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SPACE TELESCOPE COORDINATE SYSTEMS, SYMBOLS, AND NOMENCLATURE DEFINITIONS

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16. ABSTRACT

The major coordinate systems as well as the transformations and transformation angles between them, for the Space Telescope are defined in this report. The coordinate systems were primarily developed for use in pointing and control system analysis and simulation. Additional useful information (on nomenclature, symbols, quaternion operations, etc.) is contained in the appendices.

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TECHNICAL MEMORANDUM X-73343

SPACE TELESCOPE COORDINATE SYSTEMS, SYMBOLS, AND NOMENCLATURE DEFINITIONS

I. INTRODUCTION

The purpose of this document is to define and label Space Telescope (ST) reference and vehicle coordinate systems and to define nomenclature and symbols to establish a common terminology for all organizations concerned. Since the terminology was developed primarily for use in pointing and control system analysis and simulation, some necessary coordinate system definitions and other nomenclature may be missing; therefore, future revisions may be necessary.[1] This document is an extensive revision (and expansion) of Memo S& E-ASTR-SG/25-74, dated April 4, 1974, entitled "LST Symbols and Coordinate Systems Definition, Revision I."

II. COORDINATE SYSTEM DEFINITIONS

To minimize the number of subscripts, a single uppercase English letter is used whenever possible to designate a coordinate system (for example, V for the vehicle control axes system). The individual x-, y-, z-axes are then identified by numbers (as in V1, V2, V3) on the same level to avoid double subscripts (as in ω_{V2}). All coordinate systems are orthogonal and right-handed, therefore, only two of the three axes will be defined in the following. The axes of the various coordinate systems are labeled such that, for the case of all transformation angles zero, all 1-axes are parallel to each other, as well as all 2-axes and all 3-axes. Only rotational transformations will be treated, and the differences in origin location will be disregarded as immaterial to rotational dynamics. Any coordinate system which is not specifically labeled "inertial" is a rotating coordinate system. The definition of the angles for coordinate transformation are given later.

1. Any comments, questions, or contributions concerning this report are welcome (call 205-453-4718 or FTS 872-4718).

A. Vehicle Control Axes Coordinate System V (V1, V2, V3) (Fig. 1)

The origin is 6.096 m (240 in.) behind a plane through the Optical Telescope Assembly (OTA)/Support Systems Module (SSM) interface (see OTA/SSM Interface Requirements Document for a definition) on the ST centerline (optical axis). The ST centerline is identical to V1, i.e. V1 is perpendicular to the OTA primary mirror at its vertex (i.e., V1 is the ideal optical axis) and it is positive toward the OTA sunshade. The V3 axis is perpendicular to the solar array gimbal axes and it is positive along the nominal Sun direction (solar arrays in the V1, V2 plane, the Sun perpendicular to the active side of the solar arrays). The V2 axis is parallel to the solar array gimbal axes and directed to form a right-handed coordinate system.

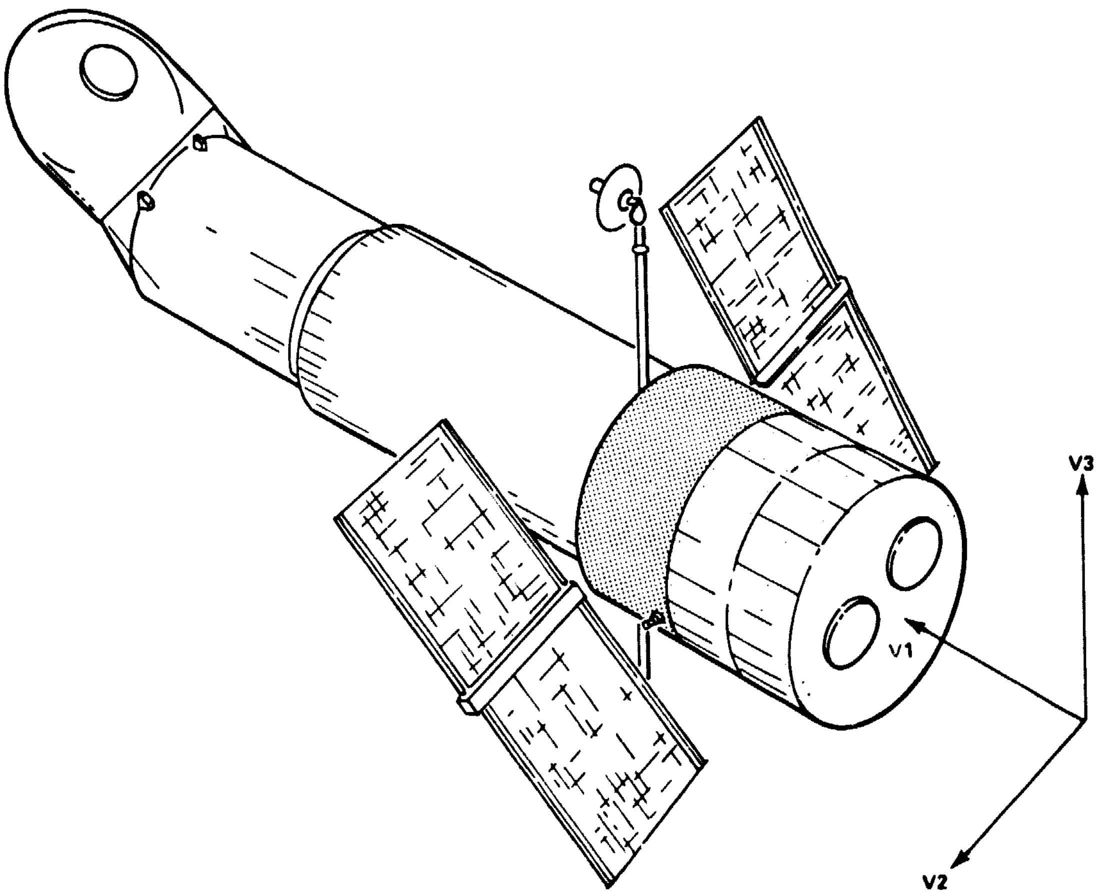

Figure 1. ST V coordinates.

During flight, the V coordinate system is defined by the relative positions of the three fine guidance sensors (FGS) (Fig. 2). V1 is defined as the direction angularly equidistant (ρ_0) from the vectors to a reference point in each of the FGS fields of view (FOV) in object space. By definition, one (and only one) FGS FOV reference point will lie in either the V1, V2 plane or the V1, V3 plane (which one and what plane depends upon the placement of the three FGS FOV). The angles between the planes defined by the V1 axis and each of the FGS FOV centers are not necessarily equal.

