Reduced State Feedback Gain Computation

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ABSTRACT

Because application of conventional optimal linear regulator theory to flight controller design requires the capability of measuring and/or estimating the entire state vector, it is of interest to consider procedures for computing controls which are restricted to be linear feedback functions of a lower dimensional output vector and which take into account the presence of measurement noise and process uncertainty. To this effect a stochastic linear model has been developed that accounts for aircraft parameter and initial uncertainty, measurement noise, turbulence, pilot command and a restricted number of measurable outputs. Optimization with respect to the corresponding output feedback gains was then performed for both finite and infinite time performance indices without gradient computation by using Zangwill's modification of a procedure originally proposed by Powell. Results using a seventh order process showed the proposed procedures to be very effective.

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1. Introduction

Because application of conventional optimal linear regulator theory to flight controller design requires the capability of measuring and feeding back the entire state vector, it is of interest to consider procedures for computing controls which are restricted to be linear feedback functions of a lower dimensional output vector. Such a procedure, however, has its limitations in that the feedback gains will be functions of the initial state vector. In addition, the presence of measurement noise and process uncertainty can lead to additional problems relating to both modelling and computation.

To this effect the stochastic linear model shown in Fig. 1 was developed to take into account turbulence, pilot command, aircraft parameter and initial uncertainty, measurement noise, and a restricted number of measurable outputs. Both finite and infinite time performance indices were considered. Optimization with respect to the output feedback gains was performed using both gradient and non-gradient based algorithms. Taking into account both timing and stability considerations, the most favorable results were obtained using Zangwill's modification of non-gradient procedure originally proposed by Powell[2]. This procedure is such that if the cost index were indeed quadratic in the gains, then the search would be along a set of conjugate directions.

The Zangwill-Powell method is especially useful for infinite time performance indices since many of the procedures proposed to date for finding output feedback gains for such indices cannot be guaranteed to converge to a solution.[3,4] Additional problems also arise if an intermediate gain perturbation result in an unstable system. This can be immediately corrected using the Zangwill-Powell procedure by setting the index itself to a very large number.

The effectiveness of the Zangwill-Powell algorithm was evaluated using sixth order linearized longitudinal equations of motion for an aircraft. Results showed the algorithms to be capable of converging to a set of gains useful for gust alleviation.

Programs and the associatedusers guide[5] are available from Mr. Ray Hood, NASA Langley Research Center.

2. Problem Statement

The system of Fig. 1 may be represented mathematically as follows:

Process: $\dot{x}_p = A_p x_p + B_p u_p + G_p x_n + \Delta A_p x_p + \Delta B_p u_p + w_p$ (1.a)

Reference system: $\dot{x}_r = A_r x_r + B_r w_r$ (1.b)

Disturbance system: $\dot{x}_n = A_n x_n + B_n w_n$ (1.c)

where x_p = (NXP x 1) plant state

x_r = (NXR x 1) reference state

x_n = (NXN x 1) external disturbance state

u_p = (NUP x 1) control vector

w_p = (NXP x 1) zero mean white plant disturbance with covariance W_p

w_r = (NWR x 1) zero mean white reference excitation noise with covariance W_r

w_n = (NXN x 1) zero mean white disturbance excitation noise with covariance W_n

$\Delta A_p, \Delta B_p$ = uncertainty in A_p, B_p respectively.

<u>Outputs</u>:

$$y_p = C_{pp} x_p + C_{pn} x_n + \gamma \quad (2.a)$$

$$y_r = C_{rr} x_r \quad (2.b)$$

$$y_n = C_{nn} x_n \quad (2.c)$$

where y_p = (NYP x 1) plant output

y_r = (NYP x 1) reference output

y_n = (NYN x 1) disturbance output

γ = (NYP x 1) zero mean white sensor noise vector

Control:

$$u_p = K_{yp}\, y_p + K_{yr}\, y_r + K_{yn}\, y_n \tag{3}$$

where any of the elements in the gains K_{yp}, K_{yr}, K_{yn} can be fixed at a given value.

$$\text{Index:}\quad J = \frac{1}{t_f}\, \mathcal{E} \int_0^{t_f} \left[(y_p - y_r)^T Q (y_p - y_r) + u_p^T R\, u_p \right] dt \tag{4}$$

In order that this problem be cast into a more succinct form, the following definitions are useful:

$$\underline{x} = \begin{pmatrix} x_p \\ x_r \\ x_n \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} y_p \\ y_r \\ y_n \end{pmatrix}$$

$$v = \begin{pmatrix} 0 \\ B_r\, w_r \\ B_n\, w_n \end{pmatrix} \qquad \underline{n} = \begin{pmatrix} \gamma \\ 0 \\ 0 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} +Q & -Q & 0 \\ -Q & Q & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} A_p & 0 & G_p \\ 0 & A_r & 0 \\ 0 & 0 & A_n \end{pmatrix} \qquad B = \begin{pmatrix} B_p \\ 0 \\ 0 \end{pmatrix}$$

$$K = (K_{y_p} \quad K_{y_r} \quad K_{y_n})$$

$$C = \begin{pmatrix} C_{pp} & 0 & C_{pn} \\ 0 & C_{rr} & 0 \\ 0 & 0 & C_{nn} \end{pmatrix}$$

This results in the following augmented system:

Process: $\dot{x} = Ax + Bu + \Delta Ax + \Delta Bu + v(t)$ (5)

Measurement: $y = Cx + n(t)$ (6)

Control: $u = Ky$ (7)

Performance index: $J = \frac{1}{t_f} \mathcal{E} \int_0^{t_f} (y^T Q_1 y + u_p^T R u_p)\, dt$

Initial State Covariance: $\mathcal{E}(x_0 x_0^T) = P_0$

were: x = (n x 1) state vector

$u = (\ell \times 1)$ control vector

y = (m x 1) output vector

K = gain matrix containing both fixed elements (most likely zero) and variable elements to be determined.

v, n = white noise vectors with respective covariance matrices V, N.

and $\Delta A, \Delta B$ = uncertainty in A and B respectively.

Two procedures were considered in order to take into account the total process uncertainty $\Delta Ax + \Delta Bu$; namely:

1) Defining

$$w(t) = \Delta Ax(t) + \Delta Bu(t) \tag{8}$$

as an additional white noise vector with zero mean and assigned covariance matrix W suitably chosen to reflect the uncertainty. The procedures developed by Joshi[6] are then applicable to solution.

2) Letting $$\Delta Ax = \sum_i x_i \, \Delta a_i \tag{9a}$$

and $$\Delta Bu = \sum_i u_i \, \Delta b_i \tag{9b}$$

where Δa_i, Δb_i, the i^{th} columns of ΔA and ΔB respectively, are in turn set equal to

$$\Delta a_i = F_i \, w$$
$$\Delta b_i = G_i \, w$$

where w is a white noise vector with covariance matrix W, and F_i, G_i are constant matrices. The optimization procedure cited by McLane[7] are then applicable to the solution.

Substituting eqs. 5 and 6 into J, eq. 7 gives

$$J = \mathcal{E} \, \frac{1}{t_f} \int_0^{t_f} \Big[x^T C^T Q_1 Cx + 2x^T C^T Q_1 n + n^T Q_1 n + x^T C^T K^T R K C x + 2 x^T C^T K^T R n + n^T K^T R K n \Big] dt \tag{10}$$

This formulation of the index can be simplified by noting that

$$\mathcal{E}(x^T C^T Q_1 n) = \mathcal{E}(x^T C^T K^T R n) = 0$$

Furthermore $\mathcal{E}(n^T Q_1 n)$ is a constant term independent of the control and therefore has no effect on the index. Thus minimization of (10) is equivalent to the minimization of

$$J = \mathcal{E} \, \frac{1}{t_f} \int_0^{t_f} x^T (C^T Q_1 C + C^T K^T R K C) \, x \, dt + \mathcal{E}(n^T K^T R K n) \tag{11}$$

Elimination of the expectation operator is now possible by recognizing that J can be rewritten as:

$$J = \text{Trace}\ (C^T Q_1 C + C^T K^T R K C)\ S + \text{Trace}\ (K^T R K N) \tag{12}$$

where
$$S = \frac{1}{t_f} \int_0^{t_f} \mathcal{E}(x\, x^T)\, dt \tag{13}$$

Thus upon computation of the integral of the state covariance matrix $\mathcal{E}(x\, x^T)$, the value of the index can be found from eq. 12.

In particular if t_f is finite, the covariance

$$P = \mathcal{E}\,(x\, x^T) \tag{14}$$

can be readily propagated, given a value for K, as follows:

. Formulation defined by eq. 8[6].

$$\dot{P} = (A + B\,K\,C)\,P + P(A + B\,K\,C)^T + B\,K\,N\,K^T B^T + (V + W) \tag{15a}$$

$$P(0) = \mathcal{E}\,(x(0)\ x^T(0)) \tag{15b}$$

. Formulation defined by eq. 9[7].

$$\dot{P} = (A + B\,K\,C)\,P + P(A + B\,K\,C)^T + M(P,\,K) + N(K) + B\,K\,N\,K^T B^T + V \tag{16a}$$

$$P(0) = \mathcal{E}\,(x(0)\ x^T(0)) \tag{16b}$$

where
$$M(P,\,K) = \sum_{ij} P_{ij}\,\hat{F}_i\,W\,\hat{F}_j^{\,T} \tag{16c}$$

$$N(K) = \sum_{ij} G_i(K_i\,N\,K_j^{\,T})\,W\,G_j^{\,T} \tag{16d}$$

$$\hat{F}_i = F_i + \sum_{\ell} G_{\ell}\,(K\,C)_{\ell i} \tag{16e}$$

$K_i = i^{th}$ row of K

and $P_{ij} = i - j^{th}$ component of P

For the case in which $t_f = \infty$, the integral S of eq. 13 will not in general converge if there is measurement noise (n) and/or process noise (v). Thus in this case (as in refs. 3, 4, 6) in the limit S will be replaced by

$$S = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} P(t)\, dt = P_{ss} \qquad (17)$$

where P_{ss} is the steady state solution to either eq. 15 or eq. 16.

It is of importance to note at this point that McLane in ref. 7 cites a potential convergence problem with the steady state solution to eq. 16 if the noise intensity is too high. This results from the need for very large feedback gains. In fact during the numerical experiments, divergence occurred in the presence of multiplicative noise standard deviations equal to 1/100 of the corresponding A_p entries.[8] Consequently the infinite time version of the formulation defined by eq. 9 was not developed beyond the experimental stages.

3. Computational Procedures

Since the performance index (eq. 12) is easily evaluated given a value for the gain matrix K, the Zangwill-Powell[1,2] method which does not require gradient computation was very attractive for optimization. This algorithm has been coded into IMSL sub-routine ZXPOWL.[9] Initial values for elements of the gain matrix K are given to this program which then makes successive perturbations in each of the variable elements and computes the corresponding value of J. Using the computed indices, perturbation directions are chosen such that if the index were quadratic in the gains, the search would be along a set of conjugate directions. One particular attractive feature of this algorithm is the ability to correct, when $t_f = \infty$, for a set of unstable gains which do not permit the determination of a steady state covariance matrix, P. This was done by computing the eigenvalues of $(A + B K C)$ for each perturbed value of K and setting J equal to a very large number (i.e., 10^{50}) whenever instability was noted.

