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STATE-SPACE FORMULATIONS FOR FLUTTER ANALYSIS

by

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1. Introduction

Presented here are various flutter-analysis techniques obtained by approximating the aerodynamic-influence-coefficient matrix (within the flutter range of the reduced frequency) in a form which is convenient to write the dynamic equations of the aircraft in a State Space format i.e., of the type

$$s\underline{x} = \underline{A}\underline{x} \tag{1}$$

where s is the complex frequency, $\underline{x}$ is the state vector, $\underline{A}$ is a frequency-independent matrix or may be treated as such (for instance if the problem is solved by iteration with $\underline{A}$ constant in each iteration).

In recent years optimal structural design and active control technology have gained an increasingly important role in the improvement of aircraft performance; sophisticated techniques are required, especially for flexible aircrafts. The optimal design as well as the control-systems design involve the interaction of numerous disciplines including structures, aerodynamics, aeroelasticity, guidance and control.

It may be noted that for the analysis of flexible aircrafts the equations of the aircraft dynamics may be cast in the format of Eq. (1) (with constant $\underline{A}$) by approximating the aerodynamic matrix with the two lowest terms of their Taylor series around k=0 (where k is the reduced frequency): This approximation is used for instance in FCAP (Flight Control Analysis Program, Refs. 1 and 2) which is a general purpose program for the analysis of flexible aircrafts of arbitrary shape with active control. The low frequency formulation however is too restrictive for analysis of flutter or even control-surface dynamics. In this case fully-unsteady-aerodynamic influence coefficients are being used in FCAP (Refs. 1 and 2).

In addition it should be noted that there exist already aerodynamic codes capable of dealing with transient response (analysis off the imaginary axis) such as SOUSSA (Refs. 3 to 8) which is the most general one presently available, since it can be used for steady, oscillatory and fully unsteady, subsonic and supersonic flows around aircrafts having arbitrary shapes. Note that SOUSSA is the only fully unsteady code for complex configurations.*

However, the currently, available flutter-analysis routines are limited to the analysis of the aerodynamic loads for purely imaginary reduced frequencies (oscillatory flow). This shortcoming is partly due to the fact that aerodynamic loads obtained from existing aerodynamics calculation methods had been limited until very recently to sinusoidal motions only and hence the classical flutter analyses (such as the V-g method) are limited to the imaginary axis. It is obvious that the flutter analysis needs to be re-formulated if one wants to take full advantage of the transient response feature of the aerodynamic routine SOUSSA.

The purpose of this paper is to present and assess various methods for approximating the aerodynamic forces so that the two basic features introduced above (State Space formulation and off-the-imaginary axis analysis) are retained. The advantages of retaining these features are considerable, not only in simplifying the flutter analysis, but especially for more advanced applications such as optimal design of active control in which the flutter is merely a contraint to the optimization problem.

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*SOUSSA is used as aerodynamic subprogram in FCAP.[1,2]

Therefore presented here are 3 procedures for the calculation of flutter frequencies and speeds. In addition to the variations of the traditional V-g method, a new formulation incorporating state space techniques of the type $\dot{\underline{x}}=\underline{A}\underline{x}$ is introduced. The advantages of the state space format are also presented. Preliminary results are briefly described.

2. V-g Method

The V-g method is briefly outlined here. Consider the Lagrange equation of motion for simple harmonic motion, $e^{i\omega t}\underline{x}$,

$$-\omega^2 \underline{M}\,\underline{x} + (1+ig)\,\underline{K}\,\underline{x} + q\,\underline{E}(k)\,\underline{x} = 0 \tag{1}$$

where $\underline{x}$ is the vector of the generalized Lagrangian coordinates, $\underline{M}$ and $\underline{K}$ are the mass and stiffness matrices, q is the dynamic pressure and $\underline{E}(k)$ is the aerodynamic influence matrix (function of the reduced frequency $k=\omega\ell/V$) and g is a fictitious damping coefficient introduced to avoid the use of complex frequencies. With some algebraic manipulations, a standard eigenvalue problem is resulted which has the form (Ref. 9)

$$(\underline{A}-\Omega\underline{I})\ \underline{x}=0$$

where $\underline{A}$ is a function of k. It should be noted that Ω has the expression

$$\Omega = \left(\frac{\omega\,\ell}{V}\right)^2 (1+ig)$$

For different values of k, a corresponding Ω results. The flutter frequency, however, is the one in which g=0, i.e. Ω is real. This yields ω and hence V (from k).

Preliminary results are presented in Figs. a and b and compared with the ones of Ref. 10.

3. Truncated Power Series Approximation to the Aerodynamic Influence Coefficients

The flutter analysis can be greatly simplified (by casting it into a space-state format) if the aerodynamic influence coefficients E_{ij} are expressed as a truncated power series in the reduced frequency k

$$\underline{E}(k) = \underline{E}_0 + ik\,\underline{E}_1 - k^2\underline{E}_2 - ik^3\,\underline{E}_3 + O(k^4) \quad (2)$$

Preliminary results for the approximation of the aerodynamic forces with three-terms power series are presented in Figs. 1 to 18, which present the real and imaginary parts of the nine aerodynamic coefficients relative to the first two bending modes (i=1 and 2 respectively) and one torsion mode (i=3) for a rectangular uniform wing.* Note that in Fig. 1, and Fig. 9 the third order approximations replicate the actual values quite accurately. Unfortunately, not all the coefficients were represented precisely by the third order power series: see in particular Fig. 5. In these particular cases, either higher order power series or an iterative scheme must be used if accurate approximations are desired.

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* These values were obtained with the procedure described in Appendix A.

4. Natural State Space Method

Combining Eqs. (1) and (2) one obtains, in the time domain (with $g = 0$)

$$\underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} + q\left(\frac{\ell^3}{V^3}\,\underline{E}_3\,\dddot{\underline{x}} + \frac{\ell^2}{V^2}\,\underline{E}_2\,\ddot{\underline{x}} + \frac{\ell}{V}\,\underline{E}_1\,\dot{\underline{x}} + \underline{E}_0\,\underline{x}\right) = 0$$

which can be recast in the State Space format as

$$\frac{d}{dt}\begin{Bmatrix} x \\ \dot{x} \\ \ddot{x} \end{Bmatrix} = -\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \hat{E}_3 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -I & 0 \\ 0 & 0 & -I \\ K+\hat{E}_0 & \hat{E}_1 & M+\hat{E}_2 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ \ddot{x} \end{Bmatrix} \qquad (3)$$

where

$$\underline{\hat{E}}_n = q\left(\frac{\ell}{V}\right)^n \underline{E}_n$$

By varying the values of V, standard root-locus techniques can be employed to determine the roots as a function of the velocity, and therefore the stability boundary (flutter speed).

Unfortunately, the validity of the State Space method is contingent on the invertibility of the $\underline{E}_3$ matrix. If $\underline{E}_3$ is ill conditioned, then small truncation or roundoff errors will yield large variations in its inverse, modifying the dynamics represented by Eq. (2).

Preliminary results were not satisfactory. The problem was traced down to the near-invertibility of the $\underline{E}_3$ matrix. For a typical evaluation of the coefficient matrices of the power series, $\underline{E}_3$ took on the following value

$$\underline{E}_3 = \begin{bmatrix} 8.15214 & -6.60933 & 10.8949 \\ -7.82938 & 6.28561 & 10.11678 \\ -31.13977 & 24.86876 & 39.64769 \end{bmatrix}$$

Note that the value of the determinant is D=.08. Note also that the product of the three diagonal terms is approximately equal to 2000. This implies a very strong elimination of significant figure, which is to be expected, since the first row is approximately equal to the second multiplied by -1.04.

5. Pade's Space-State Method

The representation of $\underline{E}(s)$ can be expressed as a matrix Pade approximant in the form

$$\underline{E}(s) = [s\underline{I} - \underline{A}]^{-1}[\underline{B}_0 + s\underline{B}_1 + s^2\underline{B}_2]$$

which could yield a better approximation at high frequencies than the power series approximation.

Introducing a new state variable:

$$\underline{y} = [s\underline{I} - \underline{A}]^{-1}[\underline{B}_0 + s\underline{B}_1 + s^2\underline{B}_2]\underline{x}$$

and modifying Eq. (3), results in the following state space model:

$$\frac{d}{dt}\begin{Bmatrix} x \\ \dot{x} \\ y \end{Bmatrix} = -\begin{bmatrix} I & 0 & 0 \\ 0 & M & 0 \\ 0 & -B_2 & I \end{bmatrix}^{-1} \begin{bmatrix} 0 & -I & 0 \\ K & G & -I \\ B_0 & B_1 & A \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ y \end{Bmatrix}$$

Matrices, $\underline{A}$, $\underline{B}_0$, $\underline{B}_1$, $\underline{B}_2$, are all dependent on $\underline{E}_3^{-1}$.* If $\underline{E}_3$ is near singular, the accuracy of the Pade approximation will be sensitive to truncation and roundoff error. The preliminary results were not satisfactory due, again, to the near singularity of the the $\underline{E}_3$ matrix.

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* Neglecting terms of order k^4 one obtains:

$A=E_3^{-1}E_2$, $B_0=-AE_0$, $B_1=E_0-AE_1$, $B_2=E_1-AE_2$

6. Iterative State Space Method

In an effort to alleviate the problem elicited by this ill conditioned matrix, an iterative State Space method is considered. In order to justify the method, consider the simple equation

$$\hat{f}(x) = \varepsilon x^3 + a x^2 + b x + c = 0 \qquad (\varepsilon \simeq 0)$$

The amplitude of one of the roots of f(x) approaches infinity as ε goes to zero. Hence, the root does not play an important role in determining the locus of roots obtained by varying the coefficients of the lower order terms, (as long as it is in the left hand side of the plane).

The other two roots may be obtained by solving by iteration the quadratic equation

$$\bar{a} x^2 + b x + c = 0$$

or

$$a x^2 + \bar{b} x + c = 0$$

or

$$a x^2 + b x + \bar{c} = 0$$

with $\bar{a}=a + \varepsilon x$, $\bar{b}=b + \varepsilon^2 x$ $\bar{c} = c+\varepsilon^3 x$ respectively. Therefore Eq. (3) is equivalent to (in the frequency domain)

$$s^2 (\underline{M} + \underline{\hat{E}}_2)\, \underline{x} + s\, \underline{\hat{\hat{E}}}_1\, \underline{x} + (\underline{G} + \underline{\hat{E}}_0)\, \underline{x} = 0$$

(with $\underline{\hat{\hat{E}}}_1 = \underline{\hat{E}}_1 + s^2\, \underline{\hat{E}}_3$) or, in space-state format

$$s \begin{Bmatrix} \underline{x} \\ \underline{x}_1 \end{Bmatrix} = - \begin{bmatrix} I & 0 \\ 0 & M + \hat{E}_2 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -I \\ G + \hat{E}_0 & \hat{\hat{E}}_1 \end{bmatrix} \begin{Bmatrix} \underline{x} \\ \underline{x}_1 \end{Bmatrix}$$

where $\underline{x}_1 = s\underline{x}$.

