ANALYSIS OF PANEL DENT RESISTANCE

Chi-Mou Ni
General Motors Research Laboratories

SUMMARY

An analytical technique for elastic-plastic deformation of panels has been developed, which may be employed to analyze the denting mechanisms of panels resulting from point projectile impacts and impulsive loadings. The correlations of analytical results with the experimental measurements are considered quite satisfactory.

The effect of elastic springback on the dent-resistance analysis is found to be very significant for the panel (122 cm x 60.9 cm x 0.076 cm) subjected to a point projectile impact at 16.45 m/sec. While the amount of springback decreases as the loading speed increases, the effect due to the strain-rate hardening of material, such as low-carbon steel, becomes more dominant and has been demonstrated in the analysis of dent resistance of a rectangular steel plate impulsively loaded.

INTRODUCTION

One of the primary concerns in exposed panels of automotive vehicles and aircraft is their ability to resist damage by denting during fabrication and in service. Generally speaking, the mechanical properties of the material, panel geometry, and loading conditions are the primary factors in determining panel dent resistance. These factors are related in a complicated way, however; therefore, it is not easy to use an intuitive approach to develop their mathematical relationship, and we must resort to an analytical approach instead.

Generally speaking, the loadings which dent the panel are somewhat random in nature. Dents may be produced in automotive panels during fabrication, for example, by the impact of one panel on another and by dropping a panel onto a holder or conveyor projection. In service, dents are commonly produced by flying stones, door impact in a parking lot, and even hail. For aerospace structures, quite often the exposed aircraft components are subjected to impact loadings, including hail and runway stones, etc. Nevertheless, in this study it is assumed that the loading conditions may be characterized with (a) projectile impact over a period of time; and (b) an impulse having a very short duration.

In this study, an analytical approach is developed to analyze the denting mechanism of panels under impact and impulsive loadings. Panel denting is usually the consequence of ductile plastic flow. The dent resistance is referred to as the panel-dent strength, measured in terms of permanent plastic deformation (not the deformation resulting from any elastic buckling). Over the past few years, the analysis and prediction of large dynamic and permanent deformations of structures caused by impact and impulsive loadings have received increasing interest (refs. 1 to 6).

Three analytical approaches to these problems are commonly used. The rigid-plastic idealization (refs. 1 to 2) has been frequently applied to analyze impulsively loaded beams, rings, flat plates, and axisymmetric cylindrical shells. There are limitations to this idealization, however. For instance, once the large deflection or geometry change is taken into account, a rigid-plastic analysis may be too complicated to use (ref. 2). Furthermore, the rigid-plastic analysis is applicable only to problems for which the initial kinetic energy is much larger than the maximum elastic strain energy. Another approach often employed for structural problems is the energy method (ref. 6) in which the energy input to the structure is equated to the plastic work done. The success of this method depends on how reasonable an estimate is made of the primary mode of deformation. For a complex structure under arbitrary transient loading, it can be difficult to make such an estimate.

The deficiencies in the two analytical methods can be skirted by various numerical methods, such as the finite difference method (refs. 3 to 5) and the finite element method (ref. 5). In this paper, a numerical scheme extended from Reference 4 is employed to analyze the panel denting as a result of being subjected to impact and impulsive loadings.

THEORETICAL FORMULATIONS

Minimum Principle

Consider a body of a continuum occupying in its natural state a region V_0 and bounded by a piecewise smooth surface, A . The body is subjected to time-dependent body force, F_m (per unit mass) and Lagranian surface traction (per unit area) T_m over that part of the initial surface area A_T . At time t , let $\{U_k\}$ be the displacement vector of a particle of the body which has an initial position of $\{X_k\}$ in a curvilinear coordinate system. The displacements are prescribed over that part of the boundary surface, A_u . The deformation of the body may be described in terms of the covariant components of the Lagrangian strain tensor, E_{kL} , defined by

$$E_{kL} = \frac{1}{2} \left(U_{k;L} + U_{L;k} + U^M_{;k} U_{M;L} \right) \tag{1}$$

