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COOLED-TURBINE AERODYNAMIC PERFORMANCE PREDICTION FROM REDUCED PRIMARY TO COOLANT TOTAL-TEMPERATURE-RATIO RESULTS

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16. Abstract

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SUMMARY

The prediction of the aerodynamic performance, for both cooled stators and turbines, at actual primary to coolant inlet total-temperature ratios T_p/T_c from the results obtained at a reduced total-temperature ratio is described in this report. Because turbine rotation affects rotor blade coolant flow, the theoretical predictions for two possible methods of turbine testing were investigated. These are (1) a constant rotor blade coolant to rotor inlet relative momentum ratio and (2) a constant rotor blade coolant to rotor inlet absolute momentum ratio. Theory and experimental results have been compared for convection-film- and transpiration-cooled stator vanes and for a film-cooled core stator and turbine stage. For these tests the total-temperature ratio T_p/T_c ranged from near 1.0 to about 2.7.

The theoretical results indicated that for a coolant to primary inlet total-pressure ratio P_c/P_p of 1, the thermodynamic efficiency is independent of the total-temperature ratio T_p/T_c for stators and is nearly independent of this ratio for turbines. For a total-pressure ratio P_c/P_p greater than 1, the primary efficiency decreases and the thermodynamic efficiency increases with increasing total-temperature ratio.

For the stator, the experimental results agreed reasonably well with the theory for the convection-film- and transpiration-cooled vanes but did not agree well with the limited data available for the film-cooled core vane. For the core turbine, the constant-relative-momentum-ratio method of testing was found to agree well with the theory for the limited amount of data available. The agreement was also reasonable for the constant-absolute-momentum-ratio method of testing. For both the stator and turbine tests, fair agreement was found, in general, with predicted coolant mass flow fractions.

INTRODUCTION

Using compressor bleed to cool turbine vanes and blades for advanced hightemperature aircraft engines has resulted in an extensive program of cooling studies at the NASA Lewis Research Center. One phase of this program is concerned with the effect of the cooling air discharged into the mainstream on both the stator vane and turbine stage aerodynamic performance. Much of the experimental results of this program have been reported in references 1 to 11. These investigations have all been conducted at primary to coolant inlet total-temperature ratios T_p/T_c near 1. This method of testing is more convenient and economical than testing at actual engine total-temperature ratios, which may be of the order of 2 or 3. However, from the limited amount of research performed at higher temperature ratios (refs. 12 and 13), it is known that the total-temperature ratio can have a significant effect on both stator vane and turbine stage aerodynamic performance.

The purpose of this report is to present a method for predicting both the stator and turbine aerodynamic performance at actual engine total-temperature ratios T_p/T_c from the results obtained at a reduced total-temperature ratio. The aerodynamic losses (total-pressure losses) due to boundary layer growth and mixing are assumed to be constant in this model, if the coolant to primary momentum ratios are maintained the same.

In this report, the prediction method is described, and calculated efficiencies and coolant to primary mass flow fractions are compared with the available experimental results. Comparisons are made for convective-film- and transpiration-cooled stator vanes (ref. 12) and for a film-cooled, single-stage core turbine (ref. 13). The total-temperature ratio T_p/T_c for these tests varied from near 1.0 to about 2.7.

METHOD OF ANALYSIS

Flow Model and Assumptions

The aerodynamic performance of cooled stators and turbines depends on both the loss in total pressure and the "loss" in total temperature caused by heat transfer between the coolants and the primary stream. It is assumed that these two loss mechanisms are essentially independent of each other.

The total-pressure loss due to boundary layer growth and mixing is assumed to be constant if the coolant to primary momentum ratios are maintained constant. The total-temperature loss due to heat transfer is calculated theoretically from the energy equation. It is also assumed that the total-pressure change due to mixing streams of different total temperatures is negligible in comparison with the total-pressure loss due to viscous effects. With these assumptions, the aerodynamic performance at actual engine total-temperature ratios T_p/T_c can be calculated from the performance obtained at reduced-total-temperature-ratio conditions. The next section describes the calculation method for stator vanes.

Stator Performance Prediction

Two definitions of stator vane performance, based on kinetic energy, are in common use at the Lewis Research Center. Primary efficiency η_p is defined as the ratio of the actual kinetic energy to the ideal kinetic energy of the primary stream only. (All symbols are defined in appendix A.) Thermodynamic efficiency η_t, on the other hand, is based on the sum of the ideal kinetic energies of the primary and coolant flows. In equation form,

$$\eta_p = \frac{(m_p + m_{sc})V_{se}^2}{m_p V_{p,id}^2} = \frac{(m_p + m_{sc})\,\Delta h_s}{m_p\,\Delta h_{p,id}} \tag{1}$$

$$\eta_t = \frac{(m_p + m_{sc})V_{se}^2}{m_p V_{p,id}^2 + m_{sc} V_{sc,id}^2} = \frac{(m_p + m_{sc})\,\Delta h_s}{m_p\,\Delta h_{p,id} + m_{sc}\,\Delta h_{sc,id}} \tag{2}$$

where

$$\Delta h_s = C_P T_{se}\left[1 - \left(\frac{p_{se}}{P_{se}}\right)^{(\gamma-1)/\gamma}\right] = \frac{V_{se}^2}{2gJ} \tag{3a}$$

$$\Delta h_{p,id} = C_P T_p\left[1 - \left(\frac{p_{se}}{P_p}\right)^{(\gamma-1)/\gamma}\right] = \frac{V_{p,id}^2}{2gJ} \tag{3b}$$

$$\Delta h_{sc,id} = C_P T_{sc}\left[1 - \left(\frac{p_{se}}{P_{sc}}\right)^{(\gamma-1)/\gamma}\right] = \frac{V_{sc,id}^2}{2gJ} \tag{3c}$$

It has been assumed, for simplicity, that the fluid is a perfect gas with constant specific heat C_P.

The stator efficiency is seen to depend on the exit total and static pressures P_{se} and p_{se} and total temperature T_{se}, as well as on the flow rates and total temperatures and pressures of the coolant and primary streams. To predict the performance at actual primary to stator vane coolant inlet total-temperature ratios T_p/T_{sc} from the results at a total-temperature ratio of 1 ($T_p^*/T_{sc}^* = 1.0$), these parameters (i.e., T_{se}, P_{se}, m_{sc}, m_p, etc.) must be related to each other for the two cases.

To accomplish this, first consider that the stator exit static-pressure to primary inlet total-pressure ratio is experimentally set the same for the two cases. That is,

$$\frac{p_{se}}{P_p} = \frac{p_{se}^*}{P_p^*} \tag{4}$$

For these test conditions it is assumed, as in reference 14, that the total-temperature ratio T_p/T_c has a negligible effect on the static-pressure distribution within the stator. Therefore, from one-dimensional flow theory, it is expected (ref. 15) that

$$\frac{m_p\sqrt{T_p}}{P_p} = \frac{m_p^*\sqrt{T_p^*}}{P_p^*} \tag{5a}$$

$$\frac{m_{sc}\sqrt{T_{sc}}}{P_{sc}} = \frac{m_{sc}^*\sqrt{T_{sc}^*}}{P_{sc}^*} \tag{5b}$$

It has been assumed (in eq. (5b)) that the coolant hole sizes are the same for the two tests. This is consistent with the test procedures used in references 12 and 13. However, it differs from the suggestionmade in reference 14 that the coolant hole sizes be altered so that the coolant flow rates are reproduced. This would result in the momentum ratios being different for the two tests.

As discussed previously, the total-pressure loss in the stator is assumed to be constant if the coolant to primary momentum ratio $m_{sc}V_{sc}/m_pV_p$ is maintained the same. For constant area, this is equivalent to maintaining $\rho_{sc}V_{sc}^2/\rho_pV_p^2$ constant. With the previous assumption of constant stator static-pressure distribution, this is accomplished if the total-pressure ratio P_c/P_p is kept constant. That is,

$$\frac{P_{sc}}{P_p} = \frac{P_{sc}^*}{P_p^*} \tag{6}$$

Under these conditions the stator total-pressure loss (as a percentage of inlet total pressure) is assumed to be constant. Therefore,

$$\frac{P_{se}}{P_p} = \frac{P_{se}^*}{P_p^*} \tag{7a}$$

or from equation (4)

$$\frac{p_{se}}{P_{se}} = \frac{p_{se}^*}{P_{se}^*} \tag{7b}$$

From equations (3a) and (7b) we find that

$$\frac{\Delta h_s}{T_{se}} = \frac{\Delta h_s^*}{T_{se}^*} \tag{8a}$$

where the stator exit total temperature T_{se} is determined from the energy equation

$$T_{se} = \frac{m_p T_p + m_{sc} T_{sc}}{m_p + m_{sc}} \tag{8b}$$

$$T_{se}^* = T_p^* = T_{sc}^* \tag{8c}$$

This relation (eq. (8a)) is in the same form as for an uncooled stator (ref. 15). Therefore, as far as the performance of the stator is concerned, the coolants can be thought of as effectively decreasing the stator inlet total temperature to the mass-averaged value T_{se}.

The primary and thermodynamic stator efficiencies are then found to be (appendix B)

$$\frac{\eta_p}{\eta_p^*} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right)}{1 + y_{sc}^*} \tag{B6}$$

$$\frac{\eta_t}{\eta_t^*} = \frac{\eta_p}{\eta_p^*}\left[\frac{1 + y_{sc}^*\left(\frac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right)}{1 + y_{sc}\left(\frac{\Delta h_{sc,id}}{\Delta h_{p,id}}\right)}\right] \tag{B12}$$

where

$$y_{sc} = \frac{m_{sc}}{m_p} = y_{sc}^*\sqrt{\frac{T_p}{T_{sc}}} \tag{B8}$$

$$\frac{\Delta h_{sc,id}}{\Delta h_{p,id}} = \frac{T_{sc}}{T_p}\left(\frac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right) = \frac{T_{sc}}{T_p}\left(\frac{\eta_p^*}{\eta_t^*} - 1\right)\frac{1}{y_{sc}^*} \tag{B14}$$

The stator efficiencies can, therefore, be calculated from these equations and from the measured efficiencies and flow rate (i.e., η_p^*, η_t^*, and y_{sc}^*) obtained at a total-temperature ratio T_p/T_{sc} of 1. The effect of the ratio T_p/T_{sc} can be seen from these equations.

