Ray-Trace Analysis of Glancing-Incidence
X-Ray Optical Systems

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FINAL REPORT

by

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I. INTRODUCTION

This report presents the results of a ray-trace analysis of several glancing-incidence x-ray optical systems. The present study was carried out under Contract No. NAS8-32115 by personnel of Montevallo Research Associates, Montevallo, Alabama during the period June 7, 1976 to August 13, 1976. Participating in the study were Dr. J. William Foreman, Jr. and Mr. Joseph M. Cardone. Only Mr. Cardone's time was supported by the present contract. Dr. Foreman was supported separately under the NASA/ASEE Summer Faculty Fellowship Program at MSFC. Computer time on the Univac 1108 at MSFC was also furnished separately to Dr. Foreman.

The object of the present study was threefold. First, following up on earlier work[1], the vignetting characteristics of the S-056 x-ray telescope were calculated using experimental data to determine mirror reflectivities. Second, a small Wolter Type I x-ray telescope intended for possible use in the GOES (Geostationary Operational Environmental Satellite) program was designed and ray traced. Finally, a ray-trace program was developed for a Wolter-Schwarzschild x-ray telescope[2] which was designed by members of Dr. A. B. C. Walker's group at Stanford University.

II. VIGNETTING CHARACTERISTICS OF THE S-056 X-RAY TELESCOPE

The S-056 solar x-ray telescope, a paraboloidal-hyperboloidal Wolter Type I instrument designed by Mangus and Underwood[3], was used as part of the Skylab instrument package to obtain numerous photographs of the sun at various soft x-ray wavelengths. In order to interpret the resulting photographic data, it is necessary to know how the energy throughput of the system varies as a function of off-axis viewing angle at various x-ray wavelengths. The reduction in energy throughput with off-axis angle (commonly referred to as vignetting) occurs for two distinct reasons: (1) Rays entering the annular aperture of the telescope begin to miss either the paraboloidal or the hyperboloidal mirror and are intercepted by the second stop, which is designed to intercept all rays which do not strike both mirrors. The second stop is also designed to limit the field of view of the telescope to approximately ± 20 arc-minutes; (2) Owing to variation of the glancing angles of incidence at both mirrors with off-axis angle, there is usually a small decrease in the overall reflection efficiency of the telescope with off-axis angle.

In a previous study[1], the vignetting characteristics of the S-056 x-ray telescope were calculated using theoretical mirror reflectivity data. However, theoretical reflectivity data invariably predict higher energy throughput than experimental reflectivity data. It was therefore decided to repeat the vignetting calculations using experimental reflectivity data. Since the S-056 mirrors were made from fused silica, it would be desirable to have experimental reflectivity data over a wide range of wavelengths for fused silica. However, there is no such data of which we are aware. Consequently, it was decided in consultation with cognizant NASA-MSFC personnel to use the data for Pyrex glass given by Stewardson and Underwood[4] for wavelengths less than 8.34 Å and to use the data for F-1 glass of Ershov, Brytov, and Lukirskii[5] for wavelengths of 8.34 Å and larger.

The reflectivity for a ray which strikes an x-ray telescope mirror at a glancing angle θ is given by the Fresnel equation,[6]

$$R = \frac{\left[(2)^{1/2} x - (A + x^2 - 1)^{1/2}\right]^2 + A - (x^2 - 1)}{\left[(2)^{1/2} x + (A + x^2 - 1)^{1/2}\right]^2 + A - (x^2 - 1)} , \qquad (1)$$

where

$$A^2 = (x^2 - 1)^2 + y^2 \qquad (2)$$

$$y = \frac{\mu_\ell \lambda}{4\pi\delta} \qquad (3)$$

and

$$x = \frac{\theta}{\theta_c} . \qquad (4)$$

In Eq. (4), θ_c is the critical angle for total external reflection. The critical angle θ_c is given by the relation

$$\theta_c = (2\delta)^{1/2}, \tag{5}$$

where $\delta = 1 - n$ and n is the real part of the index of refraction of the mirror material. For soft x-rays, n is slightly less than unity for all common materials, so that δ is a small positive quantity (typically on the order of 10^{-4}). In Eq. (3), μ_ℓ is the linear absorption coefficient of the mirror material and λ is the wavelength of the incident x-rays.

Ershov, Brytov and Lukirskii giv alues of μ_ℓ and δ for F-1 glass for wavelengths from 8.34 Å to 31.36 Å. These values can be substituted directly into Eqs. (1) through (5) to get the reflectivity R for each given wavelength. Stewardson and Underwood, on the other hand, give data for Pyrex glass at several wavelengths less than 8.34 Å, and they state the experimental values of the parameters θ_c and y rather than μ_ℓ and δ. By use of Eqs. (3) and (4), one can easily convert the values of θ_c and y into the corresponding values of μ_ℓ and δ:

$$\mu_\ell = \frac{4\pi y \delta}{\lambda} \tag{6}$$

$$\delta = \frac{\theta_c^2}{2}, \tag{7}$$

where θ_c in Eq. (7) is to be expressed in radians. A summary of the values of μ_ℓ and δ used at each wavelength is given in Table I.

The relative energy contribution from each ray which reaches the focal plane of the telescope is $R(\theta_p) \cdot R(\theta_H)$, where R is the Fresnel reflectivity function defined in Eq. (1) and θ_p and θ_H are the glancing angles of incidence at the paraboloidal and hyperboloidal mirrors, respectively. The total relative energy reaching the focal plane at a given wavelength and at a given off-axis angle is the sum of the products $R(\theta_p) \cdot R(\theta_H)$ for all rays which reach the focal plane.

The final results of the vignetting computer runs are summarized in Table II and plotted in Fig. 1. It will be observed from Fig. 1 that vignetting is roughly a linear function of off-axis angle out to approximately 20 arc-minutes. Beyond this off-axis angle, the second stop begins to come into play, and vignetting becomes more severe with increasing off-axis angle.

