NASA CONTRACTOR REPORT

NASA CR-2712

THE ELECTRON BOLTZMANN EQUATION IN A PLASMA GENERATED BY FISSION FRAGMENTS

H. A. Hassan and Jerry E. Deese

Prepared by
NORTH CAROLINA STATE UNIVERSITY
Raleigh, N.C. 27607
for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1976

1. Report No. NASA CR-2712	2. Government Accession No.	3. Re
4. Title and Subtitle THE ELECTRON BOLTZMANN EQUATION IN A PLASMA GENERATED BY FISSION FRAGMENTS		5. Report Date July 1976
		6. Performing Organization Code
7. Author(s) H. A. Hassan and Jerry E. Deese		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address North Carolina State University Raleigh, North Carolina 27607		11. Contract or Grant No. NSG 1058
		13. Type of Report and Period Covered Contractor's Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Project Manager, Frank Hohl, Environmental and Space Sciences Division, NASA Langley Research Center, Hampton, Virginia

16. Abstract

A Boltzmann equation formulation is presented for the determination of the electron distribution function in a plasma generated by fission fragments. The formulation takes into consideration ambipolar diffusion, elastic and inelastic collisions, recombination and ionization and allows for the fact that the primary electrons are not monoenergetic. Calculations for He in a tube coated with fissionable material show that, over a wide pressure and neutron flux range, the distribution function is non-Maxwellian but the electrons are essentially thermal. Moreover, about a third of the energy of the primary electrons is transferred into the inelastic levels of He. This fraction of energy transfer is almost independent of pressure and neutron flux but increases sharply in the presence of a sustainer electric field.

17. Key Words (Suggested by Author(s)) (STAR category underlined) Kinetic Theory Nuclear Pumped Lasers	18. Distribution Statement Unclassified - Unlimited Subject Category 75

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	29	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

THE ELECTRON BOLTZMANN EQUATION IN A PLASMA GENERATED BY FISSION PRODUCTS

by

H. A. Hassan and Jerry E. Deese
North Carolina State University
Raleigh, North Carolina

SUMMARY

A Boltzmann equation formulation is presented for the determination of the electron distribution function in a plasma generated by fission fragments. The formulation takes into consideration ambipolar diffusion, elastic and inelastic collisions, recombination and ionization and allows for the fact that the primary electrons are not monoenergetic. Calculations for He in a tube coated with fissionable material show that, over a wide pressure and neutron flux range, the distribution function is non-Maxwellian but the electrons are essentially thermal. Moreover, about a third of the energy of the primary electrons is transferred into the inelastic levels of He. This fraction of energy transfer is almost independent of pressure and neutron flux but increases sharply in the presence of a sustainer electric field.

INTRODUCTION

Recent experiments in CO, Xe-He and Ne-N_2 mixtures[1-3] have demonstrated direct nuclear pumping. At a time when the quest for high power lasers is at its peak, such a development is rather significant because it affords the means of uniform pumping of large volume high pressure gases.

Although the inversion mechanisms were not discussed in any detail in Refs. 1-3, it is generally believed that both the high energy heavy particles and the low energy electrons play a role in population inversion.[4,5] An estimate of the extent of the role played by the heavy particles and electrons in nuclear pumping requires, as a first step, the determination of their respective energy distributions. Using a monoenergetic source of high energy primary electrons, Wang and Miley[5] calculated the electron distribution function using Monte Carlo techniques. Later, Lo and Miley[6] employed a simplified version of the Boltzmann equation, and the results compared favorably with the Monte Carlo calculation. In their study of electric discharges in gases, Thomas[7] and Thomas and Thomas[8] have shown that predictions based on the numerical solution of the appropriate Boltzmann equations are in good agreement with those obtained using Monte Carlo methods. Moreover, computations using the Boltzmann equation formulation were carried out at a considerable saving in computer time. Because of this and because of the role played by the electrons in nuclear pumped lasers, a Boltzmann equation formulation is presented for the calculation of the electron distribution function. In this formulation, the effects of ambipolar diffusion, elastic and inelastic collisions, two and three body recombination and secondary ionization are taken into consideration. Moreover, because the primary electrons generated by the fission fragments are not monoenergetic[9], the present formulation allows for a source of primary electrons whose distribution is calculated by a procedure similar to that of Ref. 9.

The resulting nonlinear differential-difference-integral equation is solved for a He plasma generated by fission fragments in the presence and absence of externally applied electric fields over a wide range of pressure and neutron flux. The results show that the electrons are essentially thermal, but the distribution function is far from Maxwellian.

ANALYTICAL FORMULATION

The experiments of Refs. 1-3 employed tubes coated with fissionable material. Under neutron bombardment, fission fragments emerge from the coating and enter the gas. The ensuing energy transfer results in ionization and excitation of the background gas. For the high pressures of interest, the plasmas generated by the fission fragments are slightly ionized.[5] Therefore, a Lorentz gas approximation will be employed.

Letting the electron distribution function be expressed as:

$$f = f_o(v,x_i,t) + \frac{v_i f_i}{v}, \quad v^2 = v_i v_i, \quad i = 1,2,3 \tag{1}$$

where v_i is the velocity component, then the governing equations for f_o and f_i are given by[10,11]

$$\frac{\partial f_o}{\partial t} = -\frac{v}{3}\frac{\partial f_i}{\partial x_i} + \frac{eE_i}{3mv^2}\frac{\partial}{\partial v}\left(v^2 f_i\right) + \frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left[\nu\, v^2\left(vf_o + \frac{kT}{m}\frac{\partial f_o}{\partial v}\right)\right]$$

$$+ \frac{N}{v}\sum\left[v'^2 Q_s(v')\, f_o' - v^2 Q_s(v)\, f_o\right] + \left(\frac{\partial f_o}{\partial t}\right)_c \tag{2}$$

$$\frac{\partial f_i}{\partial t} = -v\frac{\partial f_o}{\partial x_i} + \frac{eE_i}{m}\frac{\partial f_o}{\partial v} - \nu f_i \tag{3}$$

where

$$\frac{1}{2}mv'^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv_s^2 \tag{4}$$

$$\nu = 2\pi v N\int^{\pi}(1-\cos\chi)\,\sigma(\chi,v)\,\sin\chi d\chi \equiv v\,N\,Q_m \tag{5}$$

and e is the electronic charge, m is the electronic mass, M is the mass of the heavy particle, T is the gas temperature, N is the gas number density,

Q_m is the momentum transfer cross section, $\frac{1}{2} mv_s^2$ is the excitation energy, Q_s is the excitation cross section and $(\partial f_o/\partial t)_c$ is the source term resulting from secondary ionization, recombination and production of electrons by fission fragments.

Steady state conditions are to be considered. Equation (3) can be solved then directly for f_i

$$f_i = - \frac{v}{\nu} \frac{\partial f_o}{\partial x_i} + \frac{eE_i}{m\nu} \frac{\partial f_o}{\partial v}. \tag{6}$$

Substituting equation (6) into equation (2) one finds

$$- \frac{v}{3} \frac{\partial}{\partial x_i}\left(- \frac{v}{\nu} \frac{\partial f_o}{\partial x_i} + \frac{eE_i}{m\nu} \frac{\partial f_o}{\partial v}\right) + \frac{eE_i}{3mv^2} \frac{\partial}{\partial v} \left[v^2 \left(- \frac{v}{\nu} \frac{\partial f_o}{\partial x_i} + \frac{eE_i}{m\nu} \frac{\partial f_o}{\partial v}\right)\right]$$

$$+ \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v}\left[\nu \, v^2 \left(vf_o + \frac{kT}{m} \frac{\partial f_o}{\partial v}\right)\right] + \frac{N}{v} \sum\left[v'^2 \, Q(v') \, f_o' - v^2 Q(v) f_o\right]$$

$$+ (\partial f_o/\partial t)_c = 0. \tag{7}$$

In general E_i has values along and normal to the axis. The component along the axis, E_x, is that due to the applied electric field and is usually given. The component normal to the axis, E_z, is obtained from consideration of the diffusion process in the tube.