For comparison purposes, gradient based procedures were also considered. Procedures searching along the directions of steepest descents and conjugate gradients were evaluated for the finite time index ($t_f < \infty$) and were observed to be no faster than the Zangwill-Powell method.[10] For the infinite time index, optimization was performed using a modification of the gradient method applied by Horisberger and Belanger to deterministic systems.[11,12] This led to unstable intermediate gain selections that resulted in divergence of the search procedure. Such instabilities are not unreasonable to expect since the computed gradient represents the rate of change with respect to K of the index defined by eq. 12 and the steady state solution of 15 or 16. If however the resulting perturbation along the computed gradient results in destabilizing gain K, then the index evaluated with eqs. 12 and 15 or 16 will be erroneously finite rather than infinite. In fact the steady state solution to 15 or 16 with a destabilizing gain could be negative definite resulting in a negative rather than infinite valued index. To more precisely illustrate this behavior, note that the gradient of the index J with respect to the gain K is equal to:[6,11]

$$\frac{\partial J}{\partial K} = \left[RKC\ P_s\ C^T + B^T\ L\ P_s\ C^T + B^T\ LBKN \right] \tag{18}$$

where

$$0 = (A + BKC)^T\ L + L(A + BKC) + (Q_a + C^T\ K^T\ RKC) \tag{19}$$

However these equations are valid only if the gain stabilizes the closed loop system $(A + BKC)$. If as a result of a perturbation, the gain K is not stabilizing, then Eq. 19 for L is no longer valid. In fact the gradient defined by Eq. 18 is in reality the gradient of Trace (L) with respect to K since J = Trace (L) if K is stabilizing. It is possible to have a value of K which in reality would result in an unstable system (i.e., $J = \infty$) but when used in Eq. 19 gives a value of $L < 0$ which in turn causes J to be negative.

To correct this behavior, the sequential unconstrained minimization technique of Fiacco and McCormick[13] was used to optimize the index with respect to K subject to the constraints that all closed loop eigenvalues be negative.[12] This procedure converged properly but required 5-10 times larger than the Zangwill-Powell method.

Thus the non-gradient based Zangwill-Powell method was recommended as the procedure for solving the stochastic reduced state feedback gain optimization problem. Specific details of the available programs are contained in the Users' Guide[5] available from Mr. Ray Hood, NASA Langley Research Center.

4. Experimental Results

4.1 System definition

A modified 6-dimensional version of the TIFS[14] aircraft with a gust input (σ = 15 fps) was used for evaluation in the presence of a zero reference command. The corresponding variable definitions were as follows:

Plant state:

$$\underline{x} = \begin{pmatrix} q \\ \Delta\theta \\ \Delta v \\ \Delta\alpha \\ \delta e \\ \delta z \\ \alpha_g \end{pmatrix} = \begin{pmatrix} \text{pitch rate} \\ \text{pitch angle} \\ \text{velocity} \\ \text{angle of attack} \\ \text{elevator deflection} \\ \text{direct lift flap deflection} \\ \text{gust induced attack angle} \end{pmatrix}$$

Plant control:

$$\underline{u} = \begin{pmatrix} \delta ec \\ \delta zc \end{pmatrix} = \begin{pmatrix} \text{elevator command} \\ \text{lift flap command} \end{pmatrix}$$

Observations:

$$\underline{Y} = \begin{pmatrix} q \\ \Delta\theta \\ \alpha \\ nz_1 \\ nz_2 \\ \dot{\gamma} \end{pmatrix} = \begin{pmatrix} \text{pitch rate} \\ \text{pitch angle} \\ \text{angle of attack} \\ \text{Point 1 vertical acceleration} \\ \text{Point 2 vertical acceleration} \\ \text{flight path angular rate} \end{pmatrix}$$

The structural matrices corresponding to climb condition, i.e., h = 1524 m, V = 106 m/s, which were used for design purposes are:

$$A_p = \begin{pmatrix} -1.686 & .000035 & .000231 & -.486 & -4.3778 & -.19948 \\ 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & -32.17 & -.0143 & 18.027 & 0. & -3.0933 \\ 1 & -0.000013 & -.000531 & -1.223 & -.1273 & -.2667 \\ 0. & 0. & 0. & 0. & -20. & 0. \\ 0. & 0. & 0. & 0. & 0. & -40. \end{pmatrix}$$

$$B_p = \begin{pmatrix} 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 20. & 0. \\ 0. & 40. \end{pmatrix} \qquad G_p = \begin{pmatrix} -.486 \\ 0 \\ .0518 \\ -1.223 \\ 0 \\ 0 \end{pmatrix}$$

$$A_r = (0) \qquad B_r = (0)$$

$$A_n = (-.2784) \qquad B_n = (.01815)$$

$$C_{pp} = \begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 64.63 & .00318 & .176 & 444.2 & 212.1 & 100.4 \\ -61.82 & .00580 & .193 & 407.8 & -116.2 & 85.5 \\ 0. & .000013 & .000531 & 1.223 & .1273 & .2667 \end{pmatrix}$$

$$C_{pn} = (0,\ 0.,\ 0.,\ 444.2,\ 407.8,\ 1.223)^T$$

$$C_{rr} = 0$$

$$C_{nn} = 0$$

Corresponding sensor noise deviations considered were:

$$\sigma_q = .5 \text{ deg/sec}$$

$$\sigma_\theta = 0.2 \text{ deg}$$

$$\sigma_{n_z} = 0.05 \text{ g}$$

Given these constraints, feedback gains for q, $\Delta\theta$, $\Delta\alpha$ were determined so as to minimize

$$J = \mathcal{E} \int_0^2 \Big[q_{11}(q)^2 + q_{22}(\Delta\theta)^2 + q_{33}(\Delta\alpha)^2 + q_{44}(n_{z1})^2 + q_{55}(n_{z2})^2 + q_{66}(\dot{\gamma})^2 + r_{11}\,\delta_{e_c}{}^2 + r_{22}\,\delta_{3_c}{}^2 \Big]\, dt$$

Values for q_{ii} and r_{ii} were chosen to be representative of the inverse maximum squared value of the weighted variables. In particular the following values were used:

$$q_{11} = 2500.$$

$$q_{22} = 50.$$

$$q_{33} = 50.$$

$$q_{44} = 4. \text{ or } 0.$$

$$q_{55} = 4. \text{ or } 0.$$

$$q_{66} = 2500. \text{ or } 0.$$

$$r_{11} = 6.$$

$$r_{22} = 3.$$

The covariance W_p of the plant disturbance was chosen to be either

$$W_{p1} = \text{Diag}\ (.2,\ 0.,\ .0007,\ .0005,\ 0.,\ 0.)$$

or $W_{p0} = 0$

These elements were chosen by computing for each component of the state equation

$$W_p(i,\ i) \approx \sum_j (\Delta a_{ij}^2) \times (\text{MAX } (x_j^2))$$

where Δa_{ij} was approximated to reflect the data in reference 14.

Considered also was the case in which process uncertainty was represented by eq. 9 with

$$(F_1,\ F_2,\ F_3,\ F_4,\ F_5,\ F_6) =$$

$$\begin{pmatrix} .4 & 0 & .002 & .2 & 2.2 & .11 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .001 & .17 & 0 & .025 \\ 0 & 0 & 0 & .24 & .024 & .16 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $G_i = 0.$

4.2 Computational results

Using the preceding data, the six gains relating q, $\Delta\theta$, $\Delta\alpha$ to δe_c and δz_c were determined.

Of interest were the following observations relative to the general computational procedures:

- Convergence time on the CDC 6600 was 40 sec for the infinite time problem and 81 sec for a two second index. This difference resulted from having to compute only the solution to a set of algebraic equations rather than a set of differential equations over a two second interval.
- Assuming that every 6 gain perturbations called for by the Zangwill-Powell method corresponds to a gradient evaluation, it was noted that the number of gradient evaluations performed by a steepest descent algorithm was comparable to the number performed by the Zangwill-Powell method.
- For a finite time index it is indeed possible to obtain gains that yield an unstable set of eigenvalues.
- For meaningful parameter uncertainties, optimization of the infinite time index subject to the steady state solution of eq. 16, led to divergence.

4.3 Simulation results

In order to illustrate the effects of the various weights and noise terms, results are presented for the following cases:

Case	Covariance eq.	q_{44}, q_{55}, q_{66}	W_p	t_f (sec)
1	15	0, 0, 0	W_{p1}	2
2	15	4, 4, 2500	W_{p1}	2
3	15	0, 0, 0	$W_{p0} = 0$	2
4	15	0, 0, 0	W_{p1}	∞
5	15	4, 4, 2500	W_{p1}	∞
6	16	0, 0, 0	$W_{p0} = 0$	2
7	16	4, 4, 2500	$W_{p0} = 0$	2

Cases 1-3 correspond to a finite time index with the uncertainties represented as additive white noise. Variations on this uncertainty and on the n_z and $\dot{\gamma}$ weighting, are incorporated. Cases 4, 5 are infinite time versions of cases 1, 2 respectively. Finally, cases 6, 7 correspond to the finite time optimization problem in which the parameter uncertainty is represented as multiplicative noise. Recall that the infinite time version of this latter case would not converge except for near negligible parameter variations.[8]

Resulting gains and the corresponding closed loop eigenvalues for these cases and the open loop system are contained in Table 1. Of interest are the following observations:

- Use of a finite time index can lead to a set of destabilizing gains. This occurred in cases 1 and 3 which did not incorporate penalties on n_z and $\dot{\gamma}$.
- Use of an infinite time index (case 4, 5) resulted in a stable closed loop system.

In order to evaluate the effectiveness of the gains given in Table 1, the following 3-second simulations were made:

- Gust input, measurement noise, initial states = 0
- Gust input, measurement noise, initial states = 0, but at the cruse flight condition (h = 3048m, v = 149.66 m/s).
- No gust, no measurement noise, q(0) = .02 r/s, $\Delta\theta(0) = .15$ r, $\Delta\alpha(0) = .15$ r.
- No gust, no measurement noise, q(0) = .02 r/s, $\Delta\theta(0) = .15$ r, $\Delta\alpha(0) = .15$ r, but at the cruise flight condition (h = 3048 m, v = 149.66 m/s).

The first two simulations were used to evaluate the effectiveness of gust alleviation at both the designed flight condition and at a perturbed flight condition. The second two simulations tested the resulting transient performance.

Tables II and III give the rms errors over the three second interval for each of the states. In interpreting the results, it should be noted that **whereas** the length of the optimization interval was either infinite on two seconds, the evaluation interval was three seconds. Of interest are the following observations:

- Best gust alleviation with respect to open loop performance results when an infinite time index is used.
- Small improvement in terms of rms errors results from weighting n_z and $\dot{\gamma}$.
- Behavior for the 2nd flight condition is comparable with the first.

Typical transiet responses for α and $\dot{\gamma}$ are given in Figs. 2-8 for the open loop system and cases 1, 2, 4, 5, 6, 7 respectively. The following observations were noted:

- Incorporation of a weighting term on $\dot{\gamma}$ significantly reduces its settling time, but possibly at the expense of a degradation in the α response (as in cases 2, 5).
- Representation of the parameter uncertainty as multiplicative noise led to slower responses.
- The influence of the weighting matrix Q is more pronounced for transient behavior than for rms gust response errors.