At each off-axis angle, 36,360 rays were entered in the telescope aperture. For a variety of reasons, not all of these rays actually reached the focal plane.[9] The number of rays actually reaching the focal plane at each off-axis angle is summarized in Table III. Since each ray entering the aperture is assumed to have an equal relative energy weight of unity, the data in Table III can be used to calculate the telescope efficiency for any given case. For example, using

Tables II and III one finds that the telescope efficiency for an off-axis angle of 5.0 arc-minutes and a wavelength of 31.36 Å is $\eta = 17,824.6/24,859 = 0.717 = 71.7\%$. All efficiencies estimated in this way are probably somewhat too high, since the ray-trace results obtained here do not take account of x-ray scattering and absorption due to mirror surface roughness, imperfect mirror figuring, possible surface contamination, etc.

Table I. Values of μ_l and δ Used at the Various X-Ray Wavelengths as Parameters in the Fresnel Reflectivity Equation

λ (Å)	δ	μ_l (cm^{-1})
6.16	9.95×10^{-5}	2.8×10^{3}
6.62	1.17×10^{-4}	3.3×10^{3}
6.86	1.18×10^{-4}	1.3×10^{3}
8.34	2.60×10^{-4}	5.4×10^{3}
9.89	3.70×10^{-4}	7.0×10^{3}
12.25	5.30×10^{-4}	1.20×10^{4}
13.34	6.30×10^{-4}	1.40×10^{4}
14.56	7.30×10^{-4}	1.60×10^{4}
15.97	8.70×10^{-4}	2.20×10^{4}
17.59	9.90×10^{-4}	2.40×10^{4}
19.45	11.50×10^{-4}	3.10×10^{4}
21.84	13.60×10^{-4}	3.80×10^{4}
23.6[illegible]	11.50×10^{-4}	2.70×10^{4}
24.7[illegible]	14.50×10^{-4}	2.50×10^{4}
27.[illegible]	18.70×10^{-4}	3.30×10^{4}
31.36	24.20×10^{-4}	3.70×10^{4}

Table II. Results of S-056 Vignetting Computer Runs

Off-Axis Angle (arc-minutes)	Wavelength (Å)	Relative Energy in Spot
0.0	6.16	379.0
	6.62	1434.9
	6.86	2086.8
	8.34	15535.1
	9.89	17335.5
	12.25	16404.2
	13.34	16784.6
	14.56	16956.9
	15.97	16115.5
	17.59	16333.1
	19.45	15618.1
	21.84	15405.5
	23.62	15244.9
	24.78	17920.2
	27.42	18142.1
	31.36	19101.1
5.0	6.16	392.8
	6.62	1332.5
	6.86	2089.9
	8.34	14414.3
	9.89	16147.4
	12.25	15293.5
	13.34	15652.8
	14.56	15816.1
	15.97	15033.2
	17.59	15237.6
	19.45	14571.6
	21.84	14374.5
	23.62	14223.5

Table II. (Continued)

Off-Axis Angle (arc-minutes)	Wavelength (Å)	Relative Energy in Spot
5.0	24.78	16721.1
	27.42	16929.1
	31.36	17824.6
10.0	6.16	428.9
	6.62	1121.7
	6.86	1663.3
	8.34	13263.0
	9.89	15051.2
	12.25	14294.6
	13.34	14644.2
	14.56	14804.8
	15.97	14076.5
	17.59	14272.1
	19.45	13651.1
	21.84	13469.5
	23.62	13324.6
	24.78	15671.2
	27.42	15869.2
	31.36	16711.1
16.0	6.16	383.5
	6.62	867.1
	6.86	1225.1
	8.34	11649.5
	9.89	13674.1
	12.25	13067.5
	13.34	13413.5
	14.56	13575.7
	15.97	12916.4
	17.59	13103.7
	19.45	12538.4

Table II. (Continued)

Off-Axis Angle (arc-minutes)	Wavelength (Å)	Relative Energy in Spot
16.0	21.84	12377.3
	23.62	12237.9
	24.78	14406.5
	27.42	14594.4
	31.36	15373.5
20.0	6.16	329.2
	6.62	720.3
	6.86	1002.6
	8.34	10126.9
	9.89	12332.0
	12.25	11851.1
	13.34	12184.8
	14.56	12343.2
	15.97	11749.8
	17.59	11925.9
	19.45	11414.9
	21.84	11272.2
	23.62	11140.8
	24.78	13125.0
	27.42	13300.2
	31.36	14013.8
25.0	6.16	218.3
	6.62	500.7
	6.86	708.3
	8.34	6784.7
	9.89	8112.0
	12.25	7772.6
	13.34	7984.6
	14.56	8084.6

Table II. (Concluded)

Off-Axis Angle (arc-minutes)	Wavelength (Å)	Relative Energy in Spot
25.0	15.97	7693.5
	17.59	7806.8
	19.45	7470.9
	21.84	7376.0
	23.62	7291.5
	24.78	8587.6
	27.42	8700.8
	31.36	9166.7

