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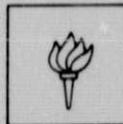
NEW YORK UNIVERSITY

FACULTY OF ARTS AND SCIENCE

DIVISION OF APPLIED SCIENCE

PROGRESS REPORT

SONIC BOOM RESEARCH

May 1, 1976 - July 31, 1976

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PROGRESS REPORT

MAY 1, 1976 - JULY 31, 1976

SONIC BOOM RESEARCH

During this period we completed our numerical program for the sonic boom
analysis including the asymmetric effect due to lift near the vertical
plane of symmetry.   Several numerical examples were computed.   The results
were presented at the third Aeroacoustic Conference in July (Two copies of
AIAA Preprints No. 76-587 are attached).

The program and a numerical example were delivered to the contractor.   A
description for the usage of the program is presented in NYU Report AA/76-11
entitled "Numerical Program for Sonic Boom Analysis-Nonlinear With Asymmetric
Correction Due to Lift, by Fanny Kung.   (Two copies are attached).

NUMERICAL PROGRAM FOR SONIC BOOM ANALYSIS*
- NONLINEAR WITH ASYMMETRIC CORRECTION DUE TO LIFT

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ABSTRACT

A computer program for CDC 6600 is developed for the nonlinear sonic boom
analysis including the asymmetric effect of lift near the vertical plane
of symmetry. The program is written in FORTRAN IV language. This program
carries out the numerical integration of the nonlinear governing equations
from the input data at a finite distance from the airplane configuration at
a flight altitude to yield the pressure signitude at ground. The required
input data and the format for the output are described. A complete program
listing and a sample calculation are given in the Appendix.[1]

LIST OF SYMBOLS

a	local speed of sound in ft/sec.
a_o	speed of sound at flight altitude
c^+, c^-	outgoing and incoming characteristics in the plane $\psi = 0$.
$\bar{f}$	shock shape, $\bar{x} = \bar{f}(\bar{r}, \psi)$
g	32.2 .. ft/sec^2
h	altitude in ft.
L	reference length (say the body length) in ft.
M	local Mach number
M_o	flig Mach number
$\Delta p = p - p_\infty$	pressure increment from ambient value in lbs./ft^2
$\bar{r}$	radical distance/L
R	gas constant
S	entropy
$\bar{S}_{sh}$	S_{sh}/R = entropy induced by shock wave/R, at $\psi = 0$
$\tilde{S}_{sh}$	$(S_{sh}/R)_{\psi\psi}$ at $\psi = 0$
u, v, w	axial velocity radial and circumferential components in ft./sec.
U_∞	flight velocity $= M_o\, a_o$ in ft./sec.
$\tilde{u}$	$u_{\psi\psi}/U$ at $\psi = 0$
$\tilde{v}$	$v_{\psi\psi}/U$ at $\psi = 0$
$\hat{w}$	w_ψ/U at $\psi = 0$
T	temperature in ^{o}Rankine
$\bar{x}$	axial coordinate (horizontal)/L
$\bar{y}$	vertical coordinate/L
Y	flight altitude in ft.

List of Symbols (Cont'd)

α   the half cone angle of the equivalent body in degree

β   shock angle with the flow direction ahead of the shock

γ   ratio of specific heat $= 1.4$

θ   flow inclination with x-axis

μ   Mach angle

ν   Prandtl Meyer angle

ψ   meridian angle, $\psi = 0$ is the vertical plane of symmetry below the airplane

Subscripts

∞   the undisturbed value along the same streamline

$\bar{x}, \bar{r}, \psi$ partial derivatives

OUTLINE OF THE NUMERICAL PROGRAM

The basic equations and the justific   ns for the approximations in the
asymmetric effects for this numerical program were described in ref. 1, 2.
The numerical analysis deals with nine variables, M, θ, $\bar{r}_\infty$, $\bar{S}_{sh}$, $\tilde{u}$, $\tilde{v}$, $\hat{w}$, $\tilde{y}_\infty$
and $\tilde{S}_{sh}$ as functions of $\bar{x}$, $\bar{r}$ in the vertical plane of symmetry $\psi = 0$.   .e
first four variables are the variables in the quasi-symmetrical analysis
(ref. 1) and the remaining five variables, which are the circumferential deriva-
tives, $u_{\psi\psi}/U_\infty$, $v_{\psi\psi}/U_\infty$ $U^{-1}\tilde{w}_\psi$, $(y_\infty)_{\psi\psi}/L$ and $(S_{sh})_{\psi\psi}/R$ at $\psi = o$, representing the
asymmetric variables (ref. 2). The nine governing equations for these two
sets of variables are coupled weakly through one variable $\hat{w}$.