For the special case of a stator vane coolant to primary inlet total-pressure ratio of 1 ($P_{sc}/P_p = 1.0$), the thermodynamic efficiency becomes (appendix B)

$$\eta_t = \eta_t^* = \text{Constant} \tag{B19}$$

and is, therefore, independent of the total-temperature ratio. This is an interesting and important result, since for engines operating with coolants supplied by compressor bleed, the total-pressure ratio P_{sc}/P_p is close to 1 (at least for the first stage). In the next section, the calculation method is extended to cooled turbine stages.

Turbine Performance Prediction

Two definitions of turbine stage performance, similar to the definitions for stator performance, are in common use. The primary efficiency $\bar{\eta}_p$ and thermodynamic efficiency $\bar{\eta}_t$ for a turbine are defined as

$$\bar{\eta}_p = \frac{(m_p + m_{sc} + m_{rc})\,\Delta H_r}{m_p\,\Delta H_{p,id}} \tag{9a}$$

$$\bar{\eta}_t = \frac{(m_p + m_{sc} + m_{rc})\,\Delta H_r}{m_p\,\Delta H_{p,id} + m_{sc}\,\Delta H_{sc,id} + m_{rc}\,\Delta H_{rc,id}} \tag{9b}$$

where

$$\Delta H_{p,id} = C_P T_p \left[1 - \left(\frac{P_{re}}{P_p}\right)^{(\gamma-1)/\gamma}\right] \tag{10a}$$

$$\Delta H_{sc,id} = C_P T_{sc} \left[1 - \left(\frac{P_{re}}{P_{sc}}\right)^{(\gamma-1)/\gamma}\right] \tag{10b}$$

$$\Delta H_{rc,id} = C_P T_{rc} \left[1 - \left(\frac{P_{re}}{P_{rc}}\right)^{(\gamma-1)/\gamma}\right] \tag{10c}$$

The enthalpy drop across the turbine ΔH_r is given by

$$\Delta H_r = \frac{\frac{\tau\omega}{J}}{m_p + m_{sc} + m_{rc}} \tag{10d}$$

Most of the assumptions necessary to predict the turbine stage performance are similar to those used for the stator. For example, it is again assumed that the rotor exit static-pressure to primary inlet total-pressure ratio is experimentally set the same for the actual and reduced total-temperature-ratio T_p/T_c cases. That is,

$$\frac{p_{re}}{P_p} = \frac{p^*_{re}}{P^*_p} \tag{11}$$

The static-pressure distribution within the turbine is also assumed to be the same for the two cases, so that equation (4) is still valid. The primary and stator coolant flow rates are, therefore, given by equation (5). Similarly, for a constant relative blade surface static-pressure to blade coolant total-pressure ratio, the rotor blade coolant flow rate is given by

$$\frac{m_{rc}\sqrt{\hat{T}_{rc}}}{\hat{P}_{rc}} = \frac{m^*_{rc}\sqrt{\hat{T}^*_{rc}}}{\hat{P}^*_{rc}} \tag{12a}$$

where the rotor blade coolant relative total conditions $\hat{T}_{rc}$ and $\hat{P}_{rc}$ are calculated (if they are not measured) by the relations for isentropic flow

$$\hat{T}_{rc} = T_{rc} + \frac{U_m^2}{2gJC_p} \tag{12b}$$

$$\hat{P}_{rc} = P_{rc}\left(\frac{\hat{T}_{rc}}{T_{rc}}\right)^{\gamma/(\gamma-1)} \tag{12c}$$

Analogous with the stator performance prediction method, it is assumed for the turbine that the relative total-pressure loss in the rotor is constant, if the rotor blade coolant to rotor inlet relative momentum ratio $m_{rc}W_{rc}/m_{se}W_{se}$ is maintained the same. As shown in appendix C, this is accomplished if

$$\frac{\hat{P}_{rc}}{P_p} = \frac{\hat{P}^*_{rc}}{P^*_p} \tag{C11}$$

Because the rotor blade coolant relative total pressure $\hat{P}_{rc}$ is usually calculated, it is not a very convenient experimental quantity. In fact, most of the data of reference 13 were obtained at a constant absolute total pressure P_{rc} (and, therefore, varying $\hat{P}_{rc}$), which corresponds to a constant rotor blade coolant to rotor inlet absolute momentum ratio. Since it is desired to compare both methods with the corresponding experimental data, both prediction techniques are presented herein. The performance prediction method based on a constant <u>absolute</u> momentum ratio is given in appendix D. The prediction method based on a constant <u>relative</u> momentum ratio is presented in this section and appendix C.

Similar to the stator prediction method, the energy equation is again used to predict the turbine performance. For a cooled turbine, the energy equation can be written as

$$(m_p + m_{sc} + m_{rc})C_p(T_m - T_{re}) = (m_p + m_{sc} + m_{rc})\,\Delta H_r = \frac{\tau\omega}{J} \tag{13a}$$

where the mass-averaged inlet total temperature T_M of all the flows is

$$T_M = \frac{m_p T_p + m_{sc} T_{sc} + m_{rc} T_{rc}}{m_p + m_{sc} + m_{rc}} \tag{13b}$$

The energy equation has purposely been put into the same form as that for an uncooled turbine operating from an inlet total temperature equal to T_M. The decrease of the total temperature to T_M has essentially accounted for the loss in total temperature due to heat transfer, which has (as discussed previously) been assumed to be independent of the loss in total pressure due to viscous effects. Therefore, analogous with uncooled-turbine aerodynamic theory (ref. 15), it is assumed that the turbine performance is given by

$$\frac{\Delta H_r}{T_M} = \frac{\Delta H_r^*}{T_M^*} \tag{14a}$$

$$T_M^* = T_p^* = T_{sc}^* = T_{rc}^* \tag{14b}$$

This is similar to equation (8) for the stator performance.

To maintain the rotor incidence angle constant and to be consistent with the analysis of appendix C, the mean-radius blade speed U_m (at the two test conditions) should be set so that

$$\frac{U_m}{\sqrt{T_{se}}} = \frac{U_m^*}{\sqrt{T_{se}^*}} = \frac{U_m^*}{\sqrt{T_p^*}} \tag{15}$$

The primary and thermodynamic turbine efficiencies for a constant relative momentum ratio are given by (appendix C)

$$\frac{\bar{\eta}_p}{\bar{\eta}_p^*} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right) + y_{rc}\left(\frac{T_{rc}}{T_p}\right)}{1 + y_{sc}^* + y_{rc}^*} \tag{C29}$$

$$\frac{\bar{\eta}_t}{\bar{\eta}_t^*} = \frac{\bar{\eta}_p}{\bar{\eta}_p^*}\left[\frac{1 + y_{sc}^*\left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*}\right) + y_{rc}^*\left(\frac{\Delta H_{rc,id}^*}{\Delta H_{p,id}^*}\right)}{1 + y_{sc}\left(\frac{\Delta H_{sc,id}}{\Delta H_{p,id}}\right) + y_{rc}\left(\frac{\Delta H_{rc,id}}{\Delta H_{p,id}}\right)}\right] \tag{C39}$$

where

$$y_{sc} = y_{sc}^*\sqrt{\frac{T_p}{T_{sc}}} \tag{C30}$$

$$y_{rc} = y_{rc}^*\sqrt{\frac{T_p}{T_{rc}}}\sqrt{\left(\frac{\hat{T}_{rc}^*}{T_{rc}^*}\right)\left(\frac{T_{rc}}{\hat{T}_{rc}}\right)} \tag{C32}$$

$$\frac{\Delta H_{sc,id}}{\Delta H_{p,id}} = \frac{T_{sc}}{T_p}\left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*}\right) = \frac{T_{sc}}{T_p}\left[\frac{1 - \left(\frac{P_{re}^*}{P_{sc}^*}\right)^{(\gamma-1)/\gamma}}{1 - \left(\frac{P_{re}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}}\right] \tag{C42}$$

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p}\left[\frac{1 - \left(\frac{\hat{T}_{rc}}{T_{rc}}\right)\left(\frac{T_{rc}^*}{\hat{T}_{rc}^*}\right)\left(\frac{P_{re}^*}{P_{rc}^*}\right)^{(\gamma-1)/\gamma}}{1 - \left(\frac{P_{re}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}}\right] \tag{C44}$$

$$\frac{\hat{T}_{rc}^*}{T_{rc}^*} = 1 + \frac{\frac{U_m^{*2}}{T_p^*}}{2gJC_p} \tag{C33}$$

$$\frac{\hat{T}_{rc}}{T_{rc}} = 1 + \frac{T_p}{T_{rc}} \left(\frac{1 + y_{sc}^* \sqrt{\frac{T_{sc}}{T_p}}}{1 + y_{sc}^* \sqrt{\frac{T_p}{T_{sc}}}} \right) \left(\frac{\frac{U_m^{*2}}{T_p^*}}{2gJC_P} \right) \tag{C36}$$

Corresponding results are given in appendix D for the constant-absolute-momentum-ratio case. The main differences are in the rotor blade coolant fraction and the ideal enthalpy change.

RESULTS AND DISCUSSION

The aerodynamic performance at actual total-temperature ratios T_p/T_c that was predicted from the experimental results obtained at a reduced total-temperature ratio is compared in this section with the available data. The stator results (ref. 12) are discussed first, followed by the turbine results (ref. 13).

Cooled Stator Performance

The experimental aerodynamic performance of two geometrically similar stator vanes with different cooling designs (convection-film cooling and transpiration cooling) is presented in reference 12. The two vane configurations are shown in figure 1, and schematic cross-sectional views of these vanes are shown in figure 2. For the convection-film-cooled vane (fig. 2(a)), the coolant was ejected from film cooling slots on the pressure and suction surfaces and from a trailing-edge slot. The transpiration vane (fig. 2(b)) ejected the coolant over most of the vane surface in a direction normal to the vane surface. The stator tests were performed at nominal total-temperature ratios T_p/T_c of 1.0, 1.75, and 2.75 and at total-pressure ratios P_c/P_p of 1.0, 1.2, and 1.5.

Convection-film-cooled vane. - The stator efficiency η and coolant mass flow fraction y_{sc} for the convection-film-cooled vane are shown in figures 3 and 4, respectively. The theoretical curves, shown by solid lines in these figures, were calculated from the results obtained at a total-temperature ratio T_p/T_c of 1 by the method given in appendix B. The calculated trends of efficiency (primary efficiency decreasing and thermodynamic efficiency generally increasing) with increasing total-temperature ratio (fig. 3) agree well with the experimental data. The agreement is slightly better for the thermodynamic efficiency than for the primary efficiency. This is due, in part, to the coolant mass flow fraction, which has a larger effect on the primary than on the thermodynamic efficiency. Also, at a total-pressure ratio P_c/P_p of 1, the experimental data agree very well with the predicted independence of the thermodynamic efficiency and the total-temperature ratio T_p/T_c.