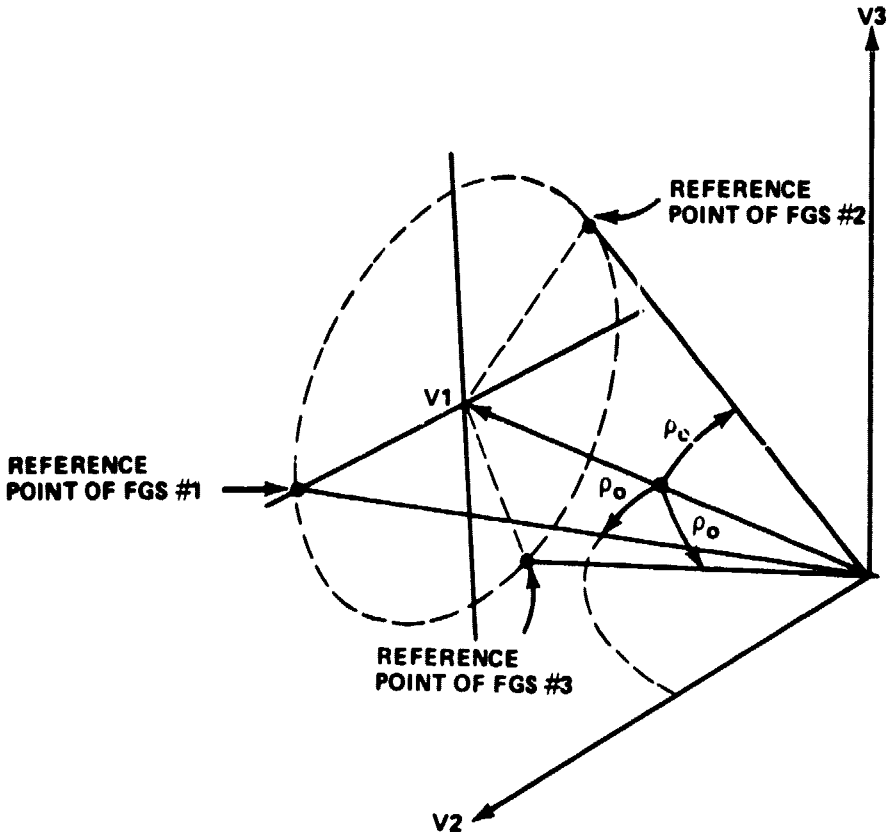

Figure 2. On-orbit V coordinates.

B. Attitude Reference Coordinate System R (R1, R2, R3)

This coordinate system is the attitude reference for ST at all times. The origin is at the ST center of mass. The orientation of the R1, R2, R3 axes will depend on the mission and/or target to be observed. For zero attitude error, the V axes are parallel to the R axes.

C. Initial Attitude Reference Coordinate System RZ (RZ1, RZ2, RZ3)

The RZ system specifies the orientation of the R system at the beginning of an attitude maneuver.

D. Terminal Attitude Reference Coordinate System RT (RT1, RT2, RT3)

The RT system specifies the desired orientation of the R system at the termination of an attitude maneuver.

E. Principal Axes Coordinate System P (P1, P2, P3)

The origin is at the ST mass center. The axes are the principal moment-of-inertia axes and are labeled such that the trace of the transformation matrix between the P and the V system is maximized (Pi is close to Vi, $i = 1, 2, 3$).

F. Body Mass Property Coordinate System B (B1, B2, B3)

The origin is at the ST mass center. The B1, B2, B3 axes are parallel to the V1, V2, V3 axes, respectively.

G. Equatorial Inertial Coordinate System E (E1, E2, E3) (Fig. 3)

This is the basic inertial coordinate system. All other coordinate systems are defined with respect to E. The origin is at the center of the Earth. The E3 axis is in the equatorial plane and it is positive toward the vernal equinox. The E2 axis is perpendicular to the equatorial plane and it is positive toward the Earth's North Pole. The vernal equinox position is defined as its mean position at 1950.0.

