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A VELOCITY–COMMAND CONTROLLER FOR A VTOL AIRCRAFT

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ABSTRACT

A technique is presented for calculating feedback and feedforward gain matrices that enable a VTOL aircraft to track input commands of forward and vertical velocity while maintaining acceptable responses to pilot inputs. Leverrier's algorithm is used for determining a set of state-variable, feedback gains that force the closed-loop poles and zeros of one pilot-input transfer function to pre-selected positions in the s-plane. This set of feedback gains is then used to calculate the feedback and feedforward gains for the velocity-command controller. The method is computationally attractive since the gains are determined by solving systems of linear, simultaneous equations. The method has been used in a digital simulation of the CH-47 helicopter to control longitudinal dynamics.

NOMENCLATURE

A	= 4 x 4 coefficient matrix
$\underline{c}$	= 2 x 1 vector of velocity commands
D	= the system characteristic equation
G	= 4 x 2 control matrix
I	= 4 x 4 identity matrix
K	= 2 x 4 gain matrix for pilot commands
K_1	= 2 x 4 feedback gain matrix for velocity commands
K_2	= 2 x 2 feedforward gain matrix for velocity commands
K_3	= 2 x 4 feedback matrix for velocity commands
$N(x_i,\delta_j)$	= the numerator of the x_i/δ_j transfer function
q	= pitch rate
u	= forward velocity perturbation
w	= vertical velocity perturbation
$\underline{x}$	= 4 x 1 state vector
$\underline{\delta}$	= 2 x 1 input vector
$\underline{\delta}_p$	= 2 x 1 pilot input vector
δ_e	= differential collective control perturbation
δ_c	= collective control perturbation
θ	= pitch angle perturbation

Subscripts

CL	= closed-loop function
OL	= open-loop function

INTRODUCTION

In recent years the overcrowding of this country's major airports has led to increased interest in the development of commercial VTOL or STOL aircraft capable of operating in the 0-500 mile range. These vehicles would operate from separate runways at existing airports or from rooftops or short runways in or near business districts. Studies have shown (1),(2) that such aircraft could reduce the total trip time providing they do not have to operate under existing take-off and landing procedures with their long delays. This will require innovations in vehicle design plus improved navigation, guidance, and control systems. One approach to this latter problem is to use on-board digital computers to handle the navigation, guidance, and control functions in an adaptive mode. Thus it is necessary to have a controller capable of following guidance commands from the computer and at the same time present acceptable flying qualities to pilot inputs.

The procedure to be presented requires a set of state-variable feedback gains that result in acceptable responses to pilot inputs. This set of gains is then used to generate feedback and feedforward gains that maintain the same response to pilot inputs, but enable the vehicle to track input commands from the guidance system in the form of changes in vertical and forward velocity. The procedure has been used in a digital simulation of the longitudinal dynamics of the CH-47 helicopter.

BASIC CONCEPTS

In considering the longitudinal dynamics of the CH-47 helicopter, the linearized equations of motion may be written in the form

$$\dot{\underline{x}} = A\underline{x} + G\underline{\delta} \quad (1)$$

where

$$\underline{x} = \begin{bmatrix} u \text{ (forward velocity perturbation)} \\ w \text{ (vertical velocity perturbation)} \\ q \text{ (pitch rate)} \\ \theta \text{ (pitch angle perturbation)} \end{bmatrix}$$

$$\underline{\delta} = \begin{bmatrix} \delta_e \text{ (differential collective perturbation)} \\ \delta_c \text{ (collective perturbation)} \end{bmatrix}$$

A is a 4 x 4 coefficient matrix
G is a 4 x 2 control matrix

Introducing state-variable feedback from all states to both inputs results in a control vector of the form

$$\underline{\delta} = K\underline{x} + \underline{\delta}_p \qquad (2)$$

where

K is a 2 x 4 matrix of feedback gains
$\underline{\delta}_p$ is a 2 vector of pilot commands

Applying this control to equation (1) yields

$$\dot{\underline{x}} = (A + GK)\underline{x} + G\underline{\delta}_p \qquad (3)$$

The Laplace transform of equation (3) is

$$[sI - (A + GK)]\underline{x}(s) = G\underline{\delta}_p(s) \qquad (4)$$

This equation leads to eight closed-loop transfer functions relating the four state variables to the two controls. These transfer functions may be obtained by employing Cramer's rule. Thus, they all have the same denominator, $|sI - (A + GK)|$, but different numerators that are obtained by taking the determinants of the matrices that result when the appropriate columns of the G matrix are substituted for columns of $[sI - (A + GK)]$. Each transfer function is a ratio of polynomials in s and the coefficients in these polynomials are functions of K. If values of K can be found to force these coefficients to take on predetermined values, then the poles and zeros of the closed-loop transfer functions can be placed anywhere in the s-plane.