Herein, a covariant derivative of a variable with respect to X_k is designated by the semicolon in the subscript position, as $(\ )_{;k}$, and the repetition of an index in a term indicates summation. The Lagrangian strain may be expressed as the sum of two parts: elastic strain, E^e_{kL} and plastic strain E^p_{kL} . It is postulated that the constitutive relationships, in terms of the symmetric Kirchhoff stress tensor S_{kL}, may be plastic-strain velocity dependent but are not influenced by strain accelerations. In other words, it is assumed that

$$S_{kL} = S_{kL}\left(E^{e}_{MN}, E^{p}_{MN}, \dot{E}^{p}_{MN}, \theta\right) \tag{2}$$

in which θ is the temperature and $\dot{E}^{p}_{MN}$ is the velocity rate of plastic straining. The contravariant components of the Kirchhoff stress tensor, S^{kL}, satisfy the boundary conditions

$$S^{kL}\left(g_{ML} + U_{M;L}\right) N_k = T_M \text{ on } A_T \tag{3}$$

in which N_k is the covariant outward unit normal to A, and g_{ML} is the metric tensor.

It has been shown (ref. 7) that the time acceleration field, $\ddot{U}_M = D^2U_M/Dt^2$, of the body, which has known or predetermined displacement and velocity fields at time t, is distinguished from all kinematically admissible ones by having the minimum value of the following functional:

$$I = \int_{V_o} S^{kL}\ddot{E}_{kL}\,dV_o + \frac{1}{2}\int_{V_o} \rho_o \ddot{U}^M \ddot{U}_M\,dV_o - \int_{A_T} T^M \ddot{U}_M\,dA - \int_{V_o} \rho_o F^M \ddot{U}_M\,dV_o \tag{4}$$

in which ρ_o is the initial mass density. The minimum principle is valid for continuous as well as sectionally discontinuous acceleration fields. Ordinarily, it is sufficient to use the first variation with respect to the acceleration, $\delta_{acc}I = 0$, to establish governing equations or to solve a problem by a direct method of variational calculus.

Kinematics

Consider a cylindrical-shell panel of mean radius R, thickness h, axial length L, and arc width $R\beta$. Let (x,y,z) be the axial, circumferential and (outward) normal coordinates, and (U_x,U_y,U_z) be the corresponding physical components of the displacement vector of a point in the shell, respectively. Then, by utilizing the Love-Kirchhoff assumption for thin shells and by neglecting wave propagation through the thickness, the displacement components of a particle can be expressed in terms of the corresponding displacement (and its derivatives) of the middle surface as

$$\left.\begin{aligned} U_r &= u - zw,_x \\ U_y &= v - \frac{z}{R}(w,_y + v) \\ U_z &= w \end{aligned}\right\} \tag{5}$$

where u, v, and w denote the axial, tangential, and (outward) normal displacement components of a point on the mid-surface. Having the displacement components, the strain accelerations can then be defined as

$$\ddot{E}_{xx} = (1 + u,_x)\ddot{u},_x + v,_x\ddot{v},_x + w,_x\ddot{w},_x - zw,_{xx} + \dot{u}^2,_x + \dot{v}^2,_x + \dot{w}^2,_x$$

$$\begin{aligned}\ddot{E}_{xy} = \frac{1}{2}\Big[&u,_y\ddot{u},_x + (1 + v,_y - \ddot{w})v,_x + (v + w,_y)\ddot{w},_x + (1 + u,_x)\ddot{u},_y \\ &+ v,_x(\ddot{v},_y - \ddot{w}) + w,_x(\ddot{v} + \ddot{w},_y) - z(2\ddot{w},_{xy} + \ddot{v},_x) + \dot{u},_x\dot{u},_y \\ &+ \dot{v},_x(\dot{v},_y - w)\Big]\end{aligned} \tag{6}$$

$$\begin{aligned}\ddot{E}_{yy} = &u,_y\ddot{u},_y + (1 + v,_y - w)(\ddot{v},_y - \ddot{w}) + (v + w,_y)(\ddot{w},_y + \ddot{v}) \\ &- z(\ddot{w},_{yy} + \ddot{v},_y) + \dot{u}^2,_y + (\dot{v},_y - \dot{w})^2 + (\dot{v} + \dot{w},_y)^2\end{aligned}$$

