTION ELLIPSES AND CIRCLES

P(%)	λ_e (ellipse)	λ_c (circle)	P(%)	λ_e (ellispe)	λ_c (circle)
0.000	0.0000	0.0000	65.000	1.4490	1.0246
5.000	0.3203	0.2265	68.268	1.5151	1.0713
10.000	0.4590	0.3246	70.000	1.5518	1.0973
15.000	0.5701	0.4031	75.000	1.6651	1.1774
20.000	0.6680	0.4723	80.000	1.7941	1.2686
25.000	0.7585	0.5363	85.000	1.9479	1.3774
30.000	0.8446	0.5972	86.466	2.0000	1.4142
35.000	0.9282	0.6563	90.000	2.1460	1.5175
39.347	1.0000	0.7071	95.000	2.4477	1.7308
40.000	1.0108	0.7147	95.450	2.4860	1.7579
45.000	1.0935	0.7732	98.000	2.7971	1.9778
50.000	1.1774	0.8325	98.168	2.8284	2.0000
54.406	1.2533	0.8862	98.889	3.0000	2.1213
55.000	1.2637	0.8936	99.000	3.0348	2.1460
60.000	1.3537	0.9572	99.730	3.4393	2.4320
63.212	1.4142	1.0000	99.9877	4.2426	3.0000

$$\lambda_e = \sqrt{2}\,\sqrt{-\ln(1 - P)}$$

$$\lambda_c = \sqrt{-\ln(1 - P)}$$

Because graphical displays of the probability ellipses are useful and informative, we have devised a convenient plotting procedure for electronic computer operations. First, rewrite equation (9) in the more recognizable conic form,

$$AX^2 + BXY + CY^2 + DX + EY + F = 0 \quad , \qquad (14)$$

where

$$A = \sigma_y^{2}$$

$$B = -2\rho\sigma_x\sigma_y$$

$$C = \sigma_x^{2}$$

$$D = 2\sigma_x\sigma_y\rho\bar{Y} - 2\sigma_y^{2}\bar{X} = -(B\bar{Y} + 2A\bar{X})$$

$$E = 2\sigma_x\sigma_y\rho\bar{X} - 2\sigma_x^{2}\bar{Y} = -(B\bar{X} + 2C\bar{Y})$$

$$F = \sigma_y^{2}\bar{X}^2 - 2\sigma_x\sigma_y\rho\bar{X}\bar{Y} + \sigma_y^{2}\bar{Y} - \sigma_x^{2}\sigma_y^{2}\lambda^2$$

or

$$F = A\bar{X} + C\bar{Y}^2 + B\bar{X}\bar{Y} - AC\lambda^2 \quad .$$

We replaced λ^2 by λ_e^{2} from equation (13) and obtained

$$F = A\bar{X}^2 + C\bar{Y}^2 + B\bar{X}\bar{Y} - AC(1 - \rho^2)\,\lambda_e^{2} \quad .$$

For graphical presentations the range of the variable is important in order to arrange the scale. The largest and smallest values of X and Y for a given probability ellipse, p, are given by

$$X_{L,S} = \bar{X} \pm \sigma_x \lambda_e \tag{15}$$

$$Y_{L,S} = \bar{Y} \pm \sigma_y \lambda_e \quad , \tag{16}$$

where, as before, $\lambda_e = \sqrt{2}\sqrt{-\ln(1 - p)}$.

Although there are several approaches to graphing the probability ellipses, we have found the following procedure most advantageous for electronic computer plotting. In establishing the computer plotting program, the sample estimates for $\bar{X}$, $\bar{Y}$, σ_x, σ_y, and ρ are constants in equation (14). The user makes the choice of probability ellipses desired. Thus, p in equation (13) is programmed as a parameter. The largest and smallest values for X and Y are computed by equations (15) and (16) for the largest probability ellipse selected. This sets the graphical scale. Values of X within the range of X smallest to X largest are obtained by incrementing X between these limits. Using the quadratic equation, a solution of equation (14) is made for Y for each value of X and plotted. The centroid $(\bar{X}, \bar{Y})$ for the family of probability ellipses is plotted as a point. Labeling and other identification completes the plotting program.

For a given probability, equation (14) defines an ellipse which contains p-percent of the points X, Y. Since the entire area under the bivariate normal density function [equation (8)] is unity, upon integration for a given probability ellipse, that given ellipse contains p-percent of the total area. In the wind analysis p-percent of the wind vectors fall within the specified probability ellipse. From this point of view, a specified probability ellipse gives the joint probability that p-percent of the U-V components lie within the given ellipse.

C. Circular Normal Distribution

When $\sigma_x^2 = \sigma_y^2 = \sigma^2$ and $\rho = 0$ in the bivariate normal distribution, the probability ellipses of equation (9) reduce to circles whose centers are at the means $\overline{X}$, $\overline{Y}$. The radii of the probability circles are $\sigma_{V1}\lambda_c$, where

$$\sigma_{V1} = \sqrt{2\sigma^2} \tag{17}$$

and

$$\lambda_c = \sqrt{-\ln(1 - p)} \quad . \tag{18}$$

Values for λ_c for selected probabilities, p, are given in Table 2.

Because this function is simple, it can be easily graphed manually. However, the generalized plotting technique for electronic computer plotters as represented by equation (14) can be advantageously used.

D. Functions of the Bivariate Normal Distribution

The very important and useful properties of the bivariate normal distribution presented in this section are:

1. Conditional distributions
2. The Rayleigh distribution
3. The sum and differences
4. Rotation of coordinates
5. Directional distribution.

The conditional distributions of bivariate normal variables are normally (univariate) distributed. The resolution of bivariate normally distributed variables (the modulus of the vector) is Rayleigh distributed. The sum and differences of bivariate normally distributed variables are normally (univariate) distributed. For bivariate normally distributed variables, the probability distribution remains bivariate normal and the marginal distributions remain normal (univariate) with the rotation of the coordinates.

Specific equations for the properties which are functions of the five parameters of the bivariate normal distribution are presented in general notation for wide applications.

1. <u>Conditional Distributions.</u>

a. Conditional Distributions of Bivariate Normally Distributed Variables. Given that two random variables X and Y are bivariate normally distributed, the conditional distribution $f(Y|X)$ is read as $f(Y)$ given X, and likewise $f(X|Y)$ is read as $f(X)$ given Y. The conditional probability distribution function $F(Y|X)$ has the mean $E(Y|X)$ and variance $\sigma^2_{(x|y)}$, where

$$E(Y|X^*) = \bar{Y} + \rho\left(\frac{\sigma_y}{\sigma_x}\right)(X^* - \bar{X}) \tag{19}$$

and

$$\sigma^2_{(y|x^*)} = \sigma_y^2 (1 - \rho^2) \quad . \tag{20}$$

The conditional standard deviation is

$$\sigma_{(y|x^*)} = \sigma_y \sqrt{1 - \rho^2} \quad . \tag{21}$$

The asterisk is placed on X (i.e., X^*) to emphasize that X is the given value which may be assigned any arbitrary value. For illustrations and many applications, it is convenient to let $X^* = \bar{X} + t\sigma_x$, where t is chosen from Table 1 to give a desired probability level for X^*.

By interchanging the variables and parameters, the conditional distribution function for $F(X|Y^*)$ has the conditional mean

$$E(X|Y^*) = \bar{X} + \rho\left(\frac{\sigma_x}{\sigma_y}\right)(Y^* - \bar{Y}) \quad , \tag{22}$$

conditional variance

$$\sigma^2_{(x|y^*)} = \sigma_x^2(1 - \rho^2) \quad , \tag{23}$$

and conditional standard deviation

$$\sigma_{(x|y^*)} = \sigma_x\sqrt{1 - \rho^2} \quad .$$

The above conditional normal probability distribution functions are univariate normal distributions for a (fixed) given value for one of the bivariate normal variables. Thus the t-values given in Table 1 are applicable for conditional probabilities statements. For example,

$$F(Y|X^*) = E(Y|X^*) + t\sigma_{(y|x^*)} \quad .$$

For $t = 1.6449$ there is a 95 percent chance that Y is less than or equal to $\bar{Y} + 1.6449\,\sigma_y$, given that $X = X^*$. In symbols this statement reads

$$P\{Y \leq E(Y|X^*) + 1.6449\,\sigma_{(y|x^*)}|X = X^*\} = 0.9500 \quad .$$

Interval probability statements can also be made; namely,

$$P\{Y_1 = E(Y|X^*) - t\sigma_{(y|x^*)} \leqq Y \leqq Y_2 = E(Y|X^*) + t\sigma_y |X = X^*\}$$

where X^* can take on any fixed value of X, but a convenient arrangement is to let $X^* = \bar{X} \pm t\sigma_x$.

The close connection of the regression function of Y on X to the conditional mean for the bivariate normal distribution is noted; namely,

$$Y = \bar{Y} + \rho\left(\frac{\sigma_y}{\sigma_x}\right)(X - \bar{X}) \quad . \tag{24}$$

Similarly, the regression function of X on Y is

$$X = \bar{X} + \rho\left(\frac{\sigma_y}{\sigma_x}\right)(Y - \bar{Y}) \quad . \tag{25}$$

These are linear functions and express the same results as would be obtained from a least squares regression line.

b. Conditional Distribution of the Quadravariate Normal Probability Distribution. The marginal and conditional distributions of the multivariate normal distribution are normal. We have a particular interest in the conditional distribution of quadravariate normal variables, taking two variables against the remaining two variables. In general functional notation, we desire that

$$f(x_1, x_2 | x_3, x_4) = \frac{f(x_1, x_2, x_3, x_4)}{f(x_3, x_4)} \quad . \tag{26}$$

This conditional distribution is bivariate normal. After computing the two conditional means, the two conditional variances, and the conditional (partial) correlation coefficient from the required 14 parameters describing the quadravariate normal distribution, the joint conditional probability distribution function is computed using these conditional parameters as inputs to the same equations used for the bivariate normal probability distribution function of Section II. B. Although the extended algebraic expressions, even for this simple case, for the conditional multivariate normal distribution parameters become very complex, the evaluations are amenable to numerical computation by a computer. For the wind analysis it is necessary to compute the bivariate normal conditional wind shear parameters given a wind vector at a reference altitude; a computer program has been devised for this purpose. This computer program together with an example is presented in the appendix.

2. The Rayleigh Distribution.

a. The Univariate Rayleigh Distribution. A property of the bivariate normal distribution that has many applications is that the distribution of the variable R, defined by

$$R = \sqrt{X^2 + Y^2} \quad , \tag{27}$$

can be derived.

The probability distribution of R will be referred to as the Rayleigh distribution. The variable R, in our applications, is recognized as wind speed, the modulus of the vector wind shear, and the modulus of the vector wind change with respect to time. The Rayleigh distribution giving special cases, moments, and applications was presented by Smith [4]. In the derivation of the probability density function, f(R), it is convenient to consider the bivariate normal variables as independent, make the change of variables to polar coordinates, and integrate over the angular variable. The required integration is a mathematical problem. Wiel [5] expressed the results in a single summation involving the products of the modified Bessel function of the first kind.

$$f(R) = \alpha_0 R e^{-\alpha_1 R^2} \Big[I_0(\alpha_2 R^2) I_0(\alpha_3 R) + 2 \sum_{k=1}^{\infty} I_k(\alpha_2 R^2)\, I_{2k}(\alpha_3 R) \cos 2k\psi \Big] \quad R \geqq 0 \quad , \tag{28a}$$

where

$$\alpha_0 = \frac{1}{\sigma_x \sigma_y} \exp\left[-\frac{1}{2}\left\{\frac{\bar{X}^2}{\sigma_x^2} + \frac{\bar{Y}^2}{\sigma_y^2}\right\}\right] ,$$

$$\alpha_1 = \frac{\sigma_x^2 + \sigma_y^2}{4\sigma_x^2 \sigma_y^2} ,$$

$$\alpha_2 = \left|\frac{\sigma_x^2 - \sigma_y^2}{4\sigma_x^2 \sigma_y^2}\right| ,$$

$$\alpha_3 = \left[\left(\frac{\bar{X}}{\sigma_x^2}\right)^2 + \left(\frac{\bar{Y}}{\sigma_y^2}\right)^2\right]^{1/2}$$

and

$$\tan \psi = \frac{\bar{Y}\sigma_x^2}{\bar{X}\sigma_y^2} .$$

The functions, $I_0(\cdot)$, $I_k(\cdot)$, and $I_{2k}(\cdot)$ are the modified Bessel function of the first kind for zero order, kth order, and 2kth order.

In 1967 Yadavalli [6] extended Weil's work to include the condition for correlated variables. For correlated variables the α coefficients in equation (28a) are replaced by the following "a" coefficients:

$$a_0 = \exp\left[-\frac{1}{2}\left\{\frac{\bar{X}^2}{\sigma_a^2} + \frac{\bar{Y}^2}{\sigma_b^2}\right\}\right] \Big/ \sigma_a \sigma_b ,$$

where σ_a^2 and σ_b^2 are the rotated variances to produce zero correlation between X and Y. σ_a and σ_b are the positive and negative roots[2] of the expression

$$\sigma^2_{(+,-)} = \frac{1}{2}\left\{\sigma_x^2 + \sigma_y^2 \pm \left[(\sigma_x^2 + \sigma_y^2)^2 - 4\sigma_x^2\sigma_y^2(1-\rho^2)\right]^{1/2}\right\} \quad ,$$

$$a_1 = (\sigma_x^2 + \sigma_y^2)/4(1-\rho^2)\sigma_x^2\sigma_y^2 \quad ,$$

$$a_2 = \frac{\left[(\sigma_x^2 - \sigma_y^2)^2 + 4\rho^2\sigma_x^2\sigma_y^2\right]^{1/2}}{4(1-\rho^2)\sigma_x^2\sigma_y^2} \quad , \qquad (28b)$$

$$a_3 = \left[\left(\frac{\bar{X}}{\sigma_a^2}\right)^2 + \left(\frac{\bar{Y}}{\sigma_b^2}\right)^2\right]^{1/2} \quad ,$$

and

$$\tan\psi = \frac{\bar{Y}}{\bar{X}}\,\frac{\sigma_a^2}{\sigma_b^2} \quad .$$

Since this density function cannot be integrated in closed form from zero to R, numerical integration is used to obtain practical results for the probability distribution function; i.e.,

---

2. This computational form is obtained from the determinant

$$\begin{vmatrix} \sigma_x^2 - K & \sigma_x\sigma_y\rho \\ \sigma_x\sigma_y\rho & \sigma_y^2 - K \end{vmatrix} \quad ,$$

where K is $\sigma^2_{(+,-)}$, and σ_a and σ_b are analogous to the standard deviation of the major and minor axes of the bivariate normal probability ellipse.

$$F(R) = \int_0^{R^*} f(R)\, dR \quad .$$

A number of special cases can be obtained from the general Rayleigh distribution [equation (28)], the most simple of which is to let $\sigma_x \equiv \sigma_y = \sigma$ and $\overline{X} = \overline{Y} = 0$ with independent variables X and Y. This gives

$$f(R) = \frac{R}{\sigma^2} e^{R^2/2\sigma^2} \qquad R \geqq 0 \quad , \tag{29}$$

which is recognized as the classical Rayleigh probability density function. This and other special cases are discussed by Smith [4] and more recently by White [7], who gives moments for some special Rayleigh functions. The density function, equation (29), can be integrated in closed form over any range of the variable R. Hence, the Rayleigh probability distribution function, $F(R)$, is

$$F(R) = 1 - e^{-\frac{R^2}{2\sigma^2}} \quad . \tag{30}$$

b. Bivariate Rayleigh Distribution. The mathematics for the generalized bivariate Rayleigh probability density function become very complex. Following the work of Miller et al. [8] for the special case where the variables X_1 and X_2 are bivariate normally distributed and independent with $\sigma_{x_1} = \sigma_{x_2} = \sigma_x$, and for the other bivariate normally distributed variables Y_1 and Y_2, which are independent with $\sigma_{y_1} = \sigma_{y_2} = \sigma_y$, and with the identical correlation $\rho(x_1, y_1) \equiv \rho(x_2, y_2) = \rho$, a solution for $f(R_x, R_y)$ has been obtained:

$$R_x = \sqrt{X_1^2 + X_2^2}$$

and

$$R_y = \sqrt{Y_1^2 + Y_2^2} \quad .$$

$f(R_x, R_y)$ will be recognized as the joint distribution of the modulus of the vector wind shear and wind speed when the special conditions are satisfied.

The following expression for the bivariate Rayleigh probability density function was obtained:

$$f(R_x, R_y) = \frac{R_x R_y e^{-s/2}}{(\det M)} \Bigg[I_0(\omega_1 R_x) I_0(\omega_2 R_y) I_0(R_x C_{12} R_y) + \sum_{k=1}^{\infty} (-1)^s I_k(\omega_1 R_x) I_k(\omega_2 R_y) I_k(R_x C_{12} R_y) \cos(dk) \Bigg] , \quad (31)$$

where R_x and R_y are the variables greater than or equal to zero. For applications for wind speed and vector wind shear, the sample input parameters are $\bar{u}$, s_u, $\bar{v}$, s_v, $\bar{u}'$, $s_{u'}$, $\bar{v}'$, $s_{v'}$, and $r(u,u')$, $r(v,v')$, where the primes refer to wind component shears and the natural letters indicate wind components at a reference height, H_o. The computations required are:

(1) $\sigma_x^2 = \frac{1}{2}[s_u^2 + s_v^2]$, $\sigma_y^2 = \frac{1}{2}[s_{u'}^2 + s_{v'}^2]$, $\rho = \frac{1}{2}[r(u,u') + r(v,v')]$

$a_1 \equiv \bar{u}$, $a_2 \equiv \bar{v}$, $b_1 \equiv \bar{u}'$, $b_2 \equiv \bar{v}'$.

(2) $(\det M) = \sigma_x^2 \sigma_y^2 (1 - \rho^2)$.

(3) $$S = \frac{1}{\sigma_x^2(1-\rho^2)}[R_x^2 + a_1^2 + a_2^2] + \frac{1}{\sigma_y^2(1-\rho^2)}[R_y^2 + b_1^2 + b_2^2] - \frac{\rho(a_1 b_1 + a_2 b_2)}{\sigma_x \sigma_y (1-\rho^2)} .$$

(4) $$\omega_1 = [\sigma_y^2(a_1^2 + a_2^2) - 2\rho\sigma_x\sigma_y(a_1 b_1 + a_2 b_2) + \rho^2\sigma_x^2(b_1^2 + b_2^2)]^{1/2} \Big/ \sigma_x^2\sigma_y^2(1-\rho^2) .$$

(5) $$\omega_2 = [\sigma_x^2(b_1^2 + b_2^2) - 2\rho\sigma_x\sigma_y(a_2b_1 + a_2b_2) + \rho^2\sigma_y^2(a_1^2 + a_2^2)]^{1/2} \Big/ \sigma_y^2\sigma_x^2(1 - \rho^2) \quad .$$

(6) $$C_{12} = \left\| \frac{\rho(a_1b_1 + a_2b_2)}{\sigma_x\sigma_y(1 - \rho^2)} \right\|$$

If ρ is negative $(-1)^s = (-1)^k$; if ρ is positive $(-1)^s = (-1)^{2k} = 1$. The double bars, $\|\ldots\|$, indicate the absolute value of the evaluation.

(7) $$\alpha = \tan^{-1} \frac{a_2\sigma_y - \rho b_2\sigma_x}{a_1\sigma_y - \rho b_1\sigma_x} \quad .$$

$$\alpha = (\theta_1 - \theta_2) \quad ,$$

where

$$\theta_1 = \tan^{-1}\left[\frac{b_2\sigma_x - \rho\, a_2\sigma_y}{b_1\sigma_x - \rho\, a_1\sigma_y}\right]$$

and

$$\theta_2 = \tan^{-1}\left[\frac{a_2\sigma_y - \rho\, b_2\sigma_x}{a_1\sigma_y - \rho\, b_1\sigma_x}\right] \quad .$$

As can be seen, the evaluation of equation (31) is no simple task, and only a few sample results have been obtained. However, these results have given considerable insight into the nature of the relationship between wind speed and the modulus of the vector wind shear. Of greatest interest was a comparison of the derived conditional probability of the wind shear given the wind speed; i.e.,

$$\Pr\{R_x \leq R_x^* | R_y = R_y^*\} \quad ,$$

which was obtained by numerical integration of

$$f(R_x, R_y) = \frac{f(R_x, R_y)}{f(R_y)} \quad .$$

The sample results compare favorably with the corresponding empirical conditional wind shears given a wind speed. Because of the complexity of equation (31) and the necessary assumptions, no general application is expected for practical problems involving the relationship of two Rayleigh distributed variables, but the few sample results that have been obtained and the theoretical inferences have given some insight into the properties of two Rayleigh distributed variables which also illustrate a property of a multivariate (four) normal distribution function.

c. Distribution of Wind Directions and Conditional Distribution of Speed Given a Wind Direction. At times there is an interest in the frequency of wind directions and the probability of wind speed for a given wind direction. The procedure to derive these two probability density functions is to first change the variables for the bivariate normal density function to polar coordinates; this gives

$$g(r, \theta) = r d_1 e^{-\frac{1}{2}(a^2 r^2 - 2br + c^2)} \quad ,$$

where

$$a^2 = \frac{1}{(1 - \rho^2)} \left[\frac{\cos^2 \theta}{\sigma_x^2} - \frac{2\rho \cos \theta \sin \theta}{\sigma_x \sigma_y} + \frac{\sin^2 \theta}{\sigma_y^2} \right] \quad ,$$

$$b = \frac{1}{(1 - \rho^2)} \left[\frac{\bar{x} \cos \theta}{\sigma_x^2} - \frac{\rho(\bar{x} \sin \theta + \bar{y} \cos \theta)}{\sigma_x \sigma_y} + \frac{\bar{y} \sin \theta}{\sigma_y^2} \right] \quad ,$$

$$c^2 = \frac{1}{(1-\rho^2)} \left[\frac{\overline{x}^2}{\sigma_x^2} - \frac{2\rho\overline{x}\overline{y}}{\sigma_x \sigma_y} + \frac{\overline{y}^2}{\sigma_y^2} \right] \quad ,$$

$$d_1 = \frac{1}{2\pi\sigma_x \sigma_y \sqrt{1-\rho^2}} \quad ,$$

and $r = \sqrt{x^2 + y^2}$ is the modulus of the vector or speed and θ is the direction of the vector. After integrating $g(r, \theta)$ over $r = 0$ to ∞, we arrive at the probability density function of θ,

$$g(\theta) = \frac{d_1}{a^2} e^{-\frac{1}{2}c^2} \left[1 + \sqrt{2\pi} \left(\frac{b}{a}\right) e^{\frac{1}{2}\left(\frac{b}{a}\right)^2} \Phi\left(\frac{b}{a}\right) \right] \quad , \qquad (33)$$

where a^2, b, c^2, and d_1 are as previously defined in equation (31) and

$$\Phi\left(\frac{b}{a}\right) \cong \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$

is taken from tables of normal distribution functions or made available through a computer subroutine.

