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SHUTTLE ONBOARD IMU ALIGNMENT METHODS

MISSION PLANNING, MISSION ANALYSIS, AND SOFTWARE FORMULATION

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1.0 SUMMARY

The current approach to the Shuttle IMU alignment is based solely on the Apollo Deterministic Method (Reference (A)). This method is simple, fast, reliable and provides an accurate estimate for the present cluster to mean of 1950 transformation matrix. This method must remain in the software as the basic estimator, however, if four or more star sightings are available, the application of least squares analysis can be utilized. The least squares method offers the next level of sophistication to the IMU alignment solution. The least squares method studied in this working paper shows that a more accurate estimate for the misalignment angles is computed, and the IMU drift rates are a free by-product of the analysis. Core storage requirements are considerably more; estimated 20 to 30 times the core required for the Apollo Deterministic Method. After the least squares analysis, the next level of solution would be the use of the Kalman filter. Kalman filtering is not discussed in this paper, but according to Reference (B), an entire orbit of star tracker data (90 minutes) is required before an accurate solution is computed. Hence, the least squares method offers an intermediate solution utilizing as much data that is available without a complete statistical analysis as in Kalman filtering.

2.0 INTRODUCTION

Due to the lack of compensation for certain IMU drift errors, the present cluster system slowly rotates away from the desired cluster system for which the stored REFSMAT is valid. Periodically the Shuttle IMU's will need realignment to provide a more accurate reference to the mean of 1950 coordinate system. With the COAS and the Star Tracker systems, line-of-sight (LOS) vectors can be computed to known stars on the celestial sphere. This data can then be used to determine the validity of the stored REFSMAT and/or when an IMU realignment is necessary. The methods discussed in this paper are; (1) The Apollo Deterministic Method, requiring two star sightings, (2) A direct matrix solution requiring three sightings, (3) and the least squares method requiring a minimum of four sightings.

3.0 DISCUSSION

Data from the COAS or Star Tracker is transformed into the actual IMU coordinate system (present cluster system) and used in conjunction with the M50 (mean of 1950) unit vector to the known star (X). The known star unit vector is read from a table based on positive identification of the star in the COAS or Star Tracker field of view. The diagram in Figure (1) relates the coordinate systems involved with the IMU alignment method:

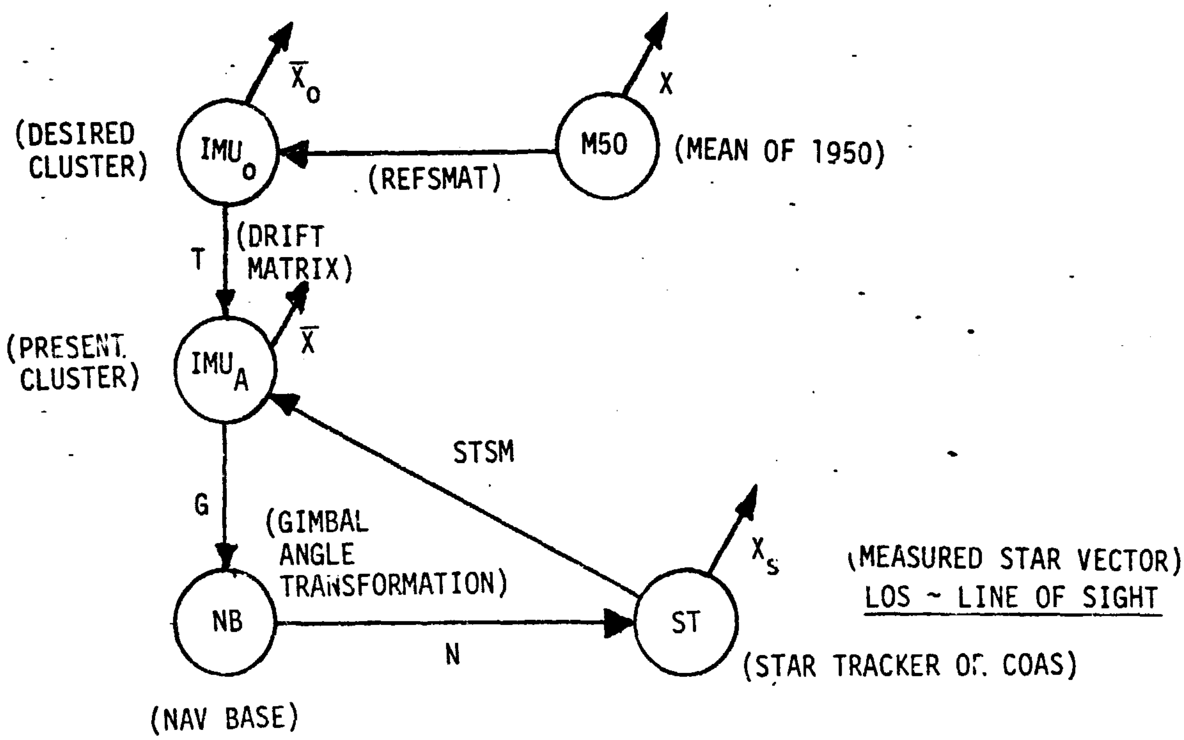

FIGURE (1)