Constitutive Relationship

For an isotropic and homogeneous material, the elastic stress-strain relationship may be reasonably expressed by

$$\dot{E}^e_{kL} = \frac{1}{E}\left[(1 + \nu)\ \dot{S}_{kL} - \nu\delta_{kL}\dot{S}_{MM}\right] \tag{7}$$

where ν is Poisson's ratio and δ_{kL} is the Kronecker symbol. The plastic stress-strain relationship based on the isothermal, incremental theories of plasticity may be derived from Drucker's postulate of positive work in plastic deformation. Drucker's postulate establishes two requirements:

(a) The loading surface is convex and (b) at a smooth point of the yield surface, the plastic strain rate vector is always directed along the normal to the loading surface or

$$\dot{E}^p_{kL} = \begin{Bmatrix} G\dfrac{\partial f}{\partial S_{kL}}\dfrac{\partial f}{\partial S_{MN}}\dot{S}_{MN} & \text{for } f = 0 \text{ and } \dfrac{\partial f}{\partial S_{MN}}\dot{S}_{MN} > 0 \\ 0 & \text{for } f < 0 \text{ or } \dfrac{\partial f}{\partial S_{MN}}\dot{S}_{MN} \leq 0 \end{Bmatrix} \tag{8}$$

where G is a scalar proportionality function depending on the state of the material and may be determined, based on the concept of isotropic hardening, as (ref. 4)

$$\left.\begin{aligned} G &= \frac{3}{4J_2}\left(\frac{1}{E_t} - \frac{1}{E}\right) && \text{for } f = 0 \text{ and } \frac{\partial f}{\partial S_{MN}}\dot{S}_{MN} > 0 \\ &= 0 && \text{for } f < 0 \text{ or } \frac{\partial f}{\partial S_{MN}}\dot{S}_{MN} \leq 0 \end{aligned}\right\} \tag{9}$$

in which E_t , a function of J_2 , is the tangent modulus which may be obtained from the uniaxial Kirchhoff stress vs Lagrangian strain curve of the material. Herein, a generalized J_2 criterion based on the Mises yield function is employed for the shell problem as

$$f = J_2 - \kappa^2 = \frac{1}{3}\left(S^2_{xx} - S_{yy} + S^2_{yy}\right) + S^2_{xy} - \kappa^2 = 0 \tag{10}$$

where κ is the strain-hardening parameter.

It has been long recognized that the strain-rate sensitivity of material may be one of the important factors affecting the dynamic responses of elastic-plastic structures. A generalized formula which accounts approximately for the multiaxial behavior of a strain-rate sensitive material is employed and expressed as

$$\frac{\sigma_e}{\sigma_o} = 1 + \left(\frac{\dot{\varepsilon}_e}{D}\right)^{\frac{1}{p}} \tag{11}$$

where σ_e (effective stress) $= \left(\frac{3}{2}\,\bar{S}'_{ij}\bar{S}'_{ij}\right)^{\frac{1}{2}}$, $\bar{S}'_{ij} = \sigma_{ij} - \frac{1}{3}\,\delta_{ij}\sigma_{kk}$

$\dot{\varepsilon}_e$ (effective strain rate) $= \left(\frac{2}{3}\,\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{\frac{1}{2}}$

D and p = material constants

Finite-Difference Energy Method

A numerical approach based on the finite-difference direct method in conjunction with the minimum principle (as shown in Eq. (4)) is developed to analyze the large dynamic responses of cylindrical shell panel under impact and impulsive loadings. To make the amount of computation tenable, an idealized sandwich shell having a number of discrete, thin load-carrying sheets made of a work-hardening material is employed. The indices i, j, and k are introduced to indicate the spatial position of a point in the shell as follows: $x = i\Delta x$, $i = 1,\ldots m$; $y = j\Delta y$, $j = 1,\ldots n$; $z = k\Delta z$, $k = 0, \pm 1,\ldots \pm \ell$; where Δx, Δy, and Δz are chosen spacings of coordinates x, y, and z, respectively. The spatial derivatives of accelerations and displacements are replaced by discrete values of accelerations and displacements through a central finite-difference scheme. The functional I , by Equation (4), may be replaced by a finite summation through using the trapezoidal rule for the integration. The explicit expressions for accelerations at any time step $t = q\Delta t$ may be obtained by minimizing the functional I^q with respect to the discrete accelerations as follows:

$$\frac{\partial I^q}{\partial \ddot{u}^q_{i,j}} = 0; \quad \frac{\partial I^q}{\partial \ddot{v}^q_{i,j}} = 0; \quad \frac{\partial I^q}{\partial \ddot{w}^q_{i,j}} = 0 \tag{12}$$

The discrete accelerations must also satisfy the boundary conditions for the clamped cylindrical shell panels which require that three displacement components and their slopes all have a value of zero.