DETERMINATION OF FEEDBACK GAINS

While it is not possible to specify all of the coefficients of all transfer functions, several techniques (3),(4),(5) have been presented for calculating the feedback gains necessary to place the poles of all transfer functions and the zeros of one pre-selected function at arbitrary locations in the s-plane. The methods referenced above are all based on Leverrier's algorithm (6) and the techniques of (4) and (5) compute the gains by solving linear simultaneous equations.

Before the gain matrices for the velocity controller can be calculated, feedback gains must be determined that yield acceptable responses to pilot inputs. To accomplish this, the procedure developed in (5) was employed. This method is based on the fact that the closed loop numerator and denominator functions of this system may be written as

$$N_{CL}(x_i,\delta_e) = N(x_i,\delta_e) - \sum_{\substack{j=1\\ j\neq i}}^{4} k_{2j} N\binom{x_i,\delta_e}{x_j,\delta_c} \qquad (5)$$

$$N_{CL}(x_i,\delta_c) = N(x_i,\delta_c) - \sum_{\substack{j=1\\ j\neq i}}^{4} k_{1j} N\binom{x_i,\delta_c}{x_j,\delta_e} \qquad (6)$$

$$D_{CL} = D_{OL} - \sum_{i=1}^{2}\sum_{j=1}^{4} k_{ij} N(x_j,\delta_i) + \sum_{i=1}^{4}\sum_{\substack{j=1\\ j\neq i}}^{4} k_{1i}k_{2j} N\binom{x_i,\delta_e}{x_j,\delta_c} \qquad (7)$$

where $N(x_i,\delta_e)$ is the open-loop numerator of the x_i/δ_e transfer function and D_{OL} is the open-loop denominator. The coefficients of these terms may be generated using Leverrier's algorithm.

$N\binom{x_i,\delta_e}{x_j,\delta_c}$ represents the determinant of $[sI - A]$ with i^{th} column replaced by $\underline{g}_1$ and the j^{th} column replaced by $\underline{g}_2$ (columns of the control matrix G). The coefficients of these terms do not come directly from Leverrier's algorithm, but can be determined using the procedures of either (4) or (5).

Equations (5) and (6) show that the numerator polynomials are linear functions of (n - 1) of the 2n feedback gains. Thus specifying the coefficients of one numerator allows direct calculation of (n - 1) elements of one row of the gain matrix. Arbitrarily picking the n^{th} element of that row and substituting into (7) allows direct calculation of the remaining n elements of K.

One should note from the above discussion that, in general, the poles and zeros of only one transfer function may be arbitrarily placed and that the gain matrix is not unique.

SELECTION OF A MODEL

While, in theory, the above technique is capable of generating feedback gains that can place the closed-loop poles and zeros of one transfer function at arbitrary locations in the s-plane, in practice, considerable care must be exercised in specifying these values. If the pole-zero locations or the arbitrary gain element are not carefully selected, either the gains will be too large or the other transfer functions of the system will have zero locations that result in unacceptable responses. For the CH-47 the poles were placed to satisfy settling time requirements on step inputs. However, little or no information was available on how to specify zero locations and the method employed was basically trial and error.

CLOSED-LOOP RESPONSE TO PILOT INPUTS

Once the elements of the gain matrix have been determined so that the poles and zeros of one transfer function are fixed, the numerators of the remaining closed-loop transfer functions must be checked. While all transfer functions have the same denominator so that if one is stable all will be stable, stability is not the only factor to be considered in evaluating step responses. Whether the steady-state change to a unit step input is too large or too small must be considered and right-half-plane zeros close to the origin may cause problems (7).