Before one can attempt the solution of equation (7), one needs to specify E_z and $(\partial f_o/\partial t)_c$. In the presence of ambipolar diffusion E_z is determined from the requirement that the ion flux is equal to the electron flux in the normal direction. The ion or electron flux, $\Gamma_{i,s}$, is defined as

$$\Gamma_{i,s} = \frac{4\pi}{3} \int f_{i,s} \, v^3 \, dv \tag{8}$$

where s refers to the ions or electrons. Using equation (6) with e replaced by $-e_s$, one obtains

$$\Gamma_{i,s} = -\frac{4\pi}{3}\int \frac{v^4}{\nu_s}\frac{\partial f_{o,s}}{\partial x_i}\,dv - \frac{4\pi}{3}\frac{e_s E_i}{m_s}\int \frac{v^3}{\nu_s}\frac{\partial f_{o,s}}{\partial v}\,dv$$

$$= -\frac{\partial}{\partial x_i}(n_s D_s) - n_s\,\mu_s\,E_i \qquad (9)$$

where

$$D_s = \frac{4\pi}{3}\frac{1}{n_s}\int \frac{v^4}{\nu_s} f_{o,s}\,dv$$

and

$$\mu_s = \frac{4\pi}{3}\frac{e_s}{m_s n_s}\int \frac{v^3}{\nu_s}\frac{\partial f_{o,s}}{\partial v}\,dv \qquad (10)$$

where D is the diffusion coefficient and μ is the mobility. The mobility is, by definition, a positive quantity; thus,

$$\mu_e = -\frac{4\pi}{3}\frac{e}{mn_e}\int \frac{v^3}{\nu}\frac{\partial f_o}{\partial v}\,dv$$

and

$$\mu_i = -\frac{4\pi}{3}\frac{e}{Mn_i}\int \frac{v^3}{\nu_i}\frac{\partial f_{o,i}}{\partial v}\,dv. \qquad (11)$$

The requirement that $\Gamma_{z,i} = \Gamma_{z,e}$ gives

$$E_z = -\frac{(D_e - D_i)}{\mu_e + \mu_i}\frac{1}{n_e}\frac{dn_e}{dz} \equiv -\frac{\gamma}{n_e}\frac{dn_e}{dz} \qquad (12)$$

and

$$\Gamma_{z,s} = -D_A\frac{dn_e}{dz},\; D_A = \frac{D_e\,\mu_i + D_i\,\mu_e}{\mu_i + \mu_e}. \qquad (13)$$

The equation governing n_e follows from equation (7) by multiplying by $4\pi v^2$ and integrating from 0 to infinity. Carrying out the indicated operation, one obtains

$$\frac{\partial}{\partial x_i}\left\{\frac{\partial}{\partial x_i}\int \frac{4\pi}{3}\frac{v^4}{\nu} f_o dv - \frac{4\pi}{3}\frac{eE_i}{m}\int \frac{v^3}{\nu}\frac{\partial f_o}{\partial v} dv\right\} + 4\pi\int \left(\frac{\partial f_o}{\partial t}\right)_c v^2 dv = 0$$

or, using equations (9), (10) and (13)

$$\frac{d}{dz}\left(D_A \frac{dn_e}{dz}\right) + 4\pi\int \left(\frac{\partial f_o}{\partial t}\right)_c v^2 dv = 0. \tag{14}$$

To carry out the integration in equation (14), one needs to determine $(\partial f_o/\partial t)_c$. This term consists of a secondary ionization term, a production term resulting from the fission fragments and a recombination term. These processes are rather complex, making it necessary to employ crude approximations for their representation. Thus, the procedure employed by Chan and Moody[12] for representing secondary ionization will be employed. Letting

$$F_o = \frac{4\pi}{m}\left(\frac{2\varepsilon}{m}\right)^{1/2} f_o, \quad \left(\frac{\partial F_o}{\partial t}\right)_c = \frac{4\pi}{m}\left(\frac{2\varepsilon}{m}\right)^{1/2}\left(\frac{\partial f_o}{\partial t}\right)_c \tag{15}$$

the contribution to $(\partial F_o/\partial t)_c$ resulting from secondary ionization can be written as[12]

$$\int G(\varepsilon,\varepsilon') F_o \, d\varepsilon' - A_i F_o \tag{16}$$

where

$$\varepsilon = \frac{1}{2} mv^2, \quad G(\varepsilon,\varepsilon') = 2 A_i(\varepsilon') D(\varepsilon,\varepsilon' - \varepsilon_i),$$

$$A_i(\varepsilon) = v Q_i N = \left(\frac{2\varepsilon}{m}\right)^{1/2} Q_i N,$$

$$D(\varepsilon, \varepsilon' - \varepsilon_i) = \begin{cases} \dfrac{1}{\varepsilon' - \varepsilon_i}, & 0 \leq \varepsilon \leq \varepsilon' - \varepsilon \\ 0 & , \varepsilon \geq \varepsilon' - \varepsilon_i \end{cases} , \tag{17}$$

ε_i is the ionization potential and Q_i is the ionization cross section. Using (17) in (16), one finds

$$\int G(\varepsilon, \varepsilon') F_o d\varepsilon' = 2 \int_{\varepsilon+\varepsilon_i}^{\infty} \frac{A_i(\varepsilon') F_o d\varepsilon'}{\varepsilon' - \varepsilon_i}$$

$$= 2 \int_{\varepsilon}^{\infty} \frac{A_i(\xi + \varepsilon_i) F_o (\xi + \varepsilon_i)}{\xi} d\xi. \tag{18}$$

To estimate the production term resulting from the fission fragments, a procedure similar to that employed by Guyot, Miley and Verdeyen[9] will be used. The fission fragments are assumed to fall into two groups: a light group with an average mass number of 96, an average initial charge of 20e and an average energy of 98 MEV; and a heavy group with an average mass number of 140, an average initial charge of 22e and an average energy of 67 MEV.

Consider the case of a tube coated with fissionable material and bombarded with neutrons. For the case where the range of the particle in the coating is less or equal to the thickness of the coating, the number of heavy particles k born with energy $E_{o,k}$ and having an energy E at location z per unit surface per unit time and per unit energy is chosen as[9]

$$F_k(y,E,E_{o,k}) = \frac{S\tau(n+1)}{2 E_{o,k}} \eta_k^n \left| 1 - \frac{\xi_k}{1 - \eta_k^{n+1}} \right|$$

$$\text{for } 0 \leq y \leq \lambda_{g,k}, \; 0 \leq E \leq E_{m,k} \tag{19}$$

where

$$\xi_k = \frac{y}{\lambda_{g,k}}, \quad \eta_k = \frac{E}{E_{o,k}}, \quad y = a - z \tag{20}$$

λ_g is the range of the particle in the gas, τ is the thickness of the coating and S is the source rate of charged particles per unit volume and is given by

$$S = N_t \; \sigma_B \; \phi/2$$

where ϕ is the neutron flux, N_t is the number density of the target material and σ_B is the fission cross section. For fission fragments $n \simeq -1/2$.