The $\overline{X}$ vector is generated from the measured LOS vector in the Star Tracker or COAS coordinate system by,

$$\overline{X} = (STSM)\ X_s, \tag{1}$$

where the (STSM) matrix is the Star Tracker or COAS to stable member (present cluster) transformation matrix. This matrix is a function of the gimbal angle transformation matrix (G) and hence is a function of time when the Shuttle is rotating in inertial space. The (STSM) matrix is defined from Figure (1) above as:

$$(STSM) = [(N)(G)]^T \tag{2}$$

where (N) is a constant matrix relating the Star Tracker or COAS to the navigation base reference frame.

The M50 star vector X from the star table is transformed into the ideal (desired) IMU system by the known REFSMAT (R), (a constant matrix) as follows:

$$\overline{X}_o = (R)\ X. \tag{3}$$

The vector $\overline{X}_o$ in the desired cluster system is transformed into the actual (present cluster) IMU system by the drift matrix T;

$$\overline{X} = (T)\ \overline{X}_o \tag{4}$$

The drift matrix is a function of time since it describes the relationship between the present cluster and desired cluster systems. These two systems drift apart at some small uncompensated drift rate ω, where the 1σ value for ω is 0.035°/hr. or about 0.972×10^{-5}°/sec. for the Shuttle IMU's.

By substituting equation (3) into (4)

$$\overline{X} = (T)(R)\ X \tag{5}$$

and defining $(R') = (T)(R)$,

the R' matrix is determined when the Apollo deterministic alignment calculation is applied to the problem. The solution for the instantaneous drift matrix then becomes;

$$T = R'\ R^T. \tag{6}$$

The misalignment Euler angles may then be extracted from the T matrix and displayed to the crew for a criteria to realign the IMU.

The numerical problem of onboard Shuttle IMU alignment reduces to the solution for the drift matrix (T) as a function of time, the known star vectors X_m, and computed $\overline{X}_m$ vectors in the present cluster system. Simplification of the diagram in Figure (1) to the basic coordinate system involved results in Figure (2).

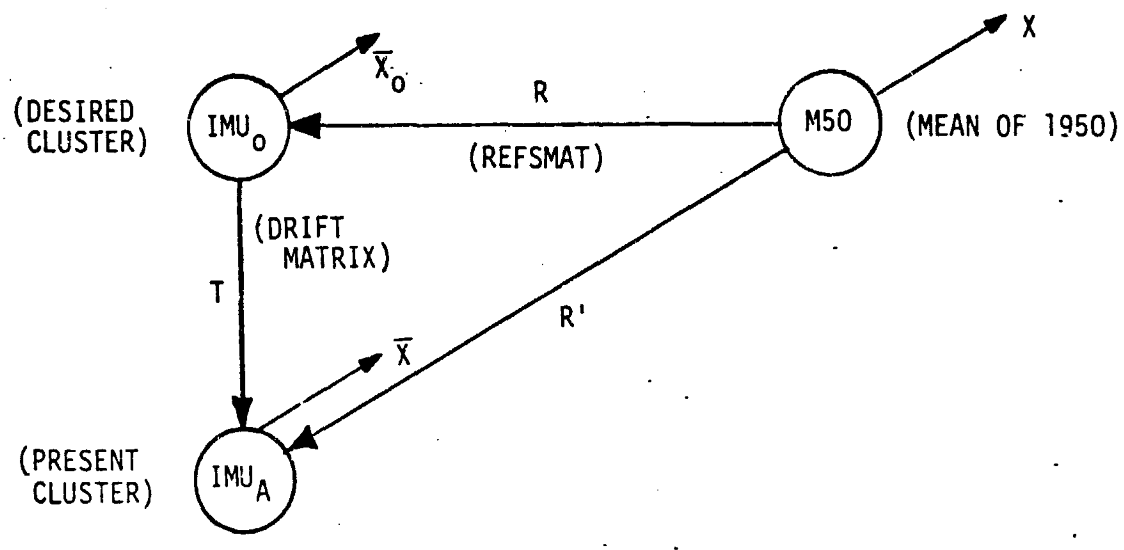

FIGURE (2)

An input data table can be made available to the onboard software for processing. The format of the input data table is shown in Table I.