5. Conclusions and Recommendation

5.1 Conclusions

Optimal output feedback control gains were determined using the Zangwill-Powell procedure which does not require gradient computation. Two stochastic formulations of the process equation were considered in order to take into account process uncertainty, process disturbances and sensor noise. Results using a seventh order system showed that the Zangwell-Powell method is very effective for control gain computation.

Significant among the conclusions resulting from this study are the following:

- Use of a finite time performance index can result in a set of gains which do not stabilize the closed loop system.
- If it is possible at all to stabilize the system with the specified feedback configuration, then the optimization of an infinite time performance index will yield a set of gains that do indeed stabilize the closed loop system.
- Optimization of infinite time indices is less time consuming than the optimization of finite time indices because of the need to solve algebraic and not differential equations.
- Gradient based algorithms when applied to the optimization of infinite time indices can result in divergence. This in particular results when the gradient is computed using the algebraic steady state Riccati solutions to the matrix covariance and co-state equations.

- Performance is very sensitive to the selection of the penalty matrices Q and R.
- Reduced state feedback control can be effective for gust alleviation.
- An initial gain matrix that stablizes the system is needed for the infinite time cases.

5.2 Recommendations

In view of the fact that the prime objective of this study was to develop a set of programs useful for reduced state feedback control computation, it is recommended that future efforts be concerned with procedures for applying these procedures towards design of flight control systems. Specifically further studies should be made to determine techniques for applying reduced state stochastic infinite time optimization procedures to the design of control systems for flexible aircraft. This would then involve the following activities.

- Develop procedures that determine initial gains for stabilizing the closed loop system. Since optimization of infinite time indices requires an initial stabilizing gain matrix, which is not always readily available, it would be desireable to include an initializing procedure that determines a set of stabilizing gains if possible with the assigned feedback structure. If this is not possible, then the program should request a modification to the structure.
- Evaluate the effectiveness of procedures for incorporating aircraft parameter uncertainty. At present either additive or multiplicative noise has been used for representing aircraft parameter uncertainty. A study needs to be made of how the corresponding noise covariances should be chosen so that control

gains designed for one flight condition may be used at other flight conditions. An additional item of interest would be the representation of the parameters as constant but unknown random quantities, rather than the time varying quantities presently being used.

- Study the sensitivity of the designs to initial conditions.
- Consider the design of controllers for (1) gust alleviation (2) initial condition response, and (3) pilot command following. Evaluate how a controller designed for one of these three conditions will perform under the other two situations.
- Consider the inclusion of gain magnitude constraints.

6. Publications and Presentations

Presented:

"Applications of the Zangwill-Powell Method in Computing Output Feedback Gains for Linear Stochastic Systems," by H. Kaufman, "Optimization Days 1976, "Montreal, Quebec, May 6, 7, 1976.

Submitted for Publication and Presentation

"Computation of Output Feedback Gains for Linear Stochastic Systems Using the Zangwill-Powell Method" by H. Kaufman, submitted to IEEE 1976 Conference on Decision and Control, December 1976.

Master Project Reports

"Reduced State Feedback Gain Computation for a Linear Stochastic Infinite Time System Using the Zangwill-Powell Method" by Paul Bay, R.P.I., Dept. of Electrical and Systems Eng., May 1976.

"Computation of Constant Optimal Output Feedback Gains for Linear Systems," by David A. Peterson, R.P.I., Dept of Electrical and Systems Eng., May 1976.

References:

1. Zangwill, W., "Minimizing a Function without Calculating Derivaties", Computer Journal, Vol. 10, 1967, pp. 293-296.

2. Powell, M.J.P., "An Efficient Method for Finding the Minimum of a Function of Several Variables without Calculating Derivatives", Computer Journal, Vol. 7, 1964, pp. 155-162.

3. Mendel, J.M., and Feather, J., "On the Design of Optimal Time-Invariant Compensators for Linear Stochastic Time-Invariant Systems", IEEE Trans. Auto. Control, Oct. 1975, pp. 653-657.

4. Basuthakur, S. and Knapp, C. H., "Optimal Constant Controllers for Stochastic Linear Systems:, IEEE Trans. Auto. Control, Oct. 1975, pp. 664-66[illegible].

5. Kaufman, H., "Reduced State Feedback Gain Computation", Users Guide to Computer Programs", Supplemental report to NASA grant NSG 1188, Sept. 1976.

6. Joshi, S.W., "Design of Optimal Partial State Feedback Controllers for Linear Systems in Stochastic Environments", 1975 IEEE Southeastcon, Charlotte, NC, 1975.

7. McLane, P.J., "Linear Optimal Stochastic Control Using Instantaneous Output Feedback", Int. J. Control, 1971, Vol. 13, No. 2, pp. 383-396.

8. Bay, P., "Reduced State Feedback Gain Computation for a Linear Stochastic Infinite Time System Using the Zangwill-Powell Method", Masters Project, R.P.I., Electrical and Systems Eng. Dept., Troy, New York, May 1976.

9. IMSL Library 1, Edition 4, 1975, IMSL, Inc., Houston, Texas.

10. Peterson, D., "Computation of Constant Optimal Output Feedback Gains for Linear Systems," Masters Project, R.P.I., Electrical and Systems Eng. Dept., Troy, New York, May 1976.

11. Harrisberger, H.P., and Belanger, P., "Solution of the Optimal Constant Output Feedback Problem by Conjugate Gradients", IEEE Trans. on Automatic Control, May 1974, pp. 434-435.

12. Stolfi, F., "Computation of Constant Output Feedback Gains Using the Sequential Unconstrained Minimization Procedure", Masters Project, R.P.I., Electrical and Systems Eng. Dept., Troy, N. Y., Sept. 1976.

13. Fiacco, A.V. and McCormick, G.P., Nonlinear Programming, Sequential Unconstrained Minimization Techniques, Wiley, N.Y., N.Y., 1968.

TABLE I

EIGENVALUES AND GAINS

Case	GAIN MATRIX K $\delta_{ec} = K_{11}\, q + K_{12}\, \Delta\theta + K_{13}\, \Delta\alpha$ $\delta_{3c} = K_{21}\, q + K_{22}\, \Delta\theta + K_{23}\, \Delta\alpha$	Eigenvalues
Open Loop	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$-1.455 \pm j\ .660$ $-\ .007 \pm j\ .0823$ $-\ 20.$ $-\ 40.$
1	$\begin{pmatrix} 6.83 & -1.66 & .283 \\ 1.95 & -2.09 & 2.27 \end{pmatrix}$	$-11.1 \pm j\ 23.0$ -39.1 -1.83 $+.214$ $-.015$
2	$\begin{pmatrix} -.366 & .0473 & .0603 \\ .0340 & -2.84 & -1.73 \end{pmatrix}$	$-.045 \pm j\ .18$ -40.4 -21.6 $-.041$ $-.725$
3	$\begin{pmatrix} -.0046 & -0143 & -.0215 \\ .644 & .126 & 3.05 \end{pmatrix}$	-39.0 -20.0 $-1.95 \pm j\ .295$ $+.006 \pm j\ .0823$
4	$\begin{pmatrix} 6.78 & .4511 & .661 \\ 2.19 & -2.077 & 3.755 \end{pmatrix}$	$-10.9 \pm j\ 22.9$ -38.7 -2.26 $-.0579$ $-.0382$
5	$\begin{pmatrix} -.219 & .143 & -.0155 \\ -.0565 & -3.56 & -1.06 \end{pmatrix}$	-40.2 -21.0 $-.35 \pm j\ .0587$ $-.926$ $-.26$

6	$\begin{pmatrix} .0781 & .0872 & .016 \\ .0433 & .0529 & .0164 \end{pmatrix}$	-39.8 -19.6 $-1.64 \pm j\ .857$ $-.0853 \pm j\ .0507$
7	$\begin{pmatrix} .0087 & .0376 & .102 \\ .0161 & .0056 & .390 \end{pmatrix}$	-39.9 -20.0 $-1.49 \pm j\ 1.04$ $-.0388 \pm j\ .0904$

TABLE II

RMS ERRORS FOR THE CLIMB FLIGHT CONDITION USING GAINS DESIGNED FOR THIS CONDITION

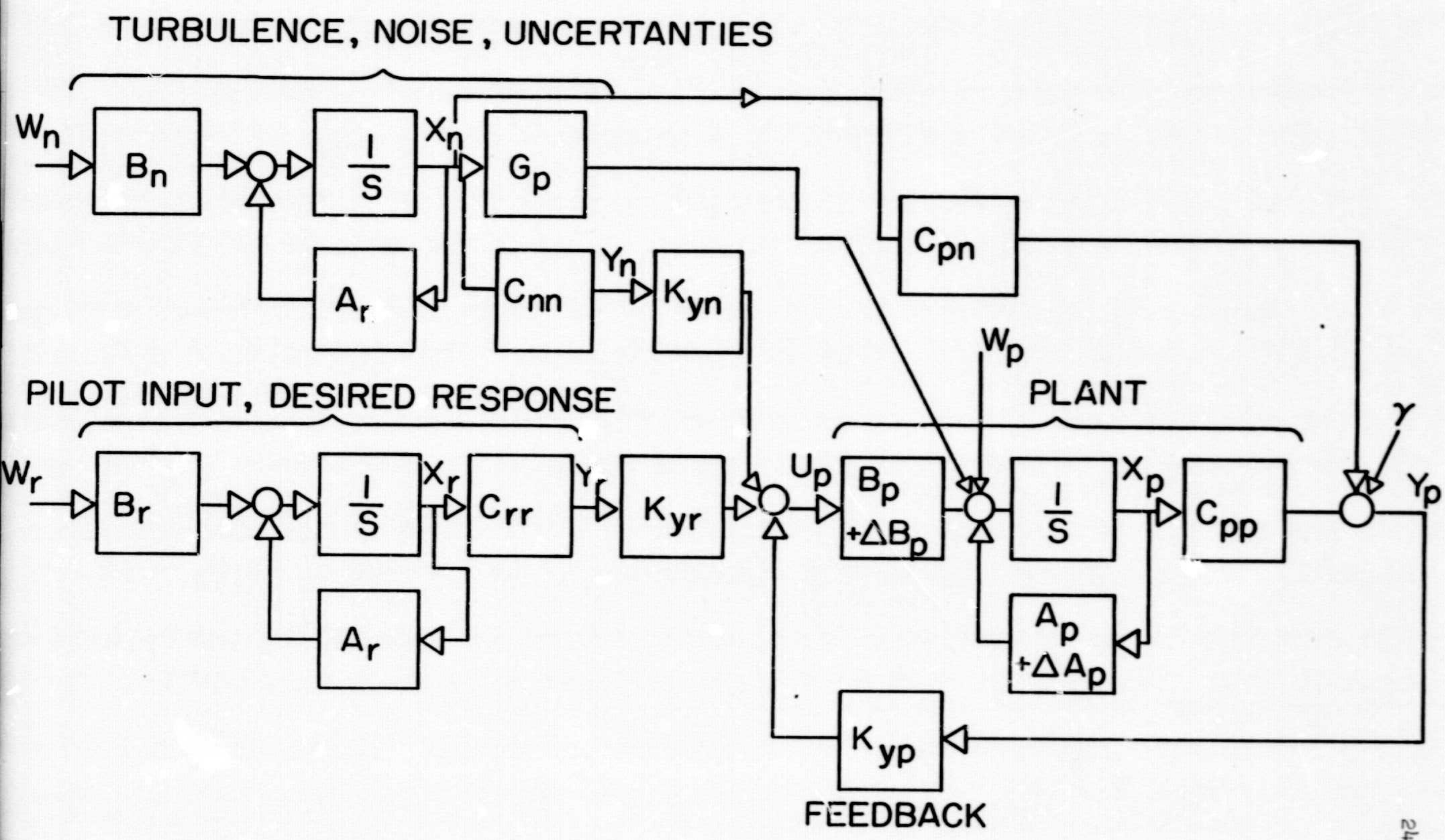
	q (r/s)	$\Delta\theta$ (r)	$\Delta\alpha$ (r)	Δn_{31} (f/s^2)	Δn_{32} (f/s^2)	$\dot{\gamma}$ (r/s)
Open Loop	.00976	.00366	.0131	8.24	7.66	.0235
1	.00966	.00412	.00987	9.22	7.27	.0218
2	.0118	.00826	.0135	8.98	8.13	.0258
3	.00920	.00350	.0101	7.23	6.67	.0203
4	.00959	.00409	.00889	9.15	6.07	.0212
5	.0112	.00713	.0165	8.69	7.92	.0249
6	.00932	.00350	.0122	7.86	7.34	.0235
7	.00927	.00414	.0123	7.71	7.24	.0222