For a slab model any quantity Q(y) at a distance y from one of the planes is given by

$$Q(y) = \sum_{k=1}^{2} Q_k(y, E_{o,k}). \tag{21}$$

The contribution from two plane parallel sources at a distance 2a apart gives

$$Q_{tot} = Q(y) + Q(2a - y). \tag{22}$$

The number of electrons created per unit volume, per unit time, and per unit energy with a kinetic energy between E and E + dE at location z by the k^{th} heavy particle is given by

$$f_k(y,\varepsilon) = \int_{\varepsilon_{i,s}}^{E_{o,k}} F_k(y,E,E_{o,k}) \; K_i(E,\varepsilon) \; dE \tag{23}$$

where

$$K_i(E,\varepsilon) = N \; \Sigma \; N_s \; \sigma_s, \quad \sigma_s = \frac{\sigma_o}{(\Delta E_s)^3} \; G_s\left(\frac{\varepsilon_{i,s}}{\Delta E_s}; \; \frac{E}{\varepsilon_{i,s}}\right)$$

$$\Delta E_s = \varepsilon + \varepsilon_{i,s}, \; \sigma_o = \pi e^4 Z_k^2, \; Z_k = Z_{o,k} \varepsilon_k^{m/2} \tag{24}$$

and N_s is the number of electrons in the s^{th} shell of the gas atom, $\varepsilon_{i,s}$ is the ionization potential, $Z_{o,k}$ = 20 or 22 and $m \simeq 1$ for fission fragments. The expression for σ_s follows from Ref. 13. The desired contribution to $(\partial F_o/\partial t)_c$ is obtained from equations (22), (23) and (24) as

$$g(y,\varepsilon) = \sum_{k=1}^{2} \{f_k(y,\varepsilon) + f_k(2a - y,\varepsilon)\}. \tag{25}$$

Two and three body recombinations are considered. For a recombination reaction of the type

$$A^+ + e \rightarrow A$$

the contribution to $(\partial F_o/\partial t)_c$ can be written as

$$v \frac{\beta_R}{v} n_{A^+} F_o = \beta_R n_{A^+} F_o \tag{26}$$

where $\frac{\beta_R}{v}$ is the recombination cross section and β_R is a constant. For a three body recombination, the contribution to $(\partial F_o/\partial t)_c$ will be chosen as

$$- 2 \int A_R D(\varepsilon, \varepsilon' - \varepsilon_i) F_o d\varepsilon' + A_{R,o} S(\varepsilon - \varepsilon_i) F_o \tag{27}$$

where

$$S = \begin{cases} 0; \; \varepsilon < \varepsilon_i \\ 1; \; \varepsilon \geqslant \varepsilon_i \end{cases} \tag{28}$$

and

$$A_R = A_{R,o} n_e^2 (\varepsilon - \varepsilon_i)$$

$$A_{R,o} = (\alpha_R\, n_e) / \int_{\varepsilon_i}^{\infty} (\varepsilon - \varepsilon_i)\, F_o d\varepsilon \tag{29}$$

where α_R is the three body recombination coefficient. The reasoning that led to equation (27) is similar to that employed in deriving equation (16). The choices indicated in equations (16), (26) and (27) satisfy both particle and energy conservation.

Using equations (16), (25), (26) and (27), the term $4\pi \int v^2 (\partial f_o / \partial t)_c\, dv$ can be written as

$$4\pi \int v^2 (\partial f_o / \partial t)_c\, dv = \alpha\, n_e + \beta - \beta_R\, n_e^2 - \alpha_R\, n_e^3 \tag{30}$$

where

$$\alpha = \frac{8\pi N}{n_e m^2} \int_{\varepsilon_i}^{\infty} Q_i(\xi)\ \xi\ f_o(\xi)\ d\xi; \ \beta = \int g d\varepsilon. \tag{31}$$

Because $g(z,\varepsilon)$ does not depend on a simple way on position and because, for the pressure and dimensions under consideration, it is well approximated by its value at the axis, it is assumed when solving equation (14) that

$$\beta(z) = \beta(0). \tag{32}$$

Although the above assumption simplifies the calculations, it is not required for carrying out the solution. The boundary conditions for equation (14) can be expressed as

$$\frac{dn_e}{dz} = 0 \text{ at } z = 0 \tag{33}$$

and, at the wall, a Schottky boundary condition is employed, i.e.

$$n_e = 0 \text{ at } z = a. \tag{34}$$

With n_e determined from equation (14), E_z follows from equation (12).

METHOD OF SOLUTION

The function $f_o(v,z) = H_o(\varepsilon,z)$ is assumed to have the representation

$$H_o(\varepsilon,x) = \sum n_j(x)\ h_j(\varepsilon) \tag{35}$$

where the n_j's represent an orthogonal set. This set may be obtained from a suitable linear combination of functions ψ_1, ψ_2, ... ψ_r any r of which are linearly independent for arbitrary r, Ref. 14. By definition, the electron number density $n_e(z)$ is

$$\begin{aligned} n_e(z) &= \int_o^\infty 4\pi\ v^3\ f_o dv \\ &= 2\pi \left(\frac{2}{m}\right)^{3/2} \sum n_j(z) \int_o^\infty \varepsilon^{1/2}\ h_j(\varepsilon)\ d\varepsilon. \end{aligned} \tag{36}$$

Thus, if one chooses

$$2\pi \left(\frac{2}{m}\right)^{3/2} \int_o^\infty \varepsilon^{1/2}\ h_o(\varepsilon)\ d\varepsilon = 1;\ 2\pi \left(\frac{2}{m}\right)^{3/2} \int_o^\infty \varepsilon^{1/2}\ h_j(\varepsilon)\ d\varepsilon = 0,\ j \geqslant 1 \tag{37}$$

then

$$n_o(z) = n_e(z) \tag{38}$$

where $n_e(z)$ is determined from equation (14). The governing equation for h_k is determined from equation (7) by substituting first for E_z and $(\partial f_o/\partial t)_c$ and then multiplying by $n_k(z)$ and integrating from zero to a. The resulting equation for h_k depends on $h_o \ldots h_{k-1}$. This means that before one can determine h_k, one has to determine h_o, $h_1 \ldots h_{k-1}$.

In this work, it is assumed that the solution may be approximated by the first term. Using the above procedure, one finds that h_o is given by the following equation,

$$\lambda_4 \left\{ - (\eta/q_m)\, h_o - (e\,\gamma/\varepsilon_i)(\eta/q_m) \frac{dh_o}{d\eta} + (e\,\gamma/\varepsilon_i) \frac{d}{d\eta} (\eta\, h_o/q_m) \right.$$

$$\left. + \{(e\,\gamma/\varepsilon_i)^2 + (e\, E_x\, a/\varepsilon_i)^2/\sigma_4\} \frac{d}{d\eta} \left[(\eta/q_m) \frac{dh_o}{d\eta}\right] \right\}$$

$$+ \frac{2m}{M} \frac{d}{d\eta} \left[q_m\, \eta^2 \left[h_o + (kT/\varepsilon_i) \frac{dh_o}{d\eta} \right] \right]$$

$$+ \Sigma\, \{\eta'\, q_s(\eta')\, h_o(\eta') - \eta\, q_s(\eta)\, h_o(\eta)\}$$

$$+ 2 \int_{\eta}^{\infty} (\xi + 1)\, q_i(\xi + 1)\, h_o(\xi + 1) \frac{d\xi}{\xi} - \eta\, q_i\, h_o$$

$$+ \lambda_1\, g(\eta) - \lambda_2\, \eta^{1/2}\, h_o + \lambda_3\, \eta^{1/2} \left[S(\eta)\, h_o(\eta)\, (\eta - 1) \right.$$

