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BY A PARTIALLY DEBONDED COMPOSITE
QUARTER-PLATE

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# A Metal Sheet Stiffened
# by a Partially Debonded Composite
# Quarter-Plate

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## ABSTRACT

An isotropic sheet stiffened by means of an orthotropic quarter-plate is considered. The quarter-plate is assumed to be perfectly bonded to the metal sheet everywhere except an area of debonding which may develop due to high stress concentrations. The adhesive which has a small constant thickness will be treated as a shear spring. The loads are applied at infinity and supposed to be transmitted through the metal plate. Shear stress distribution between the two plates are obtained from the continuity of displacements along an area where they are bonded to each other.

## INTRODUCTION

As the composite materials are used in structures at an increasing rate, the problems associated with the life prediction of such structural components become of major interest. Among others, one can mention the use of such stiffening elements as stringers, strips and composite cover plates as some of the problem areas. Hence, significant number of solutions are now available in the open literature related to these areas.

In this work, a sandwich panel will be considered where a metal sheet is assumed to be stiffened by a composite quarter-plate. Two plates are perfectly bonded to each other except an area of debonding which may develop during the life of the structure. Hence, the effect of the cover plate as a stiffening element and the way this is influenced by the debonding process will be investigated. The loads are applied at the boundary of the infinite metal sheet.

The method of solution consists of satisfying the continuity of displacements in an area where two plates are perfectly bonded to each other [1]. The solution of the resulting integral equations gives the shear stress distribution between the two plates. Application of this approach requires the knowledge of Green's functions for both isotropic ([1], [2]) and orthotropic ([3], [4]) plates which are listed in the Appendices. Although the solution is presented for a metal sheet without a crack, by using the Green's function for a cracked plate given in Appendix A, results can also be obtained for a cracked sheet. As indicated in Appendix B, the quarter-plate solution is based on an iterative process whereas the Green's function for an infinite plate with or without a crack is known in closed form ([1], [2] and Appendix A).

Here, the boundary of the debond area will be assumed known. Otherwise, it can be determined by an iteration and using an appropriate failure criterion on the boundary [1].

## FORMULATION OF THE PROBLEM

Let D denote the region where the two plates are bonded to each other and $h_c$, $h_a$ and $h_p$ represent the thicknesses of the cover plate, the adhesive and the metal sheet respectively. The elastic constants for the orthotropic quarter-plate are $E_x$, $E_y$, $G_{xy}$, and $\nu_{yx}$ and for the isotropic plate are E and $\nu$. The shear modulus of the adhesive is $\mu_a$.

$T_x$, $T_y$ are the loads per unit length applied at infinity. Due to the relatively small thicknesses of the plates and the adhesive, the problem will be solved under the assumption of generalized plane stress and the unknown shear stresses in the adhesive will be considered as body forces in the plate solutions [1]. This will satisfy the stress continuity conditions and the displacement conditions can then be written as follows:

$$u_p(z) - u_c(z) = \frac{h_a}{\mu_a} P(z)$$

$$v_p(z) - v_c(z) = \frac{h_a}{\mu_a} Q(z) \quad , \quad z \varepsilon D \quad . \tag{1}$$

where $u_c$, $v_c$ and $u_p$, $v_p$ are the displacements of the cover plate and the metal sheet in x,y directions, P and Q are the shear stresses in the adhesive (see Fig. 2) and

$$z = x + iy \tag{2}$$

The displacements can be expressed as

$$u_p(x,y) = \frac{T_x - \nu T_y}{Eh_p} \; x + \frac{1}{4\pi\mu_p h_p(1+\kappa)} \iint_D [k_1(x,y,x_o,y_o) \cdot$$

$$P(x_o,y_o) + k_2(x,y,x_o,y_o)Q(x_o,y_o)] \; dx_o dy_o \; + \; r\cdot b\cdot displ.$$

$$v_p(x,y) = \frac{T_y - \nu T_x}{Eh_p} \; y + \frac{1}{4\pi\mu_p h_p(1+\kappa)} \iint_D [k_3(x,y,x_o,y_o) \cdot$$

$$P(x_o,y_o) + k_4(x,y,x_o,y_o)Q(x_o,y_o)] \; dx_o dy_o \; + \; r\cdot b\cdot displ. \tag{3}$$