The calculated trends of coolant mass flow fraction with total-temperature ratio (fig. 4) show fair agreement with the experimental data. At the highest total-temperature ratio, the experimental results are about 10 percent lower than the calculated values. This may be due to heat transfer to the coolant, between the total-temperature-measuring station and the vane cavity. This would tend to increase the calculated coolant flow rate. A 10 percent error in the predicted value of the coolant fraction y_{sc} results in about a 0.3 percent error in the predicted primary efficiency. The agreement is, therefore, felt to be satisfactory as far as estimating the performance is concerned.

Transpiration-cooled vane. - The stator efficiency η and coolant mass flow fraction y_{sc} for the transpiration-cooled vane are shown in figures 5 and 6, respectively. The agreement between the calculated efficiencies and coolant fractions and the experimental results, although reasonable, is not as good as for the convection-film-cooled vane. The larger differences between the theoretical and experimental results may be due to the method of coolant ejection, which for these vanes is normal to the surface. However, at a total-pressure ratio P_c/P_p of 1, the thermodynamic efficiency is again seen to be fairly constant experimentally, in agreement with the theoretical prediction.

The poorer agreement between experiment and theory at a total-pressure ratio of 1.5 (fig. 5) was further investigated by estimating the efficiency from the experimental data at a total-temperature ratio of 1.78. The results are shown in figure 5 by dashed lines. The good agreement at a total-temperature ratio of 2.75 suggests that the performance measurements at a total-temperature ratio of 1.0 may be in error. Of course, many other explanations are possible.

Cooled-Core Turbine Performance

The experimental aerodynamic performance of a film-cooled, single-stage core turbine is presented in reference 13. Both stator and turbine performance data are given in the reference. The cooled vane and blade are shown in figure 7, and schematic cross-sectional views are given in figure 8. Cooling air was provided to vanes, blades, and all endwalls. The vanes and blades each have 45 and 35 rows of cooling holes, respectively. The coolant is ejected from both the vanes and the blades at approximately 35° to the surface and in line with the flow.

The stator and turbine tests were performed at nominal total-temperature ratios T_p/T_c of 1.1, 1.75, and 2.7. For these tests the total-pressure ratios P_c/P_p were maintained constant at about 1.07 (rotor blade coolant value, 0.73). For the turbine tests, as discussed previously, this results in the rotor blade coolant to rotor inlet absolute momentum ratio being kept constant. At a total-temperature ratio of 2.7, the effect of varying the rotor blade coolant flow (by changing the rotor blade coolant absolute total pressure) on performance was also investigated experimentally. From these tests, it was then possible to compare the theory and the results for the case where the rotor blade coolant to rotor inlet relative momentum ratio is constant. (This corresponds to the rotor blade coolant relative total-pressure to primary total-pressure ratios $\hat{P}_{rc}/P_p$ being constant.) The stator tests are discussed first, followed by the turbine tests.

Core stator. - The stator efficiency η and the coolant mass flow fractions y_{sc} as a function of total-temperature ratio T_p/T_c are shown in figure 9. These tests were performed at the same conditions that existed for the turbine design-point investigation. The inlet total-pressure to exit static-pressure ratio was 1.50. The agreement between the theoretical and experimental results for efficiency (fig. 9(a)) is not good. The efficiencies, at total-temperature ratios T_p/T_c of 1.77 and 2.70, are about two points higher than would be estimated from the reduced-temperature-ratio results. The estimated efficiency based on the experimental results at a total-temperature ratio of 1.77 is also shown in figure 9(a) by the dashed line. The experimental results at a total-temperature ratio of 2.70 agree well with this estimation. As for the transpiration-cooled vanes, this again indicates the possibility that the measurements at the lower total-temperature ratio (i.e., 1.14) may be in error.

In general, the coolant mass flow fractions, shown in figure 9(b), agree well with the theoretical calculations. The good agreement at the higher total-temperature ratios suggests that the heat transfer effects that may have influenced the previous stator coolant flows (ref. 12) were not significant for these tests. This is probably due to the total temperatures being measured closer to the points of coolant injection for these tests.

Core turbine. - The turbine efficiency $\bar{\eta}$ as a function of total-temperature ratio T_p/T_c is shown in figure 10. These tests were performed at the turbine design inlet to exit total-pressure ratio of 1.84. Experimental and theoretical results for both constant relative and constant absolute momentum ratios are shown in figures 10(a) and (b), respectively. As discussed previously, this corresponds experimentally to constant $\hat{P}_{rc}/P_p$ and constant P_{rc}/P_p, respectively. The theoretical curves, shown as solid lines in the figure, were calculated by the methods described in appendixes C and D.

The theoretical results for the two cases (i.e., constant relative and constant absolute momentum ratios) are quite similar. The primary efficiency $\bar{\eta}_p$ decreases somewhat faster for the constant-relative-momentum-ratio case with increasing total-temperature ratio. Also, the thermodynamic efficiency $\bar{\eta}_t$ is relatively independent of total-temperature ratio for both cases. This results from the fact that the total-pressure ratios P_c/P_p were only slightly greater than 1 for these tests (except for the rotor coolant).

For the constant-relative-momentum-ratio case (fig. 10(a)), the agreement between the theory and the single experimental point is good. However, more experimental data are needed in this mode of operation to assess its general validity. For the constant-absolute-momentum-ratio case (fig. 10(b)), the agreement between the theoretical and experimental results is reasonable, although not as good as for the constant-relative-momentum-ratio case.

The coolant mass flow fractions y_c are shown in figure 11 as a function of total-temperature ratio T_p/T_c. As discussed previously, the rotor blade coolant flow rate depends on the relative total conditions in the rotor blade cavity (see eq. (12a)). Therefore, the two different modes of operation (i.e., constant relative and constant absolute momentum ratios) result in different experimental as well as different theoretical results. In general, the agreement between the theoretical and experimental results is good. For the rotor blade coolant fraction the agreement is less satisfactory. This is probably due to the uncertainty of the calculated relative total conditions for the rotor blade coolant, which were based on isentropic flow. A 10 percent error in the estimated rotor coolant fraction results in about a 0.3 percent error in the estimated efficiency. The rotor blade coolant fraction estimation is, therefore, felt to be satisfactory, as far as estimating the efficiency is concerned.

Comparing the coolant fractions for the stator and turbine tests (figs. 9(b) and 11(b), respectively) shows that the stator vane and stator tip wall coolant fractions are very similar for the two tests. However, the stator hub wall coolant fractions are different. For the stator tests, the hub wall coolant fractions were higher at total-temperature ratios T_p/T_c of 1.14 and 1.77 and about the same at 2.70, in comparison with the turbine tests. The reason for this behavior is not known.

SUMMARY OF RESULTS

The prediction of the aerodynamic performance, for both cooled stators and turbines, at actual primary to coolant inlet total-temperature ratios T_p/T_c from the results obtained at a reduced total-temperature ratio has been described herein. Because turbine rotation affects the rotor blade coolant flow, the theoretical predictions for two possible methods of turbine testing were investigated. These are (1) constant rotor blade coolant to rotor inlet relative momentum ratio (constant ratio of rotor blade coolant relative total pressure to primary total pressure), and (2) constant rotor blade coolant to rotor inlet absolute momentum ratio (constant ratio of rotor blade coolant absolute total pressure to primary total pressure).

The theory has been compared with the available experimental results. The experimental results include stator performance data for two geometrically similar vanes with different cooling designs (convection-film cooling and transpiration cooling) and for a film-cooled core stator vane. These tests were performed for total-temperature ratios T_p/T_c from near 1.0 to about 2.7. The results for the turbine include performance data for a film-cooled, single-stage core turbine tested over the same total-temperature-ratio range. The following results were obtained:

1. The theoretical results indicated that for a coolant to primary inlet total-pressure ratio P_c/P_p of 1, the thermodynamic efficiency is independent of total-temperature ratio T_p/T_c for stators and is nearly independent of this ratio for turbines. For total-pressure ratios P_c/P_p greater than 1, the primary efficiency decreases and the thermodynamic efficiency increases with increasing total-temperature ratio T_p/T_c.

2. For the stator, the experimental results agreed reasonably well with the theory for the convection-film-cooled and transpiration-cooled vanes. The agreement was not good for the limited data available for the film-cooled core vane.

3. For the film-cooled core turbine, based on the limited data available, the constant-relative-momentum-ratio method of testing agreed well with the theory. The agreement was also reasonable for the constant-absolute-momentum-ratio method of testing.

4. For both stator and turbine tests, fair agreement was found, in general, between the predicted and measured coolant mass flow fractions. This was considered satisfactory as far as predicting the efficiency was concerned.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 26, 1976,
505-04.