$$\left. - 2 \int_{\eta+1}^{\infty} \xi^{1/2}\, h_o(\xi)\, d\,\xi \right] = 0 \tag{39}$$

where

$$\lambda_1 = \frac{m^2\, \sigma_1}{8\pi N\, Q_m(0)\, \varepsilon_i}, \qquad \lambda_2 = \frac{\beta_R\, \sigma_2}{N\, Q_m(0)} \left(\frac{m}{2\varepsilon_i}\right)^{1/2}$$

$$\lambda_3 = \frac{A_{R,o}\, \sigma_3}{N\, Q_m(0)} \left(\frac{m\, \varepsilon_i}{2}\right)^{1/2}, \quad \lambda_4 = \frac{\sigma_4}{3a^2\, N^2\, Q_m^2(0)}$$

$$q_m = Q_m(\varepsilon)/Q_m(0), \quad q_s = Q_s(\varepsilon)/Q_m(0), \quad \eta = \varepsilon/\varepsilon_i \tag{40}$$

and

$$\sigma_1 = \int_o^1 n_e \, d\zeta/\sigma, \quad \sigma_2 = \int_o^1 n_e^3 \, d\zeta/\sigma$$

$$\sigma_3 = \int_o^1 n_e^4 \, d\zeta/\sigma, \quad \sigma_4 = \int_o^1 (dn_e/d\zeta)^2/\sigma$$

$$\sigma = \int_o^1 n_e^2 \, d\zeta, \qquad \zeta = z/a. \tag{41}$$

As may be seen from equations (14) and (30), the σ's are not independent. Thus, multiplying equation (14) by n_e and integrating between o and a, one finds

$$-\frac{D_A}{a^2}\sigma_4 + \alpha + \beta\,\sigma_1 - \beta_R\,\sigma_2 - \alpha_R\,\sigma_3 = 0. \tag{42}$$

Equation (42) follows also from equation (39) by integrating with respect to ε from 0 to infinity.

Letting

$$G = \lambda_4\,(\eta/q_m)\left[(e\,\gamma/\varepsilon_i)\,h_o + \{(e\,\gamma/\varepsilon_i)^2 + (e\,E_x\,a/\varepsilon_i)^2/\sigma_4\}\frac{dh_o}{d\eta}\right]$$

$$+ \frac{2m}{M}\,q_m\,\eta^2\,\{h_o + (kT/\varepsilon_i)\,\frac{dh_o}{d\eta}\} \tag{43}$$

it is seen that equation (39) can be expressed as two first order equations

$$\frac{dG}{d\eta} = +\lambda_4\,(e\,\gamma/\varepsilon_i)\,(\eta/q_m)\,\frac{dh_o}{d\eta} + R(\eta)\,h_o - \lambda_1\,g(\eta)$$

$$-\Sigma\,\eta'\,q_s(\eta')\,h_o(\eta') - 2\int_\eta^\infty (\xi+1)\,q_i(\xi+1)\,h_o(\xi+1)\,\frac{d\xi}{\xi}$$

$$+ \lambda_3 \eta^{1/2} \int_{\eta+1}^{\infty} \xi^{1/2} h_o(\xi) \, d\xi \tag{44}$$

and

$$\frac{dh_o}{d\eta} = \frac{(G/\eta) - \{(\lambda_4/q_m)(e\,\gamma/\varepsilon_i) + (2m/M)\, q_m\, \eta\}\, h_o}{(\lambda_4/q_m)\{(e\,\gamma/\varepsilon_i)^2 + (e\,E_x\, a/\varepsilon_i)^2/\sigma_4\} + (2m/M)\, q_m\, \eta\, (kT/\varepsilon_i)} \tag{45}$$

where

$$R(\eta) = \lambda_4\ (\eta/q_m) + \eta\, \Sigma\, q_s(\eta) + \eta\, q_i + \lambda_2\, \eta^{1/2} - \lambda_3\, \eta^{1/2}\ (\eta - 1)\ S(\eta). \tag{46}$$

Equations (44) and (45) can be integrated using a Runge-Kutta or a predictor-corrector method starting at some $\varepsilon = \varepsilon_{max}$. Before one can start the integration one has to determine σ_j; these quantities depend on $n_e(z)$ which is governed by equations (14) and (30). Because some of the quantities appearing in equation (30), i.e. D_A and α, depend on h_o, while others, i.e. β_R and α_R, depend on T_e, the electron temperature which, in turn, depends on h_o, the problem under consideration is nonlinear and an iterative procedure is required to determine the solution.

As a result of equation (42)

$$G = 0 \text{ at } \varepsilon = \varepsilon_{max} \tag{47}$$

thus, to start the integration, one needs to assume α, $A_{R,o}$, μ_e, D_e and the value of h_o at $\varepsilon = \varepsilon_{max}$. For the assumed α, μ_e, D_e, and $A_{R,o}$, the value of h_o at $\varepsilon = \varepsilon_{max}$ is determined from the requirement indicated by equation (37). Using the calculated $h_o(\varepsilon)$ new values of D_e, μ_e, $A_{R,o}$ and α are calculated from equations (10), (11), (29) and (31) and are used to recalculate σ_j. The above procedure is repeated until convergence is achieved.

As is seen from the above, the calculation of h_o requires a rather lengthy iterative procedure. Unfortunately, the convergence of the above method becomes increasingly difficult as the electric field decreases and/or as the pressure increases. This is because the coefficients of the highest derivatives in equation (39) become rather small thus necessitating extremely small integration steps. Because of this, the method of composite expansions[15] was used in integrating equation (39) when E_x was set equal to zero.

In this method the function $h_o(\eta)$ is written as

$$h_o = h_o(\eta;\delta) = \psi(\eta;\delta) + \chi(\eta_o;\delta)$$

$$= \sum_{n=0}^{\infty} \delta^n \Big(\psi_n(\eta) + \chi_n(\eta_o)\Big) \simeq \psi_o + \chi_o \tag{48}$$

where δ is the largest of the two parameters $(2m/M)$ and $(e\gamma/\varepsilon_i)\lambda_4$ and $\eta_o = \eta/\delta^{\ell}$ is the inner variable with $\ell > 0$ being determined according to the procedure outlined below. The function $\chi(\eta_o;\delta)$ is negligible outside the inner region, i.e. when $\eta_o \to \infty$.

The equations for ψ_n are determined by setting

$$h_o = \Sigma\ \delta^n\ \psi_n \tag{49}$$

in equation (39) and equating equal powers of δ. When this is done ψ_o is determined from all terms in equation (39) that are not multipled by δ, i.e.

$$\psi_o = \{\Sigma\ \eta'\ q_s(\eta')\ \psi_o(\eta') + 2\int_{\eta}^{\infty} (\xi + 1)\ q_i(\xi + 1)\ \psi_o(\xi + 1)\ \frac{d\xi}{\xi}$$

$$+ \lambda_1 g - 2\ \lambda_3\ \eta^{1/2} \int_{\eta+1}^{\infty} \xi^{1/2}\ \psi_o\ d\xi\}\{\eta(q_i + \Sigma\ q_s)$$

$$+ \eta^{1/2} \left(\lambda_2 - \lambda_3 \; S(\eta) \; (\eta-1)\right)\}^{-1}. \tag{50}$$

The equations for $\chi_n(\eta_o)$ are determined by first transforming equation (39) from η to η_o and choosing ℓ so that the coefficient of the highest derivative does not depend on δ. This procedure was not used here; instead, equations (44) and (45) were integrated starting at $\eta = 0$ using the condition $G(0) = 0$ and an assumed value of $h_o(0)$. The quantity $h_o(0)$ was determined from the requirement that at some η_m where $|h_o - \psi_o| < \delta_m$

$$\left| 2\pi \left(\frac{2\varepsilon_i}{m}\right)^{3/2} \left\{\int_0^{\eta_m} \eta^{1/2} \; h_o(\eta)d\eta + \int_{\eta_m}^{\infty} \eta^{1/2} \; \psi_o(\eta)d\eta\right\} - 1\right| < \delta_n \tag{51}$$

where δ_m and δ_n are preselected small numbers.