$$u_c(x,y) = \iint_D [k_5(x,y,x_o,y_o)P(x_o,y_o) + k_6(x,y,x_o,y_o)Q(x_o,y_o)]$$

$$dx_o dy_o + r\cdot b\cdot displ.$$

$$v_c(x,y) = \iint_D [k_7(x,y,x_o,y_o)P(x_o,y_o) + k_8(x,y,x_o,y_o)Q(x_o,y_o)]$$

$$dx_o dy_o + r\cdot b\cdot displ. \tag{4}$$

where $\kappa=(3-\nu)/(1+\nu)$ and the kernels $k_j(x,y,x_o,y_o)$ $j=1,..4$ are given in Appendix A and $k_j(x,y,x_o,y_o)$ $j=5...8$ are obtained by an iterative process in Appendix B [4].*

Hence, from equations (1)-(4), the integral equations of the problem is obtained as

$$P(x,y) + \iint_D [k_{11}(x,y,x_o,y_o)P(x_o,y_o) + k_{12}(x,y,x_o,y_o)Q(x_o,y_o)]dx_o dy_o$$

$$= (T_x-\nu T_y)\frac{\mu_a}{Eh_p h_a}X$$

$$Q(x,y) + \iint_D [k_{21}(x,y,x_o,y_o)P(x_o,y_o) + k_{22}(x,y,x_o,y_o)Q(x_o,y_o)]dx_o dy_o$$

$$= (T_y-\nu T_x)\frac{\mu_a}{Eh_p h_a}y \quad , \quad (x,y)\epsilon D \tag{5}$$

*Note that the location and orientation of the cover plate with respect to the metal sheet is insignificant if the metal sheet does not have a crack. In general, the rigid body displacements for both plates should be the same.

where

$$k_{11} = -\frac{\mu_a}{h_a}(\gamma k_1 - k_5) \qquad k_{12} = -\frac{\mu_a}{h_a}(\gamma k_2 - k_6)$$

$$k_{21} = -\frac{\mu_a}{h_a}(\gamma k_3 - k_7) \qquad k_{22} = -\frac{\mu_a}{h_a}(\gamma k_4 - k_8)$$

$$\gamma = \frac{1}{4\pi\mu_p h_p(1+\kappa)} \tag{6}$$

The system of Fredholm integral equations (5) with logarithmic singularities can be solved numerically as follows

$$P(x_j,y_j) + \sum_k [k_{11}(x_j,y_j,x_{ok},y_{ok})P(x_{ok},y_{ok}) + k_{12}(x_j,y_j,x_{ok},y_{ok})$$

$$\cdot\, Q(x_{ok},y_{ok})]\,\Delta Ak = (T_x - \nu T_y)\frac{\mu_a}{Eh_p h_a}x_j$$

$$Q(x_j,y_j) + \sum_k [k_{21}(x_j,y_j,x_{ok},y_{ok})P(x_{ok},y_{ok}) + k_{22}(x_j y_j,x_{ok},y_{ok})$$

$$Q(x_{ok},y_{ok})]\,\Delta Ak = (T_y - \nu T_x)\frac{\mu_a}{Eh_p h_a}y_j \qquad i = \ldots \tag{7}$$

where $\Delta Ak$'s are the appropriately chosen area elements covering the entire domain D. Solving (7) the shear stresses P and Q at $x_{ok},y_{ok}$ locations are obtained.

## ACKNOWLEDGMENT

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## APPENDIX A

Kernels $k_j(x,y,x_0,y_0)$ $j=1,..4$: From [2] and [5]

(a) For a cracked sheet [5]:

$$k_1(x,y,x_0,y_0) = -\mathrm{Re}\,[f_1(z,z_0) + f_2(z,z_0)]$$

$$k_2(x,y,x_0,y_0) = -\mathrm{Re}\,[if_1(z,z_0) - if_2(z,z_0)]$$

$$k_3(x,y,x_0,y_0) = -\mathrm{Im}\,[f_1(z,z_0) + f_2(z,z_0)]$$

$$k_4(x,y,x_0,y_0) = -\mathrm{Im}\,[if_1(z,z_0) - if_2(z,z_0)] \qquad (A.1)$$

$$f_1(z,z_0) = -\kappa[\log(z-z_0) + \log(\bar{z}-\bar{z}_0)] + \frac{\kappa}{2}[\theta_1(z,z_0) + \theta_1(\bar{z},\bar{z}_0)] - \frac{1}{2}[\theta_1(\bar{z},z_0) + \kappa^2\theta_1(z,\bar{z}_0)] + (\frac{\kappa-1}{2})[\kappa\theta_2(z) - \theta_2(\bar{z})] + (\frac{z_0-\bar{z}_0}{\bar{z}-z_0})\theta_5(z,z_0)$$