APPENDIX A

SYMBOLS

c_p specific heat at constant pressure, J/kg-K; Btu/lbm-^{o}R

g force-mass conversion constant, 32.174 lbm-ft/lbf-sec^2

H total enthalpy, J/kg; Btu/lbm

h specific enthalpy, J/kg; Btu/lbm

J mechanical equivalent of heat, 778.0 ft-lbf/Btu

m mass flow rate, kg/sec; lbm/sec

P total pressure, N/m^2; lbf/ft^2

p static pressure, N/m^2; lbf/ft^2

T total temperature, K; ^{o}R

U_m blade speed at mean radius, m/sec; ft/sec

V absolute velocity, m/sec; ft/sec

W relative velocity, m/sec; ft/sec

y coolant to primary mass flow fraction, m_c/m_p

α absolute flow angle measured from axial direction, deg

γ ratio of specific heats

η stator efficiency based on kinetic energy

$\bar{\eta}$ turbine efficiency

ρ density, kg/m^3; lbm/ft^3

τ torque, N-m; ft-lbf

ω angular velocity, rad/sec

Subscripts:

c coolant

cr flow conditions at Mach 1

id ideal or isentropic

M mass averaged

p primary or stator inlet

r rotor

rc rotor blade coolant

re rotor exit

s stator

sc stator vane coolant

se stator exit (rotor inlet)

t thermodynamic

Superscripts:

* conditions at $T_p/T_c = 1.0$

^ relative conditions

APPENDIX B

CALCULATION OF STATOR PERFORMANCE FROM RESULTS AT A PRIMARY TO COOLANT INLET TOTAL-TEMPERATURE RATIO OF 1

The primary efficiency η_p can be written by using equations (1) and (8b) as

$$\eta_p = \frac{(m_p + m_{sc})\,\Delta h_s}{m_p\,\Delta h_{p,id}} = \frac{(m_p T_p + m_{sc} T_{sc})\dfrac{\Delta h_s}{T_{se}}}{m_p T_p\left(\dfrac{\Delta h_{p,id}}{T_p}\right)} \tag{B1}$$

or

$$\eta_p = \frac{\left(1 + y_{sc}\dfrac{T_{sc}}{T_p}\right)\dfrac{\Delta h_s}{T_{se}}}{\dfrac{\Delta h_{p,id}}{T_p}} \tag{B2}$$

where the stator coolant mass flow fraction is defined as

$$y_{sc} = \frac{m_{sc}}{m_p} \tag{B3}$$

At a total-temperature ratio T_p/T_{sc} of 1, equation (B2) becomes

$$\eta_p^* = \frac{(1 + y_{sc}^*)\dfrac{\Delta h_s^*}{T_{se}^*}}{\dfrac{\Delta h_{p,id}^*}{T_p^*}} \tag{B4}$$

From equations (3b) and (7b) we find that

$$\frac{\Delta h_{p,id}}{T_p} = \frac{\Delta h^*_{p,id}}{T^*_p} \tag{B5}$$

Dividing equation (B2) by (B4) and using equations (B5) and (8a) give

$$\frac{\eta_p}{\eta^*_p} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right)}{1 + y^*_{sc}} \tag{B6}$$

The stator coolant mass flow fraction y_{sc} is obtained from equations (5a) and (5b) as

$$y_{sc} = \frac{m_{sc}}{m_p} = \frac{m^*_{sc}\sqrt{\frac{T^*_{sc}}{T_{sc}}\frac{P_{sc}}{P^*_{sc}}}}{m^*_p\sqrt{\frac{T^*_p}{T_p}\frac{P_p}{P^*_p}}} = y^*_{sc}\sqrt{\frac{T_p}{T_{sc}}}\sqrt{\frac{T^*_{sc}}{T^*_p}}\left(\frac{P_{sc}}{P_p}\right)\left(\frac{P^*_p}{P^*_{sc}}\right) \tag{B7}$$

or from equations (6) and (8c) as

$$y_{sc} = y^*_{sc}\sqrt{\frac{T_p}{T_{sc}}} \tag{B8}$$

Therefore, equation (B6) becomes finally

$$\frac{\eta_p}{\eta^*_p} = \frac{1 + y^*_{sc}\sqrt{\frac{T_{sc}}{T_p}}}{1 + y^*_{sc}} \tag{B9}$$

The thermodynamic efficiency η_t can be written by using equations (2) and (1) as

$$\eta_t = \frac{\eta_p m_p \Delta h_{p,id}}{m_p \Delta h_{p,id} + m_{sc} \Delta h_{sc,id}} = \frac{\eta_p}{1 + y_{sc}\left(\dfrac{\Delta h_{sc,id}}{\Delta h_{p,id}}\right)} \tag{B10}$$

At a total-temperature ratio of 1, equation (B10) becomes

$$\eta_t^* = \frac{\eta_p^*}{1 + y_{sc}^*\left(\dfrac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right)} \tag{B11}$$

Dividing equation (B10) by (B11) gives

$$\frac{\eta_t}{\eta_t^*} = \frac{\eta_p}{\eta_p^*}\left[\frac{1 + y_{sc}^*\left(\dfrac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right)}{1 + y_{sc}\left(\dfrac{\Delta h_{sc,id}}{\Delta h_{p,id}}\right)}\right] \tag{B12}$$

From equations (3), (6), and (7a) we find that

$$\frac{\Delta h_{sc,id}}{\Delta h_{p,id}} = \frac{T_{sc}}{T_p}\frac{\left[1 - \left(\dfrac{p_{se}}{P_{sc}}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\dfrac{p_{se}}{P_p}\right)^{(\gamma-1)/\gamma}\right]} = \frac{T_{sc}}{T_p}\frac{\left[1 - \left(\dfrac{p_{se}^*}{P_{sc}^*}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\dfrac{p_{se}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}\right]} = \frac{T_{sc}}{T_p}\left(\frac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right) \tag{B13}$$

and using equation (B11) gives

$$\frac{\Delta h_{sc,id}}{\Delta h_{p,id}} = \frac{T_{sc}}{T_p}\left(\frac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*}\right) = \frac{T_{sc}}{T_p}\left(\frac{\eta_p^*}{\eta_t^*} - 1\right)\frac{1}{y_{sc}^*} \tag{B14}$$

So that, from equations (B8), (B14), and (B9), equation (B12) becomes finally

$$\frac{\eta_t}{\eta_t^*} = \left(\frac{1 + y_{sc}^* \sqrt{\dfrac{T_{sc}}{T_p}}}{1 + y_{sc}^*}\right)\left[\frac{\dfrac{\eta_p^*}{\eta_t^*}}{1 + \sqrt{\dfrac{T_{sc}}{T_p}}\left(\dfrac{\eta_p^*}{\eta_t^*} - 1\right)}\right] \tag{B15}$$

For the special case when the total-pressure ratio P_{sc}/P_p is 1,

$$\frac{\Delta h_{sc,id}^*}{\Delta h_{p,id}^*} = 1 \tag{B16}$$

and equation (B14) becomes

$$y_{sc}^* = \frac{\eta_p^*}{\eta_t^*} - 1 \tag{B17}$$

so that equation (B15) is

$$\frac{\eta_t}{\eta_t^*} = \left(\frac{1 + y_{sc}^* \sqrt{\dfrac{T_{sc}}{T_p}}}{1 + y_{sc}^*}\right)\left(\frac{1 + y_{sc}^*}{1 + \sqrt{\dfrac{T_{sc}}{T_p}}\, y_{sc}^*}\right) = 1 \tag{B18}$$

or

$$\eta_t = \eta_t^* = \text{Constant} \tag{B19}$$

APPENDIX C

CALCULATION OF TURBINE PERFORMANCE FROM RESULTS AT A PRIMARY TO COOLANT INLET TOTAL-TEMPERATURE RATIO OF 1 FOR A CONSTANT RELATIVE MOMENTUM RATIO

Turbine Relative and Absolute Total Conditions

The rotor blade coolant to rotor inlet relative momentum ratio $m_{rc}w_{rc}/m_{se}w_{se}$ is maintained constant if the relative total-pressure ratio is constant. Therefore,

$$\frac{\hat{P}_{rc}}{\hat{P}_{se}} = \frac{\hat{P}^*_{rc}}{\hat{P}^*_{se}} \tag{C1}$$

Under these conditions the loss in relative total pressure (as a percentage of inlet total pressure) for the rotor is assumed to be constant. This is analogous to the stator assumption (eq. (7a)). Therefore,

$$\frac{\hat{P}_{re}}{\hat{P}_{se}} = \frac{\hat{P}^*_{re}}{\hat{P}^*_{se}} \tag{C2}$$

The relations between the relative and absolute total conditions at the stator exit (or rotor inlet) for an axial turbine are (ref. 15)

$$\frac{\hat{T}_{se}}{T_{se}} = 1 + \frac{W^2_{se}}{2gJC_pT_{se}} - \frac{V^2_{se}}{2gJC_pT_{se}} \tag{C3a}$$

$$\frac{\hat{P}_{se}}{P_{se}} = \left(\frac{\hat{T}_{se}}{T_{se}}\right)^{\gamma/(\gamma-1)} \tag{C3b}$$

From equations (3a) and (8a) we have that

$$\frac{V^2_{se}}{T_{se}} = \frac{V^{*2}_{se}}{T^*_{se}} \tag{C4}$$

or

$$\left(\frac{V}{V_{cr}}\right)_{se} = \left(\frac{V^*}{V^*_{cr}}\right)_{se} \tag{C5}$$

The relative velocity W_{se} is related to V_{se} and to the wheel speed U_m by the velocity diagram at the stator exit. The wheel speed is experimentally set so that $U_m/\sqrt{T_{se}} = U^*_m/\sqrt{T^*_{se}}$ (eq. (15)). Therefore,

$$\left(\frac{U_m}{V_{cr}}\right)_{se} = \left(\frac{U^*_m}{V^*_{cr}}\right)_{se} \tag{C6}$$

If we assume that the nozzle flow angle is constant (i.e., $\alpha_{se} = \alpha^*_{se}$), the velocity diagram relations give

$$\left(\frac{W}{V_{cr}}\right)_{se} = \left(\frac{W^*}{V^*_{cr}}\right)_{se} \tag{C7}$$

or

$$\frac{W^2_{se}}{T_{se}} = \frac{W^{*\,2}_{se}}{T^*_{se}} \tag{C8}$$

Therefore, equations (C3a) and (C3b) result in

$$\frac{\hat{T}_{se}}{T_{se}} = \frac{\hat{T}^*_{se}}{T^*_{se}} \tag{C9}$$

$$\frac{\hat{P}_{se}}{P_{se}} = \frac{\hat{P}^*_{se}}{P^*_{se}} \tag{C10}$$

From equations (C1), (C10), and (7a) we find that

$$\frac{\hat{P}_{rc}}{P_p} = \left(\frac{\hat{P}_{rc}}{\hat{P}_{se}}\right)\left(\frac{\hat{P}_{se}}{P_{se}}\right)\left(\frac{P_{se}}{P_p}\right) = \left(\frac{\hat{P}^*_{rc}}{\hat{P}^*_{se}}\right)\left(\frac{\hat{P}^*_{se}}{P^*_{se}}\right)\left(\frac{P^*_{se}}{P^*_{se}}\right) = \frac{\hat{P}^*_{rc}}{P^*_p} \tag{C11}$$

Therefore, the rotor blade coolant to rotor inlet relative momentum ratio is maintained constant by experimentally setting the rotor blade coolant relative total-pressure to primary inlet total-pressure ratio constant.