The closed loop numerator and denominator coefficients are easily obtained using Leverrier's algorithm with A replaced by (A + GK). With the coefficients known, any polynomial root-finding routine can be used to determine the pole and zero locations, and any transfer function with a right-half-plane zero should be checked by plotting its step response.

If the step input responses of all transfer functions are acceptable, then a satisfactory pilot-command control system of the form shown below has been generated.

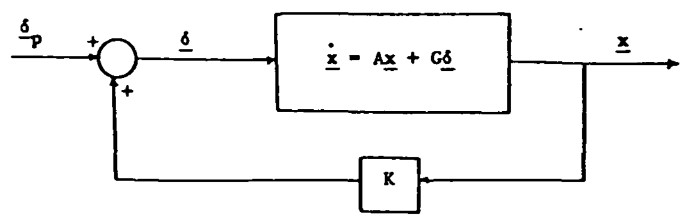

Fig. 1 Pilot-Command Controller

CLOSED-LOOP VELOCITY COMMAND CONTROLLER

With the above system it is possible, using the steady state responses of the transfer functions, to calculate the control inputs required to produce command changes in forward and vertical velocity (u_c and w_c). However, this would be an open-loop velocity controller with the inherent disadvantages of open-loop systems. To overcome this, the following system configuration is proposed.

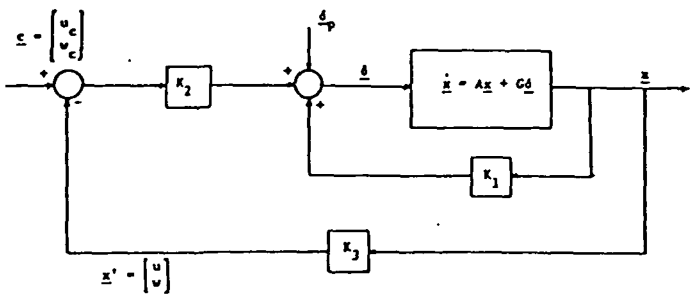

Fig. 2 Configuration for Velocity-Command Controller

where

$$K_3 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

K_2 is a 2 x 2 matrix of feed-forward gains

K_1 is a 2 x 4 matrix of feedback gains

$\underline{c}$ is a 2 vector of commanded changes in u and w

$\underline{x}' = K_3\underline{x}$ is a 2 vector of the perturbations in u and w

From figure 2 it can be seen that

$$\underline{\delta} = K_1\underline{x} + K_2[\underline{c} - K_3 x] \tag{8}$$

$$\underline{\delta} = K_1\underline{x} - K_2K_3\underline{x} + K_2 c \tag{9}$$

$$\underline{\delta} = [K_1 - K_2']\underline{x} + K_2\underline{c} \tag{10}$$

or

$$\underline{\delta} = K\underline{x} + K_2\underline{c} \tag{11}$$

Thus the closed-loop dynamics are defined by

$$\dot{\underline{x}} = (A + GK)\underline{x} + GK_2\underline{c} \tag{12}$$

Now let K be the set of feedback gains obtained by specifying the poles and zeros of one of the pilot-command transfer functions. Then it is apparent that the velocity-command and pilot-command transfer functions will have the same poles. Employing Cramer's rule and expanding the numerator determinants it can be shown that the numerators of the u/u_c, w/u_c, u/w_c, and w/w_c transfer functions may be expressed as

$$N(u,u_c) = k_{2_{11}} N_{CL}(u,\delta_e) + k_{2_{21}} N_{CL}(u,\delta_c) \tag{13}$$

$$N(w,u_c) = k_{2_{11}} N_{CL}(w,\delta_e) + k_{2_{21}} N_{CL}(w,\delta_c) \tag{14}$$

$$N(u,w_c) = k_{2_{12}} N_{CL}(u,\delta_e) - k_{2_{22}} N_{CL}(u,\delta_c) \tag{15}$$

$$N(w,w_c) = k_{2_{12}} N_{CL}(w,\delta_e) + k_{2_{22}} N_{CL}(w,\delta_c) \tag{16}$$