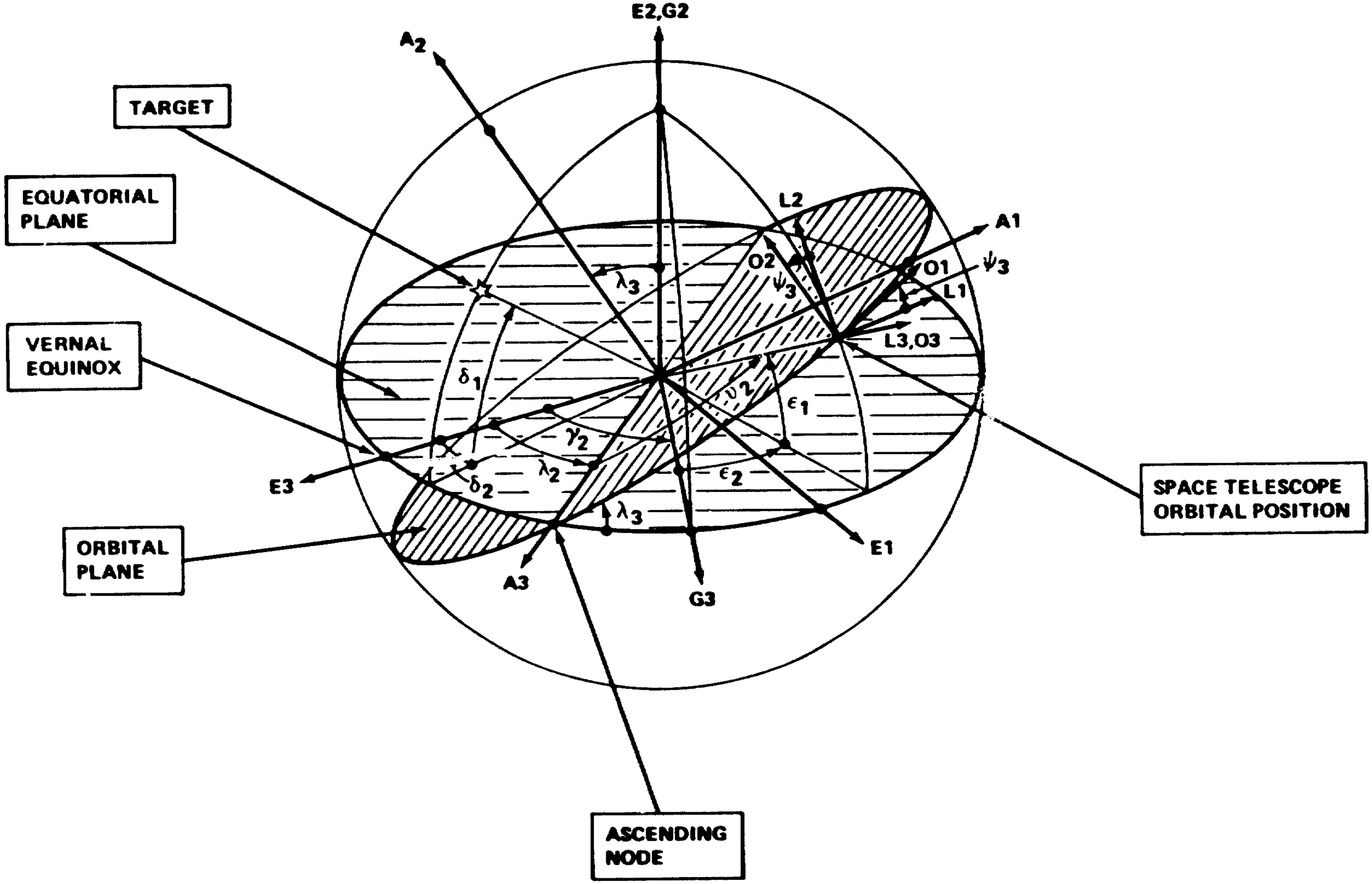

Figure 3. Coordinate systems E, G, A, O, L.

H. Equatorial Earth-Fixed Coordinate System G (G1, G2, G3) (Fig. 3)

The origin is at the center of the Earth. The G3 axis is in the equatorial plane and it is positive toward the meridian of zero longitude. The G2 axis is perpendicular to the equatorial plane and is positive toward Earth's North Pole (G2 = E2).

I. Orbital Coordinate System A (A1, A2, A3) (Fig. 3)

The origin is at the center of the Earth. The A3 axis is in the equatorial and in the orbital planes and it is positive toward the ascending node of the orbit. The A2 axis is perpendicular to the orbital plane and is positive in the direction of the orbital angular velocity vector.

J. Orbital Local Vertical Coordinate System O (O1, O2, O3) (Fig. 3)

The origin is at the center of mass of the ST. The O3 axis is parallel to the local vertical and it is positive away from the Earth's center. The O2 axis is perpendicular to the orbital plane and is positive in the direction of the orbital angular velocity vector (O2 = A2).

K. Magnetic Local Vertical Coordinate System L (L1, L2, L3) (Fig. 3)

The origin is at the center of mass of the ST. The L2 axis is parallel to the local vertical and it is positive away from the Earth's center (L3 = O3). The L1 axis is parallel to the equatorial plane and is directed positively in the same sense as Earth's tangential velocity vector, i.e., L1 points due east. The L2 axis points due north.

L. Attitude Sensor and Instrument Coordinate Systems Si (Si1, Si2, Si3); i = 1,2,3, etc. (Fig. 4)

The coordinate system for the ith attitude sensor or instrument is defined such that the Si1 axis is along the sensor boresight (in terms of object space), and Si2 and Si3 match some other characteristics (e.g., parallel to the edges of a square FOV). The term sensor includes all direction sensors, i.e., FGS (see more detailed treatment under "Fine Guidance Sensor Coordinate System"), scientific instruments, star trackers, Sun sensors, magnetometers, etc. The origin can be at any convenient point.

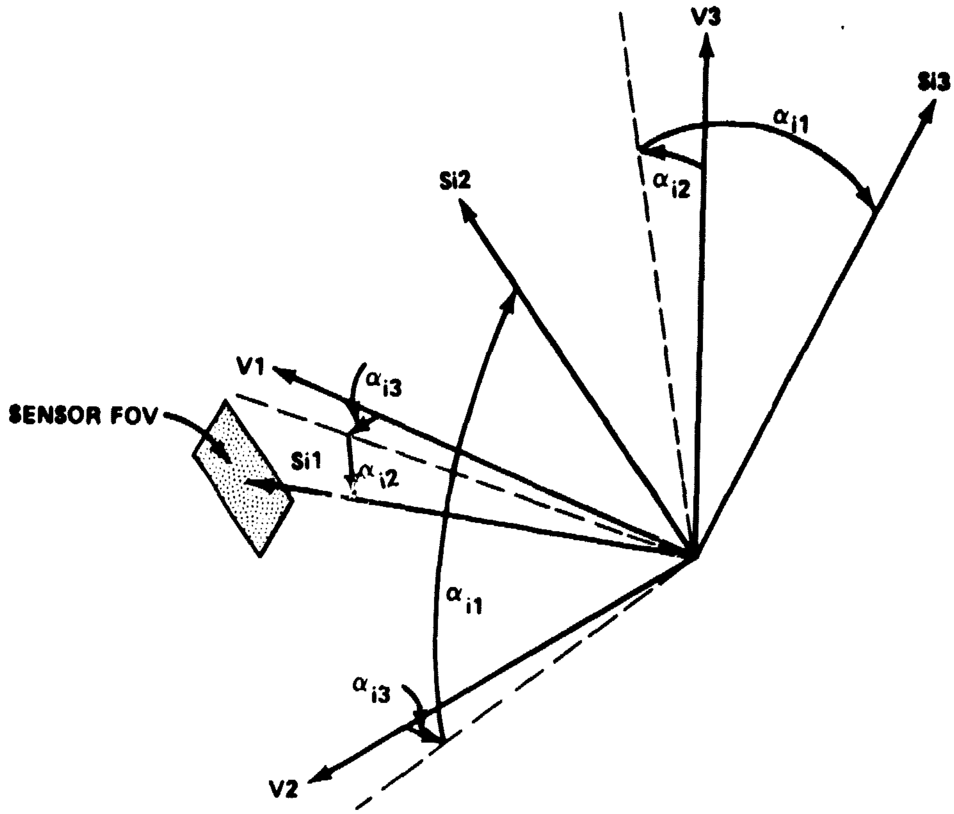

Figure 4. Attitude sensor coordinates.

The i identification is as follows:

1. Fine Guidance Sensor No. 1
2. Fine Guidance Sensor No. 2
3. Fine Guidance Sensor No. 3

4. Star Tracker No. 1
5. Star Tracker No. 2
6. Star Tracker No. 3
7. Sun Sensor
8. Magnetometer
9. Spare
10. Spare
11. Scientific Instrument No. 1
12. Scientific Instrument No. 2
13. Scientific Instrument No. 3
14. Etc.