At the rotor exit, the total temperature T_{re} is obtained from the assumed performance relation (i.e., eq. (14a))

$$\frac{\Delta H_r}{T_m} = \frac{C_P(T_M - T_{re})}{T_M} = \frac{\Delta H_r^*}{T_M^*} = \frac{C_P(T_M^* - T_{re}^*)}{T_M^*} \tag{C12}$$

or

$$\frac{T_{re}}{T_M} = \frac{T_{re}^*}{T_M^*} = \frac{T_{re}^*}{T_{se}^*} \tag{C13}$$

It is assumed that $T_M \approx T_{se}$, so that equation (C13) gives approximately

$$\frac{T_{re}}{T_{se}} \approx \frac{T_{re}^*}{T_{se}^*} \tag{C14}$$

The relative total temperature across the rotor is assumed to be constant (i.e., the effect of the rotor blade coolant is small). Therefore,

$$\hat{T}_{se} \approx \hat{T}_{re} \tag{C15}$$

$$\hat{T}_{se}^* \approx \hat{T}_{re}^* \tag{C16}$$

From equations (C15), (C9), (C14), and (C16) we find

$$\frac{\hat{T}_{re}}{T_{re}} \approx \frac{\hat{T}_{se}}{T_{re}} = \left(\frac{\hat{T}_{se}}{T_{se}}\right)\left(\frac{T_{se}}{T_{re}}\right) \approx \left(\frac{\hat{T}_{se}^*}{T_{se}^*}\right)\left(\frac{T_{se}^*}{T_{re}^*}\right) = \frac{\hat{T}_{se}^*}{T_{re}^*} \approx \frac{\hat{T}_{re}^*}{T_{re}^*} \tag{C17}$$

and

$$\frac{\hat{P}_{re}}{P_{re}} = \left(\frac{\hat{T}_{re}}{T_{re}}\right)^{\gamma/(\gamma-1)} \approx \left(\frac{\hat{T}_{re}^*}{T_{re}^*}\right)^{\gamma/(\gamma-1)} = \frac{\hat{P}_{re}^*}{P_{re}^*} \tag{C18}$$

Therefore, equations (C18), (C2), (C10), and (7a) result in

$$\frac{P_{re}}{P_p} = \left(\frac{P_{re}}{\hat{P}_{re}}\right)\left(\frac{\hat{P}_{re}}{\hat{P}_{se}}\right)\left(\frac{\hat{P}_{se}}{P_{se}}\right)\left(\frac{P_{se}}{P_p}\right) \approx \left(\frac{P^*_{re}}{\hat{P}^*_{re}}\right)\left(\frac{\hat{P}^*_{re}}{\hat{P}^*_{se}}\right)\left(\frac{\hat{P}^*_{se}}{P^*_{se}}\right)\left(\frac{P^*_{se}}{P^*_p}\right) = \frac{P^*_{re}}{P^*_p} \tag{C19}$$

The rotor exit static-pressure to total-pressure ratio is obtained from equations (11) and (C19) as

$$\frac{p_{re}}{P_{re}} = \left(\frac{p_{re}}{P_p}\right)\left(\frac{P_p}{P_{re}}\right) \approx \left(\frac{p^*_{re}}{P^*_p}\right)\left(\frac{P^*_p}{P^*_{re}}\right) = \frac{p^*_{re}}{P^*_{re}} \tag{C20}$$

which is similar to the relation found at the stator exit (see eq. (7b)). From equations (C19), (6), and (C11) we also find that

$$\frac{P_{re}}{P_{sc}} = \left(\frac{P_{re}}{P_p}\right)\left(\frac{P_p}{P_{sc}}\right) \approx \left(\frac{P^*_{re}}{P^*_p}\right)\left(\frac{P^*_p}{P^*_{sc}}\right) = \frac{P^*_{re}}{P^*_{sc}} \tag{C21}$$

$$\frac{P_{re}}{\hat{P}_{rc}} = \left(\frac{P_{re}}{P_p}\right)\left(\frac{P_p}{\hat{P}_{rc}}\right) \approx \left(\frac{P^*_{re}}{P_p}\right)\left(\frac{P^*_p}{\hat{P}^*_{rc}}\right) = \frac{P^*_{re}}{\hat{P}^*_{rc}} \tag{C22}$$

Turbine Performance

The primary turbine efficiency $\overline{\eta}_p$ can be written by using equations (9a) and (13b) as

$$\overline{\eta}_p = \frac{(m_p + m_{sc} + m_{rc})\,\Delta H_r}{m_p\,\Delta H_{p,\,id}} = \frac{(m_pT_p + m_{sc}T_{sc} + m_{rc}T_{rc})\dfrac{\Delta H_r}{T_M}}{m_pT_p\left(\dfrac{\Delta H_{p,\,id}}{T_p}\right)} \tag{C23}$$

or

$$\bar{\eta}_p = \frac{\left[1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right) + y_{rc}\left(\frac{T_{rc}}{T_p}\right)\right]\frac{\Delta H_r}{T_M}}{\frac{\Delta H_{p,id}}{T_p}} \tag{C24}$$

where the coolant mass flow fractions are defined as

$$y_{sc} = \frac{m_{sc}}{m_p} \tag{C25}$$

$$y_{rc} = \frac{m_{rc}}{m_p} \tag{C26}$$

At a total-temperature ratio T_p/T_c of 1, equation (C24) becomes

$$\bar{\eta}_p^* = \frac{(1 + y_{sc}^* + y_{rc}^*)\frac{\Delta H_r^*}{T_M^*}}{\frac{\Delta H_{p,id}^*}{T_p^*}} \tag{C27}$$

From equations (10a) and (C19) we find

$$\frac{\Delta H_{p,id}}{T_p} = \frac{\Delta H_{p,id}^*}{T_p^*} \tag{C28}$$

so that dividing equation (C24) by (C27) and using equations (C28) and (14a) give

$$\frac{\bar{\eta}_p}{\bar{\eta}_p^*} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right) + y_{rc}\left(\frac{T_{rc}}{T_p}\right)}{1 + y_{sc}^* + y_{rc}^*} \tag{C29}$$

which is similar to equation (B6) for the stator primary efficiency. The stator vane coolant mass flow fraction y_{sc} is again given by equation (B8), and the rotor blade coolant mass flow fraction y_{rc} is found from equations (12a) and (5a). That is,

$$y_{sc} = y_{sc}^* \sqrt{\frac{T_p}{T_{sc}}} \tag{C30}$$

$$y_{rc} = y_{rc}^* \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_p^*}\right)\left(\frac{T_p}{\hat{T}_{rc}}\right)\left(\frac{\hat{P}_{rc}}{P_p}\right)\left(\frac{P_p^*}{\hat{P}_{rc}^*}\right)} \tag{C31}$$

Using equation (C11) gives

$$y_{rc} = y_{rc}^* \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_p^*}\right)\left(\frac{T_p}{\hat{T}_{rc}}\right)} = y_{rc}^* \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_{rc}^*}\right)\left(\frac{T_{rc}}{\hat{T}_{rc}}\frac{T_p}{T_{rc}}\right)} = y_{rc}^* \sqrt{\frac{T_p}{T_{rc}}} \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_{rc}^*}\right)\left(\frac{T_{rc}}{\hat{T}_{rc}}\right)} \tag{C32}$$

The rotor blade coolant relative total temperature is calculated from equations (12b) and (16) as

$$\frac{\hat{T}_{rc}^*}{T_{rc}^*} = 1 + \frac{U_m^{*\,2}}{2gJC_PT_{rc}^*} = 1 + \frac{\dfrac{U_m^{*\,2}}{T_p^*}}{2gJC_P} \tag{C33}$$

and

$$\frac{\hat{T}_{rc}}{T_{rc}} = 1 + \frac{U_m^2}{2gJC_PT_{rc}} = 1 + \left(\frac{T_p}{T_{rc}}\right)\left(\frac{T_{se}}{T_p}\right)\left(\frac{\dfrac{U_m^{*\,2}}{T_p^*}}{2gJC_P}\right) \tag{C34}$$

From equations (8b) and (C30) we find

$$\frac{T_{se}}{T_p} = \frac{m_p T_p + m_{sc} T_{sc}}{(m_p + m_{sc}) T_p} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right)}{1 + y_{sc}} = \frac{1 + y_{sc}^* \sqrt{\frac{T_{sc}}{T_p}}}{1 + y_{sc}^* \sqrt{\frac{T_p}{T_{sc}}}} \tag{C35}$$

so that equation (C34) becomes

$$\frac{\hat{T}_{rc}}{T_{rc}} = 1 + \frac{T_p}{T_{rc}}\left(\frac{1 + y_{sc}^* \sqrt{\frac{T_{sc}}{T_p}}}{1 + y_{sc}^* \sqrt{\frac{T_p}{T_{sc}}}}\right)\left(\frac{\frac{U_m^{*2}}{T_p^*}}{2gJC_p}\right) \tag{C36}$$

Therefore, the rotor blade coolant mass flow fraction y_{rc} can be found from equations (C32), (C33), and (C36) and the measured values at a total-temperature ratio T_p/T_c of 1. The primary efficiency $\bar{\eta}_p$ can then be calculated from equations (C29) and (C30).