POINT NO.	TIME	M50 STAR LOS VECTOR	COMPUTED STAR LOS VECTOR IN PRESENT CLUSTER SYSTEM
1	t_1	X_1	$\overline{X}_1$
2	t_2	X_2	$\overline{X}_2$
3	t_3	X_3	$\overline{X}_3$
.	.	.	.
.	.	.	.
.	.	.	.
n	t_n	X_n	$\overline{X}_n$

TABLE I

INPUT DATA TABLE

The computed star LOS vector in the present cluster system is calculated using the following procedure:

Read vertical (ξ) and horizontal (η) deflection angles from the star tracker hardware system and compute the X_s LOS vector (see Figure (1)) as follows:

$$\ell = (1.0 + \tan^2 \xi + \tan^2 \eta)^{1/2}. \quad (7)$$

Then the components of the X_s vector are,

$$X_{S_1} = \frac{\tan \xi}{\ell} \tag{8}$$

$$X_{S_2} = \frac{\tan \eta}{\ell}$$

$$X_{S_3} = \frac{1}{\ell}.$$

And the $\overline{X}_n$ LOS vector above is computed immediately from equation (1)

$$\overline{X} = (STSM)\ X_S\ . \tag{9}$$

3.1 The Apollo Deterministic Solution for IMU Alignment

When only two points are available, (Table I, N = 2) the Apollo Deterministic Method may be used as an estimator for the R' matrix (see Figure (2)) and the misalignment (drift) matrix T. This method provides an accurate estimate for T and R' at the time of the observations, but does not estimate the drift rates, ω, of the T matrix.

The method may be described as follows:

Assuming the first two points of Table I are available, create a right-handed system from vectors X_1 and X_2

$$\begin{aligned} \vec{U} &= \vec{X}_1 & & & \vec{u} &= \vec{U}/|\vec{U}| \\ \vec{V} &= \vec{X}_1 \times \vec{X}_2 & &\text{LET} & \vec{v} &= \vec{V}/|\vec{V}| \\ \vec{W} &= \vec{X}_1 \times \vec{V} & & & \vec{w} &= \vec{W}/|\vec{W}| \end{aligned}$$

$$X = B\overline{\overline{X}} = \begin{pmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{pmatrix} \overline{\overline{X}}. \tag{10}$$

Likewise create the same system using $\bar{X}_1$ and $\bar{X}_2$ assuming these vectors are invariant with time;

$$\vec{U'} = \vec{\bar{\bar{X}}}_1 \qquad \vec{u'} = \vec{U'}/|\vec{U'}|$$

$$\vec{V'} = \vec{\bar{\bar{X}}}_1 \times \vec{\bar{\bar{X}}}_2 \qquad \vec{v'} = \vec{V'}/|\vec{V'}|$$

$$\vec{W'} = \vec{\bar{\bar{X}}}_1 \times \vec{V}_1 \qquad \vec{w'} = \vec{W'}/|\vec{W'}|$$

Then form

$$\bar{X} = C\bar{\bar{X}} = \begin{pmatrix} u'_1 & v'_1 & w'_1 \\ u'_2 & v'_2 & w'_2 \\ u'_3 & v'_3 & w'_3 \end{pmatrix}\bar{\bar{X}} . \tag{11}$$

Now solve for $\bar{\bar{X}} = C^T\bar{X}$ and substitute this in equation (10);

$$X = [BC^T]\,\bar{X} \tag{12}$$

or

$$\bar{X} = [BC^T]^T\,X = (CB^T)\,X . \tag{13}$$

Comparing this equation with equation (5) yields;

$$R' = CB^T . \tag{14}$$

If, $t_1 = t_2$ (Table I above) the deterministic method will yield an exact solution (assuming no star tracker/COAS measurement errors and no gimbal angle errors) for the R' matrix (equation 14) and for the drift matrix from equation (6). However, if $t_1 \neq t_2$, geometric errors result in the Apollo deterministic calculation. This error results from the fact that the vector $\bar{X}_1$ (t_1) is not updated to t_2 (the time of the second observation). A first order approximation for this update is,

$$\bar{X}_1\ (t_2) \cong \bar{X}_1\ (t_1) + (TW\ \Delta t_2)\ RX_1 \tag{15}$$

where $\Delta t_2 = t_2 - t$,

and

$$W = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{16}$$

The W matrix is the skew-symmetrix angular velocity matrix. The ω_i's are the IMU drift rates about each axis. The geometric error in the deterministic calculation results from truncating equation (15) to,

$$\bar{X}_1\ (t_2) = \bar{X}_1\ (t_1)\ . \tag{17}$$

and omitting the resulting error vector

$$\varepsilon_2 \cong (TW\ \Delta t_2)\ R\ X_1\ . \tag{18}$$

This error is then further complicated in the calculations forming equation (11) and again in equation (14). The final error will be directly related to the relative orientation of the two LOS vectors X_1 and X_2 and the time interval between star sightings, i.e., $\Delta t = t_2 - t_1$. Table II presents a preliminary study of the geometric errors based on the 3σ drift rates, i.e., $\omega = 0.105°/hr$ per axis, with no errors in the star tracker measurements. The norms of the errors in the Euler angles are computed to show the magnitude of the total error. The differences in the norms of the Euler angles for the M50 to present cluster and the misalignment matrix are due to computer round-off mainly in the M50 to present cluster computation. The relative magnitude of the Euler angles for the M50 to present cluster transformation is $\pm 360°$. The single precision accuracy expected would then be $\pm 1.0 \times 10^{-3}$ for six significant figures (assuming round-off and slight matrix non-orthogonalities). However, the relative magnitude of the Euler angles of the misalignment matrix are $< \pm 1.0$ and the expected accuracy is about $\pm 1.0 \times 10^{-6}$.

