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ANALYTICAL MODELS FOR ROTOR TEST MODULE, STRUT, AND BALANCE FRAME DYNAMICS IN THE 40- BY 80-FT WIND TUNNEL

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16. Abstract

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NOMENCLATURE

a	rotor blade two-dimensional lift-curve slope
C_H	rotor drag force coefficient
C_{M_x}	rotor roll moment coefficient
C_{M_y}	rotor pitch moment coefficient
C_Q	rotor torque coefficient
C_T	rotor thrust coefficient
C_Y	rotor side force coefficient
F	vector of rotor forces and moments acting on hub
G	vector of aerodynamic gust velocity components
$\vec{i}_s, \vec{j}_s, \vec{k}_s$	unit vectors of shaft axis system
R	rotor radius
R_{ST}	rotation matrix between shaft axis and tunnel axis systems
u_G	longitudinal gust velocity
v_G	lateral gust velocity
v_s	vector of support system input variables
w_G	vertical gust velocity
x_h, y_h, z_h	shaft axis components of rotor hub linear displacement
x_s	vector of support system degrees of freedom
α	vector of rotor hub linear and angular motion
$\alpha_x, \alpha_y, \alpha_z$	shaft axis components of rotor hub angular displacement
γ	rotor Lock number
ρ	air density
σ	rotor solidity ratio
Ω	rotor rotational speed

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SUMMARY

A mathematical model is developed for the dynamics of a wind tunnel support system consisting of a balance frame, struts, and an aircraft or test module. Data are given for several rotor test modules in the Ames 40- by 80-ft wind tunnel. A model for ground resonance calculations is also described.

INTRODUCTION

The wind tunnel testing of helicopter rotors requires a consideration of the dynamic characteristics of the coupled rotor and wind tunnel support system. An aeroelastic analysis of a rotor in a wind tunnel is described in reference 1. Such an analysis requires a mathematical description of the wind tunnel balance, strut, and test module. This report documents a model developed for the dynamics of a wind tunnel support system, including data for particular rotor test modules in the Ames 40- by 80-ft wind tunnel.

SUPPORT EQUATIONS OF MOTION

The required description of the rotor support system takes the form of a set of linear, constant coefficient differential equations, excited by forces and moments at the rotor hub (and also possibly by fixed system control inputs), plus the rotor hub motion produced by the support degrees of freedom (see reference 1). Let x_s be the vector of support degrees of freedom, and v_s the vector of control inputs for the support system. Let

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α be the linear and angular shaft motion at the rotor hub, F the rotor forces and moments acting on the hub, and g the vector of aerodynamic gust components. Following the definitions of reference 1, the components of α, F, and g are:

$$\alpha = \begin{bmatrix} x_h \\ y_h \\ z_h \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \qquad F = \begin{bmatrix} \gamma \frac{C_T}{\sigma a} \\ \gamma \frac{2C_H}{\sigma a} \\ -\gamma \frac{2C_Y}{\sigma a} \\ \gamma \frac{C_Q}{\sigma a} \\ \gamma \frac{2C_{My}}{\sigma a} \\ -\gamma \frac{2C_{Mx}}{\sigma a} \end{bmatrix} \qquad g = \begin{bmatrix} u_G \\ v_G \\ w_G \end{bmatrix}$$

The gust components are in a tunnel axis system (x aft, y right, and z up), while α and F are in the shaft axis system (see reference 1). These quantities are dimensionless -- g based on the rotor tip speed ΩR, the linear hub displacements based on the rotor radius R, and the hub forces and moments in rotor coefficient form. The general form considered for the rotor support equations of motion and the hub motion is thus:

$$a_2\ddot{x}_s + a_1\dot{x}_s + a_0x_s = bv_s + b_Gg + \tilde{a}F$$

$$\alpha = cx_s$$

For use in the aeroelastic analysis of reference 1, these equations are made dimensionless, based on ρ, Ω, and R. With F in rotor coefficient form it is also convenient to normalize the equations by dividing by $(N/2)I_b$ (where N is the number of blades, and I_b the characteristic inertia of the rotor blade). Note that the matrix $\tilde{a}$ may always be obtained from the matrix c (reciprocity theorem).