$$f_2(z,z_0) = \frac{z-z_0}{\bar{z}-\bar{z}_0} - 1 + \kappa\theta_3(z,z_0) - \theta_3(\bar{z},z_0) - \theta_4(z,z_0) + \kappa\theta_4(z,\bar{z}_0) \qquad (A.2)$$

$$\theta_1(z,z_0) = \log[z\, z_0 - a^2 + \sqrt{z_0^2-a^2}\ \sqrt{z^2-a^2}]$$

$$\theta_2(z) = \log(z + \sqrt{z^2-a^2})$$

$$\theta_3(z,z_0) = \frac{z_0-\bar{z}_0}{2\sqrt{\bar{z}_0^2-a^2}}[1 + f(z,\bar{z}_0)]$$

$$\theta_4(z,z_0) = \frac{(z-\bar{z})}{2\sqrt{\bar{z}^2-a^2}} f(\bar{z},\bar{z}_0)$$

$$\theta_5(z,z_0) = \frac{(z-\bar{z})}{2\sqrt{z^2-a^2}}\,[f(\bar{z},z_0) - J(z_0)] \tag{A.3}$$

$$f(z,z_0) = \frac{I(z)-I(z_0)}{z-z_0}$$

$$I(z) = \sqrt{z^2-a^2} - z$$

$$J(z) = \frac{z}{\sqrt{z^2-a^2}} - 1$$

$$z = x + iy$$

$$z_0 = x_0 + iy_0 \tag{A.4}$$

(b) <u>For a plate without a crack [2]</u>:

$$k_1(x,y,x_0,y_0) = 2\kappa \mathrm{Re}[\log(z-z_0)] - \mathrm{Re}[f_3(z)]$$

$$k_2(x,y,x_0,y_0) = -\mathrm{Im}[f_3(z)]$$

$$k_3(x,y,x_0,y_0) = -\mathrm{Im}[f_3(z)]$$

$$k_4(x,y,x_0,y_0) = 2\kappa \mathrm{Re}[\log(z-z_0)] + \mathrm{Re}[f_3(z)]$$

$$f_3(z) = \frac{z-z_0}{\bar{z}-\bar{z}_0} - 1$$

## APPENDIX B

Kernels $k_j(x,y,x_0,y_0)$ $j=5,..8$:

These kernels (Green's functions) can be obtained by an iteration (successive approximation) scheme introduced by Hetenyi and developed in [4] for orthotropic quarter-plate.

Iteration starts with a basic symmetric system as introduced in Fig. 3c. The stresses and the displacements for this system will be denoted by

$$\sigma_x^c, \sigma_y^c, \tau_{xy}^c, u^c, v^c(x,y) \tag{B.1}$$

Then the solutions to two secondary half-plate problems are found (Figs. 3a and 3b) for R=1 as

$$\sigma_x^a, \sigma_y^a, \tau_{xy}^a, u^a, v^a(x,y)$$

and

$$\sigma_x^b, \sigma_y^b, \tau_{xy}^b, u^b, v^b(x,y) \tag{B.2}$$

which are Green's functions for half-plate problems. These can be expressed either in real variables [4] or in complex variables [5].