3.2 An Alternate Solution for IMU Alignment

In the event that 3 points were available in Table I ($M = 3$), the following method offers a solution with possible extension to solve for the drift rates. Write equation (4) for each of the data points from Table I,

$$\begin{aligned} \overline{X}_1 &= T_1\, \overline{X}_{o_1} \\ \overline{X}_2 &= T_2\, \overline{X}_{o_2} \\ \overline{X}_3 &= T_3\, \overline{X}_{o_3} \quad . \end{aligned} \tag{19}$$

Assuming $T_1 = T_2 = T_3$, i.e., either the drift rates are small or the time between sightings is small, $t_1 \cong t_2 \cong t_3$, a matrix can be constructed of the form.

$$(\overline{X}_1\ \overline{X}_2\ \overline{X}_3) = T\ (\overline{X}_{o_1}\ \overline{X}_{o_2}\ \overline{X}_{o_3}) \quad . \tag{20}$$

where each vector is now a column in a matrix and in matrix form is

$$\overline{X} \cong T\, \overline{X}_o \quad . \tag{21}$$

This equation has an immediate solution,

$$T \cong (\overline{X})(\overline{X}_o)^{-1} \quad . \tag{22}$$

The restriction here is that $\overline{X}_o$ must be non-singular. Th matrix $\overline{X}_o$ is indeed non-singular if different stars are selected for each of the three sightings in Table I. This method, however, has the same types of errors as the Apollo deterministic solution. In the Shuttle star tracker system, the time between star acquisitions may be 5 to 30 minutes, depending on the stars of opportunity. With these Δt's and a 3σ IMU drift rate, the geometric error using equation

(23) is in the range of 0.01 to 0.1 degrees in the misalignment angles.

The T matrices of equation (19) are written,

$$T_n \cong T_o + T_o W \Delta t_n \tag{23}$$

as a first order approximation for the T matrix as a function of time and ω. The T_o matrix in equation (23) is defined at some starting time t_o. The time t_o is some short interval before t_1 and $\Delta t_n = t_n - t_o$. Substituting equation (23) into (19), and adding one one more star sighting, we have,

$$\begin{aligned}
\overline{X}_1 &\cong (T_o + T_o W \Delta t_1)\, \overline{X}_{o_1} \\
\overline{X}_2 &\cong (T_o + T_o W \Delta t_2)\, \overline{X}_{o_2} \\
\overline{X}_3 &\cong (T_o + T_o W \Delta t_3)\, \overline{X}_{o_3} \\
\overline{X}_4 &\cong (T_o + T_o W \Delta t_4)\, \overline{X}_{o_4} \; .
\end{aligned} \tag{24}$$

By grouping into matrix form, a number of matrix equations can be formed. However, a solution for either the T_o matrix or the W matrix is not immediately available. This is because the W matrix is singular and this first order approximation is valid only for small Δt's. It might be possible to form a closed form approximation for T and W using a group of such equations, but further analysis of the problem using these methods would be required. The absence of a closed form solution for T_o and W suggests the application of statistical modeling techniques such as least squares analysis or Kalman filtering.

3.3 Application of Least Squares Analysis for IMU Alignment

The line-of-sight vector from the present cluster as shown in Figure (2) and equation (4) for the Nth star sighting is;

$$\overline{X}_n = T_n\ (\theta_o,\ \omega,\ t_n)\ \overline{X}_{o_n}, \tag{25}$$

where $T_n\ (\theta_o,\ \omega,\ t_n)$ denotes the T_n matrix which is a function of the misalignment angles θ_o, at some time t_o, and ω represents the IMU drift rates. The star tracker measurements can be used to compute $\overline{X}_n$ and by using an initial estimate for $\hat{\theta}_o$ (from the Apollo deterministic method) and $\hat{\omega}$, the T_o matrix can be integrated (from time t_o to time t_n) to obtain an estimate for the $\overline{X}_n$ vector,

$$\hat{\overline{X}}_n = T_n\ (\hat{\theta}_o,\ \hat{\omega},\ t_n)\ \overline{X}_{o_n}\ . \tag{26}$$