Normal Mode Description

Consider a general normal mode description of the elastic wind tunnel support system. The displacement $\vec{u}(\vec{r},t)$ and rotation $\vec{\theta}(\vec{r},t)$ at an arbitrary point $\vec{r}$ are expanded in series of orthogonal vibration modes, with the generalized coordinates $q_k(t)$:

$$\vec{u}(\vec{r},t) = \sum_k q_k(t)\,\vec{\xi}_k(\vec{r})$$

$$\vec{\theta}(\vec{r},t) = \sum_k q_k(t)\,\vec{\gamma}_k(\vec{r})$$

The differential equations for the degrees of freedom q_k are then

$$M_k(\ddot{q}_k + g_s\omega_k\dot{q}_k + \omega_k^2 q_k) = Q_k$$

where M_k is the modal mass and ω_k the natural frequency; g_s is the structural damping coefficient for the mode; and Q_k is the generalized force. The hub motion is obtained from the mode shapes $\vec{\xi}$ and $\vec{\gamma}$ at the rotor hub:

$$\alpha = c\,\{q_k\}$$

where

$$c = \begin{bmatrix} & \vec{i}_s\cdot\vec{\xi}_k & \\ & \vec{j}_s\cdot\vec{\xi}_k & \\ \cdots & \vec{k}_s\cdot\vec{\xi}_k & \cdots \\ & \vec{i}_s\cdot\vec{\gamma}_k & \\ & \vec{j}_s\cdot\vec{\gamma}_k & \\ & \vec{k}_s\cdot\vec{\gamma}_k & \end{bmatrix} = \begin{bmatrix} \cdots & R_{ST}\vec{\xi}_k & \cdots \\ & R_{ST}\vec{\gamma}_k & \end{bmatrix}$$

Here $\vec{\xi}$ and $\vec{\gamma}$ are in the tunnel axis system, so R_{ST} is the rotation matrix to the shaft axes. The generalized forces due to the rotor hub forces and moments are:

$$\{Q_k\} = \tilde{a}F$$

where

$$\tilde{a} = \begin{bmatrix} & & & \vdots & & & \\ 2\vec{k}_s\cdot\vec{\xi}_k & \vec{i}_s\cdot\vec{\xi}_k & -\vec{j}_s\cdot\vec{\xi}_k & & -2\vec{k}_s\cdot\vec{\gamma}_k & \vec{j}_s\cdot\vec{\gamma}_k & -\vec{i}_s\cdot\vec{\gamma}_k \\ & & & \vdots & & & \end{bmatrix}$$

Making these equations dimensionless as appropriate produces the required support equations of motion.

Ground Resonance Model

A simple model for ground resonance calculations is obtained by describing the support by lateral and longitudinal inplane flexibility with an arbitrary number of modes. Vertical, yaw, pitch, and roll motions of the hub are neglected. It is assumed that the measured hub impedance is available from shake tests. Then the equations of motion for the generalized coordinates q_k are:

$$M_k \ddot{q}_k + C_k \dot{q}_k + M_k \omega_k^2 q_k = f_k$$

where $f_k = H$ or Y for longitudinal and lateral modes respectively. The hub motion is

$$x_h = \sum_{\text{long. modes}} q_k$$

$$y_h = \sum_{\text{lat. modes}} q_k$$

The natural frequency ω_k, generalized mass M_k, and modal damping coefficient C_k may be obtained from the hub impedance. The matrices in the support equations of motion are thus

$$a_2 = \left[\diagdown M_k^* \diagdown \right] \qquad a_1 = \left[\diagdown C_k^* \diagdown \right] \qquad a_0 = \left[\diagdown K_k^* \diagdown \right]$$