Example computations of $g(\theta)$ using sample parameters for wind statistics agree closely with the empirical frequency of wind directions. If desired, equation (33) can be integrated numerically over a chosen range of θ to obtain the probability that the vector direction will lie within the chosen range; i.e.,

$$F(\theta) = \int_{\theta_1}^{\theta_2} g(\theta)\, d\theta \quad . \qquad (34)$$

One application may be to obtain the probability that the wind will blow from a given quadrant or sector as, for example, on shore.

The conditional probability distribution function of r given θ has a special interest in the subsequent wind analysis. This function is expressed as

$$\Pr\{r \leq r^* | \theta = \theta_0\} = \frac{\int_{r=0}^{r^*} g(r,\theta)\, dr}{\int_{r=0}^{\infty} g(r,\theta_0)\, dr} \,, \tag{35}$$

which is equal to

$$\frac{g(r^*,\theta)}{g_1(\infty,\theta)} \,.$$

After carrying out the indicated processes, we arrive at

$$\Pr\{r \leq r^* | \theta = \theta_0\} = 1 - \frac{\frac{d_1}{a^2}\, e^{-\frac{1}{2}c^2}\, e^{\frac{1}{2}\left(\frac{b}{a}\right)^2} \left[e^{-\frac{1}{2}r_s^{\,2}} + \sqrt{2\pi}\left(\frac{b}{a}\right)\{1 - \Phi(r_s)\} \right]}{g(\theta)} \,, \tag{36}$$

where $r_s = (ar^* - b/a)$ and all other coefficients and $g(\theta)$ are as previously defined. For the special case when $\bar{x} = \bar{y} = 0$, equation (33) reduces to the following simple case:

$$\Pr\{r \leq r^* | \theta = \theta_0\} = 1 - e^{-\frac{r^{*2}}{2a^2}} \,. \tag{37}$$

There is a special significance of equation (37) when related to the bivariate normal probability distribution. If r^* and θ are measured from the centroid of the probability ellipse, then the probability that $r \leq r^*$ is the same as the given probability ellipse. Further, by solving equation (37) for r^*, we have

$$r^* = a\sqrt{-2 \ln (1 - P)} \quad . \tag{38}$$

If a probability ellipse P is chosen, equation (37) gives the distance of r along any θ from the centroid of the ellipse to the intercept of the specified probability ellipse. If there is an interest in conditional probability of winds for a given θ relative to the monthly means, equation (38) is applicable. If it is desired to find the magnitude of the wind along any θ relative to the monthly mean to the intercept of a given probability ellipse, equation (38) is applicable.

3. Sum and Differences. A property of the bivariate normal distribution used in the wind analysis is that the difference of two random bivariate normally distributed variables is normally distributed. Since the sum of two such variables is derived by Gnedenko [3], we will state this result first and then give the distribution of the differences of two normal variables for the same condition.

The probability density function for the sum of two variables $\zeta = (X+Y)$ for these stated conditions is normally distributed with

$$\text{mean,} \quad \bar{\zeta} = (\bar{X} + \bar{Y}) \tag{39}$$

and

$$\text{variance,} \quad \sigma_\zeta^2 = \sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2 \quad . \tag{40}$$

This gives the p.d.f. for ζ as

$$f(\zeta) = \frac{1}{\sqrt{2\pi\sigma_\zeta^2}} e^{-\frac{(\zeta-\bar{\zeta})^2}{2\sigma_\zeta^2}} \quad . \tag{41}$$

The probability density function for the difference of two variables $\eta = (X-Y)$ for the stated conditions is normally distributed with

$$\text{mean,} \quad \bar{\eta} = (\bar{X} - \bar{Y}) \tag{42}$$

and

$$\text{variance,} \quad \sigma_\eta^2 = \sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2 \quad . \tag{43}$$

The p.d.f. for the difference, η, is

$$f(\eta) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\frac{(\eta-\bar{\eta})^2}{2\sigma_\eta^2}} \quad . \tag{44}$$

The probability distribution functions for $f(\eta)$ and $f(\xi)$ are treated as any univariate normal distribution function.

4. Rotation of Coordinates for a Bivariate Normal Distribution. An expression for the rotation of variances to produce zero correlation between bivariate normal variables was presented in equation (28); namely,

$$\sigma_a^2 = \frac{1}{2}\left\{\sigma_x^2 + \sigma_y^2 + \left[\left(\sigma_x^2 + \sigma_y^2\right)^2 - 4\sigma_x^2\sigma_y^2(1-\rho^2)\right]^{1/2}\right\}$$

$$\sigma_b^2 = \frac{1}{2}\left\{\sigma_x^2 + \sigma_y^2 - \left[\left(\sigma_x^2 + \sigma_y^2\right)^2 - 4\sigma_x^2\sigma_y^2(1-\rho^2)\right]^{1/2}\right\} \quad .$$

This is the usual rotation of variances found in texts on statistics and probability. $\lambda_e\sigma_a$ and $\lambda_e\sigma_b$ give the major and minor axes for the probability ellipses $\left[\lambda_e = \sqrt{2}\sqrt{-\ln(1-p)}\right]$.

Our interest is to express the statistical parameters $\bar{X}$, $\bar{Y}$, σ_x, σ_y, and ρ for the rotation of the orthogonal axes through any arbitrary angle α. This interest is motivated from an application of wind component statistics with respect to any flight azimuth of an aerospace vehicle. Falls and Crutcher [9] derived the necessary expressions for this operation. Because of the important applications their expressions are repeated here in our notation.

a. Rotation of the means through α degrees:

$$\bar{X}_\alpha = \bar{X} \cos (90 - \alpha) + \bar{Y} \sin (90 - \alpha) \tag{45}$$

$$\bar{Y}_\alpha = \bar{Y} \cos (90 - \alpha) - \bar{X} \sin (90 - \alpha) \quad . \tag{46}$$

b. Rotation of the variances through α degrees:

$$\sigma^2_{x_\alpha} = \sigma_x^{\,2} \cos^2 (90 - \alpha) + \sigma_y^{\,2} \sin^2 (90 - \alpha) + 2\rho\sigma_x\sigma_y \cos (90 - \alpha) \sin (90 - \alpha) \tag{47}$$

$$\sigma^2_{y_\alpha} = \sigma_y^{\,2} \cos^2 (90 - \alpha) + \sigma_x^{\,2} \sin^2 (90 - \alpha) - 2\rho\sigma_x\sigma_y \cos (90 - \alpha) \sin (90 - \alpha) \quad . \tag{48}$$

c. Rotation of the linear correlation coefficient through α degrees:

$$\rho_\alpha = \frac{\mathrm{cov}\,(X, Y)_\alpha}{\sigma_{x_\alpha} \sigma_{y_\alpha}} \quad , \tag{49}$$

where $\operatorname{cov}(X, Y)_\alpha$ is the rotated covariance,

$$\operatorname{cov}(X,Y)_\alpha = \operatorname{cov}(X,Y)\,[\cos^2(90-\alpha) - \sin^2(90-\alpha)$$

$$+ \cos(90-\alpha)\sin(90-\alpha)(\sigma_y^{\,2} - \sigma_x^{\,2})]$$

and

$$\operatorname{cov}(X,Y) = \rho\sigma_x\sigma_y \quad .$$

By using these rotational equations, the bivariate normal distribution with respect to any desired rotated coordinates can be obtained from sample estimates that have been computed with respect to a specific axis. The marginal distributions after rotation are also normally (univariate) distributed. By using the rotational equations, computational efforts are greatly reduced for applications requiring statistics with respect to several coordinate axes.

III. WIND ANALYSIS

Using the general probability functions presented in the previous section, a few examples are presented to illustrate the properties of the multivariate normal probability distribution as applied to upper wind data samples. In this section it is necessary to make the distinction between the theoretical parameters of the multivariate normal probability distribution function and sample estimates for the corresponding theoretical parameters.

A. Data Sample

The wind data samples for Cape Kennedy, Florida, and Vandenberg Air Force Base, California, from 0 to 27 km altitude are the same as used by Falls [10,11]. For Cape Kennedy the sample consists of 12 years of measurements taken from January 1, 1956, to December 31, 1967. The wind data are serially complete rawinsonde observations taken twice daily following the procedure described by Vaughan et al. [12]. Wind direction and speed are tabulated at 1 km intervals for the 28 altitudes, 0 to 27 km. The serially complete, twice daily wind records for Vandenberg Air Force Base consist of 8 years of measurements taken from January 1, 1965, to December 31, 1972, and tabulated in the same manner as for Cape Kennedy.

Using the meteorological coordinate system (Fig. 1) the wind components for each altitude for each monthly reference period are computed. A grouping of all like months for the period of record is called the monthly reference period.

The five parameters for the bivariate normal probability distribution function for the zonal and meridional wind components are computed by standard statistical methods. These five parameters are designated in the tables of Volumes II and III of this report as $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u,v)$, i.e., the means and standard deviations of the zonal and meridional wind components and the correlation coefficient between the two components. These parameters are entered in the tables under the headings UBAR, SDU, R(U,V), VBAR, and SDV on the line designated as H_o, for reference altitude. These entries are identical to those tabulated by Falls in References 10 and 11.

The main body of the tables contains the remaining nine parameters for the quadravariate normal distribution for wind components at reference altitude H_o and the component shears between H_o and all altitudes H from 0 to 27 km. The component shear parameters were computed from sample component shears in the following special manner:

$$\left.\begin{aligned} U' &= \left(U_{H_o} - U_H\right) , \quad \text{if } H_o > H \\ U' &= \left(U_H - U_{H_o}\right) , \quad \text{if } H_o < H \end{aligned}\right\} \qquad (50)$$

and

$$\begin{aligned} V' &= \left(V_{H_o} - V_H\right) , \quad \text{if } H_o > H \\ V' &= \left(V_H - V_{H_o}\right) , \quad \text{if } H_o < H \end{aligned} \qquad (51)$$

Note this change of order for the sign of U' and V' as H cross H_o. This is important later in computing the vector wind profile model.

We now have the sample variables U_i, V_i, U_i', and V_i' from which the parameters $\bar{u}$, $s_{u'}$, $r(u,u')$, $\bar{v}$, $s_{v'}$, $r(v,v')$, $r(u',v')$, $r(u',v)$, and $r(v',u)$ are computed. These parameters are tabulated for each reference altitude ($H_o = 0, 1, 2, \ldots, 27$) versus altitude H = 0 to 27 km. $H_o - H$ is the shear interval. In the tabulations these nine parameters are designated as UPBAR, SD(VP), R(U, UP), VPBAR, SD(UP), R(V, VP), R(UP, VP), R(UP, V), and R(VP, U) in computer printer characters.

B. Probability Distribution of Wind Components

From the principles of the univariate normal probability distribution (Section II. A), the normal probability distribution function for wind components can be computed using the parameters $\bar{u}$ and s_u for the zonal wind component and $\bar{v}$ and s_v for the meridional component. A result comparing the normal distributions for zonal and meridional wind components with empirical percentiles is illustrated for the March winds at 12 km altitude over Cape Kennedy (Fig. 2).

By knowing the linear correlation coefficient between the zonal and meridional wind components and the means and standard deviations of the zonal and meridional wind components, these parameters with respect to any coordinate axes can be computed using equations (45) through (49). The percentiles and interpercentile range of wind components with respect to any coordinate rotation can be computed from the new parameters. This is the procedure followed by Falls [10, 11] in preparing the tables for his reports.

C. Probability Distribution of Wind Vectors

As a vector quantity, the wind components for a reference period are bivariate normally distributed. That is, the joint distribution of wind components is bivariate normally distributed. Using the five parameters at H_o for $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u,v)$, the probability wind ellipses can be computed following the procedure given in Section II. B. An example for March winds at 12 km over Cape Kennedy is shown in Figure 3. In this figure for example, 99 percent of the wind vectors emanating from the origin of the south-north, west-east coordinate axes have their extremities within the 99 percent probability ellipse.

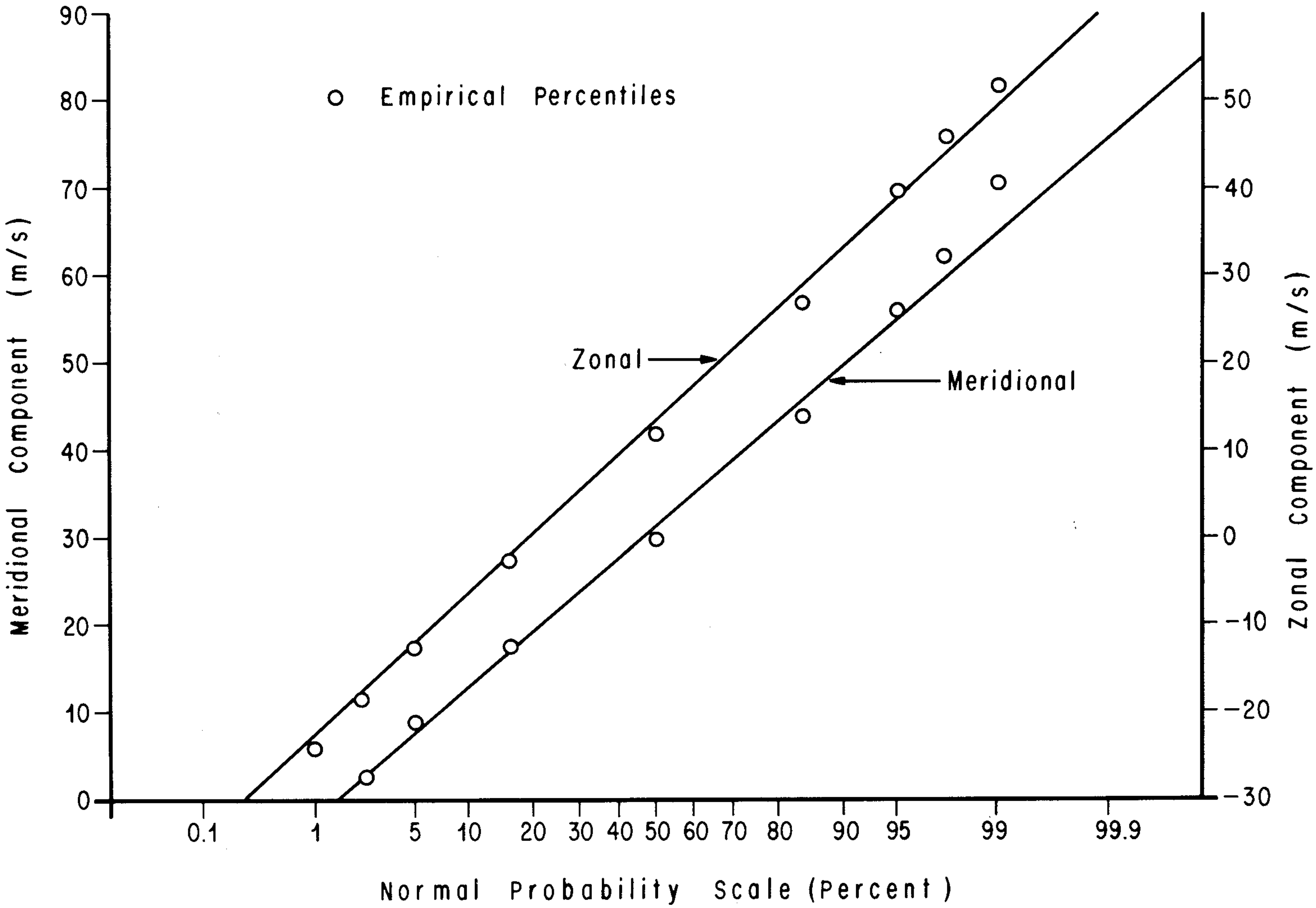


Figure 2. Normal (univariate) probability distribution of zonal and meridional wind components at 12 km altitude, March, Cape Kennedy, Florida.

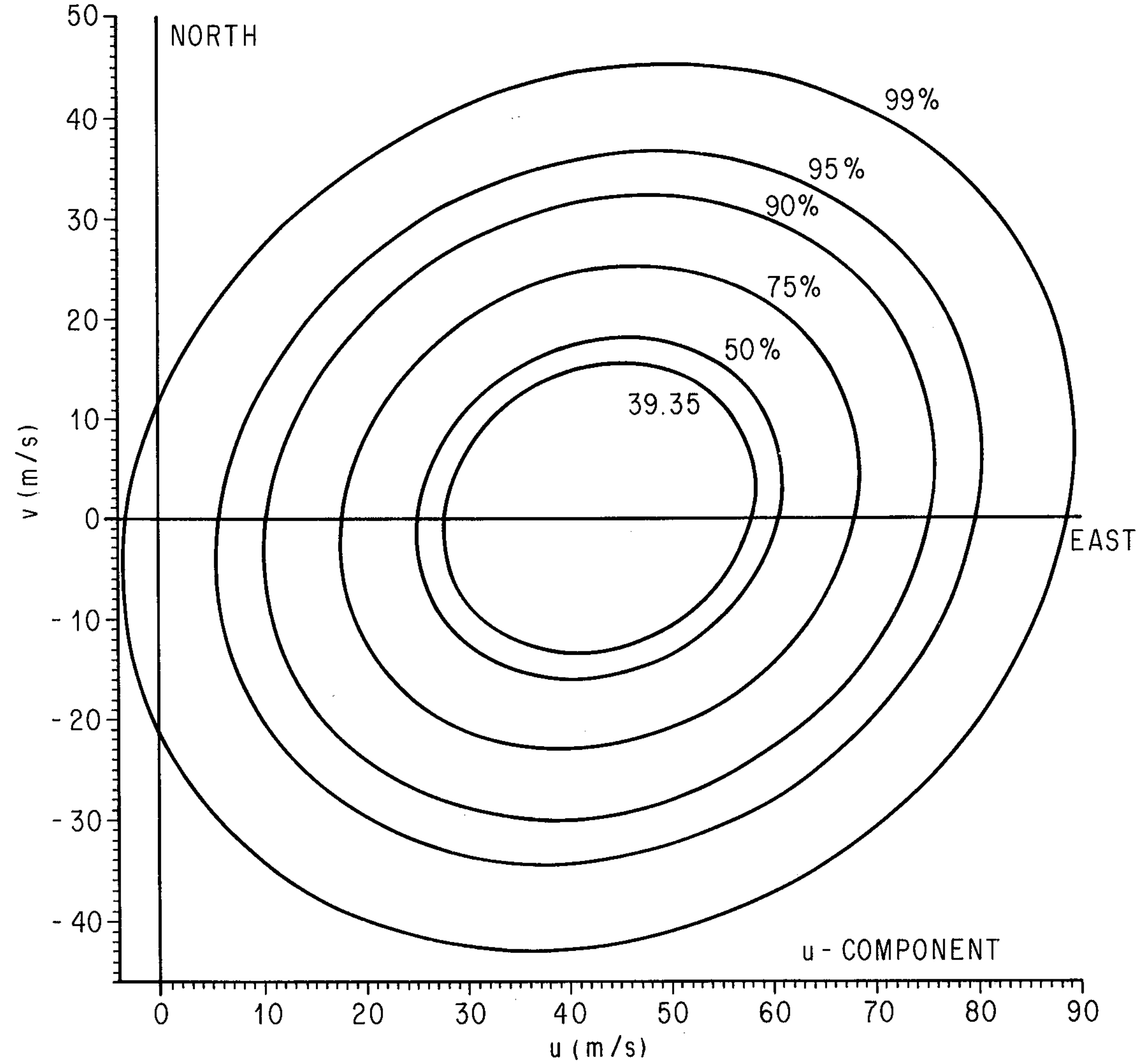


Figure 3. Bivariate normal distribution of zonal and meridional wind components at 12 km altitude, March, Cape Kennedy, Florida.

An interesting comparison between the 90th interpercentile range of wind components versus all azimuths (computed as outlined in Sections II. A and II. D) and the 95 percent wind vector ellipse (Section II. B) for Cape Kennedy for February at 12 km altitude is shown in Figure 4. A similar comparison for Vandenberg winds for December at 12 km altitude is shown in Figure 5. The graphical areas in common with the 90th interpercentile range of wind components versus all azimuths and the joint probability of wind components described by the 95 percent probability ellipse are functions of the vector mean wind, the standard deviations, and the correlation between the two components. The interpercentile range of a wind component is obtained from the integration of the univariate normal probability density function for a particular probability area. The probability ellipse is an area that contains a certain percent of the wind vectors derived from the integration of the bivariate normal probability density function. Figures 4 and 5 represent a comparison of wind components versus all azimuths and wind vectors for which there is no common basis except the same five sample parameters $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u, v)$ are used in preparing these illustrations.

D. Conditional Probability of Wind Components

In Section II. D. 1 we gave the general expression for the conditional distribution for the bivariate normal distribution. An example for Cape Kennedy winds at 12 km for March of the conditional probability of the meridional wind component given the zonal wind component is presented in Figure 6. These conditional meridional wind components are illustrated by dashed lines at the conditional mean designated as E0.00 and at the conditional means ± 1, ± 2, and ± 2.45 conditional standard deviations given that the zonal wind component is equal to the $\bar{u} \pm 1$, ± 2, and ± 2.45 illustrated by the vertical lines. Selected percentile values and interpercentile ranges of the meridional wind component are obtained given that the zonal wind is equal to the mean ± 1, ± 2, and ± 2.45 standard deviations. The wind vector probability ellipses are shown for comparison. The same procedures were used [4] for the conditional component shears given a wind component at the reference altitude, H_o, to develop a synthetic component profile.