TABLE III

RMS ERROR FOR THE CRUISE FLIGHT CONDITION USING GAINS DESIGNED FOR THE CLIMB CONDITION

	q (r/s)	$\Delta\theta$ (r)	$\Delta\alpha$ (r)	Δn_{31} (f/s^2)	Δn_{32} (f/s^2)	γ (r/s)
Open Loop	.00934	.00347	.0118	7.80	7.24	.0226
1	.00977	.00417	.00900	9.22	7.02	.0215
2	.0183	.0208	.0241	8.81	7.87	.0253
3	.0883	.00405	.00878	6.78	6.26	.0196
4	.00978	.00408	.00806	9.17	6.70	.0210
5	.0119	.0103	.0164	8.48	7.61	.0243
6	.00889	.00372	.0107	7.47	6.97	.0218
7	.0889	.00509	.0109	7.30	6.88	.0218

FIG. I REDUCED STATE CONTROL LAW SYNTHESIS



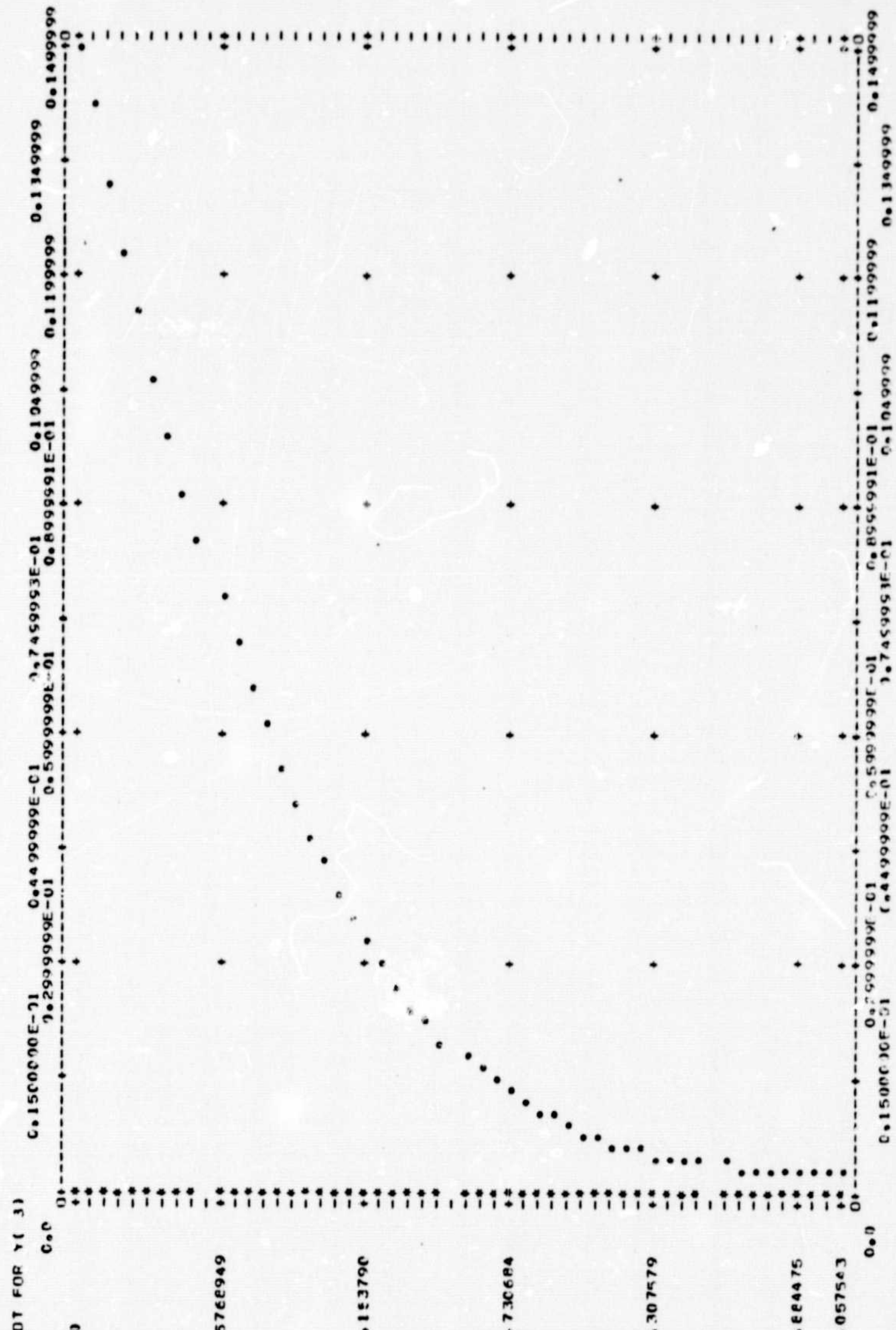


Figure 2a Open loop

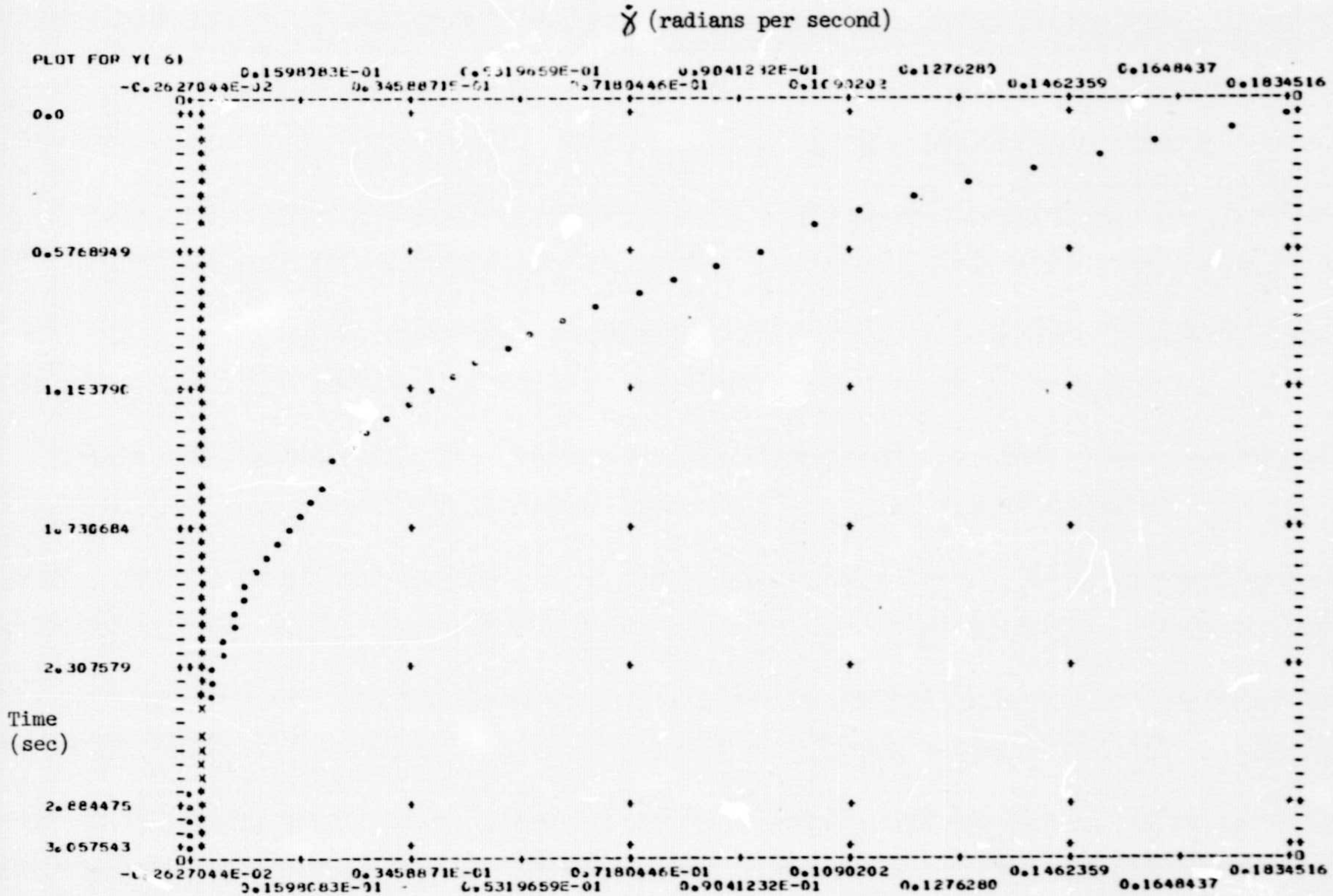


Figure 2b Open loop

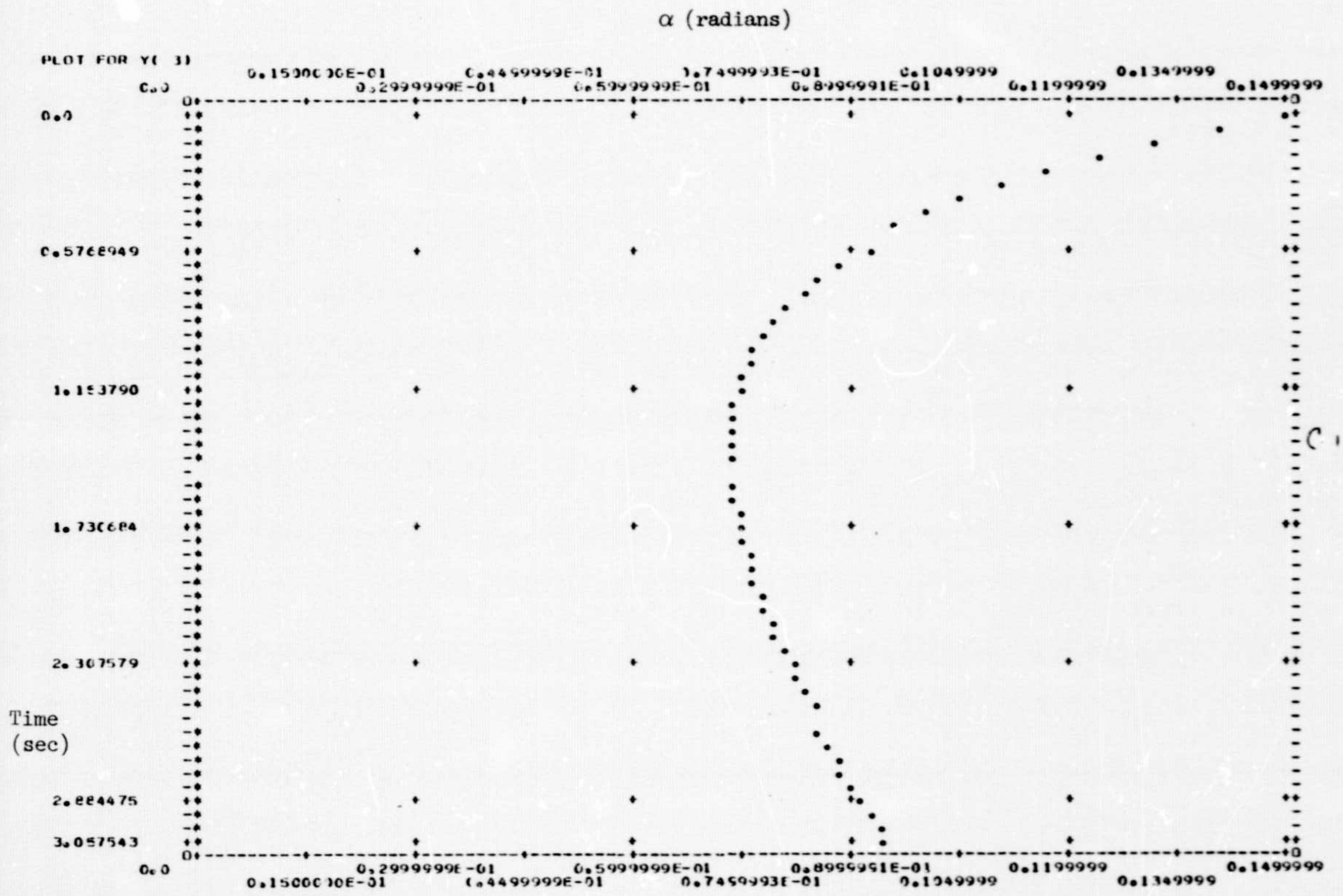


Figure 3a Case 1

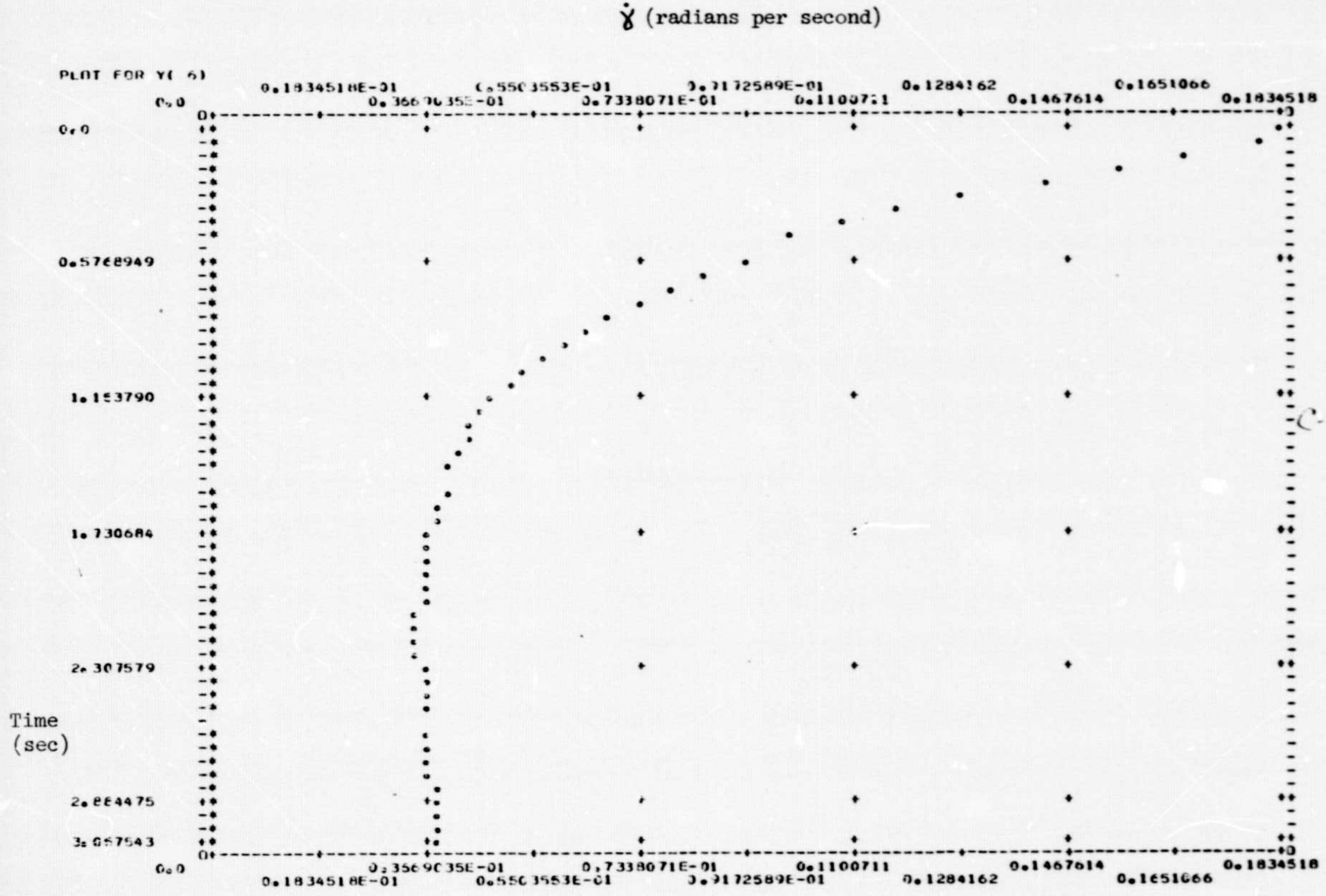


Figure 3b Case 1

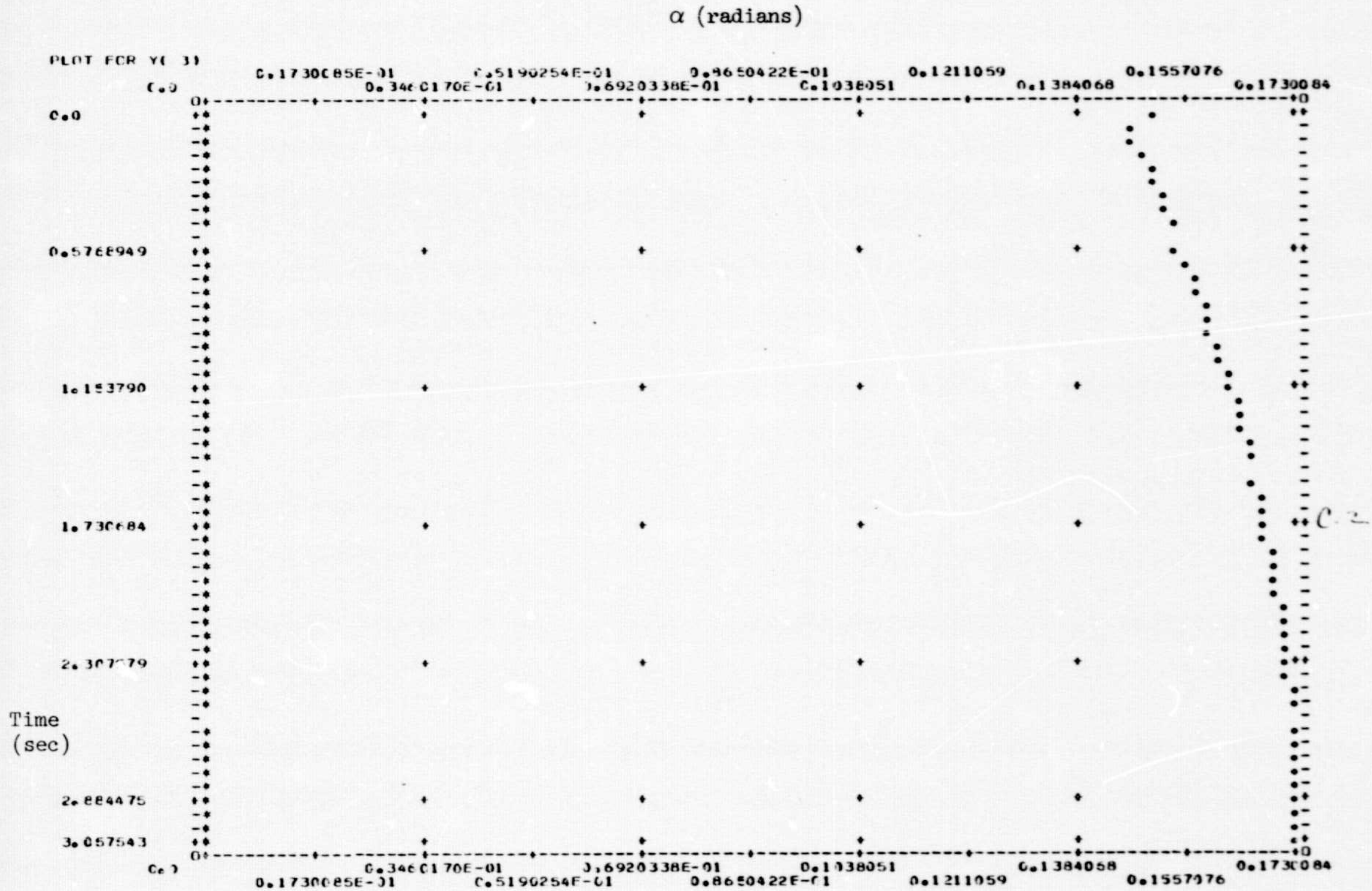


