

RESPONSE OF LONG, FLEXIBLE CANTILEVER BEAMS

TO APPLIED ROOT MOTIONS

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SUMMARY

Results are presented for an analysis of the response of long, flexible cantilever beams to applied root rotational accelerations. Maximum values of deformation, slope, bending moment, and shear are found as a function of magnitude and duration of acceleration input. Effects of tip mass and its eccentricity and rotatory inertia on the response are also investigated. It is shown that flexible beams can withstand large root accelerations provided the period of applied acceleration can be kept small relative to the beam fundamental period.

INTRODUCTION

In the design of large space structures, it is necessary to understand the dynamic response of flexible, low-frequency structures. A typical design problem is shown in figure 1, where a 100-meter beam is deployed from the space shuttle orbiter for a proposed molecular vacuum facility. The design of a lightweight boom requires a knowledge of the motion caused by input accelerations produced by control forces applied at the shuttle orbiter. The duration of these control forces is a small fraction of the first natural period of the boom. The purpose of this paper is to present results of an analysis of lightweight flexible booms to short-duration acceleration impulses and to find the permissible values of these input accelerations. Effects of tip mass magnitude, eccentricity, and rotatory inertia are included in the analysis.

DESCRIPTION OF ANALYSIS

The configuration analyzed in this paper is the cantilever beam shown in figure 2. The beam of length L , depth D , stiffness EI , and mass per unit length ρ has a tip mass M with a rotatory inertia I_M and an eccentricity B . The analysis considers a constant rotational input acceleration A which is applied for a time T_0 and is then removed. The duration of input T_0 varies over the range from an impulsive input ($T_0 \rightarrow 0$) to a step input ($T_0 \rightarrow \infty$). A nondimensional measure of the duration of input acceleration is given by the ratio T_0/T where T is the period of the first natural frequency of the cantilever beam. In the present study, the region with low values of T_0/T is of main interest.

Simple beam theory is used to obtain the differential equation of motion

$$EI \frac{\partial^4 Y(X,t)}{\partial X^4} + \rho \left[\frac{\partial^2 Y(X,t)}{\partial t^2} + X \frac{d^2 \theta(t)}{dt^2} \right] = 0 \quad (1)$$

where $\theta(t)$ is the rigid body rotation and $Y(X,t)$ is the elastic deformation of the rotating beam. The deflection $Y(X,t)$ satisfies the boundary conditions

$$\left. \begin{aligned} Y(0,t) &= 0 \\ \frac{\partial Y(0,t)}{\partial X} &= 0 \\ -EI \frac{\partial^3 Y(L,t)}{\partial X^3} + \bar{M} \left[(B+L) \frac{d^2 \theta}{dt^2} + \frac{\partial^2 Y(L,t)}{\partial t^2} + B \frac{\partial^3 Y(L,t)}{\partial X \partial t^2} \right] &= 0 \\ EI \frac{\partial^2 Y(L,t)}{\partial X^2} + B\bar{M} \left[(B+L) \frac{d^2 \theta}{dt^2} + \frac{\partial^2 Y(L,t)}{\partial t^2} + B \frac{\partial^3 Y(L,t)}{\partial X \partial t^2} \right] \\ &+ I_M \left[\frac{d^2 \theta}{dt^2} + \frac{\partial^3 Y(L,t)}{\partial X \partial t^2} \right] &= 0 \end{aligned} \right\} \quad (2)$$

and the initial conditions

$$\left. \begin{aligned} Y(X,0) &= 0 \\ \frac{\partial Y(X,0)}{\partial t} &= 0 \end{aligned} \right\} \quad (3)$$

The rigid body rotation is given by

$$\left. \begin{aligned} \theta &= \frac{1}{2} At^2 \quad \text{for} \quad 0 < t < T_0 \\ \text{and} \\ \theta &= AT_0 \left(t - \frac{1}{2} T_0 \right) \quad \text{for} \quad t > T_0 \end{aligned} \right\} \quad (4)$$

In the analysis the elastic deformation $Y(X,t)$ is given by

$$Y(X,t) = \sum_{n=1}^{\infty} a_n(t) \phi_n(X) \quad (5)$$

where $\phi_n(X)$ are the beam vibration modes for the cantilever beam and $a_n(t)$ are generalized coordinates. Results are obtained for elastic beam deflection $Y(X,t)$, slope $\frac{\partial Y(X,t)}{\partial X}$, bending moment $M(X,t)$, and shear resultant $Q(X,t)$

Modal equations for these responses were programed on a digital computer and the maximum value of each was found at several stations along the beam.