Similarly, thermodynamic efficiency $\bar{\eta}_t$ can be written by using equations (9b) and (9a) as

$$\bar{\eta}_t = \frac{\bar{\eta}_p m_p \, \Delta H_{p,id}}{m_p \, \Delta H_{p,id} + m_{sc} \, \Delta H_{sc,id} + m_{rc} \, \Delta H_{rc,id}} = \frac{\bar{\eta}_p}{1 + y_{sc}\left(\frac{\Delta H_{sc,id}}{\Delta H_{p,id}}\right) + y_{rc}\left(\frac{\Delta H_{rc,id}}{\Delta H_{p,id}}\right)} \tag{C37}$$

and

$$\bar{\eta}_t^* = \frac{\bar{\eta}_p^*}{1 + y_{sc}^*\left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*}\right) + y_{rc}^*\left(\frac{\Delta H_{rc,id}^*}{\Delta H_{p,id}^*}\right)} \tag{C38}$$

Therefore,

$$\frac{\bar{\eta}_t}{\bar{\eta}_t^*} = \frac{\bar{\eta}_p}{\bar{\eta}_p^*}\left[\frac{1 + y_{sc}^*\left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*}\right) + y_{rc}^*\left(\frac{\Delta H_{rc,id}^*}{\Delta H_{p,id}^*}\right)}{1 + y_{sc}\left(\frac{\Delta H_{sc,id}}{\Delta H_{p,id}}\right) + y_{rc}\left(\frac{\Delta H_{rc,id}}{\Delta H_{p,id}}\right)}\right] \tag{C39}$$

From equation (10) we find

$$\frac{\Delta H_{sc,id}}{\Delta H_{p,id}} = \frac{T_{sc}}{T_p}\frac{\left[1 - \left(\frac{P_{re}}{P_{sc}}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\frac{P_{re}}{P_p}\right)^{(\gamma-1)/\gamma}\right]} \tag{C40}$$

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p}\frac{\left[1 - \left(\frac{P_{re}}{P_{rc}}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\frac{P_{re}}{P_p}\right)^{(\gamma-1)/\gamma}\right]} \tag{C41}$$

Substituting equations (C19), (C21), and (C22) into equations (C40) and (C41) gives

$$\frac{\Delta H_{sc,id}}{\Delta H_{p,id}} = \frac{T_{sc}}{T_p}\left[\frac{1 - \left(\frac{P_{re}^*}{P_{sc}^*}\right)^{(\gamma-1)/\gamma}}{1 - \left(\frac{P_{re}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}}\right] = \frac{T_{sc}}{T_p}\left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*}\right) \tag{C42}$$

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p}\left[\frac{1 - \left(\frac{\hat{P}_{rc}}{P_{rc}}\frac{P_{rc}^*}{\hat{P}_{rc}^*}\frac{P_{re}^*}{P_{rc}^*}\right)^{(\gamma-1)/\gamma}}{1 - \left(\frac{P_{re}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}}\right] \tag{C43}$$

Substituting the rotor blade coolant relative to absolute total-pressure ratio from equations (12c) gives

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p}\left[\frac{1 - \left(\frac{\hat{T}_{rc}}{T_{rc}}\right)\left(\frac{T^*_{rc}}{\hat{T}^*_{rc}}\right)\left(\frac{P^*_{rc}}{P^*_{rc}}\right)^{(\gamma-1)/\gamma}}{1 - \left(\frac{P^*_{re}}{P^*_p}\right)^{(\gamma-1)/\gamma}}\right] \tag{C44}$$

where the rotor blade coolant relative to absolute total-temperature ratios are given by equations (C33) and (C36).

From equations (C33) and (C36) it is seen that

$$\frac{\hat{T}_{rc}}{T_{rc}} \neq \frac{\hat{T}^*_{rc}}{T^*_{rc}} \tag{C45}$$

so that, equation (C44) gives

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} \neq \frac{T_{rc}}{T_p}\left(\frac{\Delta H^*_{rc,id}}{\Delta H^*_{p,id}}\right) \tag{C46}$$

Of course, equations (C44), (C33), and (C36) can be used to find $\Delta H_{rc,id}/\Delta H_{p,id}$; and then equation (C39) gives the thermodynamic efficiency $\overline{\eta}_t$.

APPENDIX D

CALCULATION OF TURBINE PERFORMANCE FROM RESULTS AT A PRIMARY TO COOLANT INLET TOTAL-TEMPERATURE RATIO OF 1 FOR A CONSTANT ABSOLUTE MOMENTUM RATIO

The rotor blade coolant to rotor inlet absolute momentum ratio $M_{rc}V_{rc}/M_{se}V_{se}$ is maintained constant if the absolute total-pressure ratio is constant. Therefore,

$$\frac{P_{rc}}{P_{se}} = \frac{P_{rc}^*}{P_{se}^*} \tag{D1}$$

or substituting equation (7a) gives

$$\frac{P_{rc}}{P_{p}} = \frac{P_{rc}^*}{P_{p}^*} \tag{D2}$$

Under these test conditions, it is assumed that the rotor absolute total-pressure loss (as a percentage of inlet total pressure) is constant. That is,

$$\frac{P_{re}}{P_{se}} = \frac{P_{re}^*}{P_{se}^*} \tag{D3}$$

From equations (7a), (6), and (D1) we find

$$\frac{P_{re}}{P_{p}} = \frac{P_{re}^*}{P_{p}^*} \tag{D4}$$

$$\frac{P_{re}}{P_{sc}} = \frac{P_{re}^*}{P_{sc}^*} \tag{D5}$$

$$\frac{P_{re}}{P_{rc}} = \frac{P_{re}^*}{P_{rc}^*} \tag{D6}$$

which can be compared to equations (C19), (C21), and (C22) for the constant-relative-momentum-ratio case.

The turbine performance conditions are similar to those given in appendix C. The main difference is in the rotor blade coolant mass flow fraction y_{rc} and the ideal enthalpy change $\Delta H_{rc,id}$. The primary turbine efficiency $\bar{\eta}_p$ can again be calculated from equations (C29) and (C30) as

$$\frac{\bar{\eta}_p}{\bar{\eta}_p^*} = \frac{1 + y_{sc}\left(\frac{T_{sc}}{T_p}\right) + y_{rc}\left(\frac{T_{rc}}{T_p}\right)}{1 + y_{sc}^* + y_{rc}^*} \tag{D7}$$

$$y_{sc} = y_{sc}^* \sqrt{\frac{T_p}{T_{sc}}} \tag{D8}$$

Assuming that equation (C31) is still approximately true for the constant-absolute-momentum-ratio case, the rotor blade coolant mass flow fraction becomes (using eq. (D2))

$$y_{rc} = y_{rc}^* \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_p^*}\right)\left(\frac{T_p}{\hat{T}_{rc}}\right)\left(\frac{\hat{P}_{rc}}{P_p}\right)\left(\frac{P_p^*}{\hat{P}_{rc}^*}\right)} = \sqrt{\left(\frac{\hat{T}_{rc}^*}{T_{rc}^*}\right)\left(\frac{T_{rc}}{\hat{T}_{rc}}\right)\frac{T_p}{T_{rc}}\left(\frac{\hat{P}_{rc}}{P_{rc}}\right)\left(\frac{P_{rc}^*}{\hat{P}_{rc}^*}\right)} \tag{D9}$$

or eliminating the total-pressure ratios through use of equation (12c) gives

$$y_{rc} = y_{rc}^* \sqrt{\frac{T_p}{T_{rc}}} \left(\frac{\dfrac{\hat{T}_{rc}}{T_{rc}}}{\dfrac{\hat{T}_{rc}^*}{T_{rc}^*}}\right)^{(\gamma+1)/2(\gamma-1)} \tag{D10}$$

where the rotor blade coolant relative to absolute total-temperature ratios are given by equations (C33) and (C36).

The thermodynamic turbine efficiency $\bar{\eta}_t$ is calculated from equations (C39) and (C42) as

$$\frac{\bar{\eta}_t}{\bar{\eta}_t^*} = \frac{\bar{\eta}_p}{\bar{\eta}_p^*} \left[\frac{1 + y_{sc}^* \left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}} \right) + y_{rc}^* \left(\frac{\Delta H_{rc,id}^*}{\Delta H_{p,id}^*} \right)}{1 + y_{sc} \left(\frac{\Delta H_{sc,id}}{\Delta H_{p,id}} \right) + y_{rc} \left(\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} \right)} \right] \tag{D11}$$

$$\frac{\Delta H_{sc,id}}{\Delta H_{p,id}} = \frac{T_{sc}}{T_p} \left(\frac{\Delta H_{sc,id}^*}{\Delta H_{p,id}^*} \right) \tag{D12}$$

and from equations (10), (D4), and (D6) we find

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p} \frac{\left[1 - \left(\frac{P_{re}}{P_{rc}}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\frac{P_{re}}{P_p}\right)^{(\gamma-1)/\gamma}\right]} = \frac{T_{rc}}{T_p} \frac{\left[1 - \left(\frac{P_{re}^*}{P_{rc}^*}\right)^{(\gamma-1)/\gamma}\right]}{\left[1 - \left(\frac{P_{re}^*}{P_p^*}\right)^{(\gamma-1)/\gamma}\right]}$$

or

$$\frac{\Delta H_{rc,id}}{\Delta H_{p,id}} = \frac{T_{rc}}{T_p} \left(\frac{\Delta H_{rc,id}^*}{\Delta H_{p,id}^*} \right) \tag{D13}$$

The coolant mass flow fractions y_{sc} and y_{rc} are found from equations (D8) and (D10), respectively.

REFERENCES

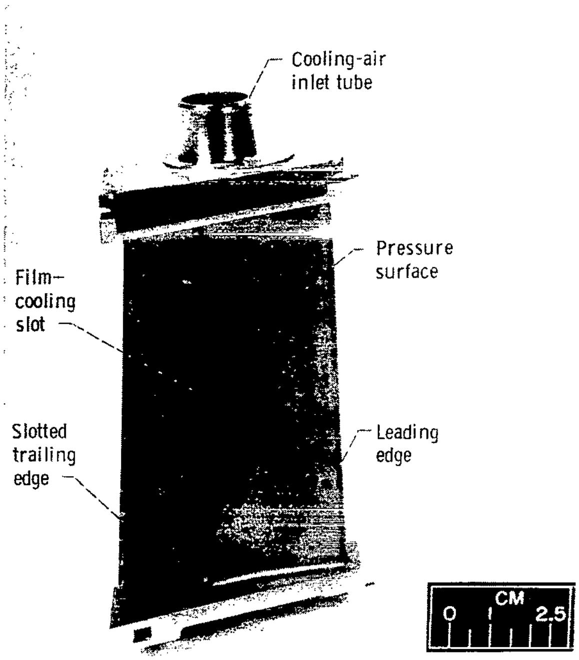
1. Whitney, Warren J.; Szanca, Edward M.; and Behning, Frank P.: Cold-Air Investigation of a Turbine with Stator-Blade Trailing-Edge Coolant Ejection. I - Overall Stator Performance. NASA TM X-1901, 1969.
2. Prust, Herman W., Jr.; Behning, Frank P.; and Bider, Bernard: Cold-Air Investigation of a Turbine with Stator-Blade Trailing-Edge Coolant Ejection. II - Detailed Stator Performance. NASA TM X-1963, 1970.
3. Szanca, Edward M.; Schum, Harold J.; and Prust, Herman W., Jr.: Cold-Air Investigation of a Turbine with Stator-Blade Trailing-Edge Coolant Ejection. III - Overall Stage Performance. NASA TM X-1974, 1970.
4. Prust, Herman W., Jr.; Schum, Harold J.; and Szanca, Edward M.: Cold-Air Investigation of a Turbine with Transpiration-Cooled Stator Blades. I - Performance of Stator with Discrete Hole Blading. NASA TM X-2094, 1970.
5. Szanca, Edward M.; Schum, Harold J.; and Behning, Frank P.: Cold-Air Investigation of a Turbine with Transpiration-Cooled Stator Blades. II - Stage Performance with Discrete Hole Stator Blades. NASA TM X-2133, 1970.
6. Behning, Frank P.; Prust, Herman W., Jr.; and Moffitt, Thomas P.: Cold-Air Investigation of a Turbine with Transpiration-Cooled Stator Blades. III - Performance of Stator with Wire-Mesh Shell Blading. NASA TM X-2166, 1971.
7. Behning, Frank P.; Schum, Harold J.; and Szanca, Edward M.: Cold-Air Investigation of a Turbine with Transpiration-Cooled Stator Blades. IV - Stage Performance with Wire-Mesh Shell Stator Blading. NASA TM X-2176, 1971.
8. Moffitt, Thomas P.; et al.: Summary of Cold-Air Tests of a Single-Stage Turbine with Various Stator Cooling Techniques. NASA TM X-52968, 1971.
9. Brown, Douglas B.; and Helon, Ronald M.: Cold-Air Aerodynamic Study in a Two-Dimensional Cascade of a Turbine Stator Blade with Suction-Surface Film Cooling. NASA TM X-2685, 1973.
10. Prust, Herman W., Jr.; and Bartlett, Wayne M.: Cold-Air Study of the Effect on Turbine Stator Blade Aerodynamic Performance of Coolant Ejection from Various Trailing-Edge Slot Geometries. NASA TM X-3000, 1974.
11. Moffitt, Thomas P.; Prust, Herman W., Jr.; and Bartlett, Wayne M.: Two-Dimensional Cold-Air Cascade Study of a Film-Cooled Turbine Stator Blade. NASA TM X-3045, 1974.