where $M_k^* = M_k/(\frac{1}{2}NI_b/R^2)$, $C_k^* = C_k/(\frac{1}{2}NI_b\Omega/R^2)$, and $K_k^* = M_k^*(\omega_k/\Omega)^2$; and the hub motion matrices are zero except for the elements:

$$\tilde{a}_{k2} = c_{1k} = 1 \qquad \text{longitudinal modes}$$

$$-\tilde{a}_{k3} = c_{2k} = 1 \qquad \text{lateral modes}$$

Cantilever Wing

A model for a wing attached to the wind tunnel with cantilever root restraint (no balance motions) is developed in reference 2 for proprotor dynamics calculations. The rotor is located on a pylon at the wing tip, with the rotor hub a distance h forward of the wing tip elastic axis. An arbitrary angle of the pylon with respect to the tunnel velocity is considered. The wing motion is described by three degrees of freedom: vertical bending, chordwise bending, and torsion. For further details of the model, see ref. 2.

BALANCE, STRUT, AND MODULE MODEL

We shall now develop a generalized coordinate description of a wind tunnel support consisting of a balance frame, struts, and an aircraft body or rotor test module. The analysis will use the free vibration modes of the aircraft or module, coupled with a simple model for the balance frame and strut system. The resulting equations in normal mode form are

$$M_k(\ddot{q}_k + \omega_k^2 q_k) + \sum_i C_{ki} q_i = Q_k = \tilde{a}_k^T F$$

$$\alpha = c_k q_k$$

Here q_k are the generalized coordinates for the complete system. The matrix $\tilde{a}$ (with rows $\tilde{a}_k^T$) may be obtained from the matrix c (with columns c_k) always.

Balance Frame

Consider a balance frame supported by a scale system. The balance has a turntable; the turntable yaw angle Ψ is defined positive to the right, $\Psi = 0$ with the main struts forward and the tail strut aft. The balance frame motion is described by the six linear and angular rigid body degrees of freedom -- x_B, y_B, z_B, α_{Bx}, α_{By}, and α_{Bz}. The elastic deflections of the balance frame are neglected. Thus the motion of an arbitrary point (x,y,z) relative to the balance frame CG is given by

$$\Delta x = x_B + \alpha_{By} z - \alpha_{Bz} y$$
$$\Delta y = y_B - \alpha_{Bx} z + \alpha_{Bz} x$$
$$\Delta z = z_B + \alpha_{Bx} y - \alpha_{By} x$$

The balance scale system is represented by springs to fixed ground: four lift scales (K_L), two side scales (K_S), and one drag scale (K_D). The balance system optionally has viscous dampers between the frame and ground -- eight dampers at the corners of the balance frame (8 working vertically, 4 longitudinally, and 4 laterally).

Struts

There are two main struts and a tail strut. It is assumed that the main struts are cantilevered at the root (the balance frame), and pinned at the tips. The tail strut is pinned at the tip, pinned at the root longitudinally, and cantilevered at the root laterally. The inertia reaction of the struts is not considered (the strut mass is included in the balance frame inertia). Only the spring restraint between the balance and module is considered -- lateral, longitudinal, and vertical for the main struts, and lateral and vertical for the tail strut. The strut deflection model is based on the modes of a uniform cantilever beam. The vertical stiffness of the strut is very high; it is only included as the simplest means of handling the vertical motion constraints. It should be noted that the assumption of pinned joints at the strut tips, and even cantilever at the roots, is probably not very good (based on the stiffnesses required to match various experimental results). The physical system is of course more complex, perhaps so complex that even a sophisticated structural dynamics model such as NASTRAN will not improve correlation much.

A prop test rig is also considered, for which only the two main struts are used. The module is constrained in pitch at the strut tips in that case.