Due to the symmetry, the basic system has zero shear stresses along the y axis, i.e., $\tau_{xy}^c(0,y)=0$, but $\sigma_x^{(c)}(0,y)=F_0(y)\neq 0$. Hence, to obtain the solution to the quarter-plate, non-zero normal stress distribution $F_0(y)$ should be erased. Using the solution to the problem (3a) and applying a symmetric normal stress distribution $\sigma_x(0,y)=-F_0(y)$ on the boundary we obtain the stress and displacement fields

$$\sigma_x', \sigma_y', \tau_{xy}', u', v'(x,y) \tag{B.3}$$

If this system is superposed with the basic system, the resulting problem yields

$$\sigma_x(0,y) = 0 \text{ but } \sigma_y(x,0) = F_1(x) \neq 0 \tag{B.4}$$

Hence, second step will be to erase these non-zero $\sigma_y(x,0)$ stresses along the x axis. This can be done using the solution of problem (3b) thus yielding non-zero normal stresses along y axis again. Since in this approach, the shear stresses along the x and y axes are always zero, repeating this process, one can obtain stress-free surfaces along these axes and the solution of the problem of a quarter-plate, within a reasonable degree of convergence [4]. The kernels ($k_5, k_7$) and $k_6, k_8$) can be obtained numerically by considering (P=1, Q=0) and (P=0, Q=1) respectively.

If one uses the expressions given in [4] for the solutions of problems (3a) and (3b) the iteration scheme will give

$$\begin{Bmatrix} \sigma_x'(x,y) \\ \sigma_y'(x,y) \\ \tau_{xy}'(x,y) \end{Bmatrix} = -\frac{k_1 k_2 (k_1+k_2)}{\pi} \int_{\eta=0}^{\infty} F_o(\eta) d\eta [\phi_{\substack{1\\2\\3}}(x,y,\eta) + \phi_{\substack{1\\2\\3}}(x,y,-\eta)] \tag{B.5}$$

where

$$\phi_{\substack{1\\2\\3}}(x,y,\eta) = \begin{Bmatrix} x^3 \\ x(y-\eta)^2 \\ (y-\eta)x^2 \end{Bmatrix} / \{[k_1^2x^2 + (y-\eta)^2][k_2^2x^2 + (y-\eta)^2]\} \tag{B.6}$$

where

$$\begin{Bmatrix} k_1^2 \\ k_2^2 \end{Bmatrix} = \frac{1}{2a_{22}} [2a_{12} + a_{66} \pm \sqrt{(2a_{12}+a_{66})^2 - 4a_{11}a_{22}}]$$

$$a_{11} = \frac{1}{E_x} \quad , \quad a_{12} = -\frac{\nu_{yx}}{E_y} = -\frac{\nu_{xy}}{E_x} \quad , \quad a_{22} = \frac{1}{E_y}$$

$$a_{66} = \frac{1}{G_{xy}} \tag{B.7}$$

And the second iteration gives

$$\begin{Bmatrix} \sigma_x^2(x,y) \\ \sigma_y^2(x,y) \\ \sigma_{xy}^2(x,y) \end{Bmatrix} = -\frac{k_1+k_2}{\pi} \int_{\xi=0}^{\infty} F_1(\xi)d\xi[\psi_{\substack{1\\2\\3}}(x,y,\xi) + \psi_{\substack{1\\2\\3}}(x,y,-\xi)] \tag{B.8}$$

where

$$\psi_{\substack{1\\2\\3}}(x,y,\xi) = \begin{Bmatrix} (x-\xi)^2y \\ y^3 \\ (x-\xi)y^2 \end{Bmatrix} / \{[k_1^2(x-\xi)^2+y^2][k_2^2(x-\xi)^2+y^2]\} \tag{B.9}$$

etc. If these steps are repeated the final stress state can be obtained as

$$\begin{Bmatrix}\sigma_x(x,y)\\ \sigma_y(x,y)\\ \tau_{xy}(x,y)\end{Bmatrix} = \begin{Bmatrix}\sigma_x^c\\ \sigma_y^c\\ \tau_{xy}^c\end{Bmatrix} + \frac{(k_1+k_2)}{\pi}\ \{-k_1k_2\int_{\eta=0}^{\infty}[(\phi_{\substack{1\\2\\3}}(x,y,\eta)+\phi_{\substack{1\\2\\3}}(x,y,-\eta))$$

$$\sum_{m=0,2,4}^{\infty} F_m(\eta)]d\eta - \int_{\xi=0}^{\infty}[(\psi_{\substack{1\\2\\3}}(x,y,\xi)+\psi_{\substack{1\\2\\3}}(x,y,-\xi))$$

$$\sum_{m=1,3,5}^{\infty} F_m(\xi)]d\xi\} \tag{B.10}$$

And the functions $F_m, m=1,2..$ are determined from the following recurrence relations