RESULTS AND DISCUSSION

The number of modes required for convergence is indicated in figure 3 for a beam without a tip mass subjected to input rotational accelerations with a large enough variation of input durations to include all possible types of responses. Although not shown, similar curves have been established for other tip mass configurations. These curves give the maximum values of nondimensional response parameters for the deflection Y_T and slope $\partial Y_T / \partial X$ at the beam tip and for bending moment M_0 and shear resultant Q_0 at the beam root. Accurate calculations of these response parameters are obtained by using only one mode for tip deflection, two modes for tip slope, and five modes for root bending moment and shear resultant. A six-mode solution is used herein as a completely converged standard of comparison.

The curves of figure 3, showing the effects of duration of acceleration input, can be divided into two regions of response types. For short-duration inputs ($T_0/T < 0.5$) the maximum responses always occur after the input root acceleration has been removed. For long-duration inputs ($T_0/T > 0.5$) the maximum responses always occur while the input acceleration is being applied and approach the values for a step input ($T_0/T \rightarrow \infty$) which have the values of two times the values for the quasi-static solution for rigid body inertia loading. The nearly horizontal curves for $T_0/T > 0.5$ show that in this region the maximum values of beam responses can be calculated by use of the simple quasi-static solution.

When the nondimensional parameters of figure 3 are used, the results for nearly impulsive input acceleration ($T_0/T \rightarrow 0$) are all compressed near the origin. Inputs in this region are of particular interest since typical control inputs are for short intervals of time while space booms have long periods. To overcome this difficulty, the results of figure 3 are repeated in figure 4 by using a different set of nondimensional parameters. These parameters have finite nonzero values for the pure impulse and are in agreement with calculated values from reference 1, which considers the instantaneous arrest of a rotating cantilever beam. These response parameters that have input acceleration impulse ($T_0 A$) in their nondimensionalizations, for short-duration inputs ($T_0/T < 0.5$), do not have the large variation with T_0/T that is obtained by using the response parameters of figure 3. For this reason, the nondimensional parameters of figure 4 are used throughout the remainder of the paper.

Effect of tip mass on maximum response is shown in figure 5 for a pure impulsive input ($T_0/T \rightarrow 0$) and for a short-duration input ($T_0/T = 0.1$). Curves are shown for the nondimensional parameters for elastic tip deflection and root bending moment. For short duration of input acceleration, the effect of duration has very little effect on the elastic tip deflection curve but has some effect on the root bending-moment curve. Note that effects of tip mass are included not only in the tip mass parameter ($\bar{M}/\rho L$) but also in the period T . Even though the nondimensional response is shown to decrease with tip mass,

the physical quantities increase as expected. For example, for a tip mass equal to the beam mass, the root bending moment increases 75 percent and the tip deflection 100 percent.

Effects of tip mass eccentricity and rotatory inertia are shown in figure 6 for a pure impulse ($T_0/T \rightarrow 0$) and for a short duration of input ($T_0/T = 0.1$). Here nondimensional tip deflection and root bending moment are shown as functions of rotatory inertia $I_M/\bar{M}L^2$ for two values of eccentricity B/L which are chosen as representative extreme values. Effects of rotatory inertia and eccentricity also appear in two parts of this figure; first, in the parameters $I_M/\bar{M}L^2$ and B/L and, second, in the period T which is used in nondimensionalizing the response parameters. Again, for short-duration inputs, the elastic tip deflection parameter is only slightly affected by duration of input but the root bending-moment parameter decreases appreciably with an increase in T_0/T .