It should be noted that the roots of Eq. (3) which are not obtained are expected to be highly damped and therefore of little interest.

Also it should be noted that a variation of the above method is obtained if the actual value of $\underline{E}$ is used (by employing obvious appropriate definitions for $\hat{\underline{E}}_0$ and $\hat{\underline{E}}_1$ in each iteration).

Results using this method are now being obtained.

7. Concluding Remarks

Several methods to solve the flutter equations have been briefly described. Preliminary results have been presented.

The state-space method has certain advantages over the V-g method. Because the approximation to the aerodynamic influence coefficients is a function of the complex variable s, the truncated power series can be evaluated at points off the imaginary axis. Hence, eigenvalues obtained which are not purely imaginary have physical interpretation. In the V-g method, eigenvalues obtained for which the value of g is not identically zero have no physical meaning, but are merely mathematically introduced.

In the V-g method, the value of k for a flutter crossing is first determined, then V is computed. If it is desired to obtain the lowest value of V for which a crossing of the imaginary axis occurs, k must sweep out to infinity, for the reduced frequency varies as 1/V. In the new method, V is directly inputed as a parameter, hence the sweep can be determined at the first crossing. In the V-g method, many flutter crossings will occur.

In addition, system theoretic concepts can be employed in the formulation of an active control solution to flutter. For example, optimal control techniques can be carried out subject to the dynamical constraints of the system. Also, expression of the aerodynamic influence coefficients in the state variable format enables the analysis of nonlinear dynamics using various methods such as multiple time scaling or the Lie transform. Dynamic stability of the wing under study can be evaluated as well as overall system performance.

APPENDIX A

EVALUATION OF MATRICES $\underline{E}_k$

Fig. 1 through Fig. 18 compare the polynomial approximations with the actual values of the aerodynamic influence coefficients obtained by SOUSSA for each element of the force matrix. The value of the coefficients are plotted as a function of the reduced frequency, k. The squares represent the actual values, while the triangles represent the polynomial approximation.

The coefficient matrices $\underline{E}_0$ and $\underline{E}_1$ were obtained as

$$\underline{E}_0 = Real\left[\underline{E}\,(k=.001)\right]$$

$$\underline{E}_1 = Imag\left[\underline{E}\,(k=.001)\right]/.001$$

$\underline{E}_2$ and $\underline{E}_3$ were computed as

$$\underline{E}_2 = \sum_{i=1}^{N}\left[\frac{Real\left[\underline{E}(k_i)\right]-\underline{E}_0}{k_i^{\,2}}\right]/N$$

$$\underline{E}_3 = \sum_{i=1}^{N}\left[\frac{Imag\left[\underline{E}(k_i)\right]-k_i\,\underline{E}_1}{k_i^{\,3}}\right]/N$$

The expression in brackets is constant if $\underline{E}(k)$ was equal to a cubic polynomial within the range of frequencies K_i (i=1,...,N).

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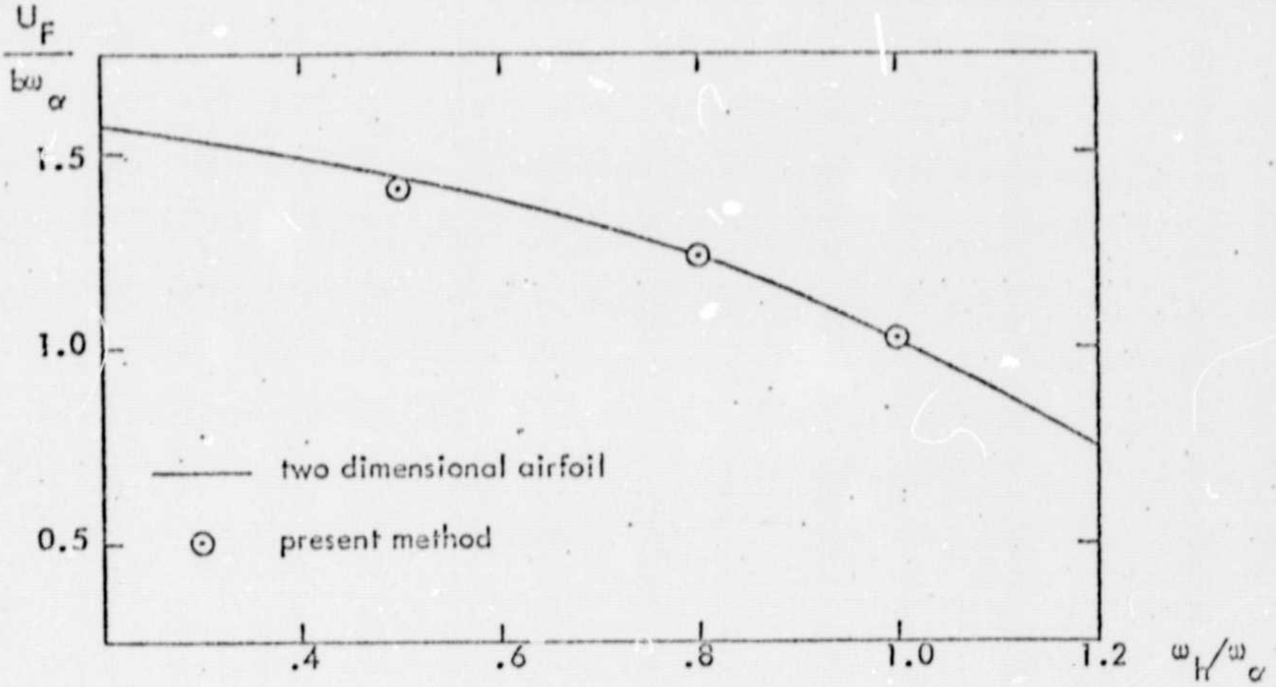


Figure a. Flutter speed as a function of ω_h/ω_α for a rectangular wing with AR = 16, M = 0, τ = 0.1%, μ = 5, X_α = 0.2, r_α = 0.5, and NX = 8, NY = 10. Results are compared with exact solution given by two dimensional airfoil theory (Ref. 10) (X_{EA} = -0.2C).

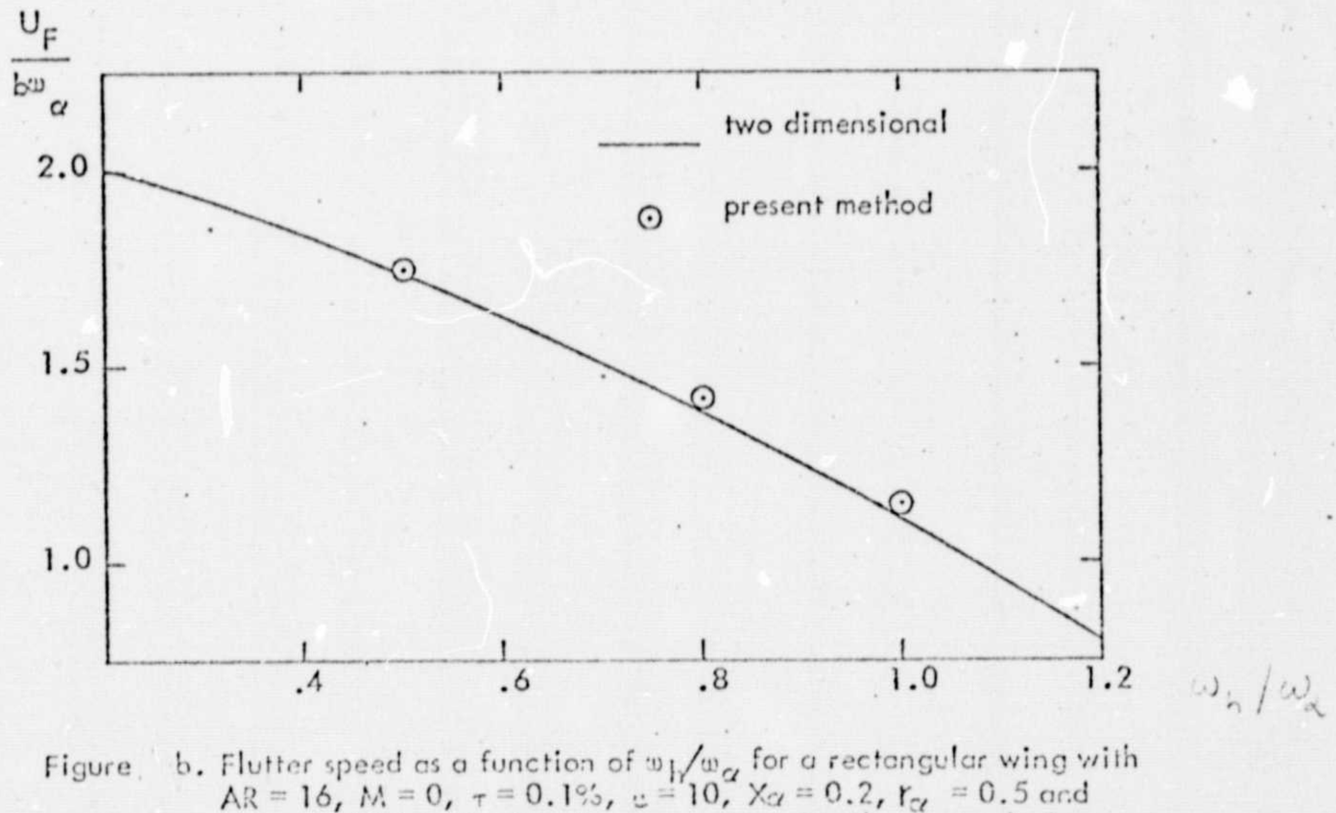


Figure b. Flutter speed as a function of ω_h/ω_α for a rectangular wing with AR = 16, M = 0, τ = 0.1%, μ = 10, X_α = 0.2, r_α = 0.5 and NX = 8, NY = 10. Results are compared with exact solution given by two dimensional airfoil theory (Ref. 10)(X_{EA} = -0.2C).