$$F_{m+1}(x) = -\frac{k_1k_2(k_1+k_2)}{\pi}\int_{\eta=0}^{\infty} F_m(\eta)d\eta\ \frac{2x\eta^2}{(k_1^2x^2+\eta^2)(k_2^2x^2+\eta^2)}$$

$$m=0,2,4....$$

$$F_{m+1}(y) = -\frac{k_1+k_2}{\pi}\int_{\xi=0}^{\infty} F_m(\xi)d\xi\ \frac{2y\xi^2}{(k_1^2\xi^2+y^2)(k_2^2\xi^2+y^2)}$$

$$m=1,3,5...,\tag{B.11}$$

$F_0(\eta)$ is obtained from the solution of problem (3c), [4].

This scheme is applied to the displacements in an identical manner by changing the functions $\phi$ and $\psi$. The use of complex variable solution [6] may prove to be much more convenient.

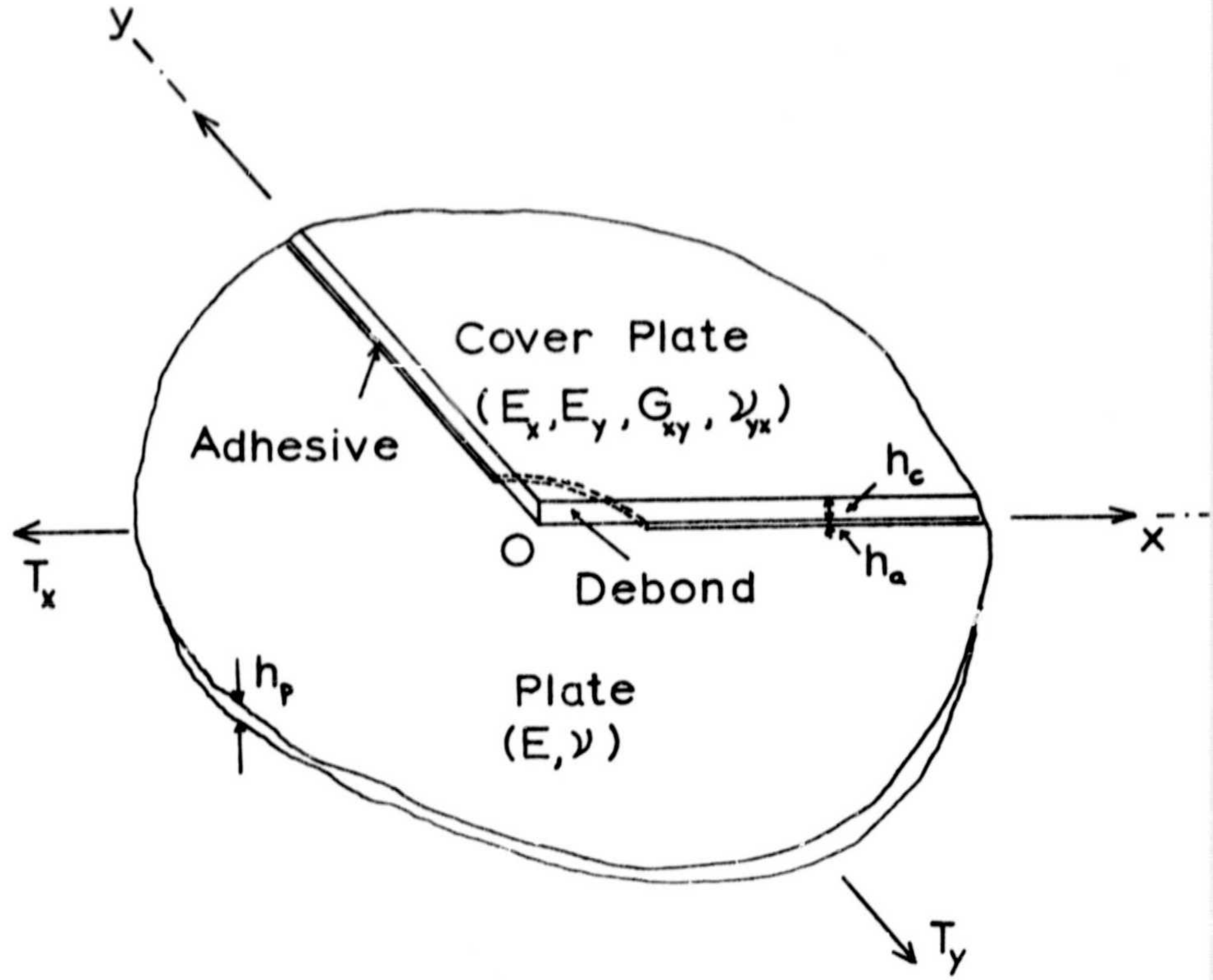


Fig. 1. Geometry of the problem.