M. Fine Guidance Sensor Coordinate System Si (Si1, Si2, Si3); i = 1,2,3

1. General. The FGS coordinate systems are considered part of the sensor coordinate system group. The FGS's get the first three (or more if necessary) values of i. The nominal boresight for the FGS's is the telescope optical axis. A reference point is identified for each FGS (Fig. 5). The vector from the origin of the V system to this reference point (in object space) is useful in the following definitions, and it is called the FGS reference vector. There are two possible methods to move the instantaneous FOV around within the total FOV, and the definitions are treated separately in the following.

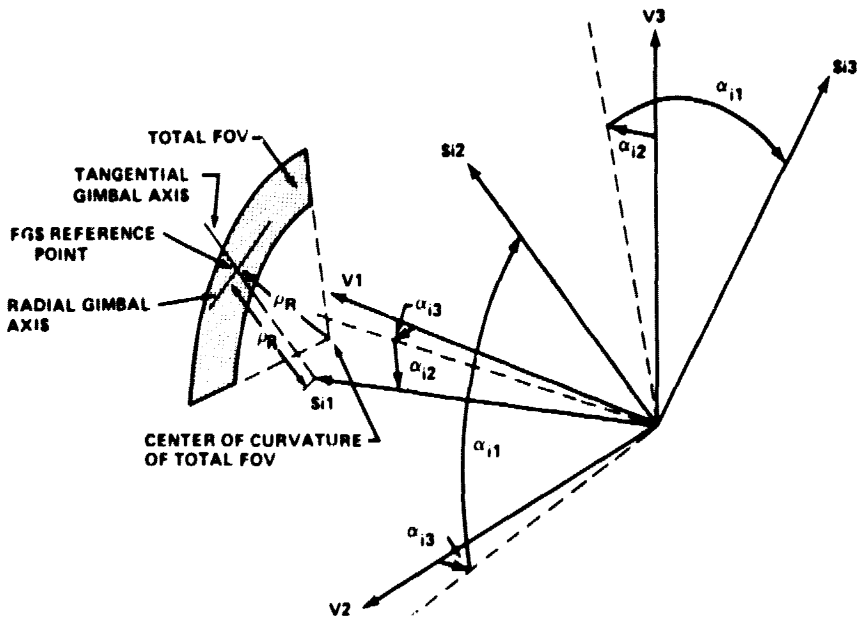

Figure 5. FGS with Euler angle movement.

2. Euler Angle Movement. One possible implementation of the Euler angle movement is by a gimbaled mirror. The zero gimbal angle position defines the FGS reference point. The Si3 axis lies in the plane formed by the radial gimbal axis and the FGS reference vector. Si1 is the nominal radius of curvature angle, ρ_R, away from the FGS reference vector (and perpendicular to Si3). For the case of no physical movement (possible implementation by a mosaic of charge coupled devices), the rectangular mosaic coordinate system is used instead of the gimbal axes.

3. Spherical Movement. Spherical coordinates characterize this movement (possible implementation by a grating plate with radial and tangential grooves). The FGS reference point is defined by the center of the grating in tangential and radial direction. The Si1 axis points to the center of curvature of the total FOV (Fig. 6). The Si2 axis is in the plane formed by the Si1 axis and the FOV center vector.

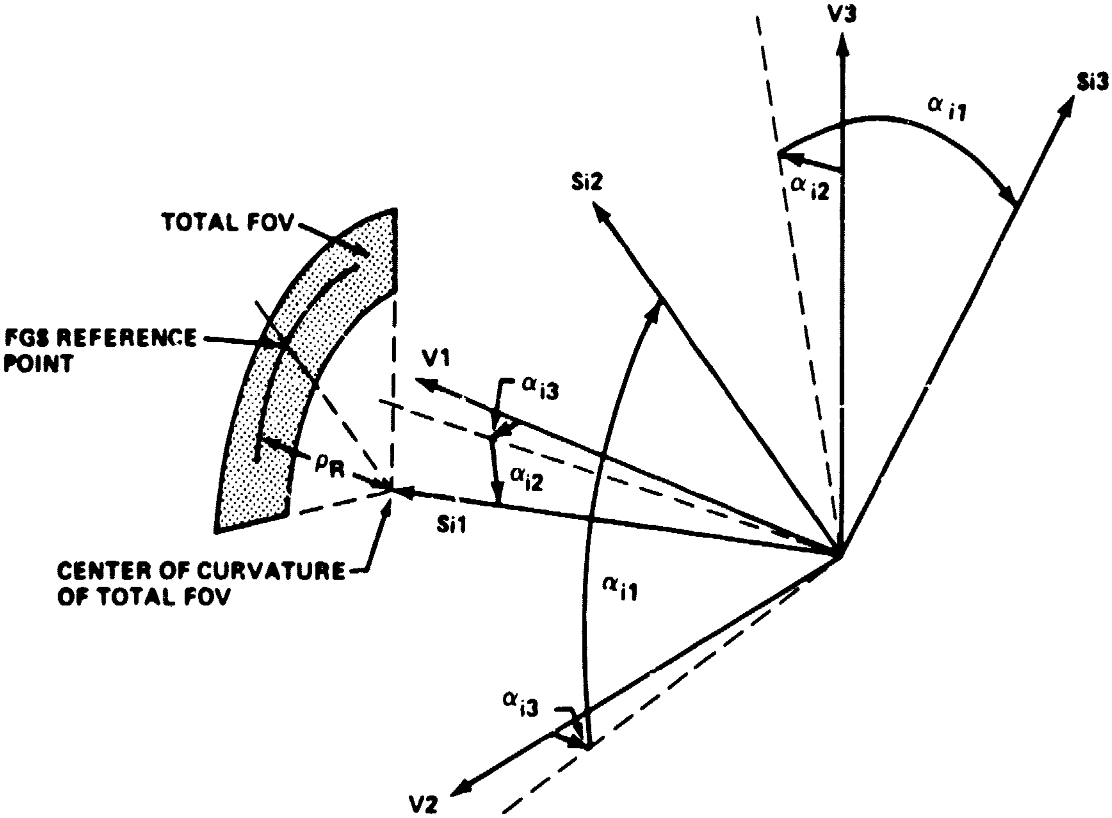

Figure 6. FGS with spherical movement.

