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ZONAL AND TESSERAL HARMONIC COEFFICIENTS FOR THE GEOPOTENTIAL FUNCTION, FROM ZERO TO 18TH ORDER

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16. Abstract

Zonal and tesseral harmonic coefficients for the geopotential function are usually tabulated in normalized form to provide immediate information as to the relative significance of the coefficients in the gravity model. The normalized form of the geopotential coefficients cannot be used for computational purposes unless the gravity model has been modified to receive them. This modification is usually not done because the absolute or unnormalized form of the coefficients can be obtained from the simple mathematical relationship that relates the two forms. This computation can be quite tedious for hand calculation, especially for the higher order terms, and can be costly in terms of storage and execution time for machine computation. In this report, zonal and tesseral harmonic coefficients for the geopotential function are tabulated in absolute or unnormalized form. The report is designed to be used as a ready reference for both hand and machine calculation to save the user time and effort.

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SUMMARY

The zonal and tesseral harmonic coefficients for the geopotential function are given in unnormalized form, from zero to 18th order.

INTRODUCTION

The coefficients of the geopotential function, both zonal and tesseral, are usually published in normalized form so that all coefficients have about the same order of magnitude. Although this practice is useful, it obscures the actual magnitude of the coefficients. This type of publication results in an unwieldy reference for computation purposes, and it requires the evaluation of lengthy factorial expressions to arrive at useful data. This report has been prepared to provide the user of the coefficients of the geopotential function with a ready reference for the zonal and tesseral harmonic coefficients. The data for this work were obtained from reference 1.

DISCUSSION

The geopotential function V is given as

$$V = \frac{g_E}{r} \sum_{n=0}^{\infty} \left(\frac{R_E}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\sin \varphi) \left[C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)\right] \qquad (1)$$

where g_E is the gravitational parameter, having units of length^3/time^2; r is the magnitude of the position vector to the point in question, having units of length; R_E is the equatorial radius of the attracting body (in this case, the Earth), having units of length; C_{nm} and S_{nm} are the harmonic coefficients; $P_{nm}(\sin \varphi)$ is the Legendre polynomial function of degree n and order m given by the expression

$$P_{nm}(\sin\varphi) = \cos^m(\varphi)\,\frac{1}{2^n}\sum_{t=0}^{\ell}\frac{(-1)^t(2n-2t)!}{t!\,(n-t)!\,(n-m-2t)!}\,\sin^{n-m-2t}(\varphi) \qquad (2)$$

where ℓ is the greatest integer equal to $(n - m)/2$ if $n - m$ is even and the greatest integer equal to $(n - m - 1)/2$ if $n - m$ is odd, φ is the latitude of the point in question, and λ is the east longitude of the point in question.

The harmonic coefficients are further classified as follows. When $m = 0$, the harmonic coefficients are called zonal; when $n \neq m$, the harmonic coefficients are called tesseral. For the particular case in which $n = m$, the harmonic coefficients are called sectorial. It may be seen from equation (1) that there are no zonal S_{n0} coefficients. Further, $C_{n0} = -J_{n0}$ where J_{n0} refers to the usually published zonal harmonic coefficients. The term $C_{00} = +1$ as the geopotential function is associated with an inverse square law of attraction. Tables I and II contain the unnormalized harmonic coefficients C_{nm} and S_{nm}, respectively. The normalized barred values are converted to unnormalized values as follows.

$$C_{nm}, S_{nm} = \left[\frac{(2 - S_{m0})(2n + 1)(n - m)!}{(n + m)!}\right]^{\frac{1}{2}} \overline{C}_{nm}, \overline{S}_{nm} \qquad (3)$$

where $S_{m0} = 1$ for $m = 0$ and $S_{m0} = 0$ for $m \neq 0$.

CONCLUDING REMARK

The information presented in this report is in a form designed for immediate use for either hand or machine computation of the geopotential function and its gradient.

Lyndon B. Johnson Space Center
National Aeronautics and Space Administration
Houston, Texas, April 21, 1976
986-16-00-00-72

REFERENCE

1. Gaposchkin, E. M.: 1973 Smithsonian Standard Earth (III). Special Rep. No. 353, Smithsonian Astrophysical Observatory (Cambridge, Mass.), Nov. 28, 1973, pp. 282-293.