Fig. 1

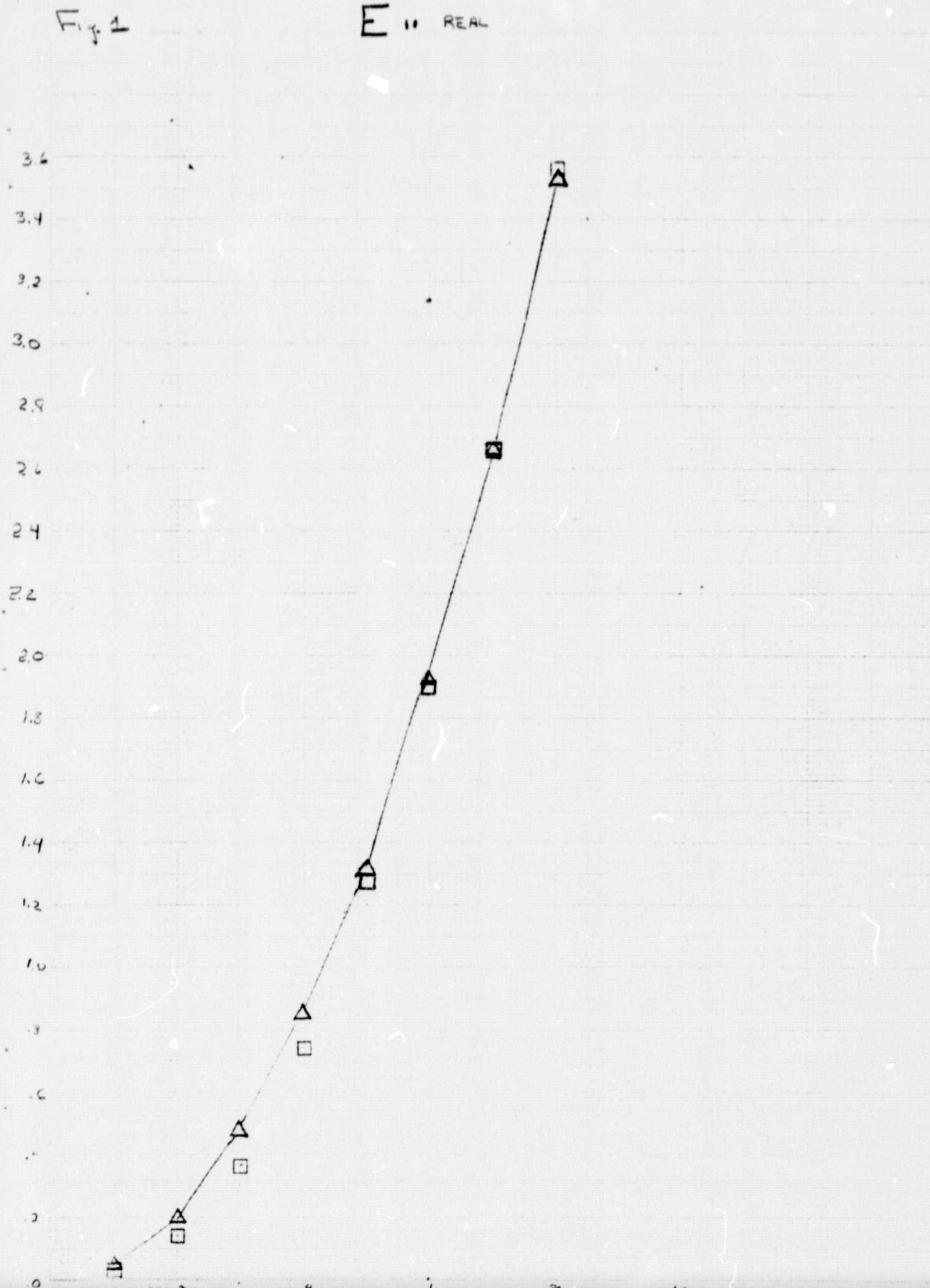


Fig. 2

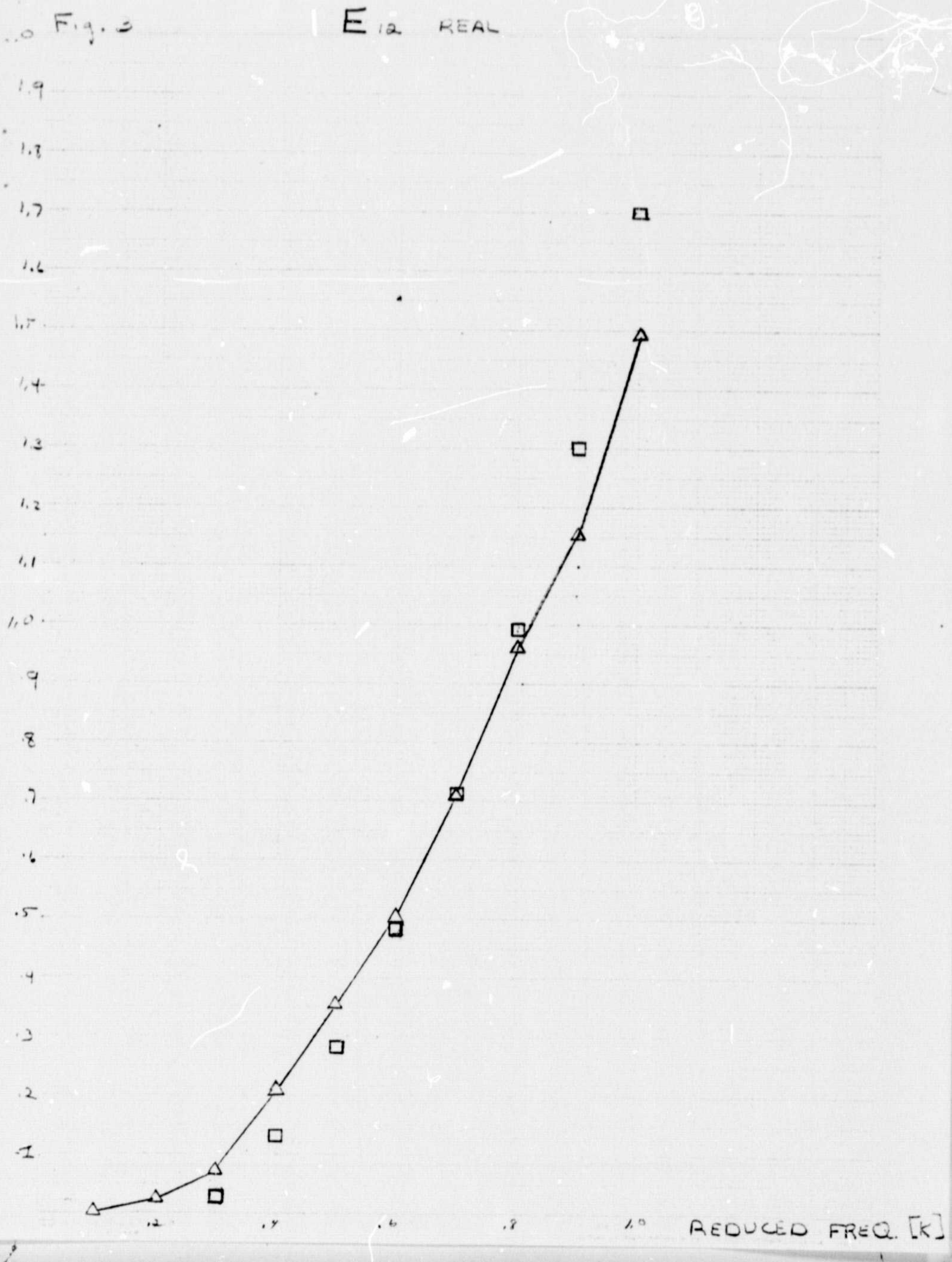


Fig. 3
E12 REAL
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.2
.4
.6
.8
1.0
REDUCED FREQ. [k]