As a check on the accuracy of the integration procedure and in order to determine the manner in which the primary electrons dispose of their energy, an electron energy equation is derived. This equation is obtained by multiplying equation (30) by η and integrating from zero to infinity. The resulting equation can be written as

$$(e \; E_x^2 \; \mu_e/\sigma_1) + \int \varepsilon \; g d\varepsilon + (\sigma_3/\sigma_1) \; \alpha_R \; \varepsilon_i =$$

$$(8\pi \; \sigma_4/a^2 \; m^2 \; \sigma_1) \left\{\frac{1}{3} \left(\frac{2}{m}\right)^{1/2} \left(\iint \frac{\varepsilon^{5/2}}{\nu} \; h_o \; d\varepsilon + e \; \gamma \int \frac{\varepsilon^{5/2}}{\nu} \; \frac{dh_o}{d\varepsilon} \; d\varepsilon\right)\right.$$

$$\left. + \frac{m^2}{8\pi} \; e \; \gamma(D_e - \gamma \; \mu_e)\right\} + \frac{8\pi}{M \; \sigma_1} \left(\frac{2}{m}\right)^{1/2} \int \nu \; \varepsilon^{3/2} \left(h_o + kT \; \frac{dh_o}{d\varepsilon}\right) d\varepsilon$$

$$+ \frac{8\pi \; N}{m^2 \sigma_1} \; \Sigma \; \varepsilon_s \int \varepsilon \; Q_s \; h_o \; d\varepsilon + \frac{3}{2} \; kT_e \; \beta_R(\sigma_2/\sigma_1) + \alpha \; \varepsilon_i/\sigma_1. \tag{52}$$

The above equation shows that the energy received by the electrons from fission fragments, electric field and three body recombination is equal to, respectively, the energy lost by diffusion, elastic collisions, inelastic collisions, two body recombination and secondary ionization.

RESULTS AND DISCUSSION

The solution of equation (39) is carried out for a He plasma. Before a solution can be obtained one needs to know the momentum transfer, excitation and ionization cross sections of He. The momentum transfer cross section up to an electron energy of 6 ev is taken from Crompton et al.[16]; above 6 ev it is assumed that the collision frequency ν is a constant equal to 2.4×10^9 sec^{-1}, Ref. 12. All helium excited states with a principal quantum number of 5 or less are included in the calculations and the excitation cross sections for these states are taken from Refs. 17-25. The ionization cross section is taken from Rapp and Englander-Golden.[26]

At the high pressures of interest, the recombination process in noble gases is complicated by the formation of molecular ions. The theory presented here allows for radiative and three body recombination. However, at high pressures the reactions

$$He^+ + 2\ He \rightarrow He_2^+ + He \tag{53}$$

$$e + He_2^+ + He \rightarrow He^* + 2\ He \tag{54}$$

where * designates an excited state play a dominant role since the forward rate for the first reaction[27] is about 10^{-31} cm^6/sec at 300^oK while the forward rate for the second reaction[28] is about 2×10^{-27} cm^6/sec at 300^oK. Because of the rapid conversion of He^+ to He_2^+ at high pressure it is seen from the second reaction that an effective two body recombination coefficient can be defined whose approximate value is

$$\beta_R \approx 2 \times 10^{-27}\, n_{He}\ \mathrm{cm}^3/\mathrm{sec}. \tag{55}$$

For reactions of the type indicated in equation (54), Biondi[29] suggested a forward rate coefficient of the form

$$k_f = 10^{-(26\pm1)}(Te/300)^{-5/2}\ \mathrm{cm}^6/\mathrm{sec}. \tag{56}$$

Because of the above uncertainties, the calculations presented here assume a value of β_R of 10^{-9} at a pressure of 100 Torr; a value which was employed in Ref. 5. At other pressures β_R is scaled according to the estimate indicated in equation (55).

The ion mobility is taken as 10.4 cm^2/V sec, which is appropriate for a He ion.[27,30] Because the ion mobility does not play a significant role in the present calculations, no attempt was made to allow for the presence of other He ions.

The calculations were carried out for a coating of U_3O_8 with the neutron flux ranging from 3.8×10^{11} to 7.6×10^{14} neutrons/cm^2 sec at a temperature of 300°K and pressures of 100 and 760 Torr. The range of parameters was chosen to ensure that the Lorentz gas approximation employed here is not violated. The spacing between the walls, i.e., 2a, is taken as 3.7 cm.

A typical plot of the distribution function of the primary electrons, $g(\varepsilon)$, is shown in Fig. 1. The plot demonstrates that the energy spectrum of the primary electrons is far from monoenergetic. Figure 2 shows the effect of the neutron flux on the electron distribution function F_o (See equation

(15)). Examination of equation (39) shows that at high energies

$$F_o \propto \lambda_1 \, g(\eta) \propto \sigma_1 \, g(\eta). \tag{57}$$

The distribution function of the primary electrons, g, is proportional to the neutron flux. On the other hand, conservation of particles, equation (42), is such that σ_1 is approximately inversely proportional to the square root of the neutron flux; thus, F_o increases with the neutron flux at high energies. In the low energy region, the distribution function approaches a Maxwellian at the gas temperature.

The effect of pressure is discussed next. As a result of increased collisions, the high energy particles are depleted at a faster rate thus resulting in a reduced distribution at higher pressures. Because F_o vs ε is presented on a log-log scale and because F_o is normalized, calculations of F_o at 100 and 760 Torr show that the effect is small and, therefore, does not warrant a separate plot.

It is evident from Fig. 2 that the distribution function is not Maxwellian. However, calculation of the electron temperature, i.e.

$$T_e = \frac{2}{3k} \int \varepsilon \, F_o \, d\varepsilon, \tag{58}$$

shows that the electrons are essentially thermal. Assuming a Maxwellian distribution function at this temperature would result in a substantial reduction of both electron excitation and ionization. This suggests that assuming the distribution function to be Maxwellian at some temperature different from the gas temperature is not a good approximation, Ref. 4.

The effect of an externally applied electric field is shown in Fig. 3. It is seen from the plot that the distribution function approaches a Druyvestyn distribution at low energies. At high energies the distribution is similar to that for a zero electric field.

Figure 4 shows the manner in which energy is transferred from the primary electrons. Over a fourth of the energy is transferred to the excited states and this fraction is essentially independent of the range of neutron flux and pressure considered here. Energy transfer from electrons in the presence of an electric field was also investigated. The results indicate that over 90% of the energy is transferred to the He excited states. Unfortunately, for the E/P considered, namely .1 V/cm Torr, the energy received by the electrons from the electric field far outweighed the energy received from the fission fragments. In spite of this, the above clearly illustrates the beneficial effect of employing a sustainer electric field in conjunction with nuclear induced plasmas for the generation of high power lasers.