TABLE I.- UNNORMALIZED HARMONIC COEFFICIENT C_{nm}

n	m					
	0	1	2	3	4	5
0	0.10000000 + 01					
1	0	0				
2	-.10826370 - 02	0	0.15362188 - 05			
3	.25410000 - 05	.21577626 - 05	.26584006 - 06	0.68342573 - 07		
4	.16179990 - 05	-.49092463 - 06	.76688186 - 07	.62092125 - 07	-0.22210654 - 08	
5	.22800004 - 06	-.45957673 - 07	.96888828 - 07	-.19301654 - 07	-.90188441 - 09	0.34362977 - 09
6	-.55199908 - 06	-.56779905 - 07	.30690303 - 08	.91517315 - 10	-.37866087 - 09	-.10898613 - 09
7	.35199996 - 06	.17223660 - 06	.20343787 - 07	.30980538 - 08	-.60769056 - 09	.12290637 - 10
8	.20499998 - 06	.75219190 - 08	.91037581 - 08	-.89552578 - 09	-.29125507 - 09	.27725347 - 10
9	.15399990 - 06	.11760482 - 06	-.15247844 - 08	-.75011669 - 09	-.34971561 - 10	-.60301570 - 11
10	.23699982 - 06	.54999323 - 07	-.22151985 - 08	-.77585383 - 09	-.12890595 - 10	-.38185997 - 12
11	-.31200001 - 06	-.71984359 - 08	-.10487030 - 08	-.50681989 - 11	-.66005425 - 11	-.73967019 - 13
12	.19200000 - 06	-.32233096 - 07	-.44445551 - 08	.43041427 - 09	-.15638137 - 10	.21782102 - 11
13	.33900010 - 06	.46918691 - 08	-.43356630 - 09	-.99035416 - 10	.93539331 - 11	.73851040 - 12
14	-.10499994 - 06	-.75462040 - 08	-.57967993 - 09	.24725700 - 09	-.54146823 - 11	-.18188947 - 13
15	-.10500024 - 06	.14921644 - 07	-.40493840 - 09	-.12690067 - 09	.21218397 - 11	.35394328 - 12
16	-.33999998 - 07	-.49056270 - 08	16513794 - 09	.99603810 - 10	.52637402 - 11	-.17544334 - 12
17	.22000007 - 06	.41416280 - 08	4899410 - 09	-.12331832 - 10	-.39931213 - 11	.22673488 - 12
18	.10200002 - 06	-.10957785 - 07	.23776071 - 09	-.48172477 - 10	.22298275 - 11	.73929837 - 14

TABLE I.- Continued

n	m					
	6	7	8	9	10	11
0						
1						
2						
3						
4						
5						
6	-0.67880982 - 11					
7	-.19084904 - 10	-0.46109211 - 12				
8	-.27242185 - 11	.10452042 - 11	0.21628914 - 12			
9	.64453161 - 12	-.37889020 - 12	.76886527 - 13	-0.26604530 - 14		
10	-.65343725 - 12	.15592804 - 12	.12470313 - 15	.14581345 - 14	0.55347647 - 15	
11	.25055278 - 12	-.14019397 - 13	-.64942727 - 15	.13071130 - 15	.49867891 - 16	0.17599280 - 16
12	-.50216354 - 13	.66073891 - 14	.89257829 - 15	-.27874032 - 15	-.13697081 - 16	-.34497846 - 18
13	-.32766339 - 13	-.97255347 - 14	-.30911939 - 16	-.12443074 - 16	.46995620 - 18	-.71747410 - 18
14	-.13600380 - 13	.53747475 - 14	-.11446413 - 15	-.12619922 - 16	.13729037 - 17	.39129532 - 18
15	.66897366 - 14	.14355000 - 14	-.23948717 - 15	-.12115734 - 16	.13606205 - 17	-.85907798 - 19
16	-.17173078 - 14	-.69378804 - 16	-.21660554 - 15	.36383409 - 17	-.43341732 - 18	-.17780157 - 19
17	-.14943152 - 13	.34272030 - 15	.52763253 - 16	-.36071852 - 17	-.61725185 - 19	-.17944767 - 19
18	.57199516 - 14	-.10769316 - 16	.43916072 - 16	-.17937162 - 19	.13184452 - 18	.51307399 - 20

TABLE I.- Concluded

n	m						
	12	13	14	15	16	17	18
0							
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12	-0.24791835 - 18						
13	-.87009645 - 19	-0.25228059 - 19					
14	.63022309 - 21	.22473701 - 20	-0.90987867 - 21				
15	.77676168 - 20	-.84003453 - 21	.25330996 - 22	-0.27247168 - 22			
16	.11482143 - 20	.16918243 - 21	-.10477119 - 22	-.69365304 - 23	-0.29359521 - 24		
17	.97587650 - 21	.63286457 - 22	-.12643725 - 23	.11328557 - 23	-.66836336 - 25	-0.43916722 - 25	
18	-.88213613 - 22	-.27730826 - 22	.74918243 - 24	-.29690427 - 24	.17594420 - 25	.29920920 - 26	-0.48942965 - 29