Fig 4

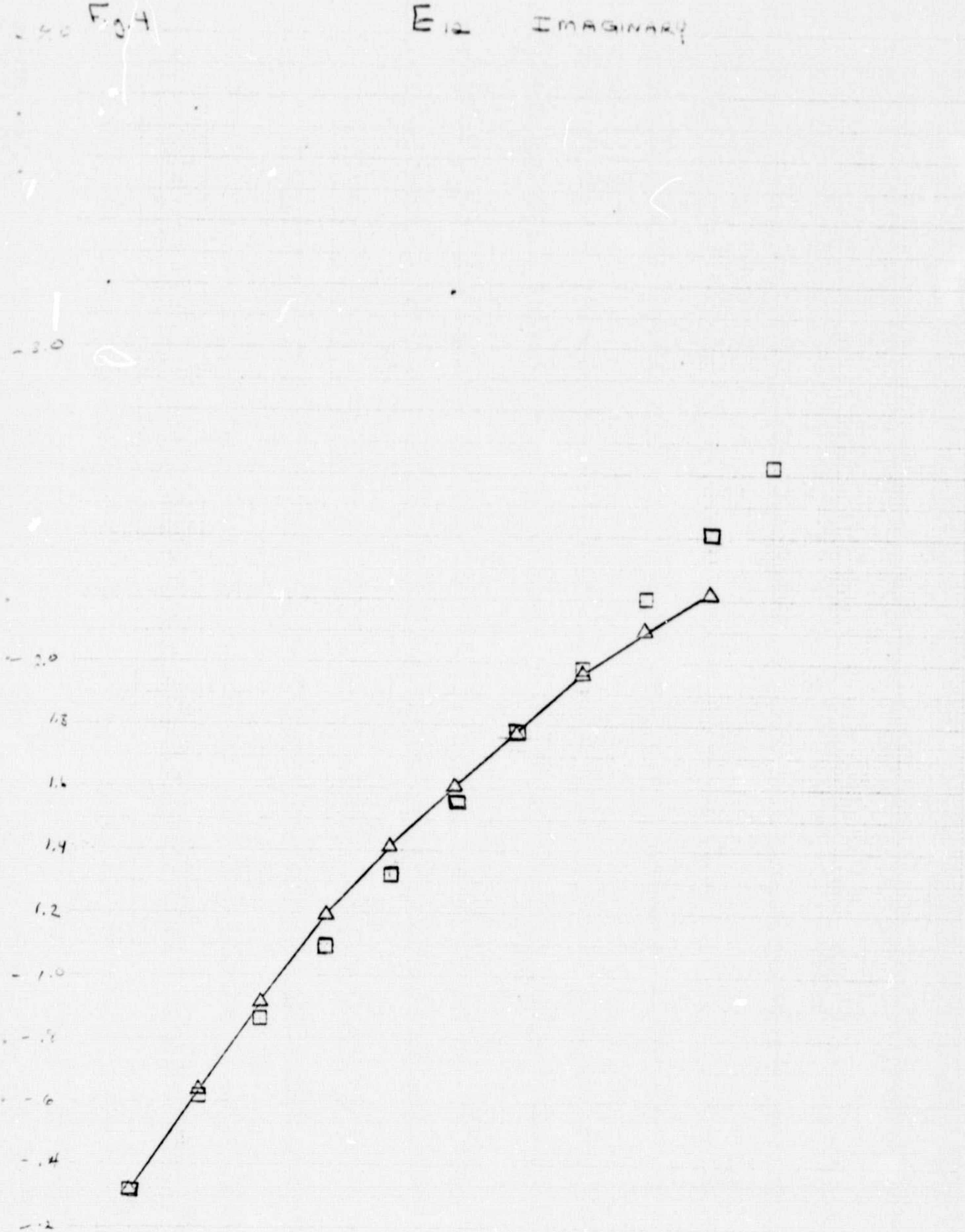


Fig. 5

E13 REAL

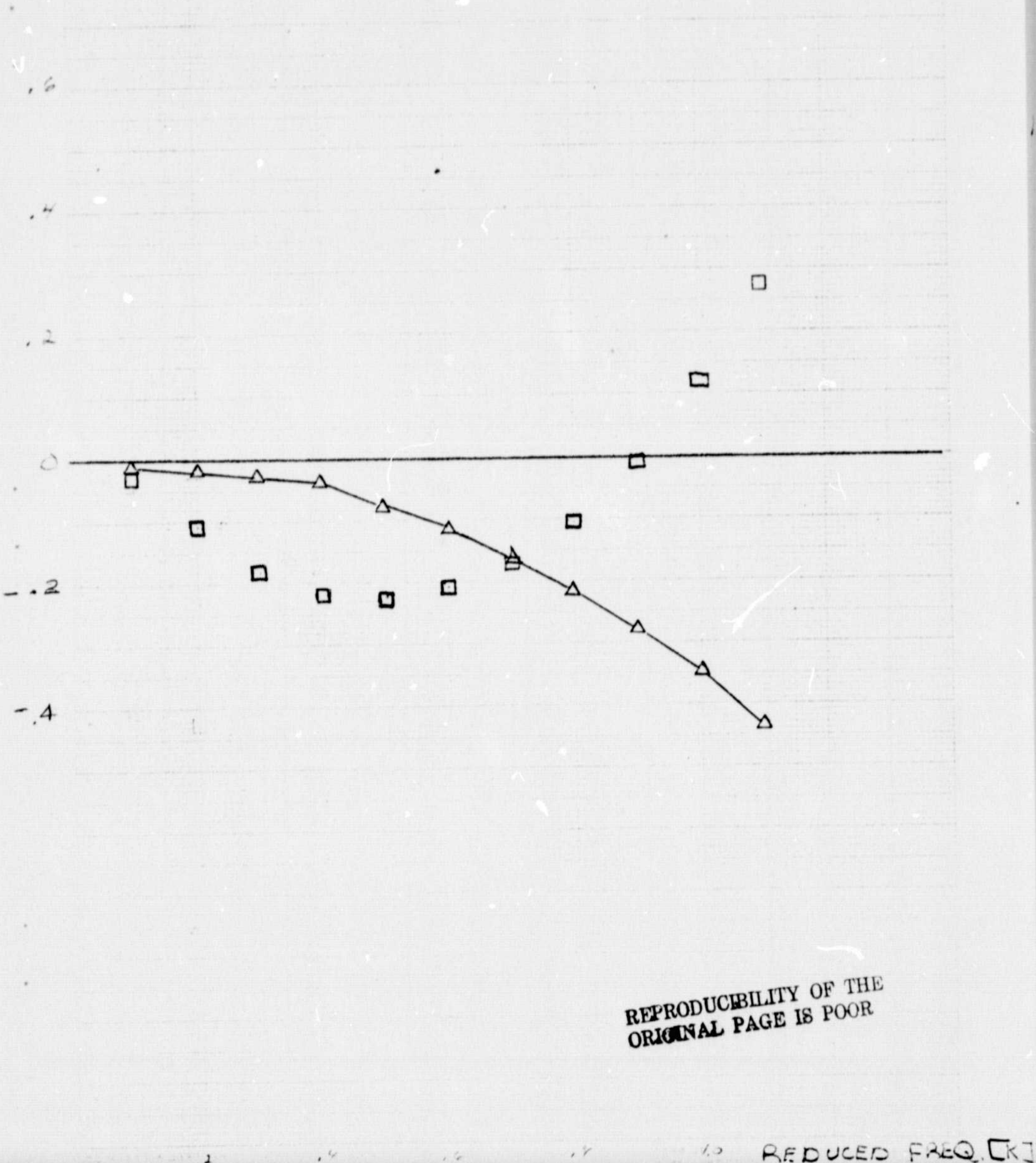


Fig 6 E_{13} IMAGINARY

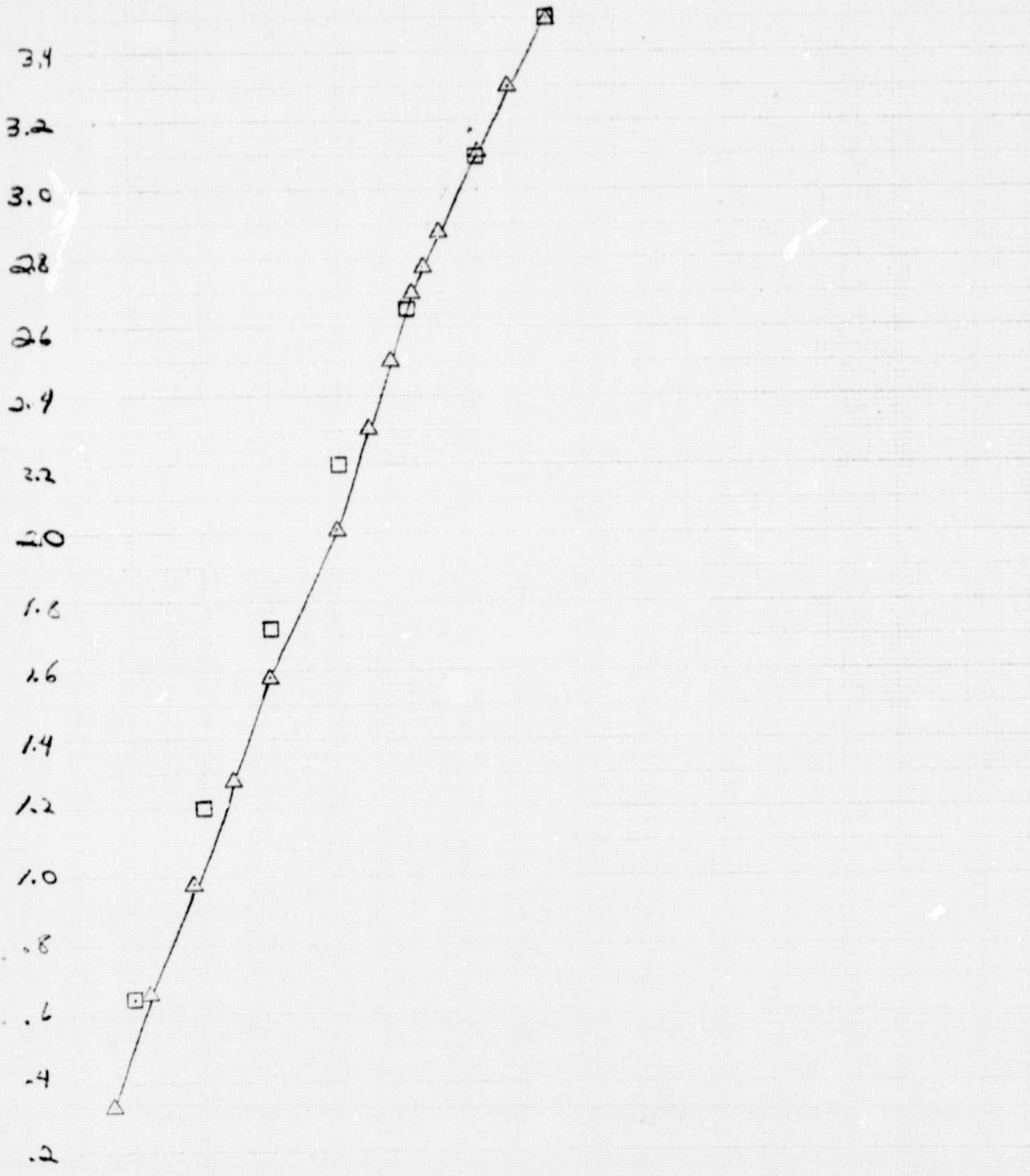


Fig. 7

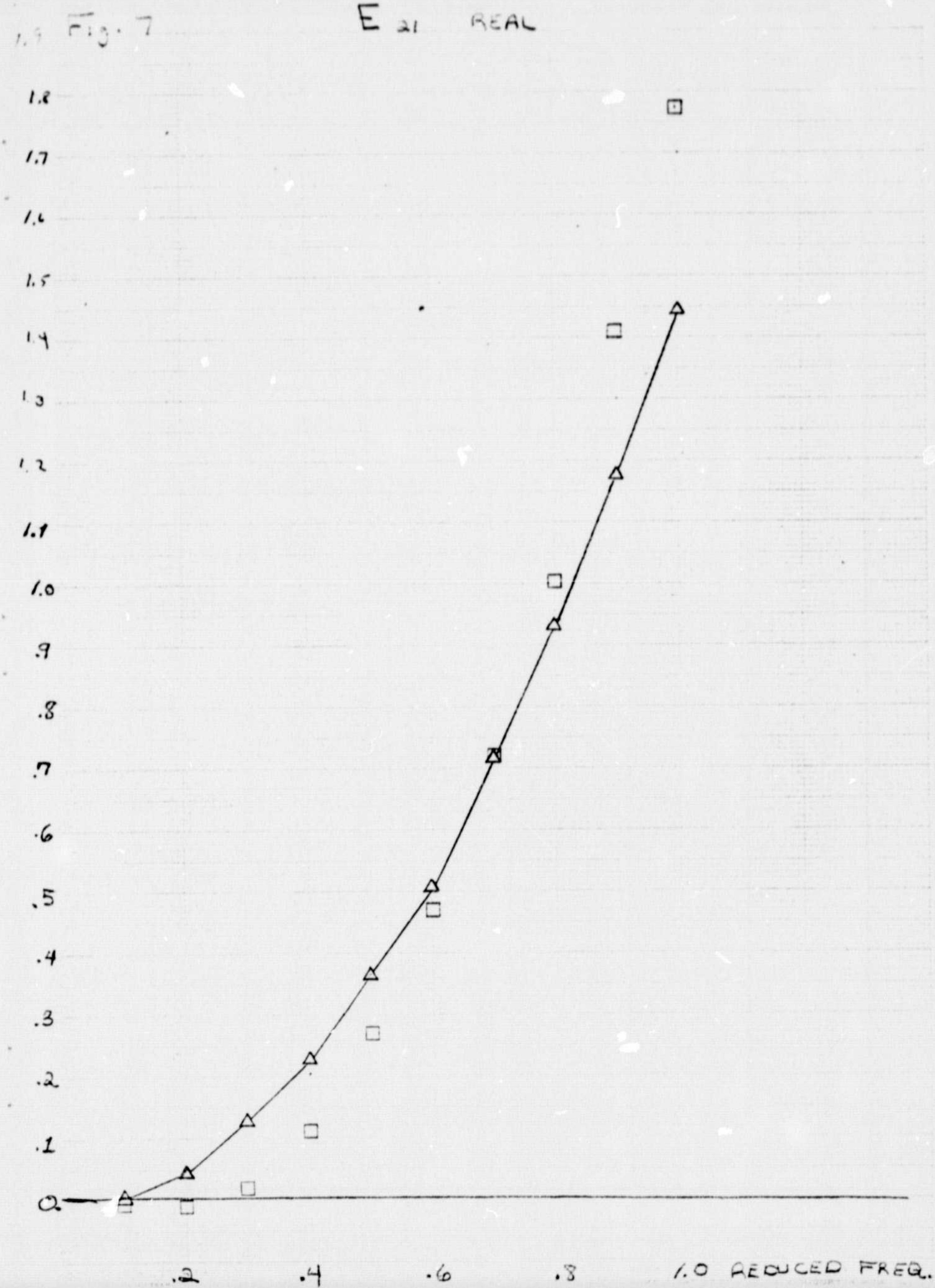

E 21 REAL
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.2