It is assumed that at time $t = q\Delta t$, the displacements, velocities, strains and stresses, have been previously determined at all nodal points of the domain. Then Equation (12) may be used to determine the accelerations $\ddot{u}^q_{i,j}$, $\ddot{v}^q_{i,j}$, $\ddot{w}^q_{i,j}$ at time t. Subsequently, the displacements at time $t + \Delta t$ may be obtained by the central difference approximation. Knowing the displacements at $t + \Delta t$, the strain increments that occurred in the time interval $(t + \Delta t - t)$ may also be determined by using the central difference scheme. Furthermore, the corresponding stress increments may be obtained by the constitutive relationships provided that the condition of loading or unloading is known. This may be accomplished by first calculating a set of stress increments corresponding to $G = 0$. Then the loading criterion, $df \gtrless 0$, may be checked and the appropriate value of G in Equation (9) is used in the calculation of the correct stress increments. By repeating the foregoing steps for each subsequent time increment, the entire history of deformation of the shell panel may be obtained.

Impact and Impulsive Loadings

In the case of projectile impact, the actual situation could be very complex and not amenable to analysis due to the irregular shapes of the panels and the indentors and their interactions during deformations. However, for simplicity the projectile is considered here to be rigid and small in size compared with the dimensions of the panel. In engineering analysis, there are, in general, two approximate methods to incorporate the impact loadings by the projectile into the mathematical system: one is termed "Collision-Imparted Velocity Method" (ref. 8) and the other, the "Collision-Force Method" (ref. 8). For the "Collision-Force Method," the contact force is included in the analysis; the contact force is neglected in the "Collision-Imparted Velocity Method," which makes it much simpler to implement. In Reference 9 it has been shown that in cases of small ratio of beam mass to impactor mass, these two methods may offer the same degree of accuracy in solutions of a simply supported beam under central impact. In this study of panel dent resistance, a point-projectile-impact loading is assumed in the analysis. Furthermore, the impactor mass is considered as rigid and attached to the panel at the impact point and then an initial velocity equal to the original impactor velocity is assumed at the panel impact point. Subsequently, the motion of the panel is then analyzed.

In the low-speed impact situation, the stress due to the impactor is dispersed continuously throughout the panel. As impact speed increases, the regions not in the immediate vicinity of the impact point will not immediately feel a stress, and it will cause more localized deformation at the impact point. In order to simplify the analysis for the very localized dents as a result of very high speed impact, a small portion of the panel around the impact point under high-intensity impulse is analyzed. In general, a solid under an impulsive loading of very short duration may be considered to be equivalent to a solid moving with a prescribed initial velocity.

RESULTS AND DISCUSSION

Since the primary concern in the analysis of panel dent resistance is the dent size, which is, in general, inversely proportional to panel resistance to loading, the dent size (or permanent set) of the impacted panel is here used as an index to calibrate its resistance to denting. As mentioned previously, the actual loading conditions are usually not deterministic and vary widely with the manufacturing and service environments of the panel. In any event, the analytical technique presented herein may be employed to analyze the panel dent sizes resulting from point projectile impacts and impulses, which should provide some fundamental understandings of panel dent resistance.