When a limiting design or maximum value is assigned to any of the calculated values of response, curves can be obtained to give maximum permissible input acceleration as a function of structural parameters. For example, if limiting values are assigned to the maximum bending strain ϵ at the root of a cantilever with a symmetrical cross section, the curves of figure 7 are obtained which give permissible nondimensional input acceleration TT_0A as a function of span to depth L/D . The $\epsilon = 0.003$ and 0.005 curves bound values of limiting bending strain that are appropriate for most isotropic and composite materials while the $\epsilon = 0.001$ curve represents a practical value of limiting bending strain that has been reduced to take into account effects such as buckling. The curves, shown for no tip mass, show that for given values of L/D and ϵ , a slightly higher value of impulse T_0A is permitted if the impulse is applied over a longer duration of time T_0 .

Sample curves with physical units are given in figure 8 for determining permissible input acceleration A . These curves are shown for a beam with no tip mass and for the reduced limiting strain condition ($\epsilon = 0.001$). The curves show the variation of permissible input rotational acceleration with the lowest natural frequency ($1/T$) and the span-to-depth ratio L/D for two values of input duration T_0/T . The $T_0/T = 0.5$ value represents the most severe case where the response approaches that of the step input and the beam behavior can be estimated from a simple quasi-static solution. The $T_0/T = 0.001$ value represents a nearly impulsive input. As the duration of input decreases, the permissible magnitude of input rotational acceleration increases. As illustrated in figure 8, a hundred-fold increase in permissible acceleration can be achieved by applying very short-duration inputs.

CONCLUDING REMARKS

A modal solution has been obtained to study the response of long, flexible cantilever beams to applied values of root rotational acceleration. Effects of tip mass with various eccentricities and rotatory inertias have been included. Results were obtained for duration of input that cover the range from near-impulsive to the step function. A set of nondimensional parameters has been

identified that facilitates looking at the response for the near-impulsive type of input accelerations. When the duration of input is more than half the period of the first natural frequency of the beam, the maximum response is nearly equal to that of the step-function input and is found to be twice the response given by simple quasi-static analysis based on rigid body inertia loading. Examples are included of application of these results to the problem of determining maximum input acceleration so that design values of maximum strain are not exceeded. These results show that large flexible booms can experience high root rotational accelerations without inducing large strains provided the duration of controlling forces are kept to a small fraction of the period of the first natural frequency.

REFERENCE

1. Stowell, Elbridge Z.; Schwartz, Edward B.; and Houbolt, John C.: Bending and Shear Stresses Developed by the Instantaneous Arrest of the Root of a Cantilever Beam Rotating With Constant Angular Velocity About a Transverse Axis Through the Root. NACA Wartime Report L-27 (ARR No. L5E25) 1945.

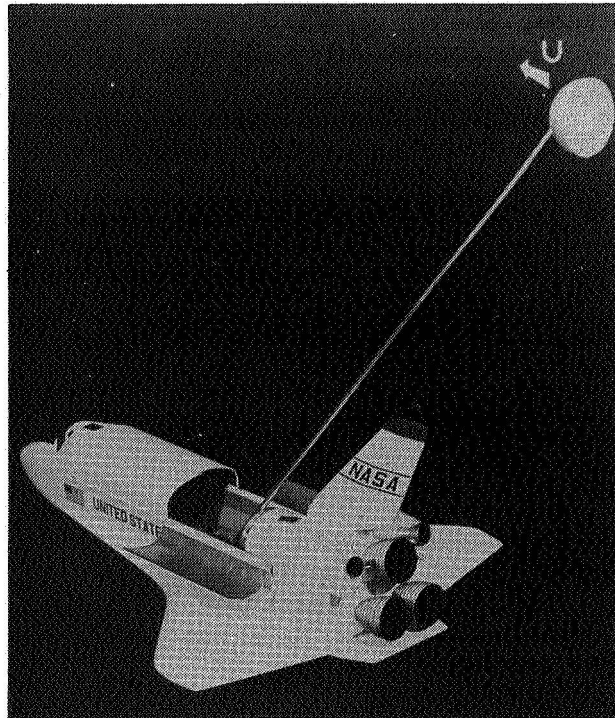


Figure 1. Long, flexible boom for molecular vacuum facility.