Because the analyses of Refs. 5 and 6 employ monoenergetic primary electrons and do not employ the experimentally measured cross sections employed here, direct comparison with the present work is not possible. Our results are, however, in qualitative agreement with their results. Based on present and some earlier unpublished calculations, we feel that the usual characterization of diffusion[6], namely, ignoring the ambipolar part and using a cosine (or Bessel function) representation to approximate the spatial derivative of the distribution function, is not adequate for fission generated plasma (See equations (14) and (30)).

In this work, the solution of the Boltzmann equation is approximated by the first term at the series solution indicated in equation (35). This approximation implies that the electron temperature is constant throughout and thus is consistent with the assumption of constant gas temperature. This explains the reason why energy losses by conduction are not included in Fig. 3. Therefore, as long as the assumption of constant temperature is adequate, which is the case at high pressures, there is no need to carry the computations beyond the first term of the assumed series.

CONCLUDING REMARKS

The formulation presented here for the calculation of the electron distribution function in plasmas generated by fission fragments is quite general and may be used for any gas or gas mixture. Results based on this formulation for He show that a large fraction of the energy of the primary electrons is transferred into the excited states. Moreover, when such plasmas are subjected to a sustainer electric field, sharp increase in the excitation rates results. Therefore, it appears that the major contribution of fission fragments is to provide the means for generating plasmas at high pressures. These plasmas can, in turn, be employed in conjunction with sustainer electric fields to generate high power lasers.

REFERENCES

1. McArthur, D. A.; and Tollefsrud, P. B.: Observation of Laser Action in CO Gas Excited Only by Fission Fragments. Applied Physics Letters, Vol. 26, February 1975, pp. 187-190.
2. Helmick, H. H.; Fuller, J. L.; and Schneider, R. T.: Direct Nuclear Pumping of a Helium-Xenon Laser. Applied Physics Letters, Vol. 26, March 1975, pp. 327-328.
3. DeYoung, R. J.; Wells, W. E.; Miley, G. H.; and Verdeyen, J. T.: Direct Nuclear Pumping of Ne-N_2 Laser. Applied Physics Letters, Vol. 28, May 1976, pp. 519-521.
4. Russell, G. R.: Feasibility of a Nuclear Laser Excited by Fission Fragments Produced in a Pulsed Nuclear Reactor. NASA SP-236, 1971, pp. 53-62.
5. Wang, B. S.; and Miley, G. H.: Monte Carlo Simulation of Radiation-Induced Plasma. Nuclear Science and Engineering, Vol. 52, September 1973, pp. 130-141.
6. Lo, R. H.; and Miley, G. H.: Electron Energy Distribution in a Helium Plasma Generated by Nuclear Radiation. IEEE Transactions on Plasma Science, Vol. PS-2, December 1974, pp. 198-205.
7. Thomas, W. R. L.: The Determination of the Total Excitation Cross Section in Neon by Comparison of Theoretical and Experimental Values of Townsend's Primary Ionization Coefficient. Journal of Physics B (Atomic and Molecular Physics), Ser. 2, Vol. 2, May 1969, pp. 551-561.

8. Thomas, R. W. L.; and Thomas, W. R. L.: Monte Carlo Simulation of Electrical Discharges in Gases. Journal of Physics B (Atomic and Molecular Physics), Ser. 2, Vol. 2, May 1969, pp. 562-570.

9. Guyot, J. C.; Miley, G. H.; and Verdeyen, J. T.: Application of a Two-Region Heavy Charged Particle Model to Noble-Gas Plasmas Induced by Nuclear Radiations. Nuclear Science and Engineering, Vol. 48, August 1972, pp. 372-386.

10. Holstein, T.: Energy Distribution of Electrons in High Frequency Gas Discharges. Physical Review, Vol. 70, September 1946, pp. 367-384.

11. Holt, E. H.; and Haskel, R. E.: Foundations of Plasma Dynamics. The Macmillan Co., New York, 1965, p. 309.

12. Chan, C. H.; and Moody, C. D.: Solution of the Classical Boltzmann Equation for He and Ne Gas Breakdown. Journal of Applied Physics. Vol. 45, March 1974, pp. 1105-1111.

13. Gryzinski, M.: Classical Theory of Atomic Collisions. I. Theory of Inelastic Collisions. Physical Review, Vol. 138, April 1965, pp. A336-A358.

14. Courant, R.; and Hilbert. D.: Methods of Mathematical Physics. Vol. I. Interscience Publishers, Inc., New York, 1953, Ch. 2.

15. Nayfeh, A. H.: Perturbation Methods. John Wiley and Sons, New York, 1973, pp. 144-150.

16. Crompton, R. W.; Elford, M. T.; and Robertson, A. G.: The Momentum Transfer Cross-Sections for Electrons in Helium Derived from Drift Velocities at 77°K. The Australian Journal of Physics, Vol. 23, October 1970, pp. 667-681.

17. St. John, R. M.; Miller, F. L.; and Lin, C. C.: Absolute Excitation Cross-Sections of Helium. Physical Review, Vol. 134, May 1964, pp. A888-A897.

18. Kieffer, L. J.: Low-Energy Electron-Collision Cross-Section Data. Part II: Electron-Excitation Cross Sections. Atomic Data, Vol. 1, November 1969, pp. 121-287.

19. Jobe, J. D.; and St. John, R. M.: Absolute Measurements of the 2^1P and 2^3P Electron Excitation Cross Sections of Helium Atoms. Physical Review, Vol. 164, December 1967, pp. 117-121.

20. Dugan, J. L. G.; Richards, H. L.; and Muschlitz, E. G.: Excitation of the Metastable States of Helium by Electron Impact. The Journal of Chemical Physics. Vol. 46, January 1967, pp. 346-351.

21. Holt, H. K.; and Krotkov, R.: Excitation of n=2 States in Helium by Electron Bombardment. Physical Review, Vol. 144, April 1966, pp. 82-93.

22. Cermak, V.: Individual Efficiency Curves for the Excitation of 2^3S and 2^1S States of Helium by Electron Impact. Journal of Chemical Physics, Vol. 44, May 1966, pp. 3774-3780.

23. Moiseiwitsch, B. L.; and Smith, S. J.: Electron Impact Excitation of Atoms. Reviews of Modern Physics, Vol. 40, April 1968, pp. 124-353.

24. Ferendeci, A. M.: Electron Excitation Cross Section of the He 2^1S State from Diffraction Losses in a He-Ne Gas Laser. Physical Review A, Vol. II, May 1975, pp. 1576-1582.

25. Showalter, J. G.; and Kay, R. B.: Absolute Measurement of Total Electron-Impact Cross Sections to Singlet and Triplet Levels in Helium. Physical Review A, Vol. 11, June 1975, pp. 1899-1910.

26. Rapp, D.; and Englander-Golden, P.: Total Cross Sections for Ionization and Attachment in Gases by Electron Impact. The Journal of Chemical Physics, Vol. 43, September 1965, pp. 1464-1479.

27. Beaty, E. C.; and Patterson, P. L.: Mobilities and Reaction Rates of Ions in Helium. Physical Review, Vol. 137, January 1965, pp. A346-A357.

28. Berlande, J.; Cheret, M.; Deloche, R.; Gonfalone, A.; and Manus, C: Pressure and Electron Density Dependence of the Electron-Ion Recombination Coefficient in He. Physical Review, Vol. A1, March 1970, pp. 887-896.

29. Biondi, M. A.: Charged-Particle Recombination Processes. Defense Nuclear Agency Reaction Rate Handbook, Bartner, M. H.; and Baurer, T., eds., Second ed., 1972, Ch. 16.