In order to validate the present analytical technique, numerical results have been obtained for the dynamic responses of a cylindrical shell panel subjected to a point projectile impact and of a rectangular plate subjected to impulsive loading:

The cylindrical shell panel clamped on its boundaries is made of aluminum alloy 6061-T6 and has the geometric properties as shown in Figure 1. The material has a mass density of 2750 kg/m^3 and a Poisson's ratio of 1/3. The uniaxial stress-strain relationship may be approximated as a bilinear relationship with E(Young's modulus) = $8 \times 10^{10} N/m^2$, E_t(tangent modulus) = $10.7 \times 10^7 N/m^2$ and σ_0(initial yield stress) = $27 \times 10^7 N/m^2$. The panel is impacted by a 0.45 kg steel ball (0.79 mm in diameter) at 16.5 m/sec. Figure 1 illustrates the analytical results of the central deflection-time relationships of the impacted panel, and the measured maximum deflection and permanent deflection in experiment. It should be noted that the maximum deflection and the permanent deflection predicted are larger than those determined experimentally. The reason for this overestimation of deflections at the impacted point may be that in this analysis, the projectile is assumed to be a point so that the predicted deformed profiles around the impact point are deeper than those observed in experiment. It is believed that the correlations can be improved by matching the contact surface instead of the point in the analysis. As one can see in Figure 1, however, the analytical results obtained can still be reasonable enough to provide understanding of the panel dent resistance. Indicated in Figure 1, the impacted point of panel with the projectile reaches its peak deflection by elastic-plastic deformation, and then springs back to its saddle point, at which time the projectile and the impacted point of panel separate, and finally it oscillates about its permanent deformation; the permanent deformation is defined as the dent. When the panel reaches peak deflection, the kinetic energy of the projectile before impact is transformed within the panel in two parts, elastic strain energy and the work of plastic strain. Since the elastic deformation is assumed to be reversible, the panel springs back, causing rebound of the projectile. The elastic strain energy released is related to the elastic stiffness of the panel, which depends on the Young's modulus, panel geometry, and on the impact speed. Evidently, for this case, the elastic springback plays an important role in determining the degree of dent resistance.

Furthermore, under certain circumstances, the impact speed can be so high that only a very localized dent occurs, with insignificant springback (Ref. 10). This phenomenon may be analyzed and reproduced by only treating the immediate impacted area being subjected to high intensity of impulse, which would simplify the analysis and still provide enough insight of the panel dent-resistance. Also, in some other circumstances, the loading conditions may be explicitly characterized as impulsive loadings with relatively short durations. To understand the denting mechanisms of panel resulting from impulsive loadings, the present analytical technique has been applied to analyze the dynamic responses of a rectangular plate subjected to a uniform impulse with equivalent initial velocity of 91.4 m/sec. The geometric dimensions of this plate are shown in Figure 2 and the material is aluminum alloy 6061-T6, whose mechanical properties are described previously. Presented in Figure 2 are the analytical results of the central deflection-time history and the experimentally measured permanent set (Ref. 11) for comparison. It is evident that the predicted dent depth of the center agrees very well with the test data. Note that the spring-back is insignificant compared with the previous case. The amount of spring-bac back generally depends on the Young's modulus, panel geometry, and the impact speed. As the springback effect decreases as the impact speed increases, the strain-rate hardening of material may become dominant when deformation rate increases. The degree of strain-rate hardening can vary with material and temper condition. For example, low-carbon mild steel generally has greater strain-rate hardening than aluminum alloy and high-strength steel.

Finally, to quantify the effect of strain-rate hardening of steel on the panel dent-resistance, the central deflection-time relationships of a rectangular steel plate (as shown in Fig. 3) subjected to a uniform impulse of 61.32 m/sec have been obtained by using the present analysis with three sets of strain-rate coefficients of Equation 11, and the test data (Ref. 11) for comparison. The steel has a mass density of 7830 kg/m^3, and a Poisson's ratio of 0.28. The uniaxial stress-strain relation may be approximated as a bilinear relationship with $E = 21 \times 10^{10} N/m^2$, $E_t = 10^3 N/m^2$ and $\sigma_0 = 21.7 \times 10^7 N/m^2$. As one can see in Figure 3, the dent depths (or permanent sets) vary significantly with the degree of strain-rate hardening.

From the aforementioned two loading conditions under which dents of panels occur, it is quite evident that how two important factors--panel elastic springback and strain-rate hardening of material--influence the panel dent-resistance. In addition to these two factors, other factors such as strain-hardening, material density, and yield stress could be important.

CONCLUDING REMARKS

An analytical technique for elastic-plastic deformation of panels has been developed, which may be employed to analyze the denting mechanisms of panels resulting from point projectile impacts and impulsive loadings. The correlations of analytical results with the experimental measurements are considered quite satisfactory.