By using the difference between the computed measurement $\overline{X}_n$ and estimated vectors $\hat{\overline{X}}_n$,

$$\Delta\overline{X}_n = \overline{X}_n - \hat{\overline{X}}_n = X_n - T_n\ (\hat{\theta}_o,\ \hat{\omega},\ t_n)\ \overline{X}_{o_n}\ , \tag{27}$$

a correction to $\hat{\theta}_o$ and $\hat{\omega}$ can be made by using least squares analysis. That is, a correction vector of the form,

$$\begin{pmatrix} \theta_{o_{i+1}} \\ \omega_{i+1} \end{pmatrix} = \begin{pmatrix} \theta_{o_i} \\ \omega_i \end{pmatrix} + \begin{pmatrix} \Delta\theta_{o_i} \\ \Delta\omega_i \end{pmatrix} \tag{28}$$

can be computed as a function of the $\Delta\overline{X}_n$ vector and applied to an iterative solution for θ_o and ω at some time t_o.

To determine the correction vector $\begin{pmatrix} \Delta\theta_o \\ \Delta\omega \end{pmatrix}$, the following analysis provides the basic least squares estimator. The vectors $\overline{X}_n$ are

$\overline{X}_n = \overline{X}_n\ (\theta_o, \omega)$ and the differential of $\overline{X}_n$ is formed by,

$$d\overline{X}_n = \begin{pmatrix} \frac{\partial \overline{X}_n}{\partial \theta_o} & \frac{\partial \overline{X}_n}{\partial \omega} \end{pmatrix} \begin{pmatrix} d\theta_o \\ d\omega \end{pmatrix} \tag{29}$$

or

$$d\overline{X}_n = P \begin{pmatrix} d\theta_o \\ d\omega \end{pmatrix} \ . \tag{30}$$

The matrix P is the partial derivative matrix whose size is 3n x 6 (n is the number of star sightings). For small changes, equation (29) becomes,

$$\Delta\overline{X}_n = \begin{pmatrix} \frac{\partial \overline{X}_n}{\partial \theta_o} & \frac{\partial \overline{X}_n}{\partial \omega} \end{pmatrix} \begin{pmatrix} \Delta\theta_o \\ \Delta\omega \end{pmatrix} \ . \tag{31}$$

The partial derivatives are formed numerically by,

$$\begin{aligned} \frac{\partial \overline{X}_n}{\partial \theta_o} &= \frac{\hat{\overline{X}}_n\ (t)_{\delta\theta_o} - \hat{\overline{X}}_n\ (t)}{\delta\theta_o} \\ \frac{\partial \overline{X}_n}{\delta\omega} &= \frac{\hat{\overline{X}}_n\ (t)_{\delta\omega} - \hat{\overline{X}}\ (t)}{\delta\omega} \end{aligned} \tag{32}$$

and

$$\begin{aligned} \hat{\overline{X}}_n\ (t)_{\delta\theta_o} &= \hat{T}\ (t_n)_{\delta\theta_o}\ \overline{X}_{o_n} \\ (t)_{\delta\omega} &= \hat{T}\ (t_n)_{\delta\omega}\ \overline{X}_{o_n} \end{aligned} \tag{33}$$

The estimates for $\hat{\overline{X}}_n\ (t_n)$, $\hat{\overline{X}}_n\ (t_n)_{\delta\theta_o}$ and $\hat{\overline{X}}_n\ (t_n)_{\delta\omega}$ require numerical integration of $T\ (\theta_o, \omega)$ from t_o to t_n the time of star sighting. The following integrals must be performed:

$$\hat{T}\ (t_n) = \int_{t_o}^{t_n} T\ (\theta_o,\ \omega)\ dt$$

$$\hat{T}\ (t_n)_{\delta\theta_o} = \int_{t_o}^{t} T\ (\theta_o + \delta\theta_o,\ \omega)\ dt \tag{34}$$

$$\hat{T}\ (t_n)_{\delta\omega} = \int_{t_o}^{t} T\ (\theta_o,\ \omega + \delta\omega)\ dt.$$

A detailed explanation of the evaluation of these integrals is given in paragraph 3.4 of this memo. $\Delta\hat{\overline{X}}_n$ is then computed by,

$$\Delta\hat{\overline{X}}_n = \hat{\overline{X}}_n\ (t) - \hat{T}\ (t_n)\ \overline{X}_{o_n} \tag{35}$$

and equation (31) is solved for $\begin{pmatrix}\Delta\theta_o\\ \Delta\omega\end{pmatrix}$ by

$$P^T\ \Delta\overline{X}_n \cong P^T P \begin{pmatrix}\Delta\theta_o\\ \Delta\omega\end{pmatrix} \tag{36}$$

and

$$\begin{pmatrix}\Delta\theta_o\\ \Delta\omega\end{pmatrix} \cong (P^T P)^{-1}\ P^T\ \Delta\hat{\overline{X}}_n \tag{37}$$

This provides the correction vector for equation (28) and with an iteration procedure offers a solution to the alignment problem. Some preliminary comparison simulations were made with the Apollo deterministic method using a maximum of 6-star sighting for the least squares solution. Table III shows the comparison results with no star tracker errors, i.e. errors in Apollo deterministic solution are those given in equation 18. Table IV shows a comparison with a 100 and 200 arc seconds of random error in the horizontal and vertical deflection angles.