The strut tip displacement is given by the sum of the bending modes of a cantilever beam. Thus for the left main strut, the tip motion due to elastic bending is:

$$\Delta x_e = \sum_n q_{MSL_x} \qquad \text{longitudinal}$$

$$\Delta y_e = \sum_n q_{MSL_y} \qquad \text{lateral}$$

$$\Delta z_e = \sum_n q_{MSL_z} \qquad \text{vertical}$$

$$\Delta \alpha_{ye} = \sum_n q_{MSL_x} (\phi_n/\ell) \qquad \text{pitch}$$

where ℓ is the strut length. These components are defined with respect to axes yawed with the balance turntable. The tip deflection for the right main strut (MSR) and the tail strut (TS) are defined similarly (only lateral and vertical deflections for the tail strut, and the main strut pitch motion is only required for the prop test rig). The potential energies of bending and extension of the strut are

$$U_{bending} = \tfrac{1}{2} K_{bend} \sum_n \beta_n q_n^2$$

$$U_{extension} = \tfrac{1}{2} K_{ext} \sum_n q_n^2$$

The modes of a uniform cantilever beam give:

n	β_n	ϕ_n
1	3.091	1.367
2	121.42	4.781
3	952.1	7.849
4	3656.	10.996

Nominally the spring constants are $K_{bend} = EI/\ell^3$ and $K_{ext} = EA/\ell$; in practice these parameters are evaluated by matching to the measured frequencies.

Module

The module motion is described by the normal modes of free vibration; the first six modes are the linear and angular rigid body degrees of freedom. The motion is defined with respect to the yawed axis system, with origin at the module CG. The linear and angular motion of the point $\vec{r} = (x,y,z)$ is thus given by the module generalized coordinates q_{M_k} as follows:

$$\vec{u}_{module} = \sum_k q_{M_k}(t)\, \vec{\xi}_{M_k}(\vec{r})$$

$$\vec{\theta}_{module} = \sum_k q_{M_k}(t)\, \vec{\gamma}_{M_k}(\vec{r})$$

For the six rigid body modes

$$\vec{\xi} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -z \\ y \end{bmatrix} \quad \begin{bmatrix} z \\ 0 \\ x \end{bmatrix} \quad \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$$

In particular, the rotor hub motion is given by

$$\alpha = \sum_k q_{M_k} \begin{pmatrix} \vec{\xi}_{M_k} \\ \vec{\gamma}_{M_k} \end{pmatrix}_{hub}$$

Here $\vec{\zeta}$ and $\vec{Y}$ are in the yawed axis system, so it is necessary to premultiply by the rotation matrix

$$R_{TM} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

to obtain the hub motion in the tunnel axis system.

Module/Strut Connection

The system has constraints imposed by the connections between the module and strut tips. Specifically, it is required that the strut tip motion (composed of balance motion plus strut bending terms) equal the module motion at the connection points. This constraint is applied to the three linear deflections at the main strut tips, and to the lateral and vertical deflection at the tail strut tip. For the prop test rig, the pitch deflection constraints at the main strut tips replace the tail strut constraints.

Degrees of Freedom

The degrees of freedom of the system consist of the six balance rigid body motions, the six module rigid body motions, and N_e module elastic modes. The strut deflections do not add degrees of freedom since the strut bending inertia is not considered. The constraint equations thus must be used to eliminate the strut deflections from the set of equations, leaving $12+N_e$ equations to be solved for the coupled normal modes of the balance/strut/module system.

Energy and Constraints

The kinetic energy of the model described above is

$$T = \frac{1}{2}\Big\{ M_{B_x}\dot{x}_B^2 + M_{B_y}\dot{y}_B^2 + M_{B_z}\dot{z}_B^2 + I_{B_x}\dot{\alpha}_{B_x}^2 + I_{B_y}\dot{\alpha}_{B_y}^2 + I_{B_z}\dot{\alpha}_{B_z}^2 + \sum_k M_{M_k}\dot{q}_{M_k}^2 \Big\}$$