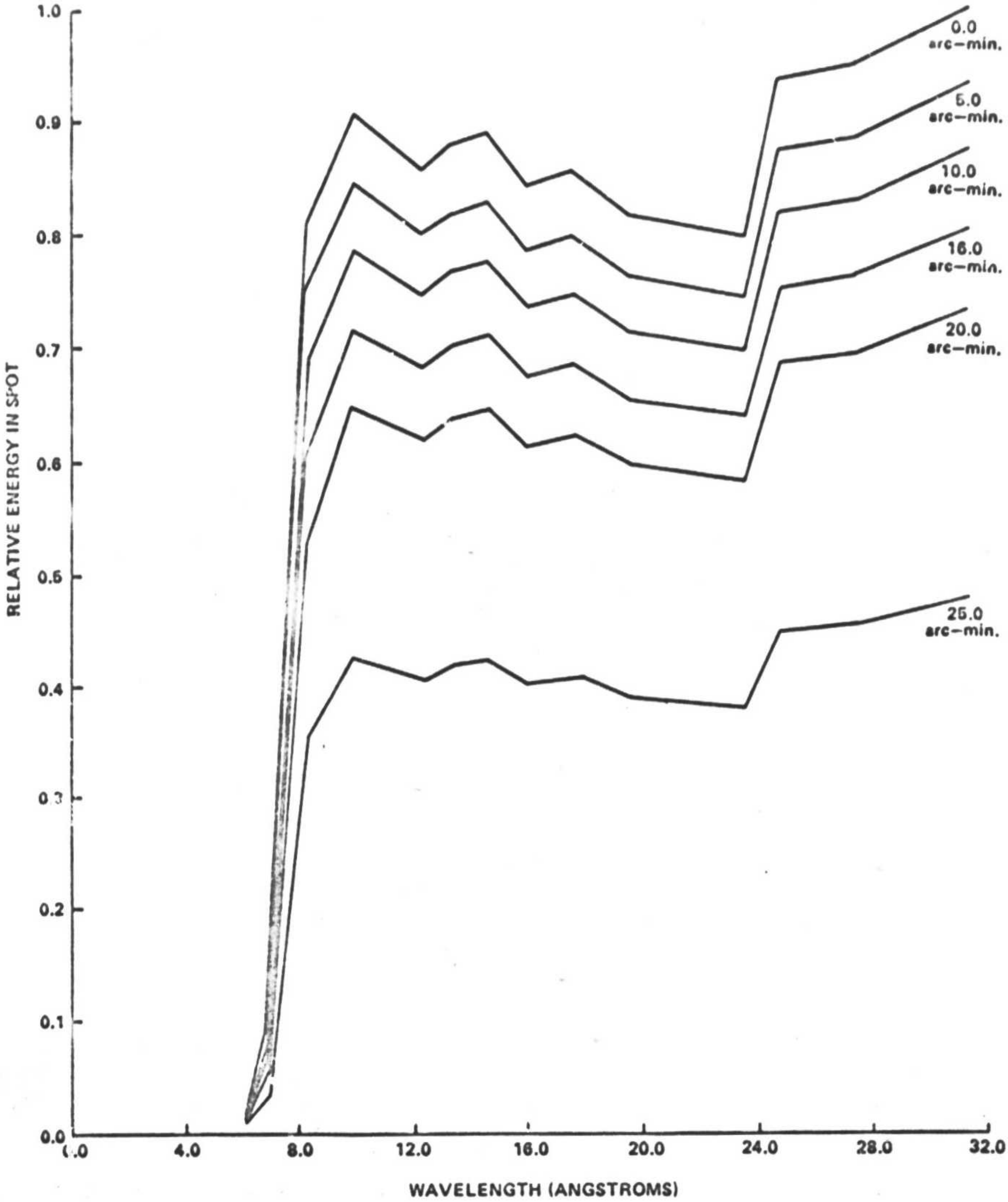

FIGURE 1. NORMALIZED RELATIVE ENERGY IN SPOT AS A FUNCTION OF WAVELENGTH FOR VARIOUS OFF-AXIS ANGLES.

Table III. Number of Rays Actually Reaching the Focal Plane at Each Off-Axis Angle

Off-Axis Angle (arc-minutes)	No. of Rays Reaching Focal Plane
0.0	26,640
5.0	24,859
10.0	23,311
16.0	21,457
20.0	19,569
25.0	12,800

III. DESIGN OF A WOLTER TYPE I X-RAY TELESCOPE FOR THE GOES PROGRAM

It is tentatively planned to place an x-ray imaging system aboard a satellite in the GOES program to observe x-ray activity on the sun and correlate such activity with global weather patterns and communications disturbances. The following data were furnished as basic design inputs for the proposed GOES x-ray imaging system:

(1) The imaging system will be a Wolter Type I paraboloidal-hyperboloidal x-ray telescope.

(2) The maximum glancing angle of incidence on the paraboloidal mirror will be 83 arc-minutes (this corresponds to a short wavelength cutoff of approximately 6.15 Å for a gold-coated mirror).

(3) The entire telescope assembly must fit in a package approximately 30 inches long.

Using these inputs, we chose the maximum glancing angle of incidence on the paraboloid, θ_{max}, to be 83 arc-minutes and the baseline focal length of the system, $f_{baseline}$, to be 25.0 inches. We also chose to make the length of the paraboloidal mirror, L_P, equal to 2.0 inches, aiming toward a paraboloidal collecting area of roughly 4.0 cm^2.

The basic design equations for a Wolter Type I x-ray telescope, rewritten slightly from the forms given in Reference 3, are:

Equation of paraboloidal mirror: $$\rho^2 = p\,(2z + p) \tag{8}$$

Equation of hyperboloidal mirror: $$\frac{(z-c)^2}{a^2} - \frac{\rho^2}{b^2} = 1. \tag{9}$$

$$\left[\begin{array}{c} \rho^2 = x^2 + y^2 \\ a^2 + b^2 = c^2 \end{array}\right]$$

$$\rho_{PMIN} = f_{BASELINE} \tan(4\theta_{MAX}) \tag{10}$$

$$p = \rho_{PMIN} \tan(\theta_{MAX}) \tag{11}$$

$$z_{PMIN} = (\rho^2_{PMIN} - p^2)/2p \tag{12}$$

$$z_{PMAX} = z_{PMIN} + L_P \tag{13}$$

$$\rho_{PMAX} = \left[p\,(2z_{PMAX} + p) \right]^{1/2} \tag{14}$$

$$c = f_{BASELINE} / 2\cos(4\theta_{MAX}) \tag{15}$$

$$a = c\left[2\cos(2\theta_{MAX}) - 1\right] \tag{16}$$

$$b = (c^2 - a^2)^{1/2} \tag{17}$$

Collecting area of paraboloid = $A_P = \pi\left(\rho^2_{PMAX} - \rho^2_{PMIN}\right)$ (18)

Collecting area of hyperboloid = $A_H = \pi\left(\rho^2_{PMIN} - \rho^2_{HMIN}\right)$ (19)

$$z_{FOCAL\ PLANE} = 2c = z_{PMIN} - f_{BASELINE} . \quad (20)$$

The geometrical meanings of the various terms are summarized in Fig. 2. Use of the input parameters θ_{max} = 83 arc-minutes, $f_{baseline}$ = 25.0 inches, and L_P = 2.0 inches in Eqs. (10) through (20) produces the following results:

p = 0.0584851950 in.

a = 12.5292425975 in.

b = 0.8570224335 in.

c = 12.5585193203 in.