E. Probability of Wind Speeds and Modules of Vector Wind Shears

By considering the winds as bivariate normally distributed in the U and V components, the wind speed (i.e., the modulus of the wind vectors) is a Rayleigh distributed variable. By defining the wind speed as

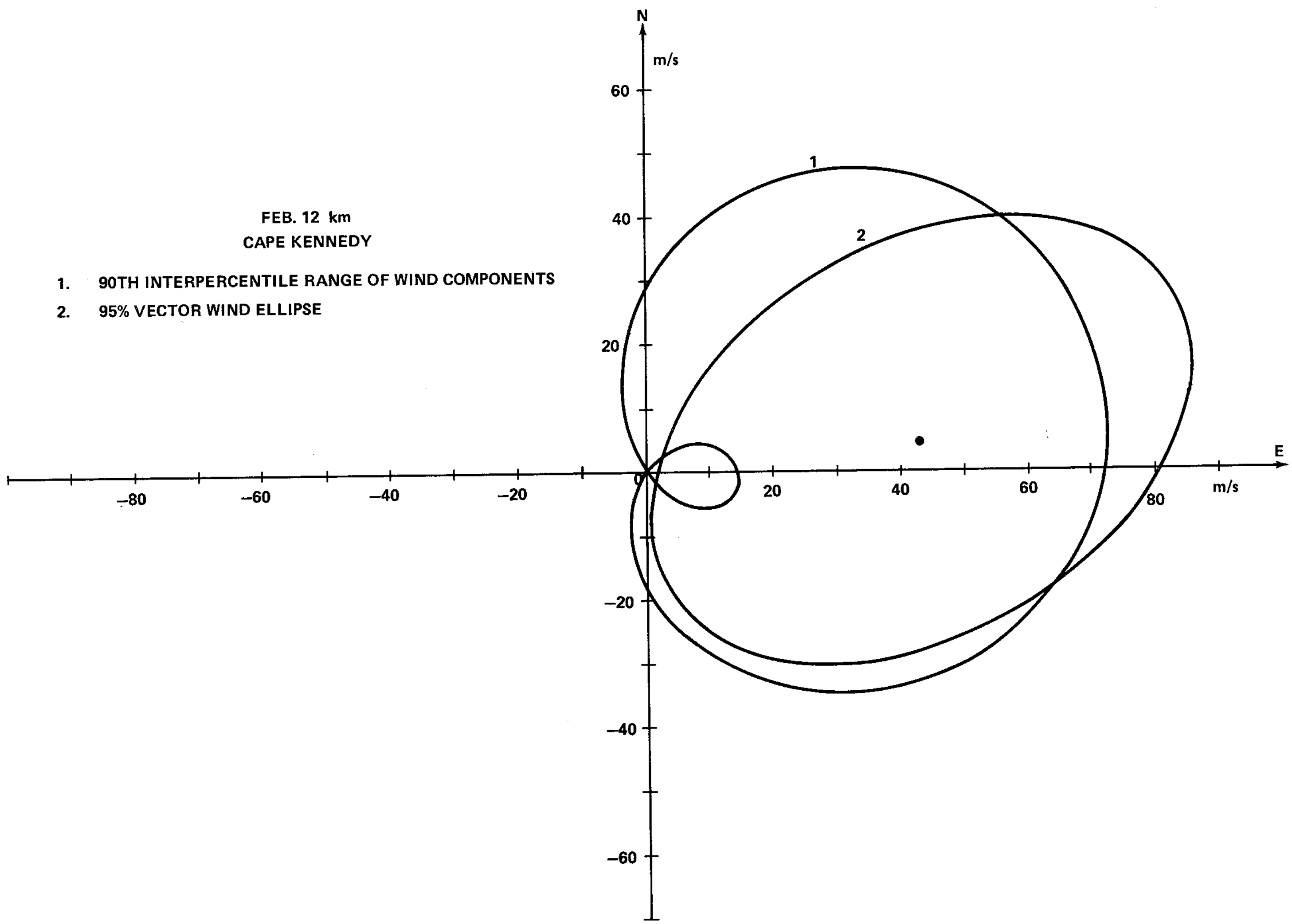


Figure 4. The 90th interpercentile range of wind components versus all azimuths and the 95 percent vector wind ellipse at 12 km altitude, February, Cape Kennedy, Florida.

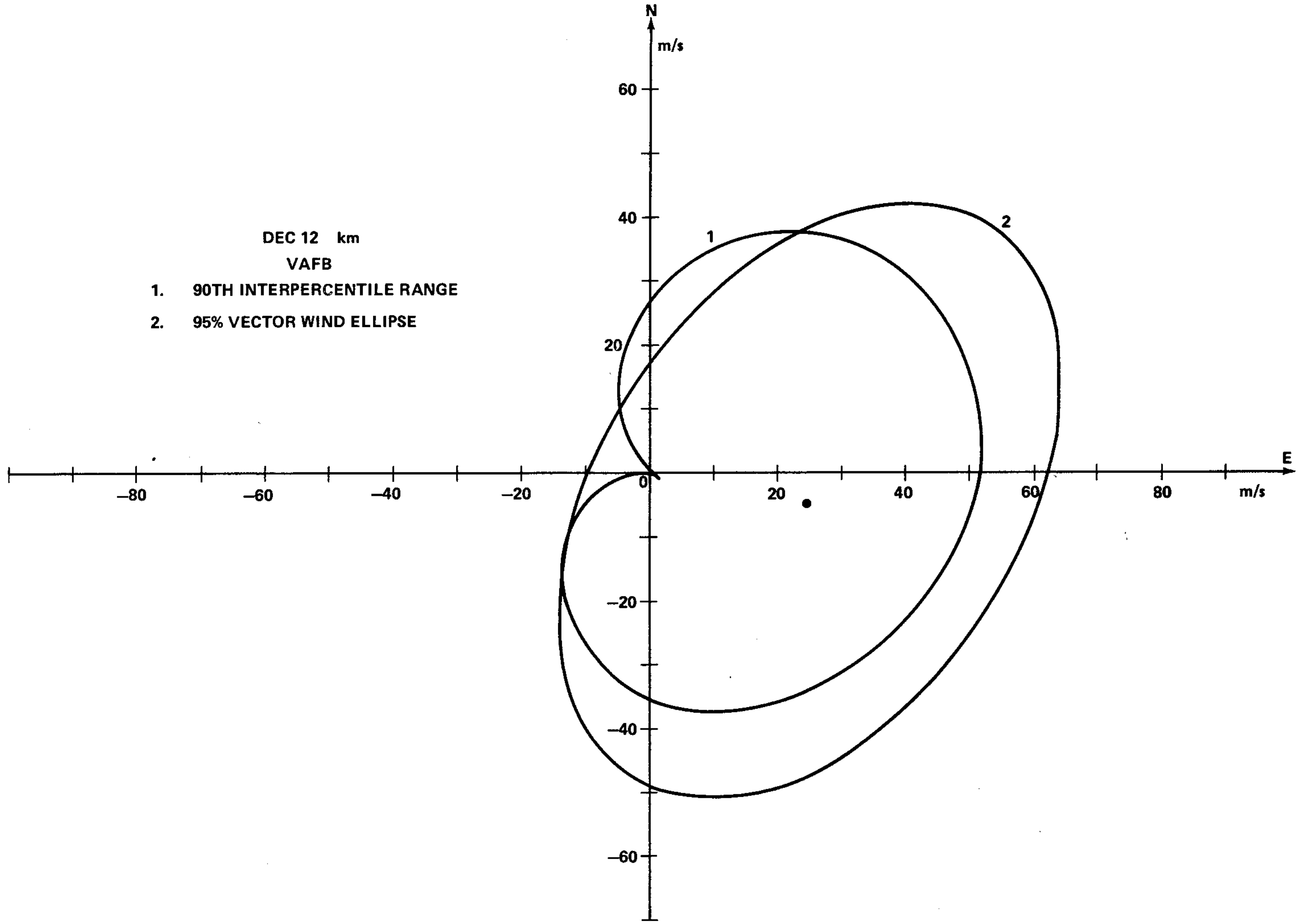


Figure 5. The 90th interpercentile range of wind components versus all azimuths and the 95 percent vector wind ellipse at 12 km altitude, December, Vandenberg Air Force Base, California.

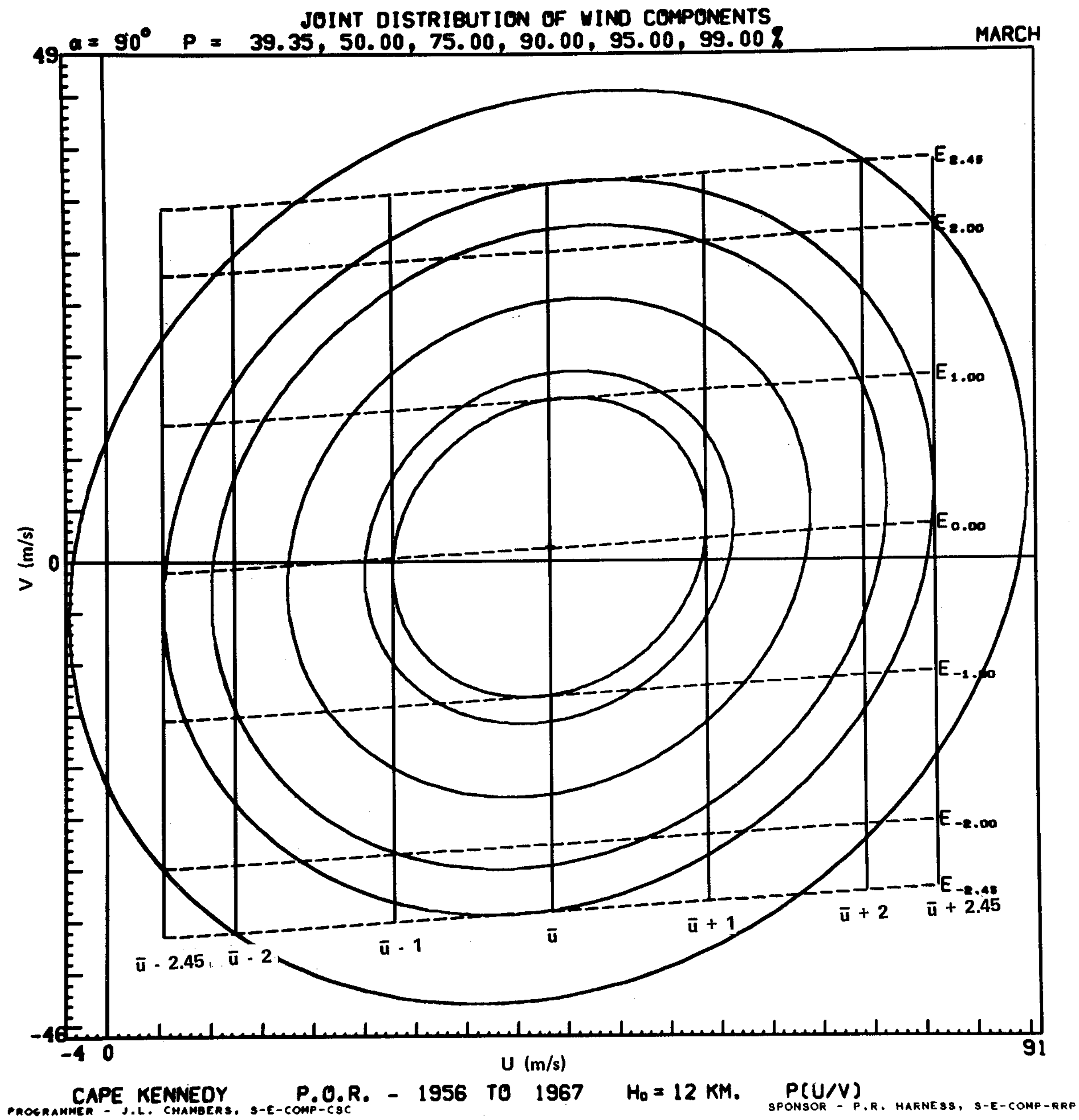


Figure 6. Conditional probability of the meridional wind component given the zonal wind component at 12 km altitude, March, Cape Kennedy, Florida.

$$W = \sqrt{U^2 + V^2} \quad ,$$

we have results for the generalized Rayleigh distribution computed by equation (28b), where the sample parameters are $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u, v)$. An example for the March wind speed over Cape Kennedy at 4, 8, and 12 km altitude compared with empirical percentiles is shown in Figure 7. Many other examples giving the results of percentiles for wind speeds derived from the generalized Rayleigh probability distribution all agree very closely with the corresponding empirical percentiles.

From statistical tests [2] on the bivariate normality of vector wind shears, it can be inferred that the wind vectors at two different altitudes can be considered as quadravariate normally distributed. For wind component shears U' and V' that are bivariate normally distributed, the moduli

$$W' = \sqrt{U'^2 + V'^2}$$

are Rayleigh distributed. Using equation (28b) and the sample parameters $\bar{u}'$, $\bar{v}'$, $s_{u'}$, $s_{v'}$, and $r(u', v')$, the Rayleigh distribution for W' has been derived and an example is shown in Figure 8 for March winds over Cape Kennedy with the reference altitude $H_o = 12$ km and the shear altitude $H = 0$ (surface = 10 m), 2, 4, 6, 8, and 10 km. As the shear interval $(H_o - H)$ becomes larger, the modulus of the vector wind shear increases.

F. Frequency of Wind Direction and Conditional Speed Given a Direction

1. Frequency of Wind Direction. From bivariate normal theory, the frequency of wind direction can be derived under the assumption that the winds are bivariate normally distributed. Using the five sample parameters $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u, v)$ in equations (33) and (34), the frequency percentages of wind blowing from each of 16 class intervals, N, NNE, etc., are compared with the observed (empirical) frequency. Two illustrations of this comparison are presented in Figures 9 and 10 for 4 km altitude and 27 km altitude in February over Cape Kennedy. An interesting theoretical analysis would be to determine the conditions of the statistical parameters that lead to bimodality for the vector direction. Examples comparing the theoretical (derived) frequency of wind direction with the observed frequency are shown in Table 3 for February winds of 12 km altitude over Cape Kennedy and in Table 4 for December winds at 10 km altitude over Vandenberg Air Force Base.

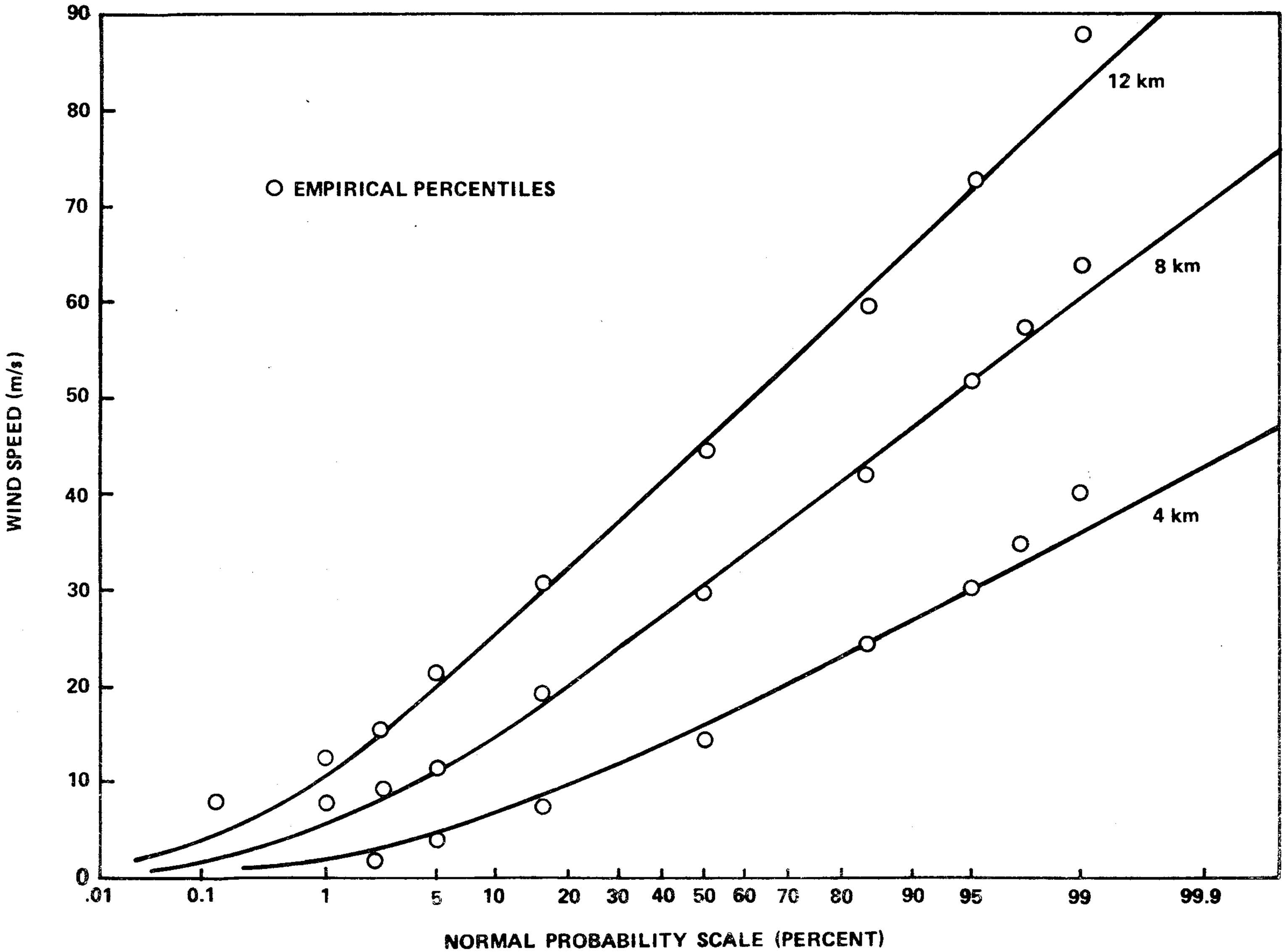


Figure 7. Generalized Rayleigh distribution of wind speed (scalar) at 4, 8, and 12 km altitude, March, Cape Kennedy, Florida.

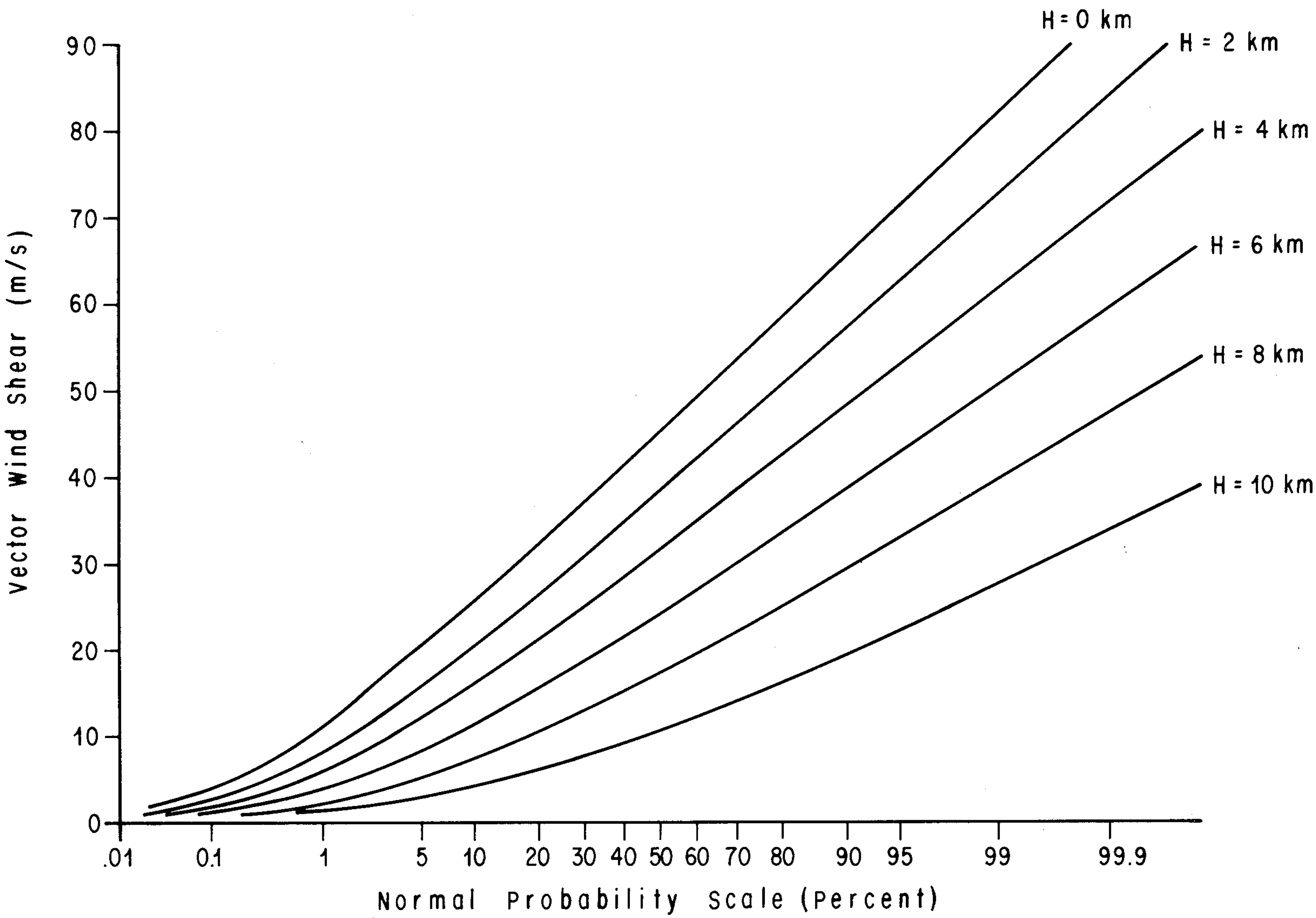


Figure 8. Generalized Rayleigh distribution of the modulus of vector wind shown for reference altitude $H_o = 12$ km and the shear altitude $H = 0, 2, 4, 6, 8,$ and 10 km, March, Cape Kennedy, Florida.

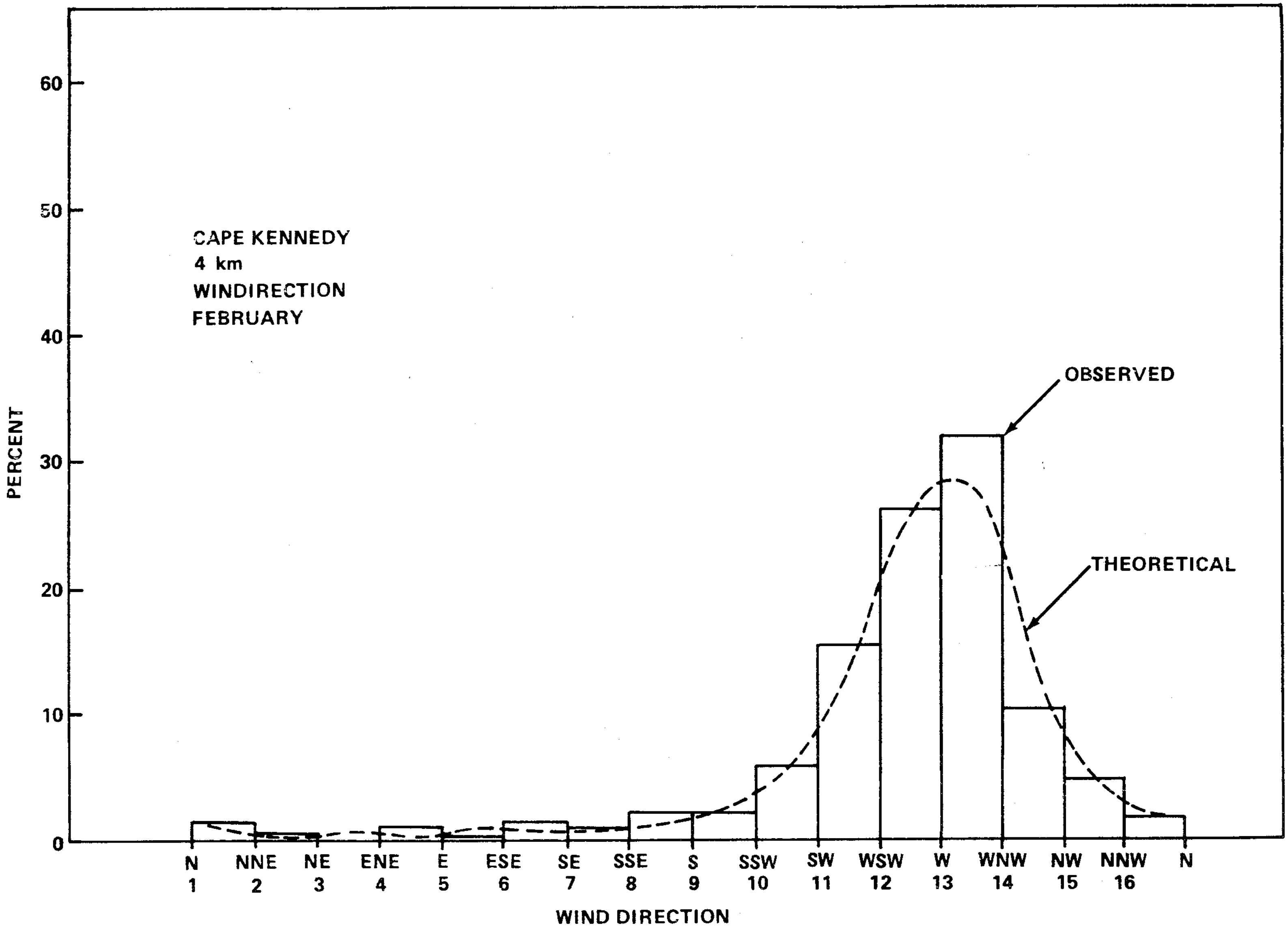


Figure 9. Comparison of theoretical (derived) wind direction frequency with observed (empirical) frequency at 4 km altitude, February, Cape Kennedy, Florida.