Figure 4a Case 2

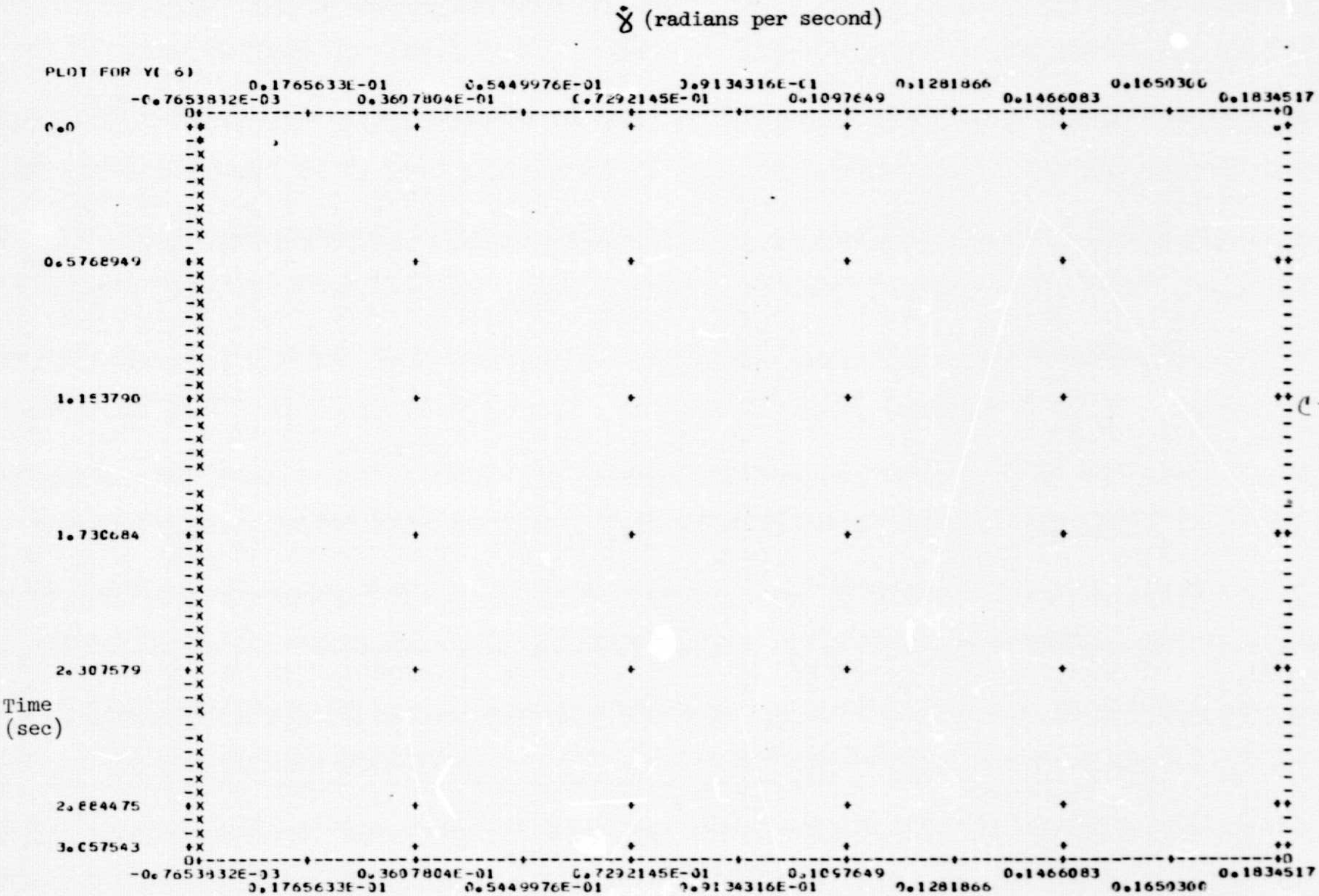


Figure 4b Case 2

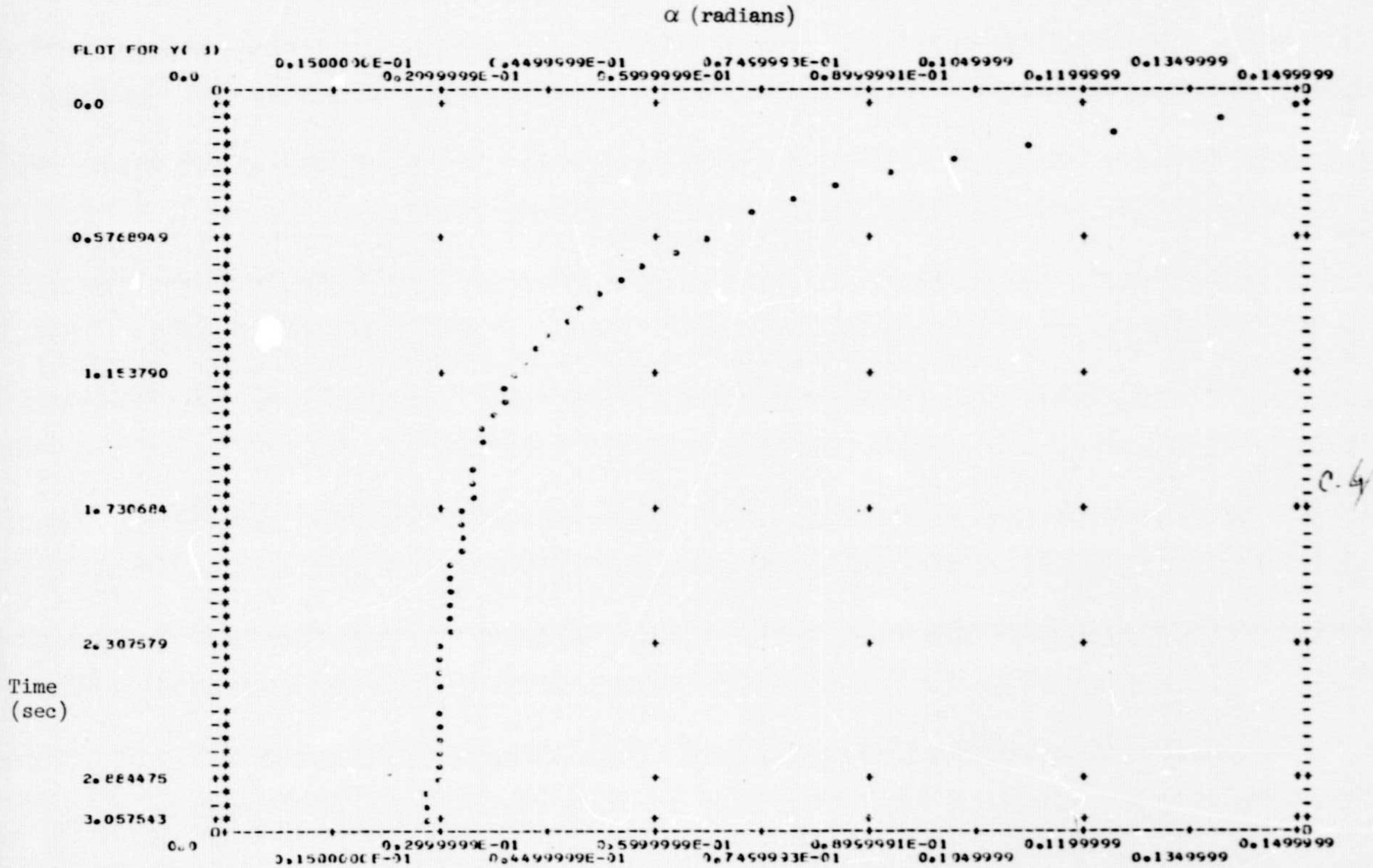


Figure 5a Case 4

C-4

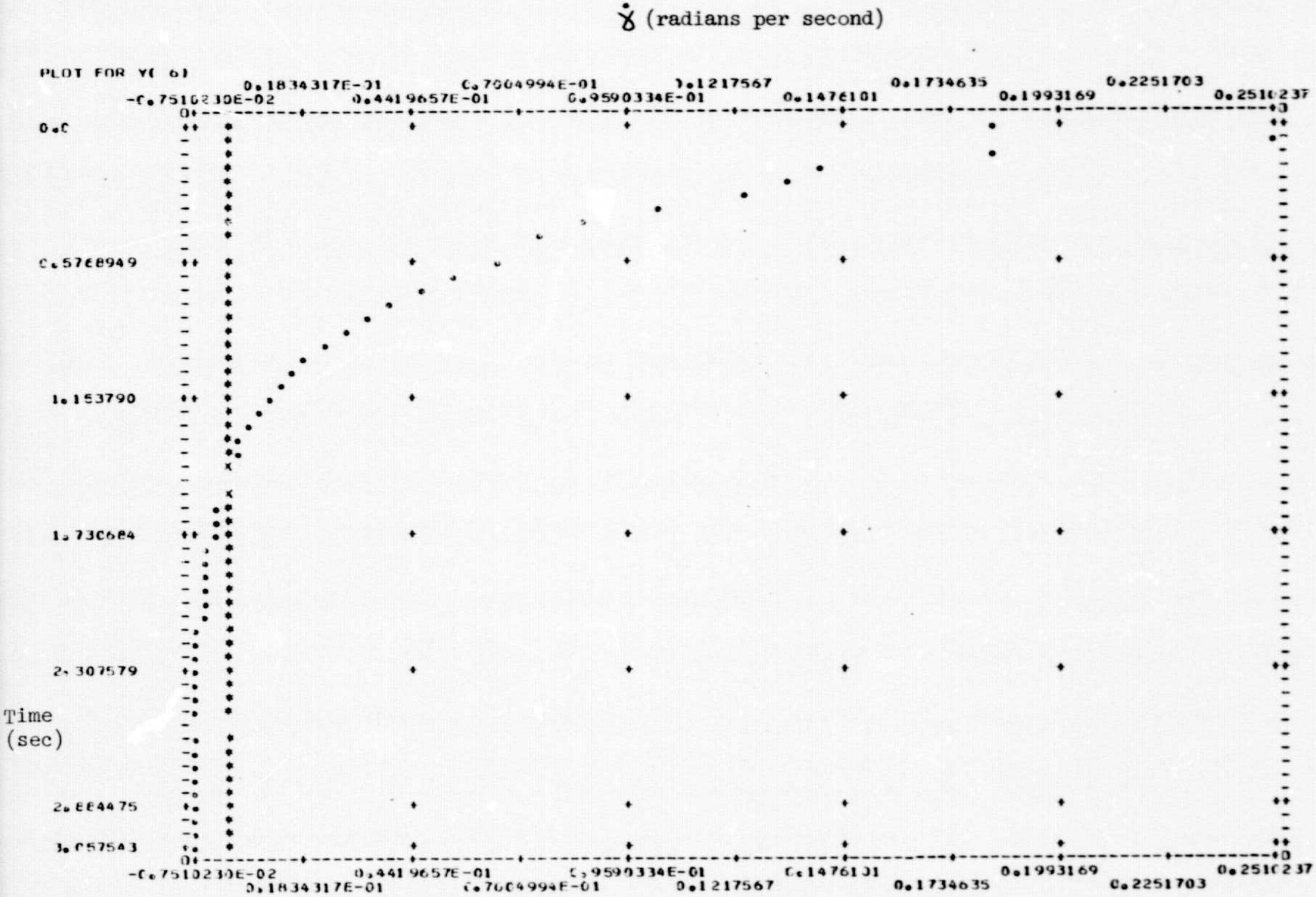


Figure 5b Case 4

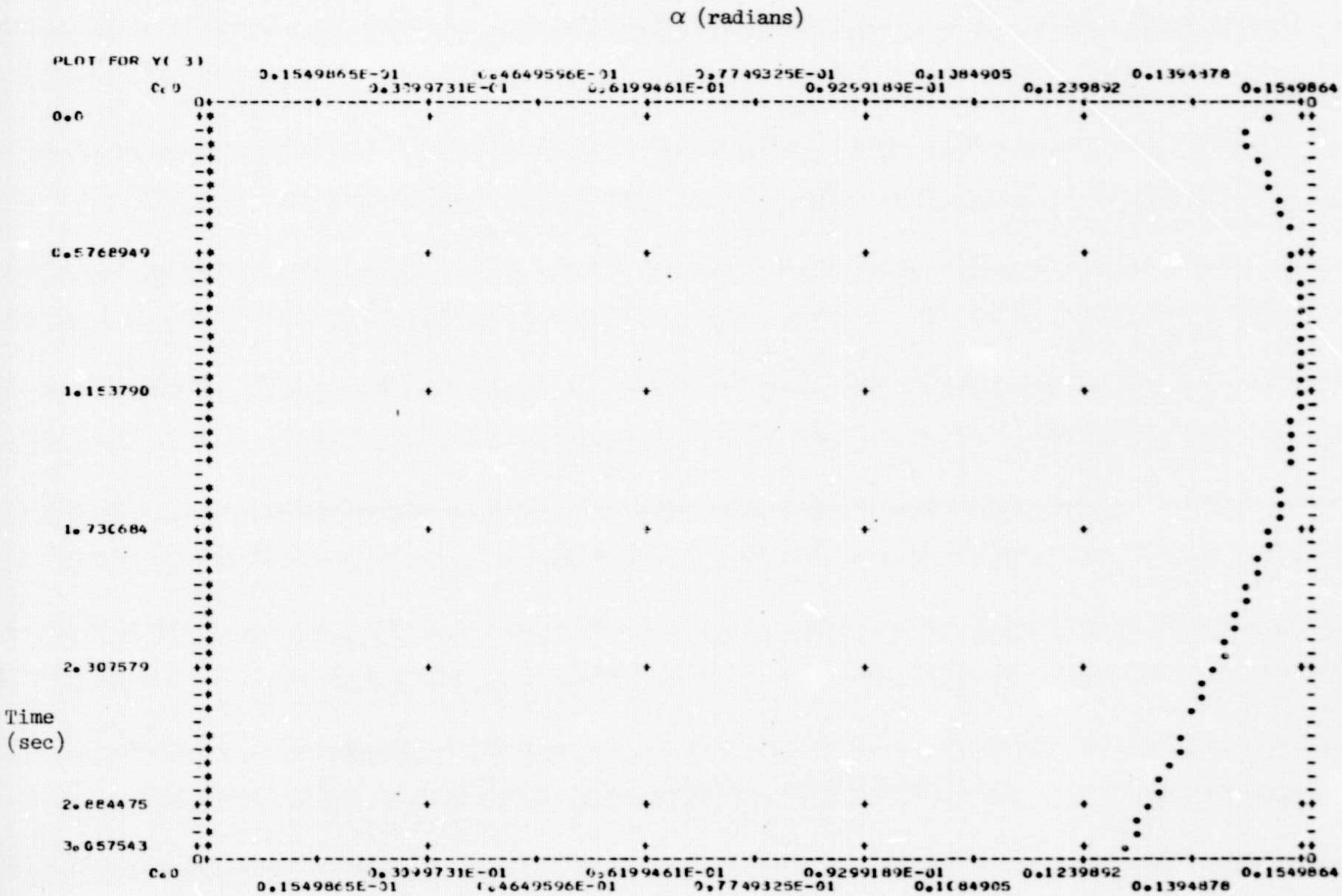


Figure 6a Case 5

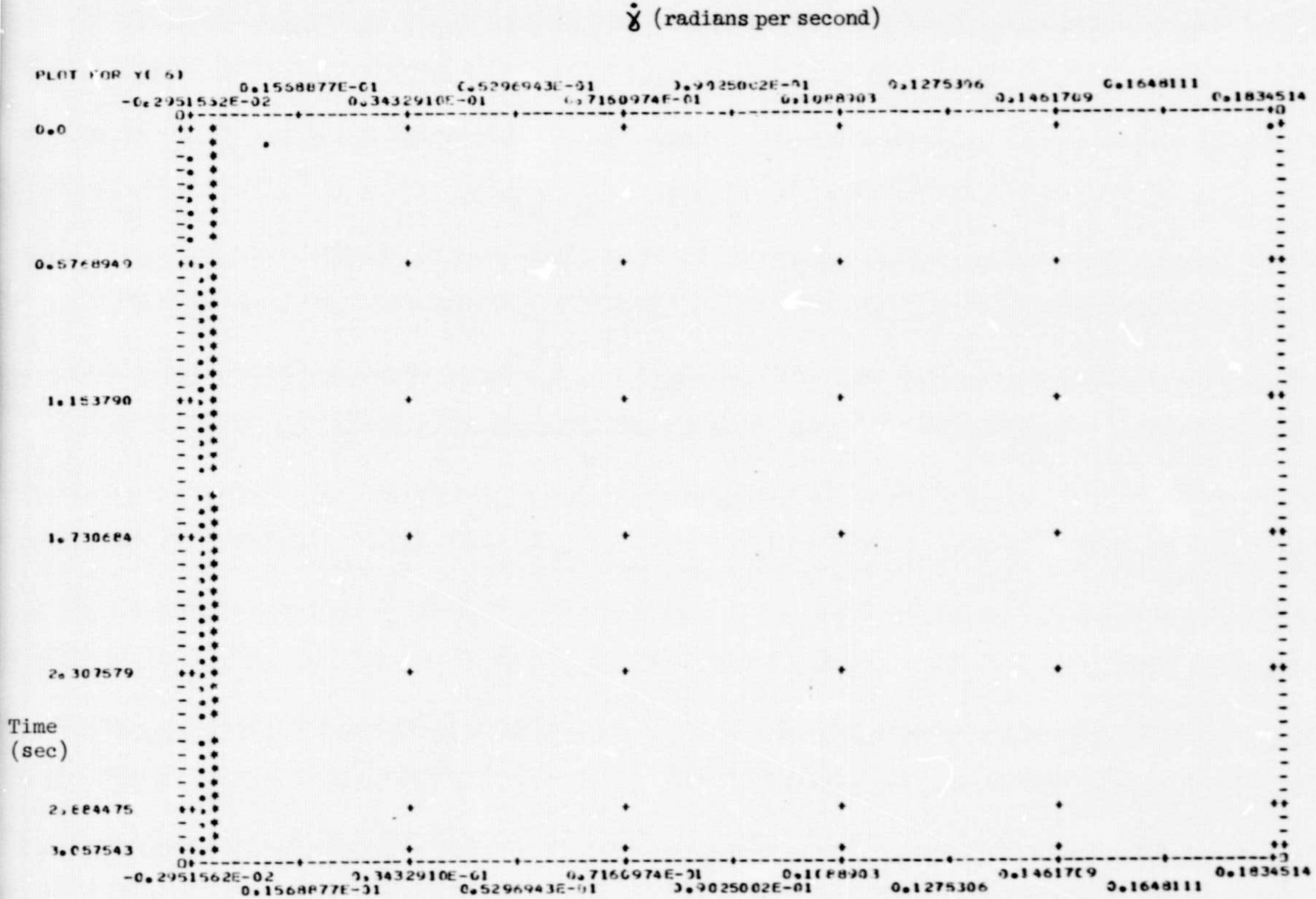