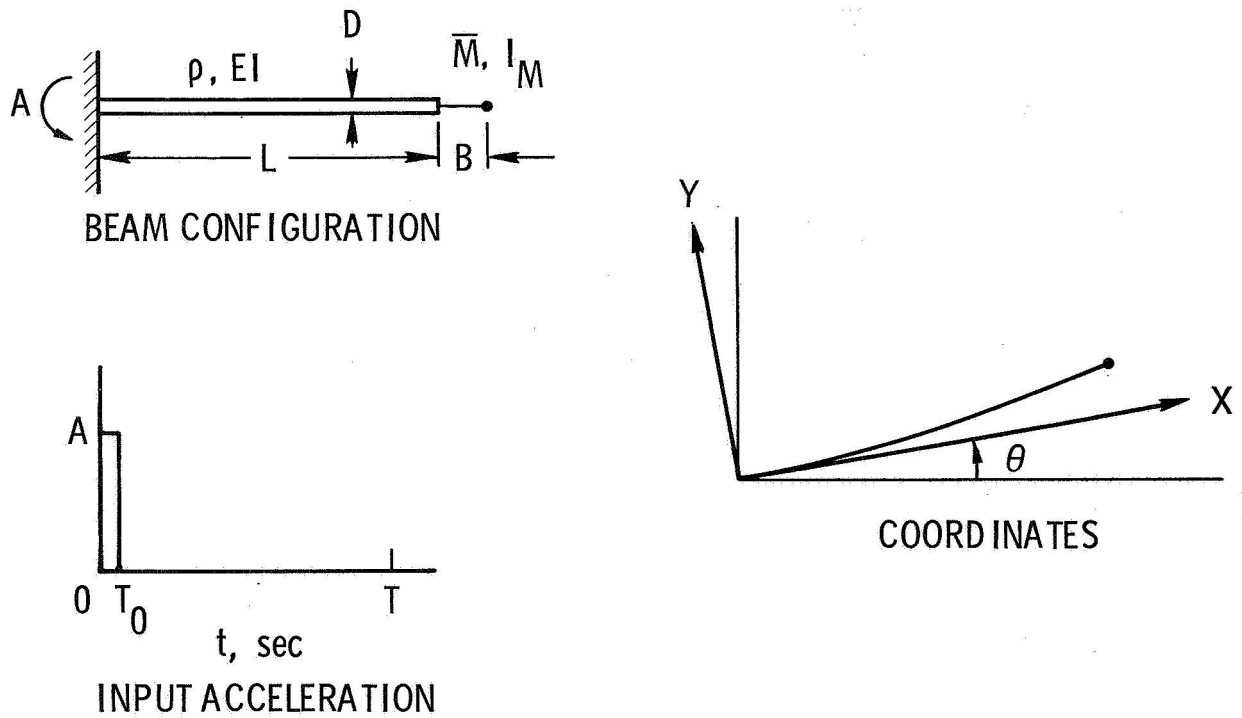


Figure 2. Flexible cantilever beam subjected to input rotational acceleration.