.4
.6
.8
1.0 REDUCED FREQ. [K]

Fig. 8

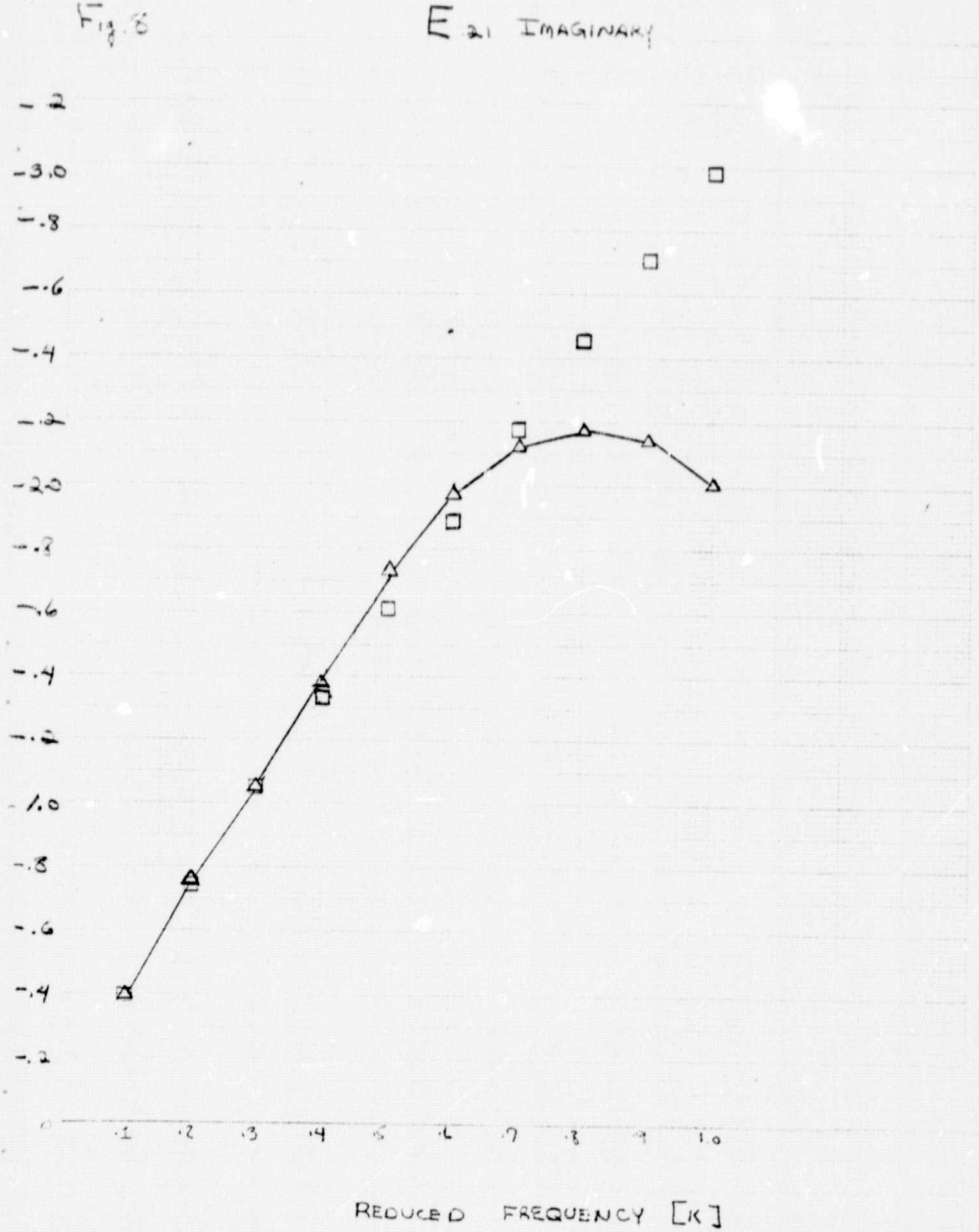

E.21 IMAGINARY
- .2
-3.0
-.8
-.6
-.4
-.2
-2.0
-.8
-.6
-.4
-.2
-1.0
-.8
-.6
-.4
-.2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
REDUCED FREQUENCY [K]