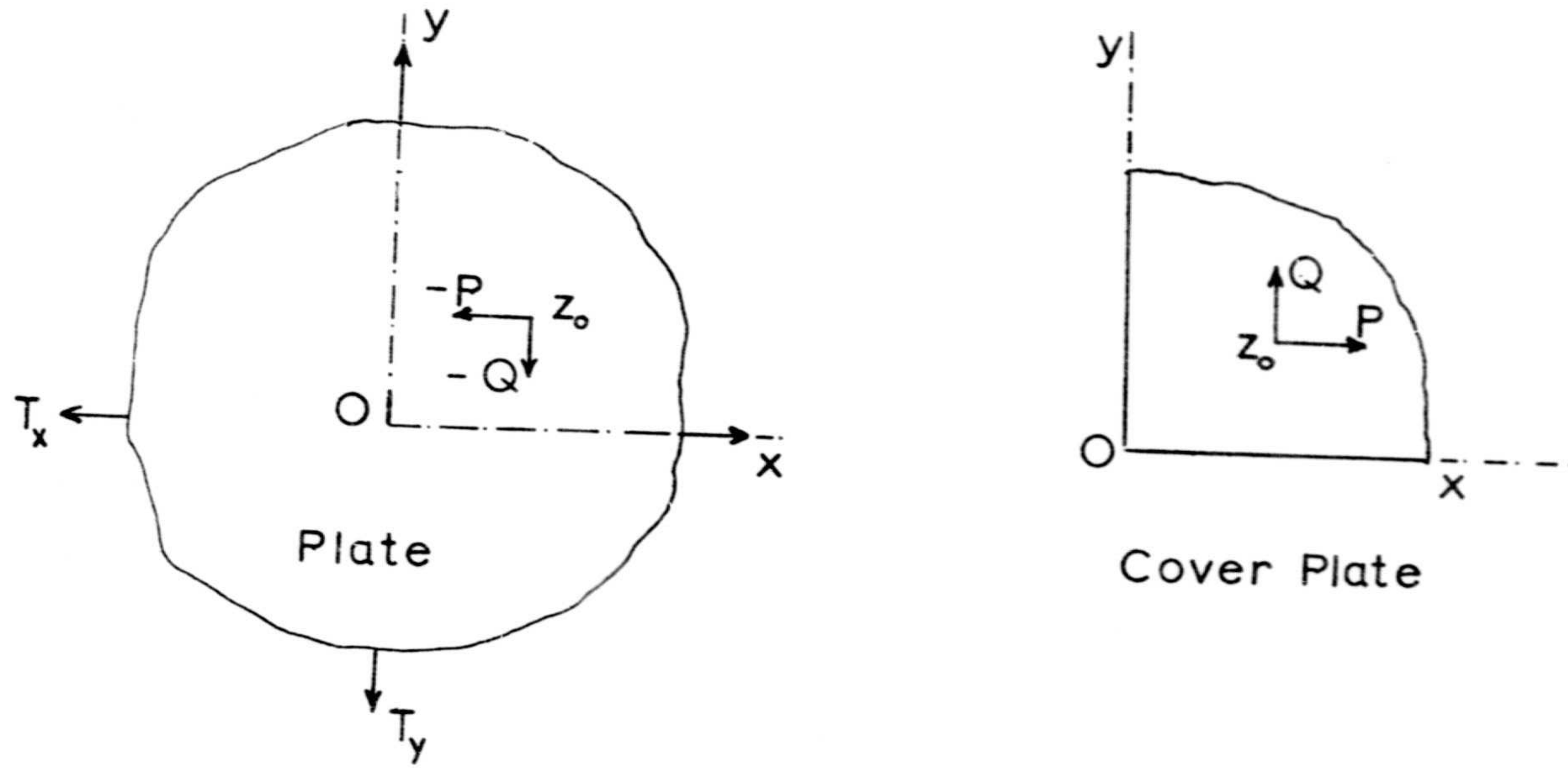


Fig. 2. Free-body diagrams.