12. Stabe, Roy G.; and Dengler, Robert P.: Experimental Investigation of Aerodynamic Performance of Cooled Turbine Vanes at Gas-to-Coolant-Temperature Ratios up to 2.75. NASA TM X-2733, 1973.

13. McDonel, J. D.; Wolfmeyer, G. W.; Hartsel, J. E.; Eiswerth, J. E.; and Trout, S. L.: Core Turbine Aerodynamic Evaluation - Test Data from Initial Turbine. NASA CR-2596, 1976.

14. Anderson, L. R.; and Heiser, W. H.: Systematic Evaluation of Cooled Turbine Efficiency. ASME Paper 69-GT-63, Mar. 1969.

15. Glassman, Arthur J., ed.: Turbine Design and Application. NASA SP-290, vol. 1, 1972.



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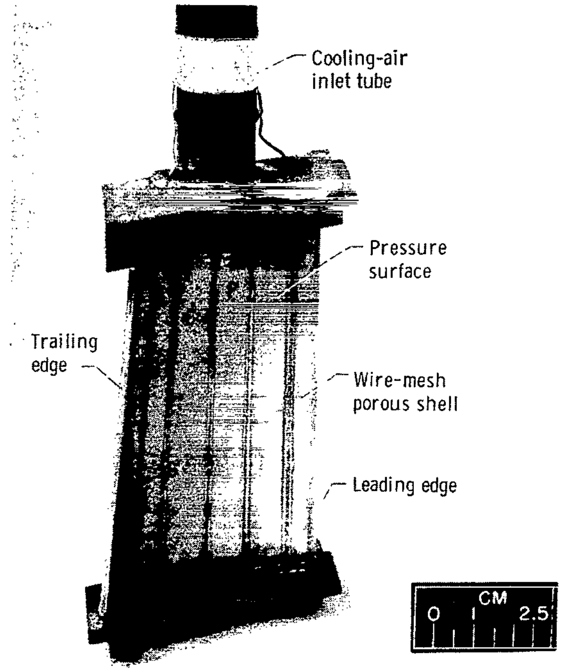
(a) Convection-film cooling.



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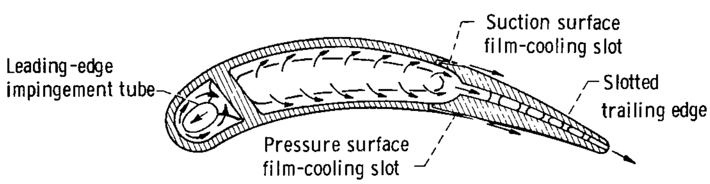
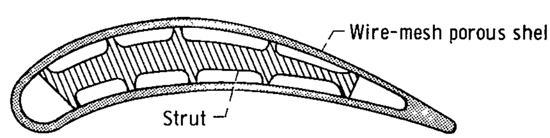
(b) Transpiration cooling.

Figure 1. - Cooled stator vanes.



(a) Convection-film cooling.



(b) Transpiration cooling.

Figure 2. - Cross-sectional view of cooled stator vanes.

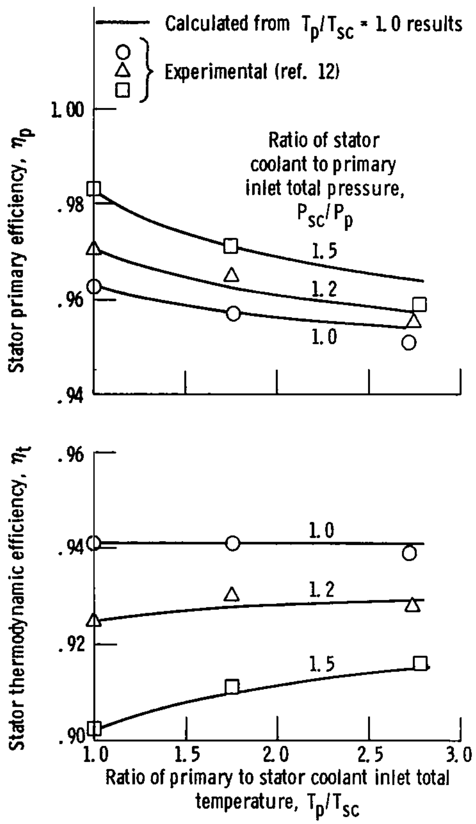


Figure 3. - Stator efficiency as function of total-temperature and total-pressure ratios for convection-film-cooled vanes.

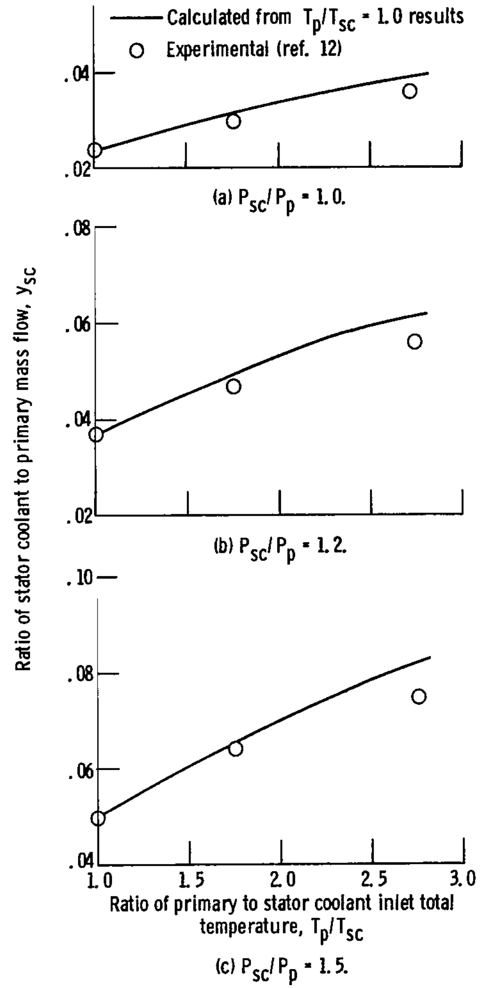


Figure 4. - Ratio of stator coolant to primary mass flow as function of total-temperature and total-pressure ratios for convection-film-cooled vanes.

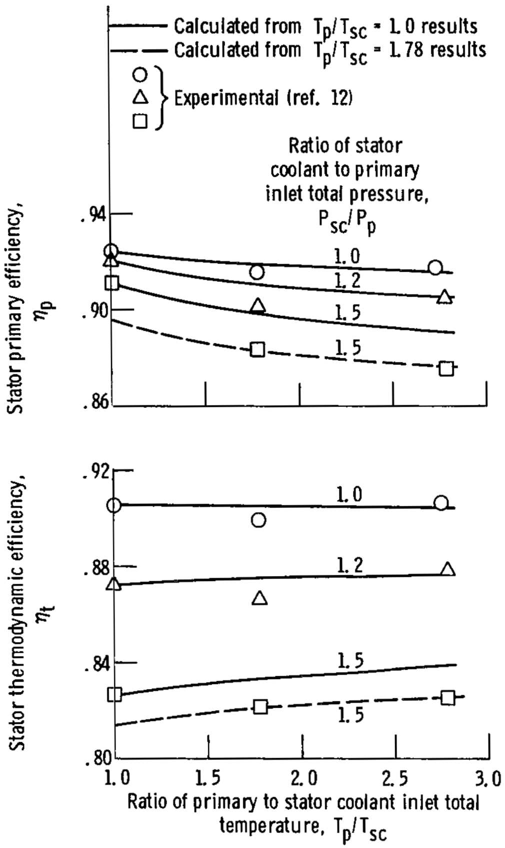


Figure 5. - Stator efficiency as function o total temperature and total-pressure ratios for transpiration-cooled vanes.