30. Oskam, H. J.; and Mittelstadt, V. R.: Ion Mobilities in Helium, Neon and Argon. Physical Review, Vol. 143, November 1963, pp. 1435-1444.

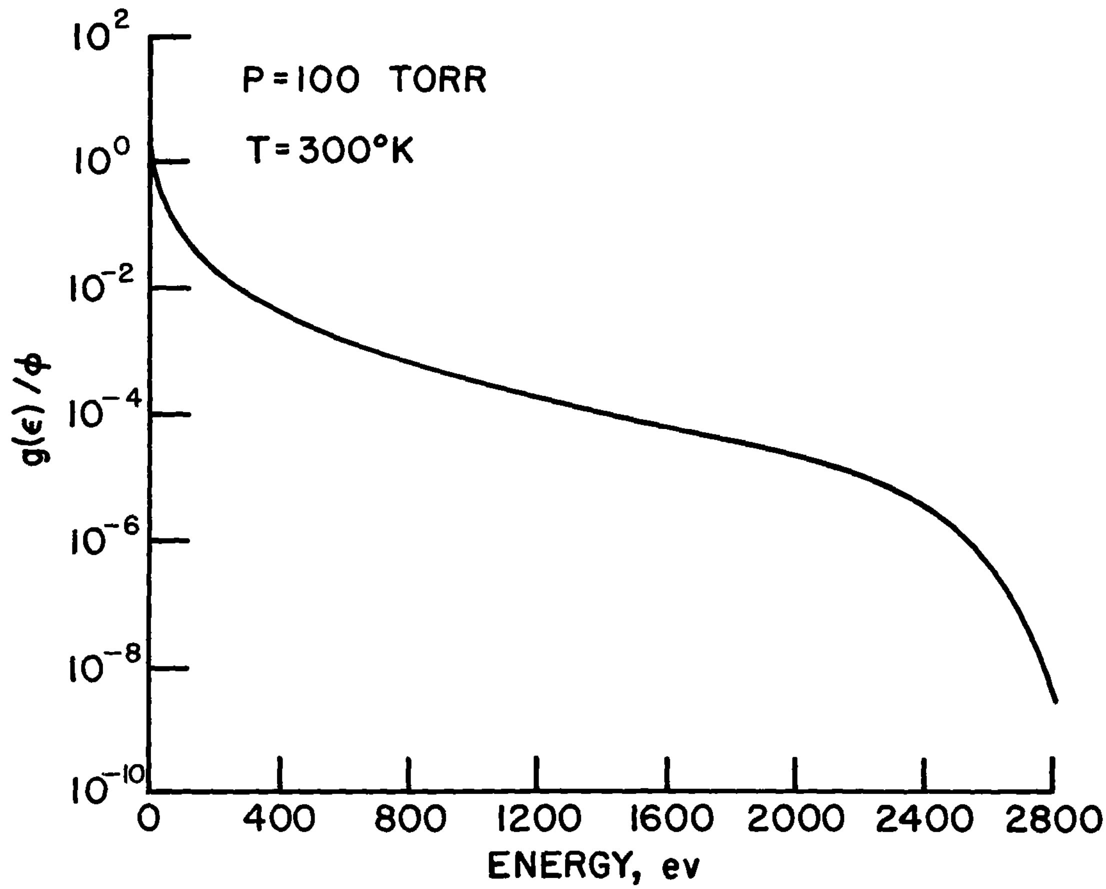

Figure 1. Energy distribution of primary electrons.

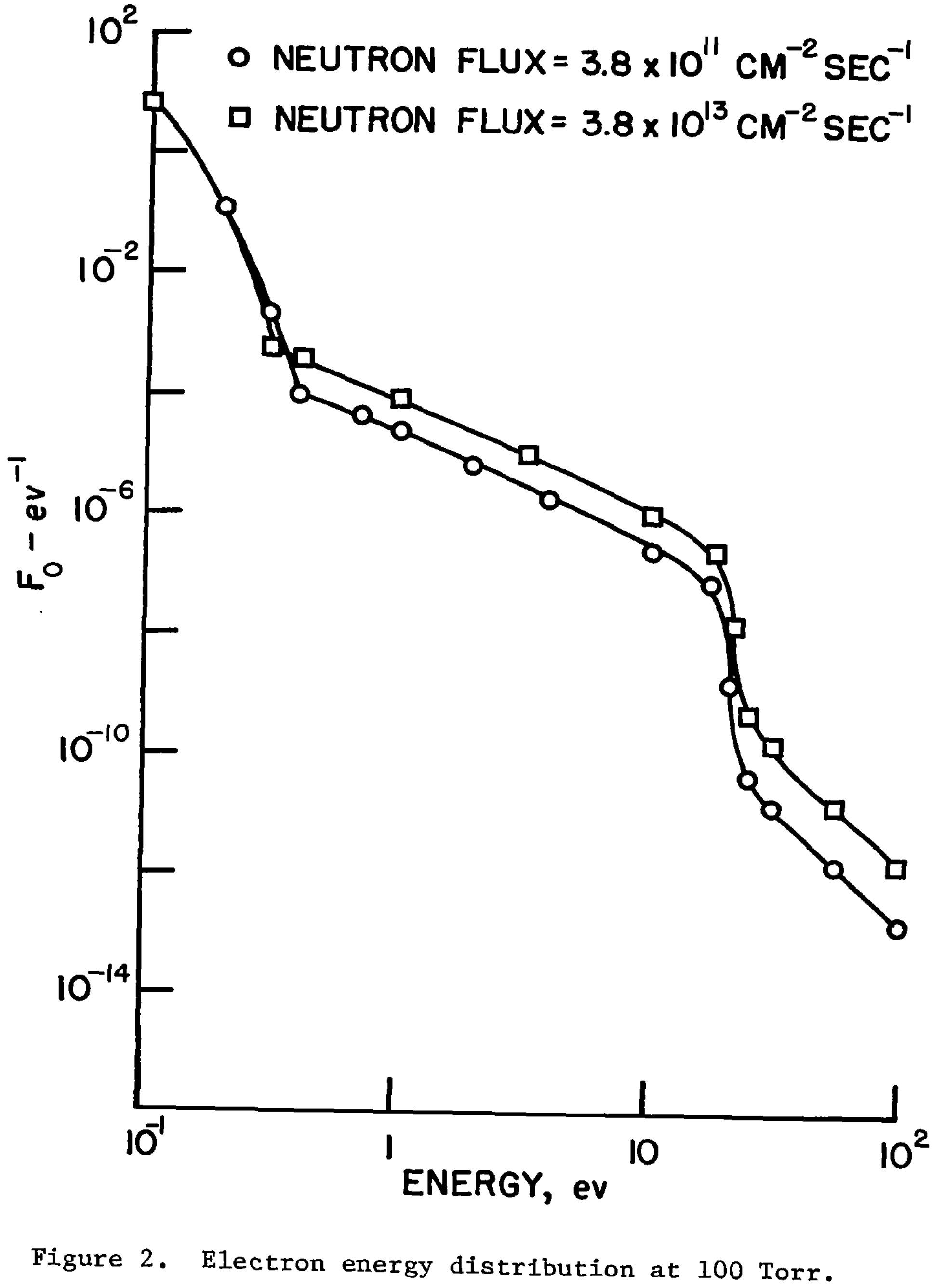

Figure 2. Electron energy distribution at 100 Torr.