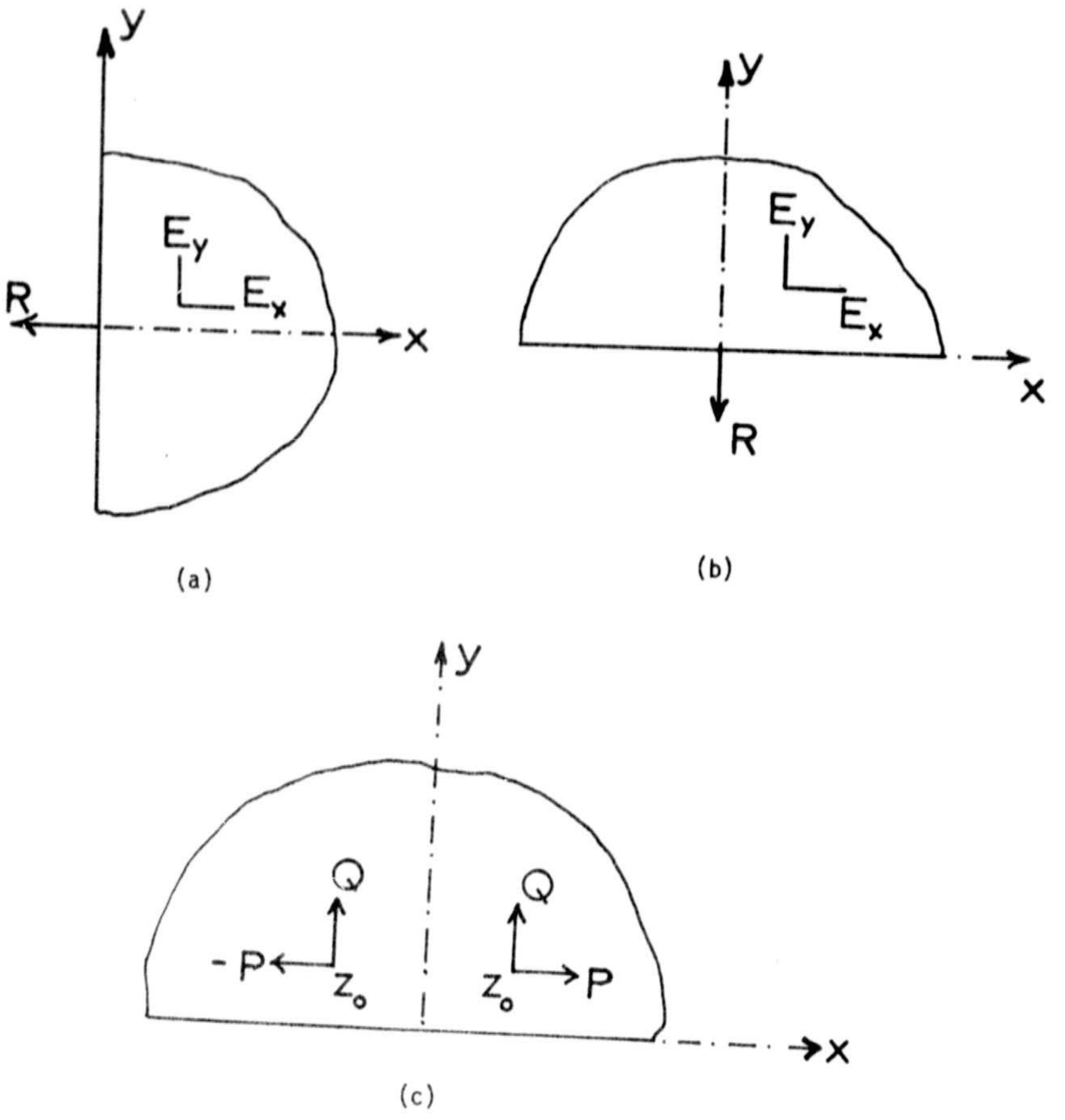


Fig. 3. Secondary half-plane problems.