The iterative procedure for the numerical integration of those nine equations
begins with the assignment of the value $\hat{w}$. The four equations for the four
variables M, θ, $\bar{r}_\infty$, $\bar{S}_{sh}$ are determined in the same manner as in the axi-sym-
metric calculations (ref. 1). The location of the new grid point $\bar{x}$, $\bar{r}$ and
the values M, θ are given by the characteristic equations with a curved C^+
line segment. The values for $\bar{r}_\infty$ and $\bar{S}_{sh}$ are obtained by integrations along
the stream line.   We will then proceed to compute the asymmetric variables.
$\tilde{u}$ and $\tilde{v}$ will be determined by the integration of the equations along C^+ and C^-
$\hat{w}$, $\tilde{y}_\infty$ and $\tilde{S}_{sh}$ are obtained by the integration of the equations along the
stream line. With this new $\hat{w}$ we continue to the next cycle of iteration until
convergence.

Similar iteration scheme is employed for the shock point as described in
ref. 2.

Since the asymmetric computations can be considered as the determination of
the value of $\hat{w}$ in the quasi-asymmetric computations. The general scheme of
the computation from the initial data to the final pressure signature at ground

will be the same as that for the axi-symmetric computations:

The calculations proceed along successions of C^- lines with index I. Along each C^- line, it will compute the new bow shock point and then the characteristic points with index J until the last C^+ line with index J_{max} or the C^+ line from a input data point. Then we proceed to the next C^- line with I increased by 1.

When two C^+ lines cross over, an embedded shock is formed. The integer KS is assigned in the order of shocks formed. For each embedded shock, its location J along the C^- line is denoted as $J(KS)$. In each sweep along a C^- line, when J equals $J(KS)$ we call the subroutine for embedded shock point (EMSHOCK) instead of $C^+ - C^-$ (CPCM) subroutines.

Whenever a shock front or a C^+ line cuts across the ground ($\bar{r} = Y/L$) the values on the ground level are obtained by linear interpolation. The pressure signature includes the reflection coefficient 1.8. When the grid point from the last C^+ line is below ground, the computation ends.

The program contains four main subroutines: (i) C^+ and bow shock [CPSH], (ii) C^+ and C^- [CPCM], (iii) formation of shock [FSHOCK] and (iv) embedded shock [EMSHOCK] . It does not have a subroutine for the intersection of two shocks therefore the program will stop when two shocks do intersect.

The input definition will be discussed in Table I, the format for input data appears in Table II, and the description of output will be in Table III.

We will discuss briefly how do we control the step sizes and prepare the input data.

CONTROL OF STEP SIZES

The step size is controlled by inequaliti , in the radial distance between
two adjacent bow shock points:
$$\bar{r}\check{-}\bar{r} < C_1 \qquad \text{for} \qquad \check{r} \le \bar{r}_1$$
$$\check{r} - \bar{r} < C_1\,\check{r} \qquad \text{for} \quad \bar{r}_1 < \check{r} \le \bar{r}_2$$
and
$$\check{r} - \bar{r} < C_2 \qquad \text{for} \quad \bar{r}_2 < \check{r}$$

In this program we have $\bar{r}_1 = 1$ $C_1 = 0.04$ $\bar{r}_2 = 100$ $C_2 = 4$. Therefore
the upper bound for the step sizes increases linearly from 0.04 at $\bar{r} = \bar{r}_1 = 1$
to 4 at $\bar{r} = \bar{r}_2 = 100$ and then remains equal to 4 all the way to the ground.

A change in step size and in the rate of increment can be made by changes
in the values of $C_1\,\bar{r}_2$ and C_2 provide that it is continuous, i.e.,
$C_1\ \bar{r}_2 = C_2$.