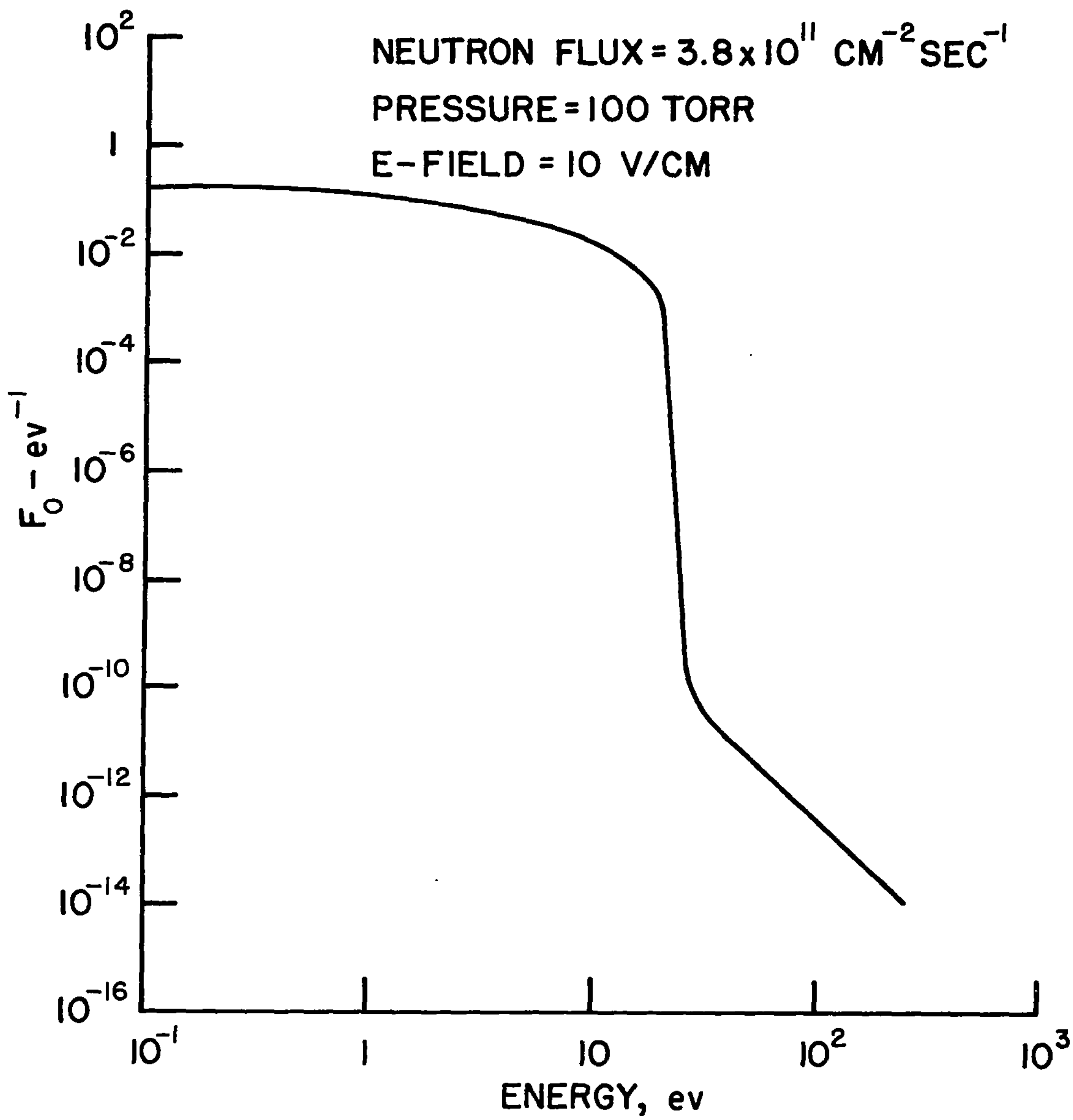

Figure 3. Effect of electric field on electron distribution function.

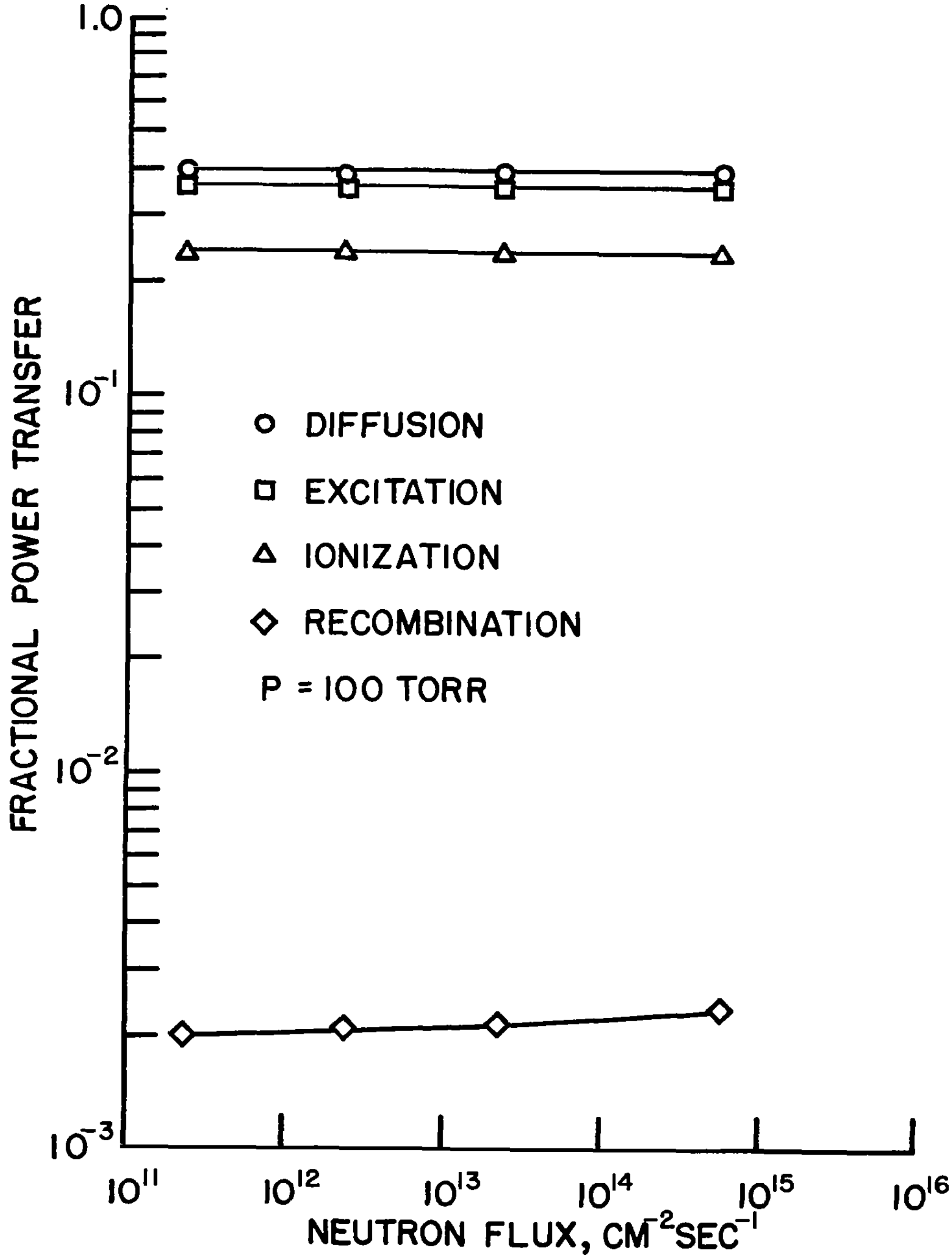

Figure 4. Fractional power transferred from the high energy electrons.