ρ_{pmin} = 2.4219062894 in.

Z_{pmin} = 50.1170386406 in.

Z_{pmax} = 52.1170386406 in.

ρ_{pmax} = 2.4697309276 in.

Z_{focus} = 25.1170386406 in.

A_P = 0.735 in^2 = 4.74 cm^2

A_H = 1.99 in^2 = 12.84 cm^2.

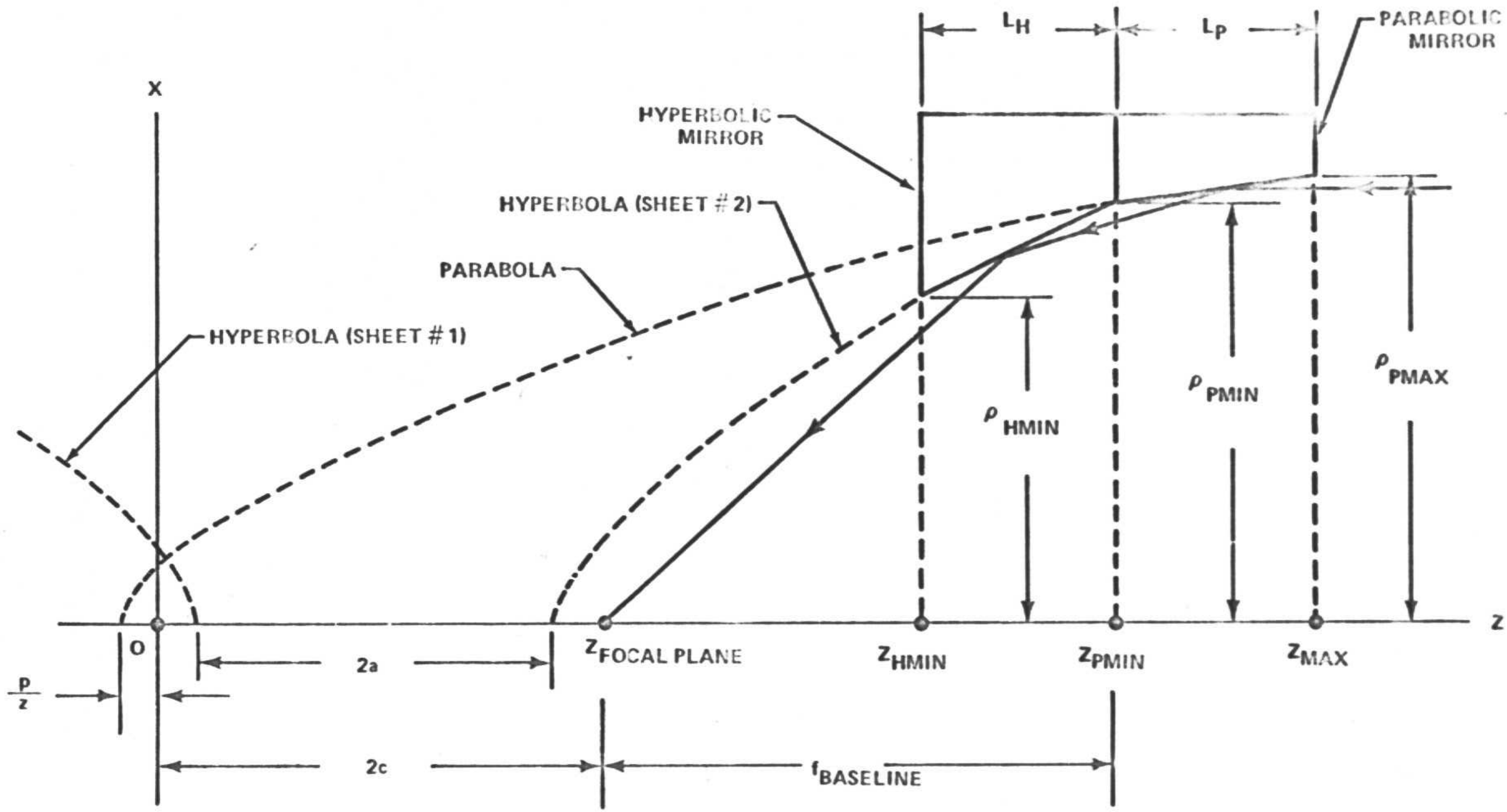

Figure 2. Geometrical parameters for a Wolter Type I x-ray telescope.

Usually the coordinates (ρ_{hmin}, Z_{hmin}) of the rearmost point on the hyperboloidal mirror in the XZ-plane are determined by requiring that the hyperboloid be just long enough to catch all on-axis rays incident on the paraboloidal mirror. If this is the case, then an incident ray parallel to the optical axis and striking the paraboloid at (ρ_{pmax}, Z_{pmax}) will, after reflection from the paraboloid, strike the hyperboloid at (ρ_{hmin}, Z_{hmin}). A straightforward application of analytic geometry to this situation gives the result

$$z_{HMIN} = \frac{1}{2\alpha}\left[-\beta + \left(\beta^2 - 4\alpha\gamma\right)^{1/2}\right], \qquad (21)$$