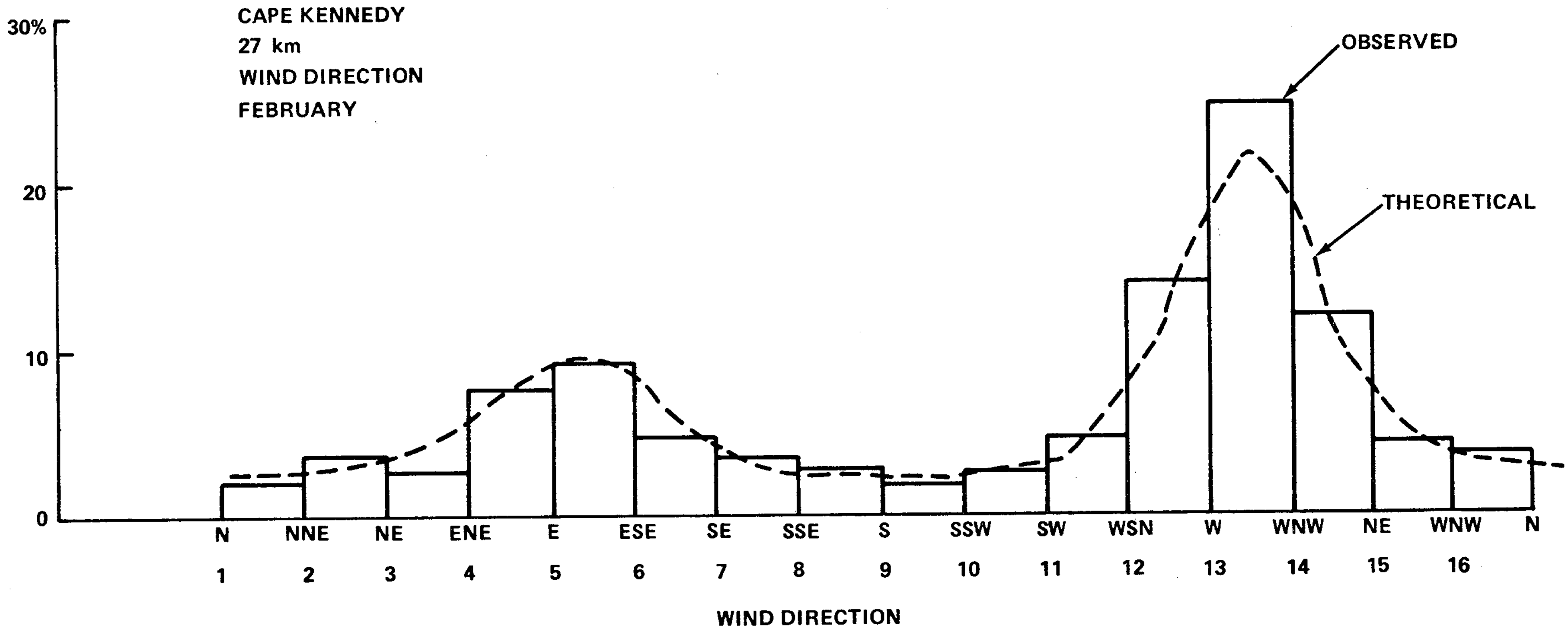


Figure 10. Comparison of theoretical (derived) wind direction frequency with observed (empirical) frequency at 27 km altitude, February, Cape Kennedy, Florida.

TABLE 3. FREQUENCY OF WIND DIRECTION AT 12 km ALTITUDE, FEBRUARY, CAPE KENNEDY, FLORIDA

Direction	$g(\theta)$, Theoretical (%)		Observed (%)	
N	0.34		0.20	
NNE	0.13		0.00	
NE	0.06		0.00	
ENE	0.03		0.10	
E	0.02		0.00	
ESE	0.02		0.10	
SE	0.02		0.10	
SSE	0.03		0.00	
S	0.08		0.00	
SSW	0.39		0.30	
SW	4.70		2.90	
WSW	34.87	= 97.87%	34.80	
W	41.13	for θ = 213.75	46.10	= 97.50%
WNW	13.70	to 326.25°	10.50	
NW	3.47		3.20	
NNW	1.00		1.60	

Note: The theoretical frequency of wind direction, $g(\theta)$, is derived from $\bar{u}$ = 43.62 m/s, $\bar{v}$ = 5.31 m/s, s_u = 16.79 m/s, s_v = 14.28 m/s, and $r(u, v)$ = 0.3510.

The period of record for, $g(\theta)$, and observed fre-frequency is from 1956 through 1967. There is excellent agreement between the derived frequency of wind direction and the observed frequency in this example.

TABLE 4. FREQUENCY OF WIND DIRECTION AT 10 km ALTITUDE, DECEMBER, VANDENBERG AIR FORCE BASE, CALIFORNIA

Direction	$g(\theta)$, Theoretical (%)		Observed (%)	
N	7.17		9.0	
NNE	3.47		3.2	
NE	1.30		2.3	
ENE	0.56		0.3	
E	0.34		0.3	
ESE	0.28		0.3	
SE	0.30		0.6	
SSE	0.42		0.0	
S	0.86	= 86.16% for θ = 168.25 to 348.75°	1.0	= 83.8%
SSW	2.63		3.2	
SW	8.83		8.7	
WSW	16.68		13.2	
W	18.01		17.7	
WNW	15.63		12.9	
NW	13.04		15.8	
NNW	10.48		11.3	

Note: From $g(\theta)$ 96.8% of the wind directions are from 168.75° through west to a direction from 33.75°. The corresponding observed frequency is 96.0%. The theoretical frequency of wind direction, $g(\theta)$, is based on the period of record from 1965 through 1972, with $\bar{u} = 23.04$ m/s, $\bar{v} = -5.73$ m/s, $s_u = 18.07$ m/s, $s_v = 21.21$ m/s, and $r(u,v) = 0.4026$. The observed frequency is from the data sample from 1965 through 1969. Considering this, the agreement between the derived frequencies of wind direction and the observed frequencies is excellent.

2. Conditional Probability of Wind Speed Given a Wind Direction. From bivariate normal theory the conditional probability distribution function of wind speed given a wind direction can be derived. Using the five sample parameters for the bivariate normal probability distribution in equation (37), the conditional probability that the wind speed will be less than or equal to a specified value, $W = W^*$, given that the wind is blowing from a direction $\theta = \theta^*$, has been computed for a few examples and is presented in Tables 5, 6, and 7. For Tables 5 and 6 the conditional probabilities for wind speed for given wind directions were held fixed for the indicated directions. For Table 7 the conditional probabilities were computed for the specified values of wind speed, $W = W^*$, where W^* is the intercept for the given wind direction to the 95 percent bivariate normal probability ellispe.

The great advantage of using a theoretical distribution function that adequately represents the data sample to compute probabilities rather than using empirical distributions is recognized. The probabilities can be derived using the moments from the total sample rather than empirical conditional probabilities computed from a subsample which is often very small. By using probability distribution functions, probability estimates for the variables can be obtained outside the observed range given by the sample.

TABLE 5. CONDITIONAL PROBABILITY OF WIND SPEED FOR $W^* \leq 75$ m/s GIVEN A WIND DIRECTION $\theta = \theta^*$ AT 12 km ALTITUDE, MARCH, CAPE KENNEDY, FLORIDA

θ^* (degrees)	$Pr\{W \leq 75 \text{ m/s} \mid \theta = \theta^*\}$, Probability (%)
0 N	99.99
45 NE	99.99
90 E	99.99
135 SE	99.99
180 S	99.99
225 SW	99.37
270 W	96.53
315 NW	99.90

TABLE 6. CONDITIONAL PROBABILITY OF WIND SPEED FOR $W^* \leq 50$ m/s GIVEN A WIND DIRECTION $\theta = \theta^*$ AT 10 km ALTITUDE, DECEMBER, VANDENBERG AIR FORCE BASE, CALIFORNIA

θ^* (degrees)	$Pr\{W \leq 50 \text{ m/s} \mid \theta = \theta^*\}$, Probability (%)
0 N	86.77
45 NE	96.89
90 E	99.97
135 SE	99.99
180 S	99.22
225 SW	76.79
270 W	85.31
315 NW	92.39

TABLE 7. CONDITIONAL PROBABILITY OF WIND SPEED FOR $W \leq W^*$ GIVEN A WIND DIRECTION $\theta = \theta^*$ AT 10 km ALTITUDE, DECEMBER, VANDENBERG AIR FORCE BASE, CALIFORNIA

θ^* (degrees)	W^* (m/s)	$Pr\{W \leq W^* \mid \theta = \theta^*\}$, Probability (%)
0	57.2	93.72
60	22.0	69.86
90	15.2	67.04
150	15.8	68.79
180	23.9	72.97
240	71.3	96.26
270	65.3	97.94
330	57.8	97.06

G. Distributions of Wind Vector Shears

The following discussion is important for a simplifying assumption in the construction of the synthetic vector wind model to be presented in Section IV. The following examples will be shown:

1. The bivariate distribution of vector wind shears.

2. The conditional bivariate distribution of vector wind shears given particular wind vectors at the reference altitude, H_o.

The first example presents the bivariate distribution of vector wind shears for March over Cape Kennedy. For this example, the reference altitude, H_o, is 12 km and the 95 percent and 99 percent vector wind shear probability ellipses, $F(u', v')$, are presented (Fig. 11). Figure 11(a) is the $F(u', v')$ for the shear altitude $H = 9$ km. This gives the shear interval 3 km below H_o. In Figure 11(b) H is 10 km, and $H_o - H$ is 2 km. In Figure 11(c) H is 11 km, and $H_o - H$ is 1 km. The 1, 2, and 3 km shear intervals above H_o are:

- Figure 11(d) — H is 13 km, and $H_o - H$ is -1 km.
- Figure 11(e) — H is 14 km, $H_o - H$ is -2 km.
- Figure 11(f) — H is 15 km, and $H_o - H$ is -3 km.

The five vector wind shear parameters, $\bar{u}'$, $\bar{v}'$, $s_{u'}$, $s_{v'}$, and $r(u', v')$, are used in the general equations for the bivariate normal probability distribution equation (14) to compute and plot these probability ellipses of vector wind shears. Note that the ellipses for small shear intervals are very nearly circular. This point can also be realized from an inspection of the shear parameters. The following test for circularity of vector wind shears, from Brooks and Carruthers [13], was applied to these sample parameters:

$$L_e = \frac{2\sigma_x \sigma_y \sqrt{1 - r(x,y)^2}}{(\sigma_x^2 + \sigma_y^2)} \quad . \tag{52}$$

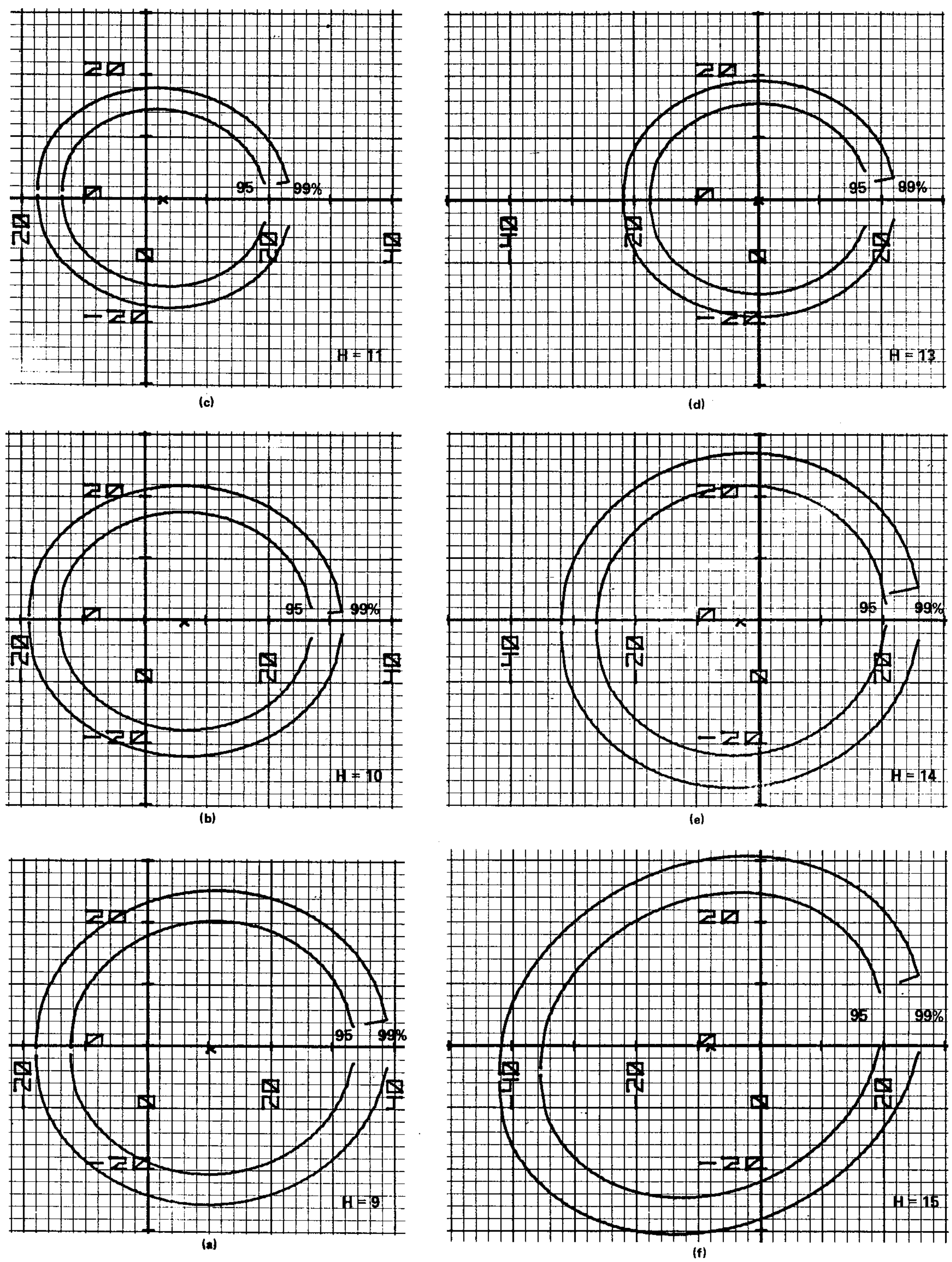


Figure 11. Bivariate normal distribution of vector wind shears (m/s), $F(u', v')$, reference altitude H_O = 12 km and H = 9, 10, 11, 13, 14, and 15 km, March, Cape Kennedy, Florida.

For an exactly circular distribtuion, $\sigma_x = \sigma_y$, $r(x,y) = 0$, and $L_e = 1$. By testing at the 5 percent level for significance, we determine that the vector wind shears for the sample size N and sample parameters can be considered as bivariate circular distributed for all shear intervals $\leq$ 5 km in all months.

Since the vector wind shears are bivariate circular distributed, it follows that the conditional vector wind shears are bivariate circular distributed for a given wind vector. An illustration of the results for Cape Kennedy March winds is given in Figures 12(a) through 12(f). For this illustration the reference altitude H_o is 12 km. The 14 sample parameters for the quadravariate normal distribution are required; i.e., $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u,v)$ at H_o = 12 km and the 9 shear parameters $\bar{u}'$, $\bar{v}'$, $s_{u'}$, $s_{v'}$, $r(u',v')$, $r(u,u')$, $r(v,v')$, $r(u,v')$, and $r(v,u')$ at the shear altitude H. The procedure is to first compute the conditional mean shears, standard deviations, and partial correlation coefficients using the computer program presented in the appendix. For this example the given wind vector was the monthly mean wind at the reference altitude H_o = 12 km. Since the conditional variances do not depend on the given wind vector, our interest in Figures 12(a) through 12(f) is only in the reduced size of these ellipses from those of the bivariate normal vector shears [Figs. 11(a) through 11(f)] and the departure from circularity. By assuming that the conditional shears are circles rather than ellipses, an error of approximately 2 m/s for the 99 percent ellipse is committed.

In the next example [Figs. 13(a) through 13(c)] for Vandenberg Air Force Base, using the December wind parameter H_o = 10 km and a given wind vector $\{\theta^* = 330°$, $W^* = 58$ m/s $\rightarrow u^* = 28.90$ m/s and $v^* = -50.06$ m/s$\}$, the conditional vector wind shear ellipses are taken over large shear intervals, and the departure from circularity becomes rather large [Figs. 13(a) through 13(c)]. However, for small shear intervals, the departure from circularity is again small.

This concludes the general wind analysis portion of this report. The reader will recognize the lack of continuity between the several illustrative examples as a weakness only in the presentation, not in the analytical formulation. There is consistent agreement between the derived (theoretical) probability results from the assumption that the wind can be treated as multivariate normal variates and the empirical (observed) frequencies as probabilities.

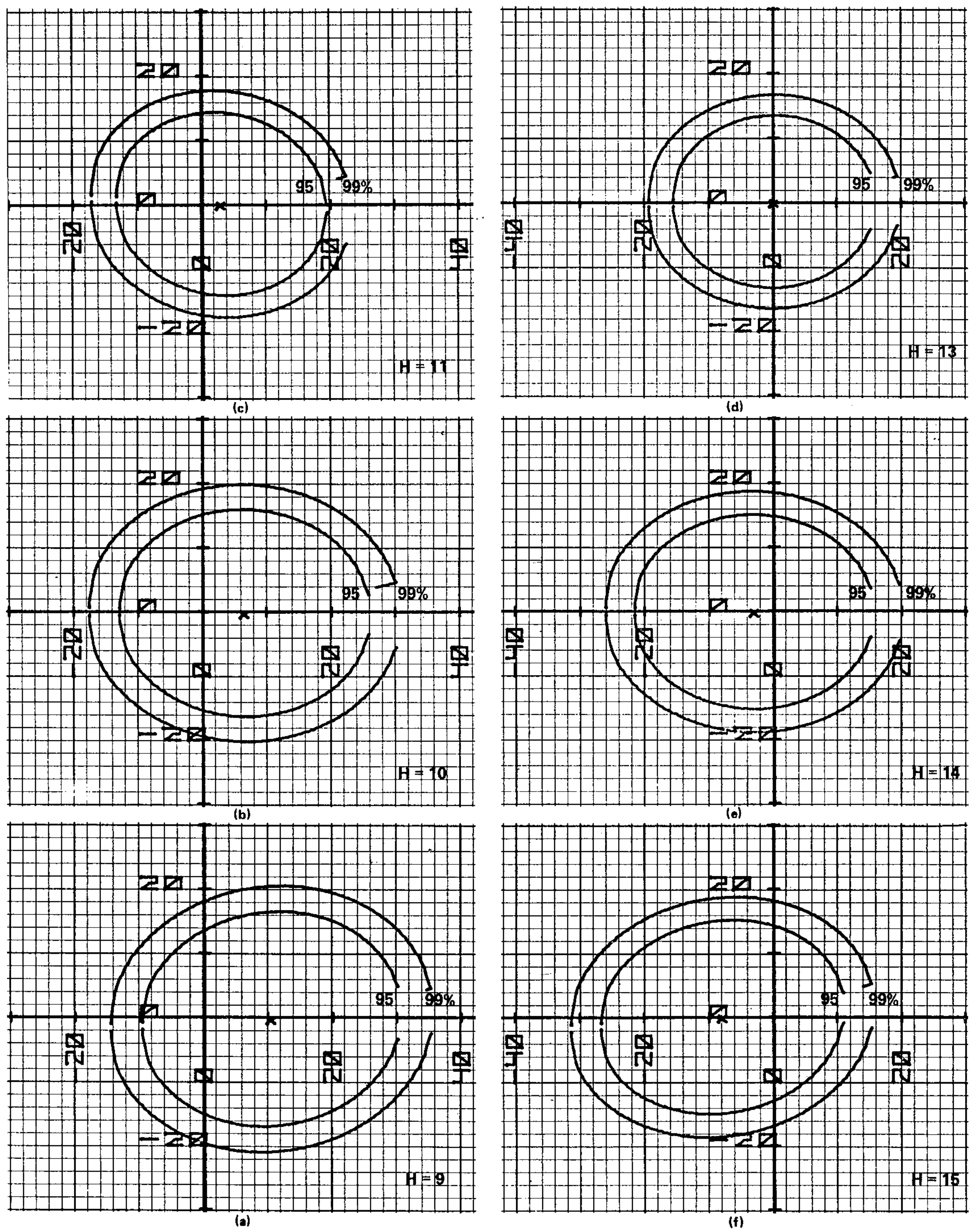


Figure 12. Conditional bivariate normal distribution of vector wind shears (m/s), given vector is the monthly mean wind vector, $F(u'_H, v'_H | u^*_{H_o} = \bar{u}_{H_o}, v^*_{H_o} = \bar{v}_{H_o})$, H_o = 12 km and H = 9, 10, 11, 13, 14, 15 km, March, Cape Kennedy, Florida.