The effect of elastic springback on the dent-resistance analysis is found to be very significant for the panel (122 cm x 60.9 cm x 0.076 cm) subjected to a point projectile impact at 16.45 m/sec. While the springback decreases as the loading speed increases, the amount due to the strain-rate hardening of material, such as low-carbon steel, becomes more dominant, which has been demonstrated in the analysis of dent resistance of a rectangular steel plate impulsively loaded.

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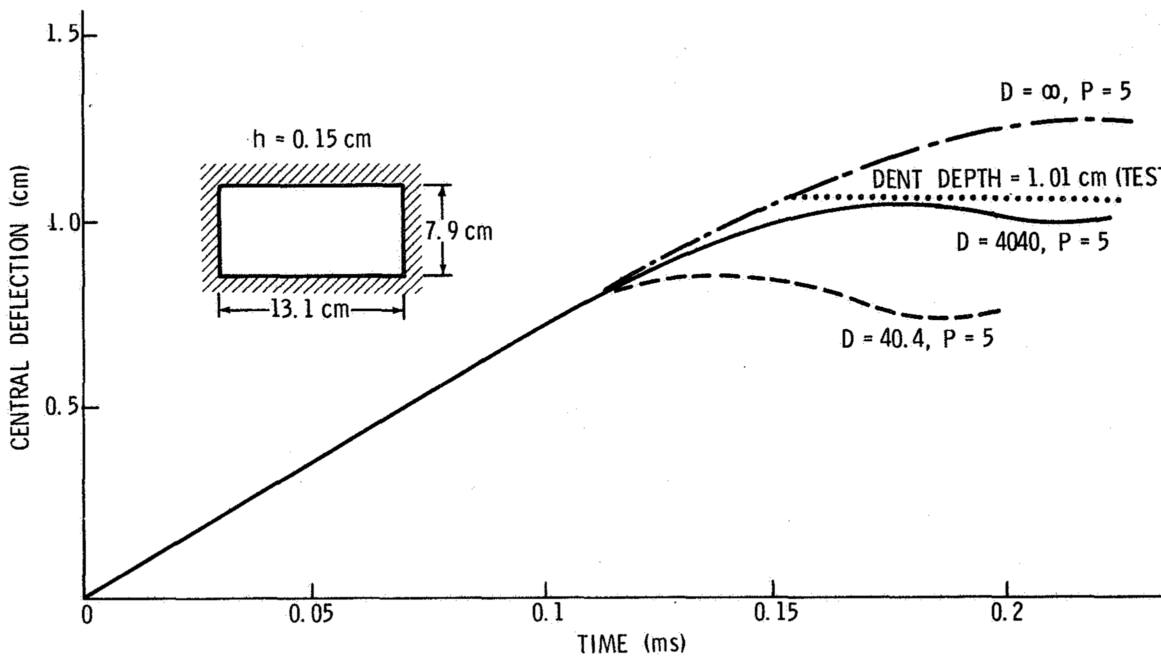

Figure 3.- Predicted deflection histories of a steel plate subjected to a uniform initial velocity of 61.32 m/sec, showing the strain-rate effect via parameter D.