III. COORDINATE SYSTEM TRANSFORMATIONS

The notation used to indicate a particular coordinate system transformation matrix is shown by the following example:

$$\underline{r}_V = [VR]\,\underline{r}_R \quad \text{with} \quad [VR] = \begin{bmatrix} VR_{11} & VR_{12} & VR_{13} \\ VR_{21} & VR_{22} & VR_{23} \\ VR_{31} & VR_{32} & VR_{33} \end{bmatrix} .$$

The vector $\underline{r}$ with components in the R system is transformed by $[VR]$ to the vector $\underline{r}$ with components in the V system. From the definition of the transformation matrix, it is obvious that $[RV] = [VR]^T$. Transformation matrices are often developed from a sequence of Euler angle rotations where each angle is specified by a Greek letter followed by a number (1, 2, or 3) corresponding to the axis of rotation about which the angle is measured, e.g.,

$$[VR] = [\theta_3]\,[\theta_2]\,[\theta_1]$$

where

$$[\theta_1] \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_1 & s\theta_1 \\ 0 & -s\theta_1 & c\theta_1 \end{bmatrix} ; \quad [\theta_2] \equiv \begin{bmatrix} c\theta_2 & 0 & -s\theta_2 \\ 0 & 1 & 0 \\ s\theta_2 & 0 & c\theta_2 \end{bmatrix} ; \quad [\theta_3] \equiv \begin{bmatrix} c\theta_3 & s\theta_3 & 0 \\ -s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} ;$$

$$c\theta_i \equiv \cos\theta_i \; ; \; s\theta_i \equiv \sin\theta_i \quad .$$

The $[\theta_i]$ matrices are single-axis, right-handed rotations. For left-handed rotations, the Greek letter is preceded by a minus sign, e.g.,

$$[-\delta_1] \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\delta_1 & -s\delta_1 \\ 0 & s\delta_1 & c\delta_1 \end{bmatrix} .$$

To avoid confusion in an Euler angle rotation sequence with repeated indices, different Greek letters have to be used for the angles with the repeated index.

The following is a list of transformation matrices which are presently defined in terms of Euler angle sequences:

$$[GE] = [\gamma_2]$$

$$[AE] = [\lambda_3]\ [\lambda_2]$$

$$[OA] = [\nu_2]$$

$$[RA] = [\zeta_1]\ [\zeta_2]\ [\zeta_3]$$

$$[VR] = [o_3]\ [o_2]\ [o_1]$$

$$[LG] = [-\epsilon_1]\ [\epsilon_2]$$

$$[OL] = [\Psi_3]$$

$$[VO] = [\xi_3]\ [\xi_2]\ [\xi_1]$$

$$[SiV] = [\alpha_{i1}]\ [\alpha_{i2}]\ [\alpha_{i3}] .$$

Other transformation matrices can be obtained from these, e.g.,

$$[OA] = [OL]\ [LG]\ [GE]\ [EA] = [\Psi_3]\ [-\epsilon_1]\ [\epsilon_2 + \gamma_2 - \lambda_2]\ [-\lambda_3] .$$

IV. RECOMMENDATIONS

It is recommended that the coordinate systems and Euler angles defined in this report be made the standards for the ST Program. To facilitate information exchange and avoid the possibility for errors and confusion inherent in the "translation" from one set of nomenclature and definitions to another, it is further recommended that the symbols, definitions, and nomenclature contained in Appendices A through F be used as much as possible.

APPENDIX A

OTHER TRANSFORMATIONS AND MATRICES

Special transformation matrices will be needed frequently. They can be divided into sensor and actuator signal transformation matrices.

SENSOR TRANSFORMATION MATRICES

These matrices are characterized by an S with a two-letter subscript (but no brackets), e.g., for the n rate gyros (ω_{Gi} is the ith rate gyro output, bias and drift neglected):

$$\underline{\omega}_G \equiv \begin{bmatrix} \omega_{G1} \\ \omega_{G2} \\ \vdots \\ \omega_{Gn} \end{bmatrix} = S_{RG} \begin{bmatrix} \omega_{V1} \\ \omega_{V2} \\ \omega_{V3} \end{bmatrix} \quad \text{with} \quad S_{RG} \equiv \begin{bmatrix} S_{RG11} & S_{RG12} & S_{RG13} \\ S_{RG21} & S_{RG22} & S_{RG23} \\ \vdots & \vdots & \vdots \\ S_{RGn1} & S_{RGn2} & S_{RGn3} \end{bmatrix} .$$

ACTUATOR TRANSFORMATION MATRICES

These matrices are characterized by an A with a two-letter subscript (but no brackets), e.g., torque on the vehicle due to torque of four reaction wheels:

$$\underline{T}_V \equiv \begin{bmatrix} T_{V1} \\ T_{V2} \\ T_{V3} \end{bmatrix} = A_{RW}^T \begin{bmatrix} T_{W1} \\ T_{W2} \\ T_{W3} \\ T_{W4} \end{bmatrix} \quad \text{with} \quad A_{RW} \equiv \begin{bmatrix} A_{RW11} & A_{RW12} & A_{RW13} \\ A_{RW21} & A_{RW22} & A_{RW23} \\ A_{RW31} & A_{RW32} & A_{RW33} \\ A_{RW41} & A_{RW42} & A_{RW43} \end{bmatrix} .$$

Another necessary transformation matrix is A_{MT} for the magnetic torquers. The definition for S_{XY} and A_{UV} was chosen such that the subscript on the matrix entries is as normally encountered and also constitutes mounting matrices for actuators as well as sensors.

PSEUDO-INVERSE

The pseudo-inverse of a nonsquare matrix (indicated by a dagger) is defined as, e.g.,

$$S^{\dagger}_{RG} = (S^T_{RG}\, S_{RG})^{-1}\, S^T_{RG} \qquad \text{or} \qquad A^{\dagger}_{RW} = A_{RW}\, (A^T_{RW}\, A_{RW})^{-1}$$

such that $S^{\dagger}_{RG}\, S_{RG}$ and $A^T_{RW}\, A^{\dagger}_{RW}$ are 3×3 unit matrices.

PHYSICAL VERSUS ESTIMATED QUANTITIES

When necessary, physical quantities should be distinguished from their computer or estimated values by appending a K subscript to the physical quantity (to keep the nomenclature of the onboard computer simple). For example,

$\underline{\omega}_{VK}$, ω_{VKi}	actual angular velocity of the vehicle and its ith component.
$\underline{\omega}_V$, ω_{Vi}	onboard estimate of $\underline{\omega}_{VK}$ and the ith component of the estimate.