Fig. 9

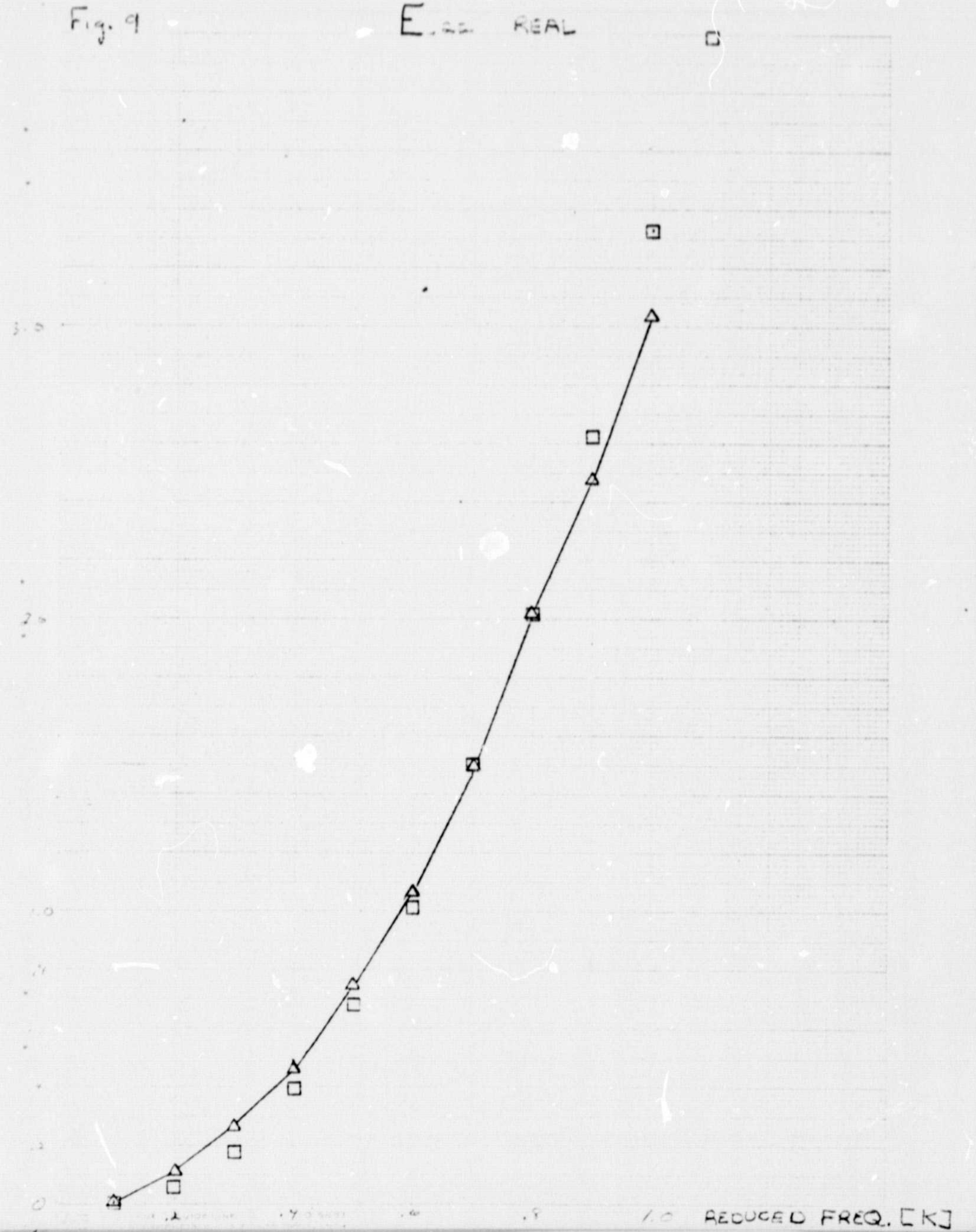


Fig. 10 E.22 IMAGINARY

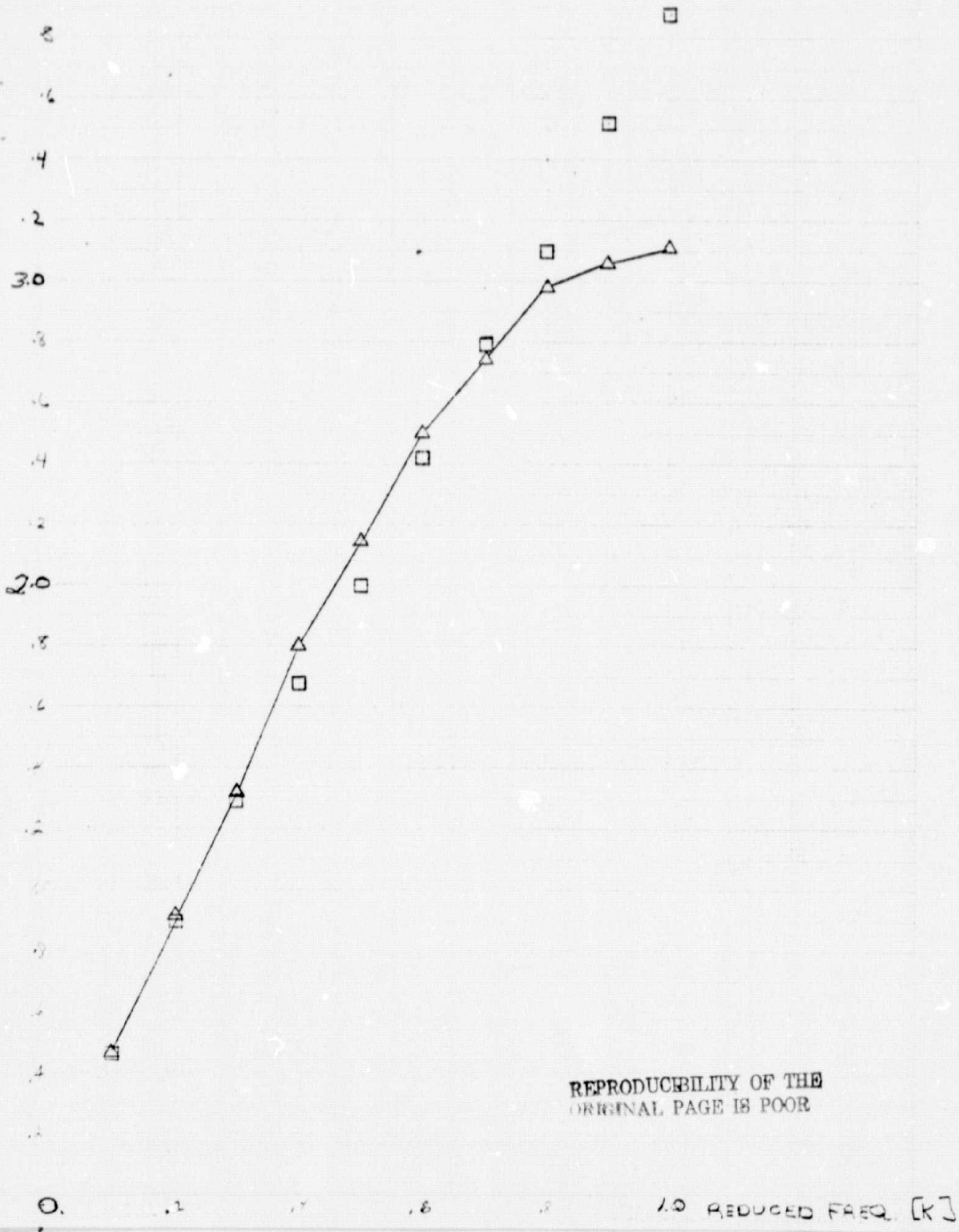


Fig. 11

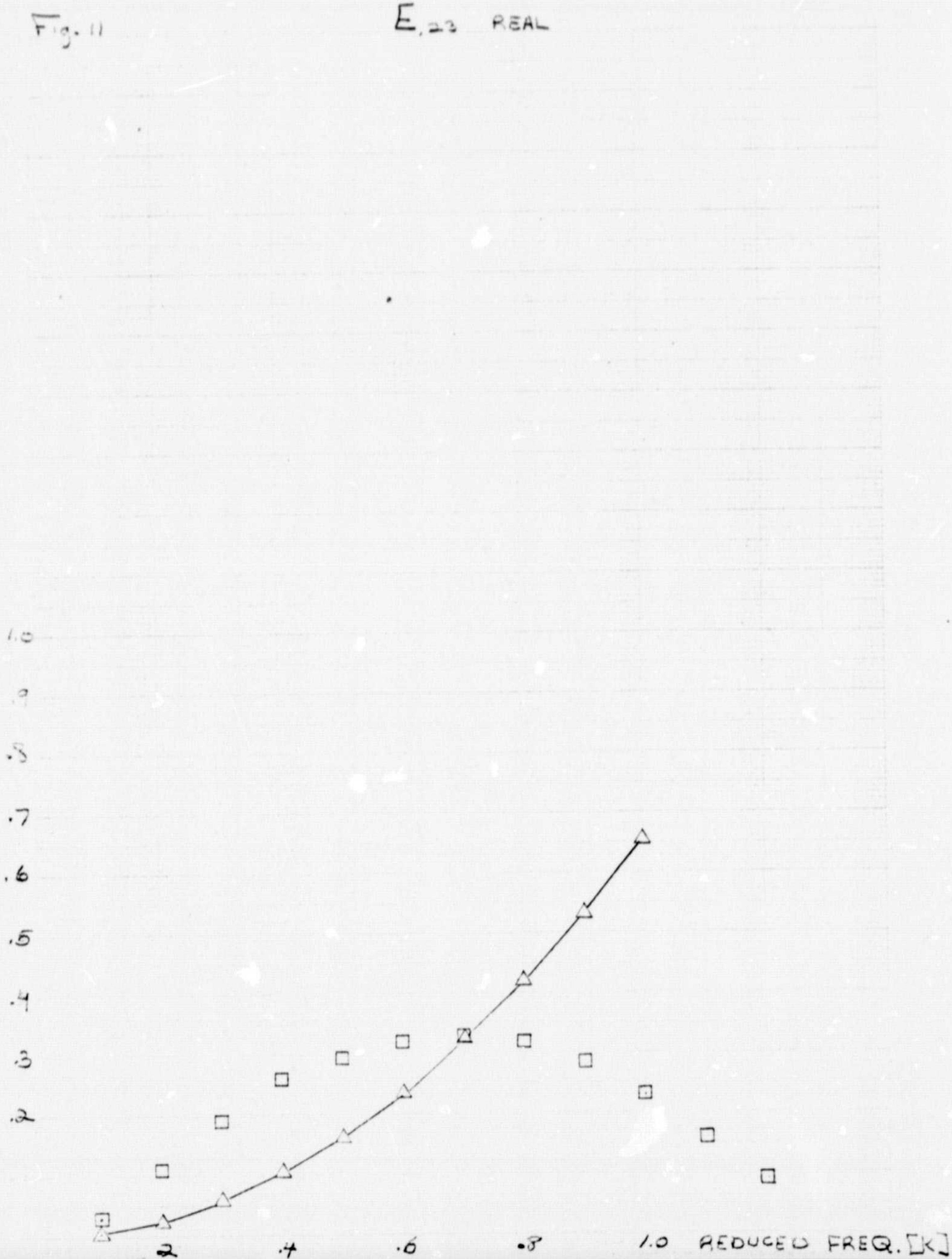


Fig. 12

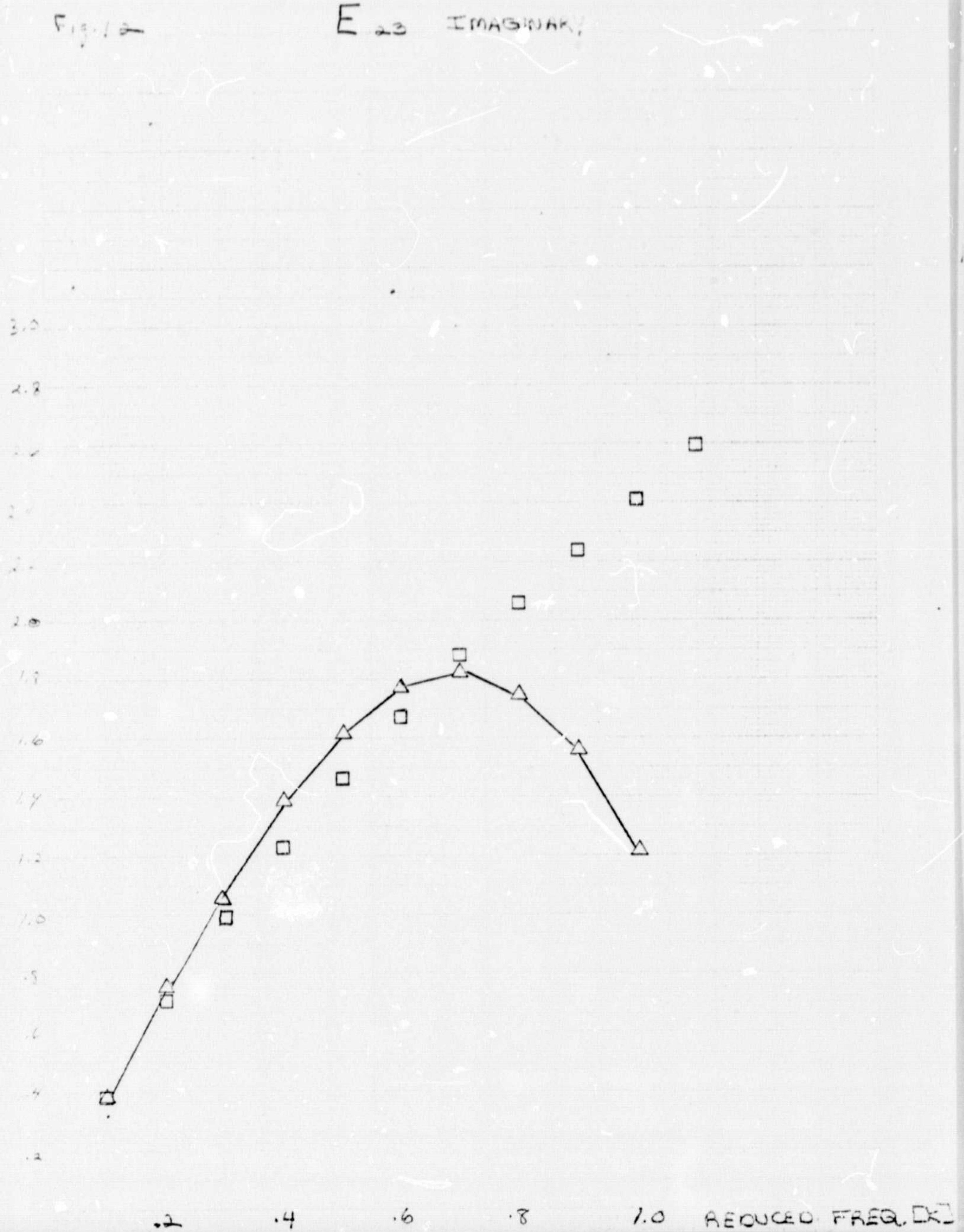