TABLE II.- UNNORMALIZED HARMONIC COEFFICIENT S_{nm}

n	m					
	0	1	2	3	4	5
0						
1		0				
2		0	-0.88149100 - 06			
3		0.24126717 - 06	-.25794649 - 06	0.21311125 - 06		
4		-.45669614 - 06	.15020563 - 06	-.71253553 - 08	0.75347614 - 08	
5		-.68484785 - 07	-.64588236 - 07	-.53971559 - 08	-.35344245 - 09	-0.21382371 - 08
6		.13970346 - 07	-.50574933 - 07	.60242316 - 09	-.11468848 - 08	-.49201944 - 09
7		.40719918 - 07	.17252129 - 07	-.48799206 - 08	-.58902474 - 09	.30796139 - 10
8		.33279647 - 07	.85082849 - 08	-.51272991 - 09	.34602120 - 09	.14689474 - 10
9		.26700368 - 07	.16773113 - 08	-.17446947 - 09	-.32967416 - 10	.37674034 - 12
10		-.37171875 - 07	-.37861010 - 08	-.42403470 - 09	-.46179273 - 10	-.73902372 - 11
11		.44547775 - 07	-.32617227 - 08	-.17572647 - 09	-.82321514 - 10	.24314703 - 11
12		-.37455158 - 07	.21141108 - 08	-.18127686 - 09	.16628034 - 10	.74347787 - 12
13		-.17648980 - 07	-.36811927 - 08	.15083154 - 09	-.24898756 - 10	.70286377 - 12
14		.27532950 - 07	.99749549 - 09	-.65391778 - 10	-.69240992 - 12	-.77185959 - 12
15		-.84834514 - 08	-.22720500 - 08	.96427367 - 10	.10035662 - 11	-.80827274 - 13
16		.26678793 - 07	.14825721 - 08	.10088679 - 10	.41345257 - 11	.21306399 - 12
17		-.19654235 - 07	-.74627732 - 09	-.28369992 - 10	.62997120 - 11	-.13901869 - 12
18		-.34671201 - 07	.76571115 - 09	-.28163345 - 10	-.33843256 - 11	-.25486394 - 13

TABLE II.- Continued

n	m					
	6	7	8	9	10	11
0						
1						
2						
3						
4						
5						
6	-0.61336581 - 10					
7	.59598680 - 11	-0.16504040 - 12				
8	.78429295 - 11	.12539595 - 11	0.11888573 - 12			
9	.14676655 - 11	-.28546366 - 12	.31494288 - 14	0.45840870 - 14		
10	-.81403752 - 13	.18228220 - 13	.81074789 - 15	.10476353 - 15	0.41066782 - 15	
11	-.10422595 - 12	-.53256350 - 13	.21543894 - 14	.41029304 - 15	-.73443528 - 16	-0.51972861 - 17
12	.83075022 - 13	.70586752 - 14	.12631793 - 14	.35155738 - 16	-.12904565 - 16	-.21047103 - 17
13	-.16933613 - 13	.14082703 - 14	.16114797 - 15	.78374830 - 16	.85943131 - 18	.17745217 - 18
14	-.27428216 - 13	.18186368 - 15	-.35810041 - 15	.31371315 - 16	-.16210543 - 17	-.92965263 - 20
15	-.20221838 - 13	.74394730 - 15	.21140956 - 15	.14901851 - 16	-.15724687 - 18	-.66059724 - 20
16	-.95949145 - 14	.93165562 - 16	-.92637750 - 16	-.12632343 - 16	-.48910332 - 19	.25361968 - 19
17	-.56786805 - 14	-.68293160 - 15	.75236040 - 17	-.13363291 - 17	.31664041 - 18	-.17532903 - 20
18	-.10747706 - 14	-.11317166 - 15	-.18038400 - 16	-.25109046 - 18	.24689642 - 19	.47613636 - 20

TABLE II.- Concluded

n	m						
	12	13	14	15	16	17	18
0							
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12	-0.15088574 - 18						
13	.14919804 - 18	0.26306428 - 19					
14	-.16608049 - 19	.34754576 - 20	0.45556690 - 22				
15	.10918469 - 20	-.11271717 - 21	-.71881359 - 22	0.16870541 - 22			
16	-.91561621 - 21	.44330079 - 22	-.57643200 - 23	-.23733371 - 23	-0.35333419 - 24		
17	.19412733 - 21	.24594818 - 22	.16408705 - 23	.73715316 - 24	-.45093232 - 25	-0.46148810 - 26	
18	.93575138 - 22	-.44165640 - 22	-.27217231 - 23	-.12592385 - 24	-.34114057 - 26	-.39914306 - 26	0.71302344 - 27