VEHICLE INERTIA MATRIX

The vehicle inertia matrix should be defined as follows (about the vehicle center of mass, in V components):

$$I \equiv \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \quad \text{with} \quad I_{ij} = I_{ji}$$

where the off-diagonal terms are defined as

$$I_{ij} = -\int dm\, x_i x_j \qquad (i \neq j) \quad .$$

APPENDIX B

VECTORS

GENERAL

Vectors should be underlined. Subscripts are used to be more specific, e.g.,

$\underline{H}_T$ total angular momentum.

H_T magnitude of $\underline{H}_T$.

Vectors referenced to a specific coordinate system are indicated (if necessary) by adding the system letter, e.g.,

$\underline{H}_{TV}$ total angular momentum in V components.

$\underline{\omega}_V$ angular velocity of the V system, V components.

$\underline{\omega}_{VR}$ angular velocity of the V system, R components.

ω_{VR1} R1 component of $\underline{\omega}_{VR}$.

ω_{VR2} R2 component of $\underline{\omega}_{VR}$.

ω_{VR3} R3 component of $\underline{\omega}_{VR}$.

UNIT VECTORS

Unit vectors are indicated by a lower-case $\underline{u}$. Subscripts are used to be more specific, e.g., the local vertical unit vector can be indicated by $\underline{u}_{L3}$. The V components are indicated by $\underline{u}_{L3V}$.

CROSS-PRODUCT MATRIX

The cross-product of two vectors can be written as

$$\underline{a} \times \underline{b} = \tilde{a}\,\underline{b}$$

where

$$\tilde{a} \equiv \begin{bmatrix} 0 & -a_3 & +a_2 \\ +a_3 & 0 & -a_1 \\ -a_2 & +a_1 & 0 \end{bmatrix}$$

and the a_i are the components of $\underline{a}$ in some coordinate system. If it is not clear which coordinate system is indicated, a subscript needs to be used, e.g.,

$$\underline{a}_V \times \underline{b}_V = \tilde{a}_V\,\underline{b}_V \;.$$

APPENDIX C

QUATERNIONS

A real rotation may be represented by quaternions. The general quaternion $Q_1 i + Q_2 j + Q_3 k + Q_4$ can be written as

$$Q \equiv \begin{bmatrix} \underline{Q} \\ Q_4 \end{bmatrix} \equiv \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} e_1 s(\phi/2) \\ e_2 s(\phi/2) \\ e_3 s(\phi/2) \\ c(\phi/2) \end{bmatrix}$$

where $\underline{e}$ is the eigen axis and ϕ is the angle of the eigen axis rotation. A quaternion representing a particular rotation from one coordinate system to another is indicated by a double subscript; e.g., a rotation from the A system to the R system is indicated by

$$Q_{RA} = [Q_{RA1} \; Q_{RA2} \; Q_{RA3} \; Q_{RA4}]^T \quad .$$

Quaternion multiplication can be cast into matrix form by the following definitions:

$$Q_{RE} = \tilde{\tilde{Q}}_{AE} Q_{RA} = \bar{\bar{Q}}_{RA} Q_{AE}$$

where the double tilde and double bar operations are

$$\tilde{\tilde{Q}} \equiv \left[\begin{array}{c|c} Q_4 U + \tilde{Q} & \underline{Q} \\ \hline -\underline{Q}^T & Q_4 \end{array}\right] = \begin{bmatrix} +Q_4 & -Q_3 & +Q_2 & +Q_1 \\ +Q_3 & +Q_4 & -Q_1 & +Q_2 \\ -Q_2 & +Q_1 & +Q_4 & +Q_3 \\ -Q_1 & -Q_2 & -Q_3 & +Q_4 \end{bmatrix}$$

and

$$\bar{\bar{Q}} \equiv \left[\begin{array}{c|c} Q_4 U - \tilde{Q} & \underline{Q} \\ \hline -\underline{Q}^T & Q_4 \end{array}\right] = \begin{bmatrix} +Q_4 & +Q_3 & -Q_2 & +Q_1 \\ -Q_3 & +Q_4 & +Q_1 & +Q_2 \\ +Q_2 & -Q_1 & +Q_4 & +Q_3 \\ -Q_1 & -Q_2 & -Q_3 & +Q_4 \end{bmatrix}$$

with

$$U \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad \underline{Q} \equiv \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} , \quad \tilde{Q} \equiv \begin{bmatrix} 0 & -Q_3 & +Q_2 \\ +Q_3 & 0 & -Q_1 \\ -Q_2 & +Q_1 & 0 \end{bmatrix} .$$

A sequence of coordinate system transformations, e.g.

$$[OA] = [OL]\,[LG]\,[GE]\,[EA] \quad ,$$

can be represented analogously by quaternions, e.g.

$$Q_{OA} = \bar{\bar{Q}}_{OL}\,\bar{\bar{Q}}_{LG}\,\bar{\bar{Q}}_{GE}\,Q_{EA} \quad .$$

Sometimes it is advantageous to have the sequence rearranged with the help of the following identities:

$$\overset{\approx}{\bar{Q}}_{xy}\,Q_{ab} = \bar{\bar{Q}}_{ab}\,Q_{xy} \qquad \text{and} \qquad \tilde{\bar{\bar{Q}}}_{xy}\,\bar{\bar{Q}}_{ab} = \bar{\bar{Q}}_{ab}\,\tilde{\bar{\bar{Q}}}_{xy} \quad .$$

The quaternion rate can be expressed as (for V, R, and $Q \equiv Q_{VR}$)

$$\dot{Q} = \frac{1}{2}\left\{\tilde{\tilde{Q}}\begin{bmatrix}\underline{\omega}_V\\ 0\end{bmatrix} - \bar{\bar{Q}}\begin{bmatrix}\underline{\omega}_R\\ 0\end{bmatrix}\right\}$$

where $\underline{\omega}_V$ is the rate of the V system (in V components) and $\underline{\omega}_R$ is the rate of the R system (in R components).[2] Explicitly we get

$$\dot{Q}_1 = \left[Q_4(\omega_{V1} - \omega_{R1}) + Q_2(\omega_{V3} + \omega_{R3}) - Q_3(\omega_{V2} + \omega_{R2})\right]\Big/2$$

$$\dot{Q}_2 = \left[Q_4(\omega_{V2} - \omega_{R2}) + Q_3(\omega_{V1} + \omega_{R1}) - Q_1(\omega_{V3} + \omega_{R3})\right]\Big/2$$

$$\dot{Q}_3 = \left[Q_4(\omega_{V3} - \omega_{R3}) + Q_1(\omega_{V2} + \omega_{R2}) - Q_2(\omega_{V1} + \omega_{R1})\right]\Big/2$$

$$\dot{Q}_4 = \left[-Q_1(\omega_{V1} - \omega_{R1}) - Q_2(\omega_{V2} - \omega_{R2}) - Q_3(\omega_{V3} - \omega_{R3})\right]\Big/2 \quad .$$