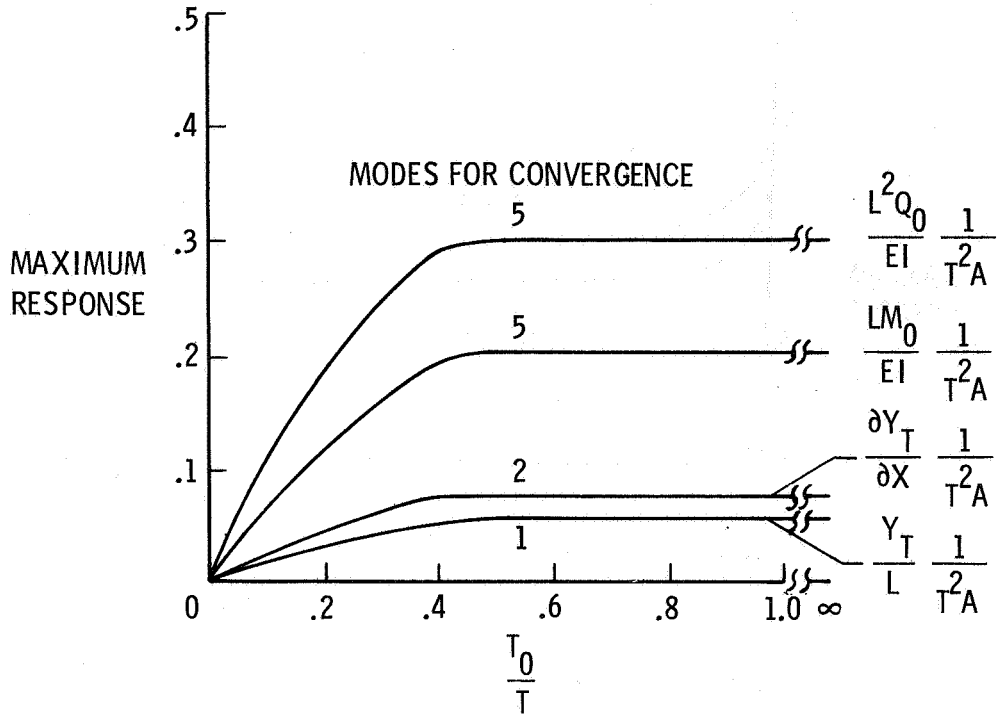


Figure 3. Effect of duration (T_0/T) of input rotational acceleration on maximum response. No tip mass ($\bar{M}/\rho L = 0$).

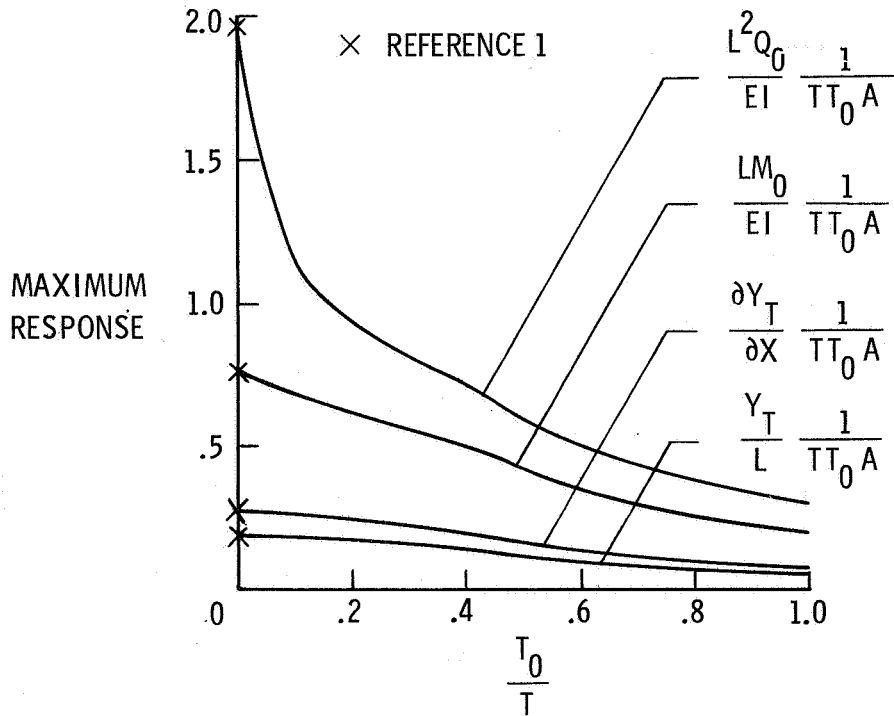


Figure 4. Response parameters appropriate for nearly impulsive input acceleration ($T_0/T \rightarrow 0$). No tip mass ($\bar{M}/\rho L = 0$).