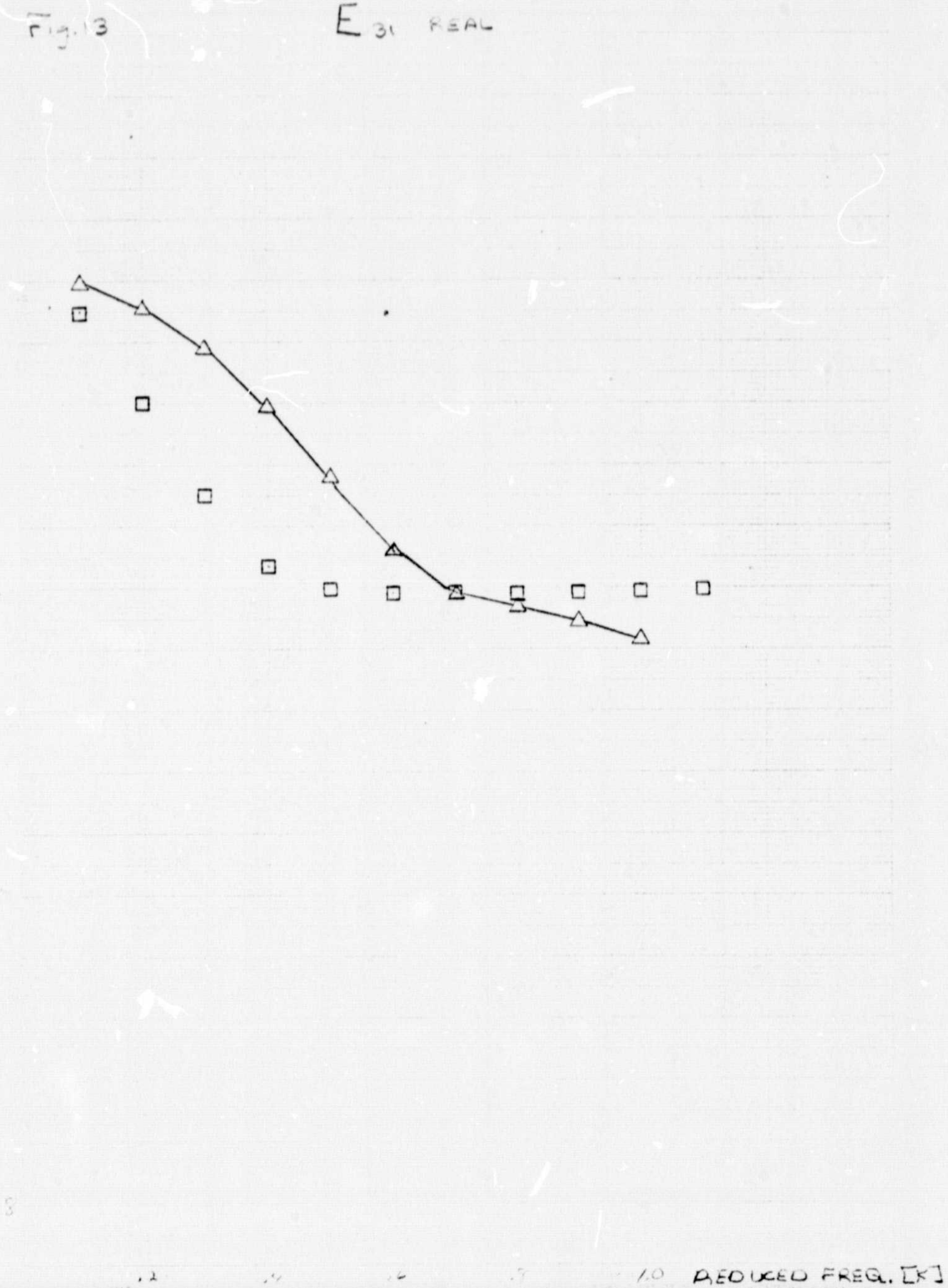


Fig. 13



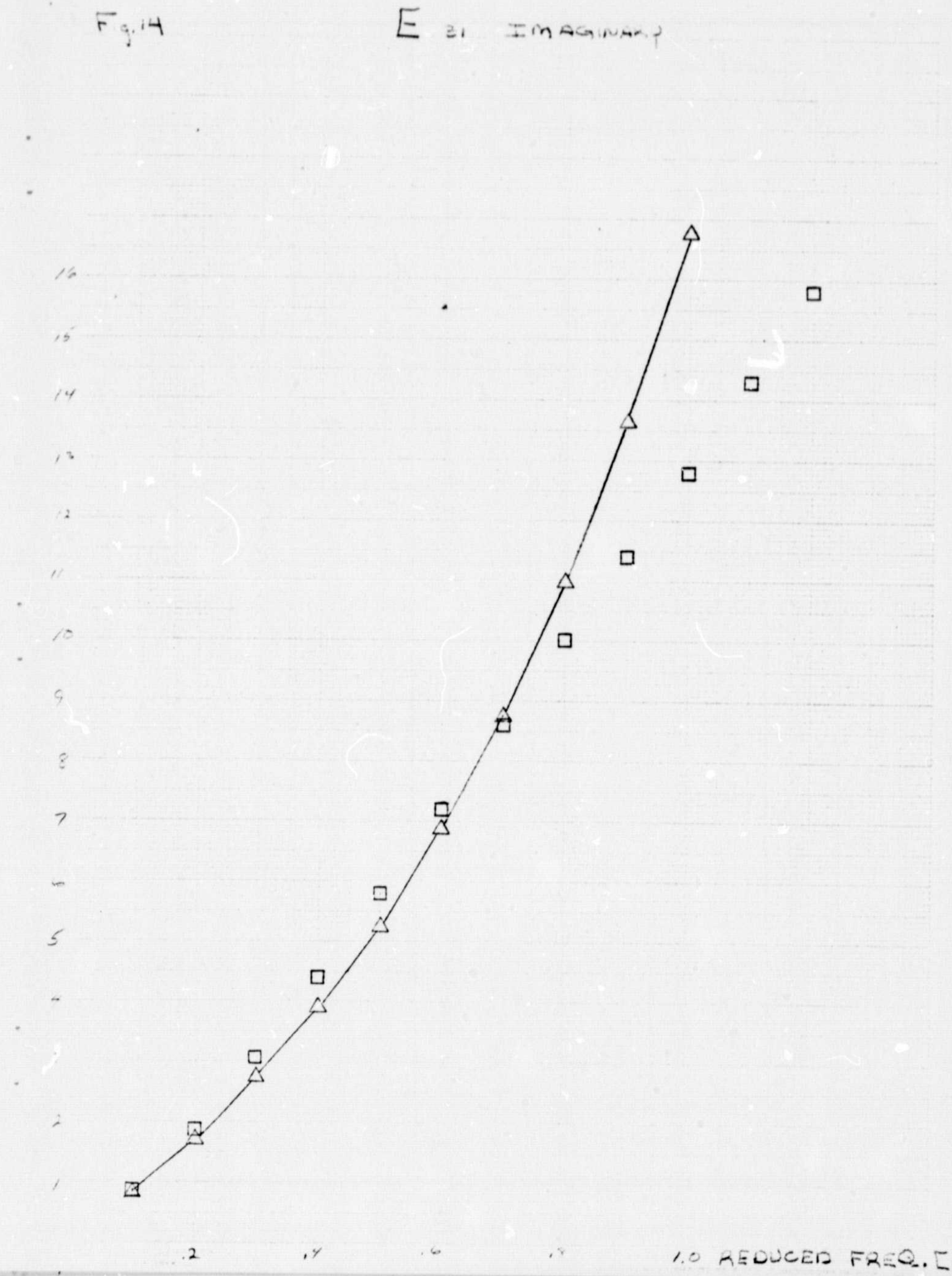


Fig. 14
E 31 IMAGINARY
16
15
14
13
12
11
10
9
8
7
6
5
4
2
1
.2
.4
.6
.8
1.0 REDUCED FREQ. [K]