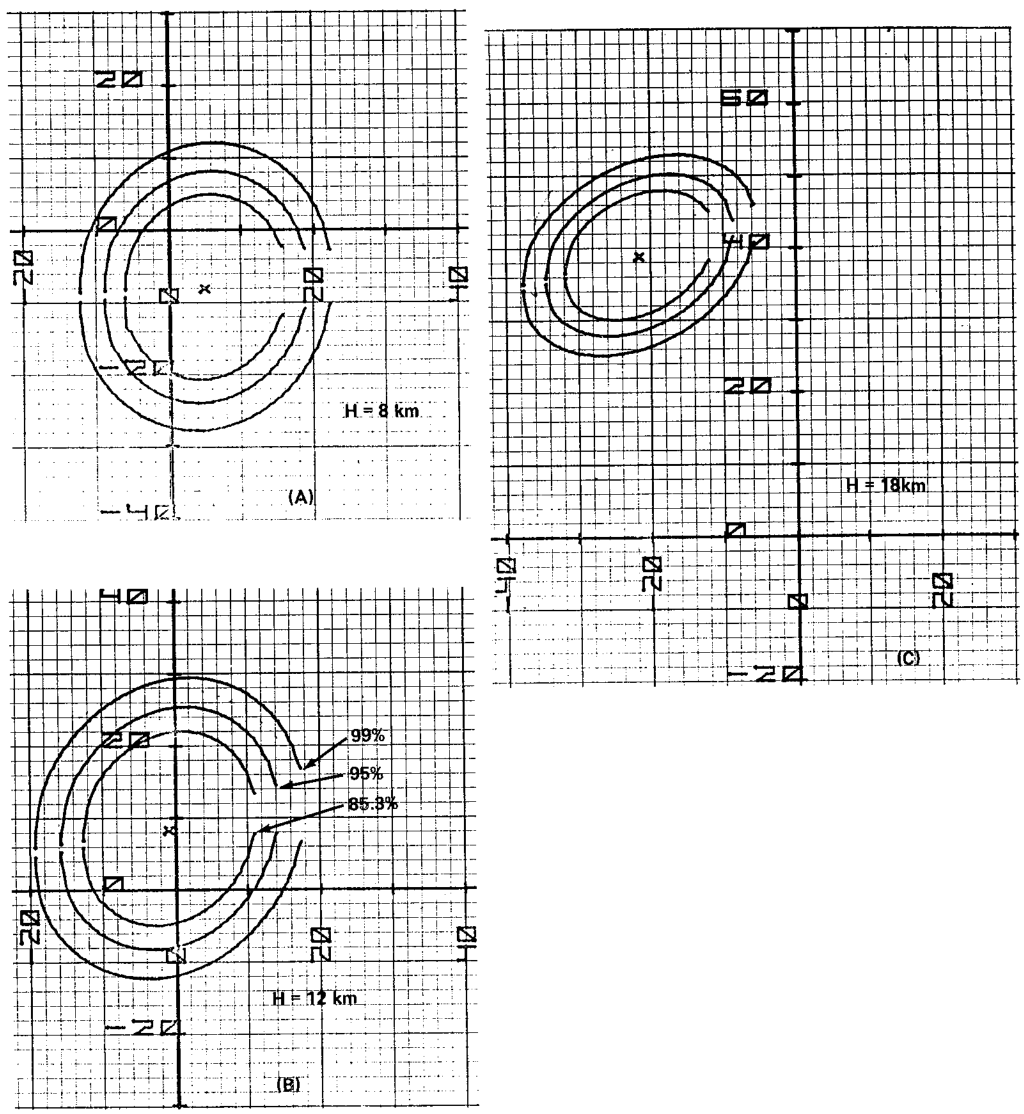


Figure 13. Conditional bivariate normal distribution of vector wind shears (m/s), given vector wind at $H_o = 10$ is 330 degrees, 58 m/s, $F(u'_H, v'_H | u^*_{H_o} = 28.9,\ v^* = -50.1)$, H = 8, 12, and 18 km, December, Vandenberg Air Force Base, California.

The powerful tools available in normal probability theory as applied to an analysis of winds-aloft data samples have been demonstrated. This introduces — on a scale heretofore not achieved — a mathematical probability rigor in the statistical analysis of winds. Many topics covered in this report could be the subject of a detailed study. Computer programs could be devised to derive the probability estimates for many special applications of wind statistics to satisfy scientific inferences and engineering applications.

IV. VECTOR WIND PROFILE MODELS

This section presents the concepts for a vector wind profile model, an outline of procedures to compute synthetic vector wind profiles (SVWP) followed by examples, and some suggestions for alternate approaches. Applications of the theoretical relationships between the variables and the parameters of the multivariate probability distribution function presented in Section II are made. The vector wind profile models presented in this section have potential applications for aerospace vehicle ascent and reentry analysis for the altitude range from 1 to 27 km.

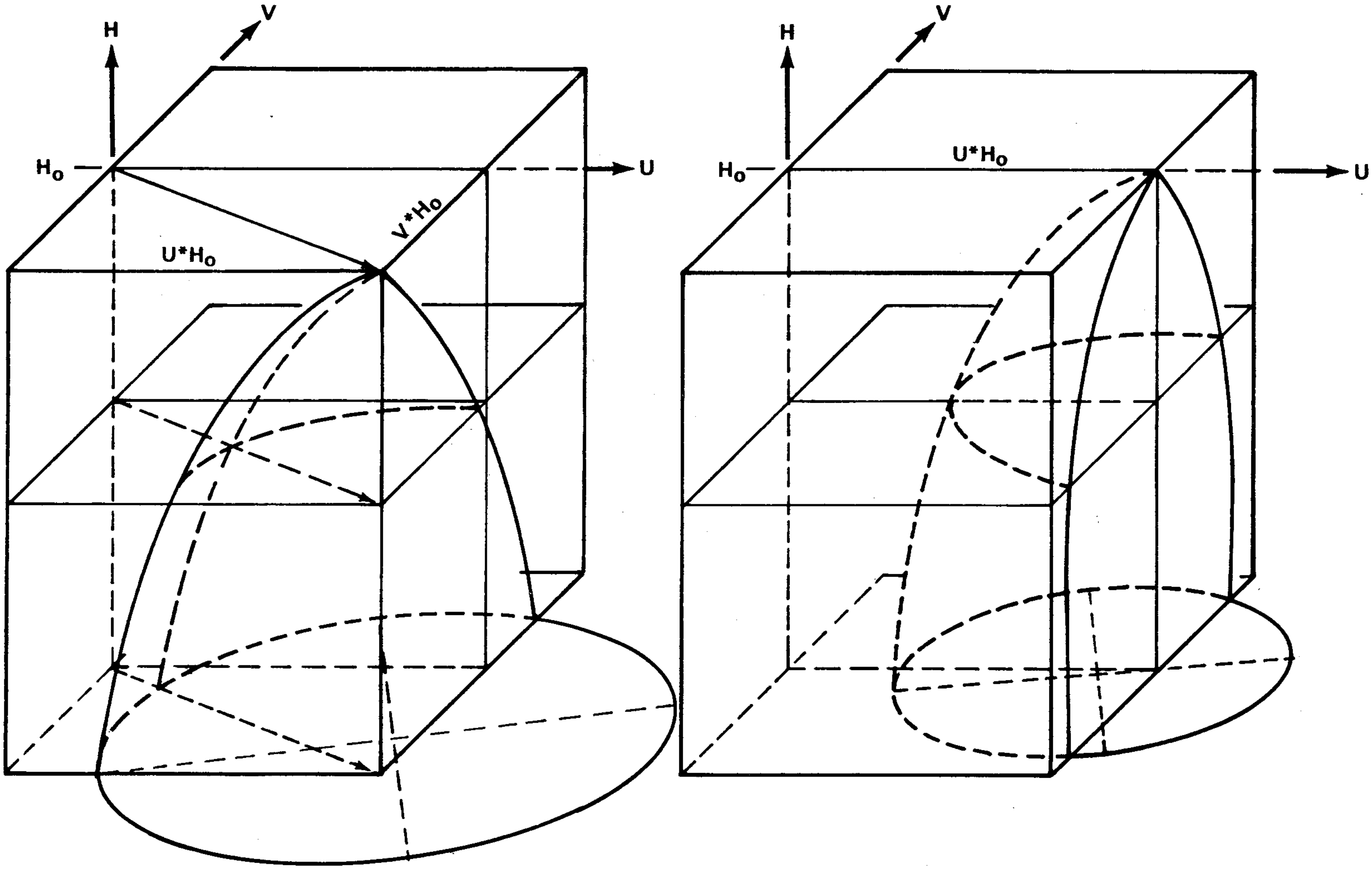
A. Vector Wind Profile Model Concepts

1. Purpose of a Model. What is a model? One definition is that a model is a representation of one or more attributes of a thing or concept. Hence, our objective in modeling the atmospheric winds is to simplify the complexity of the real wind profiles by a few attributes or characteristics to make the real wind profiles more understandable and less complicated for certain engineering applications. The modeling tools are those of mathematical probability theory and statistical analysis of wind data samples. Hopefully, through these methods, a wind model can be derived that will be a cost saving device for use in aerospace vehicle programs and still be sufficiently representative of the real wind profiles to answer engineering questions that arise in the aerospace vehicle analysis. However, the most realistic test of aerospace vehicle performance is an evaluation by flight simulations through detailed wind profiles. A sample of 150 detailed wind profiles (Jimsphere wind profiles) for each month for Cape Kennedy has been made available [14]. A sample of 150 detailed wind

profiles for each month which have all the power spectra characteristics that measured Jimsphere profiles have for Vandenberg Air Force Base has been made available for flight simulations for aerospace vehicle flights from Vandenberg Air Force Base. These two detailed wind profile data samples have the same moment statistical parameters at 1 km intervals (within statistical confidences) as the 14 parameters presented in this report. This was the basis for the selection of the 150 detailed wind profiles for each month.

2. Synthetic Vector Wind Model. In this discussion it is assumed that the reader is familiar with the synthetic scalar wind profile model presented in Reference 14. By definition, the synthetic scalar wind profile model is the locus of wind speeds versus altitude obtained from conditional wind shears given a specified wind speed at a reference altitude. The profile is constructed by subtracting the conditional wind shears from the specified wind speed. The wind shears in Reference 14 are a function of wind speed only. The SVWP extends this concept to the vector wind representation. For the SVWP the vector wind shears are a function of: (a) the reference altitude; (b) the given wind vector at the reference altitude, which makes the conditional vector wind shears wind-azimuth dependent; (c) the conditional wind shears; and (d) the monthly reference period.

For a given wind vector, the SVWP has three dimensions, whereas the synthetic scalar wind profile has two dimensions. The concept of the SVWP is illustrated in Figure 14. A wind vector is selected at the reference altitude H_o, and the conditional vector wind shears are computed for altitudes H below and above H_o. The conditional vector shears are then subtracted from the given wind vector at H_o. For two-point separation in altitude $(H_o - H)$, the cone formed by this procedure contains a specified percentage of the wind vectors at altitude H for the given wind vector at H_o. The floor of the schematic (Fig. 14) is an ellipse in which a specified percentage (usually taken as 99 percent) of the wind vectors will lie given the wind vector at H_o. The interest in modeling the wind profile is to make some logical or orderly choice to arrive at the conditional wind vectors versus altitude. It is seen from Figure 14 that there are an infinite number of paths along the surface of the conditional cone from the reference altitude H_o down to the level H. Hence, a choice of an orderly path along the surface of the conditional cone of wind vectors should be dictated



(a) From conditional shears, $F(u', v' | u_{H_o} = u^*, v_{H_o} = v^*)$.

(b) From conditional shears, $F(u', v' | u_{H_o} = u^*, v_{H_o} = 0)$.

Figure 14. Schematic of conditional bivariate normal vector winds given a wind vector at the reference altitude H_o.

by the desired scientific or engineering application. In Section IV.B a step-by-step procedure is given to compute the SVWP that is in-plane with the given wind vector. This in-plane profile has two branches: one is the smallest conditional vector wind and has the largest shears, and the other is the outer branch, which has the largest in-plane conditional wind vector but not necessarily the largest conditional shear. Also presented is the SVWP derived from the tangent intercepts to the conditional vector winds. These out-of-plane synthetic vector wind profiles have two branches: a right-turning wind direction and a left-turning wind direction with respect to altitude. The two-part in-plane SVWP and the two-part out-of-plane SVWP give a total of four synthetic vector wind profiles.

An actual example of the conditional vector winds is shown in Figure 15. The example was derived from the December wind parameters for Vandenberg Air Force Base. The reference altitude H_o is 10 km; the given wind vector at H_o is from 330 degrees at 57.8 m/s or, in terms of the components, $u^* = 28$ m/s and $v^* = -50$ m/s. Instead of conditional ellipses, 99 percent conditional circles have been computed for each altitude at 1 km intervals from 0 to 27 km altitude. The dashed line (Fig. 15) connecting the center of the conditional circles versus altitude is the conditional mean vector. The smooth curve connecting the intercepts of the conditional circles is the in-plane SVWP that has the largest conditional shears. Figure 15 is a scale plot, hence, the perspective of the three dimensions is lost.

B. Steps to Compute the Synthetic Vector Wind Profile

The following discussion is in sufficient detail for a computer program development to code the procedures to compute the SVWP. Digressions are made in the procedures to clarify some points. The primary objectives, however, are to illustrate some applications of the probability theory of Section II and to show the use of the tabulated wind statistical parameters to compute synthetic vector wind profiles.

1. Parameter Selection.

a. The first step is to select the statistical parameters for the site of interest. The tabulated wind parameters are for Vandenberg Air Force Base and Cape Kennedy.

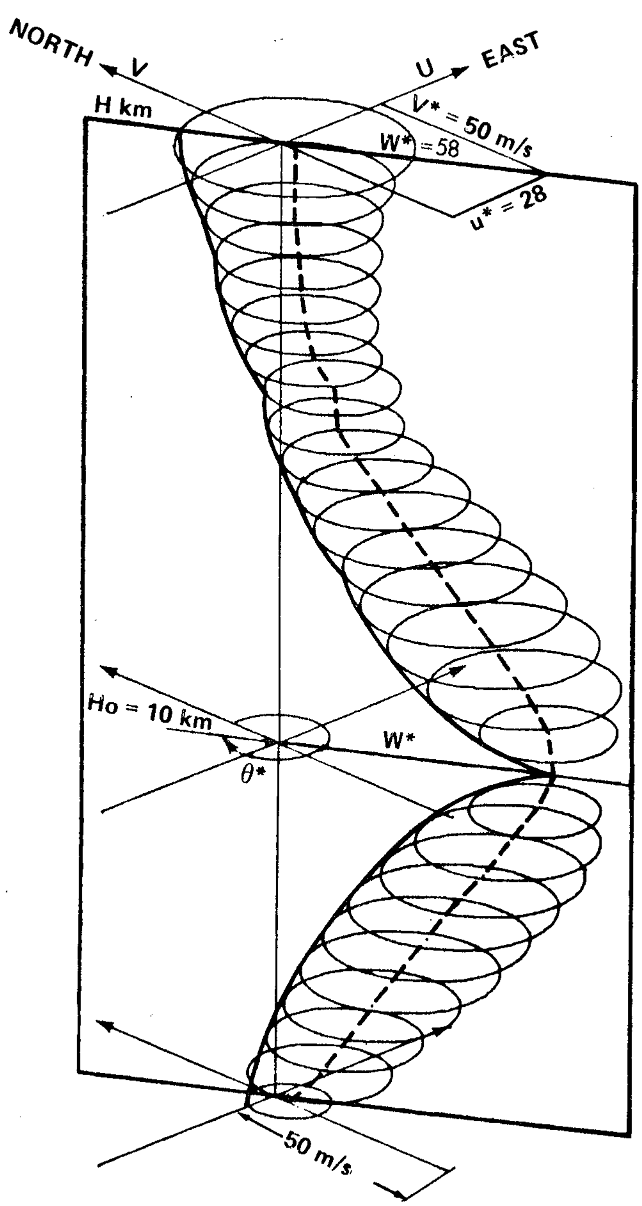


Figure 15. Conditional bivariate normal vector winds given the wind vector at $H_o = 10$ is 330 degrees at 58 m/s, December, Vandenberg Air Force Base, California.

b. Select the probability, P_E, for the probability ellipse. P_E is usually taken as 0.95 or 0.99. This selection will define the probability ellipse that contains p-percent of the wind vectors at each altitude from 0 to 27 km, including the reference altitude H_o.

c. Select the probability, P_c, for the conditional vector wind shears and the conditional vector wind circle. P_c is usually taken as 0.99. P_c defines the circle that contains p-percent of the conditional wind shears and the conditional wind vector versus altitude H, given a wind vector at the reference altitude H_o.

d. Select the reference altitude $H_o \{H_o = 0, 1, 2, \ldots, 25, 26, 27$ km$\}$. Several H_o's are to be chosen in the altitude region in which the vehicle has its greatest response to the wind.

2. Vector Wind Probability Ellipse. Using the five parameters $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u, v)$ taken from the tabulation at H_o, compute the coefficients for the probability ellipse:

$$AX^2 + BXY + CY^2 + DX + EY + F = 0 \quad , \tag{53}$$

where

$$A = s_v^{\,2} \quad ,$$

$$B = -2r(u, v)\, s_u s_v \quad ,$$

$$C = s_u^{\,2} \quad ,$$

$$D = -[B\bar{v} + 2A\bar{u}] \quad ,$$

$$E = -[B\bar{u} + 2C\bar{v}] \quad ,$$

$$F = A\bar{u}^2 + C\bar{v}^2 + B\bar{u}\bar{v} - AC[1 - \{r(u,v)\}^2]\,\lambda_e^{\,2} \quad ,$$

where $\lambda_e^{\,2} = -2 \ln (1 - P_E)$ and P_E has been chosen as 0.95, 0.99, or some other value,

$$X \equiv U - \text{wind component} \quad ,$$

$$Y \equiv V - \text{wind component} \quad .$$

3. <u>Determining the Given Wind Vector at H_o</u>. The objective here is to find the wind vectors that intercept the vector wind probability ellipse at H_o. The procedure is that of finding the intercept of a straight line (defined by the wind direction θ^*) with the ellispe. The vector wind ellipse at H_o may not cover all quadrants; however, the range of θ^* that will intercept the ellipse is required. To determine whether the probability ellipse occurs in all quadrants for a given probability ellipse, compute

$$\begin{aligned} &\bar{u} \pm \lambda_e s_u \\ &\bar{v} \pm \lambda_e s_v \end{aligned} \qquad (54)$$

and test whether $(\bar{u} - \lambda_e s_u) < 0$, $(\bar{u} + \lambda_e s_u) > 0$, $(\bar{v} - \lambda_e s_v) < 0$, and $(\bar{v} + \lambda_e s_v) > 0$. If these conditions are satisfied, the wind blowing from all directions will intercept the given probability ellipse at H_o. If the conditions for equation (54) are not satisfied, compute the range of wind directions that do intercept the probability ellipse at H_o. This operation is important if solutions to the probability ellipse, equation (53), and the given wind directions are to be obtained by assigning $\theta^* = (\theta_o + \delta)$ where $\theta_o = 0$ degrees (a wind from the north) and δ is a chosen increment, 15 degrees for example.

Figure 16 provides definitions concerning the derivation. The requirement is to obtain the lines that pass through the origin and are tangent to the ellipse, $F(x, y)$,

$$AX^2 + BXY + CY^2 + DX + EY + F = 0$$

and to obtain the straight line

$$y = mx \quad , \text{ where } m \text{ is unknown.}$$

The slope of the ellipse is

$$y' = \frac{-2AX + BY + D}{BX + 2CY + E} \quad .$$

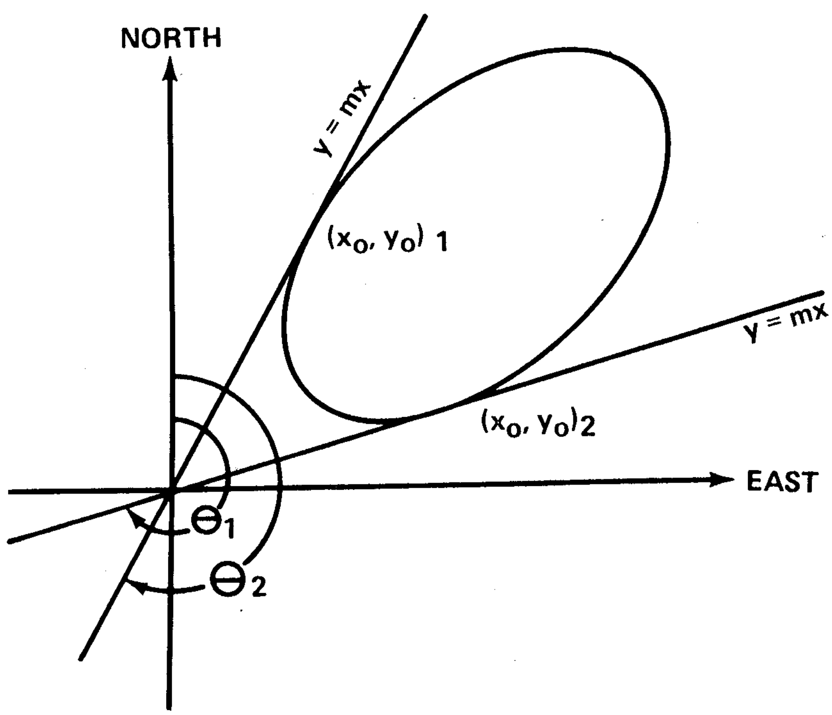


Figure 16. Definitions for line intercepts to an ellipse.

Equate

$$m = y' = F(x, y)$$

$$= \frac{-2AX + BY + D}{BX + 2CY + E} \quad .$$

Let (x_o, y_o) be points in common to the equation of the ellipse and the straight line, $m = y_o / x_o$. Then,

$$m = \frac{-2AX_o + BY_o + D}{BX_o + 2CY_o + E} \equiv \frac{y_o}{x_o} \quad .$$

To find the coordinates, substitute into the equation of the ellipse and obtain

$$Y_o = -\left[\frac{DX_o + 2F}{E}\right] \quad .$$

Find X_o as follows:

$$\underbrace{\left(A - \frac{BD}{E} + \frac{CD}{E^2}\right)}_{a} X_o^{\,2} + \underbrace{\left(\frac{-2BF}{E} + \frac{4CDF}{E^2}\right)}_{b} X_o + \underbrace{\left(\frac{4CF^2}{E^2} - F\right)}_{c} = 0 \quad . \tag{55}$$

Solve for $\hat{X}_{o1}$ and $\hat{X}_{o2}$ by the quadratic equation

$$\hat{X}_{o1}, \hat{X}_{o2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .$$

Compute

$$\hat{Y}_{o1} = -\left(\frac{D\hat{X}_{o1} + 2F}{E}\right)$$

and (56)

$$\hat{Y}_{o2} = -\left(\frac{D\hat{X}_{o2} + 2F}{E}\right) \quad .$$

Obtain the range of wind direction θ_1^* and θ_2^*:

$$\theta_1^* = \tan^{-1}\left(\frac{\hat{Y}_{o1}}{\hat{X}_{o1}}\right) + \text{quad correction}$$

$$\theta_2^* = \tan^{-1}\left(\frac{\hat{Y}_{o2}}{\hat{X}_{o2}}\right) + \text{quad correction} \quad , \tag{57}$$

where the quad correction is used to determine the meteorological convention for wind direction (Fig. 1). Compute

$$W_1^* = \sqrt{\hat{X}_{o1}^{\,2} + \hat{Y}_{o1}^{\,2}} \tag{58}$$

and

$$W_2^* = \sqrt{\hat{X}_{o2}^{\,2} + \hat{Y}_{o2}^{\,2}}$$

The wind vectors $\{\theta_1^*, W_1^*\}$ and $\{\theta_2^*, W_2^*\}$ are the tangent intercepts to the probability ellipse at H_o.