3.4 Integration of the Transformation Matrix

In the solution of the IMU alignment method, it is necessary to integrate the error matrix relating the actual IMU to the ideal IMU. Since this matrix is the state of the system, state updates are required in the analysis. Hence, the T_{t_0} matrix must be integrated forward in time to T_t. Constant IMU drift rates, ω_i are assumed. The quantity $(t - t_0)$ may take on large or small values and the drift rates, ω_i are estimated to be about 0.035 degrees per hour. The T_t matrix has the derivative;

$$\frac{dT_t}{dt} = T_t W \tag{38}$$

where the W matrix is the skew-symmetric angular velocity matrix (the coordinate axes drift rates, ω_i). There are nine elements of this matrix each of which must be integrated. Integration of the matrix using this form is not practical since truncation errors in the numerical process will cause the matrix to become non-orthogonal. Integration of the Euler angles is, however possible but this method has singularities in the transformation from coordinate axis rotation rates to Euler rates. The following method utilizes the quaternion rates which are functions of the quaternions of the matrix, W, and the coordinate axis rotation rates. The quaternion rates are given by

$$
\begin{aligned}
\dot{q}_1 &= 1/2\ (-\omega_1\ q_2 - \omega_2\ q_3 - \omega_3\ q_4) \\
\dot{q}_2 &= 1/2\ (\ \omega_1\ q_1 - \omega_2\ q_4 + \omega_3\ q_3) \\
\dot{q}_3 &= 1/2\ (\ \omega_1\ q_4 + \omega_2\ q_1 - \omega_3\ q_2) \\
\dot{q}_4 &= 1/2\ (-\omega_1\ q_3 + \omega_2\ q_2 + \omega_3\ q_1)\ ,
\end{aligned}
\tag{39}
$$

and are non-singular. Using a Taylor series expansion $q_{i_{t+\Delta t}}$ is given by,

$$\ddot{q}_{i_{t+\Delta t}} \cong q_{i_t} + \dot{q}_{i_t}\ \Delta t + \frac{\ddot{q}_{i_t}}{2}\ \Delta t^2 + \frac{1}{6}\ \dddot{q}_{i_t}\ \Delta t^3 + \cdot\ \cdot\ \cdot + \frac{1}{n!}\ q_{i_t}^{(n)}\ \Delta t^n\ . \tag{40}$$

Computation of higher order deviatives can be done by using equation (39); they are as follows:

$$\ddot{q}_i = -\frac{1}{4}\ q_i\ W^2 \tag{41}$$

where

$$W^2 = \omega_1 + \omega_2 + \omega_3\ . \tag{42}$$

and

$$\dddot{q}_i = -\frac{\dot{q}_i}{4} W^2 \tag{43}$$

$$q_i^{(4)} = \frac{1}{16}\ q_i\ W^4 \tag{44}$$

$$q_i^{(5)} = \frac{1}{16}\ \dot{q}_i\ W^4 \tag{45}$$

$$q_i^{(6)} = \cdot\ \frac{1}{64}\ q_i\ W^6 \tag{46}$$

$$q_i^{(7)} = -\frac{1}{64}\ \dot{q}_i\ W^6 \tag{47}$$

and so on for higher order derivatives. By substitution of these terms into equation (40) (for $n = 7$), and simplifying, one obtains the following:

$$q_{i_{t+\Delta t}} = q_{i_t}\left[1.0 - \frac{1}{8}W^2\,\Delta t^2 + \frac{1}{24\cdot 16}W^4\,\Delta t^4 - \frac{1}{720\cdot 64}W^6\Delta t^6\right] \qquad (48)$$

$$+ \dot{q}_{i_t}\left[1.0 - \frac{1}{24}W^2\,\Delta t^2 + \frac{1}{120\cdot 16}W^4\Delta t^4 - \frac{1}{5040\cdot 64}W^6\Delta t^6\right]\Delta t$$

Using equation (48) and selecting Δt such that $(\omega\Delta t) < 25$ (estimated) the integral of the transformation matrix can be accurately computed. Orthogonality is guaranteed since the matrix will be constructed from the final quaternion $q_{i_{t+\Delta t}}$.

4.0 CONCLUSIONS

The Apollo deterministic solution provides a basic method for the IMU alignment and produces good results even when large star tracker errors (random ±200 arc seconds in both horizontal and vertical deflection angles) are introduced. This method should remain as the main computational scheme for IMU alignment. In the event that more storage is available during the alignment mission phase, a second method such as the n point least square method (where $n \geq 4$) should be considered using the Apollo deterministic method output as a starting solution.