Fig. 15

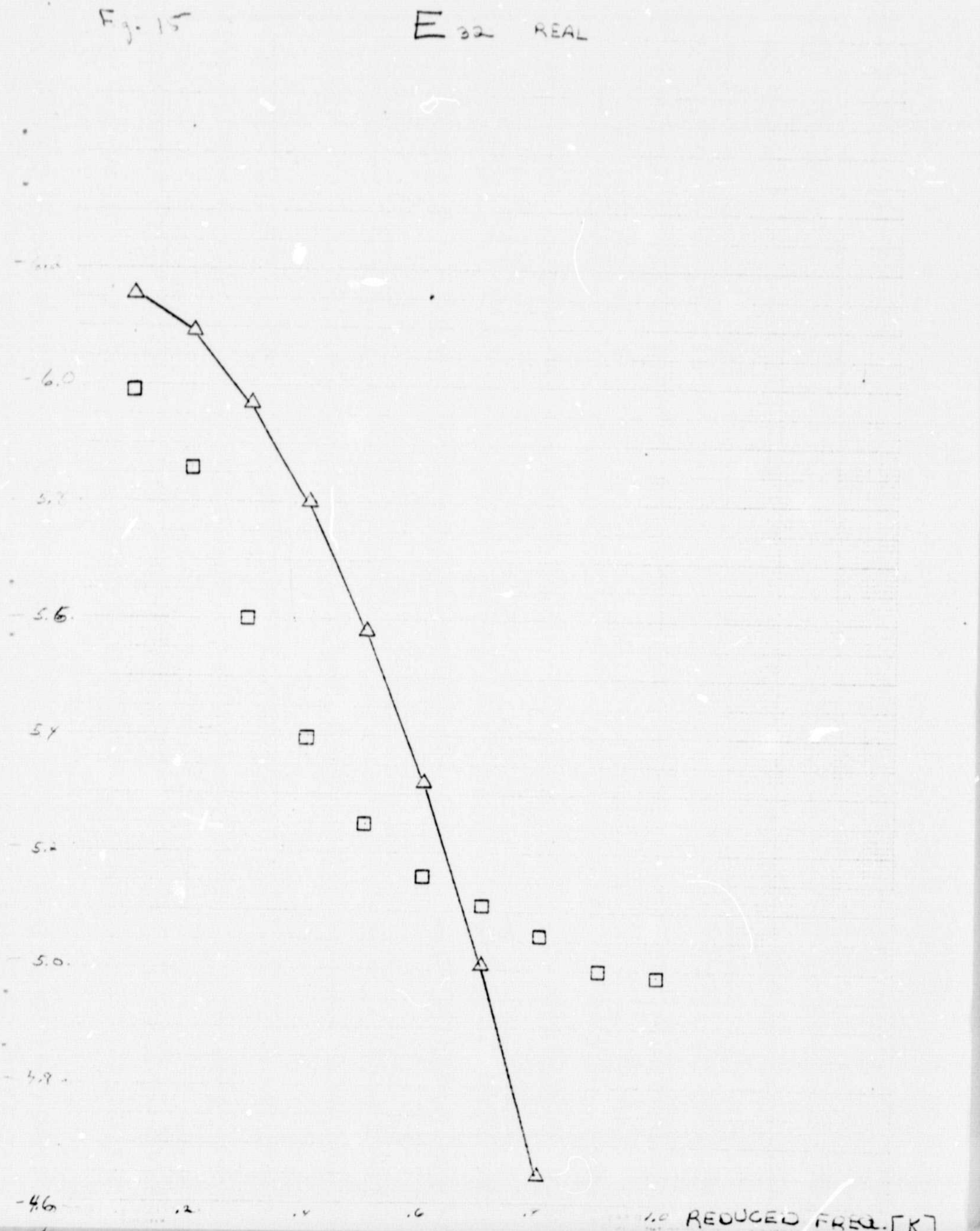
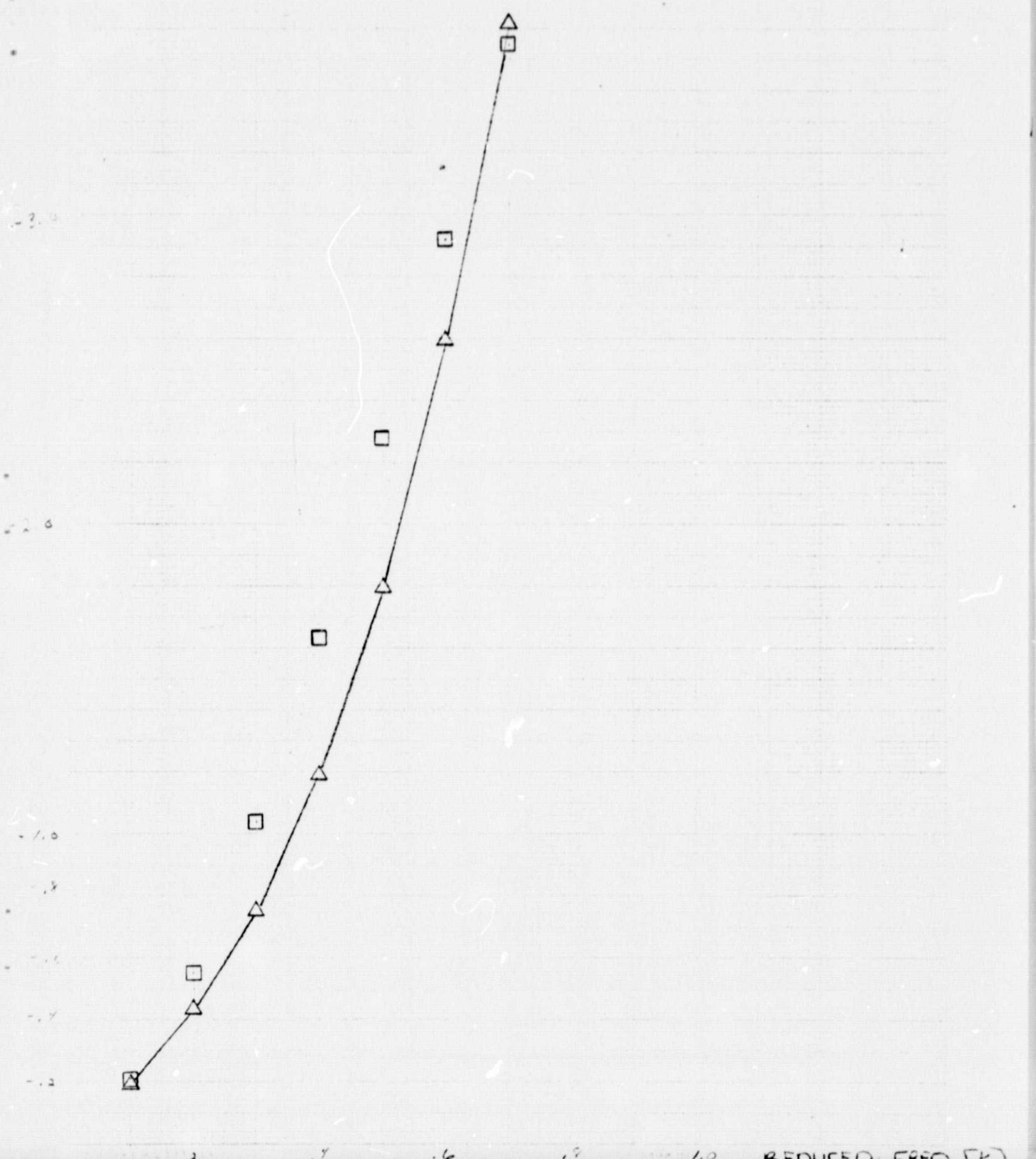


Fig. 16

E_{32} IMAGINARY



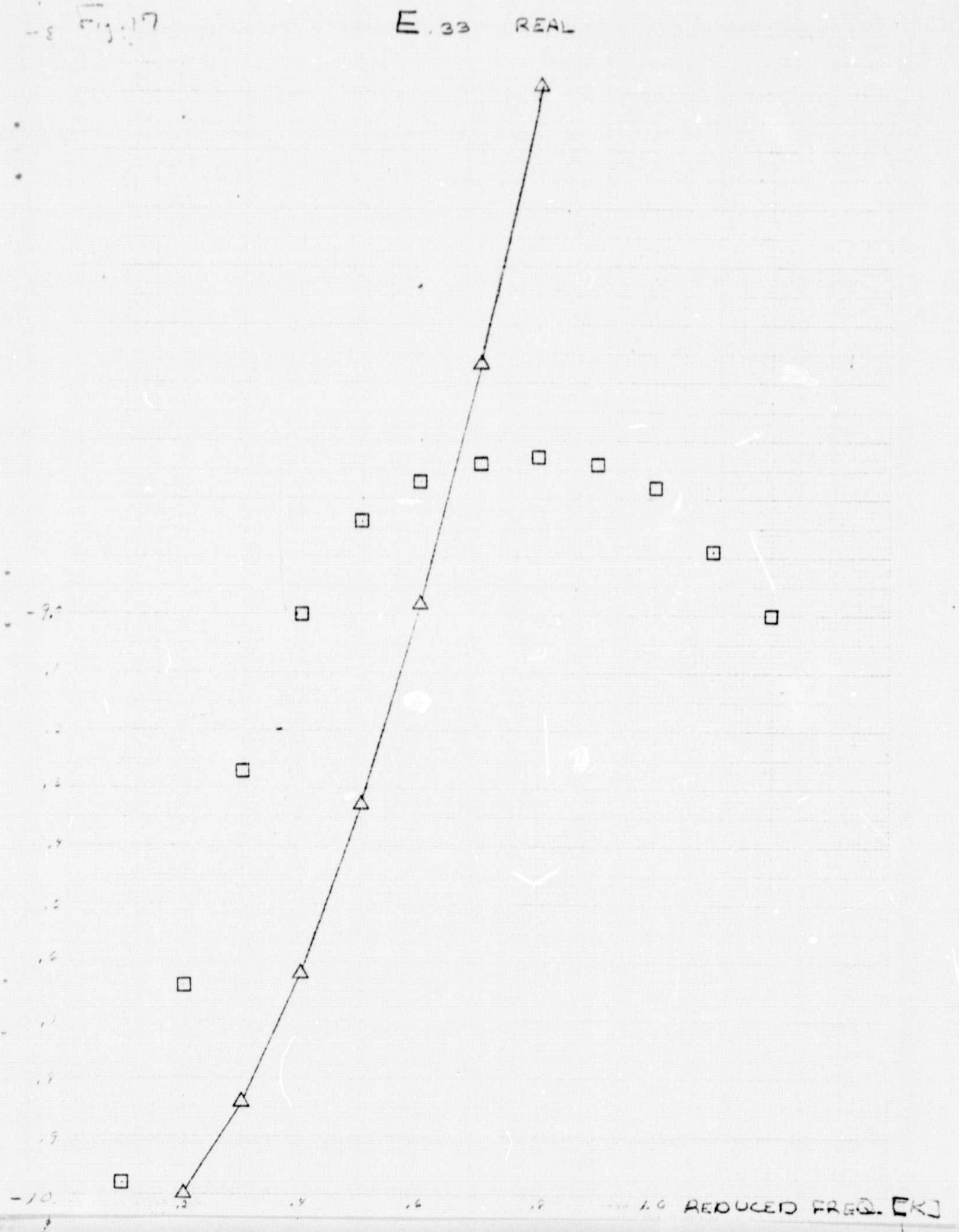


Fig. 17
E.33 REAL
-8
-9.0
-10.
.2
.4
.6
.8
1.0
REDUCED FREQ. [K]

Fig. 18

E_{33} IMAGINARY