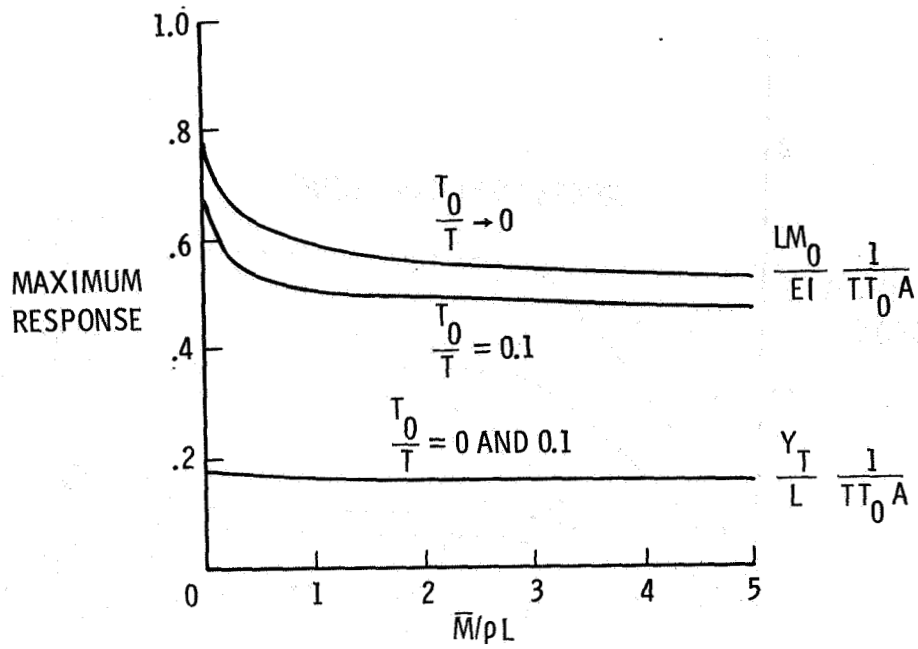


Figure 5. Effect of tip mass ($\bar{M}/\rho L$) on maximum response of beam.

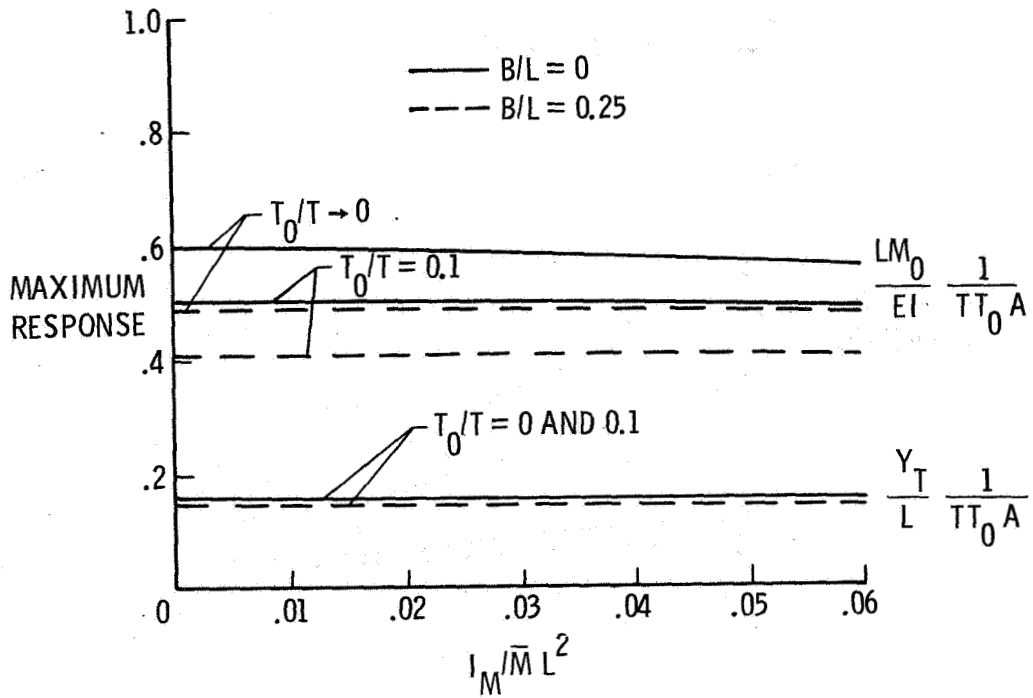


Figure 6. Effects of eccentricity (B/L) and rotatory inertia ($I_M/\bar{M}L^2$) of tip mass on maximum response of beam. $\bar{M}/\rho L = 1$.