When we want to use different control functions we should change the control
equations in the main program and in the subroutine for C^+ and shock (CPSH).

PREPARATION OF INPUT DATA

The input data along $\bar{r} = \bar{r}_0$ will be obtained from experimental data or from the full three dimensional analysis near the airplane configuration. At each point we input $\bar{x}$, M, o, $\bar{r}_\omega$, $\bar{S}_{sh}$, $\tilde{u}$, $\tilde{v}$, $\hat{w}$, $\tilde{y}_\omega$ and $\tilde{S}_{sh}$. For the bow shock we input in addition the shock angle β. It should be observed that not all the input data are independent of each other. For example: at bow shock, $\bar{r}_\omega = \bar{r}$ while $\bar{S}_{sh}$, β and M are related to o and $\bar{r}$ and $\tilde{S}_{sh}$ is related to $\tilde{y}_\omega$. When the input data are read from the data cards, we get

$$IUCAL = 0$$

If we intend to compute or modify some of the input data by some equations we set

$$IUCAL = 1$$

and make the modifications from Statement no. 107 to Statement No. 3505.

In our sample calculation, the input data will first be prepared from an axi-symmetric computation of an equivalent symmetric body to $\bar{r} = \bar{r}_0$. Therefore we have $\tilde{u} = \tilde{v} = \hat{w} = \tilde{S}_{sh} = 0$. Since $\bar{y} = Y/L - \bar{r} \cos \psi$ we have $\tilde{y}_\omega = \bar{r}_\omega$. If we set IUCAL = 0, the pressure signature will be that of a symmetric body with asymmetric effect due to the two dimensional atmosphere layer only and the difference from the pure axi-symmetric calculation is very small.

We set IUCAL = 1, as in the sample calculation and then proceed to compute the asymmetric terms $\tilde{u}$, $\tilde{v}$, $\hat{w}$, $\tilde{S}_{sh}$ and $\tilde{y}_\omega$ from the linearized theory of an assigned lift distribution (Ref. 2). Since the leading edge of wing is so located that the characteristic line hits $\bar{r} = \bar{r}_0$ at $\bar{x} = 6.85$ in the plane $\psi = o$ lying behind the bow shock, the bow shock is still symmetric with $\tilde{S}_{sh} = 0$ and $\tilde{y}_\omega = \bar{r}_\omega$. Only the input data of $\tilde{u}$, $\tilde{v}$ and $\hat{w}$ for $\bar{x} > 6.85$ are changed

by the formulas for the special lift variation in Fig. 8 of ref. 2. For a different approximation theory or a different lift variation those equations should be revised. (Statement No. 170-3505).

Finally, we want to point out that we have included the data beyond the body in order to yield the tail shock at ground. In the sample calculation we use five body lengths. An estimate of the length on the safe side can be obtained from the length required in the real atmospheric program based on Whitman's theory.

REFERENCES

1. Ferri, A.; Siclari, M. and Ting, L.: Sonic Boom Analysis for High Altitude flight at High Mach Number, AIAA Paper 73-1034. Progress in Astronautics and Aeronautics Vol. 38 pp. 301-320, AIAA, New York, 1975

2. Ferri, A.; Ting, L. and Lo, R. Nonlinear Sonic Boom Analysis Including the Asymmetric Effects. AIAA Paper No. 76 - Aeroacoustic Conference Stanford, Calif., July 26-28, 1976.

TABLE 1

INPUT DEFINITIONS

Ho   ········ Y ,   flight altitude in ft.

xL   ········ L ,   reference length (say the body length) in ft.

XXMO   ———— M_0,   flight Mach number

CRB   ——— $\bar{r}_0$,   radial distance of initial data line/L

TEST R   ⎫

TEST IM   ⎬ not used in this program

TEST X   ⎭

XXXXX1 ——————— C_1 ⎫

XXXXX2 ———·—— C_2 ⎬ constants in the control of step size

XR1 ——————— $\bar{r}_1$ ⎪

XR2 ——— —— $\bar{r}_2$ ⎭

IUCAL $\begin{cases} = 0 \\ \neq 0 \end{cases}$ all input read from data cards

                     some input data compute from formulas for each special cases

ISTOP   ————   maximum number of c^- line, for ISTOP use a large number, say 1000 to go to the ground.

             for detail information along I*th c^- line, set ISTOP = I*

HH   ———— y , altitude in ft.