5.0 REFERENCES

(A) IMU SOP Section 4.6.4.7, SD76-SH-0013, 4 June 1976.

(B) H. McQuat, "Further Data on Simulation of a Kalman Alignment Filter," CSDL Memo No. 10J-255-76, 30 June 1976.

TABLE II
GEOMETRIC ERRORS IN DETERMINISTIC METHOD

STARS USED	Δt SECONDS	ERRORS IN MISALIGNMENT ANGLES (DEGREES) $T = R'R^T$				ERRORS IN M50 TO PRESENT CLUSTER (DEGREES) R'			
		ΔYAW	ΔPITCH	ΔROLL	NORM*	ΔYAW	ΔPITCH	ΔROLL	NORM*
α-IMU/i-CEP	360	-.0056	.0092	-.017	.020	-.017	.011	.019	.027
i-CEP/β-HER	900	.010	.027	.015	.032	-.0030	-.025	.022	.033
β-HER/γ-PAV	1020	.030	.0014	.014	.033	0.033	-.013	.0062	.036
γ-PAV/α-ANT	1320	-.0026	.020	.0044	.020	-.014	-.012	-.020	.027
α-LEO/i-CEP	1500	.0059	.0088	.052	.053	.016	-.051	-.018	.056
γ-PAV/α-LEO	1920	-.0013	.029	.0081	.030	-.018	-.019	.028	.037

*NORM OF ERROR = $(\Delta YAW^2 + \Delta PITCH^2 + \Delta ROLL^2)^{1/2}$

TABLE III
COMPARISON OF APOLLO DETERMINISTIC METHOD AND 4 to 6 STAR LEAST SQUARES METHOD WITH ND TRACKER ERRORS

STARS USED	Δt SECONDS	n	ERRORS IN MISALIGNMENT ANGLES LEAST SQUARES METHOD (DEGREES)				ERRORS IN MISALIGNMENT ANGLES * APOLLO DETERMINISTIC METHOD (DEGREES)			
			ΔYAW	ΔPITCH	ΔROLL	NORM	ΔYAW	ΔPITCH	ΔROLL	NORM
α - UMI i - CEP β - HER γ - PAV	360 900 1020	4	$-.20 \text{ X}10^{-5}$	$.80 \text{ X}10^{-7}$	$-.64 \text{ X}10^{-6}$	$.21 \text{ X}10^{-5}$	$.30 \text{ X}10^{-1}$	$.14 \text{ X}10^{-2}$	$.14 \text{ X}10^{-1}$	$.33 \text{ X}10^{-1}$
α - UMI i - CEP β - HER γ - PAV α - ANT	360 900 1020 1320	5	$-.93 \text{ X}10^{-8}$	$-.89 \text{ X}10^{-6}$	$.40 \text{ X}10^{-6}$	$.97 \text{ X}10^{-6}$	$-.26 \text{ X}10^{-2}$	$.20 \text{ X}10^{-1}$	$.44 \text{ X}10^{-2}$	$.2 \text{ X}10^{-1}$
α - UMI i - CEP β - HER γ - PAV α - ANT α - LEO	360 900 1020 1320 600	6	$-.98 \text{ X}10^{-7}$	$-.67 \text{ X}10^{-6}$	$.35 \text{ X}10^{-6}$	$.76 \text{ X}10^{-6}$	$.29 \text{ X}10^{-1}$	$.18 \text{ X}10^{-1}$	$-.83 \text{ X}10^{-2}$	$.35 \text{ X}10^{-1}$
i - CEP β - HER γ - PAV α - ANT α - LEO i - CEP	900 1020 1320 600 1500	6	$.35 \text{ X}10^{-7}$	$-.69 \text{ X}10^{-6}$	$.31 \text{ X}10^{-6}$	$.76 \text{ X}10^{-6}$	$.59 \text{ X}10^{-2}$	$.89 \text{ X}10^{-2}$	$.52 \text{ X}10^{-1}$	$.53 \text{ X}10^{-1}$
β - HER γ - PAV α - ANT α - LEO i - CEP β - HER	1020 1320 600 1500 960	6	$-.20 \text{ X}10^{-6}$	$-.10 \text{ X}10^{-5}$	$.13 \text{ X}10^{-6}$	$.10 \text{ X}10^{-5}$	$.11 \text{ X}10^{-1}$	$.29 \text{ X}10^{-1}$	$.16 \text{ X}10^{-1}$	$.35 \text{ X}10^{-1}$

ERRORS IN MISALIGNMENT ANGLES ARE DUE TO GEOMETRIC ERRORS ONLY

TABLE II:
COMPARISON OF APOLLO DETERMINISTIC METHOD AND 4 TO 6 STAR LEAST SQUARES METHOD WITH ND TRACKER ERRORS
(con't)