4. Magnitude of Given Wind Vector. Determine the magnitude of wind vector W* that intercepts the probability ellipse for a given θ^* by obtaining the simultaneous solutions between the probability ellipse and the straight line:

$$Y = mx \quad , \tag{59}$$

where

$$m = \left[\frac{-\cos\theta^*}{-\sin\theta^*}\right] \quad . \tag{59}$$

The slope, m, is computed in this way to preserve the meteorological sign convention.

$$AX^2 + BXY + CY^2 + DX + EY + F = 0$$

$$Y = mX \quad .$$

Solving yields

$$\underbrace{(A + mB + m^2C)}_{a} X^2 + \underbrace{(D + mE)}_{b} X + \underbrace{F}_{c} = 0 \quad . \tag{60}$$

Using the quadratic equation, obtain $\hat{X}_1$ and $\hat{X}_2$ from equation (60). Solve for

$$\begin{aligned} \hat{Y}_1 &= m\hat{X}_1 \\ \hat{Y}_2 &= m\hat{X}_2 \quad . \end{aligned} \tag{61}$$

Define $\{\hat{X}_1, \hat{Y}_1\} \to \{u_1^*, v_1^*\}$ and $\{\hat{X}_2, \hat{Y}_2\} \to \{u_2^*, v_2^*\}$. The components $\{u_1^*, v_1^*\}$ define the given wind vector whose direction is θ^* and whose magnitude is W_1^*, where

$$W_1^* = \sqrt{u_1^{*2} + v_1^{*2}} \quad . \tag{62}$$

The wind components $\{u_2^*, v_2^*\}$ define the given wind vector that is in the opposite direction of θ^*; i.e., $(\theta^* - 180°)$ and

$$W_2^* = \sqrt{u_2^{*2} + v_2^{*2}}$$

This symmetry can be used to an advantage if efficient program coding is required; otherwise it is a problem.

In summary, we have θ^*, W^* and u^*, v^*, the wind vectors that intercept the probability ellipse at the reference altitude H_o. The characters with an asterisk always refer to values at the reference altitude H_o.

5. Intercept of the θ^*-Plane with the Probability Ellipses at all Other Altitudes. By using equations (60) and (61), the intercept of the θ^*-plane with the probability ellipses at all other altitudes is determined using the parameters $\bar{u}$, $\bar{v}$, s_u, s_v, and $r(u,v)$ taken from Falls [10,11] or from the H_o tabulations for each month included in this report. These θ^*-plane intercepts versus all altitudes will be used as an optional branch to the SVWP. If there is no θ^*-plane intercept at some altitude, this fact should be noted. A zero printout for an imaginary value is satisfactory.

6. Conditional Means and Standard Deviations of Wind Shears for the Given Wind Vector $\{u^*, v^*\}$. Compute the expected values:

$$E(u'|u^*) = \bar{u}' + r(u,u') \frac{s_{u'}}{s_u} (u^* - \bar{v})$$

$$E(v'|v^*) = \bar{v}' + r(v,v') \frac{s_{v'}}{s_v} (v^* - \bar{v}) \quad . \tag{63}$$

Compute the conditional standard deviations of wind shears:

$$s_{u'|u^*} = s_{u'} \sqrt{1 - \{r(u,u')\}^2}$$

and (64)

$$s_{v'|v^*} = s_{v'} \sqrt{1 - \{r(v,v')\}^2} \quad .$$

The sample parameters, $\bar{u}'$, $\bar{v}'$, $s_{u'}$, $s_{v'}$, $r(u,u')$, and $r(v,v')$ versus altitude H are tabulated on a single page for a given reference altitude H_o.

Note that the conditional standard deviations are a function only of the correlation coefficients and shear standard deviations, and these parameters are a function of H_o and H. We are using the simplifying assumption that the conditional vector shears can be treated as bivariate circular distributed. Therefore, compute the resultant conditional standard deviation

$$s_{V_1} = \sqrt{s^2_{u'|u^*} + s^2_{v'|v^*}} \quad . \tag{65}$$

7. <u>The Conditional Vector Wind Circle and the In-Plane SVWP.</u> Compute the conditional mean wind vector that is defined by the expected values of the component shears:

$$\left.\begin{aligned} E(u|u^*) &= u^* - E(u'|u^*) \\ E(v|v^*) &= v^* - E(v'|v^*) \end{aligned}\right\} \quad \text{for } H \leq H_o \tag{66}$$

$$\left.\begin{aligned} E(u|u^*) &= u^* + E(u'|u^*) \\ E(v|v^*) &= v^* + E(v'|v^*) \end{aligned}\right\} \quad \text{for } H > H_o \quad . \tag{67}$$

At $H = H_o$, $E(u'|u^*) = 0$, hence $E(u|u^*)$ at $H_o = u^*$. Similarly, at $H = H_o$, $E(v'|v^*) = 0$, and $E(v|v^*)$ at $H_o = v^*$. The change of sign between equations (66) and (67) is because of the convention used in computing the wind component shear sample values [see equations (50) and (51)].

The equation for the conditional vector wind circle is

$$AX^2 + BY^2 + CX + DY + E = 0 \quad , \tag{68}$$

where

$$A = 1 \quad ,$$

$$B = 1 \quad ,$$

$$C = -2E(u|u^*) \quad ,$$

$$D = -2E(v|v^*) \quad ,$$

and

$$E = [E(u|u^*)]^2 + [E(v|v^*)]^2 - 2\lambda_c^{\,2} s^2_{V_1} \quad ,$$

where $\lambda_c^{\,2} = -\ln(1 - P_c)$ and P_c is an input parameter, usually taken as 0.99.

The next step is to compute the intercept of the straight line defined by θ^* and equation (68); $Y = mX$ is as defined in equation (59). The solution is

$$(A + m^2 B) X^2 + (C + mD) X + E = 0 \quad . \tag{69}$$

$\hat{X}_1$ and $\hat{X}_2$ are obtained by the quadratic equation, and

$$\hat{Y}_1 = m\hat{X}_1$$

and (70)

$$\hat{Y}_2 = m\hat{X}_2 \quad .$$

This computation gives the in-plane SVWP.

We now have two solutions to this intercept, and it is desired to identify both solutions with respect to the θ^*-plane. Compute

$$W_A = \sqrt{\hat{X}_1^2 + \hat{Y}_1^2} \tag{71}$$

and

$$W_B = \sqrt{\hat{X}_2^2 + \hat{Y}_2^2} \quad ,$$

and take the smaller of the two values as W_1. If W_1 is from the intercept, $\hat{X}_2$, $\hat{Y}_2$ for example, which have opposite signs to u^* and v^*, the conditional wind vector is from a direction opposite that of θ^*. Therefore, assign a negative value to W_1 and compute $\theta_1 = (\theta^* - 180°)$. This conditional in-plane wind profile is noted as $SVWP_1$ and tabulated as

$SVWP_1$

$\theta_1 \quad W_1 \quad .$

When $W_1 > 0$, $\theta_1 = \theta^*$; when $W_1 < 0$, $\theta_1 = (\theta^* - 180°)$. The larger intercept, W_A from equation (71) is also an in-plane SVWP but is of the same direction as θ^*. This profile can, and often does, exceed the specified wind vector probability ellipse.

8. <u>SVWP Out-of-Plane to θ^*</u>. The equations to derive the synthetic vector wind profiles that are tangent to the conditional vector winds are presented in the following. There are three conditions for these profiles:

a. Condition A. When $W_1 > 0$ or when $\lambda_c s_{V_1} \leq W_E$, compute

$$\theta_E = \tan^{-1}\left[\frac{E(v|v^*)}{E(u|u^*)}\right] + \text{quadrant correction} \quad . \tag{72}$$

Note that

$$\theta_E = \sin^{-1}\left[\frac{E(v|v^*)}{W_E}\right]$$

and

$$\theta_E = \cos^{-1}\left[\frac{E(u|u^*)}{W_E}\right] \quad ,$$

where

$$W_E = \sqrt{E(u|u^*)^2 + E(v|v^*)^2} \quad . \tag{73}$$

W_E is the magnitude of the resultant conditional mean vector, and θ_E is the direction from which W_E is blowing, i.e., the meteorological convention. Also note the following:

(1) When $E(v|v^*) = 0$ and $E(u|u^*) > 0$, $\theta_E = 270°$.

(2) When $E(v|v^*) = 0$ and $E(u|u^*) < 0$, $\theta_E = 90°$.

(3) When $E(v|v^*) = 0$ and $E(u|u^*) = 0$, $\theta_E = 180°$.

(4) When $E(v|v^*) < 0$ and $E(u|u^*) = 0$, $\theta_E = 0°$ or $360°$, $0° \leq \theta_E \leq 360°$.

As an aid in establishing the computer logic for the meteorological convection for wind direction, the relationship

$$\theta_{E_{meteor}} = 180 + (90 - \theta_{math})$$

may be helpful. Compute

$$\Delta\theta = \sin^{-1}\left[\frac{\lambda_c s_{V_1}}{W_E}\right] ; \qquad -90° \leq \Delta\theta \leq 90° \quad . \tag{74}$$

$$\theta_2 = \theta_E + \Delta\theta \quad ,$$

and (75)

$$\theta_3 = \theta_E - \Delta\theta \quad .$$

$$W_2 = W_3 = \sqrt{W_E^{\ 2} - (\lambda_c s_{V_1})^2} \quad . \tag{76}$$

$\{\theta_2, W_2\}$ and $\{\theta_3, W_3\}$ define the synthetic vector wind profiles that are tangent vectors to the conditional vector wind circle when $\lambda_c s_{V_1} \leq W_E$.

b. Condition B. When $W_1 < 0$ or when $\lambda_c s_{V_1} > W_E$, equation (74) is invalid because the conditional vector wind circle has passed over the origin. In this case, set $\Delta\theta = 90°$ and compute

$$\theta_2 = \theta_E + 90°$$

and (77)

$$\theta_3 = \theta_E - 90° \quad .$$

The magnitudes of the conditional vectors are then computed by

$$W_2 = W_3 = \sqrt{(\lambda_c s_{V_1})^2 - (W_E)^2} \quad . \tag{78}$$

Note that the quantity $\lambda_c s_{V_1}$ is the radius of the conditional vector wind probability circle.

c. Condition C. When $W_1 < 0$, and $E(u|u^*)$ and $E(v|v^*)$ have opposite signs to u^* and v^*, and $(W_E - s_{V_1}) < 0$, the conditional probability circle has passed to the opposite quadrant from u^* and v^* and is completely contained in the opposite quadrant. This condition will be rare except for $P_E << 0.95$. The computations are performed for θ_2, θ_3, W_2, and W_3 as under condition A, but 180 degrees is subtracted from θ_2 and θ_3 because these conditional vectors are from the opposite direction of θ^*.

The SVWP out-of-plane to the given wind direction, θ^*, has been selected in this manner to give the largest turning of the SVWP direction with respect to altitude. A computer program is being established to perform the indicated operations to compute the SVWP. This computer program will have several options to permit a variety of comparisons of wind shears and the construction of SVWP.

9. Examples of Synthetic Vector Wind Profiles. Illustrated in Figures 17, 18, and 19 are examples of the SVWP's computed for Vandenberg Air Force Base with reference altitude $H_o = 10$ km, and the given wind vector at 10 km is a wind from 330 degrees at 57.8 m/s. These profiles are derived from the December statistical parameters using the procedures outlined previously.

Several curves are shown in Figure 17, for the in-plane SVWP. The application of these SVWP's for the evaluation of aerospace vehicle flight performances would be very much the same as the current practice [14]. However, more optional profiles exist than are currently used in vehicle flight analysis. For example, the flight wind loads may be evaluated by using the intercepts (curves labeled 2 in Fig. 17) of the 95 percent wind ellipses versus altitude. The in-plane SVWP may be used as labeled in Figure 17. Then, combinations of curve 1 for $H \leq H_o$ and curve 2 for $H > H_o$ may be used. For other analysis, the profile of conditional means (W_E) may be used.

The curve labeled W_A is a valid SVWP that is in-plane with θ^*, but it exceeds the 95th percent wind ellipses at all altitudes except at the reference altitude H_o. Some rationale to exclude this case based on an evaluation of the SVWP concept for aerospace vehicle response to wind should be established.

The wind shears obtained from SVWP (2) and SVWP (3) (Figs. 18 and 19) are less than those for SVWP (1) (Fig. 17). The vector wind shears for SVWP (1) for this example ($\theta^* = 330$ degrees, $W^* = 57.8$ m/s, $H_o = 10$ km, December, Vandenberg Air Force Base) are much less than those of Reference 14. However, for other given wind vectors $\{\theta^*, W^*\}$ for SVWP (1), the in-plane shears are larger than those of this example.

An example (Fig. 20) of the in-plane SVWP (1) for Cape Kennedy February winds for $H_o = 12$ km and the given wind vector, $\theta^* = 255$ degrees and $W^* = 88.09$ m/s, produces wind shears that approach those found in Reference 14. The given wind vector intercepts the 95 percent wind vector ellipse at $H_o = 12$ km.

The exact reason for the close agreement between the largest 1 km vector wind shear obtained from the SVWP and those from Reference 14 for Cape Kennedy and not for Vandenberg Air Force Base is not fully understood.

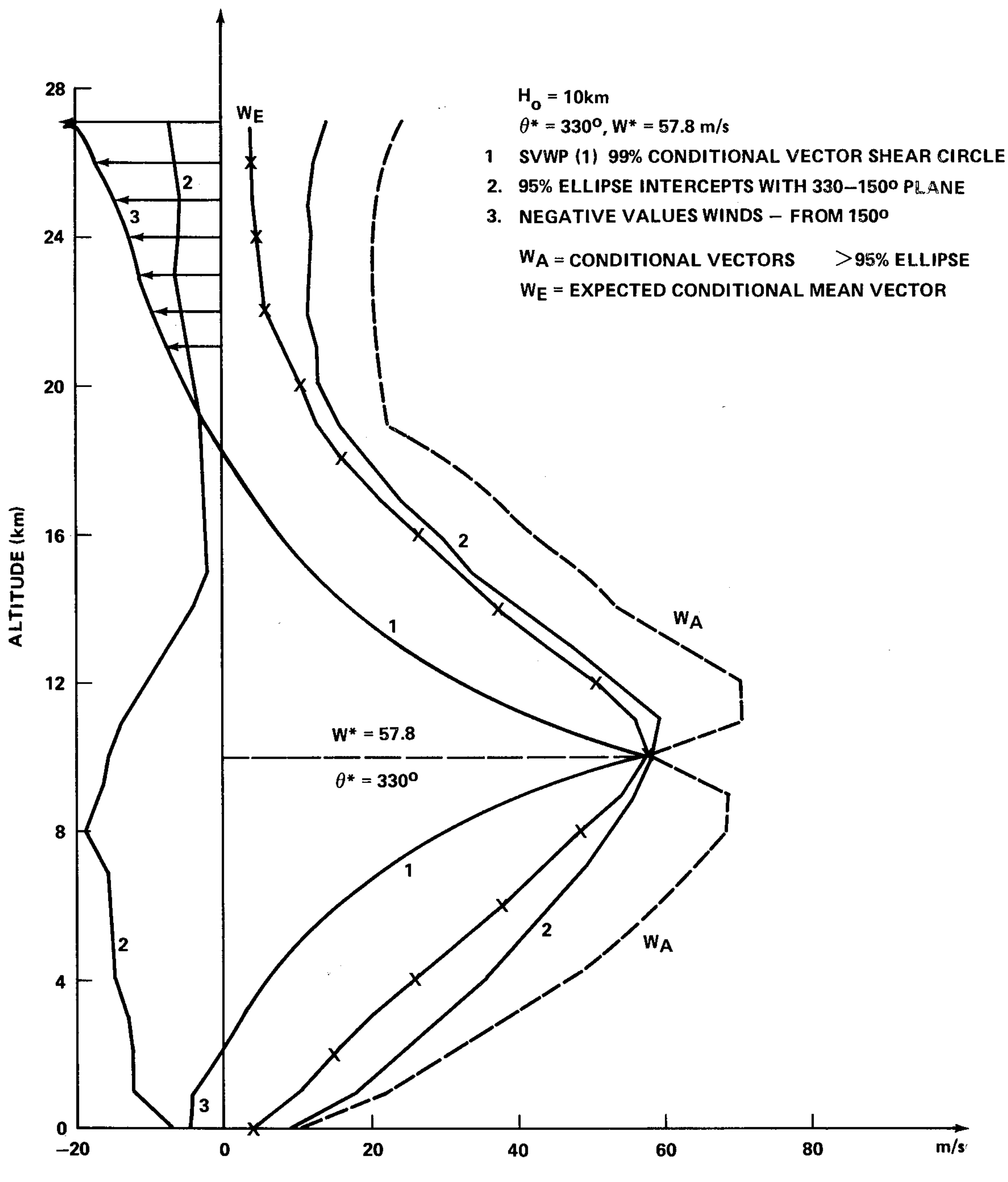


Figure 17. Synthetic vector wind profile, SVWP (1), in-plane with given wind vector, December, Vandenberg Air Force Base, California.

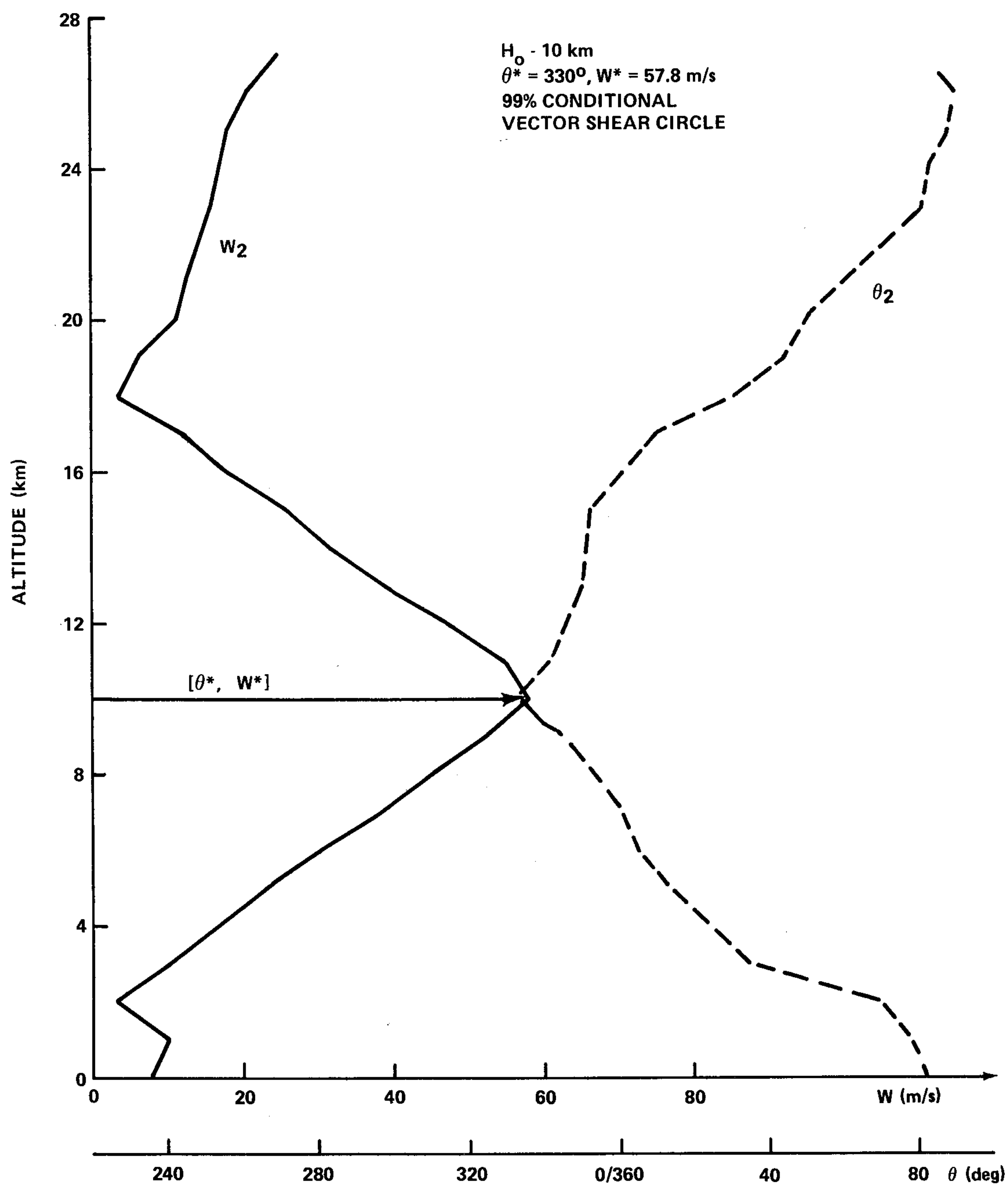


Figure 18. Synthetic vector wind profile SVWP (2), wind vectors tangent to the right of conditional wind circle, December, Vandenberg Air Force Base, California.

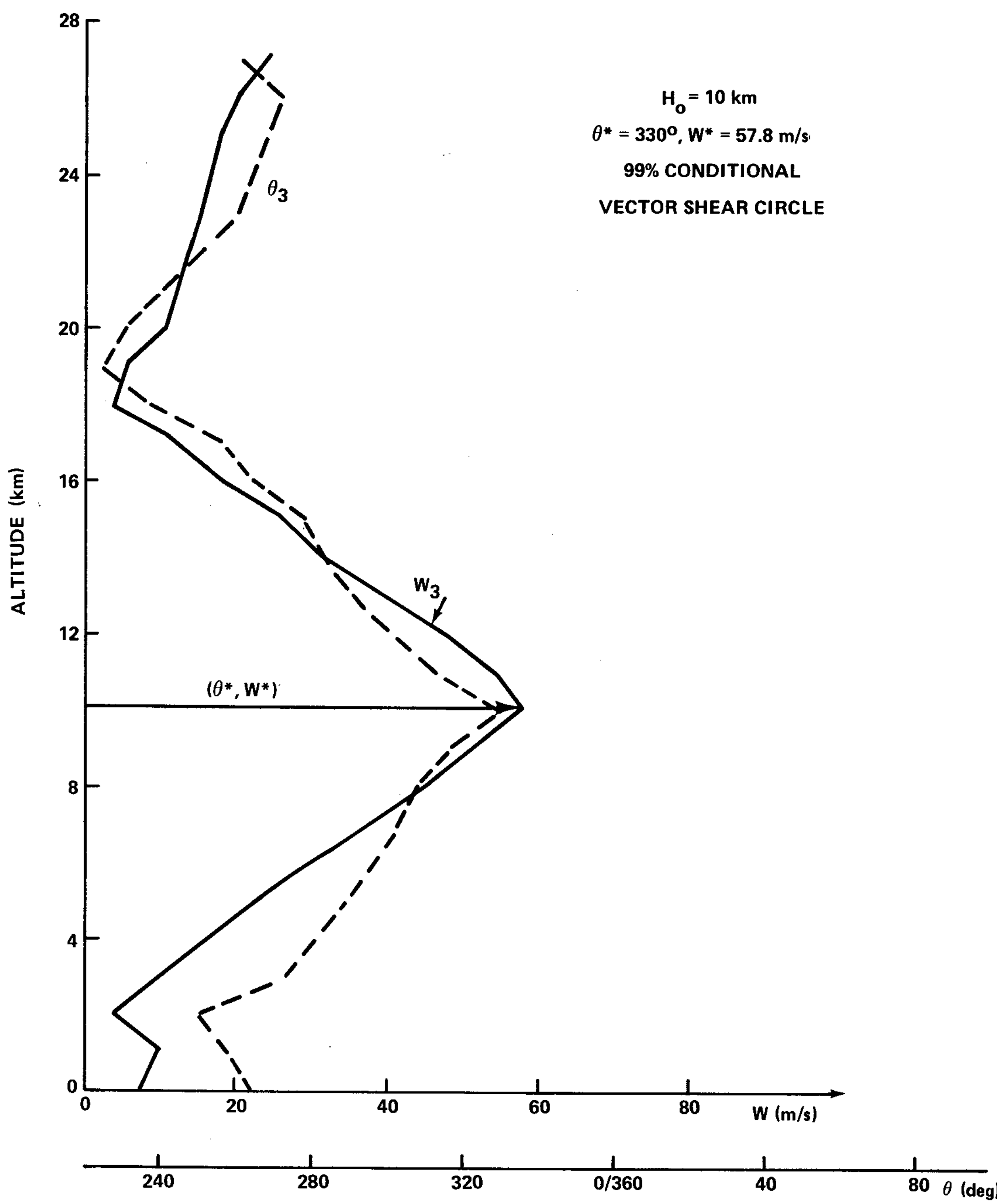


Figure 19. Synthetic vector wind profile SVWP (2), wind vectors tangent to the left of conditional wind circle, December, Vandenberg Air Force Base, California.