Another useful quaternion relation is

$$\begin{bmatrix}\underline{X}_B\\ 0\end{bmatrix} = \tilde{\tilde{Q}}^*_{BA}\,\bar{\bar{Q}}_{BA}\begin{bmatrix}\underline{X}_A\\ 0\end{bmatrix} \qquad \text{where} \quad Q^*_{BA} = \begin{bmatrix}-\underline{Q}_{BA}\\ Q_{BA4}\end{bmatrix}$$

and as a consequence

$$\tilde{\tilde{Q}}^*_{BA}\,\bar{\bar{Q}}_{BA} = \left[\begin{array}{ccc|c} & & & 0\\ & [BA] & & 0\\ & & & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

2. Since Q_{VR} is used extensively, it is convenient to drop the subscript and define $Q \equiv Q_{VR}$.

APPENDIX D

COMPUTER MNEMONICS

To simplify reading of computer programs for persons not thoroughly familiar with them, some general rules for the mnemonics are helpful. These rules are given in the following paragraphs.

ANGLES

Angles are to be indicated by a Greek letter and the mnemonic thereof by the first two letters of the Greek word for that letter. For example,

α — Alpha — AL

β — Beta — BE

γ — Gamma — GA.

Subscripts, either English letters, number, or both, are added as needed. For example,

Ω_E — OME

Ω_O — OMO

ω_{VK3} — OMVK3

ω_{R3} — OMR3 or OMR(3)

ω_{RV3} — OMRV3 or OMRV(3) .

Note that the use of parentheses is optional. Also, use is made of an extra letter to indicate that the quantity is given in a reference coordinate frame other than the one in which it is normally defined. For example, the quantity ω_{R3} is to mean the 3-axis component of $\underline{\omega}_R$ as given in its natural coordinate system R, whereas ω_{RV3} is to mean the V3-axis component of $\underline{\omega}_R$.

DERIVATIVES

The first or second derivative of a quantity is indicated by adding D or DD, respectively, after the main mnemonic, but before any numbers. For example,

$\dot{\alpha}_i$ - ALD(i)

$\ddot{\alpha}_i$ - ALDD(i) .

TRIGONOMETRIC FUNCTIONS

For the three basic trigonometric functions (sine, cosine, and tangent), the first letter of each is added as a prefix to the mnemonic of the angle. For example,

$\sin \alpha_i$ - SAL(i)

$\cos \beta_i$ - CBE(i)

$\tan \gamma_i$ - TGA(i) .

APPENDIX E

LIST OF DEFINITIONS, UNITS, AND MNEMONICS OF VARIABLES

Variable	Mnemonic	Unit	Definition
a	A	km	Orbital altitude
A, A1, A2, A3			Orbital coordinate system
A_{MT}	AMT(i,j)		Magnetic torquer mounting matrix
A_{RW}	ARW(i,j)		RW mounting matrix
$\underline{B}$	B	T	Earth magnetic field vector
$\underline{B}_L$	BL(i)	T	Earth magnetic field in L components
$\underline{B}_V$	BV(i)	T	Earth magnetic field in V components
E, E1, E2, E3			Equatorial inertial coordinate system
G, G1, G2, G3			Equatorial Earth-fixed coordinate system
g	G	m/s^2	Earth gravitational acceleration
G_I	GI(i,j)	$1/s^3$	Normalized 3 × 3 integral gain matrix
G_R	GR(i,j)	1/s	Normalized 3 × 3 rate gain matrix
G_P	GP(i,j)	$1/s^2$	Normalized 3 × 3 position gain matrix
$\underline{H}_B$	HB(i)	N·m·s	Magnetic bias momentum
$\underline{H}_D$	HD(i)	N·m·s	Magnetic desaturation momentum

Variable	Mnemonic	Unit	Definition
$\underline{H}_{RW}$	HRW(i)	N·m·s	Total RW-system momentum vector
$\underline{H}_{T}$		N·m·s	Total angular momentum vector
$\underline{H}_{TV}$	HTV(i)	N·m·s	V components of $\underline{H}_{T}$
$\underline{H}_{V}$	HV(i)	N·m·s	Vehicle angular momentum
H_{Wi}	HW(i)	N·m·s	Momentum of ith RW
I	I(i,j)	$kg \cdot m^2$	Vehicle inertia matrix
I_{Wi}	IW(i)	$kg \cdot m^2$	Inertia of ith RW
K_B	KB	V·s/rad	RW motor back-EMF constant (numerically identical)
K_B	KB	N·m/A	RW motor torque constant (numerically identical)
K_D	KD	1/[illegible]	Distribution gain
K_I	KI(i,j)	$N \cdot m \cdot s^{-1}/rad$	3 × 3 integral gain matrix
K_M	KM	1/s	Magnetic gain
K_P	KP(i,j)	N·m/rad	3 × 3 position gain matrix
K_R	KR(i,j)	N·m·s/rad	3 × 3 rate gain matrix
L,L1,L2,L3			Magnetic local vertical coordinate system
O,O1,O2,O3			Orbital Local vertical coordinate system
P,P1,P2,P3			Principal axes coordinate system
Q	Q,Q(i)		Attitude quaternion

Variable	Mnemonic	Unit	Definition
Q_{RA}	QRA, QRA(i)		Quaternion connecting coordinate system R to A
$\bar{\bar{Q}}, \tilde{\tilde{Q}}$			Q-double bar, Q-double tilde
R, R1, R2, R3			Attitude reference coordinate system
RZ, RZ1, RZ2, RZ3			Initial attitude reference coordinate system for maneuvers
RT, RT1, RT2, RT3			Terminal attitude reference coordinate system for maneuvers
$\underline{r}$	R(i)	km	Vector for Earth center to vehicle CM
r_e	RE	km	Mean radius of the Earth
Si, Si1, Si2, Si3			Attitude sensor and instrument coordinate system
S_{RG}	SRG(i, j)		Rate gyro mounting matrix
$\underline{T}_C$	TC(i)	N·m	Control torque command
$\underline{T}_{GG}$	TGG(i)	N·m	Gravity gradient torque vector
t_D	TD	s	Magnetic momentum desaturation interval
$\underline{T}_M$	TM(i)	N·m	Magnetic torque vector
$\underline{T}_{RW}$	TRW(i)	N·m	Total RW-system torque vector
T_{Wi}	TW(i)	N·m	Torque command of ith RW
$\underline{T}_V$	TV(i)	N·m	Vehicle torque
$\underline{u}_{L3V}$	UL3V(i)		Unit vector (V components)