where

$$\alpha = b^2 - a^2 \tan^2 2\theta \qquad (22)$$

$$\beta = -2b^2c \qquad (23)$$

$$\gamma = b^4 \qquad (24)$$

$$\theta = \tan^{-1}\left(p/\rho_{PMAX}\right). \qquad (25)$$

The value of ρ_{hmin} can then be determined from the relation

$$\rho_{HMIN} = z_{HMIN} \tan 2\theta. \qquad (26)$$

The values of Z_{hmin} and ρ_{hmin} found for the GOES telescope from Eqs. (21) and (26) are

$$Z_{hmin} = 48.2690250035 \text{ in.}$$

$$\rho_{hmin} = 2.2873806150 \text{ in.}$$

The length of the hyperboloidal mirror, L_H, is thus

$$L_H = Z_{pmin} - Z_{hmin} = 1.848 \text{ in.}$$

The results of a ray-trace program to compute the rms spot radius in the focal plane as a function of off-axis angle are summarized in Table IV and Fig. 3. The effective focal length of the telescope is 25.18 inches, and the plate scale in the focal plane is 3.10 microns/arc-second.

As a check on our ray-trace results, we have used the empirical formula developed by VanSpeybroeck and Chase[7] for the rms spot radius in the focal plane as a function of off-axis angle. The empirical results are plotted in Fig. 3 for comparison with our exact ray-trace results. There is reasonably good agreement between the two.

We have also determined the field curvature at the paraboloid-hyperboloid focus, using a ray-trace program which automatically locates the plane of best focus for any given off-axis angle. The plane of best focus is defined to be the plane in which the rms spot

Table IV. Root-Mean-Square Spot Radius in the Focal Plane of the GOES Telescope as a Function of Off-Axis Angle

Off-Axis Angle (arc-minutes)	RMS Spot Radius (arc-seconds)
0.0	0.00
2.0	0.32
4.0	0.79
6.0	1.49
8.0	2.43
10.0	3.62
12.0	5.05
14.0	6.67
16.0	8.54
18.0	10.73
20.0	13.06

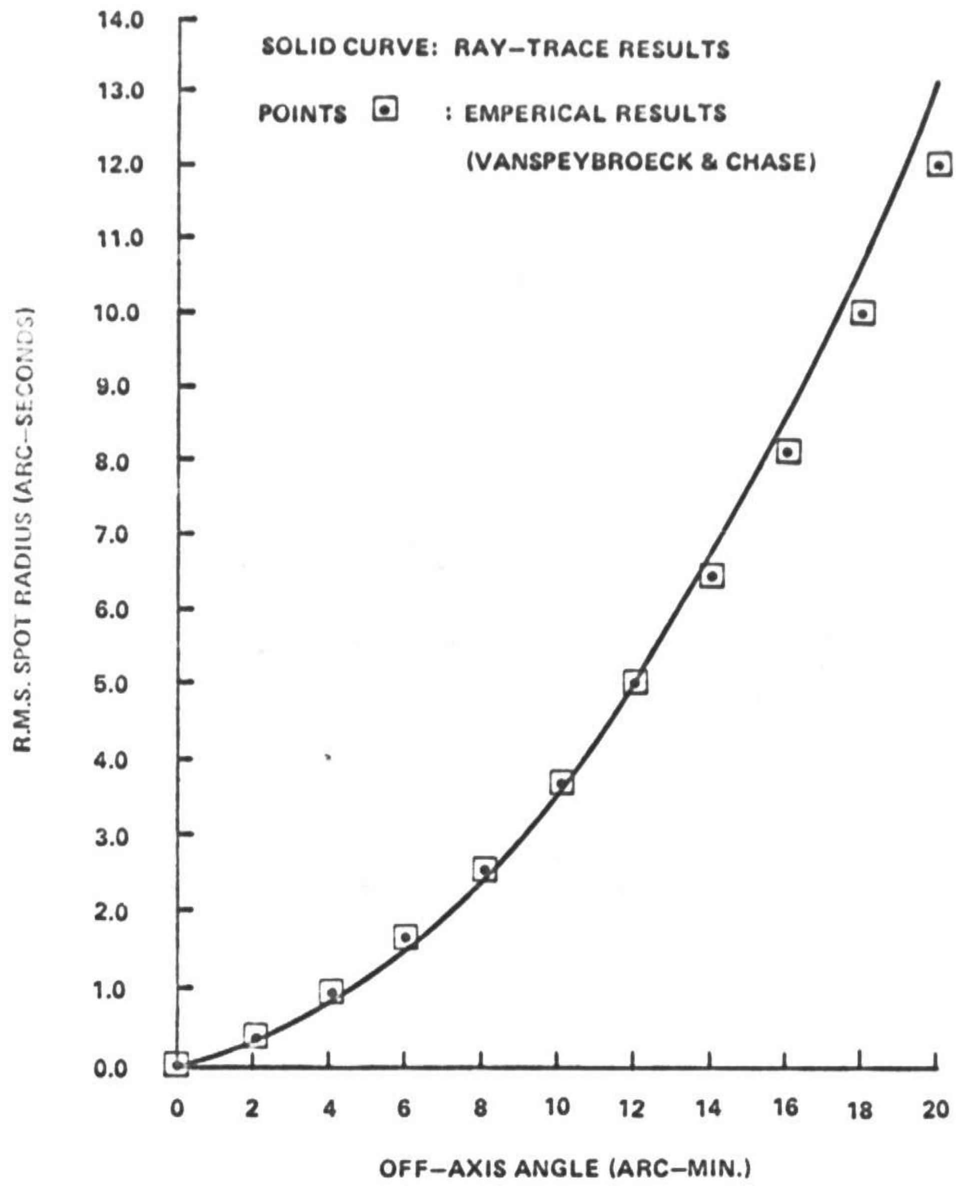

Figure 3. Root-mean-square spot radius in the flat focal plane of the GOES x-ray telescope as a function of off-axis angle.

radius is a minimum. Table V and Fig. 4 summarize the field curvature results. The empirical formula[8] for $Z_{best\ focus} - Z_{focal\ plane}$ given by VanSpeybroeck and Chase has also been used to calculate the field curvature as a check on our ray-trace results. The empirical results are plotted in Fig. 4 and show good agreement with the ray-trace results.