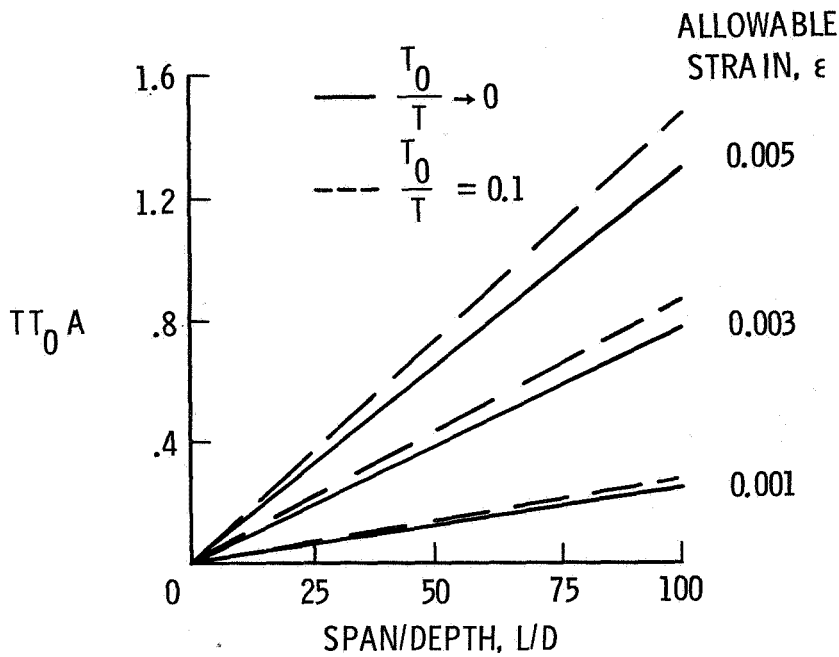


Figure 7. Nondimensional parameter ($TT_0 A$) for permissible root rotational acceleration. $\bar{M}/\rho L = 0$.

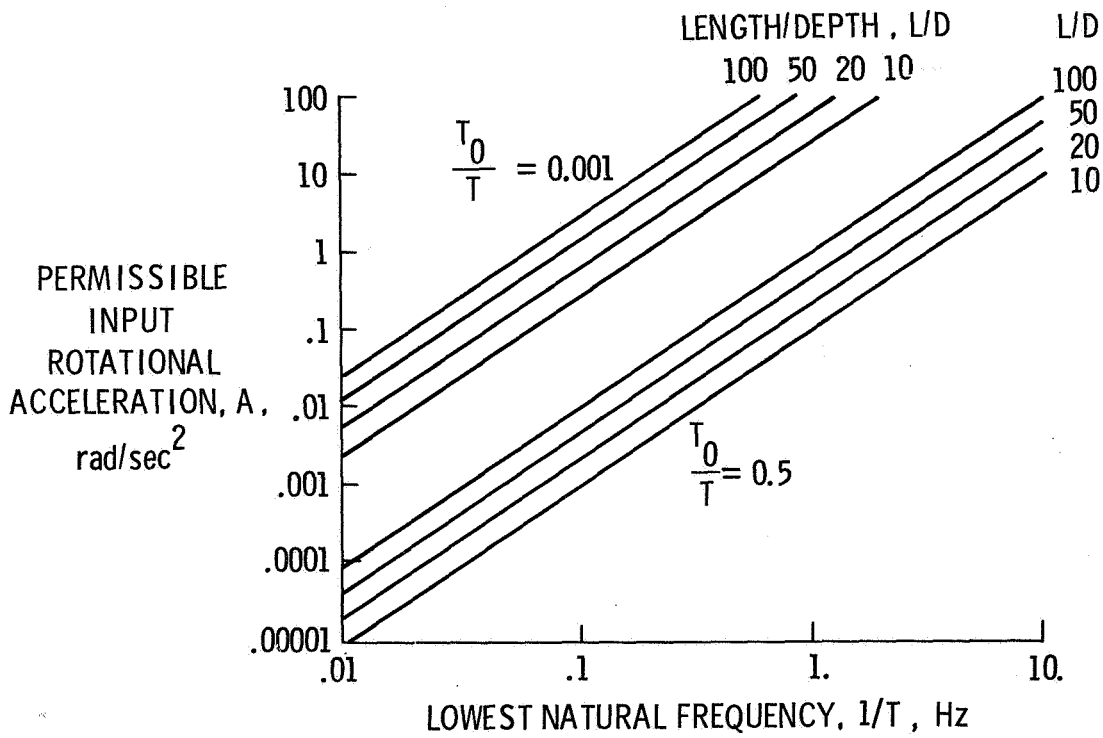


Figure 8. Permissible root rotational acceleration. $\bar{M}/\rho L = 0$, $\epsilon = 0.001$.