The potential energy is

$$U = \frac{1}{2}\Big\{ K_L\left(\Delta z^2_{RRL} + \Delta z^2_{LRL} + \Delta z^2_{RFL} + \Delta z^2_{LFL}\right)$$

$$+ K_s\left(\Delta y^2_{FS} + \Delta y^2_{RS}\right) + K_D \Delta x^2_D$$

$$+ K_{MS_x}\left(\Sigma \beta_n q^2_{MSL_x} + \Sigma \beta_n q^2_{MSR_x}\right)$$

$$+ K_{MS_y}\left(\Sigma \beta_n q^2_{MSL_y} + \Sigma \beta_n q^2_{MSR_y}\right)$$

$$+ K_{TS}\left(\Sigma \beta_n q^2_{TS_y}\right)$$

$$+ K_V\left(q^2_{MSL_z} + q^2_{MSR_z} + q^2_{TS_z}\right)$$

$$+ \sum_K M_{MK}\, \omega^2_{MK}\, q^2_{MK} \Big\}$$

The constraint equations (at the strut tips) are:

$$\Sigma q_{MSL_x} + \cos\psi\, \Delta x_{MSL} - \sin\psi\, \Delta y_{MSL} - \Sigma q_{MK}\, \vec{i}\cdot\vec{\zeta}_{MK} = 0$$

$$\Sigma q_{MSL_y} + \sin\psi\, \Delta x_{MSL} + \cos\psi\, \Delta y_{MSL} - \Sigma q_{MK}\, \vec{j}\cdot\vec{\zeta}_{MK} = 0$$

$$q_{MSL_z} + \Delta z_{MSL} - \Sigma q_{MK}\, \vec{k}\cdot\vec{\zeta}_{MK} = 0$$

$$\Sigma q_{MSL_x}(\phi_n/l) + \sin\psi\, \alpha_{B_x} + \cos\psi\, \alpha_{B_y} - \Sigma q_{MK}\, \vec{j}\cdot\vec{\delta}_{MK} = 0$$

and similarly for the right main strut and the tail strut.

Lagrange's Equations

The differential equations of motion are obtained from Lagrange's equations including constraints. The constraint equations are of the form

$$\phi_k(q_1 \ldots q_N) = 0 \qquad \text{for } k = 1 \ldots K$$

With linear constraints (as here), these equations may be written

$$\sum_n \frac{\partial \phi_k}{\partial q_n} q_n = 0$$

Then the Lagrange equations with the constraints are:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_n} - \frac{\partial T}{\partial q_n} + \frac{\partial U}{\partial q_n} = Q_n - \sum_k \lambda_k \frac{\partial \phi_k}{\partial q_n} \qquad \text{for } n = 1 \ldots N$$

Thus there are N+K equations for the degrees of freedom q_n and the Lagrange multipliers λ_k. Note that the mass and stiffness matrices are symmetrical with linear constraints.

By this procedure the equations describing the balance, strut, and module dynamic system may be constructed.

Solution

Eliminating the Lagrange multipliers and constraint equations from the system gives a set of $12+N_c$ linear differential equations, of the form:

$$A_2\ddot{x} + A_1\dot{x} + A_0x = Q$$

where A_1 is a damping matrix (balance dampers or aerodynamic damping), and Q is the generalized force vector (due to hub forces and moments; and perhaps contributions from aerodynamic gusts or support system control variables).

The homogeneous, undamped equations are

$$A_2\ddot{x} + A_0x = 0$$

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where A_2 and A_0 are real symmetric matrices. It follows that the eigenvalues are real and positive, and the eigenvectors real. Let ω_k^2 be the eigenvalues of $A_2^{-1}A_0$, and T the modal matrix (columns are the eigenvectors). Then the modal coordinates for the coupled system are defined by $q = T^{-1}x$; the natural frequencies of the modes are ω_k; and the (diagonal) generalized mass matrix is

$$\left\{ M_k \right\} = T^T A_2 T$$

The damping matrix is then

$$\left\{ C_{ki} \right\} = T^T A_1 T$$

(only the diagonal terms are usually important). Finally, the hub motion in terms of the modal coordinates of the coupled system is:

$$\alpha = R_{TM} \left[\cdots \begin{matrix} \zeta_{M_j} \\ \delta_{M_j} \end{matrix} \cdots \right] \left\{ q_{M_j} \right\}$$

so

$$c_k = R_{TM} \left[\cdots \begin{matrix} \zeta_{M_j} \\ \delta_{M_j} \end{matrix} \cdots \right] t_k$$

where t_k is the appropriate column of the modal matrix T. This completes the description of the rotor support equations of motion and the hub motion in the required form.