The rms spot radius in the surface of best focus as a function of off-axis angle is given in Table VI. Comparison of the results in Table VI with those in Table IV shows that the spot size in the surface of best focus is considerably smaller than the spot size in the flat focal plane at any fixed off-axis angle, as one would expect. The two spot sizes are compared graphically in Fig. 5.

Table V. Field Curvature Results at the Paraboloid-Hyperboloid Focus of the GOES Telescope

Off-Axis Angle (arc-minutes)	$Z_{\text{Best Focus}} - Z_{\text{Focal Plane}}$ (millimeters)
0.0	0.0000
2.0	0.0038
4.0	0.0152
6.0	0.0341
8.0	0.0602
10.0	0.0939
12.0	0.1347
14.0	0.1810
16.0	0.2346
18.0	0.2988
20.0	0.3662

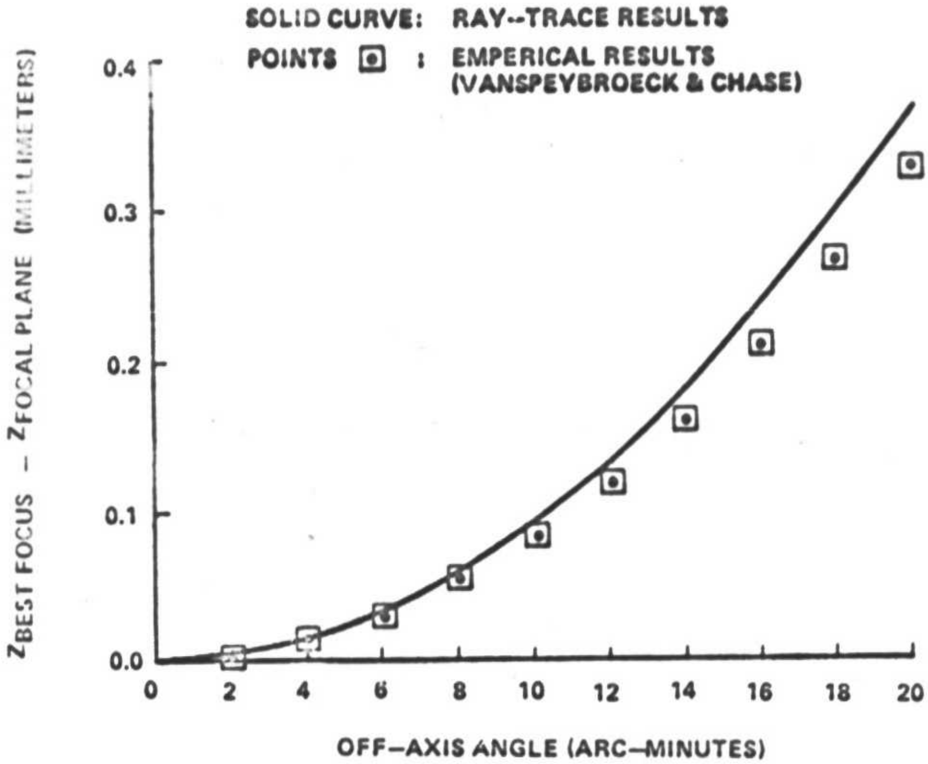

FIGURE 4. CURVATURE OF FIELD DATA AT THE PARABOLOID–HYPERBOLOID FOCUS.

Table VI. Root-Mean-Square Spot Radius in the Surface of Best Focus of the GOES Telescope as a Function of Off-Axis Angle

Off-Axis Angle (arc-minutes)	RMS Spot Radius (arc-seconds)
0.0	0.00
2.0	0.29
4.0	0.63
6.0	1.03
8.0	1.51
10.0	2.08
12.0	2.73
14.0	3.45
16.0	4.28
18.0	5.15
20.0	6.15