TT ———————   Ts, temperature in $^\circ$R

NUMBE —————   number of HH table (input card 5). In this program I use number = 63. In case the altitude is greater than 120,000 ft. We can increase this table by changing the value of NUMBE.

VV ———————— y Prandtl Mayer angle

ZMMM ————   M, Mach number

IUMBE ———   Number of entries in VV table (input card 6). In this program I use IUMBE = 172. In case the Mach number is greater than 8.5, the table has to be increased and so does the value of IUMBE.

KS ———— number of embedded shock

BETT ———— β , of the embedded shock

BET ———— β , of Bow shock

XX ———— $\bar{x}$, axial coordinate (horizontal) /L.

RR ———— $\bar{r}$, vertical coordinate /L.

THETHE ———— , flow inclination with x-axis.

XMXM ———— M, local Mach number

RINRIN ———— $\bar{r}_\infty$ value of $\bar{r}$ for a stream line far upstream

SRSR ———— $\bar{S}_{sh}$ entropy due to shock wave (s)/R

UU ———— $\tilde{u}$, $u_{\psi\psi}/U_\infty$ at $\psi = 0$

VE ———— $\tilde{v}$ $v_{\psi\psi}/U_\infty$ at $\psi = 0$

WW ———— $\hat{w}$ w_ψ /U_∞ at $\psi = 0$

yy ———— $\tilde{y}_\infty$ $Y_{\infty\psi\psi}/L$ at $\psi = 0$

SS ———— $\tilde{S}_{sh}$, $(S_{sh})_{\psi\psi}/R$ at $\psi = 0$

BIGXTEM ———— $\underline{X}$ ⎫
EPSTEM ———— $\underline{E}$ ⎬
SBIRKK ———— $\overline{kk}$ ⎱ used in the special example for creation of
asymmetric data with IUCAL = 1.
SBIR ———— S_I ⎰

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TABLE II

INPUT DATA FORMAT

Card 1   (4F 10.5)

H_o, xL, XXMO, CRB

Card 2   (3F 10.5)   (not being used, put a

TESTR, TESTIM, TESTX   blank card)

Card 3   (4F 10.5)

XXXXXX1, XXXXX2, XR1, XR2

Card 4   (4I5)

IUCAL, ISTOP

Card 5   (2F 10.5)

HH(1)   TT(1)

HH(NUMBE)   TT(NUMBE)

Card 6   (10X,   2E 20.8)

VV(1)   ZMMM(1)
VV(2)   ZMMM(2)

VV(IUMBE)   ZMMM(IUMBE)

Card 7   (5X, I5, E20.10)

MAXBOY , DELTA   (not being used, put a blank card)

Card 8   (I5, E15.8)

KS,   BETT

Card 9   (4E20.9)

BIGXTEM,   EPSTEM,   SBIRKK,   SBIR

Card 10   (3E 20.9/ 10X, 3E20.9)

    XX(1)   THETHE(1)   XMXM(1),   RINRIN(1),   SRSR(1), BET
    UU(1) , VE(1)   WW (1) ,   YY(1),   SS(1)

    XX(2)   THETHE(2),   XMXM(2),   RINRIN(2),   SRSR(2)
    UU(2)   VE(2),   WW(2)   YY(2) ,   SS (2)

TABLE III

The output for this program is divided into two parts. The first part contains a tabulation of the input parameters to the program together with three input tables, HH vs. TT; VV v.s. $ZMMM$; and the input table of $\bar{x}, 0$ M, $\dot{\bar{r}}_\infty$ $\bar{S}_{sh}$, with the given radius (CRB). The second part first contains with table of J, $\tilde{u}$, $\tilde{v}$, $\hat{w}$, $\tilde{Y}_\infty$, $\tilde{S}_{sh}$ which will be the values of the input data if IUCAL $= 0$. In case IUCAL $\neq$ 0, the values will be recalculated (see remarks in the preparation of input data) and the recalculated values of $\tilde{u}$, $\hat{v}$, $\hat{w}$, $\tilde{Y}_\infty$, $\tilde{S}_{sh}$ will be shown in the table.

