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COMMENTS ON DIFFERENCE SCHEMES FOR THE THREE-DIMENSIONAL TRANSONIC SMALL-DISTURBANCE EQUATION FOR SWEPT WINGS

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COMMENTS ON DIFFERENCE SCHEMES FOR THE THREE-DIMENSIONAL TRANSONIC SMALL-DISTURBANCE EQUATION FOR SWEPT WINGS

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SUMMARY

Certain problems arise in constructing stable finite-difference schemes for the three-dimensional transonic small-disturbance equation when crossflow terms are included to better approximate swept wings. These problems are discussed and some possible remedies are offered.

INTRODUCTION

Recently, some effort (refs. 1 to 3) has been devoted to improving the three-dimensional transonic small disturbance equation for applications to swept wings. It seems that the important crossflow term that needs to be included is $\phi_y \phi_{xy}$, where x is the longitudinal coordinate and y is the spanwise coordinate. Without this term, the small disturbance equation cannot adequately predict weak, swept shock waves. The purpose of this paper is to point out some numerical difficulties that may arise in constructing stable finite-difference approximations to equations which include this additional term.

ANALYSIS

Jameson (ref. 4) has shown that it is advantageous to examine the canonical form for the potential equation, particularly when studying appropriate methods for combining central and backward differences for various derivatives in supersonic regions of the flow. As in reference 4, then, the canonical form is written as:

$$(a^2-q^2)\ \phi_{ss} + a^2\ (\nabla^2\phi-\phi_{ss}) = 0 \qquad (1)$$

where ϕ_{ss} represents differencing in the local streamline direction, and

$$q^2 = 1 + 2\phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2 \qquad (2)$$

$$a^2 = a_\infty^2 - \frac{\gamma-1}{2}\ (q^2-1) \qquad (3)$$

$$a_\infty^2 = M_\infty^{-2} \qquad (4)$$

$$q^2\phi_{ss} = u^2\phi_{xx} + v^2\phi_{yy} + w^2\phi_{zz} + 2uv\phi_{xy} + 2uw\phi_{xz} + 2vw\phi_{yz} \qquad (5)$$

$$u = 1 + \phi_x \qquad (6)$$

$$v = \phi_y \qquad (7)$$

$$w = \phi_z \qquad (8)$$

Combining equations (2) to (4) gives:

$$\frac{a^2 - q^2}{a_\infty^2} = 1 - M_\infty^2 - \frac{\gamma + 1}{2} M_\infty^2 (q^2 - 1) \tag{9}$$

$$\frac{a^2}{a_\infty^2} = 1 - \frac{\gamma - 1}{2} M_\infty^2 (q^2 - 1) \tag{10}$$

If terms involving perturbation velocity products are ignored, equations (2), (5), (9), and (10) become:

$$q^2 \approx 1 + 2\phi_x \tag{11}$$

$$\frac{a^2 - q^2}{a_\infty^2} \approx 1 - M_\infty^2 - (\gamma+1)M_\infty^2\phi_x \tag{12}$$

$$\frac{a^2}{a_\infty^2} \approx 1 - (\gamma-1)M_\infty^2\phi_x \tag{13}$$

$$\phi_{ss} \approx \phi_{xx} + 2\phi_y\phi_{xy} + 2\phi_z\phi_{xz} \tag{14}$$

The term $\nabla^2\phi - \phi_{ss}$ is:

$$\nabla^2\phi - \phi_{ss} \approx \phi_{yy} + \phi_{zz} - 2\phi_y\phi_{xy} - 2\phi_z\phi_{xz} \tag{15}$$

Substituting equations (12) to (15) into (1) yields:

$$A(\phi_{xx} + 2\phi_y\phi_{xy} + 2\phi_z\phi_{xz}) + B(\phi_{yy} + \phi_{zz} - 2\phi_y\phi_{xy} - 2\phi_z\phi_{xz}) = 0 \tag{16}$$

where

$$A = 1-M_\infty^2 - (\gamma + 1)\ M_\infty^2\phi_x \qquad (17)$$

$$B = 1- (\gamma - 1)\ M_\infty^2\phi_x \qquad (18)$$

In the interest of further simplication, most researchers have ignored the term $\phi_z\phi_{xz}$ as unimportant for the swept wing problem. Some discussion of the relative size of this term is given in reference 2. For the purpose of this paper, $\phi_z\phi_{xz}$ will also be deleted from equation (16), yielding finally:

$$A\ (\overline{\phi_{xx}} + 2\phi_y\overline{\phi_{xy}}) + B\ (\phi_{yy} + \phi_{zz} - 2\phi_y\phi_{xy}) = 0 \qquad (19)$$

where second derivative terms belonging to ϕ_{ss} are barred to indicate that they should be represented by upwind differences in supersonic regions of flow, and terms deriving from $\nabla^2\phi - \phi_{ss}$ should be represented by central differences, in the spirit of Jameson's rotated difference scheme (ref. 4). To a close approximation, equation (19) changes from elliptic to hyperbolic type when the coefficient A changes from positive to negative. Hence, a stable numerical scheme with artificial viscosity of the correct sign is obtained by using upwind differences for $\overline{\phi_{xx}}$ and $\overline{\phi_{xy}}$ whenever $A<0$. That is, for $\overline{\phi_{xx}}$ we have:

$$\Delta x^2\ \overline{\phi_{xx}} = \phi_{i,j,k} - 2\phi_{i-1,j,k} + \phi_{i-2,j,k} \qquad (20)$$

and for $\overline{\phi_{xy}}$ we have:

$$\Delta x \Delta y \, \overline{\phi_{xy}} = \phi_{i,j,k} - \phi_{i-1,j,k} + \phi_{i-1,j-1,k} - \phi_{i,j-1,k} \tag{21}$$

if $\phi_y > 0$

and

$$\Delta x \Delta y \, \overline{\phi_{xy}} = \phi_{i,j+1,k} - \phi_{i-1,j+1,k} + \phi_{i-1,j,k} - \phi_{i,j,k} \tag{22}$$

if $\phi_y < 0$

It is important to note that such formulas always enhance diagonal dominance; that is, they produce a contribution to the diagonal (coefficient of ϕ_{ijk}) of:

$$A\left(\frac{1}{\Delta x^2} + \frac{2|\phi_y|}{\Delta x \Delta y}\right) \tag{23}$$

Such schemes are highly desirable from the viewpoint of numerical stability.