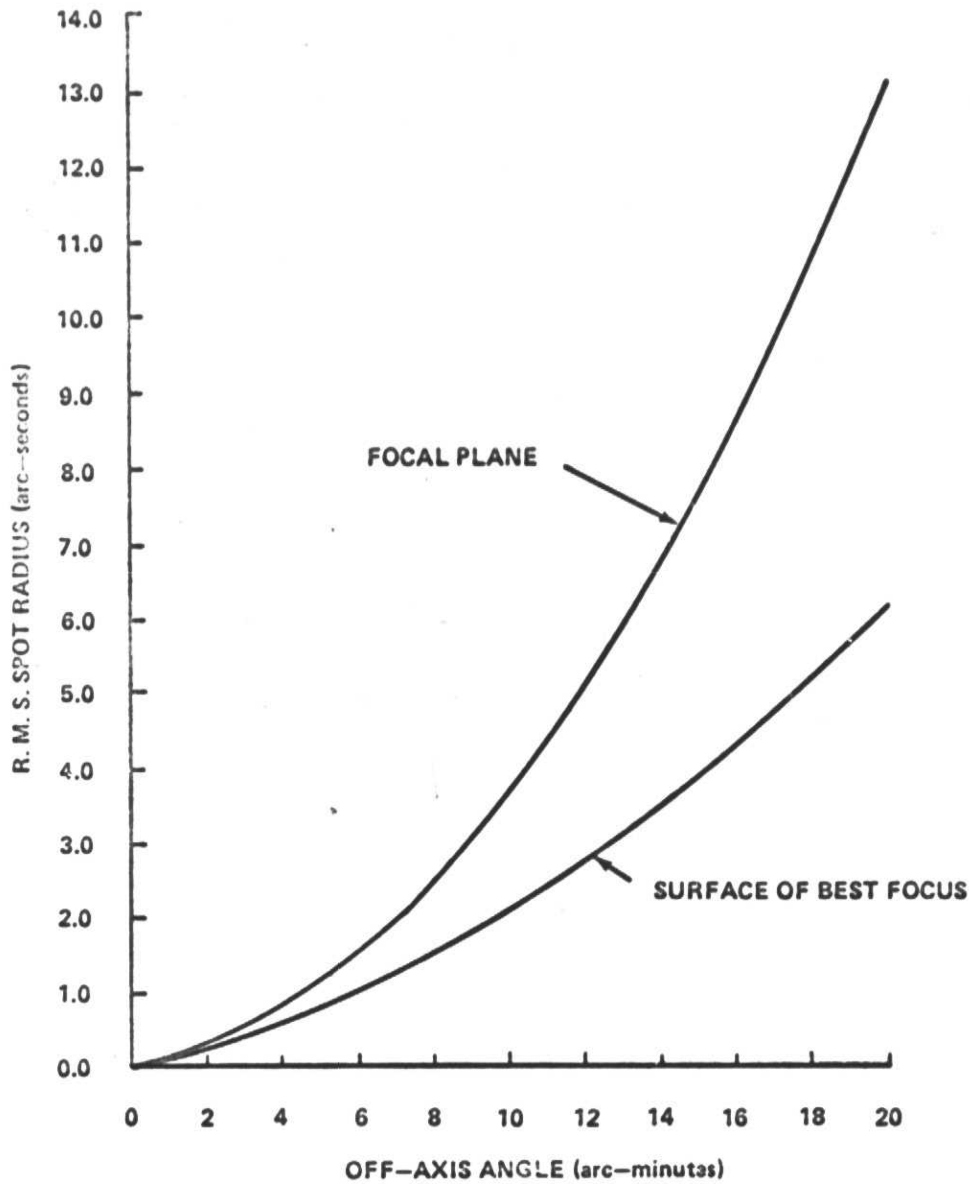

FIGURE 5. COMPARISON OF THE SPOT SIZES IN THE FLAT FOCAL PLANE AND IN THE SURFACE OF BEST FOCUS FOR THE GOES TELESCOPE.

IV. RAY-TRACE ANALYSIS OF A WOLTER-SCHWARZSCHILD X-RAY TELESCOPE

The Wolter Type I paraboloid-hyperboloid x-ray telescope, which has found such widespread use in x-ray astronomy up to the present time, is fully corrected for spherical aberration but only approximately corrected for coma. It is possible to design an aspheric-aspheric x-ray telescope which is completely corrected for both spherical aberration and coma. An aplanatic x-ray telescope of this type is called a Wolter-Schwarzschild x-ray telescope[2]. A nested array of three Wolter-Schwarzschild (W-S) telescopes has been designed by members of Dr. A. B. C. Walker's group at Stanford University. Our task was to develop a ray-trace program for this system.

Physically, a W-S x-ray telescope resembles a standard Wolter Type I telescope. However, in the W-S telescope the two mirror surfaces are general aspherics, rather than conic sections as in the Wolter Type I telescope. The equations for the mirror surfaces in the W-S telescope are expressed parametrically in terms of an angle $\hat{\beta}$:

MIRROR # 1: (Analog of the paraboloid in the Wolter Type I system)

$$x_1 = f \sin\beta \tag{27}$$

$$z_1 = -Q_1 + \frac{f^2 \sin^2\beta}{4Q_1} + \frac{f \cos^4(\beta/2)}{k^{1-k}} \left[\tan^2(\beta/2) - k\right]^{1-k}, \tag{28}$$

where

$$k = \tan^2(\beta^*/2) \tag{29}$$

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$$Q_1 = f \sin^2(\beta^*/2) \tag{30}$$

$$f = \frac{Z_0}{\cos\beta^*} \; . \tag{31}$$

The parameters Z_o, β , β^*, and f are defined in Fig. 6. It will be observed from Fig. 6 that Z_o is the baseline focal length of the system, and that

$$\beta^* = \tan^{-1}\left(\frac{X_0}{Z_0}\right) \tag{32}$$

where X_o is the radius at the intersection of the two mirrors.

MIRROR # 2: (Analog of the hyperboloid in the Wolter Type I system)

$$x_2 = d \sin\beta \tag{33}$$

$$z_2 = d \cos\beta \; , \tag{34}$$

where

$$\frac{1}{d} = \frac{1}{f}\left\{ \frac{2\sin^2(\beta/2)}{1-\cos\beta^*} + \frac{\cos^2(\beta/2)}{k^{1+k}}\left[\tan^2(\beta/2) - k\right]^{1+k} \right\} . \tag{35}$$

For the outermost set of W-S mirrors in the Stanford telescope, Z_o = 50.0 inches and X_o = 7.0 inches.