Along each $\bar{C}$ characteristic line ($I = 1, 2,\ldots$) the data of bow shock, first point of $C^+\!\!-\!\!C^-$ after the bow shock, the point of the formation of shock, and the embedded shock whenever it is present will be tabulated. In case the same I of bow shock appears twice, that means the control in the step size is activated in the bow shock calculations (see remarks in regard to the step size control). The output format is as follows:

Bow shock $I =$

$$\bar{x};\ \bar{r}^\prime\ \theta^\prime\ M^\prime\ \bar{r}_\infty^\prime\ \bar{S}_{sh}^\prime\ \beta^\prime$$

$$JMAX\quad \tilde{u}\quad \tilde{v}\quad \hat{w}\quad \tilde{Y}_\infty\quad \tilde{S}_{sh}$$

Cpcm $J =$

$$\bar{x}^\prime\ \bar{r}^\prime\ \theta^\prime\ M^\prime\bar{r}_\infty^\prime\quad \bar{S}_{sh}$$

$$\tilde{u}\ \tilde{v}\ \hat{w}\quad \tilde{Y}_\infty\ \tilde{s}$$

Shock FORMED at $J =$ $KS =$

$$\bar{x}_a^\prime\quad \bar{r}_a^\prime\ \theta_a^\prime\quad M_a^\prime\ \bar{r}_{\infty a}^\prime\quad \bar{S}_{sh_a}\quad \beta(ks)$$

$$\tilde{u}_a\quad \tilde{v}_a\ \hat{w}_a\quad \tilde{Y}_{\infty a}\ \tilde{S}_{sh_a}$$

F SHOCK FORMED CASE$\{^\Lambda_\beta$ J = , KS =

$$\bar{x}_a' \quad \bar{r}_a' \quad \Theta_a' \quad M_a' \quad \bar{r}_{\infty a}' \quad \bar{s}_{sh_a}' \quad \beta'$$

$$\tilde{u}_a \quad \tilde{v}_a \quad \hat{w}_a \quad \tilde{y}_{\infty a} \quad \tilde{s}_{sh_a}$$

$$\Theta_b' \quad M_b' \quad r_{\infty b}' \quad s_b' \quad \{^{XMCAL}_{XMDEFC}$$

$$\tilde{u}_b \quad \tilde{v}_b \quad \hat{w}_b \quad \tilde{y}_{\infty b} \quad \tilde{s}_{sh_b}$$

REMARK: Subscripts a, b for points ahead and behind the shock.

XMDEFC and XMCAL are the difference of M_b's computed from

the shock equations and from the characteristic equations.

EMSHOCK J = , KS =

$$\bar{x}_a \quad \bar{r}_a \quad \Theta_a \quad M_a \quad \bar{r}_{\infty a} \quad \bar{s}_{sh_a} \quad \beta'$$

$$\tilde{u}_a \quad \tilde{v}_a \quad \hat{w}_a \quad \tilde{y}_{\infty a} \quad \tilde{s}_{sh_a}$$

$$\Theta_b \quad \bar{M}_b \quad \bar{r}_{\infty b} \quad \bar{s}_{sh_b} \quad \tilde{f}_r$$

$$\tilde{u}_b \quad \tilde{v}_b \quad \hat{w}_b \quad \tilde{y}_{\infty b} \quad \tilde{s}_{sh_b}$$

When the calculation reaches to the ground, all the data will be inter-

polated. The output format is

EMSHOCK
REACH ground.......cpcm
cpSH

$$\bar{x} \quad \Theta \quad M \quad \bar{r}_\infty \quad \bar{s}_{sh} \quad \Delta p$$

$$\tilde{u} \quad \tilde{v} \quad \hat{w} \quad \tilde{y}_\infty \quad \tilde{s}_{sh} \quad (\Delta q)$$

REMARK: Δp = p - p_∞ in lbs./sq. ft.

Δq = p_b - p_∞ for embedded shock only