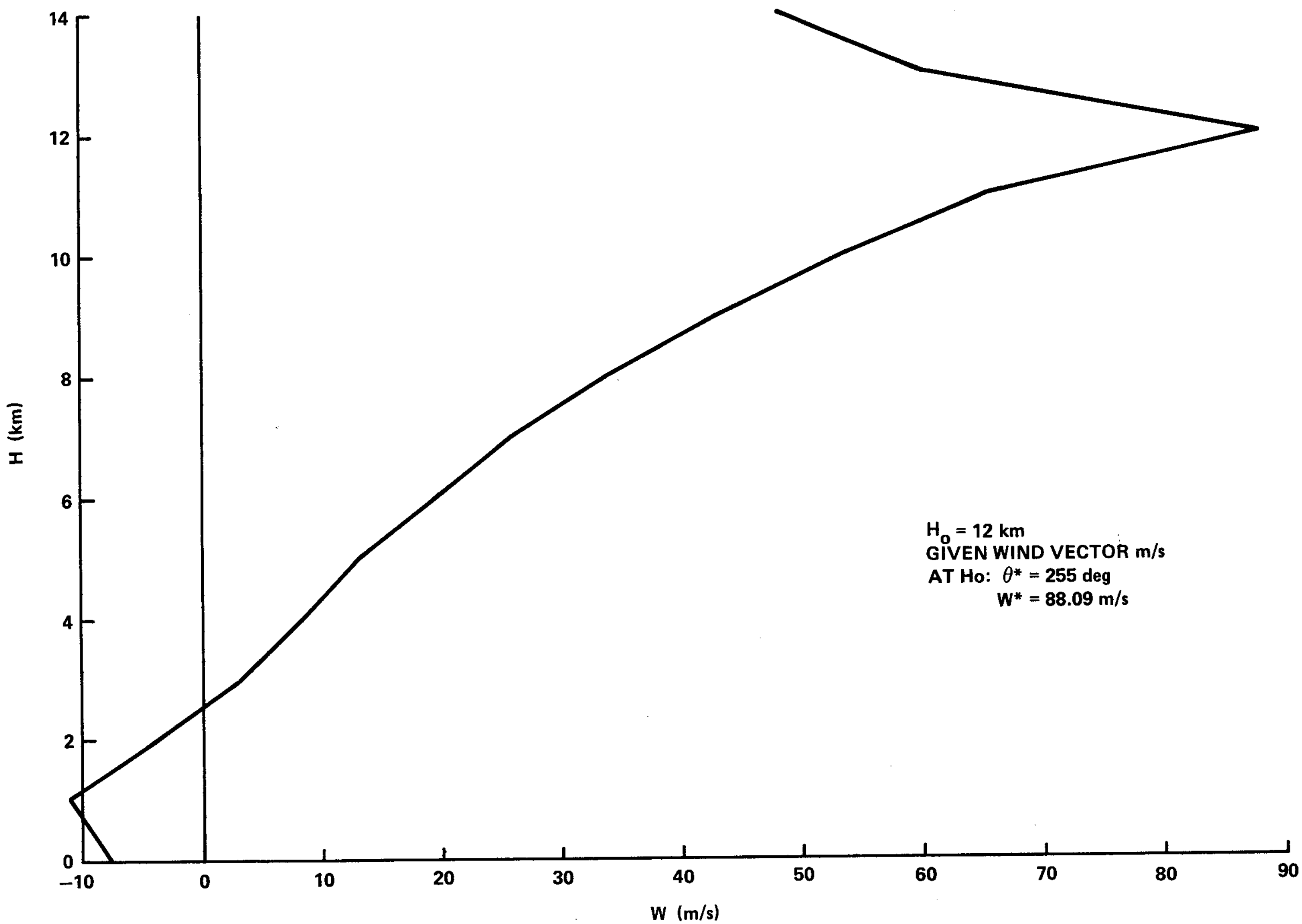


Figure 20. Synthetic vector wind profile SVWP (1), in-plane with given wind vector, February, Cape Kennedy, Florida.

It must be recognized that comparisons are being made between two different wind modeling concepts and that the statistical parameterizations are different. The wind characteristics over Cape Kennedy and Vandenberg Air Force Base are different; e.g., the monthly vector mean winds over Cape Kennedy are greater than those over Vandenberg Air Force Base, and the vector wind distributions over Vandenberg Air Force Base are more circular than those over Cape Kennedy. For the SVWP model the vector wind shear is a conditional bivariate normal probability distribution function derived from rigorous mathematical probability theory under the tested assumption that the winds can be treated as multivariate normally distributed variables. In contrast, the wind shears for Reference 14 are conditional Rayleigh variables (see Section II. D) that were derived by empirical methods. The data samples for the vector wind shears and wind speeds for Reference 14 were pooled for the entire period of record and over all reference altitudes. From this two-way (vector wind shears versus wind speed) empirical distribution, the conditional wind shears for the 99th percentiles were computed. Pooling the data samples to obtain conditional wind shears was required to increase the sample size. Probabilities derived from parameters estimated from a sample having a known probability distribution are more efficient than those derived by empirical methods. In using empirical methods it is generally understood that percentiles are computed from cumulative percentage frequencies (CPF). When the underlying probability distribution is not known or cannot be reasonably assumed and if the derived distribution from first principles (see Section II. D) becomes very complex, then empirical statistical methods have considerable merit. However, a large data sample is usually required.

From these considerations it may be concluded that the vector wind shears for the SVWP model are different from those given in Reference 14 because of the differences in the two concepts and the differences in the statistical methods.

C. Suggested Alternate Synthetic Vector Wind Profiles

A method to obtain the SVWP with respect to the monthly mean vector wind is presented here. Another alternative is to obtain the conditional wind vector probability ellipses rather than the conditional circles. A summary guide for SVWP procedures completes this section.

1. SVWP with Respect to Monthly Mean Wind and Other Alternatives. In Section IV.A it was recognized that there are an infinite number of paths one could choose in following the surface of the conditional vector wind cone from the given wind vector at a reference altitude H_o to the shear altitude H.

This alternate procedure to that of Section IV.A to obtain the SVWP begins with a different method of computing the intercepts of the given wind vector at H_o to the vector wind probability ellipse.

We saw by equation (37) that conditional probability of the residual vector magnitude for a given azimuth with respect to the vector mean has the same probability as the assigned vector ellipse. This important property of the bivariate probability distribution function is now applied using the wind parameters.

The conditional probability of the residual magnitudes of wind vectors r for a given wind azimuth φ with both taken in respect to the monthly vector mean wind (Fig. 21) is expressed as

$$\Pr\{r \leq r^* \mid \varphi = \varphi^*\} = 1 - e^{-\frac{r^{*2}}{2a^2}} , \tag{79}$$

where

$$a^2 = \frac{1}{1 - r(u,v)^2} \left[\frac{\cos^2 \varphi}{s_u^2} - \frac{2r(u,v) \cos \varphi \sin \varphi}{s_u s_v} + \frac{\sin^2 \varphi}{s_v^2} \right] .$$

Solving for r^* produces

$$r^* = a \sqrt{-2 \ln (1 - P)} \tag{80}$$

where $a = \sqrt{a^2}$ and P is the probability.

When the vector wind probability ellipse is specified, e.g., $P = 0.95$, the intercept of the conditional vector residual, $\{r^*, \varphi^*\}$ to the probability ellipse can be obtained by simple calculations (Fig. 21). Hence, with respect to the original coordinate system the wind vector that intercepts the

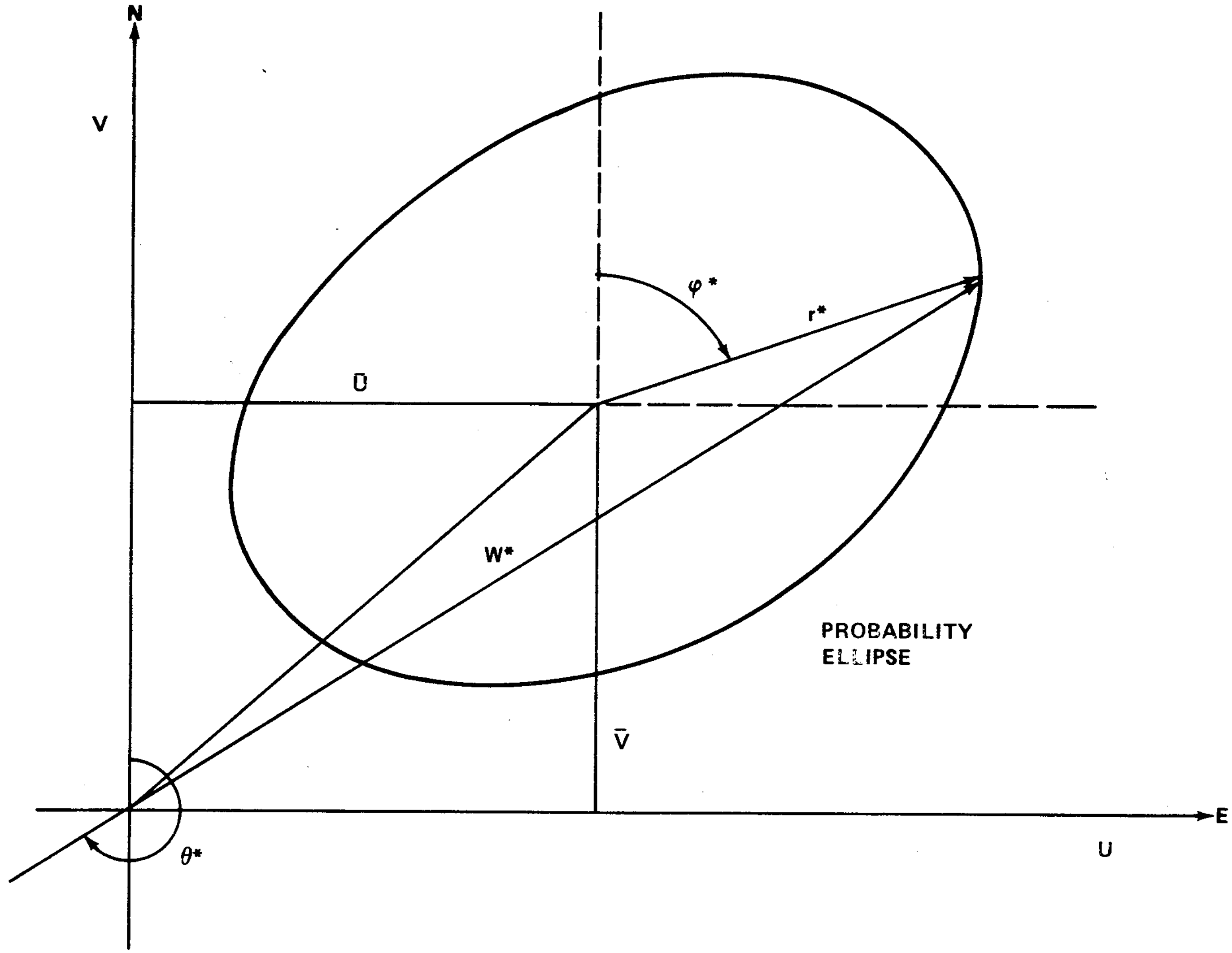


Figure 21. Relationships between the conditional Rayleigh distribution with respect to the mean vector and the bivariate normal probability ellipse.

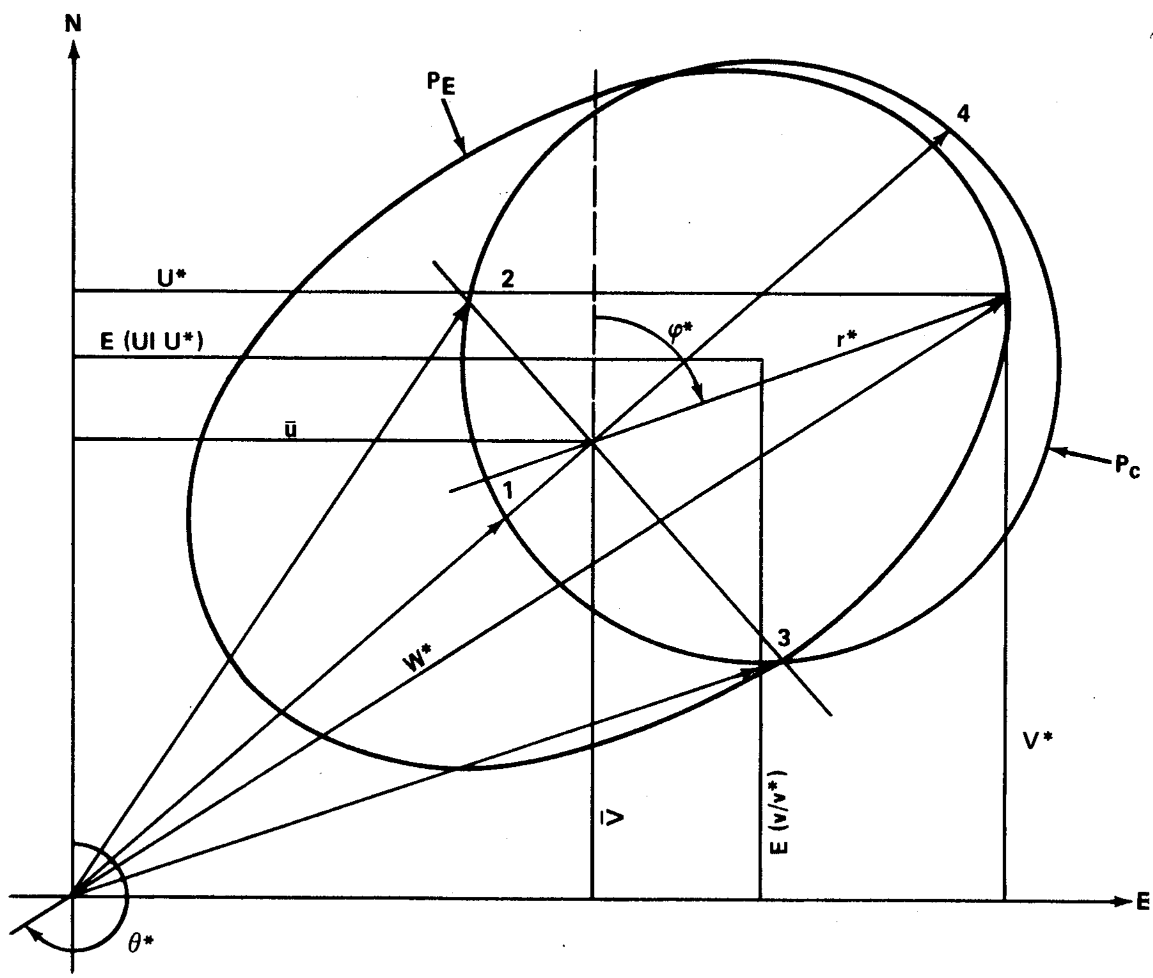
probability ellipse is readily computed in terms of the components u^* and v^*. This procedure assures that a given wind vector $\{\theta^*, W^*\}$ at the reference altitude will intercept the probability ellipse. This method is less complex than that used in Section IV. B. There is also an advantage in this procedure in the application of the SVWP for aerospace vehicle trajectory control system biasing to the monthly vector mean winds to reduce the vehicle response to wind loads.

If the residual wind azimuths, φ, are taken at equal increments to obtain the intercepts of r^* with the probability ellipse, the resulting wind directions θ^* will not have even increments. This may or may not be of concern in the applications of the subsequently derived SVWP.

After obtaining u^* and v^* for the given wind vector at the reference altitude H_o, the conditional vector wind shear circles and the conditional vector wind circles versus altitude H are computed in the same manner as presented in Section IV. B. The SVWP could also be computed in the manner of Section IV. B. However, here we have an alternative that appears attractive for wind-biased trajectory analysis. That is, at each altitude compute the intercept of the monthly mean wind vector to the conditional probability circle. Tangent intercepts or any other choice of intercepts from the given residual vector to the conditional vector wind circle may be computed (Fig. 22). In all cases the wind vectors from the original coordinate system to these intercepts are computed. The locus of these vectors versus altitude will be called SVWP's with respect to the monthly mean wind vectors.

a. Alternate 2: SVWP with Respect to Conditional Mean Wind Vector. A further alternative would be to compute the intercept of the vector from the centroid (monthly mean) for each shear altitude H to the conditional vector wind circle, and then derive the wind vector at each of these intercepts versus altitude H from the original coordinate systems. This SVWP would be referred to as the SVWP with respect to the monthly mean wind vectors versus altitude H. In the previous model alternative the in-plane SVWP wind direction remains constant, fixed by the reference altitude H_o, whereas in this second alternative the wind direction changes as a function of the monthly vector mean wind at each altitude.

b. Alternative 3: SVWP with Respect to Vehicle Flight Azimuth. The SVWP may be determined by computing the intercept of the wind vector from the origin to the conditional probability circle where the direction is



Vectors 1 and 4 are intercepts to the conditional vector wind circle in-plane with the monthly vector mean wind at the reference altitude H_o, or versus H.

Vectors 2 and 3 are intercepts to the conditional vector wind circle taken orthogonal to the monthly vector mean wind at the reference altitude H_o.

Figure 22. Alternative SVWP with respect to the monthly vector mean wind.

assigned by the vehicle flight azimuth. The choice of the intercepting vector to the conditional vector wind probability circle would be dictated by the condition that gives the largest vehicle response to the wind.[3]

There are many other options to computing SVWP. However, the objective in the wind modeling for aerospace vehicles is to use the SVWP alternative that produces the largest response to the wind profile and still remains a reasonable model of the real winds.

2. Conditional Probability Ellipses. A considerable effort has been made to show that the vector wind shears can be treated as bivariate circular distributions and, by comparison, that the conditional vector shears can be treated as bivariate circular. There would not be a great deal more computer work involved to use the conditional bivariate ellipses to obtain the SVWP. This is a further refinement that could be established if the application interest grows toward more exactitude for wind shears over larger shear intervals. For small shear intervals up to 3 to 4 km, the conditional circles are satisfactory approximations to the conditional ellipses.

The tabulations of the 14 parameters for the quadravariate normal probability distribution of wind vectors and wind vector shears have certain symmetry for altitudes above and below the reference altitude H_o. This symmetry could have been used advantageously in reducing the tabulations. A more fundamental approach to tabulating the wind parameters could have been followed also. This approach would begin by computing the variance-covariance matrix for the 14 parameters of the quadravariate normal distribution of wind components versus altitude, which is similar to the correlation matrix by Daniels and Smith [15]. Then, the normal probability theory that differences in normal variates are normal could have been used to compute the distribution of wind shear, or the conditional wind vector for a given wind vector at a reference altitude H_o could have been computed directly from the 14 parameters of the wind components.

3. Recommended Synthetic Vector Wind Profile Procedures. The following summary is the presently recommended procedure that may be followed in applying the SVWP to aerospace vehicle flight simulations to determine its response to wind loading.

---

3. Credit is due Mr. W. Norton and Mr. J. Wolf of Rockwell International, Space Division for suggestions leading to these alternative SVWP models.

a. Wind-Biased Trajectory. If the choice is made to wind bias the trajectory, then the profile of the monthly vector mean wind should be used. The option to wind bias has some significance in the computational procedures for the construction of the SVWP.

b. Selection of the SVWP alternative. Of the several alternatives given in this report to construct the SVWP, the one selected should be that which produces the desired vehicle design response to the wind profile.

c. 95 Percent Vector Wind Ellipse. The 95 percent probability vector wind ellipse is recommended for use in the construction of the SVWP. For initial aerospace vehicle flights, some lesser probability vector wind ellipse may be chosen providing a higher launch delay risk is acceptable.

d. 99 Percent Vector Wind Shear. The 99 percent vector wind shear for the conditional probability of wind vectors given a wind vector on the 95 percent probability ellipse should be used.

e. Reference Altitude and Given Wind Vector. Several reference altitudes in the altitude region suspected to give the required aerospace vehicle design parameters should be selected. A sufficient number of wind vectors that fall on the 95 percent wind vector ellipse at the reference altitudes should be selected to assure that the vehicle for the given mission will experience the appropriate design wind loading from the SVWP model.

f. Shear and Gust. When the 95 percent vector wind envelope versus altitude is used, the standard gust criteria (9 m/s) as given in Reference 14 should be used. When the shear and gust are applied to the SVWP, these should be reduced by 15 percent in accordance with the standard procedure given in Reference 14. The direction of the gust may be applied at the reference altitude in-plane with the SVWP or at any other direction that maximizes the vehicle response.

g. Interpolation for Reference Altitude. If the reference altitude of interest is not on an integer value in kilometers (H_o = 1, 2, 3, . . . , 25, 26, 27 km), the SVWP should be computed for the reference altitudes at 1 km above and 1 km below the desired reference altitude and a linear interpolation should be performed to compute the SVWP at 1 km intervals with respect to the desired reference altitude.

h. Interpolation for Shear Interval Less Than 1 km. To obtain the SVWP over the the shear interval between reference altitude and 1 km below and 1 km above the reference altitude, use interpolation equation 5.26 (page 5.96) of Reference 14.

i. Reference Altitudes Below 1 km. Although the statistical parameters and the SVWP model will produce a SVWP for reference altitudes H_o = 0 km (which in reality is 10 m above natural grade) and H_o = 1 km, the SVWP for altitudes below 1 km should not be used for the vehicle flight simulations. This is because the 14 statistical parameters are from a wind data sample taken twice daily that is not necessarily a representative sample versus all times of the day for this altitude region. An interpolation procedure should be devised to connect the SVWP at 1 km altitude to the ground wind profile up to 1 km altitude. This procedure could be similar to that used in Reference 14.

V. CONCLUSIONS

Certain objectives of this report have been met. A mathematical rigorous methodology for treating wind data sample statistics has been developed. This report, by no means, exhausts the technical alternatives for vector wind modeling. As greater insight into the wind vector and wind vector shear statistical parameters and the use of the properties of the multivariate normal probability distribution is gained, further adaptations of these statistics for specific engineering and scientific applications are envisioned.