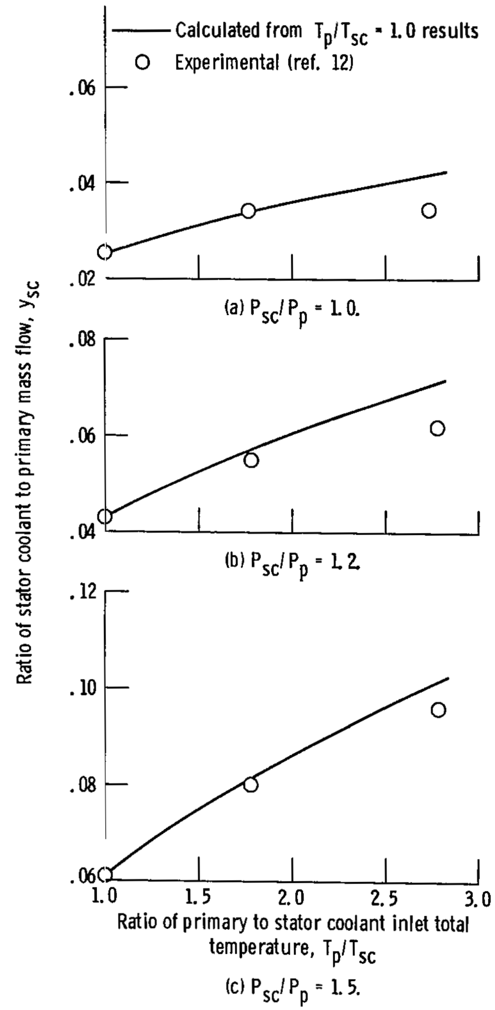
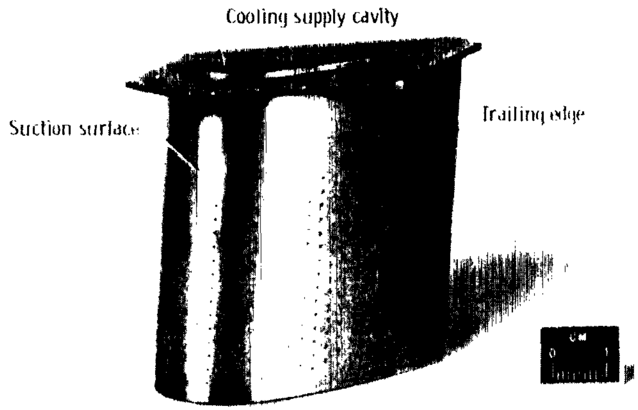
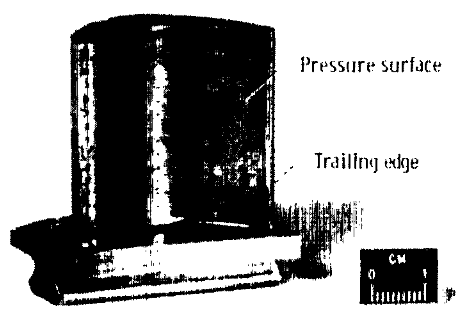


Figure 6. - Ratio of stator coolant to primary mass flow as function of total-temperature and total-pressure ratios for transpiration-cooled vanes.

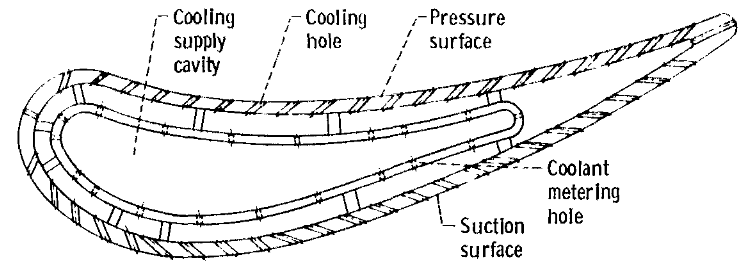


(a) Stator vane.

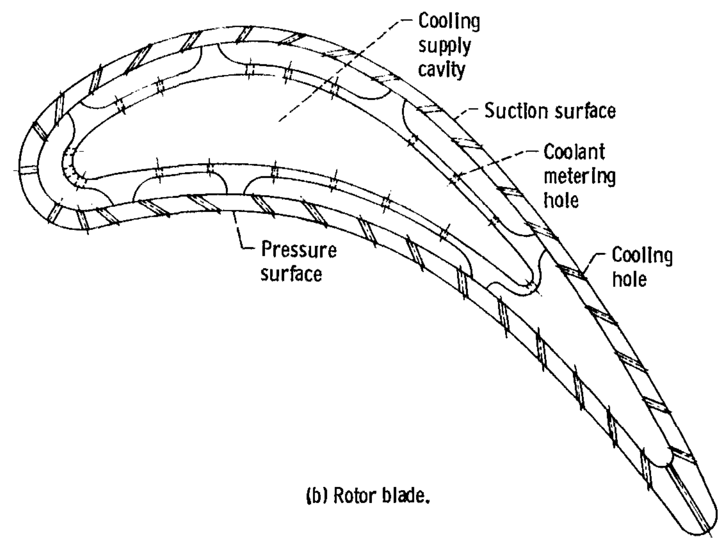


(b) Rotor blade.

Figure 7. Core turbine vane and blade.



(a) Stator vane.



(b) Rotor blade.

Figure 8. - Cross-sectional view of core turbine vane and blade.

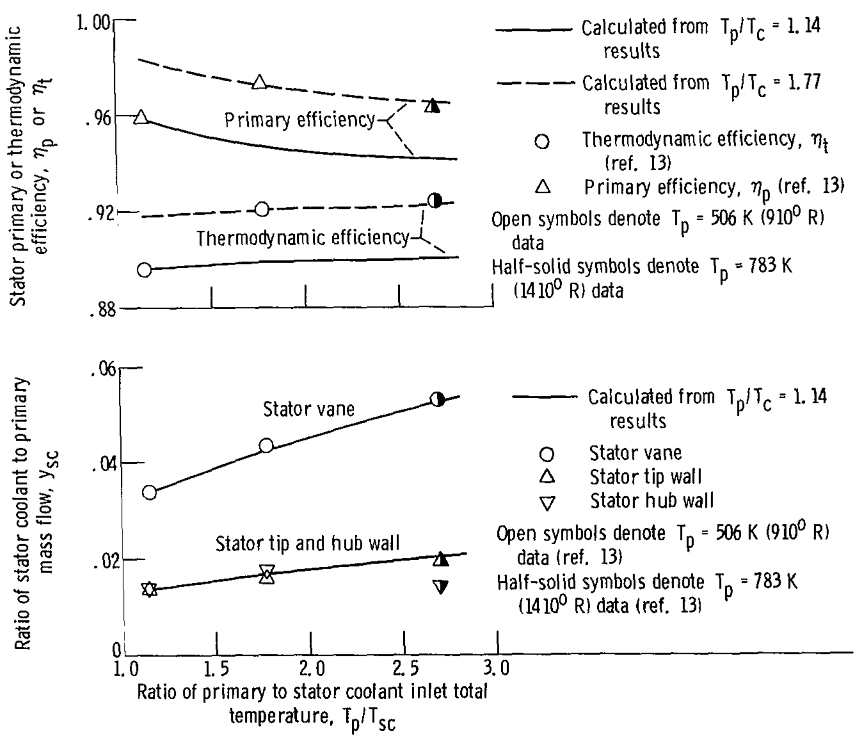


Figure 9. - Core stator efficiency and coolant mass flow fraction as function of total-temperature ratio at design overall pressure ratio of 1.50.

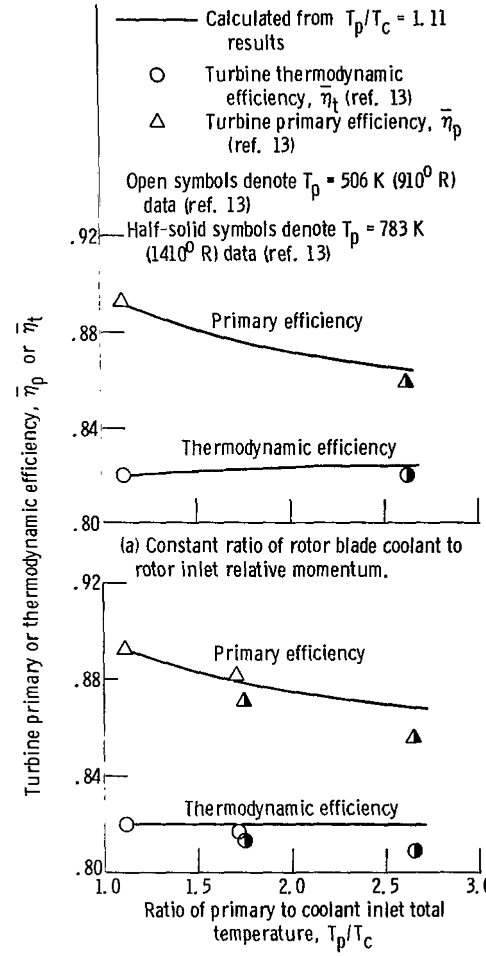
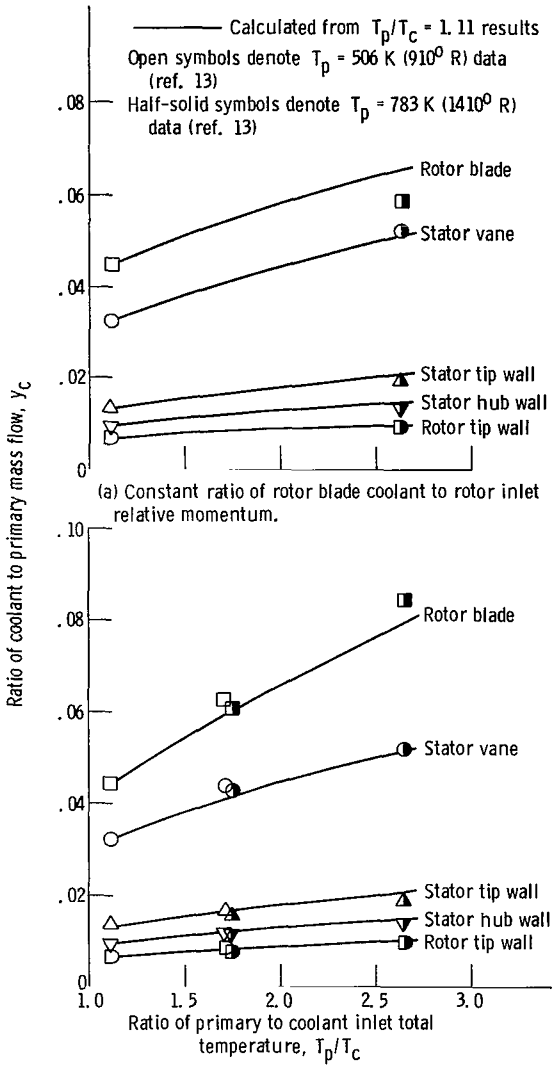


Figure 10. - Core turbine efficiency as function of total-temperature ratio at design overall total-pressure ratio of 1.84.



(a) Constant ratio of rotor blade coolant to rotor inlet relative momentum.

(b) Constant ratio of rotor blade coolant to rotor inlet absolute momentum.

Figure 11. - Ratio of core turbine coolant to primary mass flow as function of total-temperature ratio at design overall total-pressure ratio of 1.84.

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