Each of the above equations represents a third order polynomial in s with four coefficients. While it is not possible to control all of the coefficients, it is possible to control one coefficient from each equation. If the system is to track velocity commands, the steady-state response of u and w to a unit step input on u_c must be one and zero respectively and the steady-state response of u and w to a unit step input on w_c must be zero and one respectively. This requires that the constant term of the numerator of the u/u_c transfer function equal the constant term of the denominator (which is known) and the constant term of the w/u_c transfer function equal zero to provide decoupling. Placing these constraints on equations (13) and (14) yields two linear simultaneous equations to solve for $k_{2_{11}}$ and $k_{2_{21}}$. Similar restrictions on the constant terms of the w_c transfer functions result in two equations that yield $k_{2_{12}}$ and $k_{2_{22}}$. Once the values of K_2 are known K_1 can be determined using

$$K = [K_1 - K_2'] \tag{17}$$

SIMULATION

This design procedure has been applied to the linearized longitudinal dynamics of the CH-47 helicopter. The results to be presented are for nominal velocities of 150 kts forward and 250 fpm vertical descent. The A and G matrices shown below are based on stability and control derivatives provided by NASA's Langley Research Center.

$$A = \begin{bmatrix} -.05191 & -.03898 & 8.8944 & -32.176 \\ .02731 & -.57793 & 250.67 & 1.2324 \\ -.00014 & .01769 & -1.3044 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} -.14909 & -1.2698 \\ .01857 & -8.9842 \\ .31973 & .22782 \\ 0 & 0 \end{bmatrix}$$

The open-loop characteristic equation for this A matrix is

$$D_{OL} = s^4 + 1.934s^3 - 3.579s^2 - .221s + .0117$$

The sign changes in the coefficients indicate an unstable system and right-half-plane poles occur at .034 and 1.2.

Specifying the poles and zeros of the w/δ_c transfer function as

Poles = $-.75, -.8, -.8 \pm j.4$
Zeros = $-1.0, -.8 \pm j.4$

resulted in the following gain matrix:

$$K = \begin{bmatrix} .0667 & -.02 & -23.75 & -5.17 \\ -.0021 & .0034 & 28.08 & .324 \end{bmatrix}$$

when k_{12} was specified at -.02. This feedback gain matrix yields denominator and numerator polynomials for pilot-command transfer functions as shown below with the steady-state response of each listed to the right.

	Steady-state Response/inch of control
$D_{CL} = s^4+3.15s^3+3.88s^2+2.2s+.48$	
$N(\delta_e,u) = .15s^3 - 9.23s^2 - 16.1s - 6.33$	-13.2 fps
$N(\delta_e,w) = .02s^3 - .6s^2 - s - .51$	- 1.06 fps
$N(\delta_e,q) = .32s^3 + .21s^2 + .01s$	0
$N(\delta_e,\theta) = .32s^2 + .21s + .01$	.02 rad
$N(\delta_c,u) = -1.27s^3 - 10.4s^2 - 11.4s - 1.58$	- 3.29 fps
$N(\delta_c,w) = -8.98s^3 - 23.4s^2 - 21.6s - 7.19$	15.0 fps
$N(\delta_c,q) = .228s^3 + .013s^2 - .006s$	0
$N(\delta_c,\theta) = .228s^2 + .013s - .006$	- .013 rad

Thus, the system has been stabilized and the steady-state responses look reasonable. Sign changes in the u/δ_e, w/δ_e, and θ/δ_c numerators indicate right-half-plane zeros; however, in u/δ_e and w/δ_e they are located at 64 and 34 respectively and do not seriously affect the responses. The θ/δ_c right-half-plane zero is at .14 and does cause some phase reversal. However, θ is not very responsive to δ_c commands and this may not be objectionable.

To determine the feedforward matrix, equations (13) - (16) are used to form the following 2 x 2 system of equations.

$$\begin{bmatrix} -6.33 & -1.58 \\ -.51 & -7.19 \end{bmatrix} \begin{bmatrix} k_{2_{11}} \\ k_{2_{21}} \end{bmatrix} = \begin{bmatrix} .48 \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} -6.33 & -1.58 \\ -.51 & -7.19 \end{bmatrix} \begin{bmatrix} k_{2_{12}} \\ k_{2_{22}} \end{bmatrix} = \begin{bmatrix} 0 \\ .48 \end{bmatrix}$$