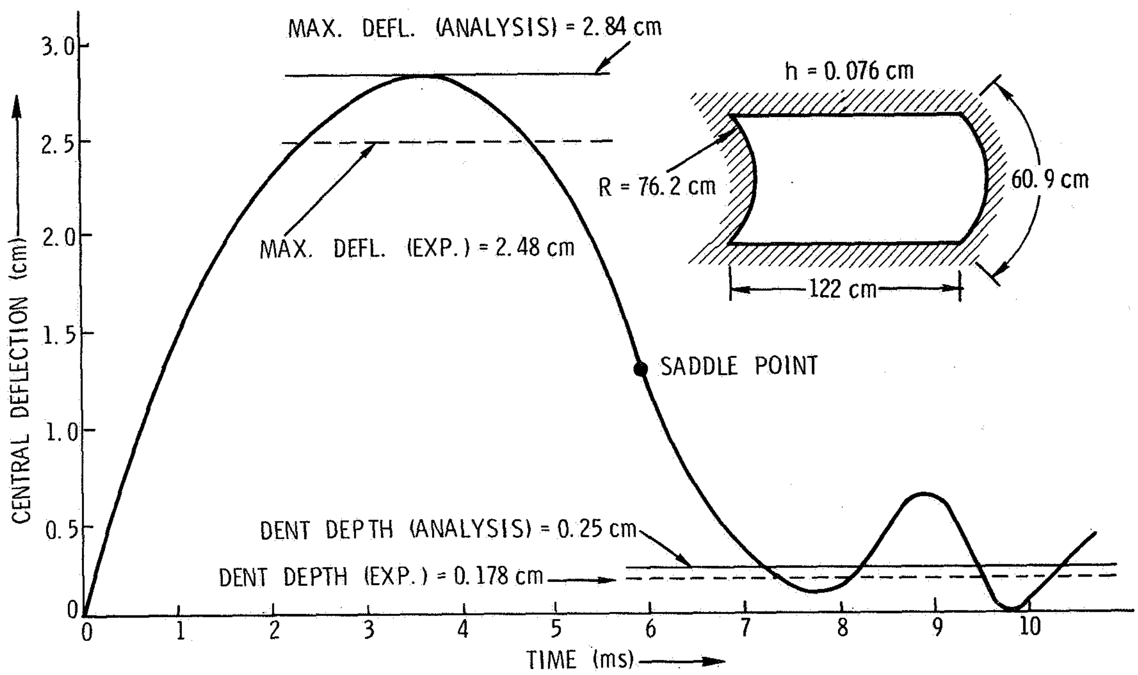

Figure 1.- Predicted deflection history of the center of a panel impacted by a 0.45-kg steel ball at 16.45 m/sec.

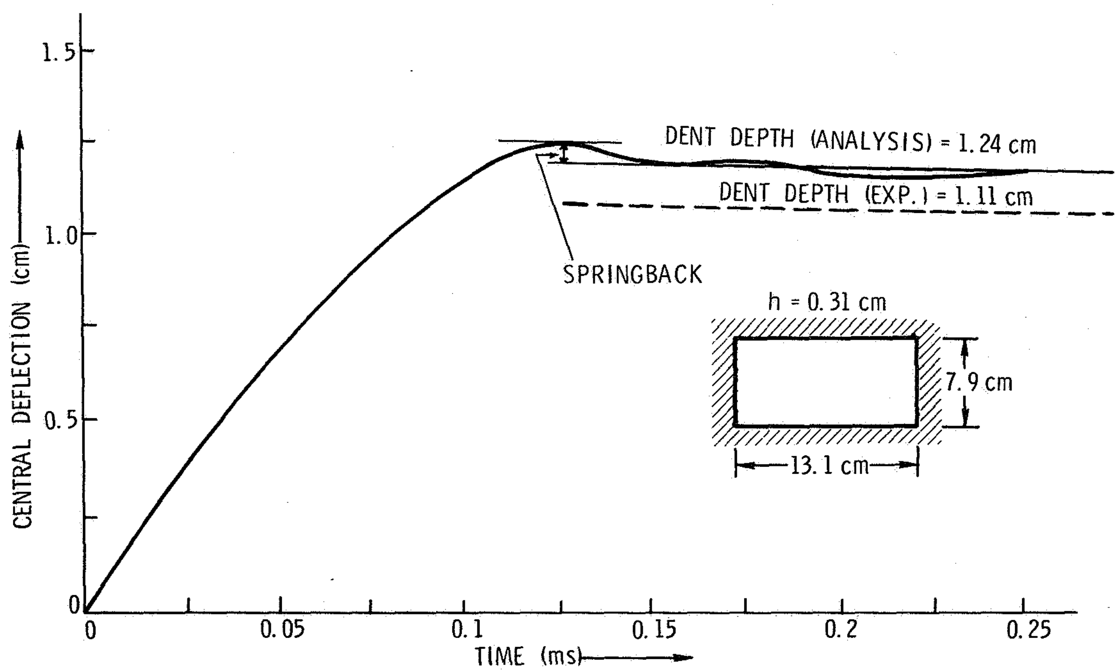

Figure 2.- Predicted deflection history of the center of a rectangular plate subjected to a uniform initial velocity of 121.9 m/sec.