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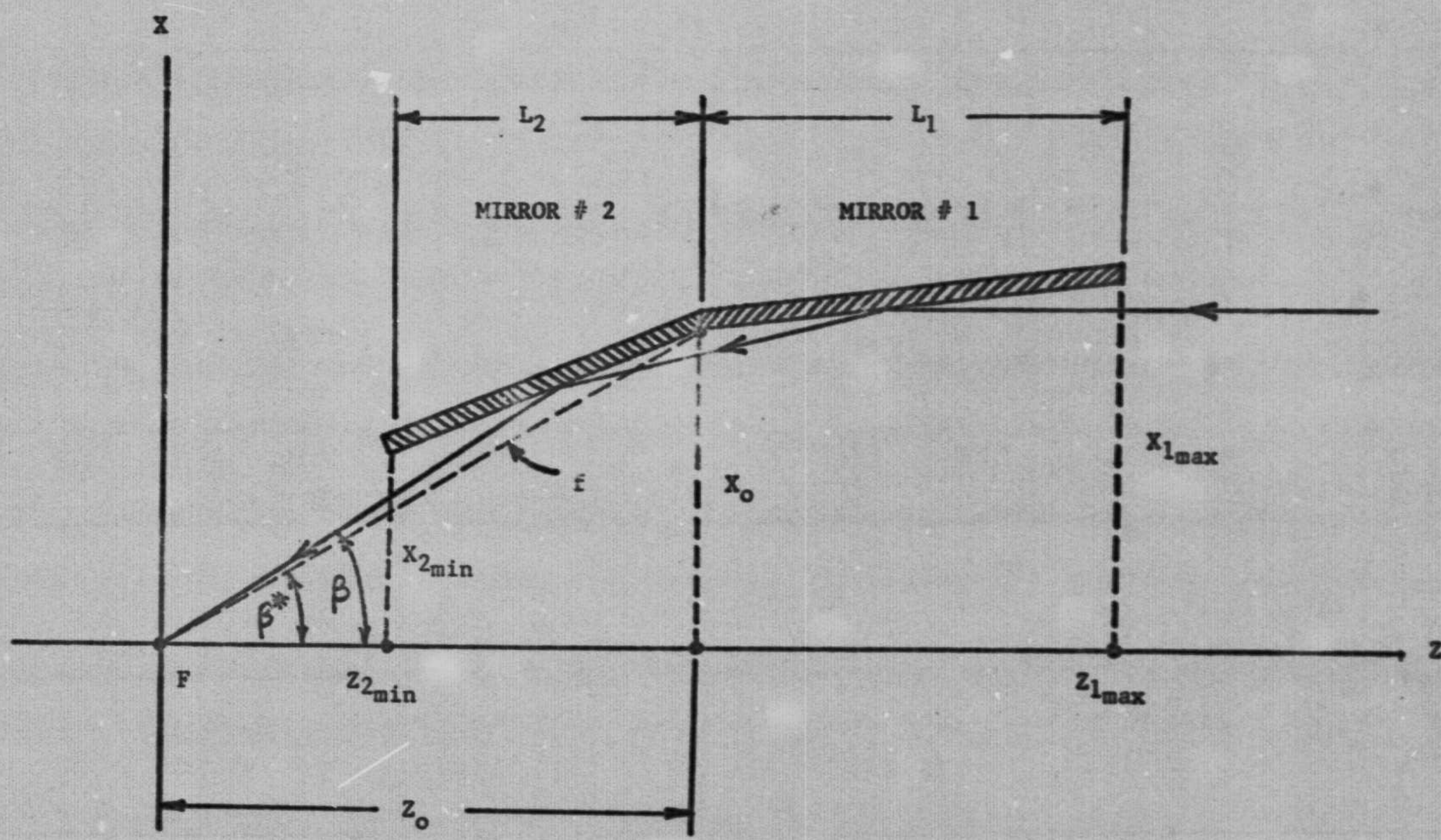

Figure 6. Definition of the basic parameters for a Wolter-Schwarzschild x-ray telescope.

The length L_1 of Mirror # 1 (see Fig. 6) was specified by the Stanford group to be 10.5 inches, making $z_{1_{max}}$ = 60.5 inches. The first job was to determine the corresponding value of β_{max}. For this purpose, an iterative computer program was set up to find the value of β corresponding to the value $z_1 = z_{1_{max}}$ = 60.5 inches. The result was

$$\beta_{max} = 8.3753315183^{o}.$$

Incidentally, the minimum value of β is

$$\beta^{*} = 7.9696103933^{o}.$$

Putting $\beta = \beta_{max}$ in Eqs. (27), (33), and (34) gives

$$x_{1_{max}} = 7.35388002 \text{ inches}$$

$$x_{2_{min}} = 6.09112390 \text{ inches}$$

$$z_{2_{min}} = 41.37225999 \text{ inches}.$$

The length of Mirror # 2 is thus

$$L_2 = z_o - z_{2_{min}} = 8.62774001 \text{ inches}.$$

In order to ray trace the outermost set of W-S mirrors, we attempted to fit Mirrors # 1 and # 2 with cubic spline functions[10] in the usual way. However, it turns out that Mirror # 1 has a horizontal tangent at the point of intersection with Mirror # 2, and this causes a singularity in the spline function fit at that point. We then had to back up and attempt a spline function fit which leaves out an infinitesimal region of both mirrors near their intersection point. Owing to extremely long turn-around times on the Univac 1108, we were just able to get a ray-trace program based on this limited spline function fit completed before our time ran out on the present contract. The results of this ray-trace program are summarized in Table VII. More work is needed to get a better spline function fit for the mirrors (especially Mirror # 1) and to double check the results in Table VII. It is also necessary, of course, to complete the ray trace for the other two sets of nested mirrors. Unfortunately, we were not able to complete this work under the present contract.

Table VII. Root-Mean-Square Spot Radius in the Focal Plane of the Wolter-Schwarzschild X-Ray Telescope (Outer Mirror Set)

Off-Axis Angle (arc-minutes)	RMS Spot Radius (arc-seconds)
0.0	0.033
2.0	0.311
4.0	1.075
6.0	2.309
8.0	4.013
10.0	6.221
12.0	8.829
14.0	11.861
16.0	15.421
18.0	19.305
20.0	23.852

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6. The Fresnel equation is written here in the form given by R. W. Hendrick, Spectral Reflectance of Solids for Aluminum K Radiation, Journal of the Optical Society of America 47, 165 (1957). Strictly speaking, one would expect two Fresnel equations to apply, one for radiation polarized perpendicular to the plane of incidence and the other for radiation polarized parallel to the plane of incidence. However, for the small glancing angles of incidence used here, the two equations give virtually identical results, so that for practical purposes the incident radiation may be considered to be unpolarized, with Eq. (1) giving the reflectivity for unpolarized incident radiation.

7. L. P. VanSpeybroeck and R. C. Chase, Design Parameters of Paraboloid-Hyperboloid Telescopes for X-Ray Astronomy, Applied Optics 11, 440 (1972), Eq. (2). The first term in this equation must be doubled to give the

rms spot radius in a flat focal plane. The equation as it stands refers to the rms spot radius in the curved surface of best focus, as the authors later mention.

8. Reference 7, Eq. (6).

9. Reference 1, Section II.

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