Variable	Mnemonic	Unit	Definition
U	U(i,j)		3×3 unit matrix
V, V1, V2, V3			Vehicle control axes coordinate system
VR_{ij}	VR(i,j)		[VR] transformation matrix components
$\underline{Y}$	Y(i)	$A \cdot m^2/T$	Lagrange multipliers for magnetic desaturation
α_{ij}	AL(i,j)	rad	Transformation angles from coordinate system V to Si (Figs. 4, 5, and 6)
γ_2	GA2	rad	* Angle from vernal equinox to prime meridian, north positive, $\gamma_2 = \gamma_{IC} + \Omega_E t$
δ_1	DE1	rad	* Latitude of target from equatorial plane, north positive.
δ_2	DE2	rad	* Longitude of target from vernal equinox, east positive
Δt	DT	s	Sample time
ϵ_1	EP1	rad	* Latitude from equatorial plane, north positive
ϵ_2	EP2	rad	* Longitude from prime meridian, east positive
ζ_i	ZE(i)	rad	Euler angle rotation about the i-axis following a 3-2-1 rotation sequence from coordinate system A to R
ζ_{Mi}	ZEM(i)		Damping ratio of the ith bending mode
η_1	ET1	rad	Solar elevation angle from orbital plane, south positive

* The angles with an asterisk can be found in Figure 3.

Variable	Mnemonic	Unit	Definition
η_2	ET2	rad	Angle from ascending node to the projection of the Sun vector onto the orbital plane
θ_i	TH(i)	rad	Euler angle rotation about the i-axis following a 1-2-3 rotation sequence from coordinate system R to V
λ_2	LA2	rad	*Angle from vernal equinox to ascending node
λ_3	LA3	rad	*Inclination of orbit
μ_C	MUC(i)	$A \cdot m^2$	Magnetic dipole moment command
μ_E	MUE	$m^3 \cdot s^{-2}$	Earth's gravitational constant
μ_{Mi}	MUM(i)	$A \cdot m^2$	Magnetic dipole moment command (ith torquer)
ν_2	NU2	rad	*Angle from ascending node to local vertical $\nu_2 = \nu_{IC} + \Omega_O t$
ρ_0	RH0	rad	Angle between each FGS reference vector and the in-flight V1 axis (Fig. 2)
ρ_R	RHR	rad	Radius of curvature (in object space) of the FGS FOV (Figs. 5,6)
τ_D	TAD	s	Distribution time constant
ϕ_3	PH3	rad	Inclination of the ecliptic plane with respect to the Earth's equatorial plane
Ψ_3	PS3	rad	*Rotation about the L3/O3 axis going from L to O

Variable	Mnemonic	Unit	Definition
Ω_E	OME	rad/s	Earth's angular rate
ω_{Gi}	OMG(i)	rad/s	Angular rate output of ith rate gyro
Ω_0	OM0	rad/s	Orbital angular rate
$\underline{\omega}_R$	OMR(i)	rad/s	Angular rate of system R
ω_{Mi}	OMM(i)	rad/s	Undamped natural frequency of ith bending mode
$\underline{\omega}_V$	OMV(i)	rad/s	Angular rate of vehicle
ω_{Wi}	OMW(i)	rad/s	Speed of ith RW

APPENDIX F

THE INTERNATIONAL SYSTEM OF UNITS

This appendix lists the basic units and a number of derived units together with their symbols as defined in the International System of Units. For further reference see, e.g., "The International System of Units," NASA SP-7012.

Physical Quantity	Name of Unit	Symbol	
	Basic Units		
Length	meter	m	
Mass	kilogram	kg	
Time	second	s	
Electric current	ampere	A	
Temperature	kelvin	K	
Luminous intensity	candela	cd	
	Derived Units		
Area	square meter	m^2	
Volume	cubic meter	m^3	
Frequency	hertz	Hz	(s^{-1})
Density	kilogram per cubic meter	kg/m^3	
Velocity	meter per second	m/s	
Angular velocity	radian per second	rad/s	
Acceleration	meter per second squared	m/s^2	
Angular acceleration	radian per second squared	rad/s^2	
Force	newton	N	$(kg\cdot m/s^2)$
Pressure	pascal	Pa	(N/m^2)
Moment of inertia	kilogram-meter-squared	$kg\cdot m^2$	$(N\cdot m\cdot s^2)$
Angular momentum	newton-meter-second	$N\cdot m\cdot s$	
Kinematic viscosity	square meter per second	m^2/s	
Dynamic viscosity	newton-second per square meter	$N\cdot s/m^2$	
Work, energy, quantity of heat	joule	J	$(N\cdot m)$
Power	watt	W	(J/s)

Electric charge	coulomb	C	(A·s)
Voltage, potential difference, electromotive force	volt	V	(W/A)
Electric field strength	volt per meter	V/m	
Electric resistance	ohm	Ω	(V/A)
Electric capacitance	farad	F	(A·s/V)
Magnetic flux	weber	Wb	(V·s)
Inductance	henry	H	(V·s/A)
Magnetic flux density	tesla	T	(Wb/m^2)
Magnetic field strength	ampere per meter	A/m	
Magnetomotive force	ampere	A	
Luminous flux	lumen	lm	(cd·sr)
Luminance	candela per square meter	cd/m^2	
Illumination	lux	lx	(lm/m^2)
Wave number	1 per meter	m^{-1}	
Entropy	joule per kelvin	J/K	
Specific heat	joule per kilogram kelvin	$J\ kg^{-1}K^{-1}$	
Thermal conductivity	watt per meter kelvin	$W\ m^{-1}K^{-1}$	
Radiant intensity	watt per steradian	W/sr	
Activity (of a radioactive source)	1 per second	s^{-1}	

<u>Supplementary Units</u>

Plane angle	radian	rad
Plane angle	degree	deg
Plane angle	arc minute	$\widehat{\text{min}}$
Plane angle	arc second	$\widehat{\text{sec}}$
Solid angle	steradian	sr

PREFIXES

The names of multiples and submultiples of SI Units may be formed by application of the prefixes:

Factor by which unit is multiplied	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

APPROVAL

SPACE TELESCOPE COORDINATE SYSTEMS, SYMBOLS, AND NOMENCLATURE DEFINITIONS

By Hans F. Kennel

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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