The resulting feedback and feedforward matrices are

$$K_1 = \begin{bmatrix} -.0105 & -.003 & -23.75 & -5.17 \\ .0034 & -.065 & 28.08 & .324 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -.0772 & .017 \\ .0054 & -.068 \end{bmatrix}$$

These gains, used in the system configuration of figure (2), yield the following velocity-command transfer function numerators and steady-state responses:

	Steady-state Response to Unit Step Command
$N(u,u_c) = -.018s^3 + .66s^2 + 1.17s + .48$	1.0
$N(w,u_c) = -.050s^3 - .08s^2 - .04s + 0$	0.0
$N(q,u_c) = -.023s^3 - .02s^2 - .0008s$	0.0
$N(\theta,u_c) = -.023s^2 - .02s - .0008$	.002 rad
$N(u,w_c) = .089s^3 + .553s^2 + .503s + 0$	0.0
$N(w,w_c) = .611s^3 + 1.58s^2 + 1.45s + .48$	1.0
$N(q,w_c) = -.01s^3 + .003s^2 + .0006s$	0.0
$N(\theta,w_c) = -.01s^2 + .003s + .0006$	.001 rad

The responses to velocity commands of $\underline{c}^T = [10,0]$ and $\underline{c}^T = [0,10]$ were plotted and are shown in figures (3) and (4) respectively.

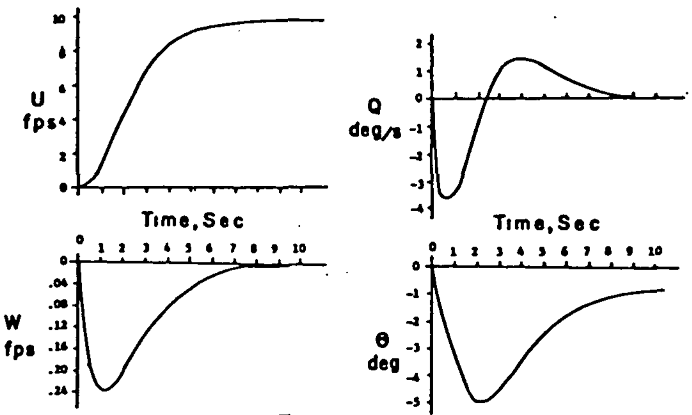

Fig. 3 Responses to $\underline{c}^T = [10,0]$

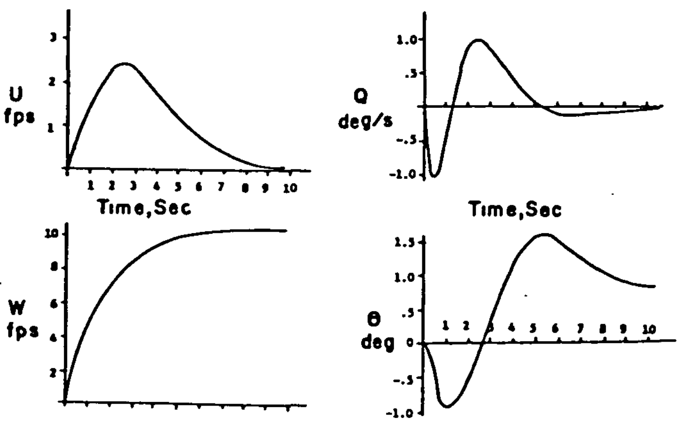

Fig. 4 Responses to $\underline{c}^T = [0,10]$

The velocities settle to within 5% of the desired steady-state changes in 5 seconds with no overshoot and the decoupling between forward and vertical velocity commands is good. The pitch angle to vertical velocity command response shows a phase reversal that reaches a peak of 0.9 degrees in 1.3 seconds, but this may not be objectionable. If these responses were unacceptable to either pilots or passengers, the gains could easily be recalculated using new pole-zero specifications.

CONCLUSIONS

A procedure has been presented that allows direct calculation of feedback and feedforward matrices for a closed-loop velocity-command controller without affecting the pilot-command transfer functions. The method has a very attractive computational characteristic in that it does not require a non-linear or iterative search technique to calculate the gains. All of the gains are determined by solving linear simultaneous equations. Numerical results are presented for the linearized longitudinal dynamics of the CH-47 helicopter.

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