DATA FOR THE AMES 40- BY 80-FT WIND TUNNEL

The following sections give the geometric, mass, and stiffness data for several rotor test modules and strut combinations in the Ames 40- by 80-ft wind tunnel. The geometry was measured directly. The inertia data were obtained from direct measurements and from NASTRAN calculations. The stiffnesses were obtained by matching the calculations with shake test results for the principal natural frequencies of the system. Experiment is the only reliable source for modal damping values, because even for the balance dampers the modal damping is very sensitive to the details of the motion. The shake test data used was from references 3 to 5.

Balance

$M = 53500$ kg
$I_x = 325000$ kg-m^2
$I_y = 340000$ kg-m^2
$I_z = 810000$ kg-m^2

$\left.\begin{array}{l} x_{CG} = -.49 \text{ m} \\ z_{CG} = 2.03 \text{ m} \end{array}\right\}$ relative to center turntable, drag link elevation; for yaw $\psi = 0$.

$K_D = 9000000$ N/m
$K_S = 9000000$ N/m
$K_L = 36000000$ N/m

$C_{damper} \doteq 15000$ N/m/sec

	Position, m (relative center turntable, drag link elevation)		
	x	y	z
scale springs			
D	3.086	0	0
RS	2.553	0	0
FS	-3.899	0	0
RRL	4.877	-5.153	1.8
RFL	-4.877	-5.153	1.8
LRL	4.877	5.153	1.8
LFL	-4.877	5.153	1.8
dampers			
long SE	-4.88	-3.05	1.4
SW	-4.88	3.05	1.4
NE	4.88	-3.05	1.4
NW	4.88	3.05	1.4
lat SE	-3.51	-5.15	1.4
SW	-3.51	5.15	1.4
NE	3.20	-5.15	1.4
NW	3.20	5.15	1.4

Rotor Test Apparatus

$M = 13800$ kg

$I_x = 2600$ kg-m^2

$I_y = 54500$ kg-m^2

$I_z = 49500$ kg-m^2

$I_{zx} = 2900$ kg-m^2

tail length = 4.521 m

tread = 2.438 m

	long struts	long struts, bal. locked	short struts	short struts, bal. locked
strut height	4.72	4.72	3.96	3.96 m
K_{MS_x}	1120000	830000	550000	450000 N/m
K_{MS_y}	1780000	920000	830000	500000 N/m
K_{TS}	730000	560000	410000	270000 N/m
K_{vert}		6×10^8 N/m		

	Position, m (relative module CG)		
	x	y	z
left main strut	-1.503	-1.219	-.15
right main strut	-1.503	1.219	-.15
tail strut	3.018	0	-.15
hub	2.442	0	1.679

Propeller Test Rig

$$\left.\begin{array}{l} M = 8600 \text{ kg} \\ I_x = 14600 \text{ kg-m}^2 \\ I_y = 950 \text{ kg-m}^2 \\ I_z = 14600 \text{ kg-m}^2 \end{array}\right\} \text{for turntable yaw } \Psi = 0$$

tread = 2.438 m

strut height = 6.07 m

	balance free	balance locked
K_{MS_x}	310000	310000 N/m
K_{MS_y}	580000	450000 N/m
K_{vert}	6×10^8 N/m	

	Position, m (relative module CG, Ψ = 0)		
	x	y	z
left main strut	0	-3.39	0
right main strut	0	-.95	0
hub	0	3.20	0

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