STARS USED	Δt SECONDS	n	ERRORS IN MISALIGNMENT ANGLES LEAST SQUARES METHOD (DEGREES)				ERRORS IN MISALIGNMENT ANGLES * APOLLO DETERMINISTIC METHOD (DEGREES)			
			Δ YAW	Δ PITCH	Δ ROLL	NORM	Δ YAW	Δ PITCH	Δ ROLL	NORM
γ - PAV α - ANT α - LEO i - CEP ε - HER γ - PAV	 1320 600 1500 960 1020	6	-13×10^{-5}	$-.24 \times 10^{-6}$	$-.40 \times 10^{-1}$	$.14 \times 10^{-5}$	$.30 \times 10^{-1}$	$.13 \times 10^{-2}$	$.14 \times 10^{-1}$	$.33 \times 10^{-1}$
α - ANT α - LEO i - CEP ε - HER γ - PAV α - ANT	 600 1500 960 1020 1320	6	$-.14 \times 10^{-5}$	$-.80 \times 10^{-6}$	$.74 \times 10^{-6}$	$.17 \times 10^{-5}$	$-.26 \times 10^{-2}$	$.20 \times 10^{-2}$	$.44 \times 10^{-2}$	$.20 \times 10^{-1}$
α - LEO i - CEP ε - HER γ - PAV α - ANT α - LEO	 1500 960 1020 1320 600	6	$-.23 \times 10^{-5}$	$-.15 \times 10^{-5}$	$.11 \times 10^{-5}$	$.29 \times 10^{-5}$	$.29 \times 10^{-1}$	$.18 \times 10^{-1}$	$-.82 \times 10^{-2}$	0.35×10^{-1}
i - CEP ε - HER γ - PAV α - ANT α - LEO i - CEP	 960 1020 1320 600 1500	6	$-.10 \times 10^{-5}$	$-.9 \times 10^{-6}$	$-.56 \times 10^{-6}$	$.14 \times 10^{-5}$	$.59 \times 10^{-2}$	$.88 \times 10^{-2}$	$.52 \times 10^{-1}$	$.53 \times 10^{-1}$

* ERRORS IN MISALIGNMENT ANGLES ARE DUE TO GEOMETRIC ERRORS ONLY

TABLE IV
COMPARISON OF LEAST SQUARES AND APOLLO DETERMINISTIC METHODS WITH RANDOM STAR TRACKER ERRORS OF 100 AND 200 ARC SECONDS

STARS USED	Δt SECONDS	n	ERRORS IN MISALIGNMENT ANGLES LEAST SQUARES METHOD DEGREES (NORM) 100 ARC SEC	200 ARC SEC	ERRORS IN MISALIGNMENT ANGLES APOLLO DETERMINISTIC METHOD DEGREES (NORM) 100 ARC SEC	200 ARC SEC
α - UMI i - [illegible] β - HER γ - PAV	360 900 1080	4	0.0562	0.1178	0.0735	0.1128
a - UMI i - CEP β - HER γ - PAV α - CRU	360 900 1080 720	5	0.0369	0.07387	0.05657	0.1149
x - UMI i - CEP β - HER γ - PAV α - CRU α - ANT	360 900 1080 720 540	6	0.0543	0.1086	0.0714	0.1518
i - CEP β - HER γ - PAV α - CRU α - ANT α - LEO	900 1080 720 540 660	6	0.0359	0.07201	0.1022	0.1653
β - HER γ - PAV α - CRU α - ANT α - LEO i - CEP	1080 720 540 660 1440	6	0.02992	0.0598	0.0671	0.1008

TABLE IV
COMPARISON OF LEAST SQUARES AND APOLLO DETERMINISTIC METHODS
WITH RANDOM STAR TRACKER ERRORS of 100 AND 200 ARC SECONDS
(CONT)

STARS USED	Δt SECONDS	n	ERRORS IN MISALIGNMENT ANGLES LEAST SQUARES METHOD DEGREES (NORM) 100 ARC SEC	200 ARC SEC	ERRORS IN MISALIGNMENT ANGLES APOLLO DETERMINISTIC METHOD DEGREES (NORM) 100 ARC SEC	200 ARC SEC
γ - PAV α - CRU α - ANT α - LEO i - CEP β - HER	 720 540 660 1440 960	6	0.02117	0.04238	.0329	0.03630
λ - CRU α - ANT α - LEO i - CEP β - HER γ - PAV	 540 660 1440 960 1080	6	.0439	0.08811	0.01664	0.04978
λ - ANT α - LEO i - CEP β - HER α - PAV α - ANT	 660 1440 960 1080 1200	6	0.0336	0.0352	0.04124	0.08146
α - LEO i - CEP β - HER γ - PAV α - ANT α - LEO	 1440 960 1080 1200 720	6	0.0134	0.0269	0.0492	0.0557
i - CEP β - HER γ - PAV α - ANT α - LEO i - CEP	 960 1080 1200 720 1440	6	0.03229	0.06469	.0671	0.1009