Figure 6b Case 5

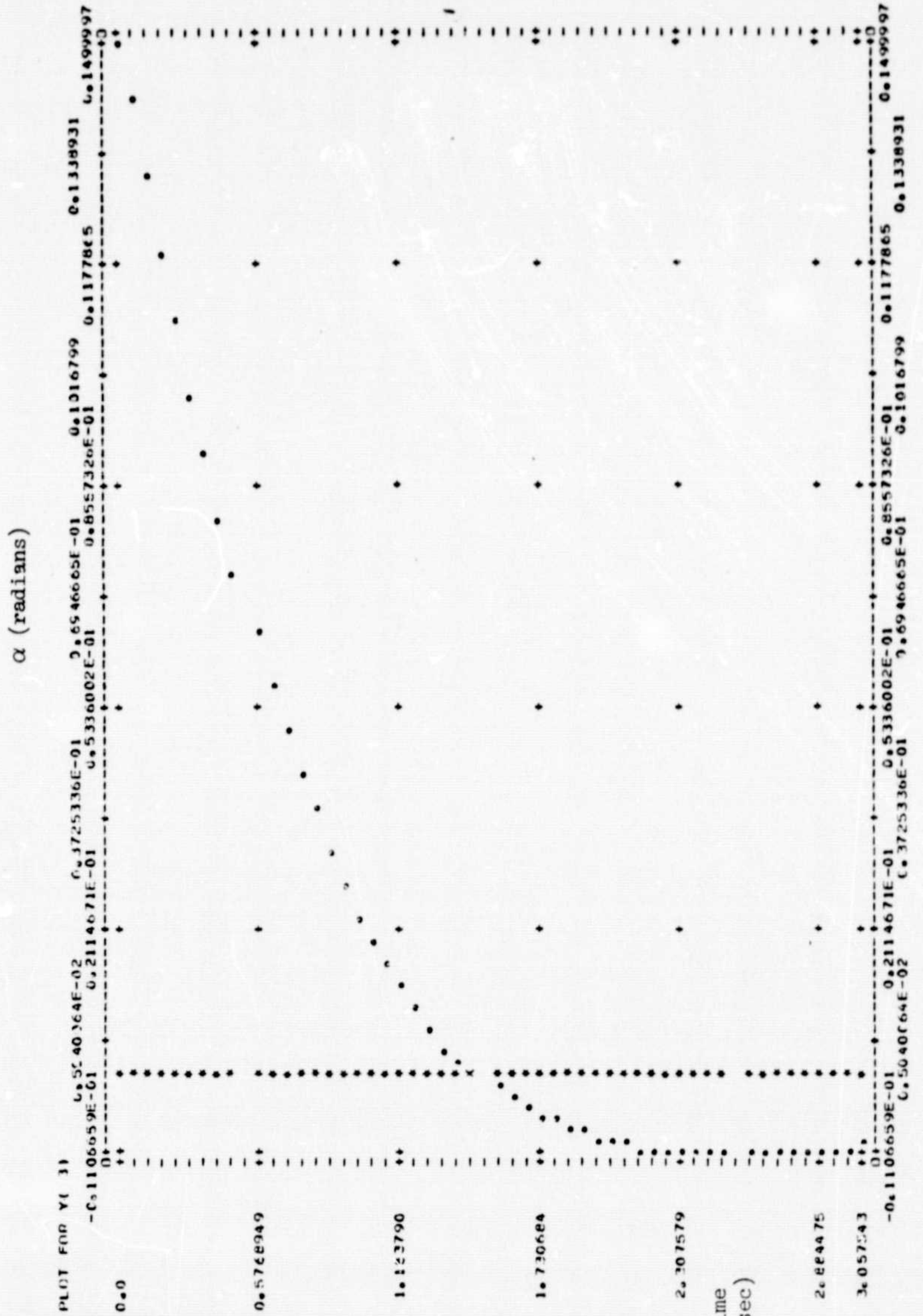


Figure 7a Case 6

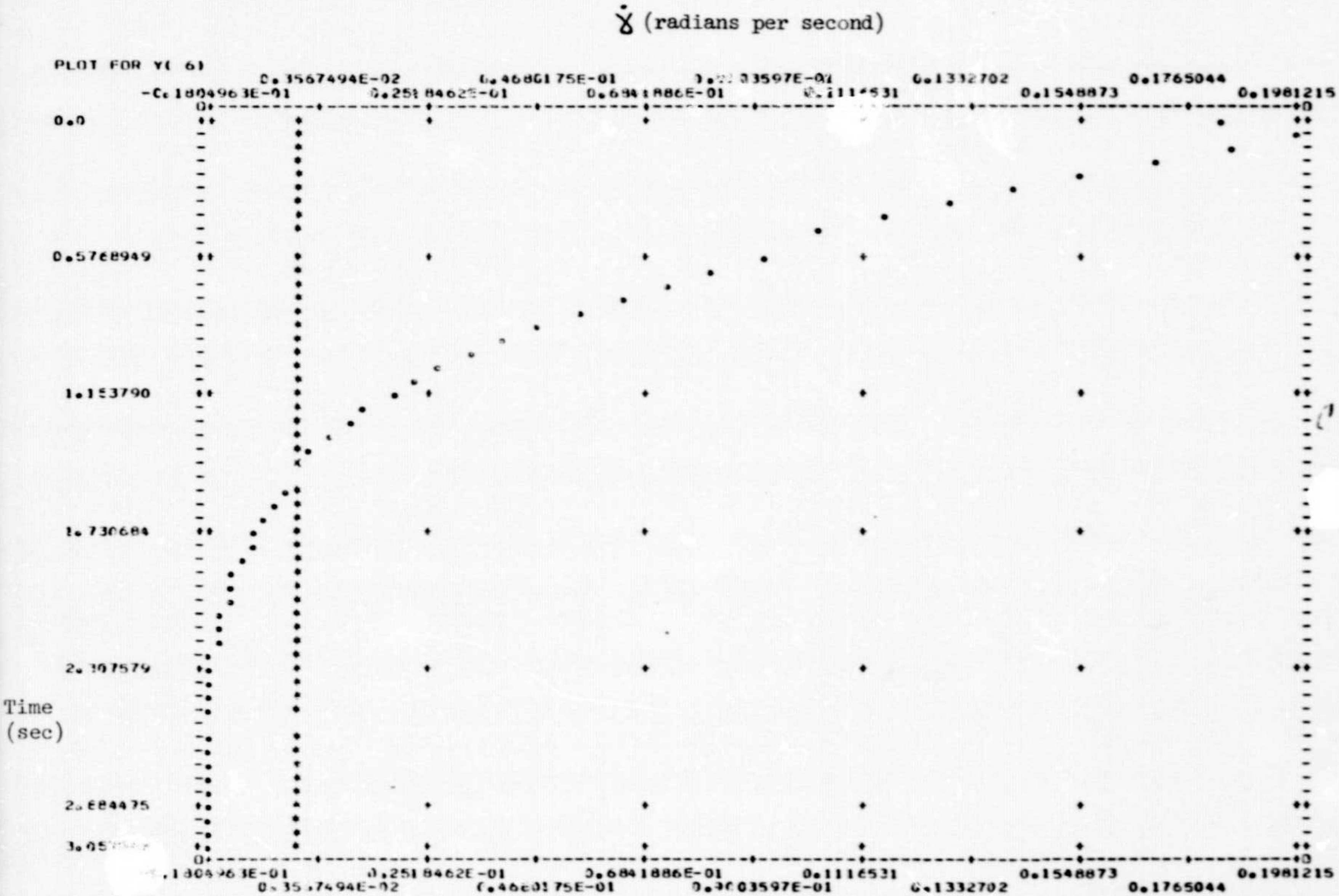


Figure 7b Case 6

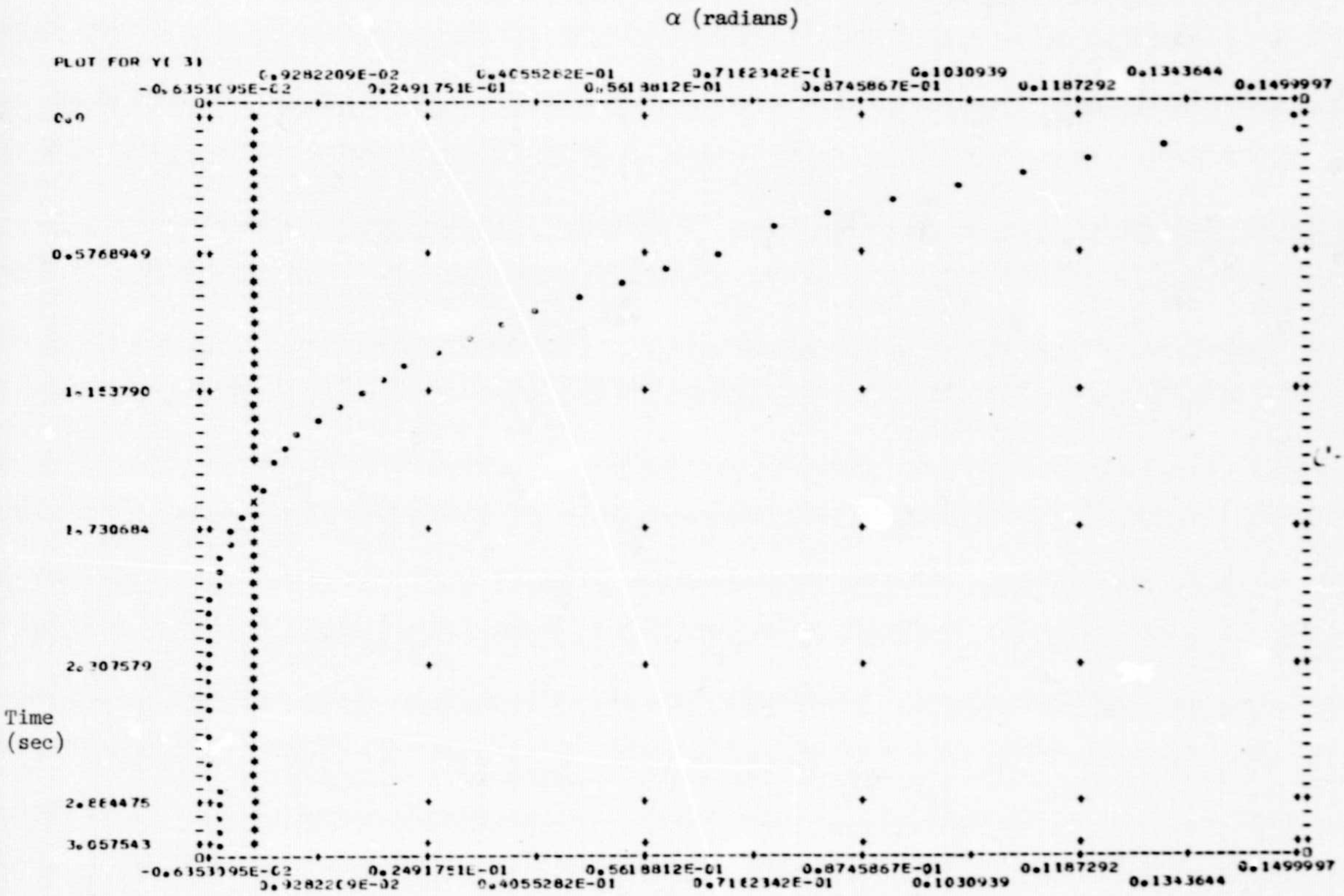


Figure 8a Case 7

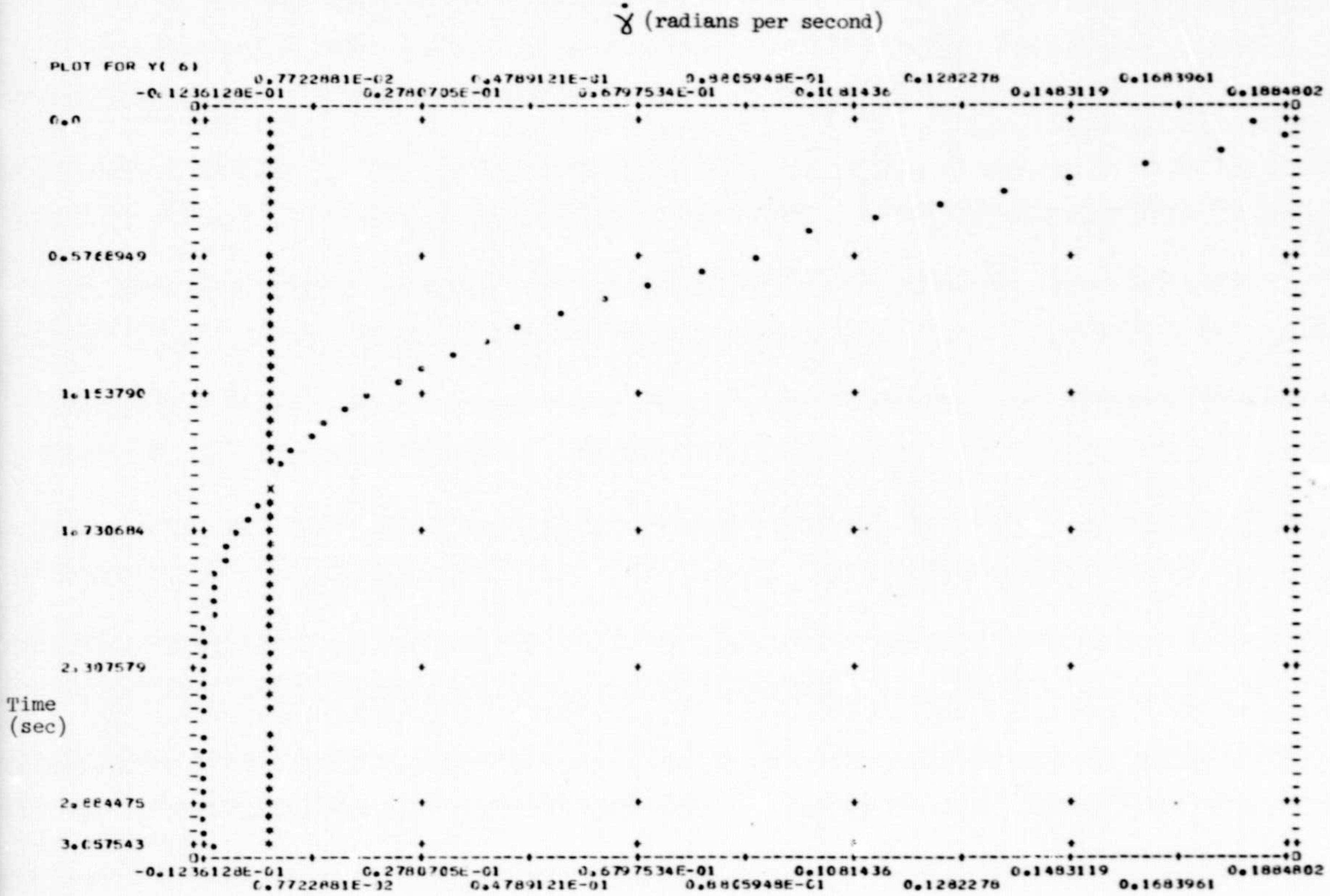


Figure 8b Case 7