Equation (19) still has an important deficiency: because of the absence of certain deleted small terms, it cannot be cast into divergence form. Further expedient approximations can be made, however, to achieve a divergence form. For example, in reference 1, the total coefficient of $\phi_y \phi_{xy}$ was collected and approximated by ignoring ϕ_x:

$$2(A - B) = -2M_\infty^2 \,(1 + 2\phi_x) \approx -2M_\infty^2 \tag{24}$$

and the ϕ_x- contribution in the coefficient of ϕ_{zz} was also ignored, yielding:

$$A\ \overline{\phi_{xx}} - 2M_\infty^2\ \phi_y\phi_{xy} + B\ \phi_{yy} + \phi_{zz} = 0 \tag{25}$$

The bar over ϕ_{xy} has been dropped in equation (25) because of the slight confusion created by combining the coefficients of $\overline{\phi_{xy}}$ and ϕ_{xy} in equation (24). Strictly speaking, to retain the identity of $\overline{\phi_{xy}}$ and ϕ_{xy} in equation (25), one would write:

$$A\ \overline{\phi_{xx}} + 2(1-M_\infty^2)\ \phi_y\overline{\phi_{xy}} - 2\phi_y\phi_{xy} + B\phi_{yy} + \phi_{zz} = 0 \tag{26}$$

However, upwind differencing for $\overline{\phi_{xy}}$, as given by equations (21) and (22), detracts from diagonal dominance! That is, application of formulas (20) to (22) in equation (26) produces a diagonal contribution of:

$$\frac{A}{\Delta x^2} + \frac{(1-M_\infty^2)\ |\phi_y|}{\Delta x \Delta y} \tag{27}$$

and the two terms compete with one another, since $A<0$. For large cross flows (large $|\phi_y|$), such a procedure would be unstable. From a numerical point of view, then, one should use an upwind difference only for $\overline{\phi_{xx}}$, and use central differences for all other terms in equations (25) or (26). An alternative is to retard $\overline{\phi_{xy}}$ in the x-direction only, for example:

$$2\Delta x\Delta y\overline{\phi_{xy}} = \phi_{i,j+1,k} - \phi_{i-1,j+1,k} + \phi_{i-1,j-1,k} - \phi_{i,j-1,k} \tag{28}$$

but then that is not a "rotated" difference scheme, since the direction of the crossflow is ignored. Furthermore, the artificial viscosity that is introduced has the wrong sign, since its coefficient is $1 - M_\infty^2$ rather than A.

In reference 2, this confusion was avoided by using a different approximation to equation (19). Namely, the term $\overline{\phi_{xy}}$ is dropped, and B approximated by 1, so that equation (19) becomes:

$$A\overline{\phi_{xx}} + \phi_{yy} + \phi_{zz} - 2\phi_y\phi_{xy} = 0 \tag{29}$$

Here there is no question about the differencing of the ϕ_{xy} term, since it may be considered as deriving entirely from $\nabla^2\phi-\phi_{ss}$, and thus would always be centrally differenced.

In reference 3, the difficulty of constructing a divergence form after neglecting certain terms is overcome by a simple but effective idea: the equation is expanded in the original conservative form,

$$[\rho(1+\phi_x)]_x + [\rho\phi_y]_y + [\rho\phi_z]_z = 0 \tag{30}$$

where ρ is the density. The terms neglected in reference 3 are essentially the same as those dropped in reference 1, and the same delicate point finally arises concerning $\overline{\phi_{xy}}$ and ϕ_{xy}. It is stated that retardation in the x-direction only is then used for the ϕ_{ss}-contributions, so a formula related to equation (28) is used. The equivalent artificial viscosity will have the wrong sign, for the same reason as before. Although the title of reference 3 includes the word "rotated difference," the claim here is that this is not a rotated difference scheme, but rather a "split" scheme, as in reference 2.

CONCLUDING REMARKS

Experience has shown that rotated difference schemes have nice stability properties, since if they are carried out correctly, each upwind difference (including cross derivatives like ϕ_{xy}) enhances diagonal dominance, as illustrated in equations (21) to (23). This feature is an indication that the artificial viscosity is correct (ref. 4). Retarding a difference formula for ϕ_{xy} in the x-direction only does not insure the correct sign to the resulting artificial viscosity unless the coefficient of ϕ_{xy} is the correct sign. Such retardation might damage stability more than a central difference formula, which produces zero viscosity.

It is felt that the Dutch approach (ref. 3) is a good one for deriving a conservative approximation for swept wings, but that more terms should be retained to force a "switching" coefficient similar to A (eq. (17)) on the equivalent $\overline{\phi_{xy}}$. In this way, a rotated scheme can be devised with correct artifical viscosity, and the concomitant diagonal dominance. Otherwise one should use a central difference formula for $\overline{\phi_{xy}}$.

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