By considering that the multivariate normal probability distribution is a reasonable model for wind data samples at 0 to 27 km altitude, the many properties of normal probability theory can be used to derive consistent probability estimates for wind speed, wind direction, wind components, vector winds, and vector wind shears. Several conditional probability estimates between several variables can also be derived. Probability estimates outside the observed sample range can be derived. The good agreement between derived probability estimates and the observed (empirical) probability estimates supports the hypothesis that the multivariate normal probability distribution is a reasonable probability model for the winds over Cape Kennedy and Vandenberg Air Force Base.

REFERENCES

1. Feller, William: An Introduction to Probability Theory and Its Applications. Volume I, John Wiley & Sons, Inc., New York, 1961.

2. Crutcher, H. L. and Falls, L. W.: Multivariate Normality. NASA TN D-8226, 1976.

3. Gnedenko, B. V: The Theory of Probability. Chelsea Publishing Co., New York, 1962.

4. Smith, O. E.: An Application of Distributions Derived from the Bivariate Normal Density Function. International Symposium on Probability and Statistics in the Atmospheric Sciences, Honolulu, Hawaii, June 1-4, 1971, pp. 162-168.

5. Weil, Herschel: The Distribution of Radial Error. Ann. of Math. Statistics, vol. 25, 1954, pp. 168-170.

6. Yadavalli, S. V.: On Applications of Some Results Related to Bivariate Gaussian Density Distribution Fuctions. Int. J. of Control, 1st Series, vol. 5, no. 2, 1967, pp. 191-194.

7. White, R. G.: Distribution and Moments of Radial Error. NASA TM X-64962, August 15, 1975.

8. Miller, K. S.; Bernstein, R. I.; and Blumenson, L. E: Generalized Rayleigh Processes. Quarterly of Appl. Math, vol. 16, no. 2, 1958, pp. 137-145.

9. Falls, L. W. and Crutcher, H. L.: Determination of Statistics for Any Rotation of Axes of a Bivariate Normal Elliptical Distribution. NASA TM X-64595, May 11, 1971.

10. Falls, L. W.: Normal Probabilities for Cape Kennedy Wind Components — Monthly Reference Periods for All Flight Azimuths — Altitudes 0 to 70 Kilometers. NASA TM X-64771, April 16, 1973.

11. Falls, L. W.: Normal Probabilities for Vandenberg Air Force Base Wind Components — Monthly Reference Periods for All Flight Azimuths, 0- to 70-km Altitudes. NASA TM X-64897, January 1975.

REFERENCES (Concluded)

12. Vaughan, W. W.; Smith, O. E.; and Canfield, N. W.: Progress in Circumventing Limitations of Upper Wind Records. J. Appl. Meteor., vol. 5, no. 3, 1966, pp. 301-303.

13. Brooks, C. E. P. and Carruthers, N.: Handbook of Statistical Methods in Meteorology. Her Majesty's Stationery Office, London, 1953.

14. Daniels, Glenn E., ed.: Terrestrial Environment (Climatic) Criteria Guidelines for Use in Aerospace Vehicle Development, 1973 Revision. NASA TM X-64757, July 5, 1973.

15. Daniels, Glenn E. and Smith, Orvel E.: Scalar and Component Wind Correlations Between Altitude Levels for Cape Kennedy, Florida, and Santa Monica, California. NASA TN D-3815, April 1968.

APPENDIX A

A PROGRAM TO COMPUTE CONDITIONAL BIVARIATE NORMAL PARAMETERS

INTRODUCTION

This appendix presents a sketch of the theory and a computer program designed to calculate the bivariate normal conditional distribution derived from the quadravariate normal distribution. The required computer inputs are described and an example is presented. The computer program is presented in Figures A-1 and A-2.

THEORY

The general multivariate normal distribution has the density

$$f(x_1, x_2, \ldots, x_k) = \frac{1}{(2\pi)^{k/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\underline{x} - \underline{\mu})' \, \Sigma^{-1} (\underline{x} - \underline{\mu})\right\} \qquad \text{(A-1)}$$

where $\underline{\mu}^1 = (\mu_1, \mu_2, \ldots, \mu_k)$, the vector of mean values and

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \cdots & \sigma_{1k} \\ \sigma_{21} & \sigma_{22} \cdots & \sigma_{2k} \\ \sigma_{k1} & \sigma_{k2} \cdots & \sigma_{kk} \end{bmatrix},$$

the symmetric variance-covariance matrix.

A property of the multivariate normal distribution is that marginal and conditional distributions are also normally distributed. The general expression for these distributions is found often in the literature.[1] Remarks are confined here to the specific case.

Assume we wish to derive $f(x_1, x_2, |x_3, x_4)$. If we define

$$\underline{\mu} = \left[\begin{array}{c} \underline{\mu_1} \\ \hline \underline{\mu_2} \end{array}\right] = \left[\begin{array}{c} \mu_1 \\ \mu_2 \\ \hline \mu_3 \\ \mu_4 \end{array}\right] \text{ and } \underline{u}' = \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right] = \left[\begin{array}{cc|cc} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \hline \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{array}\right],$$

then letting

$$\underline{x} = \left[\begin{array}{c} \underline{x_1} \\ \hline \underline{x_2} \end{array}\right] = \left[\begin{array}{c} x_1 \\ x_2 \\ \hline x_3 \\ x_4 \end{array}\right],$$

we have

$$f(\underline{x_1} \mid \underline{x_2}) = \frac{1}{2\pi \, |\Sigma^*|^{1/2}} \exp\left\{-\frac{1}{2} (\underline{x_1} - \underline{\mu}^*)' (\Sigma^*)^{-1} (\underline{x_1} - \underline{\mu}^*)\right\} \qquad \text{(A-2)}$$

1. Morrison, D. F. (1967): Multivariate Statistical Methods, Wiley, N. York.

where

$$\Sigma^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \tag{A-3}$$

and

$$\underline{\mu}^* = \underline{\mu}_1 + \Sigma_{12}\Sigma_{22}^{-1}(\underline{x}_2 - \underline{\mu}_2) \quad . \tag{A-4}$$

Computation of the parameters for this conditional distribution really reduces to computation of the quantities Σ^* and $\underline{\mu}^*$. Carefully note that the value of $\underline{\mu}^*$ includes values of $\underline{x}_2 = [x_3, x_4]'$ that must be specified before numerical values for $\underline{\mu}^*$ can be calculated.

Even for this rather easy case the actual expressions for Σ^* and $\underline{\mu}^*$ and therefore for the quadratic form in equation (A-2) are very complicated algebraically. They are, however, very amenable to numerical computation via computer. The least complicated for the expressions is that for $\underline{\mu}^*$ and the actual form is (letting $\sigma_{34} = \sigma_{43}$ for convenience)

$$\underline{\mu}^* = \begin{bmatrix} \mu_1 + \{(\sigma_{13}\sigma_{44} - \sigma_{14}\sigma_{33})(x_3 - \mu_3) + (\sigma_{14}\sigma_{33} - \sigma_{13}\sigma_{34})(x_4 - \mu_4)\}/(\sigma_{33}\sigma_{44} - \sigma_{34}^2) \\ \mu_2 + \{(\sigma_{23}\sigma_{44} - \sigma_{24}\sigma_{33})(x_3 - \mu_3) + (\sigma_{24}\sigma_{33} - \sigma_{23}\sigma_{34})(x_4 - \mu_4)\}/(\sigma_{33}\sigma_{44} - \sigma_{34}^2) \end{bmatrix} .$$

The matrix triple product $\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ makes Σ^* a complicated expression and this causes $(\Sigma^*)^{-1}$ and, therefore, the quadratic form in equation (A-2) to be almost incomprehensible in an expanded form.

COMPUTER PROGRAM AND REQUIRED INPUTS

The computer program is written to accept quadravariate data and return the conditional bivariate parameter. The conditional variance-covariance matrix and the associated standard deviations and correlations are initially calculated and printed. The program is designed to take as many pairs of "conditioning values" of x_3 and x_4 as desired and print out both the values of x_3 and x_4 plus the associated values of $\underline{\mu}^*$.

Example: The following data were input to the program

$\underline{\mu} = [21.58, -0.04, 43.35, 1.25]'$

$\sqrt{\sigma_{11}} = 11.03, \rho_{12} = 0.0503, \rho_{13} = 0.7382, \rho_{14} = -0.0199$

$\sqrt{\sigma_{22}} = 11.52, \rho_{23} = 1614, \rho_{24} = 0.8134$

$\sqrt{\sigma_{33}} = 15.47, \rho_{34} = 0.1524$

$\sqrt{\sigma_{44}} = 14.59$

$[x_3, x_4] = [43.35, 0]$.

Figure A-1 presents the output giving the calculated parameters for the bivariate conditional. Note carefully that the standard deviations and correlations are printed in matrix form for convenience — not to be confused with the variance-covariance matrix printed above it. Below the standard deviation and correlation matrix the values conditioned on and the resulting conditional means are printed. The original inputs and matrices will be printed only once but the values conditioned on, followed by the conditional means calculated using those values, will be repeated for each set of conditioning values read in.

Input to the program consists of the following cards:

Card 1	The 4 means for the quadravariate normal in 4F10.4 format.
Card 2	Standard deviation for variable 1 followed by correlations for variables 1&2, 1&3, and 1&3 in 4F10.4 format.
Card 3	Standard deviation for variable 2 followed by correlations for variables 2&3and 2&4 in 3F10.4 format.
Card 4	Standard deviation for variable 3 followed by correlation between variables 3&4 in 2F10.4 format.
Card 5	Standard deviation for variable 4 in F10.4 format.

Card 6	Number of sets of x_3, x_4 values to be conditioned on in I2 format.
Card 7	1st set of x_3, x_4 values to be conditioned on
Card 8	2nd set of x_3, x_4 values to be conditioned on
Card 9	3rd set of x_3, x_4 values to be conditioned on
Card 10	etc.

The source deck listing is given in Figure A-2.

```
MEANS VECTOR =     2[illegible]        -[illegible]        [illegible]        [illegible]
VARIANCE-COVARIANCE MATRIX
      121.6009        6.[illegible]        [illegible]        [illegible]
        6.3914      13[illegible].7.[illegible]      [illegible].7[illegible]      [illegible].7[illegible]
      123.9621        [illegible].7[illegible]     229.[illegible]        [illegible]
       -3.2025       13[illegible] 7.[illegible]        [illegible]        [illegible]
COND. VAR. COV.MATRIX
    53.17973      4.[illegible]
     4.85823     44.71[illegible]

SD&CORR,MATRIX
     7.29244      0.[illegible]
     0.09922      6.[illegible]7[illegible]

VALUES CONDITIONED ON          4[illegible]          [illegible]
CONDITIONAL MEAN*****$$$$$$$$$          [illegible].7[illegible]          -[illegible].[illegible]7[illegible]
```

Figure A-1. Variance-covariance matrix.

ORIGINAL PAGE IS
OF POOR QUALITY

```
FORTRAN IV G LEVEL  21                 MAIN                DATE = 76042          11/02/29

            DIMENSION SIGMA(2,2),XU(2),VX(2),US(2),COR(2,2),V1(2,2),
           1V2(2,2),V3(2,2),V4(2,2),U1(2),U2(2),X(2),VV24(2,2),VVV(2,2),SD(4),
           2RHO(4,4),U(4),S(4,4)
            READ(5,1) (U(I),I=1,4)
          1 FORMAT(4F10.4)
            DO 2 I=1,4
            L=I
            IF (I.LE.3) L=I+1
          2 READ(5,3) SD(I),(RHO(I,J),J=L,4)
          3 FORMAT(4F10.4)
            DO 4 I=1,4
            L=I+1
            S(I,I)=SD(I)**2
            IF (L.EQ.5) GO TO 4
            DO 4 J=L,4
            S(I,J)=RHO(I,J)*SD(I)*SD(J)
            S(J,I)=S(I,J)
          4 CONTINUE
            WRITE(6,5) (U(I),I=1,4)
          5 FORMAT('1'///' MEANS VECTOR = ',4(F10.4,2X))
            WRITE(6,6)
          6 FORMAT(' VARIANCE-COVARIANCE MATRIX'/)
            DO 7 I=1,4
          7 WRITE(6,8)(S(I,J),J=1,4)
          8 FORMAT(5X,4(F10.4,4X))
            DO 9 I=1,2
            DO 9 J=1,2
            V1(I,J)=S(I,J)
            V2(I,J)=S(I,J+2)
            V3(I,J)=S(I+2,J)
          9 V4(I,J)=S(I+2,J+2)
            D=V4(1,1)*V4(2,2)-V4(1,2)*V4(2,1)
            C=V4(1,1)
            V4(1,2)=-V4(1,2)/D
            V4(2,1)=-V4(2,1)/D
            V4(1,1)=V4(2,2)/D
            V4(2,2)=C/D
            DO 10 I=1,2
            U1(I)=U(I)
         10 U2(I)=U(I+2)
            DO 11 I=1,2
            DO 11 J=1,2
            VV24(I,J)=0
            DO 11 K=1,2
         11 VV24(I,J)=VV24(I,J)+V2(I,K)*V4(K,J)
            DO 12 I=1,2
            DO 12 J=1,2
            VVV(I,J)=0
            DO 12 K=1,2
         12 VVV(I,J)=VVV(I,J)+VV24(I,K)*V3(K,J)
            DO 13 I=1,2
            DO 13 J=1,2
         13 SIGMA(I,J)=V1(I,J)-VVV(I,J)
            COR(1,1)=SQRT(SIGMA(1,1))
            COR(2,2)=SQRT(SIGMA(2,2))
            COR(1,2)=SIGMA(1,2)/(COR(1,1)*COR(2,2))
            COR(2,1)=COR(1,2)
            WRITE(6,14)SIGMA
            WRITE(6,15)COR
         14 FORMAT('CCOND. VAR. COV.MATRIX'//2(2X,F10.5)//)
         15 FORMAT('OSD&CORR,MATRIX'//2(2X,F10.5)//)
            READ(5,16)N
         16 FORMAT(I2)
            DO 23 M=1,N
            READ(5,17) (X(I),I=1,2)
         17 FORMAT(2F10.5)
            DO 18 I=1,2
         18 XU(I)=X(I)-U2(I)
            DO 19 I=1,2
            VX(I)=0
            DO 19 J=1,2
         19 VX(I)=VX(I)+VV24(I,J)*XU(J)
            DO 20 I=1,2
         20 US(I)=U1(I)+VX(I)
            WRITE(6,21) (X(I),I=1,2)
         23 WRITE(6,22) (US(I),I=1,2)
         21 FORMAT('CVALUES CONDITIONED ON',2(5X,F10.4)/)
         22 FORMAT(' CONDITIONAL MEAN*****$$$$$$$$',2(5X,F10.4)//)
            STOP
            END

*OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = MAIN     , LINECNT =       60
*STATISTICS*     SOURCE STATEMENTS =       78,PROGRAM SIZE =      2898
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

Figure A-2. Fortran IV G level.

ORIGINAL PAGE IS
OF POOR QUALITY

C.2

APPENDIX B

DESCRIPTION OF TABLES FOR VECTOR WIND AND VECTOR WIND SHEAR STATISTICAL PARAMETERS 0 TO 27 KM ALTITUDE

This appendix describes the tabulation of the 14 sample statistical parameters for the multivariate normal probability distribution function for vector winds and vector wind shears 0 to 27 km altitude for Cape Kennedy, Florida, and Vandenberg AFB, California. The tables are for monthly reference periods, January — December. Each table has a continuation page for each reference altitude, HO, for HO = 0, 1, 2, . . . 25, 26, 27 km.

Computer characters are used for identification purposes within each table. The heading, COMPONENT STATISTICS OBSERVED DATA, is used because the computer program was used for other purposes in addition to computing these wind sample statistics.

STATION 12867 indicates that the wind data in this Volume are for Cape Kennedy, Florida.

STATION 93214 indicates that the wind data are for Vandenberg Air Force Base, California (a sample page is included in this appendix).

ALPHA = 90.0 shows that the statistics are for the zonal and meridional wind components. The meteorological coordinate system is used for these tabulations.

The month and period of record for the data sample are indicated.

The first row, designated HO, is the reference altitude in kilometers: HO = 0, 1, 2, . . . 25, 26, 27 km continued for each monthly reference period.

UBAR $\equiv \bar{u}$, the zonal mean wind component [m/s].

SDU $\equiv s_u$, the standard deviation of the zonal wind component [m/s].

R(U, V) $\equiv r(u, v)$, the correlation coefficient between zonal and meridional wind components [unitless].

VBAR $\equiv \bar{v}$, the meridional mean wind component [m/s].

SDV $\equiv s_v$, the standard deviation of the meridional wind component [m/s].

N = sample size.

Column identifications for the shear parameters are as follows (see **Section III.A** for calculations):

Col 1: H is the shear altitude 0 to 27 km or index altitude.

Col 2: HO-H is the shear interval [km]

Cols 3-11: These columns identify the component shear parameters, all of which are with respect to the shear interval.

Col 3: UPBAR $\equiv \bar{u}'$ is the zonal component mean wind shear for the zonal component between U at HO and U at H or the shear interval (HO-H) [m/s].

Col 4: SD(UP) $\equiv s_{u'}$ is the standard deviation of the zonal component shear [m/s].

Col 5: R(U, UP) $\equiv r(u, u')$ is the correlation coefficient between the zonal wind component at the reference altitude, HO, and the zonal shear over the shear interval [unitless].

Col 6: VPBAR $\equiv \bar{v}'$ is the meridional component mean wind shear [m/s].

Col 7: SD(VP) $\equiv s_{v'}$ is the standard deviation of the meridional component shear [m/s].

Col 8: R(V, VP) $\equiv r(v, v')$ is the correlation coefficient between the meridional wind component at the reference altitude, HO, and the meridional shear over the shear interval [unitless].

Col 9: R(UP, VP) $\equiv r(u', v')$ is the correlation coefficient between the zonal and meridional wind component shears [unitless].

Col 10: R(UP, V) $\equiv r(u', v)$ is the correlation coefficient between the zonal wind component shear and the meridional wind component at the reference altitude, HO [unitless].

Col 11: R(VP, U) $\equiv r(v', u)$ is the correlation coefficient between the meridional wind component shear and the zonal wind component at the reference altitude, HO [unitless].

The 5 parameters in the first row and the 9 parameters in the body of the table contain the 14 statistical parameters required for the quadravariate normal probability distribution function for the vector wind at a reference altitude, HO, and the vector wind shear at altitudes below and above the reference altitude.

TABLE 2A

COMPONENT STATISTICS

OBSERVED DATA

ALTITUDE INTERVAL = 1000 BEGIN ALTITUDE = 0 END ALTITUDE = 27000

STATION = 93214 ALPHA = 90.0 MONTH = JAN. PERIOD OF RECORD 1/65 - 1/72

HO	UBAR	SD U	R (U,V)	VBAR	SD V	N
12	24.60	19.16	.1861	-5.11	16.07	495

H	HO-H	UPBAR	SD(UP)	R(U,UP)	VPBAR	SD(VP)	R(V,VP)	R(UP,VP)	R(UP,V)	R(VP,U)
0	12	24.20	19.32	.9886	-4.53	15.28	.9776	.1674	.2088	.1583
1	11	23.69	17.76	.9759	-3.44	14.23	.9032	.0953	.1532	.1274
2	10	21.42	16.06	.9608	-2.46	13.39	.8851	.0596	.1322	.1173
3	9	18.71	15.00	.9264	-1.95	12.72	.8603	.0279	.1104	.0874
4	8	16.41	13.97	.8827	-1.83	12.04	.8283	.0225	.1010	.0774
5	7	14.15	13.17	.8210	-1.94	11.50	.7702	.0055	.0768	.0516
6	6	12.38	12.40	.7566	-1.77	11.04	.7061	.0111	.0654	.0264
7	5	10.67	11.45	.6841	-1.33	10.63	.6223	.0264	.0583	-.0016
8	4	8.85	10.42	.5687	-1.02	10.04	.5069	.0343	.0427	-.0333
9	3	6.78	9.21	.4342	-.49	9.09	.3632	.0700	.0354	-.0391
10	2	4.48	7.47	.2447	-.18	7.26	.1526	.0732	.0037	-.0774
11	1	1.64	5.48	-.0096	-.11	4.92	-.0559	.0973	-.0286	-.0948
12	0	.00	.00	.0000	.00	.00	.0000	.0000	.0000	.0000
13	-1	-.65	5.80	-.6203	.66	5.55	-.6324	.0643	-.0868	-.0101
14	-2	-2.32	8.54	-.7431	1.10	8.06	-.7574	.0369	-.0832	-.0126
15	-3	-5.06	10.59	-.8549	1.42	9.32	-.8330	.0033	-.1435	.0080
16	-4	-7.89	12.02	-.8930	1.74	10.48	-.8822	-.0010	-.1304	-.0170
17	-5	-11.15	13.54	-.9120	2.01	11.96	-.9260	.0542	-.1403	-.0675
18	-6	-14.58	15.11	-.9319	2.01	12.91	-.9500	.0725	-.1457	-.0902
19	-7	-17.72	16.28	-.9329	2.02	13.84	-.9577	.0657	-.1275	-.0977
20	-8	-20.70	16.82	-.9277	2.03	14.63	-.9666	.0634	-.1223	-.1085
21	-9	-22.65	17.16	-.9260	2.09	15.19	-.9691	.0637	-.1189	-.1154
22	-10	-24.25	17.73	-.9185	2.03	15.40	-.9702	.0888	-.1240	-.1240
23	-11	-25.30	18.14	-.9012	2.06	15.53	-.9683	.0767	-.1081	-.1194
24	-12	-25.50	18.59	-.8728	2.22	15.61	-.9636	.0762	-.0866	-.1240
25	-13	-25.64	19.28	-.8447	2.60	15.98	-.9611	.0946	-.0815	-.1374
26	-14	-25.33	19.78	-.8231	2.43	15.99	-.9553	.1049	-.0804	-.1379
27	-15	-24.46	20.13	-.7886	2.22	15.85	-.9463	.1107	-.0601	-.1376

Figure B-1. Example page of tabulation showing the 14 vector wind and vector wind shear parameters for Vandenberg AFB, California, for January for the reference altitude H_o = 12 km.

Because the inclusion of the data tables would make this report too voluminous (336 pages for each station), the tabulations are being made available in either hard copy computer printout or computer magnetic data tapes. It is considered that users of these tabulations for subsequent computations would prefer a computer tape format to that of a hard copy. A hard copy computer printout or computer data tapes for these tabulations may be obtained by referencing this report in a letter request to the following address:

Director
Space Sciences Laboratory, ES01
NASA, Marshall Space Flight Center
Marshall Space Flight Center, AL 35812

APPROVAL

VECTOR WIND AND VECTOR WIND SHEAR MODELS 0 TO 27 KM ALTITUDE FOR CAPE KENNEDY, FLORIDA, AND VANDENBERG AFB, CALIFORNIA

By O. E. Smith

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

William W. Vaughan

WILLIAM W. VAUGHAN
Chief, Aerospace Environment Division

Ray V. Hembree, for

CHARLES A. LUNDQUIST
Director, Space Sciences Laboratory

☆U.S. GOVERNMENT PRINTING OFFICE 1976–641-255/1 REGION NO. 4